**D. R. H. BEATTIE.<sup>2</sup>** The work of Professors Martin and Padmanabhan should help dispel the widely held but erroneous belief, stemming from the work of Henry, et al., [2, 3], that pressure pulse behavior in two-phase slug flow is fundamentally different from that in bubble flow. The theory of Henry, et al., assumes no mechanical coupling between the phases, a condition which was simulated in their somewhat artificial "slug flow" experiments performed to confirm their theory. The theory of Martin and Padmanabhan takes account of this mechanical coupling between the phases, which in reality is almost complete, as is the case for bubble flow. Thus slug flow pressure pulse characteristics should be similar to those expected from bubble flow. This is verified by the experimental data of Martin and Padmanabhan.

If thermal coupling between the phases is considered, a minor flow regime effect might be expected. The low thermal coupling expected for slug flow is such that the isentropic temperature change occurring during the pulse passage might reasonably be expected to be confined to the gas phase, with the result that, as supposed by Martin and Padmanabhan, the specific heat ratio  $\gamma$  in the theory is that for the gas,  $C_{gp}/C_{gv}$ . On the other hand, the more uniform dispersion of the phases in bubble flow is such that an assumption of thermal equilibrium is more reasonable. In this case, as shown by Miyazahi et al., [7], the value of  $\gamma$  then becomes

$$\frac{x C_{gp} + (1 - x) C_{l}}{x C_{gp} + (1 - x) C_{l}}$$

where x is the mass fraction of gas, and  $C_l$  the liquid specific heat. For the small gas mass fractions occurring in the described air-water slug flow experiments,  $\gamma$  might be expected to be closer to unity if bubble flow had occurred. However, this dependence of pressure pulse velocity on flow regime, which is not taken into account in the theory of Martin and Padmanabhan, is small and of the order of normal data scatter. As expected, then, the experiments of Miyazahi, et al., [6], which covered both bubble and slug flow, did not indicate any discernible influence of flow regime on pressure pulse velocity characteristics.

Martin and Padmanabhan's treatment of the drift flux model is such that the distribution parameter,  $C_0$ , is assumed to be unity. Their conclusion that the pressure pulse velocity is that given by the homogeneous model with respect to the mixture center of mass velocity applies only if the "slip" between the phases is caused only by a drift velocity  $v_{gj}$  and is not general. Distribution parameter effects are considered in [19] and its supplement [22] which give the result

$$a \simeq \sqrt{\frac{\gamma p}{\alpha (1 - C_0 \alpha) \rho_l}}$$

(which reduces to equation (15) for  $C_0 = 1$ ) with respect not to the center of mass velocity but, if  $j > v_{gj}$ , to  $j(C_0 + (1 - C_0\rho_l/\tilde{\rho}))$  where  $\tilde{\rho}$  is the homogeneous mixture density, and  $C_0$  is as calculated in [19].

The theory of [19] applies for developed flows and, as such, is not strictly applicable to the experiments of Martin and Padmanabhan where the experimental distribution parameters, which are less than unity because of the skewed

Table 1	Eı	ror	statistics	for	pressure	pulse	velocit	y data o	)f
Martin	and	Pad	manabha	n. I	Errors ar	e (calc	ulated	velocity	
	me	asur	ement vel	locit	y)/(meas	ured v	elocity)		

	Mean error	R.M.S. error
Homogeneous model	-17.4%	18.1%
Method of [19]	- 3.2\%	6.7%

voidage profiles, indicate non-developed conditions. Nevertheless, the theory of [19], as indicated in the accompanying table, predicts the pressure pulse velocity data of Martin and Padmanabhan better than the homogeneous model does, even though the developed flow void fractions predicted by the method of [19] are clearly inappropriate. This suggests that, for pressure pulse velocity prediction, the theory of [19] can be extended to nondeveloped conditions, at least when the nondevelopment is characterized by skewed voidage profiles.

## **Additional Reference**

22 Beattie, D. R. H., and Thompson, J. J., "The Effect of Flow Velocity on Two-Phase Propagation Phenomena," *Proceedings, Second International Conference on Pressure Surges*, London, Sept. 1976, supplement to paper D3.

## **Authors' Closure**

The authors wish to thank Mr. Beattie for his comments and enhancement to the paper.

The effect of flow regime on degree of thermal coupling is an important, but not necessarily well understood, consideration. As clearly stated by Mr. Beattie we implicitly assumed that slug flow is associated with an isentropic temperature change, or a low thermal coupling. It is interesting, however, to note that Miyazahi, et al., [6] were not able to discern any effect of flow regime (bubbly and slug) on pressure pulse velocity. Uncertainties in the accurate knowledge of the void fraction and in the precise measurement of the wave speed must also be kept in mind. Using the same apparatus the authors found that the pressure pulse velocity in bubbly flow was generally better predicted by isentropic behavior than by isothermal, notwithstanding the difficulty in the actual determination of the sonic velocity with a reasonable degree of confidence.

The authors agree that their model is not necessarily general for any two-phase flow system. We neglect the distribution parameter, but not the drift velocity; whereas Mr. Beattie neglects the drift velocity, but not the distribution parameter. Although it is encouraging that the inclusion of  $C_0$  in equation (15) yields an improvement over the homogeneous model ( $C_0 = 1$ ), it is somewhat surprising that the method of [19] yields incorrect developed void fraction values for our system. The authors contend that caution should be exercised in claiming that the theory of Beattie [19] can be extended to nondeveloped conditions.

Because of uncertainties associated with the measurement of  $\alpha$  and a in any pressure pulse propagation study the separation of such effects as (1) thermal coupling and (2) effect of voidage distribution in extremely difficult. The results of this investigation do demonstrate, however, that mechanical coupling must be considered in the analysis of pressure pulse propagation in slug flow.

<sup>&</sup>lt;sup>1</sup> By C. Samuel Martin and M. Padmanabhan, published in the March, 1979, issue of the ASME JOURNAL OF FLUIDS ENGINEERING, Vol. 101, No. 1, pp. 44–52.

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