Preventive War and the Balance of Power

A GAME-THEORETIC APPROACH

EMERSON M. S. NIOU
Department of Political Science
State University of New York at Stony Brook
PETER C. ORDESHOOK
Department of Government, University of Texas at Austin
Division of Social Sciences
California Institute of Technology

Preventive wars are commonly defined as wars initiated by a major power to preempt an anticipated disadvantageous position in terms of resources or military capabilities owing to the differential growth rates of countries. This essay extends a game-theoretic model of the balance of power to admit differential growth rates and also to permit countries to adjust their investments for the future. After establishing the conditions for equilibrium investment strategies, we then examine the properties of the nation-system that this equilibrium implies. Specifically, using a two-period model, we are interested in those first-period equilibria in which, because their sovereignty is subsequently threatened, countries will prefer to instigate a preventive war. We conclude by arguing that, although differential growth rates and the period 1 equilibria that initial resources imply can threaten the sovereignty of countries, there are a variety of coalitional strategies available to countries and that only some of them imply preventive war.

> What made the (Peloponnesian) war inevitable was the growth of Athenian power and the fear which this caused in Sparta. [Thucydides]

Intuition and a considerable theoretical and empirical literature concur with Thucydides's assertion that the cause of international conflict lies as much with the differential growth rates of national resources as with the relative power or resources of countries at any specific point in time. Hence, conflicts might not be attributable wholly to some current

AUTHORS' NOTE: This research was supported by a National Science Foundation research grant to the University of Texas at Austin.

JOURNAL OF CONFLICT RESOLUTION, Vol. 31 No. 3, September 1987 387-419

"imbalance" in military position, but also to the projections of future imbalances and to the corresponding perceived necessity for preventive wars (see, for example, Brodie, 1973; Fay, 1966; Organski and Kugler, 1980; Maoz, 1983; and Levy, 1986). This dynamical view is reflected also in the debate over domestic versus defense spending and the issue of whether a country should invest resources to maximize economic growth rather than appropriating those resources to maximize current military strength. Indeed, this debate shows why a static analysis based on some current distribution of national resources or power cannot capture all relevant aspects of the "balance of power" concept in particular and the causes of war in general.

In response to such realities, this essay extends a particular model of the balance of power (Niou and Ordeshook, 1986), because the extensions suggested by the literature are natural to it, and because such an extension helps us answer several important questions about the preconditions for preventive wars. First, the extensions are natural in that the model already assumes that national leaders judge coalitions and resource reallocations in terms of future as well as immediate consequences. Moreover, two concepts of stability that it uses—system and resource stability—seem to be precisely the considerations that concern national leaders in deciding between current and future defense needs. The first stability concept concerns the ability of national leaders to secure the sovereignty of their countries and the second concerns instabilities in the distribution of resources that do not threaten sovereignty.

Using these two concepts, the extended model accommodates resource growth and provides an analysis of preventive wars in two ways. First, we let growth be a function of exogenous factors, summarized by a growth rate and an initial distribution of resources. Second, we let national leaders affect growth endogenously. As part of a sequential cooperative and noncooperative game, they can vary their countries' current and future capabilities by investing into the future, thereby changing the proportion of resources subject to growth. Hence, national leaders do more than simply try to anticipate future events. In choosing their investments and in deciding whether to engage in a preventive war, they must also anticipate the anticipations of others, the actions of others based on those anticipations, and so on, with the understanding that everyone is doing the same. To learn the conditions under which countries can invest and maintain their sovereignty, and when they might initiate a preventive war because they cannot do so, our analysis describes the necessary and sufficient conditions for the existence of system and resource stability in this interactive, dynamical setting.

Our analysis does not model all factors that might operate in international systems. Rather it seeks to uncover the implications of particular assumptions about international systems, assumptions that imply stability and balance in a static context without investment (Niou and Ordeshook, 1986; Wagner, 1986). But we are also choosing our extension so that it permits us to meet our principal objective, that of addressing some specific issues that the preventive war literature raises. This literature, beginning with Thucydides's observation, provides a definition of preventive wars that our analysis calls into question in terms of its theoretical generality. Briefly, Brodie (1973) defines a preventive war as one in which a country undertakes "to destroy an already strong rival whose power one fears may grow faster than one's own." Fay's (1966) definition is "the waging of war upon a neighbor while he is still weak . . . to prevent his growing stronger," while Organski's (1968) definition is wars that are "launched by the dominant nation to destroy a competitor before it becomes strong enough to upset the existing international order." Implicit in all of these definitions and a great many others that we might gather (see Levy, 1986, for a general survey of the literature), then, is the assumption that preventive wars are not waged simply because some country or coalition acts to take advantage of a current imbalance in the distribution of power or resources. Rather preventive wars are designed to overcome a perceived future resource deficit. By examining the implications of growth rates, we implicitly adopt this assumption. But these definitions also share the view that such wars are initiated by a stronger state against a specific adversary, because its military power is declining relative to its adversary. This view is one that we question.

The specific questions about preventive war that this essay addresses then are these: Are preventive wars limited necessarily to two adversaries and are such wars initiated by a single large country? What of the possibility that preventive wars are initiated by coalitions to keep some large adversary from becoming predominant? Are preventive wars initiated only when one country grows at a faster rate than someone else or are such wars also possible when all countries grow at the same rate? Are preventive wars predetermined in the sense that given initial growth rates and resources, there is only one possible resolution to instability? That is, is there a unique and predictable response to a perceived future imbalance in the distribution of resources among countries, or are a variety of responses possible? We attempt to show with our model that the answers to these questions have not always been correctly anticipated by the authors of the previously cited definitions of preventive wars.

Before proceeding further, we should comment about our assumption that countries are unitary actors who choose without the constraints of domestic politics in mind. Clearly, we must be cognizant of the hypothesis that such politics plays an important role in determining whether preventive wars are undertaken (Lebow, 1981, 1984). Although we incorporate domestic issues in a limited way by allowing decision makers to determine what share of their country's resources is to be invested into the future and what share is to be used for strategic maneuver in the current period, because we assume that decisions are unconstrained except by their countries' resources and by the imperatives of international politics, we do not take full account of the domestic bases of foreign policy. Nevertheless, to the extent that we define what unconstrained decision makers ought to do, we provide a basis for speculating about the consequences for preventive war of constraints such as those that domestic politics can impose.

Section 1 reviews the model that we generalize, the definitions of system and resource stability, and the conclusions we deduce about each definition. Section 2 describes the model's extension, formulated as a noncooperative game in which each player's strategy is an investment decision. We show in particular that a unique equilibrium exists to this game. Section 3 analyzes equilibria that threaten to yield a predominant country, and examines the likely responses of other countries. Section 4 describes circumstances in which no country becomes predominant, but in which a subset of countries find their sovereignties threatened. Section 5 concludes with a discussion of the specific assumptions of the analysis that are most likely to be violated.

1. A STATIC MODEL

Let $S = \{1, 2, \ldots, n\}$ denote the *set of all countries*, let $C \subset S$ be a specific *coalition* of countries, and let $\mathbf{r} = (r_1, r_2, \ldots, r_n)$ be an n-tuple of *resources*, where r_i denotes the resources controlled by country i. Naturally, we suppose that $r_i \geq 0$ for all i, we let $R_i = \{r'_i \mid 0 \leq r'_i \leq r_i\}$, and for convenience we suppose that $r_1 \geq r_2 \geq \ldots \geq r_n$. Thus, $(S, \mathbf{r})$ denoted an outcome or a *state of the world*. In addition, we also make use of the notation in which $R = \Sigma_{i \in S} r_i$ denotes the total resources controlled by all countries, and $r(C) = \Sigma_{i \in C} r_i$ is the total resources controlled by the coalition C.

This notation makes no reference to time; hence, our earlier model's static quality with respect to resources. That model, though, has the simpler objective of deducing the conditions under which (S, **r**) is resource stable—the conditions under which no country or coalition will act to disrupt the current distribution of resources, **r**. Short of being resource stable, which requires that **r** satisfy a specific and restrictive condition, the model allows us to deduce the conditions under which (S, **r**) is system stable—the conditions under which no country or coalition of countries, C, will eliminate one or more members of S-C.

It might seem that the preceding notation yields a simple formulation of an "international relations game," specifically a cooperative n-person game with this characteristic function:

$$v(C) = \left\{ \begin{bmatrix} 0 \\ R \\ R/2 \end{bmatrix} \right\} \text{ if } r(C) \left\{ \begin{bmatrix} < R/2 \\ > R/2 \\ = R/2 \end{bmatrix} \right\}$$

Winning coalitions can secure any redistribution of resources, including those in which minority coalitions are eliminated, whereas minority coalitions can secure nothing. The inevitable conclusion one reaches with such a formulation is that international systems are inherently unstable. Because of their constant-sum character, such games do not possess cores (Riker, 1962). Instead, cooperative solution hypotheses such as the V-set, strong bargaining set, and the competitive solution predict resource distributions in which coalitions with a majority of resources eliminate coalitions with a minority of those resources.

But this argument misspecifies the game's characteristic function if we assume that national leaders must also be concerned with the games that ensue after other countries are eliminated. National leaders must be certain that they avoid possibilities such as the one described by Kaplan (1979): "The weakest player, by joining a nearly predominant strong player, only creates a condition in which he will be the next victim." To specify a cooperative game that takes such considerations into account, we require several assumptions about the motives of national leaders and about the "rules of the game." We list them here, beginning with the particular assumption (A1) that we generalize in the next section.

A1: R is constant

A2: R is infinitely divisible and transferable among nations.

A3: $i \in S$ is eliminated if $r_i = 0$

A4: All decision makers have complete information about the game,

A5: i prefers $\mathbf{r}$ to $\mathbf{r}'$ if $r_i > r'_i$, provided that, as a direct consequence of $\mathbf{r}$, it is not the case that i's resources can be reduced to zero.

A6: For any C and $C' \subset S$, C can defeat C' if and only if $r(C) > r(C')$. If C defeats C', $r(C')$ is transferred to C as specified by the members of C so that the resources now controlled by C equal $r(C) + r(C')$.

These assumptions are discussed elsewhere, and we will not dwell on them except to comment on A2, A3, and A5. With respect to A2, we are not under any delusion that a single transferable commodity represents and measures the military capabilities or resources of countries. But the power that such an assumption lends to a formal analysis will hopefully outweigh the distortions that it occasions. Later we can ascertain the implications of, say, situations in which resources are not freely transferable, which, in fact, is one of the ways in which we can incorporate domestic political constraints into the analysis. With the lessons of natural science in mind, we prefer to explore the implications of a "frictionless" world first, before we try to understand the consequences of friction. With respect to assumptions A3 and A5, several readers of earlier drafts have commented on the fact that nations rarely disappear. To say, though, that i is eliminated if $r_i = 0$ need not be interpreted to mean that i's sovereignty vanishes. For certain purposes, we might apply our model only to "great powers" and interpret $r_i = 0$ as i's elimination from the ranks of great powers. Alternatively, elimination can refer merely to the end of a particular regime. The important point is that, although we of necessity use the precise language of mathematics for our formal arguments, our concepts and terms are subject to a variety of interpretations, depending on the substantive situation to which the model is applied.

Assumption A5 warrants some additional comment insofar as it incorporates our conceptualization of the sequential game that nations play. Suppose that we are at some initial state of the world $(S, \mathbf{r})$ and that a subsequent state $(S', \mathbf{r}')$ is being contemplated by the decision makers in S. Moving from (S, **r**) to (S', **r**′) may involve a voluntary transfer of resources, the formation of certain coalitions, or a war. Thus the evaluation of (S', **r**′) is necessarily an evaluation of the relevant actions leading to it from (S, **r**), and predicting whether a relevant action will be chosen necessarily requires that we know how each decision maker evaluates (S', **r**′). But it is here that we must accommodate the fact that (S', **r**′) is not necessarily the "end of the game"—that other states of the world may follow from (S', **r**′). Thus, how one evaluates (S', **r**′) depends on how one evaluates its consequences. To model and make assumptions about this evaluation, we must specify the game that we suppose nations play in anarchic international systems. Briefly, we envision the following sequential situation: Beginning with the current description of the system, (S, **r**) nations are free to negotiate, make war, transfer resources, make threats, etc. Each transfer of resources and each war results, according to rules yet to be specified, in a new state (S', **r**′). If we ignore for a moment the complication that such a process might proceed indefinitely, and suppose instead that decision makers hold finite planning horizons, then, owing to the assumption that everyone shares the same information about the situation and that everyone knows that everyone shares this information, one important consequence follows: *Each decision maker can predict (up to the determinism that game theory admits) the states of the world in the sequence—including the prediction that certain states lead to its eventual elimination—and no decision maker has any advantage in making such predictions.* It also follows that, from any initial state, each decision maker can predict (again up to the determinism that game theory admits) whether a successive state will lead to its eventual elimination.

The qualification "up to the determinism that game theory admits" is intended to accommodate this possibility: If, for example, three persons, 1, 2, and 3, must divide some sum of money, say $100, among themselves using majority rule, if these three persons are identical except for their labels, and if all three are concerned solely with the amount that they possess (if they are each unconcerned about the welfare of anyone else, except what it implies about their own wealth), then we can say only that two persons will coalesce to divide the sum among themselves, excluding the third. We cannot say, however, whether this coalition will involve persons 1 and 2, or 1 and 3, or 2 and 3.

The likelihood, indeed the certainty of this indeterminism means that neither the analyst nor any decision maker can predict with certainty the outcomes that follow from a particular state (S', **r**′). But the assumption that the properties of the game being played are common knowledge implies that all participants will make the same predictions, even if those predictions merely identify a set of states that a particular initial description makes feasible. Suppose, then, that a decision maker is evaluating two alternative states (S', **r**$'$) and (S'', **r**$''$) which he can block or bring about if he adopts certain actions or strategies. The indeterminism of which we speak implies that he cannot be certain what states follow from these two, but suppose that (S'', **r**$''$) makes feasible a state of the world in which the decision maker in question is eliminated, whereas (S', **r**$'$) does not. That is, suppose that if (S'', **r**$''$) prevails and if everyone acts in accordance with the rationality principles yet to be specified, then the decision maker in question cannot preclude the possibility that he is eliminated at some point in the future if not in (S'', **r**$''$) itself—suppose the security level of (S'', **r**$''$), denoted s(S'', **r**$''$), equals zero—but if everyone acts rationally with (S', **r**$'$) as the starting point, then our decision maker knows that if everyone else responds rationally to the actions of everyone else, he can ensure his continued existence—suppose s(S', **r**$'$) is something greater than zero. Then our first assumption about preferences is that the decision maker prefers (S', **r**$'$) over (S'', **r**$''$). Second, if s(S', **r**$'$) = s(S', **r**$'$) = 0, then the decision maker is indifferent. Finally, we assume that if neither s(S', **r**$'$) nor s(S'', **r**$''$) are zero, then the decision maker prefers (S', **r**$'$) to (S'', **r**$''$) if his resource in **r**$'$ exceed his resource in **r**$''$. Thus assumption A5 requires, in effect that, *a country prefers a larger resource distribution to a smaller one, unless that larger resource distribution leads to some future distribution in which its resources are reduced to zero, and it always prefers having some resources to having no resources, that is, survival is preferred to elimination.*

Our next two assumptions require that we specify the acts available to countries. Briefly, we distinguish among the following:

a1: negotiate to cede resources to other countries,
a2: aggressively act to secure resources from other countries,
a3: negotiate to secure resources from other countries,
a4: aggressively oppose the actions of other countries,
a5: act neither to secure or to cede resources.

Then, to reflect the costs of war and aggressive action, we assume,

A7. If a2 and a3 lead to otherwise identical outcomes, a decision maker prefers a3.

Next, we want to suppose that if, say, countries i and j are in conflict, and if country k is more powerful than either of them, then k can profit from the conflict and gain some additional resources. Any one of a large number of formally stated assumptions is sufficient to secure our results, but the following strong version is the least ambiguous:

A8. If, for disjoint C, C', and C'', C attacks C' and C' attacks C'', $r(C) > r(C') > r(C'')$, then C absorbs C', leaving C'' unaffected.

Thus far, our assumptions, although specifying the rules of a cooperative n-person game, are not sufficient for defining a characteristic function, $v(C)$, to which we might apply some solution hypothesis and render a prediction. In particular, we need to specify each country's security value, given what its leaders believe will be the game that results if certain countries are eliminated. To specify $v(C)$, then, we must also model bargaining and specify the conditions under which countries can ensure their sovereignty. Elsewhere, Wagner (1986) describes this process as a noncooperative extensive-form game, whereas our approach is to modify the perspectives of bargaining set theory (Aumann and Maschler, 1964) to fit the problem at hand.

Briefly, the modifications we make incorporate the following specific considerations. First, we are not identifying a set of "stable" payoff vectors, but rather we are ascertaining whether the particular vector $(S, \mathbf{r})$ can be upset. Thus we are identifying the outcomes that can be reached from the status quo. Second, in the context of defining system stability (as against resource stability), countries are not defending $\mathbf{r}$ in particular; rather, they are defending their sovereignty. Hence, country i is not required to defend r_i, but instead must defend receiving a nonzero payoff. Third, countries prefer resources that are secured through "negotiation" rather than through conflict (assumption A7). Finally, and as an immediate consequence of our assumptions, no country should, if possible, allow another to secure a majority of resources since this implies the eventual elimination of all but the dominant country. With these considerations in mind, we offer the following notation and definitions: Letting W denote the set of winning coalitions (coalitions that control more than half the total resources), W^* denote the set of minimum-winning coalitions, and $E(\mathbf{r}) = \bigcup_{C \epsilon W^*} C$, be the set of essential countries in S, given $\mathbf{r}$. Further, let $\mathbf{C} = (C, \ldots)$ be a coalition structure that partitions the members of S into exhaustive and disjoint coalitions (the empty coalition, ϕ, is always an element of $\mathbf{C}$), and let $(\mathbf{r}, \mathbf{C})$ be a proposal consisting of a distribution of resources and a particular coalition structure, then we define

Threat: ($\mathbf{r}'$, $\mathbf{C}'$) is a threat by C against C' with respect to (S, $\mathbf{r}$), current status quo, if and only if

i. $C, C' \epsilon \mathbf{C}'$
ii. $r(C) > r(C')$
iii. $r'_i = 0$ for all $i \epsilon C'$
iv. $r'_j > r_j$ for all $j \epsilon C$.

And, in particular, ($\mathbf{r}'$, $\mathbf{C}'$) is a threat against i if $i \epsilon C'$. Surveying this definition, condition i requires that C and C' both be disjoint coalitions in the coalition structure $\mathbf{C}'$. This is only reasonable, of course, since if i attacks j, we can hardly say that i and j have coalesced to coordinate their strategies. Condition ii is borrowed from the reasonable presumption that countries will attack others only if they anticipate being able to win; hence, the requirement that C's resources exceed the resources of C'. Condition iii requires that a threat be a proposal to eliminate attacked countries. Finally, condition iv states that the members of C will coalesce to attack others only if, individually, each anticipates some immediate gain in terms of increased resources from such an act.

Counterthreat: ($\mathbf{r}''$, $\mathbf{C}''$) is a counterthreat to ($\mathbf{r}'$, $\mathbf{C}'$) by $K \subseteq C' \cap C''$ if and only if

i. either $C \subset C^*$ or $C \cap C'' \neq \phi$, where $C'', C^* \epsilon \mathbf{C}''$
ii. ($\mathbf{r}''$, $\mathbf{C}''$) is a threat to C^*
iii. r''_i preferred to r'_i for all $i \epsilon C''$

A counterthreat by the collection K, then, is, according to conditions i and ii, a proposal in which K is in both C' (the coalition that is being attacked) and C'' (the coalition that is formulating the counter) that either threatens all the members of C (the originally threatening coalition) or that coopts one or more members of C. In addition, condition iii requires that all countries in the countercoalition, C'', prefer the counter to the original threat.

Viable Counterthreat: The counterthreat ($\mathbf{r}''$, $\mathbf{C}''$) is viable for $i \epsilon$ K if and only if there is no $\tilde{C} \subseteq C''$ – {i} such that $\tilde{C}$ has a threat, ($\tilde{\mathbf{r}}$, $\tilde{\mathbf{C}}$), against C^* or C^* + {i}, with $\tilde{r}_j$ preferred to r_j'' *for all* $j \epsilon \tilde{C}$.

A counterthreat is viable for one of the threatened members of C' if and only if i's coalition partners in the counter have some incentive to coalesce with i in the sense that whenever they exclude i (to form $C'' - \{i\}$), they cannot make a counterthreat that they all prefer to the counter that they can make with i. Our final assumption now is the following:

A9: $i \in S$ will not be eliminated from the game if and only if it possesses a viable counterthreat to every threat.

It follows by definition that $(S, \mathbf{r})$ is system stable if and only if, for all $i \in S$ and for every threat against i, i has a viable counterthreat.

The logic of our model, now, is to suppose that national leaders are free to negotiate for the transfer of resources among themselves or to threaten alliances for the forced reallocation of resources. But in taking such actions, each nation must make certain, if possible, that it does not permit a reallocation from the status quo in which it, at some future stage of the process, becomes a victim (as when some other nation secures over half of the available resources). From this perspective, we prove the following in the original presentation of this model.

Theorem 1: $(S, \mathbf{r})$ is system stable if and only if $S = E(\mathbf{r})$.

To illustrate this result, consider the resource distribution (120, 50, 50, 40, 40), in which everyone is essential. For example, regardless of what threats and counters are made, country 4 or 5 can always transfer 30 units of resources to country 1. Our assumptions imply that no nation will be able to secure more than half the resources, and because nations prefer receiving resources "peacefully" rather than "aggressively," an allocation such as (150, 50, 50, 40, 10) represents an ideal point for 1. Notice, in particular, then, that the security levels of countries 4 and 5 are not zero (as represented in the usual simple-game characteristic function representation of this situation), but are, instead, 10.

These developments permit the definition of a more appropriate characteristic function for the analysis of resource stability. Briefly, if C is "winning" (if $r(C) > R/2$), and if ϵ is any arbitrarily small positive number, then the characteristic function of a system-stable game is as follows:

$$v(C) = r(C) + (R/2 - \max_{i \in C} [r_i]) \qquad [1a]$$

if $r(S-C) > R/2 - \max_{i \in C} r_i$, otherwise $v(C) = R - \epsilon$; and

$$v(S-C) = r(S-C) - (R/2 - \max_{i \in C} [r_i]) \qquad [1b]$$

if $r(S-C) > R/2 - \max_{i \in C} r_i$, otherwise $v(S-C) = \epsilon$.

Resource stability now, means that there exists an allocation of resources that, given the preceding characteristic function, cannot be upset by any country or coalition, which requires that the game has a core. The following result shows that this occurs only in a special limiting case:

Theorem 2: If $r_i < R/2$ for all $i \in S = E(\mathbf{r})$, then the corresponding cooperative n-person game is an essential constant sum game, and, thus, has no core. But if $r_i = R/2$, then the game is inessential and its core corresponds to $\mathbf{r}$.

Hence, resource stability is possible, but only if one country controls precisely half the resources. Otherwise, the game has no core and countries can cycle indefinitely, negotiating and renegotiating agreements (but without threatening the sovereignty of any player).

2. DYNAMIC RESOURCES: A PERIOD 1 EQUILIBRIUM

In the preceding model, national leaders are forward-looking in that they will not form coalitions that, although perhaps temporarily useful, lead to their eventual elimination. Nevertheless, the analysis is static because R is constant, and because countries can increase their resources only by securing them from other countries. But now we assume a two-period structure that admits differential resource growth rates among countries and that also permits decision makers to choose the amount of resources invested in the future. We suppose that country *i*'s resources at the beginning of period 1 is r_i, that *i* can invest any part of r_i, say p_i $(0 \leq p_i \leq r_i)$, and that this investment will grow to the amount $(w_i + 1)p_i$ in period 2, where $w_i \geq 0$ for all $i \in S$. That is, after making this investment, *i* will have $r_i - p_i$ resources available in period 1 and, ceteris paribus, $r_i - p_i + (w_i + 1)p_i = r_i + w_i p_i$ resources in period 2.

That is not to say that we predict that i will have $r_i + w_i p_i$ resources in period 2, since this prediction ignores the assumption that everyone's information is complete. To see what we mean, we can think of our 2-period situation in terms of four stages, where period 1 consists of three stages, and period 2 consists of a single stage. Assuming that $(S, \mathbf{r})$ emerges as the prevailing state from whatever has transpired beforehand, preventive wars, if they occur at all, occur in the first stage, as do resource reallocations induced by other means in anticipation of what will transpire subsequently. In the second stage national leaders make their investment decisions. We conceptualize the third stage as one in which, with the state being $(S, \mathbf{r}')$, countries can be eliminated unless they can defend their sovereignty. Period 2 consists of a single stage in which the prevailing state is $(S, \mathbf{r}'')$, after which coalition formation, redistribution and the like reoccur. Notice, then, that we cannot predict that $\mathbf{r}''$ ultimately equals $(r_1 + w_1 p_1, \ldots, r_n + w_n p_n)$, since the actual distribution of resources that countries use as the basis of their investment decisions may be modified in the first and third stages of period 1. More complicated still is the fact that, with complete information, everyone should anticipate (at least up to the predictions of cooperative solution theory) the distributions that emerge from each stage of period 1, as well as period 2, and should plan accordingly.

Our objective, though, is not a prediction about the final distribution of resources. Rather, it is to measure the anticipations of national leaders so that we can identify the root causes of conflict and the forces that operate to yield particular coalitions in the first period. So rather than build coalitional considerations directly into the determination of ultimate resource distributions, we initially ignore such considerations, taking $\mathbf{r}''$ as the basis whereby countries judges the implications of their investment decisions and the necessity for a preventive war, *ceteris paribus*. Coalitions and preventive wars may arise to redistribute resources before $\mathbf{r}''$ prevails, but it is the anticipation of this post-investment vector that dictates events.

We require, nevertheless, additional assumptions about preferences for alternative investments. Because we are interested in the conditions for the emergence of preventive wars and since we suppose that the survival of a country depends on its resources *relative* to other countries, rather than assuming that national leaders maximize the *absolute* magnitude of their countries' resources, it seems appropriate to assume that, conditional on being able to defend one's sovereignty in period 1, i maximizes its proportion of resources in period 2,

$$(r_i + w_i p_i) \Big/ \sum_{j=1}^{n} [r_j + w_j p_j] .$$

In noncompetitive situations, if all growth rates are positive, then this assumption implies that countries should invest all of their resources. But the qualification that this assumption makes about being able to defend one's sovereignty in period 1 implies that, in deciding how much to invest, *i* must consider the possible investment decisions of other countries, keeping the following two factors in mind: (1) It cannot invest "too much," or, given the decisions of others, it might find itself eliminated in the third stage of period 1, in which case its current and future resources are divided among the remaining countries; and (2) It cannot invest "too little," since that may make it vulnerable to being eliminated in period 2.

These two factors give rise to two questions that we must answer. First, under what conditions (in terms of **r** and **w**) does each country possess an investment strategy that guarantees that it will not be eliminated in both periods? Second, under what conditions will a country find it impossible to secure being essential in period 2 and thus prefer a preventive war in period 1?

To see why we must ask and answer the first question, notice that the two-period scenario is in part an n-person noncooperative game in which each nation's strategy is its investment decision and in which its payoff is 0 if it is eliminated in either stage, and the proportion of resources it controls in period 2, $(r_i + w_i p_i) / \Sigma_{j=1}^{n} (r_j + w_j p_j)$, otherwise. To illustrate, let the initial distribution be $\mathbf{r}$ = (120, 80, 60, 40) and let the vector of growth rates for this 4-country system be $\mathbf{w}$ = (1, 1, .5, .5). Suppose country 1 invests 100 units. Notice that, with 20 units of resources remaining, country 1 cannot be rendered inessential in the first period unless the actions of one or two of the other countries render the third predominant. Given current growth rates, this means that in the second period, 1's resources total 20 (the amount withheld in the first period) plus 2 times 100, which totals 220. But if country 2 believes that investing 100 units is indeed 1's strategy, then it can invest 70 units without fearing the possibility that it will be rendered inessential, short of "irrational actions" by countries 3 and 4. However, if 1 believes that 2 intends to invest 70 units, then it should invest nothing instead and become predominant in the first period. This, of course, is the familiar "he-thinks-that-I-think" regress, and one of our tasks is to ascertain the conditions under which such regresses terminate because the game possesses a pure strategy Nash equilibrium.

We must be cautious in trying to resolve such regresses in our formulation of a noncooperative game, however, because we must first decide how to accommodate one important feature of reality, namely that real investment decisions are not made at a specific instant. Rather, investment strategies evolve gradually over time, during which other countries can observe decisions and adjust accordingly, or can adjust to the adjustments of others, and so forth. The United States, for example, cannot act to shift resources from, say, procurement of weapons (the expenditure of current resources) to research and development (an investment) without the Soviet Union perceiving this shift and adjusting to it, and without us anticipating the possibility of making a readjustment, long before any funds are actually spent. But because we have rendered decision points discrete, even a three-stage conceptualization of period 1 abstracts from this reality.

A model that allows for the possibility of continuous adjustments and readjustments seems too complex to specify or to analyze. A differential game analysis might be appropriate, or a very elaborate description of an extensive form, but analysis would become unduly complex without any guarantee of generality. Hence, to accommodate the possibilities of secondary, tertiary, etc., adjustments within period 1, we try a different and simpler approach. Recall that game theory traditionally defines the security value of a strategy such as an investment decision in terms of what it guarantees a person should everyone else make decisions in that person's worst interests. Calculated in this way, a strategy's security value represents a pessimistic view of possibilities. Without supposing that people actually choose in accordance with this calculation (although such a choice can be justified if the game is a two-person zero sum game), the justification for this calculation is that if everyone chooses their strategies simultaneously, coordination and readjustments are impossible, so security value numbers represent real possibilities. But suppose instead that we calculate security values under the presumption that other people respond to a particular choice in their own best interest, which presumably should occur if they choose after the person in question acts, or if they are allowed an opportunity to readjust their decisions. To be more specific, consider the following definition of the security value of the period 1 post-investment resource level for country i, r_i', which defines the security value of a strategy after considering certain responses and adjustments by other decision makers. Letting

$E(\mathbf{r})$ denote the set of countries in S that are essential when the distribution of resources is $\mathbf{r}$, then

> The *security value of* r'_i, denoted $s_i(r'_i)$ equals 1 if and only if $i \in E(\mathbf{r}^o = (r^o_1, \ldots, r'_i, \ldots, r^o_n))$ for all $\mathbf{r}^o$ satisfying: for no $j \in S, j \neq i$, is it the case that $j \notin E(\mathbf{r}^o)$ but $j \in E(\mathbf{r}^*)$ where $r^*_k = r^o_k$ *for all* $k \neq j$ and $r_j \geq r^*_j > r^o_j$. Otherwise, $s_i(r'_i) = 0$.

To see what this definition entails and to understand how we can use it to accommodate investment adjustments within a period, notice first that in a quite reasonable fashion, our definition set $s_i(r'_i) = 1$ if i is essential, regardless of the investment decisions of others. But now suppose that country i is inessential at r'_i, given the particular distribution $\mathbf{r}'$. Then there are two possibilities: Everyone else is essential, or someone else is inessential. If everyone else is essential, then if attacked, i cannot suppose that its sovereignty will be "protected" by other countries defending themselves, since, from the definition of inessential, no other country requires i's support or existence to ensure its own survival. Hence, in this instance, our definition of security value sets $s_i(r'_i)$ equal to 0. That is, the security value of r'_i is zero if there exists some pattern of investments among the remaining countries that renders i uniquely inessential.

But for the possibility that our definition is intended explicitly to accommodate, notice that our definition does not require that i be essential with $\mathbf{r}'$ in order for us to set $s_i(r'_i)$ equal to 1. To understand why we do this, suppose that i is not alone being inessential, and suppose that whenever one (or more) of these countries responds to make itself essential by reducing its investment, such a move necessarily renders i essential as well. That is, suppose that other inessential countries, in pursuing their own self interests, as a byproduct necessarily aid country i by rendering it essential as well. Then we set $s_i(r'_i) = 1$. The rationale for this attribution of security levels is this: If i, and j are inessential but if each is made essential when the other one acts to become essential, then everyone knows that one or both countries cannot be eliminated from the game. Owing to the interdependence of their security, if one country, say j, is attacked, then the other, i, must respond to make itself essential—otherwise i will become the next victim, because without the possibility of any rational response from j, i cannot hope to become essential and maintain its sovereignty. That is, to accommodate collapsing a dynamic investment adjustment process into a static one-shot decision stage, we suppose that countries, if faced with an attack because they are inessential, attempt to adjust their investments so as to render themselves essential, and we further suppose that all countries, in evaluating the security levels of alternative strategies, assume that other countries respond rationally if someone attempts to take advantage of a temporary vulnerability.

Before we use this notion of security value to define preferences over alternative investment decisions, we must first consider the possibility that everyone computes the security value to them of their strategy in $\mathbf{r}'$ to be 1, but if everyone adopts the strategy specified by $\mathbf{r}'$, then a subset of S all find themselves inessential and unable, owing to the strategies of the remaining countries, to extricate themselves from this situation. If this possibility arises, then there may be no basis for supposing, as we do shortly, that countries ought to maximize their investments subject to the constraint that, whenever possible, they not choose strategies with a security value of 0. To see that this situation cannot arise, notice that i alone cannot be inessential; otherwise $s_i(r_i') = 0$. So suppose that all $i \in X \subset S$ are inessential with $\mathbf{r}'$, that all $i \in S - X$ are essential, and that $s_i(r'_i) = 1$ for all $i \in S$. That the strategies of the countries in $S - X$ cannot "trap" two or more countries in X, say the subset Y, into being inessential follows from the fact that by setting $s_i(r_i') = 1$, we require that for every $j \in Y$, there exists an adjustment by $Y - \{j\}$ that renders all members of Y essential. Hence, the members of X cannot be so "trapped."

We are now in a position to specify completely the investment game extension of our basic model. Recall that a noncooperative game is characterized by three things: the set of play, the strategies available to each player, and a payoff function, which either specifies the payoffs (utility) to each player from each vector of joint strategy choices or which specifies a rank order of preferences for each player over these vectors. The set of players, of course, is S. Country i's strategy, in turn, is the amount of resources that it fails to invest $r_i' = r_i - p_i$, so its strategy space is $R_i = \{r_i' \mid 0 \leq r_i' \leq r_i\}$. What remains, then, is the definition of the payoff function.

To see how decisionmakers might reasonably evaluate alternative strategies, let $\mathbf{r}' = (r_i', \ldots, r_n')$ and $\mathbf{r}^* = (r_i^*, \ldots, r_n^*)$ be any two alternative resource distributions describing what each country holds in the first period after it makes its investments. Our first assumption about preferences is that whenever possible, a country, say i, will prefer to restrict its strategy choices to those that have a security value of 1, and that within this set, it will prefer to maximize its investment. After all, maximizing its investment can only place a country in a better position (vis-à-vis its security) in the next period. Formally, suppose $s_i(r_i') = s_i(r_i^*) = 1$. Then i prefers $\mathbf{r}'$ to $\mathbf{r}^*$ whenever $r_i' > r_i^*$ and is indifferent whenever $r_i' = r_i^*$. Now suppose that a country does not have a strategy with a security value of 1. In this instance the specification of preferences is more complicated in that we must assume that preferences are lexicographic—that the relevant decisionmaker evaluates the alternatives by one criterion (namely, the criterion of remaining essential in period 1), and only if the strategies are equivalent there, by a second criterion (that it maximize its investment). Specifically, suppose that j is such a country and let $s_j(r_j') = s_j(r_j^*) = 0$. Then j prefers $\mathbf{r}'$ to $\mathbf{r}^*$ if j is essential in $\mathbf{r}'$ but not in $\mathbf{r}^*$. If j is essential with both resource distributions, then it prefers the vector that maximizes its investment. Finally, to ensure that preferences are complete, we assume that in the event that j is not essential in either $\mathbf{r}'$ or $\mathbf{r}^*$, then it prefers to maximize its investment (although it will certainly be eliminated from the game by some coalition).

To restate these assumptions about preference formally (merely as a bookkeeping convenience), let $u_i(\mathbf{r})$ denote a utility function for country i that summarizes its preferences (that is, i prefers $\mathbf{r}$ to $\mathbf{r}'$ if and only if $u_i(\mathbf{r}) > u_i(\mathbf{r}')$). Then our assumption about preference is:

All: $u_i(\mathbf{r}) > u_i(\mathbf{r}')$ if:

(1) $s_i(r_i) = 1$ and $s_i(r_i') = 0$ or

(2) $s_i(r_i) = s_i(r_i') = 0$ and $i \in E(\mathbf{r})$ and $i \notin E(\mathbf{r}')$,

and $u_i(\mathbf{r}) \geqslant u_i(\mathbf{r}')$ if:

(3) $s_i(r_i) = s_i(r_i') = 1$ and $r_i \leqslant r_i'$ or

(4) $s_i(r_i) = s_i(r_i') = 0$, and $i \in E(\mathbf{r}), E(\mathbf{r}')$, and $r_i \leqslant r_i'$, or

(5) $s_i(q_i) = s_i(q_i') = 0$, $i \notin E(q), E(q')$, and $q_i \leqslant q_i'$.

with strict inequality holding for utility whenever strict inequality holds between r_i and r_i'.

We want to show that a unique Nash equilibrium exists for the game with these preferences. That is,

Theorem 3: If $(S, \mathbf{r})$ is system-stable, then there exists a unique period 1 equilibrium.

Proof: The validity of the result follows almost directly from our definitions. Let r_{imin} be the minimum resource level that country i can choose such that, $s_i(r_{imin}) = 1$. That is, $s_i(r_{imin}) = 1$, whereas $s_i(q < r_{imin}) = 0$ for all q satisfying $r_{imin} > q \geq 0$. Temporarily, let $r_{imin} = r_i$ whenever no such level exists for i. By definition, then, $\mathbf{r}' = (r_{1min}, \ldots, r_{nmin})$ exists. It follows from this specification of r_{imin} and the description of preferences embodied in the definition of a period 1 equilibrium that i will not invest more. Further, i will not invest less since this only reduces its resources in period 2 without increasing its security value in period 1. Finally, for those countries for which $s_i(q) = 0$ for all $q \in R_i$, i is either inessential in $\mathbf{r}'$, in which case we reset r_{imin} equal to 0, or i can find a maximum investment (0 if necessary) such that it is essential in $\mathbf{r}'$. Q.E.D.

The most illuminating way to convey the meaning of this theorem and the definitions that precede it is with some examples that we generalize, in the case of three- and four-country situations, with some remarks.

Example 1: Let $(S, \mathbf{r}) = (\{1, 2, 3\}, (120, 100, 80))$. To see that the unique period 1 equilibrium is the distribution $\mathbf{r}' = (20, 40, 20)$, notice that even if country 1 reduces its investment to 0 so that (120, 40, 20) prevails, the definition of security value and the requirement that $s_i(r_i') = 1$ whenever possible requires that 2 needs only consider those distributions in which no other country acts to render itself inessential, given what everyone else does. Hence, we assume that 3 responds by reducing its investment also, and at (120, 40, 80) 2 is essential. That is, at 40, country 2, by supposing that everyone else responds rationally to each others' decisions, remains essential. But if 2 increases its investment so that (20, 30, 20) prevails, then even if 3 responds to 1's decision, 2 is inessential in (120, 30, 80). Similarly, if 1 increases its investment so that the distribution (10, 40, 20) prevails, then it becomes inessential, and no rational response by anyone else will render it essential. For example, even if 3 responds to the fact that it also is inessential in (10, 40, 20), 1 cannot ensure its survival with 10 units of resources if countries 2 and 3 reduce their investments to zero. As a generalization of this example, we offer the following remark (recalling our convention of letting $r_1 \geq r_2 \geq \ldots \geq r_n$):

Remark 1: For any three-country system $(\{1, 2, 3\}, (r_1, r_2, r_3))$ that is system-stable, the equilibrium values of p_i, i = 1, 2, 3, satisfy

$$p_1 = r_1 + r_3 - r_2 \quad \text{and} \quad p_2 = p_3 = r_2 + r_3 - r_1.$$

Proof: It follows from the definition of essential that either all countries are essential in a three-person situation or one country is predominant. Since $(S, \mathbf{r})$ is system stable, all countries are essential. To invest, country 1 must ensure that the next largest country, 2, cannot become predominant, which requires that $(r_1 - p_1) + r_3 \geq r_2$. To maximize p_1, we let equality hold, which implies the first equality in the remark. The remaining equalities follow in the same way. Q.E.D.

Example 2: Let $(S, \mathbf{r}) = (\{1, 2, 3, 4\}, (120, 80, 60, 40))$. To see that $\mathbf{r}' = (40, 40, 40, 40)$ is the unique period 1 equilibrium, notice first that if 1 alone invests to reduce its period 1 investment to 80, then country 4 is inessential in (80, 80, 60, 40). Since 4 is inessential if it controls any lesser total of resources and since everyone else is essential in (80, 80, 60, 40)—and thus has no incentive to adjust in a way that might make 4 essential—and since the security value of every strategy for 4 is zero, 4's payoff is nonzero only if it chooses 40. On the other hand $s_i(40) = 1$ for $i = 1, 2, 3$ whereas $s_i(40) = 1$ for $i = 1, 2, 3$ whereas $s_i(q < 40) = 0$ otherwise (e.g., 1 is inessential in (30, 40, 40, 40), but no one else is inessential and thus 1 cannot be assured that it will be rendered essential by the rational responses of others). Hence r′ is the unique period 1 equilibrium.

This example should not be interpreted to mean that, in four-country systems, everyone should invest down to the smallest country. But before we present an example to that effect, we offer the following remark:

Remark 2: If $(S, \mathbf{r})$ is a four-country system that is system-stable, then the period 1 equilibrium investments satisfies two conditions:

(1) $p_i \leq r_i - r_4$ for all i, and

(2) $p_1 \leq R - 2r_2$ and $p_j \leq R - 2r_1$, $j = 2, 3, 4$.

Proof: There are three possibilities: (i) Country i invests too much so that it alone becomes inessential, (ii) i invests too much and renders one country predominant, and (iii) i invests too much and renders two countries, including itself, inessential. Condition 1 avoids the first possibility. If the inequality is reversed, then the other three countries can invest down to r_4, in which case i is inessential, and none of the other countries has any incentive to change its strategy because they are all essential. Condition 2 avoids the second possibility. In the case of country 1, 2 can become predominant unless $[r_1 - p_1] + r_3 + r_4 \geq r_2$.

Algebraic manipulation yields the constraint on p_1 that condition 2 specifies, and a parallel argument establishes the remainder of the condition. Suppose now that case (iii) applies. If 3 is the country in question, then 4 must be inessential also, as must 3 and 4 if they combine. But this can occur only if 1 is predominant, and condition 1 avoids that possibility. On the other hand, if the country in question is 2, it cannot be that only 2 and 4 are inessential, so suppose that 2 and 3 are inessential. But again, this means that 4 is inessential, and that 1 is predominant. Hence, case (iii) is indistinct from (ii), and condition 2 avoids this possibility. Q.E.D.

Example 3: To illustrate a four-country system in which the equilibrium investments, unlike those in example 2, do not require that all countries invest to the smallest country, let $(S, \mathbf{r}) = (\{1, 2, 3, 4\}, (145, 80, 65, 10))$. Notice first that if country 3 invests more than 10 units, then country 1 becomes predominant. So let $p_3 = 10$. Now country 2 can invest 10 units also, down to 70; although this makes 1 predominant, 2 knows that 3 can reduce its investment and 3 knows that 2 can reduce its investment as well to offset this possibility. With the investments just described, country 1 knows that it cannot use its temporary predominance to attack 2 or 3, so, in accordance with the assumption that countries perfer to maximize the proportion of resources controlled in period 2 while at the same time choosing strategies that afford a security level of 1, country 1 invests 135 units. Finally, since 4 is inessential if it alone invests, $s_4(q) = 0$ for all q, and since 4 is inessential in (10, 70, 55, q) for all $q < 5$ but essential for $q \geq 5$, the unique period 1 equilibrium is one in which the countries, after making their investments, are left with (10, 70, 55, 5).

Stating general results that parallel Remarks 1 and 2 for $n > 4$ requires unwieldy algebra, since the number of possible cases that must be considered grows geometrically with n. We conclude this section, then, with a single five-country example to illustrate the method of analysis.

Example 4: Let $(S, \mathbf{r}) = (\{1, \ldots, 5\}, (100, 80, 60, 30, 30))$. In this instance the unique equilibrium is (30, 30, 30, 30, 30). Notice first that every country remains essential, regardless of the actions taken by the remaining countries. For example, if 1 and 2 reduce their investments to zero, countries 3, 4, and 5 are essential in (100, 80, 30, 30, 30). To see that no country can invest more, suppose 1 invests down to 20. But then at (20, 70, 40, 30, 30), 1 is inessential while everyone else is essential.

3. THE EMERGENCE OF A PREDOMINANT COUNTRY

Theorem 3 is important because it tells us that as long as a system is system-stable initially, there exists a unique investment pattern and hence a future system that national leaders can anticipate. But this theorem also points to the necessity of a dynamical view in which preventive wars are waged in the first period to forestall elimination at some subsequent time. That is, the only way in which an equilibrium just described can be upset is if some coalition of countries has an incentive to threaten or to indeed initiate a preventive war, given what will occur in the second period.

Recall that motivation of this study is to identify the conditions for preventive wars. Having accepted the proposition that preventive wars, if they occur at all, are caused by the threat that one or more countries will be rendered inessential in a subsequent (the second) period, there are two ways in which this precondition can be met: (1) One country threatens to become predominant in the second period, and (2) one or more countries (but less than $n - 1$) are faced with becoming inessential. The previous section tells us what investments countries will make, but notice that those investments are independent of the w_i's. Here we will look more closely at these growth rates to determine the circumstance under which the first presumed precondition is met.

Our approach permits us to address other issues, namely whether preventive wars are necessarily initiated by a single country, and whether preventive wars require differential growth rates among countries. Because it is easy to imagine circumstances under which one country becomes predominant if its growth rate is sufficiently large, consider the situation in which all w_i's are equal. Now let $(S, \mathbf{r})$ correspond to our second example, $(\{1, 2, 3, 4\}, (120, 80, 60, 40))$, in which the respective investments of the country are 80, 40, 20, and 0, and the period 1 equilibrium is (40, 40, 40, 40). Hence, if $w_i = w = 4$ for all i, then the distribution (440, 240, 140, 40) prevails in period 2. Notice that countries 2, 3, and 4 all become inessential in the second period, in which case it is in the collective interests of countries 2, 3, and 4 to coalesce against 1 before 1 becomes predominant. This example, then, shows that differential growth rates are not required for the first presumed precondition of preventive war—that one country threatens to become predominant—to be met.

To generalize this example, let I denote the total resources invested by all countries in period 1. Assuming that I corresponds to the period 1 equilibrium, the following remark defines the necessary and sufficient condition for meeting or avoiding the first precondition:

Remark 3: If $w_i = w$ for all $i \epsilon S$, then no country is predominant in period 2 when the period 1 equilibrium prevails if and only if, for all $i \epsilon S$, (i) $p_i - I/2 \leq 0$, (ii) otherwise $w \leq (R/2 - r_i)/(p_i - I/2)$.

Proof: By definition, no country is predominant if and only if no country controls more than half the resources. Formally, this means that, for all $i \epsilon S$, $r_i + wp_i \leq (R + wI)/2$, which, after some manipulation, is equivalent to

$$w(p_i - I/2) \leq R/2 - r_i. \qquad [2]$$

To prove the sufficiency of the first inequality in the remark, notice that if this inequality is satisified, then $wp_i \leq wI/2$, and because $(S, \mathbf{r})$ is stable in the first period by assumption, $r_i \leq R/2$. Adding these two inequalities yields expression 2, so, regardless of w, i is not predominant. Condition (ii) is simply a restatement of expression 2 when $(p_i - I/2) > 0$. Then the conditions stated in the remark are necessary follows since, if $(p_i - I/2) > 0$, then expression 2 is satisfied automatically, whereas if condition (ii) is violated, then the equality in expression 2 is reversed and the system is unstable in period 2. Q.E.D.

The difficulty with interpreting Remark 3 is that it treats the p_i's and the r_i's are functions of the r_i's. Remarks 1 and 2, however, provide a means for interpreting Remark 3, at least for three- and four-country situations. First, to illustrate the connecting of Remarks 1 and 3 with respect to three-country systems we can establish the following result:

Remark 4: If $w_i = w$ for all i, and if $(S, \mathbf{r})$ is system stable, then regardless of w, no country becomes predominant whenever the period 1 equilibrium prevails in a three-country system if $r_1 < r_2 + r_3/3$.

Proof: From Remark 1, we get $I = 3r_3 + r_2 - r_1$. Then no country is predominant regardless of w—condition (i) of Remark 3—if $r_1 \leq r_2 + r_3/3$, if $r_2 \leq r_1 + r_3$, and if $r_3 \geq r_2 - r_1$. The last two inequalities are obviously always satisfied since $r_1 \geq r_2 \geq r_3$. Hence, all three-country systems that are system stable (that do not have a predominant country) cannot give rise to a predominant country after investment if $r_1 \leq r_2 + r_3/3$. Q.E.D.

If the condition of this remark is violated (for example, if $\mathbf{r}$ = (140, 100, 60)), then condition (ii) of Remark 3 can be used in conjuction with Remark 1 to show that the period 1 equilibrium does not imply the emergence of a predominant country if w is less than or equal to $(r_2 + r_3 - r_1)/(3r_1 - 3r_2 - r_3)$. Rather than attempt to interpret this constraint, we turn to systems with four essential countries, in which case we can use Remarks 2 and 3 to establish the following result:

> *Remark 5:* If $w_i = w$ for all i, and if $(S, \mathbf{r})$ is system stable, then the only four-country system that cannot give rise to a predominant country whenever the period 1 equilibrium prevails, regardless of w, is a system in which $r_i = R/4$ for all i.

Proof: First, to show that if $r_i = R/4$ for all $i \in S$, notice that from the symmetry of $(S, \mathbf{r})$ and the fact, from Theorem 3, that the period 1 equilibrium is unique, we must have $p_i = p_j$ for all i and j, in which case $r_i'' = r_j''$ for all i and j. Hence, no one is predominant in period 2. Second to show tht the only four-country system in which the period 1 equilibrium necessarily implies a stable system in period 2 regardless of w, notice that from Remark 2 and from the definition of I, that $I = R - 4r_4$. Thus, from Remark 3, no country becomes predominant regardless of w if and only if $r_i \leq R/2 - r_4$, $i = 1, 2, 3$. Summing for $i = 1, 2$, and 3 yields $\Sigma r_i = R - r_4 \leq 3R/2 - 3r_4$, which requires that $R/4 \leq r_4$. But since $\mathbf{r}$'s are equal and since the choice of country 4 as the base is arbitrary, we must have $r_i = R/4$ for all i. Q.E.D.

Four-country systems in which all resources are equal, however, is a relatively uninteresting case, because no country should make any investment in this event. For example, if $\mathbf{r}$ = (75, 75, 75, 75), and if any country invests anything, then that country is inessential while all others remain essential. Thus $\mathbf{r}$ is itself the unique period 1 equilibrium. Suppose, then, that this restrictive equality condition is not satisfied. In particular, let $\mathbf{r}$ = (120, 80, 60, 40), which corresponds to example 2 in Section 2. The period 1 equilibrium is (40, 40, 40, 40), so country 1's investment—80 units—exceeds the combined investments of 2, 3, and 4—60 units—and 1 will become predominant if w is sufficiently large. Specifically, from condition (ii), 1 becomes predominant if $w > (150 - 120)/(80 - 70) = 3$. For instance, if $w = 4$, then the resources of the countries in period 2 become (440, 240, 140, 40), and 1 is predominant.

The issue such examples raise is: Is the emergence of a dominant country avoidable, and is the formation of specific coalitions such as

{2, 3, 4} in the example required to avoid it? We proceed by addressing the first part of this question, beginning with the following remark:

Remark 6: If $(S, \mathbf{r})$ is system stable and if $r_i = R/2$ for only one $i \in S$, then the unique period 1 equilibrium satisfies $r'_j = r_j$ for all $j \neq i$ and $r'_i < r_i$.

Proof: First, to show that no $j \in S, j \neq i$, can invest, notice that if $p_j > 0$, then $r_i > r'(S - \{i\})$. So j, by investing any positive amount renders itself inessential and the result cannot be a period 1 equilibrium. Now to show that i can invest, we first establish this lemma: If $r_j \geq r_k$ and j is inessential in $(S, \mathbf{r})$ then k is inessential in $(S, \mathbf{r})$. If k is essential, then by definition, there exists a $C \in W^*$, $k \in C$, such that $r(C - \{k\}) + r_k > R/2$ and $r(C) - r_k \leq R/2$. Because $r_j \geq r_k$, it must be the case that $r(C - \{k\}) + r_j > R/2$ and $r(C) - r_j \leq R/2$. So j is essential, which contradicts the assumption of the lemma that j is inessential. Turning to the proof of the remark, it must be true that $r_j < R/2$ for all $j \neq i$. It follows from the lemma just stated that i can invest to reduce its resources in period 1 to r_2, the second largest country: If i becomes inessential, all countries are inessential, which is impossible, so by A10, the unique period 1 equilibrium is an $\mathbf{r}'$ in which i invests at least $r_i - r_2$. Q.E.D.

The preceding remark is concerned solely with a property of the period 1 equilibrium. Our next remark, however, is concerned with both periods and thereby takes us part way to addressing the issue of whether any country will become predominant ultimately, given our assumptions.

Remark 7: If there exists an $i \in S$ such that $r_i = R/2$ and $w_i > 0$, then no country invests owing to the threat of preventive war, in which case $(S, \mathbf{r})$ is resource (and system) stable in both periods.

The logic of this remark is straightforward: Elsewhere we show that if $r_i = R/2$ for some $i \in S$, then $(S, \mathbf{r})$ is resource stable and thus system stable (see Remark 5 in Niou and Ordeshook, 1986). By remark 6, no country can invest except i. But if $w > 0$, then $r''_i > \Sigma_{j=1}^{n} r''_j/2$ in period 2, which is to say that i becomes predominant. Hence, the only strategy available to $S - \{i\}$ is to threaten i in period 1 if i invests. Since $r(S - \{i\}) = r_i > r'_i$, $S - \{i\}$ can keep i from investing. Since no one invests, it follows that $(S, \mathbf{r})$ describes period 2.

Our final result, now, addresses the central question, namely, can any country pursue an investment strategy whereby it becomes predominant in the second period? Equivalently, do countries always possess strategies that keep others from becoming predominant?

Remark 8: If (S, **r**) is system stable, then no $i \in S$ can become predominant in period 2.

The logic of Remark 8 is this: Remark 7 handles the case in which $r_i = R/2$ for one $i \in S$. Suppose then that $r_i < R/2$ and that w_i is great enough so that i becomes predominant in period 2. Unless the resources of some country other than i are increased or unless i's are decreased in period 1, each $j \in S - \{i\}$ is inessential in period 2. But if $r_i < R/2$, there exists at least one winning coalition, $S - \{i\}$, and generally several minimal winning coalitions, such that the members all strictly prefer a reallocation that they can secure to the eventual result of the period 1 equilibrium. It follows that outcomes in which i becomes predominant cannot be in any main-simple V-set. Because the cooperative game in question involves transferable utility, the main simple V-set exists and corresponds to both the strong bargaining set and the competitive solution (Ordeshook, 1986), which is to say that cooperative solution theory predicts the statement of the result.

The argument for Remark 8 suggests that the coalition $S - \{i\}$ can form to block i. Recall our earlier example, though, in which **r** = (120, 80, 60, 40) and w = 4 so that **r**″ = (440, 240, 140, 40). To see that, since no country controls precisely half the resources, {2, 3, 4} is not the only possible coalition, notice that countries 1, 2, and 3 can coalesce against 4, eliminate 4, and agree to a redistribution that results in the new distribution (120, 100, 80). Now the period 1 equilibrium is (20, 40, 20) and the period 2 distribution is (420, 340, 320). Clearly, country 1 prefers this alternative since it is not forced to cede any resources, and countries 2 and 3 meet their objective of remaining essential in period 2. It is because of this possibility that earlier we referred to the anticipated rise of a predominant country as a "presumed" precondition for preventive war. This new possibility, then, admits some indeterminacy into the analysis. First, notice that a coalition of 1, 2, and 3 against 4 in which 2 and 3 share 4's resources is feasible because w is not so great as to imply that 2 and 3 nevertheless become inessential. And even if w is much larger than 4, 1 can transfer resources to 2 and 3 in period 1 in the coalition {1, 2, 3} against 4. Indeed, in nearly any circumstance, 1, 2, and

4, or 1, 3, and 4, as well as 1, 2, and 3 can coalesce. Thus, in its present form, this "cooperative" game does not have a core and any number of outcomes might prevail (as predicted by the cooperative solution concept such as the V-set or the competitive solution).

This example suggests that *if the largest country threatens to become predominant in period 2, then in general any number of coalitions are possible–including coalitions in which the largest country is a member.* We have refrained to this point from offering snippets of history to support our assumptions or the implications of our models. Without a careful and exhaustive review, it is too easy to find examples that support particular arguments, and any such approach is to be mistrusted. Nevertheless, this conclusion about the role of the largest country in preventive wars seems wholly unanticipated by the literature of preventive wars and we cannot resist the temptation to cite one historical example. During the Warring States Period of China, Ch'in, the most powerful of the seven states, sought in 302 B.C. to overcome the state of Ch'u (Ssuma, 80 B.C.). However, rather than permit the balance-of-power policy of Ho Tsung to assert itself in the form of a coalition of Ch'u and some number of the other five states, Ch'in judiciously ceded territory to negotiate a temporary coalition with the states of Wei and Han in its ultimately successful campaign against Ch'u. Indeed, owing to a combination of diplomatic and military skill, it was not until 247 B.C. that the armies of Wei, Han, Chao, Ch'u and Yen, reasserting the Ho Tsung policy for the last time, blocked Ch'in. But by then it was too late to halt the eventual unification of China in 221 B.C. under its first emperor, Ch'in Wang Cheng.

The rationale for anticipating any of several coalitions, in fact, follows from our model generally. Specifically, if $C \epsilon W$, then the characteristic function value for C, $v(C)$, is given by expression (1a) whereas $v(S - C)$ is given by (1b). The earlier rationale for this characteristic function is that if one or more countries are threatened with becoming inessential and if

$$r(S-C) > R/2 - \max_{i \epsilon C} r_i,$$

then $S - C$ could transfer resources to the largest country, set $r_1 = R/2$, and ensure both resource and system stability. Remark 7 implies that the same rationale applies here: $S - C$ can increase the resources of country 1 to $R/2$, in which case no country can invest. But for this characteristic function, there is no core unless $r_1 = R/2$, so, as asserted, any number of coalitions are possible.

Thus, despite the ultimate unification of China that occurred under Ch'in, if the assumptions of our theory are satisfied (perfect divisibility and the transferability of resources, complete information, and the ability to anticipate fully and react to the choices of others), predominant countries ought not to arise, and a variety of coalitional possibilities might be observed as a consequence of one country threatening to render all others inessential.

4. INESSENTIAL COUNTRIES

It follows from Remark 8 that we should attribute the eventual emergence of a predominant country, such as what occurred at the end of the Warring States Period in China, only by referring to violations of our assumptions. But before we pursue this idea, we should consider the second possible precondition for preventive wars, namely that a subset of countries (less than $n - 1$) is threatened with becoming inessential. To illustrate this possibility, let $(S, \mathbf{r}) = (\{1, \ldots, 5\}, (60, 60, 60, 30, 30)$, so the period 1 equilibrium is (30, 30, 30, 30, 30). If $w_i = 2$ for all i, then the distribution in the next period becomes (120, 120, 120, 30, 30), in which case countries 4 and 5 are inessential. Thus it appears that these two countries have an incentive to commit to some coalition in the first period for the purpose of eliminating or reducing the resources of a third.

Before discussing these coalitional considerations, it is useful to describe the conditions under which countries remain essential in the second period. Letting $p(C) = \Sigma_{i \epsilon C} p_i$ be the total investment of the coalition C, recall that $i \epsilon S$ is essential if and only if there exists a $C \epsilon W^*$, the set of minimal winning coalitions, such that $i \epsilon C$. Formally, this requires that

$$r(C) + wp(C) - r_i - wp_i \leq (R + wI)/2 \qquad [3]$$

and

$$r(C) + wp(C) > (R + wI)/2 \qquad [4]$$

Expression 3 requires that $C - i$ is losing, and expression 4 requires that C be winning. Rearrangement of terms yields

$$w[I/2 - p(C - i)] \geq -[R/2 - r(C - i)]$$

and

$$w[I/2 - p(C)] < -[R/2 - r(C)].$$

There does not appear to be a simply algebraic identity that we can use to state when these conditions will be satisfied. However, for completeness, we state a result that can be arrived at using some simple algebra (which we do not present here), in conjunction with this notation: $I(C) = I/2 - p(C)$; $I(C - i) = I/2 - p(C - \mathrm{i})$; $R(C) = R/2 - r(C)$; and $\mathrm{R}(C - i) = R/2 - r(C - i)$. Then,

Remark 9: if $w = w_i$ for all $i \epsilon S$, then $i \epsilon S$ is essential in period 2 if and only if there exists a $C \subset S$, $i \epsilon C$, such that,

1. if $I(C-i) < 0$, $R(C-i) > 0$, and $-R(C)/I(C) < w \leqslant -R(C-i)/I(C-i)$
2. if $I(C-i) = 0$ then: (i) $I(C) < 0$ and $w > -R(C)/I(C)$, or (ii) $I(C) > 0$, $R(C) < 0$, and $w < -R(C)/I(C)$, or (iii) $I(C) = 0$, $R(C) < 0$ and $w \geqslant -1$
3. if $I(C-i) > 0$, and (i) $I(C) > 0$, $R(C) < 0$, and $-R(C)/I(C) > w \geqslant -R(C-i)/I(C-i)$, or (ii) $I(C) = 0$, $R(C) < 0$, $w \geqslant -1$, or (iii) $I(C) < 0$, $w > -R(C)/I(C)$, and $w \geqslant -R(C-i)/I(C-i)$.

Our final result takes Remark 9 and defines a necessary and sufficient condition for the system that prevails in period 2 to be system stable. First, letting $w_i(C)$ denote the range of values of w associated with C such that i is essential in period 2, as deduced from the conditions set forth in Remark 9, and letting W_i^* denote the set of minimal winning coalitions of which i is a member, then i is essential if and only if

$$w \in \bigcup_{C \in W_i^*} w_i(C).$$

For the system to be stem stable, this must be true for all $i \epsilon S$, which establishes the following remark:

Remark 10: If $w = w_i$ for all $i \in S$, then a system is system stable in period 2 if and only if

$$w \in \bigcap_{i \in S} \bigcup_{C \in W_i^*} w_i(C).$$

Suppose that this condition is violated in such a way that a minority of countries is rendered inessential in period 2. Specifically, consider again the initial distribution (60, 60, 60, 30, 30) in which, with $w = 2$, the period 1 equilibrium, $\mathbf{r}' = (30, 30, 30, 30, 30)$, leads to $\mathbf{r}'' = (120, 120, 120, 30, 30)$, so that 4 and 5 are inessential. The question, then, is whether there are any coalition possibilities open to 4 and 5 that will render them essential in period 2. Ignoring the ϵ that each country can assure itself in the first period (that is, without assuming that it implies elimination, let $\epsilon = 0$), suppose, in particular, that they coalesce with 1 and 2 to force 3 to transfer 30 units each to 1 and 2, in which case $\mathbf{r}$ becomes (90, 90, 0, 30, 30). Then $\mathbf{r}' = (30, 30, 0, 30, 30)$ and $\mathbf{r}'' = (210, 210, 0, 30, 30)$, and 4 and 5 are essential. The problem with this proposal, however, is that 1, 2, and 5 alone can attack 3 and 4 to force the period 1 distribution to (105, 105, 0, 0, 30), in which case 3 and 4 are eliminated, 5 is essential in period 2 (Remark 2), and 1, 2, and 5 prefer the eventual outcome $\mathbf{r}'' = (165, 165, 0, 0, 90)$ to any other proposal. Thus, rather than a preventive war in which 4 and 5 secure their survival, 4 is eliminated and a war occurs in which two of the major powers secure additional resources.

This example suggests that, *although countries becoming inessential might seek coalitions that avoid this possibility, there is no guarantee that they will be successful in the attempt.* Indeed, the discussion in the previous section, which extends the relevance of the characteristic function given by expressions (1a) and (1b) to the current model, pertain here as well. The cooperative game that national leaders play in anticipation of the consequences of period 1 equilibria, in general, does not have a core. Hence, any number of coalitions are possible, including ones in which some but not all of the countries threatened with becoming inessential are included.

5. CONCLUSIONS

The word "war" implies some recognizable use of force, but, except for the distinction between redistributions that occur "voluntarily" and those that are forced by some form of "aggression," countries can be required to transfer resources by opposing coalitions in any number of ways, from peaceful negotiation to armed conflict. That is, our analysis focuses on strategic imperatives—on when a country must cede power, on when to anticipate coalitions, on who will be members of those coalitions, and on the resource distributions that prevail after the coalition process ends—but it leaves aside consideration of tactics such as the form of aggression. Thus we can say that, if a coalition C prevents $i \in S - C$ *from becoming predominant by requiring* i to cede resources to the members of C, then some form of "preventive action" has occurred. But we cannot say that action is necessarily a "war." Hence our analysis merely provides us with a means for identifying the circumstances under which alternative investment strategies are likely to threaten the future sovereignty of countries, whether any country is likely to threaten becoming predominant, and which countries are threatened with becoming inessential.

Even with this qualification, our analysis establishes that the presumed preconditions for preventive war are not sufficient conditions. If one country threatens predominance, then although we predict a redistribution in which this predominance is prevented, the coalition affecting this change may include the largest country. That is, threatened predominance does not imply a coalition of n - 1 countries against the one. Alternatively, if fewer than n - 1 countries are threatened with becoming inessential, then rather than a preventive war in which those so threatened join, we might also observe the threatened countries attacking each other in conflictual coalitions.

An armchair review of historical events would presumably support much of what we outline as possibilities in the previous section. But perhaps the most evident counterfactual result derived from our model is Remark 8. Despite the prediction that no country will become predominant—that countries threatened by the predominance of another always possess strategies to counteract this threat—countries have succeeded in gaining ascendancy over large systems. Our only explanations for such events, then, must lie in the possible violations of our assumptions.

One likely violation concerns the assumption that resources are perfectly transferable among countries (assumption A2). Although convenient mathematically, domestic political considerations, such as when the internal stability of a regime would disappear if it ceded territory to another state, might preclude such transfers. Another likely violation concerns the assumption that information is complete. This assumption incorporates a number of "sins," such as the supposition that national leaders can calculate resources with precision and that they can identify exactly which coalitions are winning and which are losing. It certainly would not take much historical research, though, to uncover dramatic instances of miscalculation. Third, we assume implicitly that the countries in S are well defined so that we can judge whether there are n, or $n + 1$, or $n - 1$, etc. players in the game. But it is not always possible to judge whether a country is an independent actor, or merely an extension of some larger state. Finally, we do not suppose that there are any domestic constraints on investment decisions. Indeed, internal political considerations doubtlessly play a significant role here as well. And, by precluding certain investment patterns, they may inhibit as well as exacerbate the forces leading to preventive wars.

Thus considerations exogenous to the model, such as the flexibility of national leaders to negotiate and to invest, determine the ultimate implications of a threatened predominance and the details of action. But we have learned that the "seeds of preventive war" are not sown necessarily in international systems—that the mere existence of nation-states, of differential growth rates, and of national leaders who act in the interests of their countries does not imply instability and war. The absence of diplomatic skill, a lack of domestic flexibility in the determination of foreign policy, and misinformation are also essential ingredients. Presumably, more refined models will detail the precise circumstances of military conflict and the role of variables currently exogenous to our model. Hopefully, though, our model provides a clearer view of the character of stable versus unstable international systems, and will prove amenable to these refinements.

REFERENCES

AUMANN, R.J. and M. MASCHLER (1964) "The bargaining set for cooperative games," in Dresher et al. (eds.) Advances In Game Theory. Princeton, NJ: Princeton Univ. Press.

BRODIE, B. (1973) War and Politics, New York: Macmillan.

FAY, S. B. (1966) The Origins of the World War. New York: Free Press.

KAPLAN, M. (1979) Towards Professionalism in International Theory: Macrosystem Analysis. New York: Free Press.

LEBOW, R. N. (1981) Between War and Peace. Baltimore: Johns Hopkins Univ. Press.

LEBOW, R. N. (1984) "Windows of opportunity: do states always jump through them?" International Security (Summer).
LEVY, J. S. (1986) "Declining power and the prevention motivation." World Politics, forthcoming.
MAOZ, Z. (1983). "Resolve, capabilities, and the outcome of interstate disputes, 1816-1976." J. of Conflict Resolution (December).
ORDESHOOK, P. C. (1986) Game Theory and Political Theory. New York: Cambridge Univ. Press.
ORGANSKI, A. F. K. (1968) World Politics. New York: Knopf.
ORGANSKI, A. F. K. and J. KUGLER (1980) The War Ledger. Chicago: Univ. of Chicago Press.
RIKER, W. H. (1962) The Theory of Political Coalitions. New Haven, CT: Yale Univ. Press.
SSUMA, C. (80 B.C.), The Historical Records (Shih Chi).
THUCYDIDES (1954) The Peloponnesian War (R. Warner, trans.) New York: Penguin.
WAGNER, R. (1986) "The theory of games and the balance of power." World Politics (July).