Preview estimation and control for (semi-) active suspensions

Citation for published version (APA):
Huisman, R. G. M. (1992). *Preview estimation and control for (semi-) active suspensions*. [Pd Eng Thesis]. Technische Universiteit Eindhoven.

Document status and date:
Published: 01/01/1992

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

Preview Estimation and Control for (Semi-) Active Suspensions

Rudolf G.M. Huisman

Abstract

The dynamic behaviour of a vehicle (e.g. the dynamic tire forces, the suspension deflection, and the vertical acceleration of the sprung mass) can be improved significantly if the passive suspension is replaced by an active suspension with preview. Preview means that a priori information of the road surface is used in the control of the suspension.

In this report, the rear wheels of the tractor of a tractor-semitrailer combination are actively suspended with preview. The active suspension is tested for realistic road surfaces, i.e. rounded pulses and stochastic road surfaces, and is compared with a representative passive suspension. Moreover, an observer is designed to reconstruct the preview information (here: the road surface) from simple measurements at the front side of the tractor. Finally, a strategy is described to test the combination of the controller and the observer.

It appears that the spectacular results obtained from tests of the active suspension on a step function as the road input can not be generalized for more realistic road surfaces. However, the performance of the active suspension is still significantly better than that of the passive suspension. Compared with the minimum available preview time of 1/8 second, a preview time of 1 second improves the dynamic behaviour of the vehicle especially for low-frequent road-excitations.

To reconstruct the preview information, a model of the road surface is not necessary and the measurements have not to be differentiated. The estimation error due to model errors is reduced quickly. It is yet not clear whether or not this is fast enough to guarantee a good closed loop behaviour. The observer reduces the drift in the estimated road surface. The main disadvantage of the observer is that not all the measurements are filtered.

The combination of the controller and the observer has yet not been tested thoroughly.

Contents

Chapter 1

Introduction

In the design of a passive suspension for road vehicles, increasing the comfort of the occupants, improving the manoeuvrability, and decreasing the required suspension working space are conflicting demands (Sharp and Crolla [20]). In case of a tractor-semitrailer, especially the maximum acceleration of the cargo and of the pitch of the cabin should be reduced, and the required suspension working space and the dynamic tire force variation - note that road damage is to a great extent caused by heavy trucks - should be minimized. Again, these wishes are conflicting.

If (semi-) active suspensions are used, it is possible to improve all these performance quantities. However, the degree of improvement is often disappointing. Nevertheless, a significant performance improvement can be obtained if a priori information of the road surface (*preview*) is used in the control of the active suspension (Crolla and Abdel-Hady [5], Foag and Grübel [7], Louam et al. [16]).

In this report, two subjects related to (mainly active) suspensions are considered. One is the performance of an active suspension with preview for realistic road surfaces and the other is the determination of the necessary preview information.

Huisman et al. [10] have developed a control strategy for an active suspension with preview. The suspension has been tested for a step function as the road input. For this road surface, the performance improvement compared with a representative passive suspension is spectacular. In this report, the active suspension is tested for rounded pulses and for stochastic road surfaces. Also, the influence of the *preview time* (i.e. the time over which the road information is known in advance) is investigated.

At the moment, we are only interested in the vertical dynamic behaviour of the tractor-semitrailer. Manoeuvrability and roll are outside the scope of this report. Because of this, a two DOF vehicle model, containing a sprung and unsprung mass, is used to test the active suspension. In this test, the preview information is supposed to be known.

Foag and Grübel [7] use sensors to reconstruct the road surface. The main advantage of this method is that preview is available for both front and rear wheels. A disadvantage is that what the sensors detect is not always an obstacle (e.g. an empty paper box). Other obstacles might not be detected properly (e.g. a pothole filled with water).

Another strategy to obtain the preview information is to assume that the excitation of the rear wheels is a time-delayed version of that of the front wheels (Crolla and Abdel-Hady [5], Louam et al. [16]). Then, the problems with pseudo-obstacles do not occur. On the

other hand, preview is only available to control the suspension at the rear wheels. A more comprehensive comparison of the two strategies mentioned has been made by Van Rijn [22].

We have decided to use the latter strategy, among other things, because especially for tractor-semitrailers, still a significant performance improvement can be obtained if preview is available to control only the rear wheels of the tractor. Moreover, it is a cheap solution.

The road surface is reconstructed from measurements at the front side of the tractor. First, we determine *which* quantities have to be measured. This problem is solved, more in general, for systems with unknown inputs. Therefore, the *observability* of systems with unknown inputs is determined.

Next, an observer is described which reconstructs the road surface from the measurements. The observer resembles a Luenberger observer [17] but also uses Kalman filter theory [12]. The observer is tested for both deterministic and stochastic road surfaces. Also the influence of model and measurement errors is examined.

Finally, a strategy is presented to combine the controller of the active suspension and the observer.

In Chapter 2, the control strategy of [10] is reviewed briefly. The results from the tests of the active suspension with preview for realistic road surfaces is described in Chapter 3. Some topics about observability are presented in Chapter 4. The method to reconstruct the road surface is described in Chapter 5 and tested in Chapter 6. In Chapter 7, the combination of the controller of the active suspension and the observer is described. Conclusions are drawn in Chapter 8 and ideas for further investigation are presented in Chapter 9.

Chapter 2

Active suspension with preview

2.1 Introduction

In the paper presented by Huisman et al. [10], a control strategy is derived for an active suspension with preview. The active suspension is used to control the rear axle of the tractor of a tractor-semitrailer combination. To save fuel, this will be done only incidentally. Because of this, the performance of the suspension has been investigated for deterministic road surfaces at first.

A reason to start with the investigation into *active* suspensions has been that its performance forms an upper limit of the performance achievable with a *semi-active* suspension. Because the first results of the active suspension are satisfactory [10], the use of further investigation into semi-active suspensions is confirmed.

In this chapter, the control strategy of the active suspension with preview is described briefly. In [10], a two DOF vehicle model and a step function as the road input are used to test the control strategy. The most important results from this test will also be presented here.

2.2 Description of the control strategy

In general, a linear time-invariant dynamic system can be described by the state equation

$$\dot{\mathbf{x}}(\tau) = \mathbf{A}\mathbf{x}(\tau) + \mathbf{B}\mathbf{u}(\tau) + \mathbf{E}\mathbf{w}(\tau), \quad \mathbf{x}(t) = \mathbf{x}_t, \tag{2.1}$$

with state $\mathbf{x}$, input $\mathbf{u}$ (actuator forces, etc.), and disturbance $\mathbf{w}$ (road surface, etc.). The quantity $\mathbf{y}$ to be controlled is related to $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{w}$ by the output equation

$$\mathbf{y}(\tau) = \mathbf{C}_y\mathbf{x}(\tau) + \mathbf{D}_y\mathbf{u}(\tau) + \mathbf{F}_y\mathbf{w}(\tau). \tag{2.2}$$

For active suspensions, $\mathbf{y}$ contains, for example, the dynamic tire load variation, the suspension deflection, and the acceleration of the sprung mass.

The input $\mathbf{u}$ is determined from the requirement that the quadratic performance criterion

$$J = \frac{1}{2} \int_{t}^{t+t_p} [\mathbf{y}(\tau)^T\mathbf{Q}\mathbf{y}(\tau) + \mathbf{u}(\tau)^T\mathbf{R}\mathbf{u}(\tau)]d\tau \tag{2.3}$$

3

is minimized under the constraints (2.1) and (2.2). The integral is defined over $[t, t + t_p]$ because the disturbance (here: the road surface) is supposed to be known t_p seconds, the so-called *preview time*, in advance. The input which minimizes J is [10]

$$\mathbf{u}(\tau) = -\mathbf{K}_1\mathbf{x}(\tau) + \mathbf{K}_2\mathbf{r}(\tau) - \mathbf{K}_3\mathbf{w}(\tau), \tag{2.4}$$

with $\mathbf{K}_1 = \mathbf{R}_d^{-1}[\mathbf{B}^T\mathbf{P} + \mathbf{D}_y^T\mathbf{Q}\mathbf{C}_y]$, $\mathbf{K}_2 = \mathbf{R}_d^{-1}\mathbf{B}^T$, $\mathbf{K}_3 = \mathbf{R}_d^{-1}\mathbf{D}_y^T\mathbf{Q}\mathbf{F}_y$, $\mathbf{R}_d = \mathbf{R} + \mathbf{D}_y^T\mathbf{Q}\mathbf{D}_y$, and $\mathbf{r}$ determined by

$$\dot{\mathbf{r}}(\tau) = -\mathbf{A}_g^T\mathbf{r}(\tau) + \mathbf{K}_4\mathbf{w}(\tau), \quad \mathbf{r}(t + t_p) = \mathbf{P}\mathbf{x}(t + t_p), \tag{2.5}$$

with $\mathbf{A}_g = \mathbf{A}_d - \mathbf{B}_d\mathbf{P}$, $\mathbf{K}_4 = \mathbf{P}\mathbf{E} - \mathbf{P}\mathbf{B}\mathbf{R}_d^{-1}\mathbf{D}_y^T\mathbf{Q}\mathbf{F}_y + \mathbf{C}_y^T\mathbf{Q}_d\mathbf{F}_y$, $\mathbf{A}_d = \mathbf{A} - \mathbf{B}\mathbf{R}_d^{-1}\mathbf{D}_y^T\mathbf{Q}\mathbf{C}_y$, and $\mathbf{Q}_d = \mathbf{Q} - \mathbf{Q}\mathbf{D}_y\mathbf{R}_d^{-1}\mathbf{D}_y^T\mathbf{Q}$. Moreover, $\mathbf{P}$ is the solution of the algebraic Riccati equation

$$\mathbf{A}_d^T\mathbf{P} + \mathbf{P}\mathbf{A}_d - \mathbf{P}\mathbf{B}_d\mathbf{P} + \mathbf{C}_d = \mathbf{0}, \tag{2.6}$$

with $\mathbf{C}_d = \mathbf{C}_y^T\mathbf{Q}_d\mathbf{C}_y$.

To calculate the input $\mathbf{u}$ at time t, only the current state $\mathbf{x}_t$ and the solution $\mathbf{r}(t)$ of Eq. (2.5) are needed.

2.3 Results from earlier work

In [10], the control strategy is tested for the two DOF vehicle model shown in Fig. 2.1, which represents the rear side of a tractor. A step function is chosen as the road input and the preview time t_p is 1/8 [s] which is the preview time that is at least available if the road surface at the front side of the tractor is reconstructed without time delay and if the maximum speed of the truck (v in Fig. 2.1) is 100 km/h. The model parameters used are given in Appendix A.
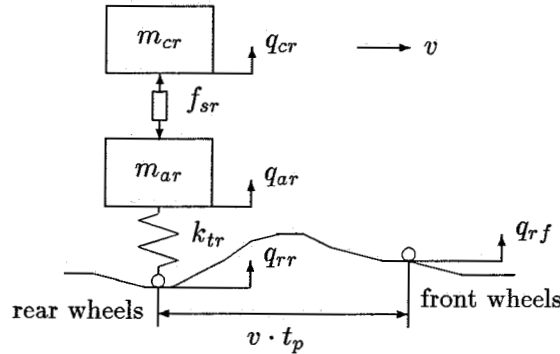
Figure 2.1: Two DOF vehicle model used to test the control strategy.

The performance of the active suspension is compared with that of a representative passive suspension with a linear spring and damper (which parameter value is given in Appendix A) in stead of the actuator which generates the force f_{sr}. The chosen output quantities to be controlled are given by $\mathbf{y} = [q_{ar} - q_{rr}, q_{cr} - q_{ar}, \ddot{q}_{cr}]^T$.

For this situation, either a 65% reduction of the required suspension working space or a 55% reduction of the maximum acceleration of the sprung mass is possible without increase of the dynamic tire force variation.

Chapter 3

Performance of an active suspension with preview

3.1 Introduction

As seen in Section 2.3, the results of the active suspension for a step function as the road input are promising. However, a step function is not a very realistic road surface. Therefore, in this chapter, the suspension is tested for more realistic road surfaces: rounded pulses (in Section 3.2) and stochastic road surfaces (in Section 3.3). Moreover, the influence of the preview time is examined. Conclusions are drawn in Section 3.4.

3.2 Rounded pulses

Alanoly and Sankar [2] and Marcelissen [19] use rounded pulses as the road input. They are described by the equation (see Fig. 3.1)

$$q_{rf}(t) = q_{max} \cdot \frac{e^2}{4} \cdot (2\pi \frac{t}{t_d})^2 \cdot e^{(-2\pi \frac{t}{t_d})} , \tag{3.1}$$

where q_{rf} is the vertical position of the road surface at the front wheels of the tractor (see Fig. 2.1). These signals are far more realistic than a step.

The suspension behaviour can be determined by calculating the response of the vehicle for a range of the pulse-heights q_{max} and the pulse-widths t_d. Here, this range is chosen such that for the two DOF vehicle model with the passive suspension, described in Section 2.3, either the available suspension working space or the maximum allowable tire force is reached. The combinations of q_{max} and t_d for which the response of the active suspension is determined are given in Appendix A. Note that the response of the active suspension has been calculated only up to $t_d = 11.8$ [s] because of numerical problems.

In Figs. 3.2 and 3.3, the performance of the active suspension with preview is shown for the minimum available preview time $t_p = 1/8$ [s] and for a "long" preview time $t_p = 1$ [s], respectively, and for three combinations of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$. One combination minimizes the required suspension working space, another minimizes the maximum acceleration of the sprung mass, and the third one results in a good "overall" performance. A good overall performance means that the required suspension working space, the acceleration
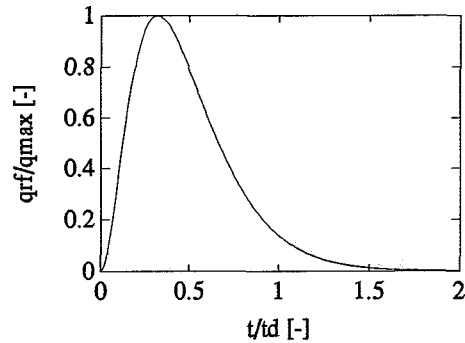
Figure 3.1: Rounded pulse.

of the sprung mass, and the dynamic tire force are reduced significantly compared with the passive suspension. The three combinations of Q and R have been selected in [10] from a large number of combinations for which the response of the active suspension was calculated using a step function as the road input.

When we look at Figs. 3.2 and 3.3, the following attracts the attention:

- The performance improvement possible for a step function as road surface does not hold for the wide range of rounded pulses though a significant performance improvement is still possible:

  - For the active suspension which minimizes the required suspension working space, the reduction of the required working space is remarkable. However, especially the accelerations of the sprung mass but also the dynamic tire forces are inadmissible.

  - The active suspension which minimizes the maximum acceleration of the sprung mass indeed reduces this maximum value significantly (see $t_d \approx 0.3$ [s] in the figures showing the maximum positive acceleration of the sprung mass). This improvement compared with the passive suspension, however, is not that big for all t_d's. Moreover, the dynamic tire forces are too high for $t_d \approx 0.2$ [s]. It must be possible to avoid this by slightly altering the combination of Q and R. Note that this active suspension reduces the required suspension working space with about 20%.

  - The "best overall" suspension indeed shows the best overall performance. The slight increase of the dynamic tire force compared with the passive suspension is acceptable.

- The influence of increasing the preview time to $t_p = 1$ [s] becomes significant from $t_d = 0.2$ [s]. This is plausible because a preview time $t_p = 1/8$ [s] is not enough anymore to overlook the pulse which will enter the vehicle if $t_d \geq 0.2$ [s].

- For $t_d < 0.08$ [s], the maximum negative dynamic tire force increases strongly. This is not realistic: at a certain force, the tire will burst. It might be useful to incorporate a maximum permissible negative tire force.

Figure 3.2: Performance of an active suspension with preview compared with a passive suspension for rounded pulses as the road input. Preview time $t_p = 1/8$ [s]; (——) passive; (o) active, best overall performance; (x) active, required suspension working space minimized; (+) active, maximum acceleration sprung mass minimized.
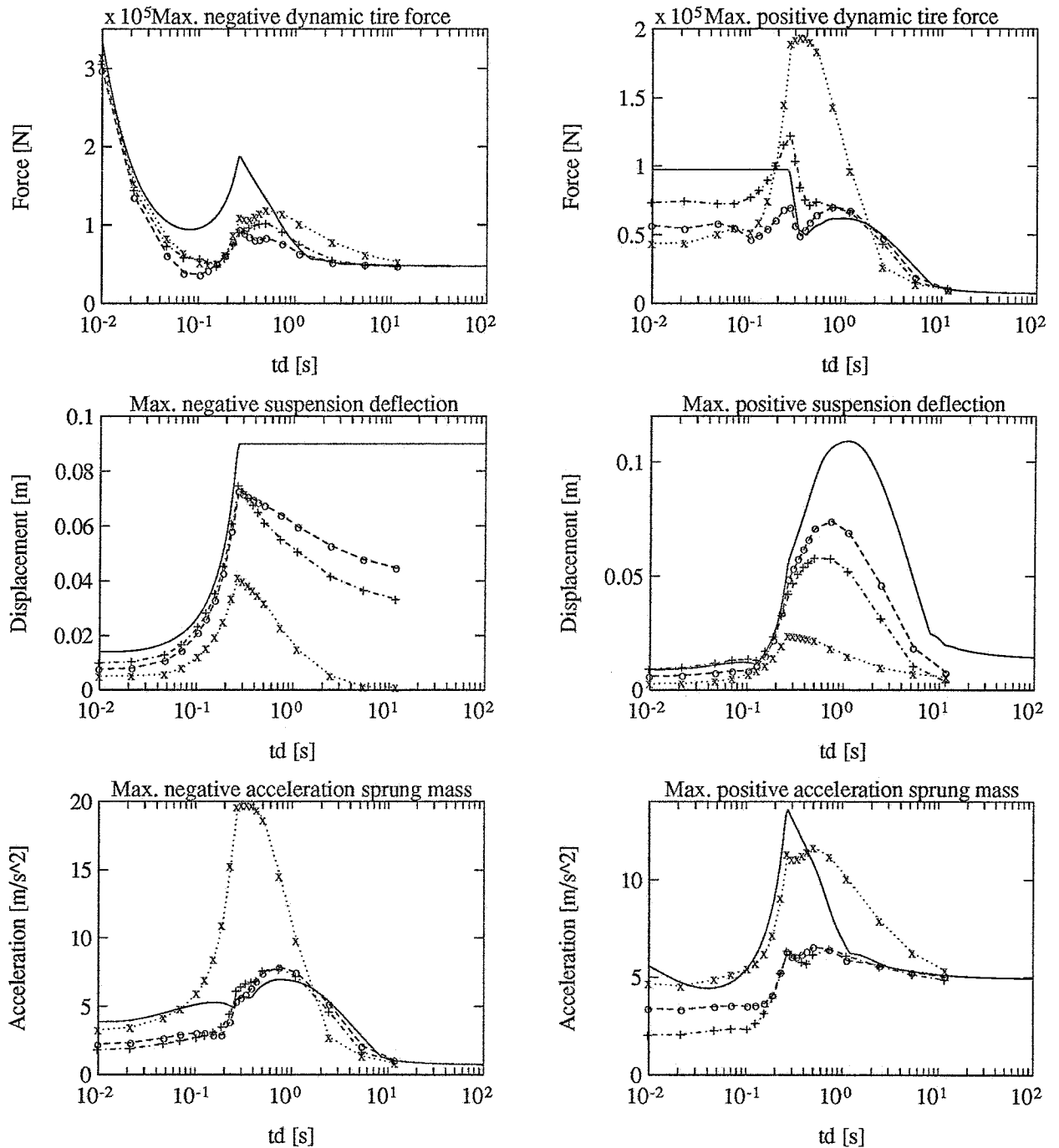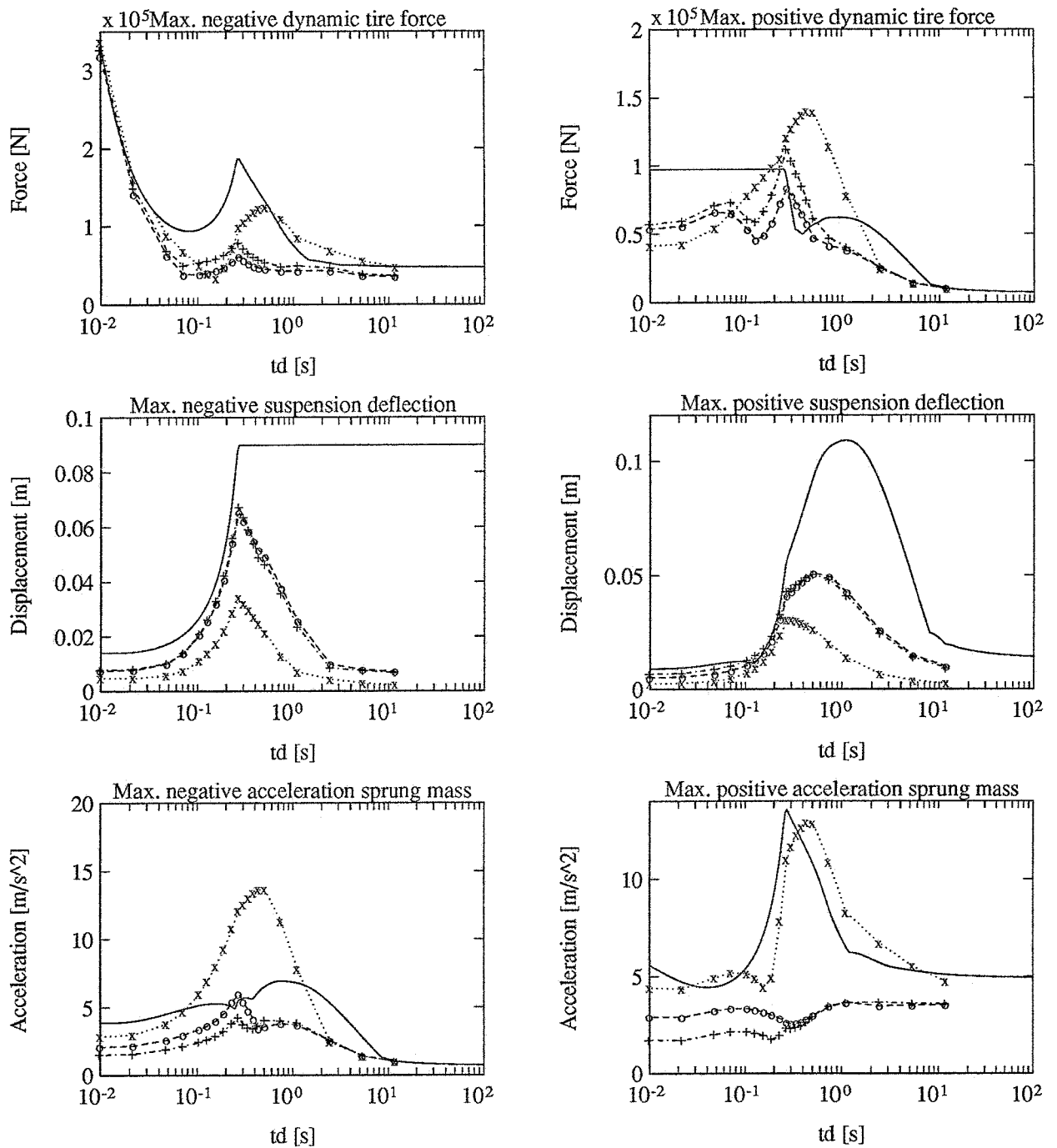
Figure 3.3: Performance of an active suspension with preview compared with a passive suspension for rounded pulses as the road input. Preview time $t_p = 1$ [s]; (——) passive; (o) active, best overall performance; (x) active, required suspension working space minimized; (+) active, maximum acceleration sprung mass minimized.

3.3   Stochastic road surface

In literature, the performance of a suspension is often demonstrated for a stochastic road surface (e.g. Crolla and Abdel-Hady [5], Crolla and Aboul Nour [6], El Madany [18], and Sharp and Crolla [20]). Though it is not likely that DAF will choose for an active suspension which operates continuously, the active suspension with preview is also tested for a stochastic road surface to test the robustness.

The stochastic road surface is represented by the power spectral density [5]

$$\text{PSD}(\eta) = \frac{R_c}{\eta^\kappa}, \tag{3.2}$$

where η is the wavenumber and R_c and κ determine the road type. For low wave numbers $(\eta < \eta_0)$, the PSD of the road surface is supposed to be constant:

$$\text{PSD}(\eta < \eta_0) = \frac{R_c}{\eta_0^\kappa}. \tag{3.3}$$

Here, we used $R_c = 3 \cdot 10^{-6}$ [m^2(cycle/m)$^{\kappa-1}$], $\kappa = 2.5$, and $\eta_0 = 0.01$ [cycle/m] which represent a slightly worse than average minor road [5]. The power spectrum can also be written as a function of the frequency f and the vehicle speed v if we use the relation $f = v \cdot \eta$:

$$\text{PSD}(f) = \frac{R_c \cdot v^{\kappa-1}}{f^\kappa}. \tag{3.4}$$

The active suspension is tested for two vehicle speeds. If we suppose that the road surface can be reconstructed at the front wheels, then, for a vehicle speed $v = 25$ [m/s] and a wheelbase of 3.125 [m], the preview time t_p is 1/8 [s]. For $v = 3.125$ [m/s] the preview time t_p is 1 [s].

The RMS values (not frequency weighted) of the dynamic tire force variation, the suspension deflection, and the acceleration of the sprung mass are shown in Tables 3.1 and 3.2.

Table 3.1: Performance of an active suspension with preview compared with a passive suspension for a stochastic road surface. Preview time $t_p = 1/8$ [s]; Active 1 = best overall performance; Active 2 = required suspension working space minimized; Active 3 = maximum acceleration sprung mass minimized.

RMS values	Passive	Active 1	Active 2	Active 3
Dynamic tire force [10^4 N]	4.0	2.6	3.9	3.6
Suspension deflection [10^{-2} m]	2.3	1.6	0.67	1.5
Acceleration sprung mass [m/s^2]	2.6	2.1	3.9	1.9

When we look at Tables 3.1 and 3.2, the following attracts the attention:

• Most of the remarks made on the performance of the active suspension for the rounded pulses can also be made for the stochastic road surface:

Table 3.2: Performance of an active suspension with preview compared with a passive suspension for a stochastic road surface. Preview time $t_p = 1$ [s]; Active 1 = best overall performance; Active 2 = required suspension working space minimized; Active 3 = maximum acceleration sprung mass minimized.

RMS values	Passive	Active 1	Active 2	Active 3
Dynamic tire force [10^4 N]	0.86	0.56	0.77	0.74
Suspension deflection [10^{-2} m]	0.58	0.25	0.11	0.25
Acceleration sprung mass [m/s^2]	0.56	0.33	0.71	0.29

- A remarkable reduction of the required suspension working space is possible, however, at the expense of an inadmissible increase of the acceleration of the sprung mass.

- The accelerations of the sprung mass can be reduced significantly. Again, the required suspension working space is reduced as well. Note that the dynamic tire forces are not greater than that of the passive suspension.

- The "best overall" suspension shows the best overall performance for stochastic road surfaces too.

- The improvement of the dynamic tire force of the active suspension with $t_p = 1$ [s] compared with the passive suspension is almost the same as for $t_p = 1/8$ [s]. This sounds reasonable because an increase of the preview time mainly improves the response for low-frequent road-excitations (in terms of rounded pulses: for large t_d's), while the dynamic tire force is mainly determined by high-frequent (about 10 Hz) road-excitations. The suspension deflection is mainly a result of low-frequent road excitations and indeed, the reduction of the required suspension working space is significantly greater for $t_p = 1$ [s]. The same remarks as for the dynamic tire force should hold for the accelerations of the sprung mass. However, the performance improvement is significantly greater (not as great as for the suspension deflection but still significant) for $t_p = 1$ [s]. It might be useful to determine the frequency response of the active suspension in order to explain this.

3.4 Conclusions

The following conclusions can be drawn:

- The results obtained by testing the active suspension for a step function as road surface can not be generalized for other, more realistic, road surfaces like rounded pulses and stochastic road surfaces. However, the performance of the active suspension is still significantly better than that of the passive suspension: the accelerations of the sprung mass can be reduced substantially without increase of the dynamic tire force and the required suspension working space. This result is also true for the minimum available preview time $t_p = 1/8$ [s].

- A preview time $t_p = 1$ [s] improves the performance especially for low-frequent road-excitations. For the rounded pulses, improvement is visible from $t_d = 0.2$ [s].

- In determining the height of the rounded pulses, it is not only necessary to look at the dynamic tire force which causes tire lift off. The dynamic tire force which causes the tire to burst should be regarded as well.

- Though it might be more difficult (because of the very large number of simulations to be done), it is recommended to choose the weighting matrices in the optimization of the active suspension with preview on the basis of the dynamic behaviour of the vehicle for rounded pulses and not for a step function as the road input.

Chapter 4

Observability

4.1 Introduction

Before we can find out *how* to reconstruct the road surface, we have to determine first *how many* and *which quantities* have to be measured. In this chapter, a method is described to check whether or not the state and/or the unknown input (in our case: the road surface) can be reconstructed from a given set of measurements. In other words, the *observability* of the system is determined.

Two cases of observability are dealt with, one in which the initial state is *unknown* and one in which the initial state is *known*. As will be shown in Chapter 5, the latter case is of particular interest.

The observability of the state and the unknown input is determined only for linear time invariant (L.T.I.) systems. This is not a problem yet because the vehicle models used so far belong to this class of systems. The criteria for observability are given in terms of state space matrices.

In Section 4.2, some definitions with respect to observability are given. Criteria for observability are given in Section 4.3. Finally, conclusions are drawn in Section 4.4.

4.2 Some definitions on observability

The next two definitions with respect to observability of a system are given by Basile et al. [3]. These definitions define the observability of the state and do also hold for nonlinear systems.

Definition 1 *A system with* unknown *initial state and unknown input is observable if and only if the state trajectory in a finite time interval is a function of the measured output in the same interval.*

Definition 2 *A system with* known *initial state and unknown input is observable if and only if the state trajectory in a finite time interval is a function of the initial state and the measured output in the same interval.*

Less formally, this means that a system is called *observable* if the state can be reconstructed only from the measured output and, eventually, knowledge of the initial state. Because we also want to reconstruct an unknown input, it is useful to give two definitions with respect to the observability of the unknown input.

Definition 3 *A system with* unknown *initial state and unknown input is unknown input observable if and only if the unknown input in a finite time interval is a function of the measured output in the same interval.*

Definition 4 *A system with* known *initial state and unknown input is unknown input observable if and only if the unknown input in a finite time interval is a function of the initial state and the measured output in the same interval.*

Less formally again, this means that a system is called *unknown input observable* if the unknown input can be reconstructed only from the measured output and, eventually, knowledge of the initial state.

4.3 Observability of linear time invariant systems

In this section, criteria for the observability of L.T.I. systems are presented. These criteria are illustrated by an example in Appendix B. To avoid any misunderstanding, the meaning of some mathematical expressions is given first.

4.3.1 Some mathematical notations

Suppose that $\mathbf{X}$ is a set of vectors $\mathbf{x}_i$:

$$\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_s\}, \quad \mathbf{x}_i \in R^n, \quad i = 1, ..., s, \tag{4.1}$$

and let $\mathcal{R}(\mathbf{X})$ be the space spanned by these vectors, i.e.

$$\mathcal{R}(\mathbf{X}) = \{\sum_{i=1}^{s} \lambda_i \mathbf{x}_i \mid \lambda_i \in \mathbb{R}\}. \tag{4.2}$$

The space perpendicular to $\mathcal{R}(\mathbf{X})$ is denoted as $\mathcal{R}(\mathbf{X})^{\perp}$ and is defined by

$$\mathcal{R}(\mathbf{X})^{\perp} = \{\mathbf{y} \in R^n \mid \forall_{\mathbf{x} \in \mathcal{R}(\mathbf{X})} (\mathbf{y}, \mathbf{x}) = 0\}, \tag{4.3}$$

where $(\mathbf{y}, \mathbf{x})$ is the inner product of $\mathbf{y}$ and $\mathbf{x}$.

Consider the linear transformation $\mathbf{A} : R^n \rightarrow R^m$. Then, the image of $\mathcal{R}(\mathbf{X})$ under $\mathbf{A}$, denoted as $\mathbf{A}\mathcal{R}(\mathbf{X})$, is defined by

$$\mathbf{A}\mathcal{R}(\mathbf{X}) = \{\sum_{i=1}^{s} \lambda_i \mathbf{A}\mathbf{x}_i \mid \lambda_i \in \mathbb{R}\}. \tag{4.4}$$

4.3.2 Observability of a system

Consider the state equation (2.1)

$$\dot{\mathbf{x}}(\tau) = \mathbf{A}\mathbf{x}(\tau) + \mathbf{B}\mathbf{u}(\tau) + \mathbf{E}\mathbf{w}(\tau), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \tag{4.5}$$

where $\mathbf{x} \in R^n, \mathbf{u} \in R^p$, and $\mathbf{w} \in R^q$. Remind that $\mathbf{u}$ is the *known* input and $\mathbf{w}$ is the *unknown* input (or disturbance). Suppose that the measured output $\mathbf{z}$ can be written as

$$\mathbf{z}(\tau) = \mathbf{C}_z \mathbf{x}(\tau), \tag{4.6}$$

where $\mathbf{z} \in R^m$. Then, the observability of the state $\mathbf{x}$ can be divided in two cases:

1. The initial state is *unknown*.

   According to Basile and Marro [4], the observable subspace $\mathcal{O}_1 \subseteq R^n$ is equal to

   $$\mathcal{O}_1 = \mathcal{Y}_{n-1}, \tag{4.7}$$

   where $\mathcal{Y}_{n-1} \subseteq R^n$ is defined by the recursive relationship

   $$
   \begin{aligned}
   \mathcal{Y}_0 &= \mathcal{R}(\mathbf{C}_z^T); & (4.8) \\
   \mathcal{Y}_i &= \mathcal{R}(\mathbf{C}_z^T) + \mathbf{A}^T(\mathcal{Y}_{i-1} \cap \mathcal{R}(\mathbf{E})^\perp), & i = 1, ..., n-1. & (4.9)
   \end{aligned}
   $$

   The unobservable subspace $\mathcal{Q}_1 \subseteq R^n$ is equal to

   $$\mathcal{Q}_1 = \mathcal{O}_1^\perp. \tag{4.10}$$

   According to Definition 1, the system is observable if the space $\mathcal{O}_1$ has dimension n.

2. The initial state is *known*.

   First, assume for the moment that the initial state is unknown and determine the unobservable subspace $\mathcal{Q}_1$ according to Eq. (4.10). According to Basile et al. [3], the unobservable subspace $\mathcal{Q}_2 \subseteq R^n$ is equal to

   $$\mathcal{Q}_2 = \mathcal{Z}_n, \tag{4.11}$$

   where $\mathcal{Z}_n \subseteq R^n$ is defined by the recursive relationship

   $$
   \begin{aligned}
   \mathcal{Z}_0 &= \emptyset; & (4.12) \\
   \mathcal{Z}_i &= (\mathbf{A}\mathcal{Z}_{i-1} + \mathcal{R}(\mathbf{E})) \cap \mathcal{Q}_1, & i = 1, ..., n. & (4.13)
   \end{aligned}
   $$

   The observable subspace $\mathcal{O}_2 \subseteq R^n$ is equal to

   $$\mathcal{O}_2 = \mathcal{Q}_2^\perp. \tag{4.14}$$

   The system is observable according to Definition 2 if the space $\mathcal{O}_2$ has dimension n.

   Note that the term $\mathbf{B}\mathbf{u}$ in Eq. (4.5) has no influence on the observability criteria (see Appendix C).

4.3.3 Unknown input observability of a system

In Section 4.3.2, we presented criteria to check the observability of a L.T.I. system. When we want to reconstruct the road surface which enters the front wheels of the tractor, the *unknown input observability* of the system has to be checked as well.

There are two ways to determine the unknown input observability:

1. Determine the observability of the state according to the strategy described in Section 4.3.2. Then, the system is unknown input observable if the system is observable and the matrix $\mathbf{E}$ has full column rank.

   This result can easily be verified. If the system is observable, then the terms $\dot{\mathbf{x}}$, $\mathbf{A}\mathbf{x}$, and $\mathbf{B}\mathbf{u}$ in Eq. (4.5) are known so the unknown input satisfies

   $$\mathbf{E}\mathbf{w} = \dot{\mathbf{x}} - \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{u}, \tag{4.15}$$

Hence, $\mathbf{w}$ can be determined if the matrix $\mathbf{E}$ has full column rank (which is almost always the case).

If the system is not observable, we have to determine whether or not the unknown input $\mathbf{w}$ can be reconstructed from the observable part of the state. This has not been worked out in detail.

2. Include the unknown input in the state and determine the observability of the extended system according to the strategy presented in Section 4.3.2. If the unknown input is a part of the observable subspace, the system is unknown input observable.

The extended system is built up in the following way. Define a new state

$$\mathbf{x}' = \begin{bmatrix} \mathbf{x} \\ \mathbf{w} \end{bmatrix}, \qquad (4.16)$$

which results in the state- and measured output equation of the extended system:

$$\dot{\mathbf{x}}' = \mathbf{A}'\mathbf{x}' + \mathbf{E}'\mathbf{w}', \quad \mathbf{x}'(t_0) = \mathbf{x}'_0, \qquad (4.17)$$

$$\mathbf{z} = \mathbf{C}'_z\mathbf{x}', \qquad (4.18)$$

with

$$\mathbf{A}' = \begin{bmatrix} \mathbf{A} & \mathbf{E} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{C}'_z = \begin{bmatrix} \mathbf{C}_z & \mathbf{0} \end{bmatrix}, \quad \mathbf{E}' = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}. \qquad (4.19)$$

In fact, to form the extended system, only an integrator has been added to the system. This is illustrated in Fig. 4.1 where S1 is the original system described by Eq. (4.5) (the known input $\mathbf{u}$ has been omitted) with unknown input $\mathbf{w}$. S2 is the extended system described by Eq. (4.17) with unknown input $\mathbf{w}' = \dot{\mathbf{w}}$ and extra initial condition $\mathbf{w}(t_0) = \mathbf{w}_0$. Note that we are not interested in whether the new unknown input $\mathbf{w}'$ can be determined or not.
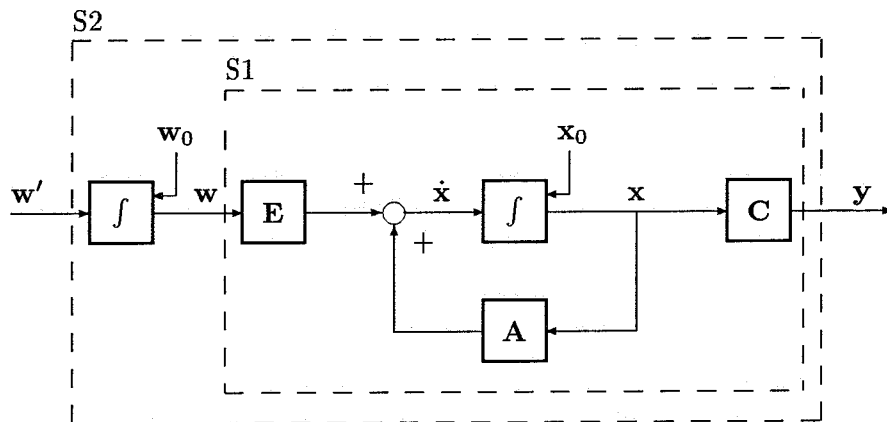
Figure 4.1: Block diagram of the original and the extended system.

An advantage of using the extended state is that it is possible to include measurements described by

$$\mathbf{z} = \mathbf{C}_z\mathbf{x} + \mathbf{F}_z\mathbf{w}, \qquad (4.20)$$

because this equation can be written in the form of Eq. (4.18) by choosing $\mathbf{C}'_z =$ [$\mathbf{C}_z$ $\mathbf{F}$]. In the example shown in Appendix B, we show that a measurement equation in the form of Eq. (4.20) is useful in the reconstruction of the road surface.

4.4 Conclusions

We can draw the following conclusions:

- Using the methods described in Section 4.3, it is possible to determine, given a set of measurements and, eventually, knowledge of the initial conditions, whether or not a linear time invariant system is observable and/or unknown input observable. By doing this, also the (un-)observable subspaces are determined. A state space model of the system must be available. Note that it is not possible to determine directly how many and which quantities have to be measured.

- It is useful to extend the state of the system with the unknown input because in that case, with the methods described in Section 4.3, the observability can be checked given a larger set of measurements which, for example, also include accelerations.

Chapter 5

Observer which reconstructs the road surface

5.1 Introduction

In Chapter 4, we described criteria to determine the observability of an unknown input. In this chapter, we deal with the problem of *how* to reconstruct the road surface - which is an unknown input - at the front wheels of the tractor. It is not our intention to present a general applicable observer for systems with unknown inputs.

In this report, the reconstructed road surface is used as preview for the active suspension at the rear wheels of the tractor. However, this preview information can also be used in the control strategy of a semi-active suspension (Hać [9]) and for a suspension that is incidentally active. That is why the road surface itself is reconstructed and not, for example, its time derivative (which could also be used in the calculation of the actuator force of the active suspension). For (semi-) active suspensions without preview, the reconstruction method can also be applied to determine the state quantities needed to feed back.

Some observers from literature for systems with unknown inputs are presented in Section 5.2. Our method to reconstruct the road surface is described in Section 5.3. Finally, conclusions are drawn in Section 5.4.

5.2 Observers from literature

In literature, several methods are proposed to determine an unknown input. Here, we describe some of them briefly, just to show the variety of strategies available to tackle the reconstruction problem.

Johnson [11] describes an observer based on a Luenberger observer [17]. The unknown input must be the output of an extra dynamic model with standard inputs (impulses, etc.). Only the state of the combined system is reconstructed but this is not a problem since the unknown input is included in the state (see Section 4.3.3).

Konik [15] and Stein and Park [21] describe an unknown input observer which reconstructs both the state and the unknown input of the original system. Konik demonstrates the observer in the reconstruction of the road surface using a two DOF vehicle model. In both observers, *redundant* measurements cannot be taken into account. Moreover, to reconstruct the unknown

17

input, it is necessary to differentiate the measurements. This deteriorates the signal to noise ratio significantly.

In contrary to [11], the observer described by Yang and Wilde [23] does not need a model of the unknown input. However, for systems with many degrees of freedom, the choice of the parameters in this observer is not clear. The observer only reconstructs the state.

A disadvantage of these observers is that no attention is paid to measurement and model errors. This is one of the reasons why we try to find in the next section an observer which has some features of a Kalman filter [12] [13].

5.3 Reconstruction of the road surface

Before we come to the reconstruction of the road surface, the vehicle model and the measurements used in the reconstruction are described.

5.3.1 Model used in the reconstruction

To reconstruct the road surface from measurements at the front wheels, the front side of the tractor is modelled as shown in Fig. 5.1. Here, m_{af} represents the unsprung mass (front axle, brakes, etc.), k_{tf} represents the tire stiffness, q_{rf} the vertical position of the road surface at the front wheels of the tractor, q_{af} the vertical position of the unsprung mass, and q_{cf} the vertical position of the chassis straight above the front axle. The force f_{sf} represents the force from the secondary suspension acting on the unsprung mass.

A vehicle model containing only the unsprung mass is chosen because, for a tractor-semitrailer, a more complex and accurate model will contain many degrees of freedom in order to model the cabin and the motor, leading to excessive computational requirements.

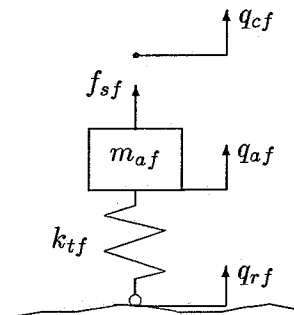Figure 5.1: Model used for the observer.

5.3.2 Measurements used in the reconstruction

The criteria presented in Section 4.3 to determine the observability of a system and/or the unknown input can be used to find the quantities which have to be measured to reconstruct the road surface. Especially the criteria for systems with unknown inputs and *known* initial state will appear to be opportune now.

If we suppose that the initial conditions are known (which does not form a practical problem), it is sufficient to measure the suspension force f_{sf} and the vertical acceleration $\ddot{q}_{af}$.

To reduce the effect of measurement errors, it is useful to have redundant measurements. For this reason, also the vertical acceleration of the chassis $(\ddot{q}_{cf})$ and the suspension deflection $(q_{cf} - q_{af})$ are measured.

The force f_{sf} is either measured directly or determined from measurements of the suspension deflection $(q_{cf} - q_{af})$ and of the suspension deflection velocity $(\dot{q}_{cf} - \dot{q}_{af})$, using a model of the secondary suspension.

5.3.3 Reconstruction of the road surface from the measurements

The reconstruction of the road surface is carried out in the following way:

1. A first estimate for the vertical position of the axle $(\hat{q}_{af1})$[1] and of the chassis $(\hat{q}_{cf1})$ is made by integration of $\ddot{q}_{af}$ and $\ddot{q}_{cf}$ using a simple integration scheme. Integration is possible because the initial conditions are supposed to be known.

2. A Luenberger observer [17] is used to update the estimated vertical position of the axle $(\hat{q}_{af})$ using the measurements $\ddot{q}_{af}$, $\ddot{q}_{cf}$, and $q_{cf} - q_{af}$, and the estimates $\hat{q}_{af1}$ and $\hat{q}_{cf1}$:

$$\dot{\hat{x}}_l(\tau) = A_l\hat{x}_l(\tau) + B_l u_l(\tau) + K_l(z_l(\tau) - \hat{z}_l(\tau)), \quad \hat{x}_l(t_0) = \hat{x}_{l0}, \tag{5.1}$$

$$\hat{z}_l(\tau) = C_{zl}\hat{x}_l(\tau), \tag{5.2}$$

with

$$\hat{x}_l = \begin{bmatrix} \hat{q}_{af} \\ \dot{\hat{q}}_{af} \\ \hat{q}_{cf} \\ \dot{\hat{q}}_{cf} \end{bmatrix}, \quad u_l = \begin{bmatrix} \ddot{q}_{af} \\ \ddot{q}_{cf} \end{bmatrix}, \quad z_l = \begin{bmatrix} \hat{q}_{af1} \\ \hat{q}_{cf1} \\ q_{cf} - q_{af} \end{bmatrix}, \tag{5.3}$$

and

$$A_l = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_l = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_{zl} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}. \tag{5.4}$$

The observer matrix K_l has to be determined yet.

3. Finally, the road surface $(\hat{q}_{rf})$ is estimated from

$$\hat{q}_{rf} = \hat{q}_{af} + \frac{m_{af}\ddot{q}_{af} - f_{sf}}{k_{tf}}. \tag{5.5}$$

The following remarks have to be made on this observer:

- Step 1 is necessary to make system (5.1) - (5.2) observable for the case that the initial state x_{l0} is *unknown*. This is required when we want to use a Luenberger observer.

---

[1] The symbol $\hat{a}$ denotes an estimate of a.

- The observer matrix $\mathbf{K}_l$ is chosen using a Kalman filter approach [12] [13]. For this purpose, model and measurement noise are added to the equations which describe "reality":

$$\dot{\mathbf{x}}_l(\tau) = \mathbf{A}_l\mathbf{x}_l(\tau) + \mathbf{B}_l\mathbf{u}_l(\tau) + \mathbf{E}_l\xi_l(\tau), \quad \mathbf{x}_l(t_0) = \mathbf{x}_{l0}, \tag{5.6}$$

$$\mathbf{z}_l(\tau) = \mathbf{C}_{zl}\mathbf{x}_l(\tau) + \zeta_l(\tau), \tag{5.7}$$

where ξ_l and ζ_l are white noise disturbances with zero mean and intensity matrices $\mathbf{Q}_\xi$ and $\mathbf{R}_\zeta$ respectively. $\mathbf{R}_\zeta$ is supposed to be invertible.

With the intensity matrices $\mathbf{Q}_\xi$ and $\mathbf{R}_\zeta$, reliable measurements ($\ddot{q}_{af}, \ddot{q}_{cf}$, and $q_{cf} - q_{af}$) can be given more emphasis than less reliable measurements ($\hat{q}_{af1}$ and $\hat{q}_{cf1}$ contain drift).

The observability matrix $\mathbf{K}_l$ is calculated from

$$\mathbf{K}_l = \mathbf{P}_l\mathbf{C}_{zl}^T\mathbf{R}_\zeta^{-1}, \tag{5.8}$$

where $\mathbf{P}_l$ is determined by the algebraic Riccati equation

$$\mathbf{A}_l\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l^T + \mathbf{E}_l\mathbf{Q}_\xi\mathbf{E}_l^T - \mathbf{P}_l\mathbf{C}_{zl}^T\mathbf{R}_\zeta^{-1}\mathbf{C}_{zl}\mathbf{P}_l = \mathbf{0}. \tag{5.9}$$

Note that the use of Kalman filter theory is not obvious because the model and measurement noises will not be white in practice.

- Measurement errors on $\ddot{q}_{af}$ and f_{sf} have a direct influence on $\hat{q}_{rf}$ (see Eq. (5.5)). Hence, bandpass filters may be necessary. Alternatively, because the estimated road surface is only used for the control of the rear suspension, a smoother (e.g. Gelb [8]) can be used to improve the measurements.

  It seems to be possible to redesign the observer such that all the measurements are filtered by the observer. Then, the use of extra filters and smoothers is superfluous. This is an idea for further investigation.

5.4 Conclusions

In this chapter, a method is presented to reconstruct the road surface from simple measurements. The method is based on a Luenberger observer but also uses Kalman filter theory to incorporate model and measurement errors. A vehicle model only containing the unsprung mass and the vertical position of the chassis is used.

The following conclusions can be made:

- To reconstruct the road surface, a model of the road surface is not necessary and the measurements have not to be differentiated. However, not all the measurements are filtered by the observer. Thus, extra filters might be necessary.

- The stability of the observer has not been investigated yet.

Chapter 6

Performance of the observer

6.1 Introduction

The observer described in Section 5.3 is tested in simulations for several road surfaces. Especially the influence of measurement noise and model errors on the performance of the observer is investigated. This is described in Section 6.2. Conclusions are drawn in Section 6.3.

6.2 Testing the observer

To test the observer, the "measurements" are taken from simulations with the "real vehicle" represented by the four DOF model shown in Fig. 7.1, which is described in Chapter 7. For this purpose, the actuator is replaced by passive suspension with a linear spring and damper (which parameter value is given in Appendix A). The suspension force f_{sf} (see Fig. 5.1) is measured directly.

A "good" combination of the intensity matrices $\mathbf{Q}_\xi$ and $\mathbf{R}_\zeta$ (see Section 5.3) has been determined by trial and error. This combination is used for all simulations. The value of $\mathbf{Q}_\xi$ and $\mathbf{R}_\zeta$ is given in Appendix A.

If both the measurements and the vehicle model used in the reconstruction are perfect, the reconstruction of the road surface is also perfect. Of course, this result is not surprising.
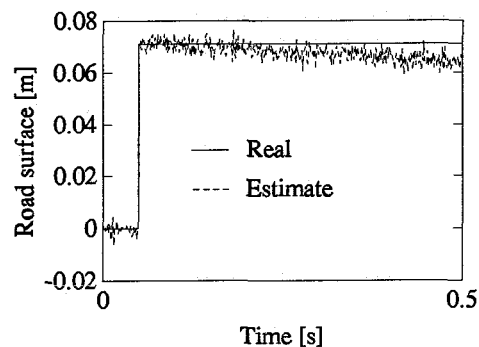

Figure 6.1: Reconstruction for a step function as the road input. All measurements are disturbed by measurement noise.

6.2.1   Influence measurement noise

In Figs. 6.1 and 6.2, the reconstruction of the road surface is shown for a step function and a stochastic road surface for the case that all measurements are disturbed by noise. The measurement noise is supposed to be white (Gaussian probability distribution function with a zero mean and a standard deviation of 2.5% of the required measurement range which has been determined for a step function as the road input with a height of 7.1 cm).
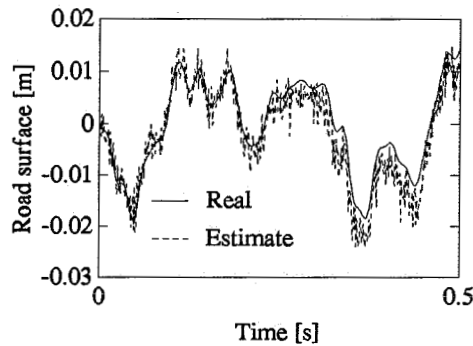
Figure 6.2: Reconstruction for a stochastic road surface. All measurements are disturbed by measurement noise.

The value of the standard deviation of the measurement noise is chosen equal for all measurements because, at the moment, we do not know exactly what measurement instruments will be used. Therefore, only a first impression of the influence of measurements noise is obtained.

We can conclude that the reconstruction is satisfactory for both deterministic and stochastic road surfaces. However, the measurement noise has a direct influence on the estimated road surface as already mentioned in Section 5.3.

Fig. 6.3 shows the estimated vertical position of the axle. In this figure, we can see that observer reduces the influence of the measurement noise on the estimated position of the axle significantly.

Figure 6.3: Reconstruction of the vertical position of the axle for a step function as the road input. All measurements are disturbed by measurement noise.

Moreover, if we compare the first estimate of the vertical position of the axle ($\hat{q}_{af1}$) with

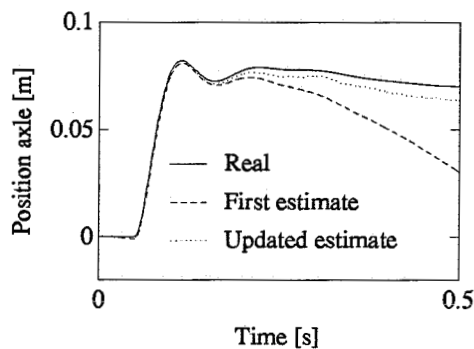the updated one ($\hat{q}_{af}$), we can see that the observer reduces the drift significantly. However, because the drift in $\hat{q}_{af1}$ and $\hat{q}_{cf1}$ are independent, it is not guaranteed that the drift is always reduced that much. For example, if both $\hat{q}_{af1}$ and $\hat{q}_{cf1}$ drift to the same direction, the measurement of the relative displacement $q_{cf} - q_{af}$ can not eliminate this drift.

6.2.2 Influence model errors

In Fig. 6.4, the influence of model errors on the reconstruction is shown. The road input is a step function and either the tire stiffness (k_{tf}) or the sprung mass (m_{af}) used in the reconstruction is different from that in "reality". Realistic model errors are chosen: 30% error in the tire stiffness (tire stiffness is nonlinear in practice) and 5% error in the sprung mass.
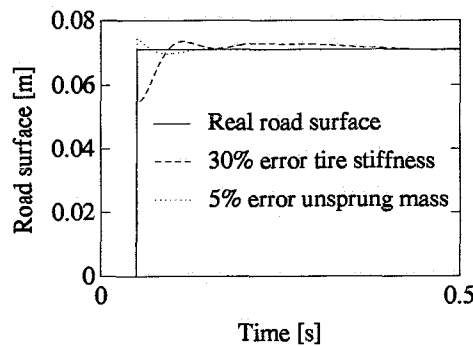
Figure 6.4: Influence of model errors.

The estimation error vanishes within 0.5 second. The influence of the error at the beginning of the step on the closed loop behaviour will only become clear when the estimated road surface is used for the controller of the active suspension at the rear wheels of the tractor. In Chapter 7, this combination of observer and controller is described. However, only a few simulations with the combination have been carried out yet.

6.3 Conclusions

The following conclusions can be drawn concerning the performance of the observer:

- If both the model used for the reconstruction of the road surface and the measurements are perfect, the observer reconstructs the road surface perfectly.

- The observer reduces the drift in the estimated road surface.

- The influence of measurement noise on the estimated vertical position of the axle is reduced significantly. However, as already mentioned in Section 5.3, the measurement noise has still a direct influence on the estimated road surface.

- The estimation error due to model errors is reduced quickly. Whether or not this is fast enough to guarantee a good closed loop behaviour of the combination of observer and controller has not been investigated yet.

Chapter 7

Combination of controller and observer

7.1 Introduction

In Chapter 2, we described a control strategy for an active suspension with preview. The reconstruction of the preview information is possible with the method described in Chapter 5. It is interesting to investigate whether the combination of the controller and the observer performs well.

In this chapter, we describe the combination of the controller and the observer. Moreover, a strategy to test this combination is presented.

Though the first results obtained with the combination are promising, it is too premature to include them in this report. So, unfortunately, no conclusions can be drawn yet concerning the performance of the combination of the controller and the observer.

7.2 Combining the controller and the observer

In practice, it is not possible to use a perfect model of the vehicle we want to control because it is impossible to make such a model and the computational effort would be excessive. Therefore, the controller and the observer have to use simple models. In this report, different models are used to control the suspension, to reconstruct the road surface, and to simulate the dynamic behaviour of the "real" vehicle. The models used for the controller and for the observer have already been described in Section 2.3 and Section 5.3.

In the model used by the controller (see Fig. 2.1), a value of the sprung mass is needed. Here, this value is chosen equal to the static load on the rear axle, divided by the gravity acceleration. Moreover, the state has to be determined to calculate the actuator force. Because the unknown input (i.e. the road surface) is reconstructed, a simple Kalman filter can be used now to reconstruct this state.

The motion of the front side of a passenger car is more or less decoupled from that of the rear side (e.g. Sharp and Crolla [20]). Therefore, quarter car models are suitable to describe the dynamic behaviour of the vehicle (remind that we are only interested in the *vertical* dynamic behaviour). For tractor-semitrailers, the front and rear side are *not* decoupled at all. Here, at least a half truck model is required to simulate the dynamic behaviour.

24

In this report, the linear, four DOF vehicle model shown in Fig. 7.1 is used to simulate the dynamic behaviour of a tractor-semitrailer. The chassis of the tractor is supposed to be rigid although this is a rather strong simplification. The mass M_c represents a very simple model of the semitrailer resting upon the tractor. The cabin and the motor are taken into account in the chassis of the tractor. The model parameters used are given in Appendix A.
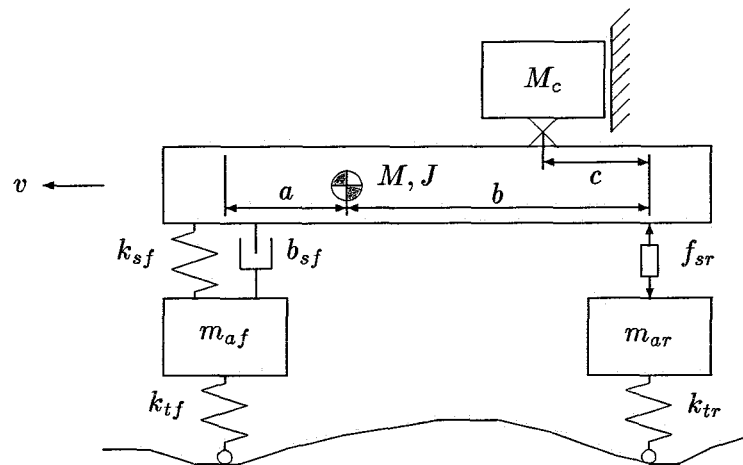

Figure 7.1: Four DOF vehicle model used to simulate the dynamic behaviour of the "real" vehicle.

The four DOF vehicle model is still a strong simplification of "reality". Nevertheless, it gives an opportunity to illustrate the consequences of using simple models to control the vehicle and to reconstruct the road surface.

Chapter 8

Conclusions

The main conclusions for the active suspension with preview on realistic road surfaces are:

- The results obtained from the tests on a step function as the road input can not be generalized for other, more realistic road surfaces like rounded pulses and stochastic road surfaces. However, the performance of the active suspension is still significantly better than that of a representative passive suspension: the accelerations of the sprung mass can be reduced substantially without increase of the dynamic tire force and required suspension working space. This result also holds for the minimum available preview time $t_p = 1/8$ [s].

- Compared with the minimum available preview time, a preview time $t_p = 1$ [s] improves the dynamic behaviour of the vehicle especially for low-frequent road-excitations.

A method has been presented to determine, given a set of measurements, the *observable subspace* of a system with unknown inputs and unknown/known initial conditions. The observable subspace contains that part of the state that can be reconstructed from the measurements and, if available, knowledge of the initial conditions. Moreover, with this method we can determine whether or not the unknown input is observable.

The main conclusions concerning the presented observer are:

- To reconstruct the road surface, a model of the road surface is not necessary and the measurements have not to be differentiated. However, not all measurements are filtered by the observer. Thus, extra filters might be necessary.

- The estimation error due to model errors is reduced quickly. Whether or not this is fast enough to guarantee a good closed loop behaviour of the combination of controller and observer has not been investigated yet.

- The observer reduces the drift in the estimated road surface.

Finally, the combination of the controller and the observer has not been tested thoroughly yet. So, unfortunately, it is too premature to present results in this report.

Chapter 9

Future investigation

9.1 Introduction

One of the striking things in carrying out research is that every time you have solved a problem there will arise ten new ones. This appeared to be true in my case too.

In this chapter, some possible subjects of future investigation are described. Moreover, some new ideas are presented. The most important subjects and ideas are described in Section 9.2, the less important ones in Section 9.3.

9.2 Most important investigation

The presented strategy to reconstruct the road surface has to be elaborated in more detail. The use of more advanced integration schemes may reduce drift further. As explained in Chapter 1, the estimated road surface is used to control the suspension at the rear wheels of the tractor. Because of this, a smoother can be used to filter the measurements, thus avoiding phase delays. Finally, the stability of the observer has to be investigated.

As shown in Section 6.2, the presented observer reduces the influence of measurement noise on the estimated vertical position of the front axle significantly. It seems to be possible to redesign the observer such that also the estimated road surface is filtered by the observer. Then, the use of extra filters or smoothers is superfluous.

For tractor-semitrailers, it turns out that the bending motion of the chassis is very significant and should be controlled. To control this motion, a half car model, including an elastic chassis, has to be used in the controller. In the simulation model, the cabin and the motor plus their suspension have to be included.

Finally, it is important to test the combination of the controller and the observer. In this report, a control strategy for an active suspension with preview is presented. However, the power consumption of this suspension will form a problem. Therefore, we must also investigate the possibilities of a semi-active suspension with preview and an incidentally active suspension. In the latter case, the preview information is also used to detect when it is necessary to switch on the active suspension, for example when a pot-hole will enter the rear wheels.

9.3 Less important investigation

One of the less important, but not less interesting, subjects for future investigation is the determination of the frequency response of the active suspension with preview. Frequency responses are often used to describe the performance of a suspension (e.g. Alanoly and Sankar [1], Karnopp [14]).

All simulations have been carried out with MATLAB. The simulation time for the active suspension with preview was rather excessive (about 1 hour on a SUN Sparc station for 2 simulation seconds) to avoid numerical problems. Especially because several "FOR-NEXT" loops are used, it is recommendable to write some parts of the programs in C or FORTRAN and link them to MATLAB. Moreover, the application of advanced numerical integration routines should be investigated.

Besides, it might be interesting to determine the "best" combination of the weighting matrices, used in the control of the active suspension, on the basis of the results for rounded pulses as the road input.

Appendix A

Parameter values

The parameter values for the vehicle model shown in Fig. 2.1 are taken down in Table A.1. For the model with a passive suspension, the actuator is replaced by a linear spring with stiffness $k_{sr} = 4.4 \cdot 10^5$ [N/m], and a linear damper with damping constant $b_{sr} = 4.31 \cdot 10^4$ [Ns/m].

Table A.1: Model parameters for model shown in Fig. 2.1.

Parameter	Value	Unit
tire stiffness k_{tr}	$6.5 \cdot 10^6$	N/m
unsprung mass m_{ar}	$1,350$	kg
sprung mass m_{cr}	$8,650$	kg

The parameter values for the vehicle model shown in Fig. 7.1 are taken down in Table A.2. For the model with a passive suspension, the actuator is replaced by a linear spring with stiffness $k_{sr} = 5.2 \cdot 10^5$ [N/m], and a linear damper with damping constant $b_{sr} = 3.5 \cdot 10^4$ [Ns/m].

Table A.2: Model parameters for the model shown in Fig. 7.1.

Parameter	Value	Unit	Parameter	Value	Unit
sprung mass M	$4,778$	kg	tire stiffness k_{tr}	$4.4 \cdot 10^6$	N/m
mass semitrailer M_c	$13,268$	kg	suspension stiffness k_{sf}	$6.9 \cdot 10^5$	N/m
unsprung mass m_{af}	815	kg	damping constant b_{sf}	$3.5 \cdot 10^4$	Ns/m
unsprung mass m_{ar}	$1,439$	kg	length a	0.518	m
mass inertia J	$9,090$	kgm^2	length b	2.732	m
tire stiffness k_{tf}	$2.2 \cdot 10^6$	N/m	length c	0.593	m

The active suspension is tested for three combinations of the weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ (see Section 3.2). The values of these weighting matrices are

1. for the case that the "best overall" performance is achieved:

$$\mathbf{Q} = \begin{bmatrix} 8 \cdot 10^{12} & 0 & 0 \\ 0 & 1 \cdot 10^{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 1 \end{bmatrix}, \tag{A.1}$$

2. and for the case that the required suspension working space is minimized:

$$\mathbf{Q} = \begin{bmatrix} 5 \cdot 10^{12} & 0 & 0 \\ 0 & 3.5 \cdot 10^{13} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 1 \end{bmatrix}, \tag{A.2}$$

3. and for the case that the maximum acceleration of the sprung mass is minimized:

$$\mathbf{Q} = \begin{bmatrix} 2 \cdot 10^{12} & 0 & 0 \\ 0 & 1 \cdot 10^{12} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 1 \end{bmatrix}. \tag{A.3}$$

The values of the intensity matrices $\mathbf{Q}_\xi$ and $\mathbf{R}_\zeta$ used to determine the observer matrix $\mathbf{K}_l$ are (see Section 6.2)

$$\mathbf{Q}_\xi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R}_\zeta = \begin{bmatrix} 10^6 & 0 & 0 \\ 0 & 10^{-3} & 0 \\ 0 & 0 & 10^{-3} \end{bmatrix}. \tag{A.4}$$

The values of t_d and q_{max} for which the response of the active suspension with preview is calculated (see Section 3.2) are taken down in Table A.3.

Table A.3: Values of t_d and q_{max} for which the response of the active suspension with preview is calculated.

t_d	q_{max}	t_d	q_{max}
[s]	[m]	[s]	[m]
$1.00 \cdot 10^{-2}$	$5.20 \cdot 10^{-2}$	$3.06 \cdot 10^{-1}$	$1.02 \cdot 10^{-1}$
$2.20 \cdot 10^{-2}$	$2.64 \cdot 10^{-2}$	$3.44 \cdot 10^{-1}$	$1.09 \cdot 10^{-1}$
$4.82 \cdot 10^{-2}$	$1.87 \cdot 10^{-2}$	$3.87 \cdot 10^{-1}$	$1.17 \cdot 10^{-1}$
$7.14 \cdot 10^{-2}$	$1.93 \cdot 10^{-2}$	$4.36 \cdot 10^{-1}$	$1.26 \cdot 10^{-1}$
$1.06 \cdot 10^{-1}$	$2.36 \cdot 10^{-2}$	$5.10 \cdot 10^{-1}$	$1.40 \cdot 10^{-1}$
$1.29 \cdot 10^{-1}$	$2.82 \cdot 10^{-2}$	$7.56 \cdot 10^{-1}$	$1.76 \cdot 10^{-1}$
$1.57 \cdot 10^{-1}$	$3.56 \cdot 10^{-2}$	$1.12 \cdot 10^{0}$	$2.29 \cdot 10^{-1}$
$1.91 \cdot 10^{-1}$	$4.82 \cdot 10^{-2}$	$2.46 \cdot 10^{0}$	$4.85 \cdot 10^{-1}$
$2.32 \cdot 10^{-1}$	$7.08 \cdot 10^{-2}$	$5.40 \cdot 10^{0}$	$1.34 \cdot 10^{0}$
$2.72 \cdot 10^{-1}$	$9.60 \cdot 10^{-2}$	$1.18 \cdot 10^{1}$	$4.65 \cdot 10^{0}$

Appendix B

An example of observability

Consider the simple one DOF model shown in Fig. B.1. If we choose $\mathbf{x} = [\ q_1, \quad \dot{q}_1\ ]^T$, then according to Eq. (4.5) $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ become

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}, \quad \mathbf{B} = \mathbf{0}, \quad \mathbf{E} = \begin{bmatrix} 0 \\ k/m \end{bmatrix}. \tag{B.1}$$

The unknown input is q_0. Suppose that we can measure the absolute velocity $\dot{q}_1$. Then,
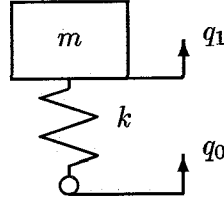

Figure B.1: Simple one DOF model.

according to Eq. (4.6), $\mathbf{C}_z = [\ 0 \quad 1\ ]$. Using the strategy descibed in Section 4.3.2, the observability subspace $\mathcal{O}_1$ for the case that the initial state is *not known* can be calculated as follows:

$$\mathcal{Y}_0 = \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right),$$

$$\mathcal{Y}_1 = \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) + \begin{bmatrix} 0 & -k/m \\ 1 & 0 \end{bmatrix} \left(\mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \cap \mathcal{R}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)\right)$$

$$= \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right),$$

Now, according to Eq. (4.7),

$$\mathcal{O}_1 = \mathcal{Y}_1 = \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right). \tag{B.2}$$

$\mathcal{O}_1$ has dimension 1 which means that the system is not observable. This is not surprising because it is not possible to reconstruct q_1 from measurements of $\dot{q}_1$ only. The unobservable

subspace is (see Eq. (4.10))

$$\mathcal{Q}_1 = \mathcal{O}_1^\perp = \mathcal{R}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right). \tag{B.3}$$

Next, suppose that the initial state is *known*. Then, the observability subspace $\mathcal{O}_2$ can be found as follows:

$$\begin{aligned}
\mathcal{Z}_0 &= \emptyset, \\
\mathcal{Z}_1 &= \left(\begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}\emptyset + \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right) \cap \mathcal{R}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \\
&= \emptyset, \\
\mathcal{Z}_2 &= \left(\begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}\emptyset + \mathcal{R}\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right) \cap \mathcal{R}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) \\
&= \emptyset.
\end{aligned}$$

According to Eq. (4.11), the unobservable subspace is equal to

$$\mathcal{Q}_2 = \mathcal{Z}_2 = \emptyset, \tag{B.4}$$

and therefore, the observable subspace $\mathcal{O}_2$ is equal to (see Eq. (4.14))

$$\mathcal{O}_2 = \mathcal{Q}_2^\perp = \mathcal{R}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right). \tag{B.5}$$

This is a space with dimension 2. Hence, the system is observable. This is not surprising: if the measurement $\dot{q}_1$ is integrated using the initial condition, then the absolute position q_1 can be obtained.

Finally, suppose that the model shown in Fig. B.1 is a very simple vehicle model where m represents the sprung mass and k is the stiffness of the suspension. In this situation, it is easy to measure the acceleration $\ddot{q}_1$. However, this measurement cannot be written in the form of Eq. (4.6). Fortunately, it is possible to use the measurement equation in the form of Eq. (4.20), which shows the advantage of using the extended state formulation in combination the methods presented to check the observability.

Appendix C

Influence of a known input on the observability

Consider the state and output equations (4.5) and (4.6)

$$\dot{\mathbf{x}}(\tau) = \mathbf{A}\mathbf{x}(\tau) + \mathbf{B}\mathbf{u}(\tau) + \mathbf{E}\mathbf{w}(\tau), \quad \mathbf{x}(t_0) = \mathbf{x}_0, \tag{C.1}$$
$$\mathbf{z}(\tau) = \mathbf{C}_z \mathbf{x}(\tau). \tag{C.2}$$

The solution of these equations is

$$\mathbf{z}(t) = \mathbf{C}_z \Phi(t, t_0) \mathbf{x}_0 + \mathbf{C}_z \int_{t_0}^{t} \Phi(t, \tau) \mathbf{B}\mathbf{u}(\tau) \mathrm{d}\tau + \mathbf{C}_z \int_{t_0}^{t} \Phi(t, \tau) \mathbf{E}\mathbf{w}(\tau) \mathrm{d}\tau, \tag{C.3}$$

where $\Phi(t, \tau)$, the *state transition matrix*, is defined by

$$\Phi(t, \tau) = e^{\mathbf{A}(t - \tau)}. \tag{C.4}$$

Because the term $\mathbf{B}\mathbf{u}$ is known, we can define a new measured output

$$\mathbf{z}'(t) = \mathbf{z}(t) - \mathbf{C}_z \int_{t_0}^{t} \Phi(t, \tau) \mathbf{B}\mathbf{u}(\tau) \mathrm{d}\tau. \tag{C.5}$$

If we also define a new state

$$\mathbf{x}'(t) = \mathbf{x}(t) - \int_{t_0}^{t} \Phi(t, \tau) \mathbf{B}\mathbf{u}(\tau) d\tau, \tag{C.6}$$

then,

$$\dot{\mathbf{x}}'(t) = \dot{\mathbf{x}}(t) - \Phi(t, t) \mathbf{B}\mathbf{u}(t) - \int_{t_0}^{t} \dot{\Phi}(t, \tau) \mathbf{B}\mathbf{u}(\tau) d\tau \implies$$

$$\dot{\mathbf{x}}'(t) = \dot{\mathbf{x}}(t) - \mathbf{B}\mathbf{u}(t) - \mathbf{A} \int_{t_0}^{t} \Phi(t, \tau) \mathbf{B}\mathbf{u}(\tau) d\tau \iff$$

$$\dot{\mathbf{x}}'(t) = \dot{\mathbf{x}}(t) - \mathbf{B}\mathbf{u}(t) - \mathbf{A}(\mathbf{x} - \mathbf{x}') \iff$$

$$\dot{\mathbf{x}}'(t) = \mathbf{A}\mathbf{x}'(t) + \mathbf{E}\mathbf{w}(t). \tag{C.7}$$

Now we can see from Eqs. (C.5), (C.6), and (C.7) that a state and output equation remain with only the *unknown* input:

$$\dot{\mathbf{x}}'(\tau) = \mathbf{A}\mathbf{x}'(\tau) + \mathbf{E}\mathbf{w}(\tau), \quad \mathbf{x}'(t_0) = \mathbf{x}_0, \qquad (C.8)$$

$$\mathbf{z}'(\tau) = \mathbf{C}_z\mathbf{x}'(\tau). \qquad (C.9)$$

This means that the *known* input has no influence on the observability.

Bibliography

[1] J. Alanoly and S. Sankar. A new concept in semi-active vibration isolation. *ASME Journal of Mechanics*, 109:242–247, June 1987.

[2] J. Alanoly and S. Sankar. Semi-active force generators for shock isolation. *Journal of Sound and Vibration*, 126(1):145–156, 1988.

[3] G. Basile, F. Hamano, and G. Marro. Some new results on unknown input observability. In *Proceedings of the 8th IFAC Triennial World Congress*, volume 1, August 1981. Kyoto, Japan.

[4] G. Basile and G. Marro. On the observability of linear, time-invariant systems with unknown inputs. *Journal of Optimization Theory and Applications*, 3(6):410–415, 1969.

[5] D.A. Crolla and M.B.A. Abdel-Hady. Active suspension control; performance comparisons using control laws applied to a full vehicle model. *Vehicle System Dynamics*, 20:107–120, 1991.

[6] D.A. Crolla and A.M.A. Aboul Nour. Theoretical comparisons of various active suspension systems in terms of performance and power requirements. In *Advanced Suspensions*, pages 1–9. The Institution of Mechanical Engineers, 1988. International Conference, London, 24-25 October, 1988.

[7] W. Foag and G. Grübel. Multi-criteria control design for preview vehicle-suspension systems. In *Proceedings of the 10th IFAC World Congress*, volume 3, pages 190–196, 1987.

[8] Arthur Gelb, editor. *Applied Optimal Estimation*. The M.I.T. Press, 1974.

[9] A. Haĉ and I. Youn. Optimal semi-active suspension with preview based on a quarter car model. *ASME Journal of Vibration and Acoustics*, pages 84–92, January 1992.

[10] R.G.M. Huisman, F.E. Veldpaus, H.J.M Voets, and J.J. Kok. An optimal continuous time control strategy for active suspensions with preview. *Vehicle System Dynamics*, 199? Accepted for publication.

[11] C.D. Johnson. On observers for systems with unknown and inaccessible inputs. *International Journal of Control*, 21(5):825–831, 1975.

[12] R.E. Kalman. A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82:35–45, March 1960.

[13] R.E. Kalman and R.S. Bucy. New results in linear filtering and prediction theory. *Journal of Basic Engineering*, 83:95–108, March 1961.

[14] D. Karnopp. Active damping in road vehicle suspension systems. *Vehicle System Dynamics*, 12:291–316, 1983.

[15] D. Konik. Berechnung unbekannter Eingangssignale aus Meßsignalen am Beispiel der Unebenheitsermittlung. *Automatisierungstechnik*, 39(6):205–210, 1991.

[16] N. Louam, D.A. Wilson, and R.S. Sharp. Optimal control of a vehicle suspension incorporating the time delay between front and rear wheel inputs. *Vehicle System Dynamics*, 17:317–336, 1988.

[17] D.G. Luenberger. Observing the state of a linear system. *IEEE Trans. Mil. Electron.*, 8:74–80, April 1964.

[18] M.M. El Madany. Performance of actively suspended cabs in highway trucks - evaluation and optimization. *Journal of Sound and Vibration*, 126(3):423–435, 1988.

[19] A.P.M. Marcelissen. Analysis of the pros and cons of the secondary suspension system of a tractor-semitrailer excited by deterministic signals. Master's thesis, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, 1991. Report No. WOC/VT/R/91.04, In Dutch.

[20] R.S. Sharp and D.A. Crolla. Road vehicle suspension system design - a review. *Vehicle System Dynamics*, 16:167–192, 1987.

[21] J.L. Stein and Y. Park. Measurement signal selection and a simultaneous state and input observer. *Journal of Dynamic Systems, Measurement, and Control*, 110:151–159, June 1988.

[22] W. Van Rijn. Limits to low working spaces - an investigation into feasible working spaces of controllable truck suspensions with preview. Master's thesis, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands, June 1992. Report No. WOC/VT/R/92.62, Confidential.

[23] F. Yang and R.W. Wilde. Observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control*, 33(7):677–681, July 1988.