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PRICE AND ADVERTISING SIGNALS OF PRODUCT QUALITY

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ABSTRACT

We present a signalling model, based on ideas of Phillip Nelson, in which both the introductory price and the level of directly "uninformative" advertising or other dissipative marketing expenditures are choice variables and may be used as signals for the initially unobservable quality of a newly introduced experience good. Repeat purchases play a crucial role in our model. J.E.L. Classification numbers: 026, 611, 530

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Although economists have included advertising and other selling expenses in various of our models at least since the 1930's, it is only within the last decade or so that we have begun to offer explanations of why advertising might affect customers' choices and thus of why firms might choose to advertise.

The most successfully developed of these models involve firms' using advertising to inform potential customers about the existence, characteristics, and prices of the commodities they offer. This work has obvious relevance to the huge volume of advertising that is directly informative on these dimensions. Most newspaper advertisements (including especially want ads) would seem to be of this sort.

However, a non-trivial amount of advertising (especially on television) has little or no obvious informational content. A recent example is the ad that was shown when Diet Coca-Cola was introduced: a large concert hall full of people, a long chorus line kicking, a remarkable number of (high-priced) celebrities over whom the camera pans, and a simple announcement that Diet Coke is the reason for this assemblage. Another example from late 1983 is the advertising campaign for the 1984 Ford Ranger truck, which featured these trucks being thrown out of airplanes (followed by half-a-dozen sky-divers) or being driven off high cliffs. These ads carry little or no direct information, other than that the product in question exists. But if that is the message being sent, these ads seem an inordinately expensive way to transmit the information. Indeed, the clearest message they carry is "We spent an astronomical amount of money on this ad."

In a series of provocative articles, Phillip Nelson (1970), (1974), (1978) has suggested that the latter is, in fact, the primary message of such ads and, moreover, that this is a useful, positive message to prospective customers. Nelson differentiated between products on a "search good" versus "experience good" basis. With the former, the relevant characteristics of the product are evident on inspection, and, because there is little gain to misrepresentation, ads for them can be directly informative. With the latter, crucial aspects of the product's quality are impossible to verify except through use of the product. Thus, unless the product is given away, one must buy without really knowing what one is getting. In such a circumstance, a seller's claims to be offering high quality are unverifiable before purchase and thus are freely copied. They are consequently meaningless and consumers will rationally ignore them. As a result, ads for such a product cannot credibly convey much direct information about the product. Yet it remains in the interests of consumers to identify high quality goods and of the producers of these "best-buys" to make themselves known.

Nelson's crucial insight was that the mere fact that a particular brand of an experience good was advertised could be a signal to customers that the brand was of high quality. It is clear that if high quality brands advertise more, and if advertising expenditures are observable (even if not perfectly so), then rational, informed consumers will respond positively to advertising, even if the ads cannot and do not have much direct informational content. What then is needed to complete the explanation is a reason why advertising should be differentially advantageous for high-quality sellers, so that they will be willing to advertise at levels which low quality sellers will not mimic.

The factor on which Nelson focuses to provide this linkage is repeat purchases. He argued that, because a high quality product is more likely to attract repeat purchases, an initial sale is, *ceteris paribus*, more valuable to a high-quality producer, and such a firm would be willing to expend more—on advertising or whatever—to attract an initial sale. This relationship would then provide the basis for the correlation of quality with the net benefits of signalling that is needed in the standard Spence- type analysis to obtain a separating equilibrium.

Nelson's analysis is very insightful and appealing, and although not worked out in terms of a formal model, it seems to fill a major gap in our understanding of advertising. We already knew from the classic analysis by Dorfman and Steiner (1954) that, *given an advertising response function*, a firm will advertise more the higher is its markup of price over marginal cost. What was lacking was a satisfactory explanation of the existence of the advertising response function, especially for experience goods and the uninformative ads associated with these products. Nelson's suggestion provides a basis for such an explanation. Unfortunately, there are major gaps in his analysis.

In particular, Nelson's signaling story focuses exclusively on advertising as the means to stimulate sales, ignoring the role of pricing. Yet the pricing issue is central. For example, might not the firm prefer to stimulate sales through its pricing rather than by advertising? Or, if advertising convinces customers of a product's high quality, shouldn't the firm raise its price? Note that if prices do vary systematically with quality, then Nelson's explanation of advertising is undercut. Customers can infer quality from prices and so have no need to look at advertising as a signal. In this circumstance, why should firms waste money on ads?

Nelson thus provides an explanation of the advertising response function, *given the size of the markup*, but this explanation is incomplete because it ignores the issues of pricing and of the determination of the markup. Making the markup endogenous, however, raises new problems.

Nelson's "*ceteris paribus*" condition includes the assumption that the margins earned on high-quality goods are the same as those on low-quality goods. That assumption is central to the argument; if margins were sufficiently higher on lower quality goods, then the value of an initial sale would be negatively correlated with quality: *Low* quality producers would then be more willing to spend resources on advertising than would high quality producers. *A priori*, there would seem to be little reason to suppose that optimal margins should be the same for high and low quality goods. Certainly, costs can vary, and we just saw above that there are also good reasons why prices might vary, even if costs were the same.

It thus becomes important to attempt to formulate Nelson's basic ideas in a complete, formal model. In fact, a number of authors since Nelson have investigated the relationship between quality and the use of the non-informative or image advertising on which he focused, and some have been explicitly interested in formalizing his ideas. However, in our opinion none of these latter efforts has been totally satisfactory. In particular, the issues of both prices and quantities being choice variables and possible signals and of repeat sales being a key phenomenon have not, to our knowledge, been satisfactorily incorporated into a formal analysis.[1]

In this paper we offer a modelling based on the repeat sales mechanism in which both price and advertising are decision variables that may potentially be used as signals of quality.[2] We show that in equilibrium both may simultaneously be used as signals, with the chosen levels of both prices and advertising differing between high- and low-quality firms (and, moreover, usually differing for the high-quality firms from the levels that would be chosen in the absence of the informational asymmetry about quality). This means, in particular, that customers could in fact infer product quality from observing either price or advertising volume. However, if the firm were to cut back on either dimension—price or advertising—of its signalling and move the relevant variable toward its full information optimal level, then a low quality firm would be willing to mimic, the signal would no longer be credible, and customers would ignore it. Meanwhile, in such an equilibrium the firm uses both variables to signal (rather than just one) since this achieves the desired differentiation at minimal cost. A corollary of this result is that an effective ban on purely dissipative signals (such as advertising is here) may lead to a Pareto-worsening in the allocation of resources.

Two points are worth noting here. First, while we will consistently refer to advertising, the analysis clearly applies to any observable expenditure that does not directly provide information or otherwise improve demand or costs. A shop in a high-rent location and corporate social responsibility activities are obvious examples. Second, the analysis is strictly applicable only to new products, whose quality is not generally known. Thus, it

[1] Kihlstrom and Riordan (1984) present a model in which advertising is a choice variable while price is competitively determined. Their analysis is insightful, but does not explicitly address the possibility that both price and advertising volume may, in equilibrium, be correlated with quality. Schmalensee (1978) offers a model in which consumers follow a rule of thumb. In it, *low* quality producers may do the advertising because markups are negatively correlated with quality and customers do not recognize the negative advertising-quality relationship. Johnsen (1976) directly attempts to formalize Nelson's argument, but does not obtain existence of equilibrium when both prices and ad budgets are choice variables. (We are grateful to Ed Prescott for this reference.)

[2] Klein and Leffler (1981) offer an alternative, complementary explanation for introductory advertising. In their formulation, unlike ours, quality is a choice variable and the problem is to motivate firms not to cheat by cutting quality. The incentive to maintain quality comes through positive markups and repeat sales, which are lost once cheating is discovered. However, these profits must be reconciled with free entry. This is achieved by requiring new firms to sink resources on ads in an amount equal to expected operating profits before they can attract any business. See also Shapiro (1981).

says little about advertising for established brands.

While this paper may be of value in clarifying the role of pricing and advertising for newly introduced experience goods, it may also offer some methodological contribution through providing the analysis of multiple variables being used simultaneously to signal for a single unobservable variable and though illustrating a method of obtaining a unique equilibrium in a signalling model.

The second of these actually underlies the first. Models based on games of incomplete information, and signalling models in particular, have typically suffered from an embarrassing plethora of (Nash) equilibria. Not only are there often both pooling and separating equilibria (as well as partial-pooling ones), but also there are typically a horde of each of these types. The source of this multiplicity is the indeterminacy of the inferences that individuals draw "off the equilibrium path," that is, when they see a level of the signal that they can tell ought not to have arisen in equilibrium. Bayes' rule gives no guidance in such situations, and the usual equilibrium notions are unspecific about how such inferences should be made. Yet these beliefs that are formed off the putative equilibrium path and the actions that they generate determine what individuals can accomplish by deviating from the prescribed strategies. They are thus crucial determinants of what will, in fact, be equilibrium behavior. The assumption of equilibrium thus places relatively few restrictions on behavior, and, consequently, many different behavior patterns can be supported in equilibrium.

Numerous authors have addressed the problem of paring down the set of equilibria in situations with incomplete information. This work has progressed on two fronts. On the one hand are arguments presented in the context of specific models and based on economic intuition relating to the particular situation being studied.[3] On the other hand is the systematic effort of game theorists to refine the Nash equilibrium concept for general games.[4]

Our approach involves something of a mixing of the two: our arguments are based on the game-theoretic notions of sequential elimination of (weakly) dominated strategies

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[3] See, e.g., Riley (1979).

[4] See, e.g., Selten (1975), Kreps and Wilson (1982), Kohlberg and Mertens (1982), and McLennan (1983).

(Moulin 1979, Pearce 1982) and of "useless" strategies[5] (McLennan 1983), as well as on the concept of strategic stability (Kohlberg and Mertens 1982), but we have specifically developed them in the economic context of signalling.[6]

In particular, we will define a locus of price-advertising pairs beyond which a low quality producer would not be willing to venture, even if by so doing it would be taken initially to be high quality. We then specify that a choice from the region on or outside this locus will be treated as having been made by a high quality firm, while the choice from the region inside the locus is treated as that of a low quality producer. The logic underlying this specification is based on (sequential) elimination of dominated and/or useless strategies: given that customers do not use dominated or useless strategies (and that the firm will not itself use such strategies in the future), a choice from outside the region is dominated for a low quality firm. Meanwhile, a choice from inside this locus is one that a low-quality firm would gladly make if doing so caused it to be thought to be high quality. We then treat this locus as a constraint for high quality firms: if they wish to distinguish themselves, they must not select a price-advertising pair from the region inside the locus, while any choice on or outside of the locus identifies them as being high quality. Conditions on the relationships between the isoprofit curves for the different types of firms then (generically) yield a unique solution to the resultant constrained optimization problem for the high quality firm. These conditions are closely related to the usual sorting conditions in signalling models. They also play a central role in establishing that the solution to this maximization problem is the unique equilibrium satisfying the ideas underlying strategic stability.

Once we are able to view the signalling problem as one of constrained optimization, the use of several signals emerges naturally. The high quality firm can be thought of as making it costly for a low quality firm to mimic it, and as seeking to do so at least cost to itself. Then, as is common in such multivariate optimization problems, the solution may easily involve non-zero use of all variables.[7]

[5] A strategy is "useless" if it is not played in any sequential equilibrium.

[6] Kreps (1984) uses these same methods to obtain uniqueness in a Spence-type model of job market signalling. The relationship between the economic and game-theoretic arguments for eliminating various outcomes is made very clear in this highly recommended paper.

[7] See Hughes (1983), Kohlleppel (1983a,b) and especially Johnsen (1976) for other models with multiple signals.

A DIAGRAMMATIC EXPOSITION

The key ideas underlying our analysis are contained in the following diagrams. We present these now, without being explicit about whether they can be generated by a fully specified model. In the next section we present one such model, show existence and uniqueness, obtain comparative statics results, and compute some numerical examples.

Consider a firm that has just developed a new product which may be of either high or low quality. The firm knows the actual quality, but consumers cannot observe quality until after purchase. The firm's immediate decision variables are the price, P, at which it will introduce the product and the amount, A, that it will spend on introductory advertising over and above whatever level is optimally used to inform potential customers of the good's existence, its price, and its verifiable characteristics. These two variables are shown on the axes of the diagrams below.

Without being specific about the exact nature of quality, we assume that the number of people who buy initially is increasing in perceived quality and that the fraction of these who make repeat purchases is increasing in actual quality. For simplicity, we assume too that quality can take on only two levels, L or H, $L < H$.

Consumers will form conjectures relating the product's quality and its introductory price and advertising level, and will attempt to infer quality, q, from the observed values of P and A.[8] Equilibrium will be a situation where these conjectures are borne out: what customers expect is what the firm, contingent on its true quality, actually chooses to do. Since these choices (and the customers' conjectures about them) may naturally depend on the true value of q, we must specify the choices corresponding to both $q = H$ and $q = L$, even though only one of these will actually obtain. (We refer to a firm producing quality q as being of "type q", $q = L, H$.)

Assume that however potential customers may form their initial estimates of quality,

[8] They may also attempt to forecast the price that will prevail in the post-introductory period, since that could bear on the value of the information they gain about product quality. In the equilibrium of the example that we offer in the next section, consumers will purchase if and only if the expected utility from consuming the product in the introductory period exceeds the introductory price. Thus, in this example, optimal and myopic behavior coincide. This arises because of the pricing policy followed by the firm, which involves raising prices in the post-introductory period so that the marginal consumer in the introductory period obtains no consumer surplus after the price is raised.

they follow utility-maximizing behavior given these estimates and their forecasts of future pricing. Further, assume that once they have gained knowledge through experience of the actual quality of the good, both they and the firm act optimally from that point forward, and that both the firm and consumers anticipate such behavior. Then, let $\pi(P, q, Q)$ denote the present value of the profits gross of advertising accruing to a firm which: (1) charges price P, (2) produces actual quality q, and (3) is initially perceived to be producing quality Q.

If actual quality were known by potential customers before purchase, then $\pi(P, q, q) - A$ would be the relevant profit function net of advertising expenditure for a firm known to be producing quality q. Clearly, the optimal advertising budget in these circumstances is $A = 0$. Denote the optimal value of P for firm known to be of type q as P_q, i.e.,P_q is P_H or P_L.

Under the actual information conditions that initially obtain with experience goods, q and Q may differ. Define B to be the locus of points $(P, A) \geq 0$ such that

$$\pi(P, L, H) - A = \pi(P_L, L, L).$$

Thus, along B, a firm producing low quality but initially perceived to be a high-quality producer earns the same profit as it would if it were recognized as an L and optimized accordingly. Assume that B has the shape shown in the diagrams, so that it defines a function $A(P)$ that coincides with the P axis for $P \leq \underline{P}$ and $P \geq \overline{P}$ and is strictly concave and positive on $(\underline{P}, \overline{P})$. Let P^0 maximize $A(P)$ or, equivalently, $\pi(P, L, H)$.

Note that $\pi(P, L, H) - A$ is strictly less than $\pi(P_L, L, L)$ for (P, A) pairs above B, and that the inequality is reversed below B.

Given the assumed behavior of consumers, a choice of (P, A) from the region above B is dominated for a firm actually producing L. The best that can happen is that such a choice leads to its product initially being perceived to be of quality H and thus to its temporarily receiving the higher demand that an H enjoys. But even if this occurs, the L firm earns less than if it selects $(P, A) = (P_L, 0)$ and is thereby taken to be an L. (If $(P_L, 0)$ leads to its being considered an H, so that it receives the high demand, so much the better.) Thus, customers can justifiably expect that a choice of (P, A) from above B cannot have been made by low-quality producer, i.e., that it is the choice of an H. On the other hand, they cannot rationally regard a choice of (P, A) from below B as surely indicating an H, for if they did so and made their initial purchases accordingly, then an L would gladly mimic this choice in order to be thought an H. Given this, we will actually assume that a choice below B is taken as surely indicating that the firm's quality is L.[9] Finally, for choices on B, we assume that these are taken by consumers as indicating an H. The justification for this is again a (weak) domination argument: in the best of circumstances, such a choice does no better than $(P_L, 0)$ for an L, and it may yield a worse outcome.

Given these beliefs, if an H firm wishes to signal its true quality in a credible fashion, it must select a (P, A) pair from on or above $A(P)$. If it happens that P_H is at or above $\overline{P}$, the point where B hits the price axis, this is no problem: the "natural", full-information price differential is enough to render mimickery unprofitable. (The same is true if P_H is less than $\underline{P}$). However, if P_H is strictly between $\underline{P}$ and $\overline{P}$, then the constraint is binding, and an H that chooses to distinguish itself will select a point on B.

Assume now that the isoprofit lines $\pi(P, H, H) - A = k$ are strictly concave. Note that they are vertically parallel and have slope zero at $P = P_H$ for any value of A, since A enters the net profit function linearly. Then, unless the optimum (P^*, A^*) for $\pi(P, H, H) - A$ subject to $A = A(P)$ occurs at $\underline{P}$ or $\overline{P}$, it will involve $A^* > 0$. Further, unless $P^* = P^0 = P_H$, it will involve the firm deviating from its full-information pricing as well.

In particular, suppose that the isoprofit lines for $\pi(P, H, H) - A$ are "more concave" than those for $\pi(P, L, H) - A$, i.e., that $\partial^2\pi(P, H, H)/\partial P^2 < \partial^2 A(P)/\partial P^2 < 0$ for $P \in (\underline{P}, \overline{P})$. (This condition on the relative slopes of isoprofit lines is a "sorting" condition: in particular, the more common condition ordering the slopes of the level curves for the two types holds on either side of P^*.) Further, assume that P_H is not "too close" to $\overline{P}$, but does exceed P^0. Then the solution involves a tangency where $A^* > 0$ and $P^* > P_H$. This possibility is illustrated in Figure 1. Note that equilibrium will typically involve signalling

[9] This specification obviously facilitates our obtaining a separating equilibrium, but we must emphasize that it is an assumption, *not* a consequence of our techniques of sequential elimination of dominated or useless strategies. Thus, it is consistent with these restrictions for customers to assign positive probability (but less than certainty) to the firm being a high quality producer if they observe a (P, A) pair from below B. However, such beliefs could alter the actual equilibrium choices only if they led to a pooling equilibrium, i.e., a (P, A) pair that both types of firms select with positive probability. In the context of the specific model in the next section, we show that if any such pooling equilibria exist, they fail to meet the Kohlberg-Mertens criterion of structural stability.

Figure 1: Signalling Equilibrium with $A^*>0$, $P^*>P_H$

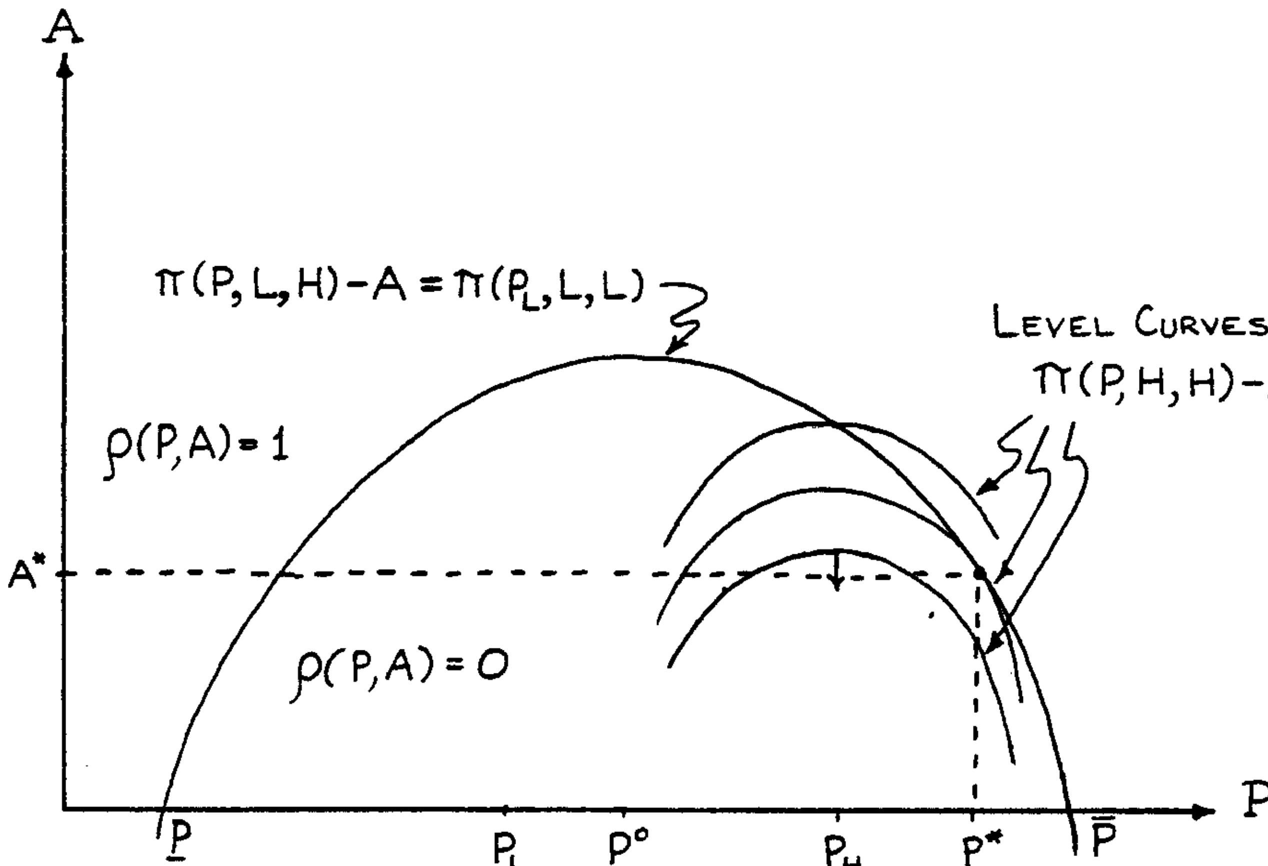
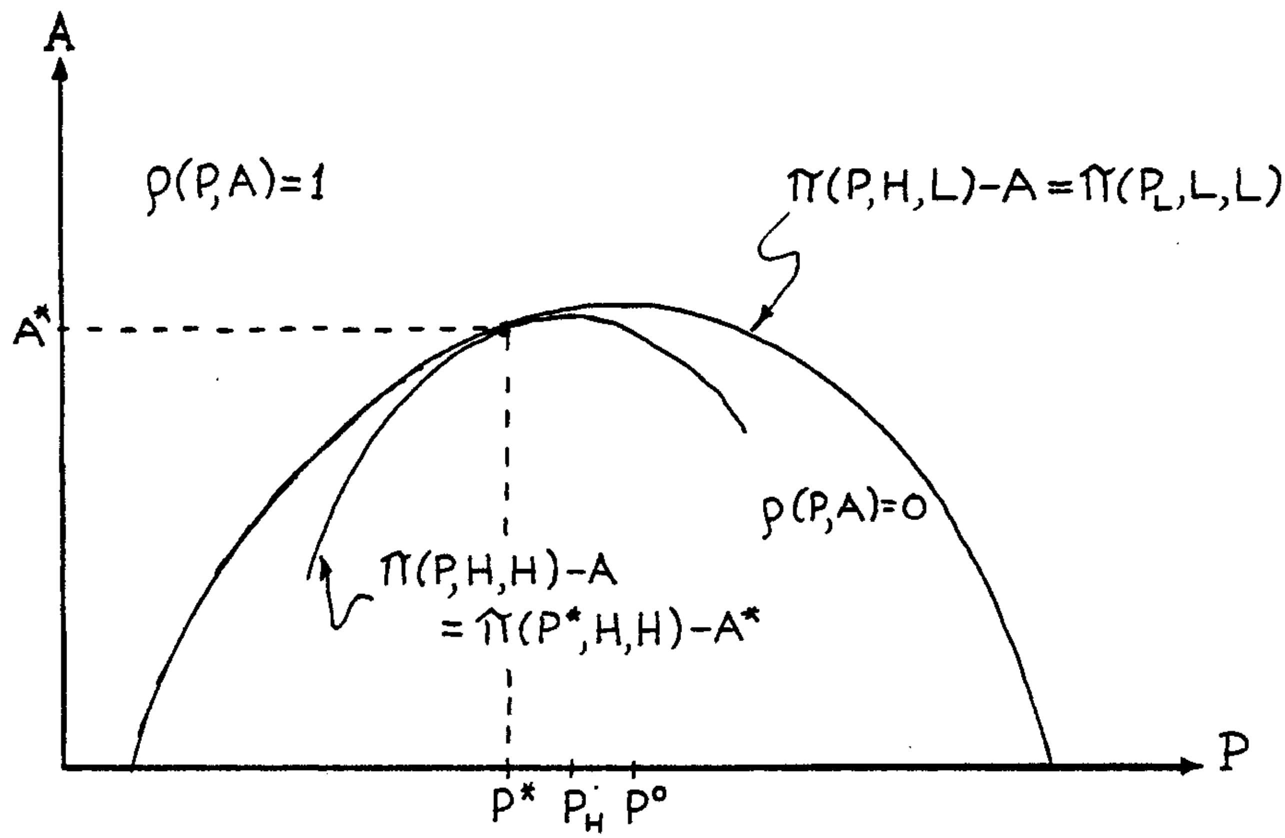


Figure 2: Signalling Equilibrium with $A^*>0$, $P^*<P_H$



with both variables. The high-quality firm employs "image" advertising while a low-quality firm does not, and, so long as $P_H \geq P_L$, we will have $P^* > P_L$, so observation of either price or advertising expenditure reveals the firm's quality. (Of course, the low-quality firm selects $(P, A) = (P_L, 0)$, since it is not worthwhile to try to mimic an H, and the firm thus "chooses" to be revealed as an L, in which case, $(P_L, 0)$ is optimal.) Moreover, even if $P_L > P_H$, we can still have P^* different from P_L.

A second possibility, corresponding to $P_H < P^0$, is illustrated in Figure 2. Here, the high-quality firm advertises, but lowers its price from its full-information level.

We should also note here the possibility that the solution will occur at $P^* = \underline{P}$ or $\overline{P}$ with $A = 0$. The case $P^* = \underline{P}$ shows up as particularly significant in the next section, where it can be interpreted in terms of overcoming the informational asymmetry by giving free samples of a high quality good.

Of course, for this methodology actually to yield an optimum, we also need to have

$$\pi(P^*, H, H) - A^* \geq \sup_{(P,A)} \{\pi(P, H, L) - A\},$$

so that the H firm prefers to distinguish itself rather than be taken initially as an L. We show in the next section that this does obtain in the model developed there, so the preceding analysis is not vacuous.

Note that, in the case where $P^* > P_H$, were excessive advertising (or other publicly observable forms of burning money) to be successfully forbidden, and if the high quality producer were to continue to differentiate itself, then it would do so by selecting $P = \overline{P}$. This involves a Pareto- worsening in the resultant outcome: low quality firms are unaffected, while high quality firms receive lower profits and their customers pay higher prices!

The foregoing analysis is incomplete in at least two ways. First, we need to establish the logical possibility that the isoprofit curves have the shapes assumed in our diagrams. Second, in view of the variety of possible equilibria in the diagrammatic analysis, a comparative equilibrium analysis needs to be done to specify the circumstances that make each type of solution most likely. To do so will require being much more explicit regarding the model.

AN EXPLICIT ALGEBRAIC MODEL

In this section we present a detailed exploration of a model leading to the diagrammatic analysis in Section 2. This model is rather special, but its analysis does establish the logical possibility of simultaneous signalling with both price and advertising. We also obtain some comparative statics results, as well as spelling out more fully our equilibrium concept. We find that producers with a new product that is of high quality and has a low marginal cost of production would introduce the product at very low price, e.g., by mailing out free samples or by handing out coupons. Such a firm will not advertise. Firms which incur a higher marginal cost to produce high quality should mix introductory pricing with an advertising campaign. As the marginal cost goes up, the price should go up and, ultimately, the advertising expenditure down, leading to an expected negative correlation between price and advertising in new product introductions over this range. Finally, if marginal cost for the high quality firm is very much higher than that for the low quality producer, no advertising will be used.

We consider a monopolist producing a newly developed experience good.[10] The quality of good is already determined and is known to the firm. Potential customers know that the product is of one of two possible quality levels, L or H, and they assign prior probability $\bar{p} \in (0,1)$ that it is of high quality, H, and the complementary probability that it is of low quality.

We suppose that the set of potential customers corresponds to an interval $[0, R]$ with a uniform distribution[11] and total mass R. Quality is operationalized as the probability that a randomly selected customer will find the product satisfactory. Thus, $1 \geq H > L > 0$. We also assume that customers' payoffs are linear in income, that the probability of the good's being satisfactory to consumer r is independent of r, and that customers have no use for more than one unit of the good per period. A customer with valuation r who buys in a particular period at price P receives utility in that period that we normalize to be $r - P$ if the good turns out to be satisfactory and of $-P$ if it does not. A customer who

[10] The assumption of monopoly seems appropriate in treating a new product, and it also facilitates the analysis.

[11] The uniform distribution gives rise to a linear demand which, in turn, facilitates computation. However, our basic results hold so long as the distribution of reservation prices gives rise to a demand curve showing decreasing marginal revenue.

does not buy receives 0.

The costs of production for a firm producing x units of quality q are $C_q x$.[12] Although one might expect that $C_H > C_L$, we do not assume this: see below. Thus, if the firm sets price P and advertising level A in some period and produces and sells x units of quality q, its profit in that period is $(P - C_q)x - A$.

Both the firm and customers maximize the expected present value of their payoffs up to a common (finite or infinite) horizon, T, using a common discount factor $\delta \in (0,1)$. Let $\Delta = \Sigma_1^T \delta^t$.

In this context, the firm initially selects a price, P, at which it will introduce the good and an introductory advertising level, A. Having observed these choices, the customers update their beliefs about q and, in light of these, decide whether to make an initial purchase. The firm produces to fill these orders, and then each customer who purchased learns whether the good is satisfactory for his or her purposes or not. In each succeeding period the firm then makes a new choice of its post-introductory price and advertising level, and each customer decides whether or not to purchase in that period, knowing whether the good is satisfactory for him or her and knowing all previous and current choices by the firm. The firm then learns the aggregate demand forthcoming at the announced price and produces to meet demand.

For simplicity, we assume that customers who do not purchase in the first round can never purchase the good. This assumption turns out to be easily relaxed, even if we also allow customers to observe the firm's post-introductory production, from which in equilibrium they could infer quality. We also assume that learning the quality takes only a single period's purchase. This too can also be relaxed, although it complicates the analysis. As well, we can allow the firm to produce before demands are realized in the first period, although this will introduce a new potential signalling variable if output is observed.[13]

To analyze this game, we assume that each decision-maker acts in a sequentially rational fashion, following a strategy from each point forward that maximizes his or her expected payoff given his or her current information and beliefs and the conjectured behavior of the

[12] We ignore any fixed costs because their inclusion would have no effect on the solution.

[13] Wilson (1983) has developed a general treatment along the lines suggested here that allows for an arbitrary (finite) number of signalling variables and a continuum of possible qualities.

others. We further require that these beliefs be formed in a manner consistent with certain principles we will discuss below (including revision according to Bayes' Rule when possible) and that the conjectured behavior be consistent with the actual choices. Thus, we are employing a refinement of the Nash equilibrium concept in the spirit of the game-theoretic criteria of perfectness (Selten 1975, Kreps and Wilson 1982).

We will be looking for an equilibrium where the price-advertising pairs chosen in the introductory period are different for the two types of firms (a "separating" or "signalling" equilibrium). Let $\rho(P, A)$ be the probability that customers ascribe to the firm being an H if the choice (P, A) is observed. Then, in such a separating equilibrium, if (P_L^S, A_L^S) and (P_H^S, A_H^S) are the choices of the two types of firms, then $\rho(P_L^S, A_L^S) = 0$ and $\rho(P_H^S, A_H^S) = 1$. However, there still remains the issue of specifying values of ρ away from these two points. This issue is crucial, since these beliefs will guide decision-making and behavior and thereby will strongly influence what is, in fact, an equilibrium.

As suggested above, we will specify ρ to be either 0 or 1 for all (P, A), with customers never entertaining the possibility that any given (P, A) could have been chosen by both firm types. This sort of specification is standard, since it makes obtaining a separating equilibrium relatively simple, but the actual specification of the regions $\rho^{-1}(0)$ and $\rho^{-1}(1)$ must still be made with some care. In particular, it must be sufficiently expensive for an L to select a point with $\rho(P, A) = 1$. Otherwise, the L will try to be taken as an H by making such a choice. Beyond this, however, there is still generally some significant latitude: as long as it is not too costly for an H to make a choice with $\rho(P, A) = 1$, then we get a signalling equilibrium with the H choosing the least expensive such (P, A) pair. Thus there are many potential separating equilibria.

The key to our analysis is to make the set on which $\rho(P, A) = 1$, i.e., on which customers believe they are facing a high-quality producer, as large as possible given the constraint that an L not want to pick one of these points. This yields a unique outcome which also enjoys a certain efficiency. The methods we use to do this are to specify that customers never assign $\rho(P, A) < 1$ to choices (P, A) having the property that, independent of the value assigned for $\rho(P, A)$, an L would earn less by this choice than by some other. This method yields the locus B of the previous section as the boundary between the regions $\rho(P, A) = 0$ and $\rho(P, A) = 1$. The method's basis in sequential elimination of dominated and useless strategies will come clearer in the sequel.

Consider then a customer with valuation r who bought in the first period and thus has already discovered whether the good is satisfactory or not. Suppose he or she now faces a price $p > 0$ in some period. With a continuum of customers, this individual's decisions whether to buy or not in any period have no impact on aggregate demand and thus cannot influence the future behavior of the firm or of other customers. Thus, it is a dominant strategy for the customer to make a repeat purchase in this period if $r - p$ is non-negative and the good is satisfactory for him or her, and not to buy if $r - p < 0$ or if the good is worthless to him or her. Thus, eliminating dominated strategies for customers, after the introductory period the demand function for the good is the same in every period and is independent both of the prices in other post-entry periods and of any post-entry advertising. Further, given the set of customers who actually purchased the good in the first period, demands in later periods are independent of the initial price and advertising. (Of course, these variables determine the set of initial purchasers.) Given the above, any equilibrium strategy must involve the firm charging a uniform price in each post-entry period and setting $A = 0$ in each of these periods. Any other strategy is thus useless. In fact, given consumer behavior and the firm's first period choices, any strategy involving varying post-introductory prices or positive post-entry advertising is dominated by some constant-price, zero advertising strategy.

Now consider a customer in the first period who has observed the price and advertising choice (P, A) of the firm. Recall that $\rho(P, A)$ is the probability that he or she now assigns to the firm being of quality H and $\big(1 - \rho(P, A)\big)$ the perceived probability that quality q is equal to L. Further, suppose the customer anticipates that the post-introductory price in each period will be $\tilde{p}_L$ if the firm is an L and $\tilde{p}_H$ if it is an H. (This involves the customer's never believing that the firm will use useless or dominated strategies.) Then the customer will purchase the good initially if and only if

$$[1 - \rho(P, A)]\{L[r - P + \Delta \max(r - \tilde{p}_L, 0)] + (1 - L)(-P)\}$$

$$+\rho(P, A)\{H[r - P + \Delta \max(r - \tilde{p}_H, 0)] + (1 - H)(-P)\} \geq \; 0.$$

The first term on the left-hand-side is the probability of getting low quality times the expected payoff to making an initial purchase and henceforth acting optimally conditional on the good being of low quality: the second term gives the parallel expression if quality turns out to be high. The expected payoffs conditional on the actual quality reflect sequential rationality in that a customer anticipates a constant price in all post-introductory periods and plans to buy in these if and only if the good is satisfactory and not too expensive.

Let $r(P, \tilde{p}_L, \tilde{p}_H, \rho)$ denote the value of r at which the above expression equals zero: $r(P, \tilde{p}_L, \tilde{p}_H, \rho)$ is the marginal customer. Once the introductory period is completed, the potential customers are those with $r \geq r(P, \tilde{p}_L, \tilde{p}_H, \rho)$ who were satisfied, i.e., demand for a type-q firm that sets price p_q is $q[R - p_q]$ if $p_q \geq r(P, \tilde{p}_L, \tilde{p}_H, \rho)$ and $q[R - r(P, \tilde{p}_L, \tilde{p}_H, \rho)]$ otherwise. In these circumstances, p_q will clearly be set at $\max(r(P, \tilde{p}_L, \tilde{p}_H, \rho), m(q))$, where $m(q) = \frac{R+C_q}{2}$ is the simple monopoly price for a firm of type q. Thus, the actual price will always (weakly) exceed $r(P, \tilde{p}_L, \tilde{p}_H, \rho)$. Then, under rational expectations (i.e., $\tilde{p}_q = p_q$, $q = L, H$) the marginal customer must always assume that both of the possible post-introductory prices will weakly exceed his or her valuation for the good. Consequently, with rational expectations the terms $max(r(P, p_L, p_H, \rho) - p_q, 0)$ are both zero, and the marginal customer is defined by

$$(1-\rho)[Lr(P, p_L, p_H, \rho) - P] + \rho[Hr(P, p_L, p_H, \rho) - P] = 0,$$

or, defining $Q(\rho) = (1-\rho)L + \rho H$,

$$r(P, p_L, p_H, \rho) = P/Q(\rho).$$

Thus, the profits $\pi(P, q, Q(\rho))$ of a firm of type q charging an introductory price of P and regarded as being a high-quality producer with probability ρ are

$$[R - \frac{P}{Q(\rho)}][P - C_q] + \Delta q[R - \max(\frac{P}{Q(\rho)}, m(q))][\max(\frac{P}{Q(\rho)}, m(q)) - C_q].$$

When $\rho = 0$ (respectively, 1) $Q(\rho)$ is L (respectively, H), and π corresponds with the notation in the previous section.

The first step in finding the equilibrium is to determine the locus B of pairs (P, A) such that an L which charged P initially and advertised at level A would earn the same amount as if it were perceived to be an L and maximized accordingly. To do this, we first calculate the maximal profits of a firm which follows this latter course.

Consider then a low quality firm which is initially believed to be low quality. If it cannot influence this perception, it will surely set $A = 0$. Its problem is then to select P to maximize its profits. Under rational expectations and sequentially rational behavior, profits for such a firm are:

$$(R - P/L)(P - C_L) + \Delta L\left[R - \max(P/L, m(L))\right]\left[\max(P/L, m(L)) - C_L\right].$$

As is intuitively clear, and readily established formally, the solution involves $P/L \geq m(L)$, since there is no point to attracting initial purchasers if one is later going to price them out of the market. The solution for the optimal P is then

$$P_L \equiv L\left[m(L) + \frac{(1-L)}{2(1+\Delta)L}C_L\right],$$

i.e., $\pi(P_L, L, L) = \max \pi(P, L, L)$. Note that the price that will be set in the post-introductory periods is indeed $p_L \equiv P_L/L \geq m(L)$. The set of customers who purchase initially and become informed is thus $[P_L/L, R]$ and a fraction L of these buy again at p_L.

If an L is to find it desirable to pretend to be an H by selecting some $(P, A) \neq (P_L, 0)$, it must thereby achieve profits $\pi(P, L, H) - A$ that exceed the value $\pi(P_L, L, L)$. Suppose then that there is some (P, A) value such that $\rho(P, A) = 1$, i.e., customers take the choice of (P, A) as certainly indicating that the firm is a high-quality producer, and suppose that an L selects this price-advertising pair.

Customers, seeing an initial price of P and believing the firm to be an H, will expect the post-introductory price p to be set at $\max\{P/H, m(H)\}$. Thus, as we saw above, customer r will make an initial purchase if $r \geq P/H$. Of these, the fraction L will be satisfied and thus be willing to consider repeat purchases. Then, the profit of an L mimicking an H by initially choosing (P, A) is

$$(R - P/H)(P - C_L) + \Delta L\{\max_{p \geq P/H}(R - p)(p - C_L)\} - A.$$

The solution to the maximization problem is $p = \max\{P/H, m(L)\}$. Thus, two cases arise. If $P/H \geq m(L)$, then the locus of points yielding $\pi(P, H, L) - A = \pi(P_L, L, L)$ is defined by

$$A = (R - P/H)(P - C_L) + \Delta L(R - P/H)(P/H - C_L) - \pi(P_L, L, L) \equiv \overline{F}(P).$$

If $m(L) = (R + C_L)/2 > P/H$, the constraint becomes

$$A = (R - P/H)(P - C_L) + \Delta L\left(\frac{R - C_L}{2}\right)^2 - \pi(P_L, L, L) \equiv \underline{F}(P).$$

Now, let $\underline{P}$ be defined as the solution to $\underline{F}(P) = 0$ and let $\overline{P}$ be the solution to $\overline{F}(P) = 0$. Then define

$$A(P) = \begin{cases} 0 & P < \underline{P} \\ \underline{F}(P) & \underline{P} \leq P \leq Hm(L) \\ \overline{F}(P) & Hm(L) \leq P \leq \overline{P} \\ 0 & \overline{P} < P. \end{cases}$$

As indicated before, we now specify ρ by

$$\rho(P, A) = \begin{cases} 1 & \text{if } A \geq A(P) \\ 0 & \text{otherwise.} \end{cases}$$

With this specification, it becomes a straightforward—if tedious—matter to calculate the equilibrium.

Given the specification of ρ, it is clearly optimal for an L to select $(P, A) = (P_L, 0)$. We now must determine the optimal choice for an H. We do this in two steps. First, we find the optimal choice under the assumption that the high-quality firm chooses to distinguish itself by selecting (P, A) with $\rho(P, A) = 1$. Then we verify that this is truly optimal by checking that the profits accruing from this choice exceed those obtained from any choice in the region $\rho(P, A) = 0$, recognizing that such a choice leads the firm to be initially perceived to be an L.

The first step then is to solve the problem: $\max \pi(P, H, H) - A$ s.t. $A \geq A(P)$.

Note first that $A(P_L) > 0$, since the firm gets more demand and higher profits at any price if believed to be an H rather than an L. (Thus, $\rho(P_L, 0) = 0$, as required.) Further, it is easily checked that $A(P)$ is strictly concave on $(\underline{P}, \overline{P})$, with its continuous first derivative being given by

$$\frac{\partial \pi(P, L, H)}{\partial P} = \begin{cases} R - 2P/H + C_L/H & P \leq Hm(L) \\ R - 2P/H + C_L/H + \Delta(L/H)(R - 2P/H + C_L) & P \geq Hm(L). \end{cases}$$

This derivative is zero uniquely at $P^0 = Hm(L) + \big(H(1 - H)C_L\big)/2(H + \Delta L)$. Note that this maximum of $A(P)$ occurs in the region $P > Hm(L)$, where $A(P)$ is $\overline{F}(P)$. Further,

A has a continuous, negative second derivative everywhere except at $P = Hm(L)$. This second derivative is $-2/H$ for $P < Hm(L)$ and $-2(H + \Delta L)/H^2$ for $P > Hm(L)$. Thus, the constraint set is very non-convex (see Figures 1 and 2), but is piecewise well-behaved. Two cases then arise, depending on whether $P_H \in (\underline{P}, \overline{P})$ or not.

If $P_H \notin (\underline{P}, \overline{P})$, then the solution is $(P_H, 0)$: the unconstrained optimal price is a sufficient signal and no advertising is used. If $P_H \in (\underline{P}, \overline{P})$, then the constraint becomes effective and the solution will involve $A > 0$ unless the optimum occurs at $\underline{P}$ or $\overline{P}$.

Thus we are led to examine the level curves of $\pi(P, H, H) - A$ on $[\underline{P}, \overline{P}]$ and to compare them with $\overline{F}$ and $\underline{F}$.

Let us first consider the case in which C_H exceeds C_L, so that $Hm(H) > Hm(L)$. One can show by straightforward calculation that, for $P < Hm(H)$, both the constraint and the level curves have positive slope, with the slope of the level curves being strictly greater than those of the constraint. Thus, in particular, the solution cannot be at $\underline{P}$ and must lie in the region $P > Hm(H)$. In this region there is a unique tangency, which occurs at

$$
\begin{aligned}
P^* &= \frac{HR}{2} + \frac{H}{2\Delta(H-L)}\left[C_H(1+\Delta H) - C_L(1+\Delta L)\right] \\
&= \left(\frac{H}{H-L}\right)\left(Hm(H) - Lm(L) + \frac{C_H - C_L}{2\Delta}\right).
\end{aligned}
$$

Further, on this region the second derivative of the level curves is strictly more negative than that of the constraint. As a consequence, so long as $P^* < \overline{P}$, i.e., $A(P^*) > 0$, the solution is for the high quality firm to price at P^* and to spend $A(P^*) > 0$ on advertising, while if $P^* \geq \overline{P}$, the solution is for the high quality firm to do all of its signalling via high prices. (This is the case illustrated in Figure 1.)

Note that P^* may occur at a point where both the constraint and the level curves have positive slope, as suggested by Figure 2. This happens if $C_H > C_L$ but $C_L > C_H(1 + \Delta H)(H + \Delta L)/(H + \Delta H)(1 + \Delta L)$, i.e., $C_H > C_L$ and $P_H < P^0$. In this case, the high quality firm lowers its price in signalling: $P_H > P^*$. (In fact, we may even have $P^* < P_L$ in this case.) Otherwise, $P^* > P_H$ and, if signalling occurs, the informational asymmetry leads to higher prices as well as to the use of uninformative advertising.

We still have to check that the high-quality firm does actually prefer to signal than to be taken initially to be a low-quality producer.

If the high quality firm does not signal, its profits will be

$$\pi(P,H,L) = (R - P/L)(P - C_H) + \Delta H\{\max_{p \geq P/L}(R-p)(p-C_H)\},$$

since it clearly will do no advertising. The solution for this problem involves

$$\tilde{P} = L\left(m(H) + \frac{(1-L)C_H}{2(L+\Delta H)}\right).$$

Verifying that the profits at this price, $\pi(\tilde{P},H,L)$, are less than those earned in signalling then concludes the analysis.

First assume that $P^* < \overline{P}$. Then the requisite inequality is $\pi(P^*,H,H) - A(P^*) > \pi(\tilde{P},H,L)$, which, in view of the definition of $A(P^*)$, we can write as

$$\pi(P^*,H,H) - \pi(\tilde{P},H,L) > \pi(P^*,L,H) - \pi(P_L,L,L).$$

Now define

$$P(\rho) = Q(\rho)\left[\frac{R}{2} + \frac{(1+\Delta H)C_H - (1+\Delta L)C_L}{2\Delta(H-L)}\right],$$

and note that the level curves of $\pi(P,H,Q(\rho))$ are tangent to those of $\pi(P,L,Q(\rho))$ uniquely at $P = P(\rho)$ and that $P^* = P(1)$. As well, note that $\partial^2\pi(P,H,Q(\rho))/\partial P^2 < \partial^2\pi(P,L,Q(\rho))/\partial P^2$ for all ρ and all P different from $Q(\rho)m(L)$ and $Q(\rho)m(H)$ (where these second derivatives are undefined). Finally, observe that

$$\partial\pi(P,H,Q(\rho))/\partial\rho \gtreqless \partial\pi(P,L,Q(\rho))/\partial\rho$$

as $P \gtreqless P(\rho)$.

Given this, rewrite the inequality as

$$\begin{aligned}&[\pi(P(1),H,H) - \pi(P(0),H,L)] + [\pi(P(0),H,L) - \pi(\tilde{P},H,L)]\\ &> [\pi(P(1),L,H) - \pi(P(0),L,L)] + [\pi(P(0),L,L) - \pi(\tilde{P},L,L)]\\ &\quad + [\pi(\tilde{P},L,L) - \pi(P_L,L,L)].\end{aligned}$$

The first bracketed term on the left-hand side is equal to $\int_0^1 [\partial\pi(P(\rho),H,Q(\rho))/\partial\rho]d\rho$ while that on the right is $\int_0^1 [\partial\pi(P(\rho),L,Q(\rho))/\partial\rho]d\rho$. As noted above, these integrals are equal. The second term on the left-hand side is larger than the corresponding term on the right because $\partial\pi(P(0),H,L)/\partial P = \partial\pi(P(0),L,L)/\partial P$ and $\partial^2\pi(P,H,L)/\partial P^2 < \partial^2\pi(P,L,L)/\partial P^2$. Finally, the third bracketed term on the right-hand side is negative, so the inequality holds. Thus, if $C_H > C_L$ and $P^* < \overline{P}$, we actually have a separating equilibrium.

Further, it is easy to see that if $P^* > \overline{P}$, then we need only run the integration to the point $P(\rho) = \overline{P}$; the inequality will still be valid.

If $C_L > C_H$, then the solution to the maximization problem is easily shown to be $\underline{P}$. This can be seen intuitively by considering the high quality firm's problem of picking price and advertising to maximize its profits subject to the L firm's profits when mimicking being less than or equal to $\pi(P_L, L, L)$. The first order conditions here involve changing price from P_H up to the point where $\partial\pi(P,H,H)/\partial P$ equals $\partial\pi(P,L,H)/\partial P$, then increasing A until the constraint is met. But with $C_H < C_L$, lowering price is always cheaper (in terms of profits) for the H firm than for the L. Thus, the solution is as claimed.

To see that $(\underline{P}, 0)$ is actually an equilibrium, i.e., that $\pi(\underline{P},H,H) > \pi(\hat{P},H,L)$, note that $\pi(\underline{P},L,H) = \pi(P_L,L,L) \geq \pi(\hat{P},L,L)$, so that $\pi(\underline{P},L,H) - \pi(\hat{P},L,L) \geq 0$. Now rewrite the required inequality as

$$[\pi(\underline{P},H,H) - \pi(\hat{P},H,H)] + [\pi(\hat{P},H,H) - \pi(\hat{P},H,L)] > 0.$$

Note that the first bracketed term exceeds the non-negative expression $[\pi(\underline{P},L,H) - \pi(\hat{P},L,L)]$ because $C_H < C_L$ and $H > L$, while the second bracketed term is also non-negative because an improvement in reputation increases profits.

This case of $C_H < C_L$ is not as unlikely as it might at first seem. First, recall that quality is not a choice variable, so that the issue of why a firm would decide to produce a high-cost/low-quality product does not arise. Moreover, recall that quality in this model is the probability that a customer will find that a product meets his or her tastes; this need not be correlated with production costs. As an example, consider a strongly flavored, European-style beer and a standard American brew: the former may well cost more to produce but be less likely to appeal to a randomly chosen American consumer.

Finally, in the case $C_H = C_L$, the level curve corresponding to the highest attainable profits consistent with successful signalling coincides with the constraint between $\underline{P}$ and

$Hm(H) = Hm(L)$, then lies under the constraint for higher prices. Thus, any price in this interval and the corresponding level of advertising can constitute an equilibrium. Note that some of these prices will be less than P_L, while all are lower than P_H.

It is a simple matter to do some comparative statics on the equilibrium. First, as we increase C_H alone, we see P initially below and then at $\underline{P}$ until $C_H = C_L$. At this point we get the interval solution of the preceding paragraph. For $C_H > C_L$, P^* increases with C_H until it hits $\overline{P}$, where price remains until C_H is so great that P_H exceeds $\overline{P}$. This result continues to hold with non-linear demand if marginal revenue is decreasing. Correspondingly, A increases or decreases with C_H as P_H is less or greater than P^0. In particular, for $P_H > P^0$, we obtain the negative correlation of price and advertising that has been noted empirically. Increasing C_L, given C_H, has the mirror effect. Further, if we assume $C_H > C_L$, then increases in Δ, the present value of future sales, decrease price, as can be seen from examining the expression for P^*. Thus, if repeat sales occur more frequently, the introductory price is lowered.

The rather complicated relationship between P and A for different levels of C_H is shown in Figures 3 and 4.

Thus, except when $C_H = C_L$, there exists a unique separating or signalling equilibrium. However, it is also possible that there might be pooling equilibria, in which there is some (P', A') such that both types of firms select (P', A') with positive probability. Note that, given our successive elimination of dominated and useless strategies, we must have $A' < A(P')$ and $\rho(P', A') \in (0, 1)$ at any such pooling equilibrium. Thus, the separating equilibrium we have already identified is the only equilibrium consistent with the specification $\rho(P, A) = 0$ for $A < A(P)$. But, as noted earlier, this specification is not an implication of successive elimination and sequential equilibrium. Thus, the possibility of pooling remains.

However, there is a further natural restriction to place upon beliefs that rules out pooling. This is that there should be no price-advertising pair $(\hat{P}, \hat{A})$ such that (i) if an H firm made this choice and were thereby taken to be an H, it would earn more than it does at the pooling equilibrium, and (ii) an L making the same choice and thereby being thought to be an H would be worse off than at the equilibrium. Algebraically, this condition is that there not exist $(\hat{P}, \hat{A})$ with $\pi(\hat{P}, H, H) - \hat{A} > \pi\big(P', H, Q(\rho')\big) - A'$ and

Figure 3: Price as a Function of C_H in Equilibrium

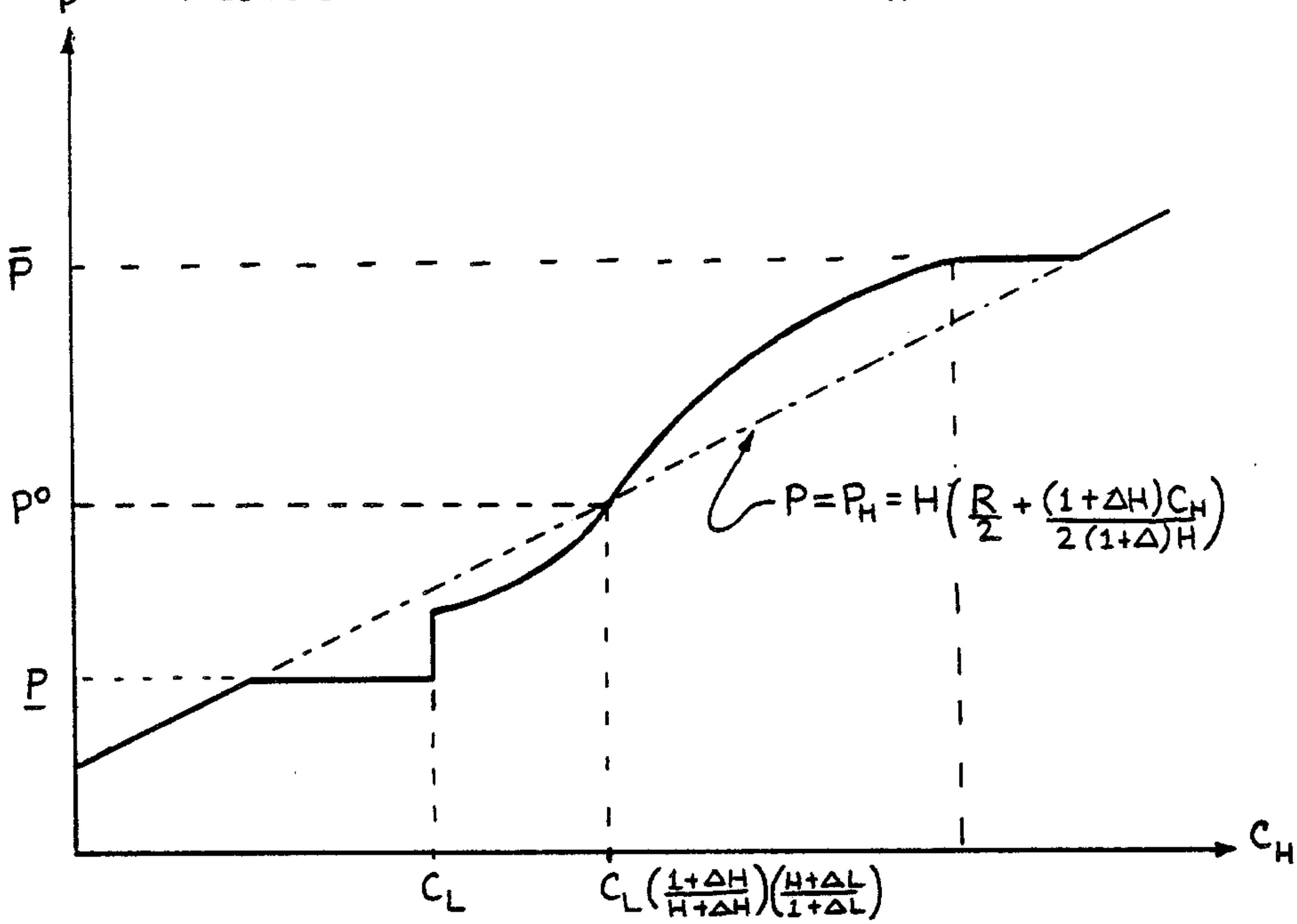
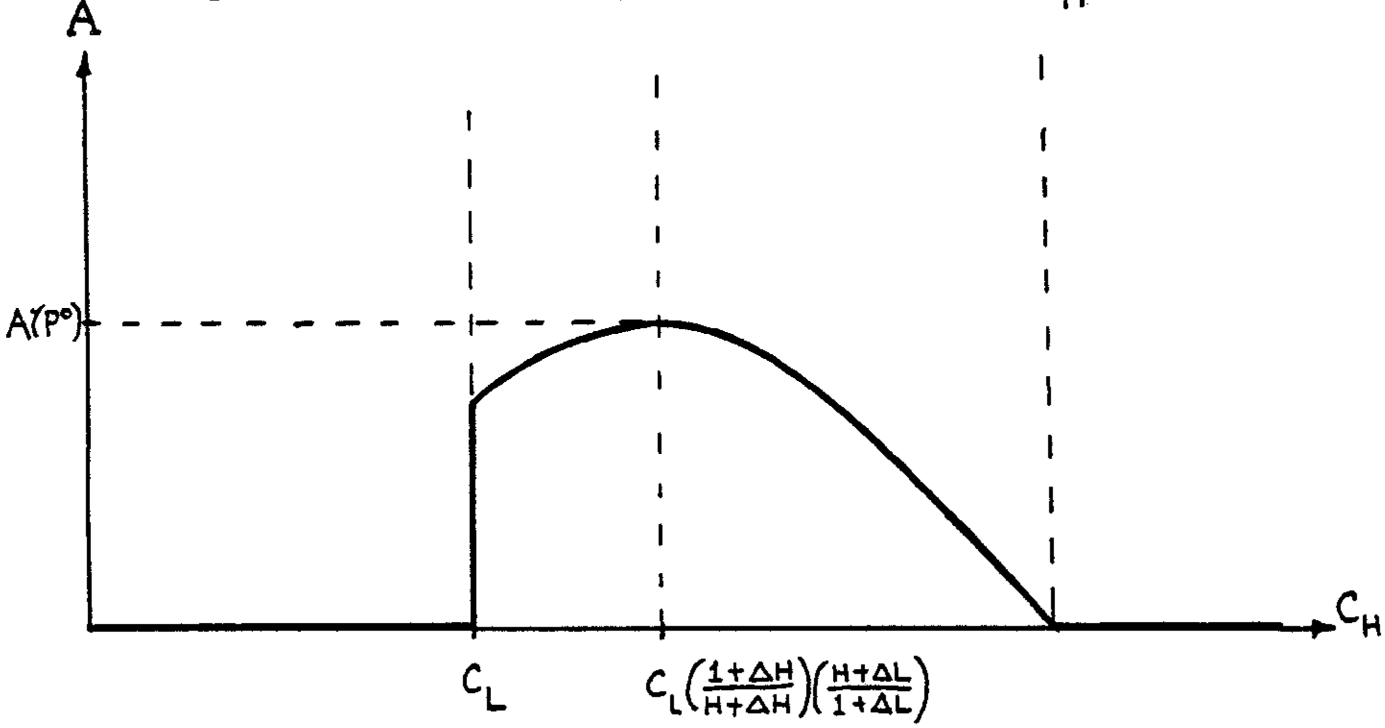


Figure 4: Advertising as a Function of C_H in Equilibrium



$\pi(\hat{P}, L, H) - \hat{A} < \pi(P', L, Q(\rho')) - A'$, where $\rho' = \rho(P', A')$ is the probability that the firm is an H given that (P', A') is the observed choice. Were such a choice to exist, a high-quality producer would have an incentive to adopt it (while perhaps making an explanatory announcement), while a low-quality producer would not. Thus, such a choice "ought" to be interpreted as signalling that the firm is high quality, and so existence of such a point $(\hat{P}, \hat{A})$ would eliminate the pooling equilibrium.

Kreps (1984) has shown that this requirement essentially follows from the Kohlberg-Mertens (1982) concept of strategic stability of equilibrium. He also shows that stability identifies the (uniquely) efficient separating equilibrium and, in this context, subsumes the successive elimination arguments we have employed.

We now show that, in fact, there are no pooling equilibria when customers' beliefs meet this requirement, i.e., we show that for any pooling equilibrium (P', A') there does exist $(\hat{P}, \hat{A})$ meeting the inequalities $\pi(\hat{P}, H, H) - \hat{A} > \pi(P', H, Q(\rho')) + A'$ and $\pi(\hat{P}, L, H) - \hat{A} <$ $\pi(P', L, Q(\rho') - A'$.

Note first that the second inequality defines a locus given by $A'(P) = \pi(P, L, H) - \pi(P', L, Q(\rho')) + A'$. This locus, B', is simply the locus B defined in the context of identifying the separating equilibrium shifted down by the amount $\pi(P', L, Q(\rho')) - A' - \pi(P_L, L, L)$, which is non-negative by the assumption that (P', A') is a pooling equilibrium. Define $\underline{P}'$ and $\overline{P}'$ analogously with $\underline{P}$ and $\overline{P}$ as the points where $A'(P) = 0$. Then, in view of our earlier analysis, in searching for $(\hat{P}, \hat{A})$, we can assume that $\hat{P} = \underline{P}'$ and $\hat{A} = 0$ if $C_H \leq C_L$ while, if $C_H > C_L$, then $\hat{P} = \min(P^*, \overline{P}')$ and $\hat{A} = A'(\hat{P})$. Thus, sufficient conditions for the required inequalities are, for $C_H \leq C_L$,

$$\pi(\underline{P}', H, H) > \pi(P', H, Q(\rho')) - A'$$

and

$$\pi(\underline{P}', L, H) \leq \pi(P', L, Q(\rho')) - A'$$

while, for $C_H > C_L$ sufficient conditions are

$$\pi(P^*, H, H) - A'(P^*) > \pi(P', H, Q(\rho')) - A'$$

and

$$\pi(P^*, L, H) - A'(P^*) \leq \pi(P', L, Q(\rho')) - A'$$

if $A'(P^*) > 0$, and

$$\pi(\overline{P}', H, H) > \pi(P', H, Q(\rho')) - A'$$

and

$$\pi(\overline{P}', L, H) \leq \pi(P', L, Q(\rho')) - A'$$

for $A(P^*) \leq 0$. We will treat each of these cases in turn. Note that in each case, the second inequality holds as an equality by construction.

First, suppose $C_H \leq C_L$ and rewrite the conditions as

$$[\pi(\underline{P}', H, H) - \pi(P', H, H)] + [\pi(P', H, H) - \pi(P', H, Q(\rho'))]$$

$$\geq A' = [\pi(\underline{P}', L, H) - \pi(P', L, H)] + [\pi(P', L, H) - \pi(P', L, Q(\rho'))].$$

The first bracketed term on the left exceeds that on the right because, with $C_H < C_L$, the level curve of $\pi(P, H, H)$ through $\underline{P}'$ lies under that for $\pi(P, L, H)$ for all $P > \underline{P}'$. Thus, it is sufficient that the second bracketed term on the left exceed that on the right. This will hold if $\partial\pi(P', H, Q(\rho))/\partial\rho \geq \partial\pi(P', H, Q(\rho))/\partial\rho$ for $\rho \geq \rho'$. Using the defining formulae this is easily established. Thus, when $C_H < C_L$, no pooling equilibrium is stable.

For $C_H > C_L$, the argument is very similar to that made in connection with showing that signalling was worthwhile for the high-quality producer. For $\hat{P} = P^*$, we note that $P^* = P(1)$, where $P(\rho) = Q(\rho)\left(\frac{R}{2} + \frac{[(1+\Delta H)C_H - (1+\Delta L)C_L]}{2\Delta(H-L)}\right)$, and then write the inequality as

$$[\pi(P(1), H, Q(1)) - \pi(P(\rho'), H, Q(\rho'))] + [\pi(P(\rho'), H, Q(\rho')) - \pi(P', H, Q(\rho'))]$$

$$\geq [\pi(P(1), L, Q(1)) - \pi(P(\rho'), L, Q(\rho'))] + [\pi(P(\rho'), L, Q(\rho')) - \pi(P', L, Q(\rho'))].$$

Recalling that $\partial\pi(P(\rho), H, Q(\rho))/\partial\rho = \partial\pi(P(\rho), L, Q(\rho))/\partial\rho$ and applying the fundamental theorem of calculus shows that the first bracketed terms on each side are equal. Then, since $\partial\pi(P(\rho'), H, Q(\rho'))/\partial P = \partial\pi(P(\rho'), L, Q(\rho'))/\partial P$, while the second derivative of $\pi(P, H, Q(\rho'))$ is strictly less than that of $\pi(P, L, Q(\rho'))$, the second bracketed term on the left exceeds the corresponding one on the right and the inequality holds. Finally, if $\hat{P} = \overline{P}'$, we simply run the integration to that value of ρ at which $P(\rho) = \overline{P}'$. Thus, no pooling equilibrium is stable.

We close this section with some simple numerical examples.

Example 1:

Let $L = 1/2$, $H = 1$, $\Delta = 2$, $C_L = 0$, $C_H = 1$ and $R = 8$. Then we easily calculate $m(L) = 4$, $m(H) = 4.5$, $P_L = 2$, $P_L/L = 4$, and $P_H = 4.5 = P_H/H$. Thus, under full information, an L would initially set price at 2, then later raise it to 4, while an H would price at 4.5 in all periods (because there is no uncertainty to be resolved about whether an H's product will be satisfactory).

The constraint $A(P)$ is given by

$$A(P) = \begin{cases} (8-P)P - 8 & P < 4, \\ 2(8-P)P - 24 & P \geq 4, \end{cases}$$

and $\pi(P, H, H)$ is given by

$$\pi(P, H, H) = \begin{cases} (8-P)(P-1) + \left(\frac{49}{2}\right) & P < 4.5, \\ 3(8-P)(P-1) & P > 4.5. \end{cases}$$

Note $P^0 = 4 < P_H$.

The tangency of the constraint and the level curve occurs at $P^* = 5.54$, and $A(P^*) = 3.5$.

It is easy to check that the H firm would not prefer to select a price-advertising pair with $\rho(P, A) = 0$ and be initially thought to be an L. The optimal such point is $(P, A) = (2.3, 0)$, which yields profits of $28.9 < 30.25 = \pi(5.5, H, H) - A(5.5)$. Thus, the solution indeed is for the L type to set price initially at $2 = P_L$, not to advertise, and to raise price later to $4 = P_L/L$, while the high quality firm sets price at $5.5 > P_H = 4.5$ in all periods and spends 3.5 on introductory advertising.

Example 2:

In this example we will have $C_H = C_L$. In the resulting equilibrium, the H firm advertises, but lowers its price from the full information level, P_H.

Set $L = 1/2$, $H = 3/4$, $\Delta = 2$, $C_L = C_H = 1$, and $R = 8$. Thus, the example is the same as the previous one except that C_L is higher and H is lower. Again, simple calculation gives $m(L) = m(H) = 9/2$, $Hm(L) = Hm(H) = 27/8$, $Lm(L) = Lm(H) = 9/4$, $P_L = 7/3$, $P_H = 41/12$, $P_L/L = 14/3$ and $P_H/H = 41/9$. The constraint $A(P)$ is given by

$$A(P) = \begin{cases} \left(8 - \frac{4}{3}P\right)(P-1) - \frac{53}{12} & P \leq 27/8, \\ \left(8 - \frac{4}{3}P\right)\left(\frac{7}{3}P - 2\right) - \frac{50}{3} & P \geq 27/8, \end{cases}$$

which yields $P^0 = 24/7 > P_H$, while $\pi(P, H, H)$ is

$$\pi(P,H,H) = \begin{cases} \left(8 - \frac{4}{3}P\right)(P-1) + \frac{147}{8} & P < 27/8, \\ \left(8 - \frac{4}{3}P\right)\left(3P - \frac{5}{2}\right) & P \geq 27/8. \end{cases}$$

For $P < 27/8$, the level curve $\pi(P, H, H) - A = \pi(P^*, H, H) - A(P^*)$ and $A(P)$ coincide, while for higher prices, the level curve is under $A(P)$. Thus, any pair $(P, A(P))$ with $\underline{P} \leq P \leq 27/8 < P_H$ is a potential solution. All that remains is to check that the H would rather signal its true identity than be taken initially as an L. However, the most profits it can earn if it fails to signal is 22.78, which is less than the 22.79 it earns by signalling.

Note that were C_H just slightly larger, we would have $C_H > C_L > C_H\left(\frac{1+\Delta H}{H+\Delta H}\right)\left(\frac{H+\Delta L}{1+\Delta L}\right)$ and we would have a unique solution at $P^* < P_H$.

Example 3:

Here we assume $C_H < C_L$. Let $L = 1/2$, $H = 1$, $\Delta = 2$, $C_L = 0$, $C_H = 1$, $R = 8$. Then $m(L) = 4.5$, $m(H) = 4$, $P_L = 2.33$, $P_L/L = 4.67$, and $P_H = P_H/H = 4$. The constraint $A(P)$ is

$$A(P) = \begin{cases} (8-P)(P-1) + \frac{49}{4} - \frac{50}{3} & P \leq 4.5 \\ 2(8-P)(P-1) - \frac{50}{3} & P \geq 4.5 \end{cases}$$

while $\pi(P, H, H)$ is

$$\pi(P,H,H) = \begin{cases} (8-P)P + 32 & P \leq 4 \\ 3(8-P)P & P \geq 4. \end{cases}$$

The lower limit $\underline{P}$ is 1.7 in this example, yielding $\pi(1.7, H, H) = 42.71$. For $P > 1.7$, the level curve of $\pi(P, H, H)$ lies everywhere under $A(P)$, so this is, in fact, the solution. Thus, the low quality firm prices initially at 2.33, then raises its price to 4.67. The high quality firm prices initially at 1.7, then raises price to its monopoly level, 4.

CONCLUSION

We have constructed a model to formalize Nelson's insight that apparently uninformative advertising for an experience good could be a signal for product quality. In doing so, we have also treated the pricing decision of the firm and allowed for the possibility that price itself might be a signal. Our analysis has confirmed and extended Nelson's fundamental point: advertising may signal quality, but price signalling will also typically occur, and the extent to which each is used depends in a rather complicated way on the difference in costs across qualities.

Interestingly, while our analysis confirms Nelson's fundamental point concerning the role of advertising, its inclusion of the pricing decision upsets the intuition that a high-quality producer will have a higher marginal benefit from attracting an initial sale and that this would provide the basis for the high-quality firm's being willing to advertise more. Initially, starting from P_H, signalling via changing price is costless for a high quality firm (because $\partial\pi(P_H, H, H)/\partial P = 0$). Thus, *all* the requisite signalling takes place through the price unless the point P^*, where $\partial\pi(P^*, H, H)/\partial P = \partial\pi(P^*, L, H)/\partial P$, is reached before the necessary differentiation has been achieved. Only in this case is advertising used as a signal. But, at P^*, the present value to a firm perceived to be high quality of an additional sale achieved through pricing is independent of the firm's actual quality!

The essential difficulty is that the notion of the "marginal benefit to attracting another initial sale" is not well-defined: in particular, it depends on who the marginal customer is, i.e., on the price being charged and on the beliefs that customers hold. Moreover, once one allows that price might be a choice variable, then ambiguity remains even after the price and beliefs are specified because there are several ways one might generate the sale. If one imagines somehow obtaining an extra sale without changing perceived quality or price, then, at price P^* and perceived quality H, the marginal profit for a firm of type q from an extra sale is $[P^*(1 + \Delta q/H) - C_q(1 + \Delta q)]$, and this expression is greater for $q = H$ than for $q = L$. Similarly, if the additional sales are generated through extra advertising, then the marginal benefit is greater for the high-quality producer so long as price always exceeds $P(\rho)$, and, in particular, if $P \geq P^*$. However, if price is the means used to increase sales, then the net marginal benefits from an additional sale are a negative constant, $(L - H)$, times $\partial\pi/\partial P$. As we have seen, if $C_H > C_L$ then, in equilibrium, this marginal benefit is always at least as large for the low-quality producer.

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