

# Price Commitments with Strategic Consumers: Why it can be Optimal to Discount More Frequently ... Than Optimal

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## Abstract

In many markets consumers incur search costs and firms choose a long-run pricing strategy that determines how they respond to market conditions. A pricing strategy may involve commitments to take actions that do not optimize short-term revenue given the information the firm learns about demand. For example, as already suggested in the literature, the firm could commit to a single price no matter whether demand is strong or weak. We introduce a new strategy - charge a “high” price only if demand is indeed “high”, otherwise offer a discount. This strategy discounts more frequently than would maximize revenue conditional on demand. Nevertheless, the frequent discounts attract consumers. We show that (i) the discount-frequently strategy is optimal (whether capacity is adjustable or not), (ii) discount-frequently is often much better than other pricing strategies, especially if no price commitment is made and (iii) “overbuying” capacity (e.g., inventory) to attract consumers (by signaling availability and the likelihood of discounts) is a poor strategy. Contrary to some recommendations in the literature to limit markdowns and to purchase ample capacity, our results provide support for a strategy that embraces frequent discounts and moderate capacity.

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# 1 Introduction

In many markets consumers incur costs to search/visit a firm, so they search only if it is worth the effort. In particular, consumers care about (1) what price they pay and (2) do they get a unit. A great price does no good for an item that is out of stock or a service that cannot be offered (e.g., no appointments or seats available). And availability isn't useful if the price is too high. So in these environments the firm needs to attract consumers with a good deal (price and availability). The firm can do this with two levers: a pricing strategy and a capacity choice.

A pricing strategy communicates to consumers how the firm behaves. It often involves commitments to act in certain ways that incur some cost to the firm in the short term. For example, a firm often has better information about overall demand for a specific product or service than consumers do. A clothing retailer may know that a dress is more popular than usual or a movie theater may learn that the latest movie isn't pulling in audiences as was expected. The dress retailer may be tempted to raise its price (or not discount it), knowing that it can sell all dresses even at a higher price. The theater may be tempted to drop its price in an effort to attract bargain hunters. In either case, failing to make the price adjustment in response to the firm's updated information is costly in the short term because changing the price can increase revenue conditional on what the firm knows, as has been shown in many settings with non-strategic consumers (i.e., consumers whose search decisions do not depend on the firm's pricing or capacity choices): e.g., Gallego and van Ryzin (1994), Talluri and van Ryzin (2004), Elmaghraby and Keskinocak (2003).

This paper studies a model with two time horizons. In the long term the firm, armed with an uncertain demand forecast, chooses a pricing strategy and possibly a capacity. In the short term the firm learns useful information about demand and then chooses a price that is consistent with the adopted pricing strategy. We consider three strategies. With the no-commitment strategy, the firm charges a price that maximizes revenue conditional on observed demand - if demand is "high", the firm charges a high list price, leaving consumers with no residual value, whereas if demand is "low", the firm charges a discounted price to ensure that all inventory is sold. With a static-pricing strategy, the firm commits to a single price no matter what it learns about demand. While a static-pricing strategy does not allow a firm to exploit new information, it has been found that committing to a fixed price (or set of prices) can help to reduce the propensity of consumers to strategically

wait for discounts (e.g., Besanko and Winston (1990), Aviv and Pazgal (2008)).

We introduce a third strategy, called discount frequently, in which the firm charges either the high (not discounted) list price or a discounted price, just as in the no-commitment strategy. The commitment with this policy is with the frequency the firm chooses to discount - in some cases, the firm discounts off the list price even if it is not in the interest of the firm to discount conditional on what it observes about demand. With the discount-frequently strategy, a consumer knows that sometimes she doesn't get a good deal (the price is high), but the firm nevertheless offers a discount often enough to justify the effort to visit the firm. For example, given demand, the firm may be indifferent between selling a portion of its capacity at the high price or discounting to sell all of its capacity. In those cases, the firm chooses to discount - it loses nothing but makes customers happier. It also chooses to discount in some cases in which it prefers the higher price. While this is costly (again, conditional on demand) this commitment attracts more consumers to the firm, which is clearly beneficial.

For a fixed capacity, we find that (i) the static-pricing strategy can perform well, and in many cases much better than the no-commitment strategy, (ii) but the discount-frequently strategy is optimal among all possible strategies. Thus, the best strategy is to charge both high and low prices, but also to avoid the temptation to maximize the short term (by not offering a discount often enough) at the expense of long-term profitability. Interestingly, given the same potential demand and the same capacity, a firm implementing the no-commitment strategy discounts with the same actual frequency as the firm that implements the discount-frequently strategy. The difference is that the discount-frequently firm attracts a higher fraction of potential demand, which makes the discount-frequently strategy more profitable. A naive manager may conclude that a discount-frequently firm is discounting too frequently given the demand it receives, not recognizing that the firm receives that demand precisely because it maintains the same frequency of discounting as the no-commitment firm. However, this can be a costly conclusion - we find that the short term profit gain from avoiding discounts is generally considerably less than the loss in long-run profit if the firm loses its reputation for discounting frequently.

If the firm can choose its capacity, then it can also use capacity as a "carrot" to attract consumers. Dana and Petruzzini (2001) show that if consumers care about availability - they don't like incurring the cost to search the firm only to learn that they cannot purchase the good or service - then

the firm should invest in more capacity than it would if consumers were non-strategic. (Gaur and Park (2007) find an analogous result in a model with consumer learning and multiple firms.) The additional capacity is meant to reassure consumers that the product will be available for them so they become more likely to search the firm - the firm cannot sell to a customer that does not visit. Similarly, adding capacity increases the likelihood that a clearance sale is justified, thereby exciting consumers with the greater prospect of a good deal. However, Dana and Petruzzi (2001) did not consider the possibility that the firm could choose its pricing strategy, in particular the frequency of clearance sales to help draw consumers. We find that it is ineffective to only use excess capacity to attract consumers. For example, the firm that is unable or unwilling to make price commitments to attract consumers purchases much more capacity and earns substantially lower profit than the firm that makes price commitments. As with a fixed capacity, the discount-frequently strategy is optimal. In fact, when capacity is expensive, discount frequently is the only strategy that makes a positive profit.

## 2 Related Literature

We are not the first to study price commitments even though we introduce a novel type of price commitment. A number of authors consider models in which consumers decide whether to purchase at the current price or to wait to purchase at a future price (e.g., Besanko and Winston (1990), Aviv and Pazgal (2008), Liu and van Ryzin (2008), Cachon and Swinney (2009), Feng and Gallego (1995), Su and Zhang (2009); Cachon and Swinney (2011); Swinney (2011); Ovchinnikov and Milner (2012); Swinney (2013); Whang (2014)). To combat this strategic behavior, it has been suggested that a firm commit to restrict discounts. This commitment is costly because it limits the firm's ability to react to updated demand information. However, a price commitment can detract consumers from strategically waiting for a discount. In our model consumers are offered a single price, so they do not consider whether to "buy now or wait for the discount". Hence, our motivation for price commitments is exclusively to attract consumers rather than to prevent strategic waiting.

A critical feature of our model is that consumers incur search costs - consumers choose to visit the firm only if they anticipate that the reward for doing so (i.e., purchasing a product at a good price) justifies the effort. Others have incorporated similar search costs, generally in a single firm

setting: Baye and Morgan (2001), Dana and Petruzzi (2001), Cil and Lariviere (2012), Alexandrov and Lariviere (2012) and Su and Zhang (2009). Dana and Petruzzi (2001) have fixed prices and focus instead on how search costs influence the firm’s capacity choice. Baye and Morgan (2001) studies a marketplace for price information for which customers may pay to subscribe or, alternatively, incur search costs to learn the price set by their local firm only. Cil and Lariviere (2012) studies the allocation of capacity across two market segments and Alexandrov and Lariviere (2012) study why firms may offer reservations. Prices are exogenously fixed in both of those papers. Su and Zhang (2009) focus on capacity commitments and availability guarantees.

Pricing and availability is considered in a number of papers that model competition across two or more firms: e.g., Deneckere and Peck (1995), Bernstein and Fedegruen (2004), Cachon and Harker (2002), Gaur and Park (2007), and Allon and Fedegruen (2007). These models assume firms choose a price without the benefit of updated demand information. Several papers empirically document that consumers do value higher availability: e.g., Masta (2011), and Cachon et al. (2013).

Other papers that compare different pricing schemes when consumers are strategic include single versus priority pricing (Harris and Raviv (1981)), subscription versus per-use pricing (Barro and Romer (1987); Cachon and Feldman (2011)), and markdown regimes with and without reservations (Elmaghraby et al. (2009)). In all of these papers the firm selects its pricing strategy before learning some updated demand information, whereas in our study we allow the firm to choose a price after potential demand is observed. Thus, consumers in our model are not sure what is the firm’s price or the product’s availability before they choose whether to search the firm.

### 3 Model Description

A single firm with  $k$  units of capacity sells to two types of consumers, high types and low types, all of whom require one unit of capacity to be served and are indistinguishable to the firm. There is a potential number of  $X$  high-value consumers, where  $X$  is a non-negative random variable that is drawn from a cumulative distribution function  $F(\cdot)$ , probability density function  $f(\cdot)$ , and mean  $\mu = \mathbb{E}[X]$ . Let  $\bar{F}(\cdot) = 1 - F(\cdot)$ . The high-value consumers have zero “mass” as each is unable to have any influence on the market individually. They have value  $v_h$  for the firm’s service. As in Dana and Petruzzi (2001) (and other papers), high-value consumers must incur a positive cost,  $c < v_h$ ,

**Table 1.** Summary of Consumer Types.

Segment	Number	Value	Search cost
High type	$X \sim F(\cdot)$	$v_h$	$c$
Low type	$\infty$	$v_l$	0

to search the firm to purchase the good or service. These search costs include the time and effort to physically travel to the firm and the mental effort associated with a purchasing decision. They receive 0 value if they choose not to search. They can implement mixed strategies: let  $\gamma \in [0, 1]$  be the probability that a high-type consumer searches the firm. Mixed strategies have also been used to describe consumer behavior in the context of joining a service, modeled as a queue (e.g., Edelson and Hildebrand (1975) and Lariviere and van Mieghem (2004)), paying to have access to a list of prices from multiple firms (e.g., Baye and Morgan (2001)), and whether to visit a restaurant (e.g., Cil and Lariviere (2012)).

There is an ample number of low-value consumers, and each has  $v_l$  value for the firm’s service. These consumers have zero (or low) search costs. We assume throughout  $v_l < v_h - c$ : a high-type consumer who visits the firm generates more value than a low type (net of search cost). If  $v_l > v_h - c$ , then the firm prefers selling exclusively to low-type consumers, which is not interesting. Table 1 summarizes the consumer types.

The firm seeks to maximize revenue and consumers seek to maximize their net value, the value of the service minus search costs and the price paid to the firm.

Events can be divided into two periods. In the first, or “long-term”, period the firm chooses a pricing strategy. The strategy determines how the firm behaves in the second, or “short term” period. All consumers observe the firm’s pricing strategy. At the start of the second period the number of high-type consumers,  $X$ , is realized. The firm observes  $X$  but consumers do not. Next, the firm chooses a price,  $p$ , and at the same time the high-type consumers choose to search the firm or not (i.e., they select  $\gamma$ ). At the end of the short-term period consumers who visit the firm purchase an item, if available. If there are more than  $k$  high-type consumers who want to purchase, the  $k$  units are randomly rationed among them (our allocation rule). While we model the “short term” period as a single period, results are equivalent to a model that has one “long-term” period followed by multiple but independent “short-term” periods (hence the names).

We consider three pricing strategies. With the first, called “no commitment”, the firm chooses a

price in the short-term period that maximizes revenue conditional on observed demand,  $X$ , capacity,  $k$ , and the firm’s expectation of consumer behavior. There is no commitment with this strategy because the firm is maximizing its short term revenue conditional on all of the information it knows. The second pricing strategy is called “static pricing” because in the long-term period the firm commits to charge a single price,  $p_s$ , in the short-term period. This requires a commitment because  $p_s$  may not be the price that maximizes the firm’s revenue in the short-term period. However, by committing to  $p_s < v_h$ , the firm potentially increases the return a high type receives from search, thereby increasing the number of high-type consumers who search. Finally, we consider a “discount frequently” strategy in which the firm chooses the same prices as in the no-commitment strategy ( $v_l$  or  $v_h$ ), but chooses the discount price with a higher likelihood (i.e., more frequently) than with the no-commitment strategy. Hence, the firm sometimes does not maximize revenue in the short-term period conditional on its information, but this commitment also encourages high types to search the firm because they expect to receive the discounted price with a higher probability.

There are additional pricing strategies beyond the three we consider. For example, instead of committing to the frequency of the two focal prices ( $v_l$  or  $v_h$ ), the firm could commit to always offer a discount by either choosing to maximize short term revenue with an intermediate price  $v_m$ ,  $v_l < v_m < v_h$ , or the deep discount,  $v_l$ . (If  $v_m = v_h$ , then this is the no-commitment strategy. If  $v_m = p_s$ , then this strategy dominates static pricing.) Using the set of scenarios described in our numerical study (Sections 5 and 6), we find this “discount always” strategy is reasonably effective when capacity is fixed (it achieves on average 96.9% of the revenue earned with discount frequently), but less effective when capacity can be chosen (it earns on average only 79% of the profit earned by discount frequently). (A complete analysis of the discount always policy is available from the authors.)

There are several important features of our model, which we discuss next.

*The firm is able to commit to a pricing strategy.* Like Aviv and Pazgal (2008), Liu and van Ryzin (2008), Elmaghraby et al. (2008), Yin et al. (2009), Liu and Shum (2013) and Whang (2014), (i) we allow the firm to commit to a pricing strategy that is not always sub-game perfect and (ii) we presume these commitments are credible. Credibility is generally achieved through repeated interaction (e.g., Fudenberg and Levine (1989)). Although we consider a model with only one short-term period, we have in mind a situation in which the firm interacts with consumers over multiple

short-term periods (e.g., multiple months or quarters). Consequently, the firm is able to establish a longrun reputation for how it conducts business. For example, in January 2012, JC Penney, a large U.S. department store chain, announced a new, simplified pricing strategy that involved far fewer discounts. However, by the spring of 2013, the company decided that the strategy did not work and they returned to a more aggressive use of price promotions (Clifford and Rampell (2013)). Thus, a firm commits to a pricing strategy through advertising and subsequent behavior - consumers who visited JC Penney after the announcement indeed noticed that they were not promoting as frequently.

*Consumers must incur a search cost before observing availability and price.* Search costs associated with product availability are likely to be inconsequential only if a consumer knows exactly which item they want (at the brand/model level) and they have the ability to find availability information quickly and accurately. Those conditions are likely to apply only in specialized situations. It is more common that consumers might not know exactly the item they want, or shop at retailers that are unable to easily provide accurate availability information. A department store, such as JC Penney, fits this description - a consumer might know that they want to purchase a blender from their housewares department, but they don't know the exact brand and model, and even if they did, the website (assuming they have easy Internet access) may not provide timely and accurate inventory information for their local store. (See DeHoratius and Raman (2008) for evidence that firms struggle to maintain accurate inventory records, even for their own internal use.) It is also likely that consumers incur significant search costs for price. Again, for the search cost to be inconsequential, the consumer must know the precise item they intend to purchase, which doesn't always apply. But even if that is known, search costs can remain. For example, a search on a retailer's website may provide the price of an item at one store near the consumer, but retailers do not always charge the same price across all stores. To find the full list of prices in the nearby stores may require calling the individual stores, which is clearly time consuming (i.e., a costly activity). Finally, Hann and Terwiesch (2003) discover that consumers act as if price search is costly even when intuition suggests it shouldn't be, possibly due to cognitive effort or limitations (e.g., Miller (1956), Roberts and Lattin (1997), Kuksov and Villas-Boas (2010)). In sum, it is likely that in many markets consumers behave as if searching for price and availability are consequential (i.e., costly).



*Low types have no search costs.* In contrast to the high types that are limited in number and incur search costs, low types are ample and search is inconsequential to them. One interpretation is that the low-type consumers are bargain hunters who visit the store without the intention to purchase in the category of interest but nevertheless are willing to make a purchase if they notice a very good deal (e.g., the price is no more than  $v_l$ ). Alternatively, even if low-type consumers have a search cost, the qualitative results of the model remain. To explain, as long as their search costs are sufficiently low, a certain fraction of them will be willing to visit the store (like the high types, they would make a tradeoff between the cost of visiting and the potential value of visiting). Thus, the firm would continue to make the tradeoff between pricing “high” to sell only to the high types and pricing “low” to sell to both types. The firm might not always be able to clear remaining inventory with the “low” price, but the firm could still generate a considerable sales boost by discounting its price.

*The firm observes demand before selecting its price.* The firm may use early season sales to quickly determine if the product has excess demand or not (e.g., Raman and Fisher (1996), Iyer and Bergen (1997), Caro and Gallien (2012)). The firm uses this information when it chooses its price, constrained by its pricing strategy (and capacity) commitments. Although we assume the firm observes a perfect signal of demand, we suspect our qualitative results continue to hold (though the analysis becomes more cumbersome) if the firm is imperfectly informed but remains better informed than consumers - when the firm has more information than consumers, consumers know that the firm may be tempted to use that information to increase its revenue, and, thus, price-commitments can still be used to attract more demand

*High types receive priority in the allocation rule.* This allocation rule is most favorable to the firm because it encourages high-type consumers to visit the firm (they know that they have priority). (Su and Zhang (2008) and Tereyagolu and Veeraraghavan (2012) also adopt this allocation rule.) Alternatively, as in Cachon and Swinney (2009), high-type and bargain hunting consumers could form a queue in which every  $1/\theta$  customer is a high type until there are no more high types, where  $\theta \in [0, 1]$ . If  $\theta = 1$ , high types are given full priority, which is the allocation rule we consider. As  $\theta$  decreases, high types are more likely to be rationed. Nevertheless, all our results apply even if the high-type consumers are not given full priority. (Details available from the authors.)

## 4 Analysis

This section analyzes the three pricing strategies already discussed: no commitment, static pricing, and discount frequently.

### 4.1 No-Commitment Pricing

Under the no-commitment strategy, the firm chooses either  $v_h$  or  $v_l$ . Given  $\{v_l, v_h\}$ , the firm can price at  $p = v_l$  and earn revenue  $v_l k$ . Alternatively, it can price at  $p = v_h$  and earn revenue  $v_h \min\{\gamma x, k\}$ . Consequently, the firm chooses  $p = v_l$  when

$$x \leq \frac{v_l k}{v_h \gamma}, \quad (1)$$

which occurs with probability  $F(v_l k / (v_h \gamma))$  and chooses  $p = v_h$ , otherwise.

The high-type consumers only earn positive utility if the price is  $v_l$  and they are able to obtain the unit. In all other cases, consumers get zero surplus. Thus, to find the high-type consumer surplus from visiting the firm, let  $\psi$  be the high-type consumer's expectation for the probability that the firm charges  $v_l$  and he is able to get a unit. A high-type consumer is indifferent between searching the firm or not if

$$\psi (v_h - v_l) = c. \quad (2)$$

In equilibrium, the belief about the probability  $\psi$  must be consistent with the actual probability. Given the rationing rule, because  $v_l$  is charged only when  $\gamma x \leq \frac{v_l}{v_h} k < k$  (from (1)), high-type consumers are *guaranteed* to get the unit when the price is  $v_l$  - the firm discounts the product only when demand is sufficiently low, which means that a unit is available for everyone. They may not be able to get a unit if the price is  $v_h$ , but in this case, their surplus is zero either way. Thus, high-type consumers do not face a rationing risk if their utility from getting a unit is positive.

From (1), the high-type consumer knows that the firm charges a low price when demand is sufficiently low. From the perspective of a consumer, the probability density function of  $x$  is  $\tilde{f}(x) = x f(x) / \mu$ . (See Deneckere and Peck (1995) for a detailed derivation of the the demand density conditional on a consumer's presence in the market,  $\tilde{f}(x)$ .) Therefore, this consumer anticipates

that the price is  $v_l$  with probability

$$\psi = \int_0^{\frac{v_l k}{v_h \gamma}} \tilde{f}(x) dx = \frac{\int_0^{\frac{v_l k}{v_h \gamma}} x f(x) dx}{\mu}. \quad (3)$$

Given  $v_l < v_h - c$ , there exists some  $\gamma$  that satisfies (3). A symmetric equilibrium strategy for high-type consumers is a  $\gamma \in [0, 1]$  such that  $\gamma$  is optimal for each customer given that all other consumers choose  $\gamma$  as their strategy.

Let  $\gamma_0$  be the fraction of consumers who visit the firm in equilibrium under the no-commitment policy. The following lemma characterizes  $\gamma_0$ . (Proofs are in the online supplement.)

**Lemma 1.** *With the no-commitment pricing strategy, the fraction of high-type consumers who visit the firm in equilibrium,  $\gamma_0$ , is unique. Furthermore,  $\gamma_0 = 1$ , if*

$$\int_0^{\frac{v_l k}{v_h}} x f(x) dx \geq \frac{\mu c}{v_h - v_l},$$

Otherwise  $\gamma_0$  is implicitly defined by

$$(v_h - v_l) \int_0^{\frac{v_l k}{v_h \gamma_0}} x f(x) dx = \mu c. \quad (4)$$

The firm's revenue under no-commitment,  $R_0(\gamma_0)$ , is

$$\begin{aligned} R_0(\gamma_0) &= F\left(\frac{v_l k}{v_h \gamma_0}\right) v_l k + v_h \gamma_0 \int_{\frac{v_l k}{v_h \gamma_0}}^{\frac{k}{\gamma_0}} x f(x) dx + \bar{F}\left(\frac{k}{\gamma_0}\right) v_h k \\ &= v_l k + v_h \gamma_0 \left( S\left(\frac{k}{\gamma_0}\right) - S\left(\frac{v_l k}{v_h \gamma_0}\right) \right). \end{aligned} \quad (5)$$

## 4.2 Static Pricing

With a static-pricing strategy, the firm commits to a single price,  $p$ , before observing demand, so consumers know that the price will indeed be  $p$  before deciding whether or not to search the firm. All high-value consumers who search the firm receive a net value equal to  $v_h - p - c$  if they obtain a unit, and if they do not obtain a unit, their net value is  $-c$ . A customer searches the firm if net

utility is not negative, i.e., if

$$\phi(v_h - p) \geq c, \quad (6)$$

where  $\phi$  is the customer's expectation for the probability of getting a unit conditional on searching the firm. The underlying potential demand distribution,  $X$ , the high-value customers' strategy,  $\gamma$ , and the rationing rule used to allocate scarce capacity determine  $\phi$ . All else being equal, as  $\gamma$  increases, more high-type customers search the firm, thereby reducing the chance that any one of them gets a unit. Consequently, the probability she gets a unit is

$$\begin{aligned} \phi &= \int_0^\infty \frac{\min\{\gamma x, k\}}{\gamma x} \tilde{f}(x) dx = \int_0^\infty \frac{\min\{\gamma x, k\}}{\gamma x} \frac{x f(x)}{\mu} dx = \frac{S_{\gamma x}(k)}{\gamma \mu} \\ &= \int_0^\infty \frac{\min\{x, k/\gamma\}}{\mu} f(x) dx = \frac{S(k/\gamma)}{\mu}, \end{aligned} \quad (7)$$

where  $S_D(q) = \mathbb{E}_D[\min\{D, q\}]$  is the sales function given demand  $D$  and  $S(\cdot)$  is shorthand for  $S_X(\cdot)$ . (Note,  $S_{\gamma X}(k) = \gamma S(k/\gamma)$ .) Given a static price,  $p$ , a symmetric equilibrium strategy for high-type consumers is a  $\gamma(p) \in [0, 1]$  such that  $\gamma(p)$  is optimal for each consumer given that all other consumers choose  $\gamma(p)$  as their strategy. If  $p$  is low enough, there is an equilibrium in which all high-type consumers visit the firm, i.e.,  $\gamma(p) = 1$ . From (6) and (7), that occurs if

$$\frac{S(k)}{\mu}(v_h - p) \geq c$$

or

$$p \leq v_h - \frac{\mu c}{S(k)} \equiv \bar{p}.$$

If  $p > \bar{p}$ , the unique symmetric equilibrium has  $\gamma(p) < 1$ , where  $\gamma(p)$  is the unique solution to

$$S\left(\frac{k}{\gamma(p)}\right) = \frac{\mu c}{v_h - p}. \quad (8)$$

Using (8), the firm's revenue function can be written as a function of  $\gamma$  alone. Define  $R_s^h(\gamma)$  as the firm's revenue function from only high-type consumers:

$$\begin{aligned} R_s^h(\gamma) &= S_{\gamma X}(k) \left( v_h - \frac{\mu c}{S(k/\gamma)} \right) \\ &= \gamma S \left( \frac{k}{\gamma} \right) v_h - \gamma \mu c. \end{aligned} \tag{9}$$

The next lemma finds the equilibrium fraction of high-type consumers who search the firm under static-pricing,  $\gamma_s$ .

**Lemma 2.** *Define  $\tilde{k}$  implicitly as*

$$v_h \int_0^{\tilde{k}} x f(x) dx = \mu c.$$

*With static-pricing, the firm's revenue function from high-type consumers,  $R_s^h(\gamma(p))$ , is concave. Let  $\gamma_s = \arg \max R_s^h(\gamma)$ . The price charged to the high types is  $p_s^h$ . If  $k \geq \tilde{k}$ , then  $\gamma_s = 1$  and  $p_s^h = \bar{p}$ . Otherwise  $\gamma_s = k/\tilde{k}$  and*

$$p_s^h = v_h - \frac{\mu c}{S(\tilde{k})}.$$

Instead of choosing  $p_s^h$  and selling only to high-type consumers, the firm also has the option to choose  $p_s = v_l$ , in which case the firm sells all its capacity and its revenue is  $p_s k$ . Finally, the firm chooses the static price,  $p_s \in \{p_s^h, v_l\}$  to maximize revenues. When  $\gamma_s < 1$ ,  $R_s^h(\gamma_s) = k v_h \bar{F}(\tilde{k})$ . Thus, under static-pricing, the optimal price is  $p_s = p_s^h$  when  $v_h \bar{F}(k/\gamma_s) \geq v_l$ , otherwise  $p_s = v_l$ .

### 4.3 Discount Frequently

Static pricing commits to charge some price that is sub-optimal once demand is observed. Although it is costly, it may be done to increase demand from high-type consumers - they do not visit if the expectation of what they can receive is too low. However, there is another way to make searching the firm attractive to consumers. Instead of always providing an intermediate discount (as in static pricing), the firm could commit to provide the deep discount (to  $v_l$ ) more frequently than would be optimal given the realization of demand. In particular, with the discount-frequently strategy the firm chooses to price at either  $v_l$  or  $v_h$  (the two optimal prices *ex-post*), but commits to markdown to  $v_l$  whenever potential demand is  $\delta k/\gamma_f$  or lower and charge  $v_h$  otherwise, where  $\delta \in [v_l/v_h, 1]$ .

This implies that the firm charges  $v_l$  with probability  $F(\delta k/\gamma_f)$ . Under this policy the firm does not commit to limit its prices - the firm charges the same prices as in the no-commitment strategy. Rather, it commits to discount often and deeply, thereby encouraging high-type consumers to search. The following lemma characterizes the fraction of high-type consumers who search the firm under the discount-frequently strategy.

**Lemma 3.** *With the discount-frequently strategy, the fraction of high-type consumers who visit the firm in equilibrium,  $\gamma_f$ , is unique. Furthermore,  $\gamma_f = 1$ , if*

$$(v_h - v_l) \int_0^{\delta k} x f(x) dx > \mu c \quad (10)$$

Otherwise  $\gamma_f$  is implicitly defined by

$$(v_h - v_l) \int_0^{\delta k/\gamma_f} x f(x) dx = \mu c. \quad (11)$$

The revenue function is

$$R_f(\delta, \gamma_f) = v_l k F(\delta k/\gamma_f) + \gamma_f v_h \int_{\delta k/\gamma_f}^{k/\gamma_f} x f(x) dx + v_h k \bar{F}(k/\gamma_f).$$

If the firm chooses  $\delta = v_l/v_h$ , then it replicates the no-commitment strategy. (Hence, discount frequently cannot be worse than no commitment.) However, if  $\delta > v_l/v_h$ , then the firm discounts more frequently than would be optimal to maximize revenue conditional on realized demand (i.e., holding  $\gamma_f$  fixed), which may entice enough high-type consumers to visit to justify the cost of discounting even though demand is high enough to keep the price high. The next theorem characterizes the optimal discount frequency,  $\delta^*$ , and the corresponding fraction of high-type consumers who visit given that discount frequency,  $\gamma_f^* = \gamma_f(\delta^*)$ .

**Theorem 1.** *Define  $\hat{k}$  implicitly as*

$$(v_h - v_l) \int_0^{\hat{k}} x f(x) dx = \mu c \quad (12)$$

*With the discount-frequently strategy, let  $\delta^*$  and  $\gamma_f^*$  be the optimal discount frequency and the result-*

ing high-type consumer search probability, respectively:

$$\delta^* = \begin{cases} 1 & k \leq \hat{k} \\ \hat{k}/k & \hat{k} < k \leq \hat{k}v_h/v_l \\ v_l/v_h & \hat{k}v_h/v_l < k \end{cases} \quad \gamma_f^* = \begin{cases} k/\hat{k} & k \leq \hat{k} \\ 1 & \hat{k} < k \leq \hat{k}v_h/v_l \\ 1 & \hat{k}v_h/v_l < k \end{cases}$$

According to Theorem 1, the actual probability of a discount is  $F(\hat{k})$  when  $k \leq \hat{k}v_h/v_l$  (and  $F(v_l k/v_h)$  otherwise), which is exactly the same probability of a discount with the no-commitment strategy. The difference is that with discount frequently, the firm enjoys higher demand (more high types choose to search), precisely because the firm maintains frequent discounts even with the higher demand. For example, when  $k < \hat{k}$ , the firm discounts whenever realized high-type demand is less than capacity - the firm charges the higher price,  $v_h$ , only when it is able to sell its entire capacity to high-type consumers. Consequently, unlike with static pricing, the firm always clears its entire inventory with the best discount-frequently strategy. Hence, it could be named the “everything must go” strategy.

The firm’s revenue function with the optimal discount-frequently strategy is

$$R_f^* = \begin{cases} k \left( v_l F(\hat{k}) + v_h \bar{F}(\hat{k}) \right) & k \leq \hat{k} \\ v_l k F(\hat{k}) + v_h \int_{\hat{k}}^k x f(x) dx + v_h k \bar{F}(k) & \hat{k} < k < \hat{k}v_h/v_l \\ v_l k F\left(\frac{v_l}{v_h}k\right) + v_h \int_{\frac{v_l}{v_h}k}^k x f(x) dx + v_h k \bar{F}(k) & \hat{k}v_h/v_l \leq k \end{cases}$$

Our main result, reported in Theorem 2, is that discount frequently is optimal for the firm considering all possible policies. This is a strong motivation for the use of the discount-frequently strategy. By offering discounts sufficiently often, and in some cases when the firm’s short term demand does not justify the discount, the firm provides the motivation needed for consumers to visit, thereby increasing the firm’s demand, which in turn leads to the maximum revenue.

**Theorem 2.** *The discount frequently strategy maximizes the firm’s revenue.*

Why is discount frequently optimal? Commitments are made to make searching the firm more attractive to consumers, but they are also costly to the firm. Thus, the best commitment substantially improves the attractiveness of the firm relative to its cost. Suppose the firm observes

**Table 2.** Parameter Values Used in the Numerical Study.

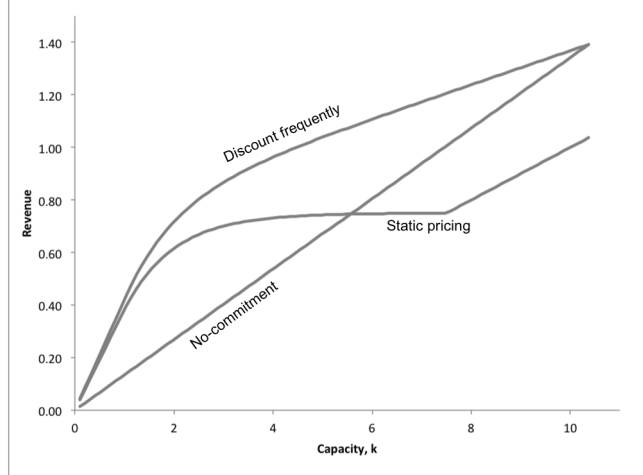
Parameter	Values
Demand Distribution	Gamma
$\mu$	1
$\sigma$	$\{0.25\mu, 0.5\mu, \mu, 1.5\mu, 2\mu\}$
$v_h$	1
$v_l$	$\{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.6\}$
$c$	$\{0.01, 0.02, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3\}$
$F(k)$	$\{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.99\}$

that realized demand,  $x$ , equals  $v_l k / (v_h \gamma)$ . The firm is indifferent between charging  $v_h$  and selling  $\gamma x$  units or charging  $v_l$  and selling  $k$  units. As the revenue function is *flat* at that point, a slight increase in the demand level that triggers a discount, say to  $v_l k / (v_h \gamma) + \epsilon$ , causes essentially no loss in revenue, yet it generates an immediate benefit in that it attracts more high-type customers to visit. Thus, this slight distortion from the short-term optimal causes little cost initially in terms of revenue conditional on  $\gamma$  but provides an immediate benefit in that it strictly increases  $\gamma$ , the fraction of high types that search. As the distortion increases, the cost increases, but this distortion in the short-term decision provides the greatest benefit relative to its cost. Cachon and Lariviere (2001) observe a similar finding in the context of signaling capacity in a supply chain - some signals are cheaper to make than others and the best signal distorts a decision near the “flat area” of a maximum.

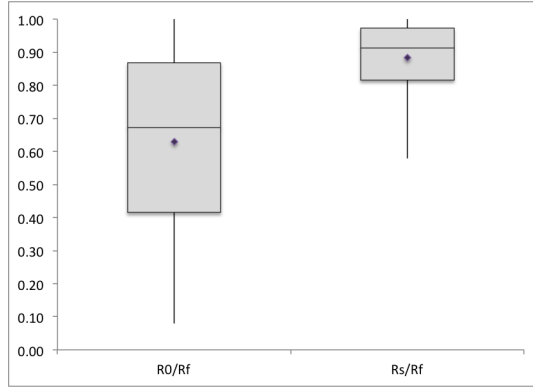
## 5 Comparison of Strategies with Fixed Capacity

Table 2 lists the parameters we use in a numerical study to compare the pricing strategies discussed in the previous section. To compare revenue, it is sufficient to manipulate  $k$ ,  $\sigma$ ,  $\mu c / v_h$  and  $v_l / v_h$ . Hence, we fix the values of  $v_h$  and  $\mu$  to 1. To represent different levels of demand uncertainty, with the Gamma distribution, the coefficient of variation ranges from a low level of uncertainty (e.g.,  $\sigma / \mu = 0.25$ ) to a high level of uncertainty (e.g.,  $\sigma / \mu = 2$ ). The  $v_l$  and  $c$  parameters span the range  $[0, v_h]$  while also satisfying  $v_l \leq v_h - c$ . There are 360 combinations of these parameters. We choose capacity to correspond to ten different fractiles of the potential demand distribution,  $k = \{F^{-1}(0.1), \dots, F^{-1}(0.99)\}$ . This yields a total of 3,600 scenarios. However, if capacity is greater than  $v_h \hat{k} / v_l$ , then the no commitment and discount-frequently strategies are both optimal. Thus,





**Figure 1.** Comparison of expected revenue with three pricing strategies for one of the considered scenarios ( $v_h = 1, v_l = 0.1, c = 0.25, X \sim \text{Gamma}(1, 1)$ )



**Figure 2.** Revenue Ratios. Diamond = average; the bottom, middle and upper horizontal lines correspond to the 1st quartile, median and 3rd quartile, respectively; the bottom and top points of the vertical lines are the minimum and the maximum, respectively.

we remove scenarios that have capacity above this threshold, leaving 3,146 scenarios.

Figure 1 illustrates the revenue achieved with each policy for a representative scenario and a wide range of capacities. (The 99th percentile of capacity is  $k = 4.6$ .) . Although no commitment performs better than static pricing for very large capacities, for most reasonable capacities, static pricing performs best and potentially much better. Overall, the figure suggests that the firm sacrifices a considerable amount of revenue if it does not make a price commitment (i.e., no commitment performs poorly). Furthermore, some types of commitments perform much better than others (i.e., discount frequently is best). These specific findings extend to our larger sample, as we discuss next.

Figure 2 shows a box-plot of the revenue ratios of the pricing strategies relative to the revenue obtained from discount frequently. As observed in Figure 1, the discount-frequently strategy

performs substantially better than no commitment or static pricing. Though both schemes can approach the discount-frequently strategy in some cases (with maximum ratios of  $R_0^*/R_f^* = 1$  and  $R_s^*/R_f^* = 99.99\%$ ), in most cases they perform poorly in comparison: the average  $R_0^*/R_f^*$  ratio is 63.0% and the average  $R_s^*/R_f^*$  ratio is 88.3%.

The results in Figure 2 emphasize two points. First, failing to make a price commitment can substantially reduce a firm's revenue. Second, it is important to make the right price commitment. In particular, a commitment to a static price is often better than no-commitment, but not effective relative to discount frequently. Hence, our results do not support a static-pricing strategy. This runs counter to the results in the literature that suggest a firm should in some cases try to commit to either not discount, or not discount deeply, or both (e.g. Besanko and Winston (1990), Aviv and Pazgal (2008), and others). In those papers the motivation for a static-pricing strategy is to mitigate the negative consequences of consumers strategically waiting for a price discount. As already mentioned, that motivation is not present in our model, so our results in no way contradict those findings. Instead, our results provide a counter-argument for the adoption of static pricing. Specifically, we find that static pricing is not the best strategy for attracting consumers to the firm when search is costly. Whether it is best to adopt an aggressive discounting strategy (as our model recommends) or to commit to not offer discounts probably depends on the importance of attracting consumers to search a firm relative to the desire to prevent consumers from strategically timing their purchases. In the example of JC Penney, it appears that JC Penney's decision to limit promotions prevented consumers from even visiting their stores and the negative effect of this loss in demand was larger than the benefit of preventing consumers from waiting for sales.

Although discount-frequently maximizes expected profit over all strategies assuming the firm adheres to its commitment, in the short-term period the firm may have an incentive (depending on the demand realization) to deviate from its commitment by charging the higher price rather than a discount. To be specific, based on (5), let  $R_0(\gamma_f)$ , be the firm's maximum expected revenue conditional that consumers choose  $\gamma_f$  as their visit strategy. Hence, deviating from the discount frequently strategy can increase the firm's expected revenue by  $R_0(\gamma_f) - R_f^* \geq 0$ . If the firm and consumers indeed interacted over only a single period, then the discount frequently commitment would not be credible. However, in the settings we consider (e.g., a department store), the firm is likely to interact with consumers over multiple periods, each of which is like our short-term

period. In such case, the commitment to follow the discount frequently strategy could be credible if consumers implement a trigger strategy that penalizes the firm for deviating. With a trigger strategy, consumers follow the discount-frequently equilibrium until they detect a deviation, at which point they switch to the no-commitment equilibrium in all subsequent periods. If the trigger is implemented, the firm's revenue loss in each period is  $R_f^* - R_0^* \geq 0$ . Thus, the discount-frequently strategy is credible if the profit from deviation,  $R_0(\gamma_f) - R_f^*$ , is less than the discounted future loss of revenue. To determine if this is likely, we evaluate the ratio of the short term gain from deviation to the single period loss from detection:

$$\frac{R_0(\gamma_f) - R_f^*}{R_f^* - R_0^*}.$$

On average, in our sample, this ratio is 0.51 and in 90% of the scenarios the ratio is 1.0 or lower - on average the gain in a single period deviation from discount frequently is only 51% of the loss that is incurred in each period after the trigger is activated. Unless the firm heavily discounts future revenue, the short term gain is unlikely to be justified by the loss of future revenue, suggesting that discount frequently can be credible.

The credibility of any pricing strategy also hinges on the ability of consumers to detect a deviation. This cannot be achieved with a single period, but it can be achieved with several periods. As the required length of the detection period increases, the benefit to the firm from a deviation increases -  $R_0(\gamma_f) - R_f^*$  is earned over multiple periods. Nevertheless, given that the single period benefit is generally less than even a single period of loss, credibility can be achieved as long as the firm cares enough about future revenue. See Radnor (1986) and Abreu et al. (1990) for detailed studies of repeated games with imperfect monitoring. (This issue is also relevant in papers, such as Liu and van Ryzin (2008), that assume a firm can credibly commit to a fill rate in future periods.)

## 6 Comparison of Strategies with Capacity Investment

In this section we determine the optimal capacity decisions under each of the pricing strategies and compare these capacity investments to a “myopic” benchmark that assumes all high types always search, i.e.,  $\gamma = 1$  for sure. The next theorem characterizes the optimal capacity levels under the

**Table 3.** Optimal capacity levels relative to the myopic consumers optimal capacity. All statistics apply only in the scenarios in which it is profitable to invest in capacity, i.e.,  $k > 0$ .

	$k_0/k_m$	$k_s/k_m$	$k_f/k_m$
# of observations with $k > 0$	662	1254	1800
Average	1.90	0.89	1.24
SD	0.77	0.17	0.30
Minimum	1.000	0.08	1.00
1st Quartile	1.22	0.84	1.03
Median	1.79	0.98	1.13
3rd Quartile	2.43	1.00	1.33
Maximum	4.25	1.00	2.94

different policies.

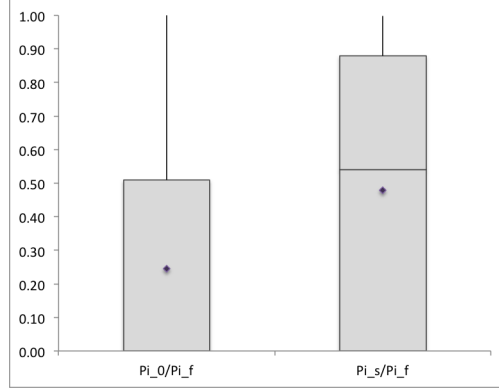
**Theorem 3.** Let  $c_k$  be the marginal cost of capacity. Then:

1. *Myopic benchmark:* The optimal capacity,  $k_m$ , is given implicitly by  $v_h \bar{F}(k) + v_l F\left(\frac{v_l k}{v_h}\right) = c_k$ .
2. *No commitment:* If  $c_k < c_k^0 = v_l + v_l \left(S\left(\frac{v_h \hat{k}}{v_l}\right) - S(\hat{k})\right) / \hat{k}$ , then the optimal capacity is  $k_0 = \max\left\{v_h \hat{k} / v_l, k_m\right\}$ . Otherwise,  $k_0 = 0$ .
3. *Static-pricing:* If  $c_k < c_k^s = v_h \bar{F}(\tilde{k})$ , then the optimal capacity,  $k_s$ , is given implicitly by  $\bar{F}(k) = c_k / v_h$ . Otherwise,  $k_s = 0$ .
4. *Discount-frequently:* If  $c_k < c_k^f = v_l + (v_h - v_l) \bar{F}(\hat{k})$ , then the optimal capacity is  $k_f = \max\{k'', k_m\}$ , where  $k''$  is given implicitly by  $v_h \bar{F}(k'') + v_l F\left(\frac{v_l k''}{v_h}\right) = c_k$ . Otherwise,  $k_f = 0$ .

**Theorem 4.** Assume  $c_k < c_k^f$ . The following statements hold:

1.  $k_s < k_m \leq k_f$
2. Let  $\underline{c}_k = v_l F(\hat{k}) + v_h \bar{F}\left(\frac{v_h \hat{k}}{v_l}\right)$ . If  $c_k \leq \underline{c}_k$ , then  $k_m = k_o = k_f$ . If  $\underline{c}_k < c_k \leq c_k^0$ , then  $k_f < k_o$ .

To compare the performance of the pricing strategies when capacity can be selected, we again considered the 360 parameter combinations detailed in Table 2 (excluding  $k$ ). As  $c_k \in (v_l, c_k^f)$  is the interesting range for capacity costs (capacity levels greater than  $c_k^f$ , result in zero profit for any policy), for each parameter combination we select 5 evenly distributed values from this range, i.e.,  $c_k^{(i)} = v_l + i(c_k^f - v_l) / 6$ ,  $i = 1, \dots, 5$ . This yields 1800 scenarios. Table 3 summarizes the ratios of the optimal capacity relative to the optimal capacity with myopic consumers.



**Figure 3.** Optimal profit ratios considering all 360 scenarios.

Table 3 demonstrates that the capacity investment under no commitment can be large (e.g., on average  $k_0/k_m = 1.90$ ). As in Dana and Petruzzi (2001), when the firm does not make a price commitment, then it can attract strategic consumers only by over investing in capacity. However, this can be inefficient and in only a minority of scenarios (662 out of 1800 scenarios) is the no-commitment strategy able to earn a positive profit. Static pricing performs better and of course discount frequently performs the best - as it is optimal, it yields a positive profit in all 1,800 scenarios. Relative to the myopic benchmark capacity, discount frequently invests in more capacity, 23.8% more on average. This “overinvestment” helps to attract consumers, because it provides additional availability and reduces the distortion needed in the frequency of discounts to attract the high-type consumers. However, discount frequently invests in far less capacity than the no-commitment strategy (assuming it is profitable to invest in capacity with no-commitment).

Figure 3 reports on the profits earned by each strategy relative to the discount-frequently strategy. We denote the optimal profit under each scheme by  $\Pi_i^* = R_i - c_k k_i$   $i = \{f, s, 0\}$  and consider all 1800 scenarios. Figure 3 confirms that when the firm has the flexibility to choose both its pricing strategy and its capacity, discount frequently outperforms the others, often by a large margin. In particular, overinvesting in capacity without a price commitment is not an effective strategy - it earns on average only 25% of what can be earned with discount frequently. Static-pricing also struggles - it earns on average only 48% of the optimal profit. Overall, the option to choose capacity accentuates the importance of making a price commitment and in particular, the correct price commitment.

As with a fixed capacity, we investigate the firm’s short term incentive to deviate from discount

**Table 4.** Average ratio of short term profit gain,  $\hat{\Pi}_f - \Pi_f^*$ , to long-run per period profit loss,  $\Pi_f^* - \Pi_0$ , as a function of the search cost,  $c$ , and the cost of capacity,  $c_k^{(i)}$ .

$$c_k^{(i)} = v_l + i \left( c_k^f - v_l \right) / 6$$

$c$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
0.01	0.01	0.02	0.04	0.14	0.23
0.02	0.03	0.05	0.12	0.18	0.35
0.05	0.11	0.20	0.22	0.30	0.70
0.10	0.28	0.28	0.34	0.53	1.28
0.15	0.35	0.37	0.48	0.9	1.89
0.20	0.44	0.48	0.66	1.10	2.56
0.25	0.54	0.63	0.87	1.48	3.35
0.30	0.68	0.81	1.16	1.95	4.33

frequently. Let  $\hat{\Pi}_f$  be the single period profit the firm earns if it deviates from discount frequently even though consumers play the discount frequently equilibrium. On average the ratio of the short term gain relative to the potential single period loss of profit,

$$\frac{\hat{\Pi}_f - \Pi_f^*}{\Pi_f^* - \Pi_0^*},$$

is 0.76. This is higher than in the fixed capacity sample (0.51), but still suggests that the gain from deviating is potentially considerably lower than the loss in future discounted profit. Table 4 reveals that the consumer's search cost and the cost of capacity strongly influence this average. When search is not costly (first three rows), the short term gain from deviating is small compared to the potential long run loss in profit, and in some cases very small (upper left corner). Given the low search cost in these scenarios, one might assume that discount frequently does not provide much of an advantage relative to no commitment. But Table 5 reveals that even with low search costs, failing to make a price commitment reduces profit by a substantial amount (between 18-100%). Table 4 also indicates that there are conditions in which credibility might be harder to achieve - when search is costly and capacity is expensive (lower right corner), the gain from a deviation is a reasonable amount of the potential loss (up to 4.33 periods). In these cases, credibility requires the firm to be sufficiently far sighted (which remains possible, just harder). Nevertheless, in these cases the firm's bigger problem is that the product is barely profitable (and generally not profitable with the no-commitment strategy). Overall, even when the firm can choose its capacity, the firm loses a substantial amount of profit if it either chooses not to make a commitment or if it loses the

**Table 5.** Average ratio of no-commitment profit,  $\Pi_0^*$ , to discount frequently profit,  $\Pi_f^*$ .

$$c_k^{(i)} = v_l + i \left( c_k^f - v_l \right) / 6$$

$c$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
0.01	0.82	0.73	0.61	0.43	0.14
0.02	0.77	0.65	0.51	0.30	0.06
0.05	0.67	0.51	0.32	0.13	0.00
0.10	0.54	0.35	0.07	0.04	0.00
0.15	0.44	0.24	0.10	0.01	0.00
0.20	0.35	0.17	0.05	0.00	0.00
0.25	0.28	0.11	0.02	0.00	0.00
0.30	0.22	0.07	0.00	0.00	0.00

credibility of its commitment.

## 7 Conclusion

Firms that do not make a price commitment can optimally select a price to respond to available demand information to maximize revenue. This is the best pricing strategy for the firm if consumers are not strategic. However, with strategic consumers, even though price commitments are costly to the firm in the short term, they are useful for attracting demand.

With fixed capacity, we show that a firm can do much better by committing to static-pricing relative to no price commitment, despite the fact that the commitment reduces the firm’s ability to match supply with demand. However, static pricing is not the best price commitment strategy. Discount frequently is the optimal strategy, and it performs substantially better than the other strategies we study. Thus, while it has been suggested that a static-pricing strategy can mitigate the negative effects of consumers strategically waiting for end-of-season discounts, we do not recommend that strategy when it is important to attract consumers to the firm (due to search costs).

When the firm can choose capacity, we show that a firm without a price commitment overinvests in capacity to attract consumers and earns substantially lower profit, if it can even earn a profit. As with fixed capacity, the discount-frequently strategy is optimal, and much better than the other policies.

According to our model, adopting a “simplified pricing policy” or curtailing discounts, as done by JC Penney in 2012 (Clifford and Rampell (2013)), can backfire considerably. Thus, we conclude that in the presence of strategic consumers and search costs, (i) price commitments are generally

necessary, (ii) the right commitment is discount frequently - the firm should give consumers a deep discount even if doing so lowers revenue conditional on demand because the higher frequency of discounts attracts consumers to the firm - and (iii) the firm should not exclusively use excess capacity as a tool to attract consumers.

Our results theoretically justify the implementation of a discount-frequently strategy. Additional work could seek to determine if firms currently utilize some form of discount frequently and if discount frequently is better in practice than alternatives. A firm could be said to be using discount frequently if they choose deeper discounts than would be optimal given existing inventory and current unbiased estimates of demand. Although not definitive, further evidence could be provided via surveys and interviews with managers - managers may intuitively be using a strategy like discount frequently if they say they are willing to provide discounts at the expense of short term profits to avoid losing their long-term reputation for offering “good deals” or “competitive prices”. Finally, JC Penney’s experience provides some support for the effectiveness of discount frequently, but additional, controlled experiments would offer more direct evidence.

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# Online Supplement: “Price Commitments with Strategic Consumers: Why it can be Optimal to Discount More Frequently...Than Optimal”

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## A Proofs

*Proof of Lemma 1.* Under the “no commitment” policy, the indifferent consumer solves

$$\frac{\int_0^{\frac{v_l k}{v_h \gamma}} x f(x) dx}{\mu} (v_h - v_l) = c. \quad (1)$$

As the left-hand-side (LHS) strictly decreases with  $\gamma$  and the right-hand-side (RHS) is constant, there either exists a unique  $\gamma \in [0, 1]$  which solves (1), or, if  $(v_h - v_l) \int_0^{\frac{v_l k}{v_h}} x f(x) dx > \mu c$ , there does not exist a  $\gamma$  which solves (1), in which case  $\gamma_0 = 1$ .  $\square$

*Proof of Lemma 2.* First, note that the expected sales function is given by

$$S\left(\frac{k}{\gamma}\right) = \int_0^{k/\gamma} x f(x) dx + \frac{k}{\gamma} \bar{F}\left(\frac{k}{\gamma}\right)$$

and that

$$S'\left(\frac{k}{\gamma}\right) = \frac{dS(k/\gamma)}{d\gamma} = -\frac{k}{\gamma^2} \bar{F}\left(\frac{k}{\gamma}\right).$$

Differentiating  $R_s^h(\gamma)$  with respect to  $\gamma$ , we get:

$$\begin{aligned} \zeta_s(\gamma) &= \frac{dR_s^h(\gamma)}{d\gamma} = v_h (S(k/\gamma) + \gamma S'(k/\gamma)) - \mu c \\ &= v_h \int_0^{k/\gamma} x f(x) dx - \mu c. \end{aligned}$$

$R_s^h(\gamma)$  is concave because  $\zeta_s(\gamma)$  is decreasing in  $\gamma$ . The optimal  $\gamma_s$  may be 1 (a corner solution) if  $\zeta_s(1) \geq 0$  or interior, in which case solving the first-order condition  $\zeta_s(\gamma) = 0$  gets the desired result. Note that  $\gamma_s \neq 0$ , because we assume that  $v_h > c$ .  $\square$

*Proof of Lemma 3.* Under the discount-frequently policy, the indifferent consumer solves

$$(v_h - v_l) \int_0^{\delta k / \gamma_f} x f(x) dx = \mu c. \quad (2)$$

As the left-hand-side (LHS) strictly decreases with  $\gamma$  and the right-hand-side (RHS) is constant, there either exists a unique  $\gamma \in [0, 1]$  which solves (2), or, if

$$(v_h - v_l) \int_0^{\delta k / \gamma_f} x f(x) dx > \mu c,$$

there does not exist a  $\gamma$  which solves (2), in which case  $\gamma_f = 1$ .  $\square$

*Proof of Theorem 1.* It is useful to think of  $\gamma_f$  as a function of  $\delta$ .  $\gamma_f$  can be rewritten in terms of  $\hat{k}$  by comparing (2) with the definition of  $\hat{k}$ . There are three regimes that specify the relationship:

$$\gamma_f = \begin{cases} \delta k / \hat{k} & k \leq \hat{k} \\ \begin{cases} \delta k / \hat{k} & \frac{v_l}{v_h} \leq \delta \leq \frac{\hat{k}}{k} \\ 1 & \frac{\hat{k}}{k} \leq \delta \leq 1 \end{cases} & \hat{k} < k \leq \hat{k} v_h / v_l \\ 1 & k > \hat{k} v_h / v_l \end{cases}$$

Whenever  $\gamma_f < 1$ ,  $(v_h - v_l) \int_0^{\delta k / \gamma_f} x f(x) dx = \mu c$  holds and whenever  $\gamma_f = 1$ ,  $(v_h - v_l) \int_0^{\delta k} x f(x) dx > \mu c$  holds. Define  $\hat{\delta}$  as follows:

$$\hat{\delta} = \begin{cases} 1 & k \leq \hat{k} \\ \frac{\hat{k}}{k} & \hat{k} < k \leq \hat{k} v_h / v_l \\ \frac{v_l}{v_h} & \hat{k} v_h / v_l < k. \end{cases}$$

Thus, if the firm chooses  $\delta \in [v_l / v_h, \hat{\delta}]$ , then  $\gamma_f = \delta k / \hat{k}$ . But if the firm chooses  $\delta \in [\hat{\delta}, 1]$ , then  $\gamma_f = 1$ . The revenue function can be written in terms of  $\delta$  and  $\hat{k}$ . As  $\hat{k}$  is fixed, the revenue function can be expressed exclusively in terms of  $\delta$  (without an implicit function defining another term):

$$R_f(\delta; \hat{k}) = \begin{cases} v_l k F(\hat{k}) + v_h \left(\frac{\delta k}{\hat{k}}\right) \int_{\hat{k}}^{\hat{k}/\delta} x f(x) dx + v_h k \bar{F}\left(\frac{\hat{k}}{\delta}\right) & v_l / v_h \leq \delta \leq \hat{\delta} \\ v_l k F(\delta k) + v_h \int_{\delta k}^k x f(x) dx + v_h k \bar{F}(k) & \hat{\delta} < \delta \leq 1. \end{cases}$$

Differentiate the revenue function:

$$\frac{dR_f}{d\delta} = \begin{cases} v_h \left(\frac{k}{\hat{k}}\right) \int_{\hat{k}}^{\hat{k}/\delta} x f(x) dx & v_l / v_h \leq \delta \leq \hat{\delta} \\ -v_h k^2 f(\delta k) \left(\delta - \frac{v_l}{v_h}\right) & \hat{\delta} < \delta \leq 1 \end{cases}$$

It is immediately clear that  $\delta > \hat{\delta}$  is not optimal - in this case the firm is marking down more frequently than optimal, but there is no benefit in terms of increased high type demand (they are already all visiting). It is also apparent that it is optimal to choose  $\delta = \hat{\delta}$ .  $\square$

*Proof of Theorem 2.* To search for the optimal pricing policy, start by fixing  $\gamma$ , the high-type's search strategy. For a given  $\gamma$ , find a set of prices, one for each possible demand realization, such that the firm's revenue is maximized and  $\gamma$  is the optimal strategy for high-type consumers. The strategy  $\gamma$  is optimal if the high-types' expected value of search equals their cost of search conditional that  $\gamma$  fraction of high-type consumers visit. Next, inspect the set of chosen prices to confirm that the set can be implemented with discount-frequently. Finally, if discount-frequently is optimal for any given  $\gamma$ , then it must be the optimal policy overall.

Begin with some definitions. There exists a threshold demand realization,  $\hat{x} = k/\gamma$ , such that for each  $x < \hat{x}$ , high-type demand,  $\gamma x$ , is strictly less than capacity. For  $x < \hat{x}$ , let  $s(r, x)$  be the sales function for demand realization  $x$  and price  $r$ :

$$s(r, x) = \begin{cases} k & r \leq v_l \\ \gamma x & v_l < r \leq v_h \end{cases} \quad (3)$$

For  $x \geq \hat{x}$ , the firm sells  $k$  units for every  $r \leq v_h$ .

Let  $\mathcal{P} = \{p : p : \mathbb{R}^+ \rightarrow \mathbb{R}^+\}$  be the class of price functions that maps each demand realization  $x$  to its price  $p(x)$ . The firm's objective is to choose a function  $p \in \mathcal{P}$  that maximizes revenue conditional on a search constraint that stipulates that the expected value a high-type receives from search is at least as great as the cost of search:

$$\begin{aligned} \max_{p \in \mathcal{P}} R(p) &= \max_{p \in \mathcal{P}} \int_0^{\hat{x}} p(x) s(p(x), x) f(x) dx + k \int_{\hat{x}}^{\infty} p(x) f(x) dx \\ \text{s.t.} \quad &\int_0^{\hat{x}} (v_h - p(x)) \frac{x f(x)}{\mu} dx + \frac{k}{\gamma} \int_{\hat{x}}^{\infty} (v_h - p(x)) \frac{f(x)}{\mu} dx \geq c \end{aligned} \quad (4)$$

The search constraint (4) can be rewritten as:

$$\int_0^{\hat{x}} p(x) x f(x) dx + \frac{k}{\gamma} \int_{\hat{x}}^{\infty} p(x) f(x) dx \leq g(\gamma),$$

where

$$g(\gamma) = v_h S\left(\frac{k}{\gamma}\right) - \mu c.$$

Note that  $g(\gamma)$  is independent of the chosen prices. Define the slack in the search constraint as:

$$g(\gamma) - \int_0^{\hat{x}} p(x) x f(x) dx - \frac{k}{\gamma} \int_{\hat{x}}^{\infty} p(x) f(x) dx.$$

An increase in any price  $p(x)$  has two effects: (i) it increases revenue and (ii) it decreases the slack in the constraint. Thus, an optimal  $p(x)$  can be found by continuously increasing the set of prices so as to maximize the ratio of the marginal increase in revenue to the marginal decrease in the slack.

Define  $p_l \in \mathcal{P}$  as the constant function  $p(x) = v_l \forall x$ . With this policy the firm generates  $v_l k$  in revenue, which is a lower bound on the revenue that can be achieved with the optimal policy. Given  $k$ , if the search constraint is not satisfied with this pricing policy, then  $\gamma$  cannot be the equilibrium search strategy in the optimal policy (because it does not generate at least  $v_l k$  in revenue). Thus, it is sufficient to consider values of  $\gamma$  such that the search constraint is satisfied with  $p_l$ : i.e., it must be that  $\gamma$  is sufficiently small so that

$$(v_h - v_l) S\left(\frac{k}{\gamma}\right) \geq \mu c.$$

In other words, with the pricing strategy  $p_l$ , there must be some slack in the search constraint.

Starting with  $p_l$ , we next increase prices for some values of  $x$  so as to increase revenue while not violating the search constraint. For the most part, revenue increases and the slack decreases smoothly in price for each  $x$ , except for the very first increase in the price above  $v_l$ . The first incremental price increase above  $v_l$  yields a discontinuous decrease in revenue (because all low-type shoppers abandon their purchases). Thus, for all  $x \leq \hat{x}$ , the first incremental price increase is particularly costly - it decreases the slack without increasing revenue. In fact, the firm generates the same revenue with price  $v_l$  as it does with price  $\hat{p}(x) = v_l k / (\gamma x)$ .

Thus, for all  $x \leq \hat{x}$ , the optimal policy either charges  $v_l$ , or some price  $\hat{p}(x) \leq p(x) \leq v_h$ . In that range, additional increases in price generate smooth increases in revenue and smooth decreases in the slack. In particular, there are two cases to consider:

Case 1:  $x \leq \hat{x}$ ,  $\hat{p}(x) \leq p(x) \leq v_h$ . The marginal increase in revenue is  $\partial R(p)/\partial p(x) = \gamma x f(x)$  and the marginal decrease in the slack with respect to price is  $x f(x)$ . Thus, the increase in revenue per unit of decrease in slack is:

$$\gamma x f(x) / x f(x) = \gamma.$$

Case 2:  $\hat{x} < x$ . The marginal increase in revenue is  $\partial R(p)/\partial p(x) = k f(x)$  and the marginal decrease in the slack with respect to price is  $(k/\gamma) f(x)$ . Thus, the increase in revenue per unit of decrease in slack is:

$$k f(x) / (k/\gamma) f(x) = \gamma.$$

To repeat, for all  $x \leq \hat{x}$ , the first incremental price increase above  $v_l$  actually decreases revenue and slack. Hence, any price increase above  $v_l$  should first be done in the  $\hat{x} < x$  demand states. In these states, all price increases generate the same constant increase in the ratio of revenue to slack (case 2), the optimal price is the maximum price so long as the search constraint is satisfied. Therefore, starting with the highest demand states in the range  $\hat{x} < x$ , increase the price from  $v_l$  to the maximum price,  $v_h$ , until either the search constraint binds (i.e., all slack is consumed) or the price is increased in all of these demand states. This yields the following price function:

$$p(x) = \begin{cases} v_l & x \leq \tilde{x} \\ v_h & \text{otherwise} \end{cases}$$

for some  $\tilde{x} \geq \hat{x}$ . If  $\tilde{x} > \hat{x}$ , then no slack remains and the above pricing strategy is the optimal solution. If  $\tilde{x} = \hat{x}$ , then some slack remains in the search constraint and price increases in the  $x \leq \hat{x}$  can be considered, which is done next.

For  $x \leq \hat{x}$ , an increase in price from  $v_l$  to  $\hat{p}(x)$  increases revenue by  $(\gamma x p - k v_l) f(x)$  and decreases slack by  $(p - v_l) x f(x)$ . The relative increase in revenue to slack consumed is the ratio:

$$\frac{(\gamma x p - k v_l) f(x)}{(p - v_l) x f(x)} = \frac{\gamma}{p - v_l} \left( p - \frac{k v_l}{\gamma x} \right)$$

which is increasing in  $x$ . This implies that, for  $x \leq \hat{x}$ , if the price is increased above  $v_l$ , then it should be increased for the highest demand state with the price still at  $v_l$ . Furthermore, because the marginal increase in revenue to consumed slack equals  $\gamma$  for all price increases above  $\hat{p}(x)$  (case 1) and the initial price increase from  $v_l$  to  $\hat{p}(x)$  is costly, if a price is increased above  $\hat{p}(x)$ , then it should be increased all the way to  $v_h$ . This leads to a pricing policy in which the firm charges either  $v_l$  or  $v_h$ , and the search constraint is binding. In particular,

$$p(x) = \begin{cases} v_l & x \leq x' \\ v_h & \text{otherwise} \end{cases}$$

where  $x' \leq \tilde{x}$ . The above can be implemented as a discount-frequently policy. Therefore, for a given  $\gamma$ , discount-frequently maximizes revenue, which implies it is an optimal policy.  $\square$

*Proof of Theorem 3.* (1) Conditional on observing  $\gamma$ , the firm's revenue is maximized by discounting to  $v_l$

when realized high-type demand is less than  $(v_l k)/(v_h \gamma)$ , otherwise the firm charges the high price  $v_h$ . If the firm follows this policy and all high-type consumers are myopic (i.e., they surely visit the firm), then the firm's profit is  $\Pi_m = v_l k + v_h \left( S(k) - S\left(\frac{v_l k}{v_h}\right) \right) - c_k k$ . Differentiate  $\Pi_m$  with respect to  $k$ :  $\eta(k) = d\Pi_m/dk = v_l F\left(\frac{v_l k}{v_h}\right) + v_h \bar{F}(k) - c_k$ .  $\Pi_m$  is quasi-concave because  $\eta(0) = v_h - c_k > 0$ ,  $\lim_{k \rightarrow \infty} \eta(k) = v_l - c_k < 0$  and  $\eta(k)$  is decreasing when  $\eta(k) = 0$  (which implies there is a unique  $\hat{k}$  such that  $\eta(k) = 0$ ). To demonstrate the latter, note that there is a unique solution to  $\eta(k)/v_h \bar{F}(k) = 0$  because  $\eta(k)/v_h \bar{F}(k)$  is increasing in  $k$ :

$$\eta(k)/v_h \bar{F}(k) = \frac{v_l F\left(\frac{v_l k}{v_h}\right) - c_k}{v_h \bar{F}(k)} + 1. \quad (5)$$

(2) Differentiate the profit function,  $\Pi_0 = R_0 - c_k k$  with respect to  $k$ :

$$\frac{d\Pi_0}{dk} = \begin{cases} v_l + \frac{v_l}{k} \left( S\left(\frac{v_h \hat{k}}{v_l}\right) - S(\hat{k}) \right) - c_k & k < \frac{v_h \hat{k}}{v_l} \\ v_l F\left(\frac{v_l k}{v_h}\right) + v_h \bar{F}(k) - c_k & k \geq \frac{v_h \hat{k}}{v_l} \end{cases}$$

When  $k < \frac{v_h \hat{k}}{v_l}$ ,  $\Pi_0(k)$  is linear and increasing if  $c_k < v_l + \frac{v_l}{k} \left( S\left(\frac{v_h \hat{k}}{v_l}\right) - S(\hat{k}) \right)$ . If  $k \geq \frac{v_h \hat{k}}{v_l}$ ,  $\Pi_0(k)$  is quasi-concave with a unique solution given by  $k_m$  (see part (1)). Therefore,  $k_0 = \max\left\{v_h \hat{k}/v_l, k_m\right\}$ , if  $c_k < c_k^0$  and 0 otherwise. (3) If  $v_h \bar{F}(k/\gamma_s) \leq v_l$ , the firm charges  $v_l$  and will choose  $k_s = 0$  because  $c_k > 0$ . Assume that  $v_h \bar{F}(k/\gamma_s) > v_l$ , so the firm charges  $p_s > v_l$ . Differentiate the profit function,  $\Pi_s = R_s - c_k k$  with respect to  $k$ :

$$\frac{d\Pi_s}{dk} = \begin{cases} v_h \bar{F}\left(\frac{k}{\gamma_s}\right) - c_k & k < \tilde{k} \\ v_h \bar{F}(k) - c_k & k \geq \tilde{k} \end{cases}$$

Observe that for  $k < \tilde{k}$  the function is linear and increasing if  $c_k < v_h \bar{F}\left(\frac{k}{\gamma_s}\right)$ . If  $k \geq \tilde{k}$ ,  $d\Pi_s(k)/dk$  is decreasing and hence  $\Pi_s(k)$  is concave, which guarantees a solution exists, is unique and is given by  $\bar{F}(k') = c_k/v_h$  if  $k' > \tilde{k}$  and by  $\tilde{k}$  otherwise. (4) Differentiate the profit function  $\Pi_f = R_f - c_k$  with respect to  $k$ :

$$\frac{d\Pi_f}{dk} = \begin{cases} v_l + (v_h - v_l) \bar{F}\left(\hat{k}\right) - c_k & k \leq \hat{k} \\ v_l F\left(\hat{k}\right) + v_h \bar{F}(k) - c_k & \hat{k} < k \leq \hat{k} v_h / v_l \\ v_l F\left(\frac{v_l k}{v_h}\right) + v_h \bar{F}(k) - c_k & \hat{k} v_h / v_l < k \end{cases} \quad (6)$$

From the  $k < \hat{k}$  part, it follows that  $k_f > 0$  if and only if  $c_k < c_k^f$ .  $d\Pi_f/dk$  is continuous and decreasing (strictly decreasing for  $k \geq \hat{k}$ ). This implies that there exists a unique capacity level  $k$  that maximizes profits. Solving  $v_l F\left(\hat{k}\right) + v_h \bar{F}(k) - c_k = 0$ , we get the first candidate  $k''$ .  $k'' \geq \hat{k}$  if  $c_k < c_k^f$  and comparing it with  $k_m$ , we get the desired result.  $\square$

*Proof of Theorem 4.* (1) Proof of  $k_s < k_m$ : Let  $\tilde{\tau}_s = v_h \bar{F}(\tilde{k})$ ,  $\tau_s(k) = v_h \bar{F}(k)$ , and  $\tau_m(k) = v_l F\left(\frac{v_l k}{v_h}\right) + v_h \bar{F}(k)$ . If  $\tilde{\tau}_s \leq c_k$ , then  $k_m > k_s = 0$ . Suppose now that  $\tilde{\tau}_s > c_k$ . Since  $\tau_s(k) < \tau_m(k) \forall k$  and  $\tau_s'(k) < 0$  and  $\tau_m'(k) < 0$ , it must be that  $k_s < k_m$ . Note that  $k_s = F^{-1}(c_k/v_h) < \tilde{k}$  since  $\tilde{\tau}_s > c_k$ . Proof of  $k_m \leq k_f$ : This immediately follows from Theorem 3. If  $k_m \geq k''$ , then  $k_f = k_m$ . Otherwise,  $k_f > k_m$ . (2) If  $c_k = c_k$ , then  $k_m = v_h \hat{k}/v_l = k_0 = k'' = k_f$ . Thus, for  $c_k \leq c_k$  we have  $k_m = k_0 = k_f$ . For  $c_k > c_k$ ,  $k'' > k_m$  and  $k''$  is decreasing in  $c_k$ . Thus,  $k_f = k'' < v_h \hat{k}/v_l = k_0$ .  $\square$