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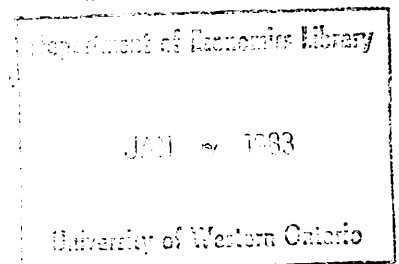
PRICE DISPERSION, PRICE FLEXIBILITY, AND  
CONSUMER SEARCH\*

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ABSTRACT

This paper presents a search model in which consumers purchase a commodity repeatedly. To economize on search costs, buyers and sellers rationally develop implicit contracts. Even though sellers can freely enter the industry, in equilibrium some sellers earn positive profits while others earn zero profit. The repetition of purchases generates a kink in the demand function facing each seller. The model provides a novel source of price dispersion: the multiplicity of equilibria inherent in the kinked demand model. Despite the kink, prices are not sticky, in that some, and possibly all, sellers change their prices whenever there is a change in costs.

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## 1. Introduction

In the standard model of consumer search, consumers are modelled as making only a single purchase of the commodity whose price is uncertain. In fact, many commodities are purchased repeatedly; for such commodities, information acquired in the past can be expected to affect current search behavior (as Stigler (1961) pointed out).

This paper presents a model of consumer search with repeated purchases. In addition to extending the range of economic situations to which search theory can be applied, there are three reasons for studying such a model. The model shows that, with imperfect information, free entry of firms may not drive all firms' profits to zero in equilibrium; it explains persistent price dispersion as being due to implicit contracts rationally developed between buyers and sellers; and it gives insights into the relationship between uncertainty and price stickiness.

Adam Smith stated that one of the necessary conditions for perfect competition is that economic agents be well informed of the opportunities available to them: "This equality [of remuneration] can take place only in those employments which are well known, and have been long established in the neighbourhood." (Smith (1976, p. 128). Note Smith's hint about the significance of the repetition of transactions.) Buyers' ignorance gives some market power to sellers. This paper addresses the question of how well informed buyers must be for some or all sellers to charge the competitive price in equilibrium, as well as the question of how costly price information must be for all sellers to be able to charge the monopoly price. It is commonly claimed that free entry of firms into the industry ensures that all firms earn zero profits. This model produces the unusual

result that it is possible in equilibrium for some sellers to earn strictly positive profits while other sellers earn zero profits; this situation persists even though entry is free.

If the standard model of search is extended to include sellers' decisions then the dispersion of prices, which is the very reason for search, often collapses (Rothschild, 1973). This raises the question of what minimal additions must be made to the standard search model in order to generate a price dispersion which persists in equilibrium. The answers to this question can be put into four categories. One source of price dispersion is differences among the sellers' cost functions; a second is the use by the sellers of mixed strategies, varying prices over time; a third is the use by buyers of non-sequential search rules; and a fourth is differences among buyers' search costs. In those models where price dispersion is generated by differences among the buyers (see Salop and Stiglitz (1977), Braverman (1980), and von zur Muehlen (1980)), it is assumed that the sellers operate under increasing returns to scale, or at least operate only on the negatively-sloped portion of a U-shaped average-cost curve. This ensures that the sellers charging low prices simultaneously have high sales and low average costs and, therefore, that all sellers earn equal (zero) profit in the free-entry equilibrium. In these models, there cannot be price dispersion if all sellers are identical or if there are non-increasing returns to scale in production when buyers search sequentially (Burdett and Judd, 1979; Carlson and McAfee, 1983). In contrast, the model of this paper shows that this conclusion is overturned if purchases are made repeatedly rather than just once: the repetition of purchases, combined with differences among buyers' search costs, can generate equilibrium price dispersion amongst identical sellers producing under constant returns

to scale.

Consider a firm selling a repeatedly-purchased commodity. The firm knows that its current customers must have reservation prices at least as high as its current price, otherwise they would have purchased elsewhere. Suppose the firm makes the Nash assumption that the other stores will maintain their current prices and considers the effects of changing its own price. If the firm were to raise its price, it would sell less to the customers who continue to purchase from it; moreover, it would lose those customers whose reservation prices are lower than the firm's new price. If the firm were to lower its price, it would sell more to its current customers; but it would not gain any new customers because none of the other firms' customers have any incentive to search. Thus there is an asymmetry between price cuts and price rises; each seller's perceived demand function is kinked at the current price. Negishi (1979), Stiglitz (1979) and Okun (1981, Ch. 4) presented this argument (in no more detail than just given) and concluded that, because the kink in the demand function gives rise to a discontinuity in the marginal-revenue function, the firm's price will remain constant in the face of small fluctuations in costs. Thus, they argued, search costs provide a rationale for the assumption of price stickiness, which is fundamental to much of macroeconomic theory: prices will be sticky in markets with repeated purchasing and imperfect information.<sup>1</sup> Each of the sellers in the present model does have a kink in his demand function that is partly induced by search costs. This paper shows, however, that it is a non sequitur to go from the demonstration that the demand function is kinked to the assertion that prices are sticky. Because of the particular way in which search costs generate the kink, some sellers will change their prices whenever there is a change in costs.

If price dispersion exists, the low-priced sellers will have perfectly flexible prices; higher-priced sellers will exhibit some degree of price stickiness. If there is no price dispersion, prices will be completely flexible.

## 2. Search with Repeated Purchases

This paper modifies the standard search model by having consumers make purchase decisions repeatedly.

The model depicts a single market in which there exists a continuum of sellers, each with the same constant average cost of production,  $c$ , and selling a homogeneous commodity. Potential sellers have free entry to the market. The assumption of a continuum of sellers formalizes the idea that there are so many firms that any one firm is an insignificant part of the total industry. Thus one firm's changing its price does not measurably alter the distribution of prices. In each period, buyers can search sequentially amongst the sellers before deciding whether and where they wish to purchase the commodity. Potential buyers all share the same tastes but may have different constant marginal costs of search (perhaps reflecting differences in their opportunity costs of time). To avoid the complications introduced by the income effects usually present in price search (see Manning and Morgan (1982) for a description) it is assumed that the buyers' marginal utility of income is constant with respect to income and the price of the searched-for commodity. While this is restrictive, it can be justified as an approximation if expenditure on the particular commodity is a small fraction of the buyers' budgets; this is likely to be the case for the repeatedly purchased commodities considered in this model. Additionally, one objective of this paper is to discover a minimal set of requirements for equilibrium price dispersion when firms have

identical constant returns to scale technologies, conditions under which other models have predicted no price dispersion in equilibrium. Relaxing the assumption of constant marginal utility of income provides another possible source of equilibrium price dispersion. The commodity is non-durable, so that consumers cannot store it from one period to the next.

In this model, it is necessary to specify explicitly the nature of the search costs. Repeated purchasing offers buyers the opportunity to economize on the costs of locating trading partners as well as the costs of learning about prices. (Diamond (1982) emphasized the costs of locating sellers.) Transportation costs due to travelling to sellers cannot be economized on by repeated purchasing and are excluded from the search costs considered here.

Buyers' preferences are represented by the indirect utility function  $V(p, p_0, y)$  where  $p$  denotes the price of the searched-for commodity,  $p_0$  is a known vector of other commodities' prices and  $y$  is disposable income.  $V(\cdot)$  is assumed to be differentiable with respect to  $y$  and  $p$  so that, by Roy's Identity, each buyer's demand function for the commodity is  $q(p) = -\partial V / \partial p$ , where the marginal utility of income is normalized to be unity. Unlike many search models,  $q(p)$  is not restricted to being own-price inelastic.

Denote by  $p_m$  the industry monopoly price (that is, the minimum of the price at which industry marginal profit equals zero and the lowest price at which any buyer would refuse to purchase the commodity).

Both buyers and sellers have complete knowledge of the cumulative density function (c.d.f.)  $F(p)$  of prices offered by sellers and of  $G(k)$ , the c.d.f. of marginal search costs over buyers.  $F$  and  $G$  may be absolutely continuous, discrete, or mixed distribution functions. Denote the supports of  $F$  and  $G$  by  $\bar{F}$  and  $\Gamma$  respectively. Denote by  $p_l$  the infimum of  $\bar{F}$ , that is,

the lowest price charged.<sup>2</sup>

Buyers use their information about  $F$  and  $G$  to compute their personal reservation prices, and sellers use this information to judge if their individual expected profits in the next period can be improved by altering their selling prices. Consider each of these decisions in turn.

Buyers search sequentially within each period. Temporarily, suppose each buyer believes that each seller will continue to charge his current price in each future period. (Later in this section it will be shown that, in equilibrium, this is a rational expectation.) This, and complete knowledge of  $F(p)$ , gives the expected utility maximizing strategy of a buyer the form of a reservation price strategy; that is, there is a reservation price  $p^r$  such that the buyer should continue to search until a selling price  $p \leq p^r$  is discovered. Once  $p$  is discovered, the buyer should accept  $p$  and stop searching (see McCall (1970, p. 116) for more details). Denote the c.d.f. of reservation prices over buyers by  $H$ , with support  $\mathcal{K}$ . The reservation price  $p_i^r$  for a buyer with a constant marginal search cost  $k_i$  is defined by

$$k_i = \frac{d_i}{1-d_i} \int_{p_\ell}^{p_i^r} [V(x, \cdot) - V(p_i^r, \cdot)] dF(x) . \quad (1)$$

The integral is Riemann-Stieltjes; it shows the expected single-period benefit of one more search given that  $p_i^r$  is the lowest price found so far. The buyer's discount rate is  $d_i$ ; thus the right-hand side of (1) shows the expected present value of searching once more given the buyer's belief that prices are constant over time. (Here it is assumed that the time horizon is infinite; if the time horizon is finite, the right-hand side of (1) changes but nothing else in the model is affected.) Equation (1) implicitly defines the inverse reservation-price function  $\theta: \mathcal{K} \rightarrow \Gamma$  such that  $k_i = \theta(p_i^r)$ .



It follows that  $H(p^r) \equiv G(\theta(p^r))$ . It is shown in the appendix that  $\theta$  is continuous and strictly monotone increasing.

Since sellers know  $F$  and  $G$ , they can deduce  $H(p^r)$  from (1). Consider the typical seller's use of this information in an equilibrium state (supposing that such a state exists). His price in the current period is  $p'$ . Those buyers who have accepted his current price  $p'$  must each possess reservation prices at least as great as  $p'$ . Temporarily, suppose each seller believes that each buyer begins his next period search by sampling the seller he patronized in the current period. (Again, later in this section this will be shown to be a redundant restriction.) This induces an asymmetry in the seller's assessments of the effects of price increases and price decreases on his next period's expected profits.<sup>3</sup>

First, consider the seller's assessment, in this period, of the effects of setting his next-period price at  $p'' < p'$ . He knows this action will retain his current buyers since, when they return and sample him first, they will discover a price  $p'' < p' \leq p^r$  for each of these buyers. Consequently they will accept his price  $p''$ . However, our Nash seller does not believe he will attract any new buyers by lowering his price to  $p''$ . This is because he believes that every other seller will continue to charge their current prices next period as well. He also knows that changing only his price results in no measurable change in  $F$ . This leads him to conclude that other sellers' current buyers will return to them at the start of their next period searches, be offered the same acceptable prices and search no further. Consequently, our seller believes he can dislodge no other seller's buyers by lowering his own price. The next period profit levels achievable from not increasing his price from  $p'$  are, therefore, in his opinion proportional to

$$E \pi (p | p', \lambda) = \lambda(p-c)q(p) \quad \text{for } p \leq p' \quad (2)$$

where  $\lambda$  is the fraction of the total number of buyers who are customers of the particular seller this period.

Now consider the seller's assessment, in this period, of the effects of setting his next period price at  $p'' > p'$ . There is now a chance that, when his current buyers begin their next period searches with him, some will find the new selling price  $p''$  is greater than their reservation prices  $p_1^r$ . These buyers will reject his offer of  $p''$  and pass on to sample other sellers. How many customers will a price increase drive away? Denote by  $f$  the generalized probability density function (g.p.d.f.) of the c.d.f.  $F$  (that is, a probability function which may contain both discrete and absolutely continuous parts). Similarly, denote  $h$  by  $g$  and the g.p.d.f.'s of the c.d.f.'s  $G$  and  $H$ . Denote the supremum of  $\mathcal{K}$  by  $\sup \mathcal{K}$ . Let

$$T(p | p') = \begin{cases} 0 & ; \text{ for } p < p' \\ \int_{p'}^p \frac{h(x)}{F(x)} dx & ; \text{ for } p' \leq p \leq \sup \mathcal{K} \\ \int_{p'}^{\infty} \frac{h(x)}{F(x)} dx & \\ 1 & ; \text{ for } p > \sup \mathcal{K} \end{cases} \quad (3)$$

The derivation of  $T(p | p')$  is explained in the appendix.  $T(p | p')$  is the c.d.f. of the reservation prices of the buyers currently patronizing our seller. He therefore knows there is a probability of  $T(p | p')$  that he will drive away any one of his current buyers by increasing his price to  $p$  in the next period. The expected next-period profit levels achievable by increasing his price are proportional to

$$E \pi (p | p', \lambda) = \lambda(p-c)q(p)(1-T(p | p')) \quad , \quad \text{for } p' < p \leq \sup \mathcal{K} \quad (4)$$

Together (2) and (4) define the seller's next-period expected-profit function.

Let  $p'$  denote the solution (assuming it exists) to the risk-neutral seller's problem of choosing price  $p$  to maximize  $E\pi(p|p', \lambda)$ . Comparing (2) and (4) shows that the current price  $p'$  maximizes expected profit if

$$(p-c)q(p) \leq (p'-c)q(p') \quad \text{for } p \leq p' \quad (5)$$

and

$$(p-c)q(p)(1-T(p|p')) \leq (p'-c)q(p') \quad \text{for } p > p' \quad (6)$$

Condition (5) requires  $p' \leq p_m$ , the monopoly price. Condition (6) requires  $T(p|p')$  to increase sufficiently rapidly to at least offset the increments to profits that would be achieved by increasing  $p$  towards  $p'$  if buyers had no alternative sellers.

At some points (in Sections 5 and 6) for the sake of simplicity it will be assumed that the c.d.f.  $G$  is differentiable for  $k > k_\ell$ . From the continuity and strict monotonicity of the function  $\theta$ , it follows that  $H$  and therefore  $T$  are then also differentiable. Inequalities (5) and (6) can then, in this special case, be rewritten as showing the marginal effect on the firm's profit of a small increase in price:

$$(p'-c) \frac{\partial q(p')}{\partial p} + q(p') \geq 0 \quad (7)$$

and

$$[(p'-c) \frac{\partial q(p')}{\partial p} + q(p')][1-T(p'|p')] - (p'-c)q(p')t(p'|p') \leq 0. \quad (8)$$

Inequality (7), like (5), says that  $p'$  is no greater than the monopoly price. In (8)  $t(p|p')$  is the right-hand derivative of  $T(p|p')$ , the probability density function of reservation prices of customers of a seller charging the price  $p'$ ; let  $t(p'|p') \equiv \lim_{p \rightarrow p'} t(p|p')$ . From (A8) (see the appendix),  $T(p'|p') = 0$  since the distributions are continuous. Thus (7) and (8) say that the seller will continue to charge the price  $p'$  only if  $p' \leq p_m$  and

$$t(p'|p') \geq \frac{(p'-c) \frac{\partial q(p')}{\partial p} + q(p')}{(p'-c)q(p')} \quad (9)$$

This inequality says that the rate at which the seller expects customers to leave whenever the price is raised slightly above  $p'$  is large enough to outweigh the extra profits gained from the customers who remain with the seller. Note also, from the appendix, that

$$t(p' | p') = \frac{h(p')/F(p')}{\int_{p'}^{\infty} [h(x)/F(x)] dx} . \quad (10)$$

Hence  $t(p' | p') > 0$  if and only if  $h(p') > 0$ . The price  $p'$  is charged by some firm only if it is some buyer's reservation price or it is the monopoly price.

Now reconsider the restriction that buyers begin their next-period searches by sampling their current sellers first. At an equilibrium, this is a redundant restriction. Buyers know how sellers make pricing decisions, and sellers know buyers know this. Moreover, all agents know when the market is at a time-invariant equilibrium. At an equilibrium, therefore, buyers know that each seller has set a price which, for that seller, satisfies (5) and (6). Consequently, no seller will choose to change his price so each buyer knows, costlessly, that if he returns to his current seller in the next period he will, with certainty, be offered the same price as before.<sup>4</sup> This price is no greater than his reservation price, so each buyer's expected-utility-maximizing search strategy is to revisit his current seller first in the next period and accept the unchanged price. Thus in equilibrium, there is a particular asymmetry of information. Each buyer knows, without incurring any further search costs, the price of one seller; moreover, the price that he knows is acceptable to him in that it is no greater than his reservation price.<sup>5</sup>

### 3. No Buyers Fully Informed

In this section we show that if each buyer's search cost is strictly bounded above zero (that is, there exists  $\epsilon > 0$  such that  $g(k) = 0$  for every  $k \in [0, \epsilon]$ ) then, in the only equilibrium state, all firms charge the monopoly price, even though firms have free entry.

Suppose there exists a buyer with marginal search cost  $k_\ell$  (where  $k_\ell$  denotes the infimum of  $\Gamma$ ). Since  $k_\ell \geq \epsilon > 0$ , it follows from (1) that there exists  $\delta > 0$  such that this buyer's reservation price  $p_i^r \geq p_\ell + \delta$ ; that is,  $h(p^r) = 0$  for every  $p^r \in [0, p_\ell + \delta)$ . This implies (from (3))  $T(p|p') = 0$  for  $p_\ell < p' \leq p < p_\ell + \delta$  which, in turn, implies (6) cannot hold unless  $p' = p_m$ ; the seller will increase his profit by increasing his price unless his current price  $p'$  is the monopoly price.

What has been shown is that if all searchers have strictly positive search costs, there cannot be price dispersion in equilibrium; at the only equilibrium, all firms charge the monopoly price. (A similar result was obtained for the single-purchase case by Burdett and Judd (1979).) The reason is straightforward. If there were a price dispersion, the lowest-price firm, knowing that all its customers face a positive cost of finding an alternative seller, knows it can raise its price by some amount and not lose any customers. Thus there cannot be price dispersion in equilibrium. Suppose there were a single-price equilibrium at a price less than the monopoly price. Then, once again, each seller knows that its customers face a positive cost of finding an alternative seller. Even though the customers know with probability one the price they would find elsewhere, the fact that they must incur a positive cost to find an alternative seller means that they would be prepared to pay a slightly higher price to their current seller. Thus the seller can raise his price slightly without losing

any customers; it will be in his interest to do so whenever the current price is less than the monopoly price.

This equilibrium and the equilibria to be described in Sections 4 and 5 are not destroyed by the entry of new sellers, even though some of the existing sellers make positive profit in equilibrium. Any potential entrant knows that, since there is a continuum of sellers, its entering the market does not measurably alter the distribution of prices. The probability of any buyer finding a new entrant is zero. Thus, since each buyer has found a seller whose price is no greater than his reservation price, each buyer rationally decides not to search and so an entrant attracts no customers.

#### 4. Some Buyers Fully Informed I

This section and the next describe equilibrium when some buyers have zero marginal search costs.<sup>6</sup> It is convenient to consider two cases separately, where each case is distinguished from the other by the nature of the g.p.d.f.  $g(k)$  of marginal search costs for values of  $k$  close to zero.

Consider how each firm's profit function is changed from (2) and (4) by the presence of buyers with zero marginal search costs. Buyers with zero marginal search costs will search until they locate a seller offering the lowest price  $p_\ell$ . This adds a Bertrand-type discontinuity at  $p_\ell$  to each seller's expected-profit function (Bertrand, 1883, p. 503). To see why, note that any seller whose current price is  $p' > p_\ell$  will not be patronized by any of the zero-search-cost buyers. If his current price is  $p' = p_\ell$ , however, then the seller will obtain some fraction,  $\alpha > 0$ , of the zero-search-cost buyers. This gives his expected-profit function the form

$$E \pi (p | p', \lambda) = \begin{cases} \lambda (p-c)q(p) (1-T(p | p')) & ; \text{ for } p > p' \\ \lambda (p-c)q(p) & ; \text{ for } p_\ell < p \leq p' \\ (\lambda + \alpha G(0)) (p-c)q(p) & ; \text{ for } p \leq p_\ell \end{cases} \quad (11)$$

(The question of how  $\alpha$  is determined, that is, how customers are allocated across firms charging prices equal to  $p_\ell$ , need not concern us.)

Consider the case in which  $G(0) > 0$ , that is, there is a mass of zero-marginal-search-cost buyers. If  $G(0) = 1$ , that is, all buyers have zero search costs, then, by the standard Bertrand argument, in equilibrium all sellers charge the perfectly competitive price. There is no price-dispersed equilibrium. Suppose, instead, that  $0 < G(0) < 1$ ; that is, a set of buyers with positive probability measure have zero marginal search costs, but some buyers have positive marginal search costs. In this case all equilibria exhibit price dispersion. To see why, note that in equilibrium the lowest price  $p_\ell$  cannot be greater than the marginal production cost  $c$ ; if  $p_\ell > c$  then, since  $G(0) > 0$ , the expected profit of any entrant charging  $p'$  such that  $c < p' < p_\ell$  is strictly positive. Either new entrants or existing firms would engage in a Bertrand-type rivalry, forcing  $p_\ell$  down to marginal cost  $c$ . However, in equilibrium some firms must offer prices greater than  $c$ . To see this, suppose all sellers charge a price equal to marginal cost,  $c$ . Each firm is then selling to a typical sample of buyers. With  $p' = c$ , (6) becomes

$$(p-c)q(p)(1-T(p|c)) \leq 0 \quad , \text{ for } p > c \quad . \quad (12)$$

Inequality (12) implies  $T(p|c) = 1$ , which requires all buyers to have reservation prices equal to  $c$ ; that is, all buyers must have zero search costs. This contradicts  $G(0) < 1$ , so some sellers must charge prices strictly greater than  $c$  in equilibrium. The reason for this is that, with all sellers charging the competitive price, if one seller raises his price, he will keep some of

his positive-search-cost customers (who will not find it worthwhile to look for another seller if the price increase is small enough), and he will lose all of his zero-search-cost customers. From the former customers, the seller now extracts positive profit; he loses no profit from the loss of the latter customers because, at price equal to marginal cost, profit was zero. Thus the price rise increases the seller's profit; the original situation (all sellers charging a price equal to marginal cost) must not have been an equilibrium.

The above remarks establish that the equilibrium will be one of price dispersion if  $0 < G(0) < 1$ . The lowest price charged is  $c$ . A fraction  $G(0)$  of the buyers will purchase from the competitively priced sellers. In equilibrium, all other customers will purchase from sellers with price strictly greater than marginal cost. (Since if a competitive-priced seller had some positive-search-cost customers then, knowing the local elasticity of demand, he would know that he could increase his profits by raising his price above marginal cost.)

The range of prices above marginal cost has some degree of indeterminacy. The conditions for equilibrium (conditions (5) and (6)) imply that each firm's marginal-expected-profit function is discontinuous: multiple equilibria can occur.

##### 5. Some Buyers Fully Informed II

Two cases have been considered: the case in which there is a positive lower bound on buyers' search costs, and the case in which a significant fraction of buyers have zero search costs ( $G(0) > 0$ ). For the sake of completeness, we now consider the intermediate case, in which  $G(0) = 0$  but  $G(k) > 0$  for every  $k > 0$ . The set of buyers with zero marginal search costs has zero probability measure but there is a positive density  $g(k) > 0$  for  $k$



arbitrarily close to zero. This disciplines sellers, for any price increase from the current value must generate a strictly positive probability of losing some buyers. (Compare this to the case discussed in Section 3 where no such discipline exists and joint monopoly is the only equilibrium.) However, this discipline is ineffective for any seller currently charging the marginal-cost price,  $c$ . The equilibrium condition (6) cannot be satisfied if  $p' = c$  for this would require

$$(p-c)q(p)(1-T(p|c)) \leq 0 \text{ for } p > c . \quad (13)$$

However,  $G(k) > 0$  for  $k > 0$  implies  $H(p) \equiv G(\theta(p)) > 0$  for  $p > p_\ell \geq c$ .  $H(p) > 0$  for  $p > c$  implies  $h(p) > 0$  for some  $p > c$  which, from (3), requires  $T(p|c) > 0$  for  $p > c$ . This ensures that (13) does not hold. Any seller currently setting a price of  $c$  would increase his price. In this case, therefore, marginal-cost pricing cannot be part of any equilibrium.

Further examination of this case is simplified by considering now the special case in which the functions  $G$ ,  $H$ , and  $T$  are differentiable. By the continuity and strict monotonicity of the function  $\theta$ ,  $g(k) > 0$  for all  $k > 0$ ,  $k \in \Gamma$  implies  $h(p') > 0$  for all  $p' > p_\ell$ ,  $p' \in \mathcal{K}$ . From (10), this implies  $t(p'|p') > 0$  for all  $p' > p_\ell$ . Interpret the monopoly price  $p_m$  as the price at which industry marginal profit is zero (rather than the price at which buyers leave the market). As  $p'$  increases towards  $p_m$ , the numerator of the right-hand side of (9) (marginal profit) approaches zero. Hence, with  $t(p'|p') > 0$ , it is possible for (9) to be satisfied at prices strictly less than the monopoly price; (9) is of course also satisfied at the monopoly price.

Like the case analyzed in Section 3, therefore, the system is in equilibrium when all sellers charge the monopoly price. Unlike the case of Section 3, however, this is not the only equilibrium. Other single-price

equilibria, as well as price-dispersed equilibria, are possible, provided no firm charges a price less than the lowest price which satisfies (9).

(This price lies strictly between marginal cost and the monopoly price.)

Unlike the case analyzed in Section 4, no firm's price is equal to marginal cost in equilibrium. The equilibria of this intermediate case thus lie between the equilibrium of the case of Section 3 and the equilibria of the case of Section 4.

## 6. Price Flexibility

Negishi (1979), Stiglitz (1979), and Okun (1981) argued that kinked-demand curves of the sort modelled here generate price stickiness. The results of this paper, in contrast, show that the kinked nature of the demand function does not necessarily imply that prices are inflexible.

Consider first the case in which all buyers' search costs are strictly positive, so that at equilibrium all sellers charge the monopoly price. This price will change whenever there is a change in costs or market demand.

Suppose, on the other hand, a significant fraction of buyers have zero marginal search costs, so that the equilibrium has dispersed prices. Consider the effect of a change in the sellers' marginal cost function. Those sellers charging a price equal to marginal cost will change their price in response to any change in costs. Only the higher-priced sellers will possibly have inflexible prices: for such sellers, the marginal-cost function lies within the discontinuity in the marginal-expected-revenue function. Even some of these sellers, however, might change their prices when costs change.

First, if there are enough buyers with sufficiently high search costs, then some stores might charge the monopoly price in equilibrium. If the monopoly price is the price at which industry marginal profit is zero (rather than the price at which buyers leave the market), then sellers charging the monopoly price will lower their price whenever their marginal cost falls. They will not necessarily raise their price when marginal cost increases; sellers charging the monopoly price may exhibit upward price stickiness but not downward price stickiness.

Second, any equilibrium selling-price distribution has a gap between the lowest price (equal to marginal cost) and the second-lowest price. To see this, reconsider the sellers' profit function. Make the special assumption for simplicity that the distributions  $F$  and  $H$  are differentiable (so that any price charged must satisfy (7) and (9)). Suppose the current price  $p'$  is strictly greater than marginal cost. There are two possibilities. First, (9) may not be satisfied for any price between marginal cost and the monopoly price. In this case  $t(p' | p')$  is too small to dissuade the sellers from raising their prices provided  $c < p' < p_m$ . In equilibrium there will be two prices charged: zero-search-cost buyers will buy at price equal to marginal cost, and positive-search-cost buyers will pay the monopoly price. These prices will be completely flexible in the face of cost changes. Second, suppose (9) is satisfied for some  $p'$ ,  $c < p' < p_m$ . The right-hand side of (9) is monotonic decreasing in  $p'$ , from infinity when  $p' = c$  to zero when  $p'$  is such that marginal profit (that is, the numerator) is zero. Given the continuity of  $F$  and  $G$ , and given that, with a price dispersion, there must be a significant fraction of sellers charging the price  $c$  so that  $F(p') > 0$  for  $p' > c$ , the left-hand side of (7) is positive and finite for  $p' > c$  in the support of  $F$ . Thus, as  $p'$  approaches  $c$ , the inequality in (9) will eventually be reversed. Define  $p_s$  to be the lowest price for which (9) holds as an equation;  $p_s > c$ . Then no seller charging a price greater than marginal cost will charge a price less than  $p_s$ ; there is a gap in the selling-price distribution between  $p = c$  and  $p = p_s$ . (Although, given the multiplicity of equilibria,  $p_s$  itself need not be charged.) From (9),  $p_s$  changes as  $c$  changes. If there is a change in marginal cost, then sellers charging  $p_s$  and slightly above  $p_s$  may change their prices in response to a cost change.

Consider now the effects on prices of a change in demand. Suppose each buyer's demand curve shifts. Sellers charging price equal to marginal cost will not change their prices. The monopoly price  $p_m$  and the second-lowest price  $p_s$  will change. If there are sellers charging either of these prices, they may change their prices. Thus, in contrast to the price response to cost changes, the lowest-priced sellers will exhibit price inflexibility while the higher-priced sellers will exhibit partial price flexibility in the face of demand changes.

Sellers with prices strictly between  $p_s$  and  $p_m$  will not change their prices in response to every small change in either costs or demand. To see this, rewrite (7) and (8) as

$$\frac{p' \frac{\partial q(p')}{\partial p} + q(p')}{\frac{\partial q(p')}{\partial p}} \leq c \leq \frac{p' \frac{\partial q(p')}{\partial p} + q(p') - p' q(p') t(p' | p')}{\frac{\partial q(p')}{\partial p} - q(p') t(p' | p')} \quad (14)$$

Condition (14) says that marginal cost lies between the bounds of the discontinuity in marginal revenue. An upper bound on how large a cost or demand change must be before it generates a price change is the size of this discontinuity. The difference between the upper bound and the lower bound in (14) is

$$D = \frac{(p')^2 t(p' | p')}{\eta^2 - \eta p' t(p' | p')} , \quad (15)$$

where  $\eta$  denotes the price elasticity of demand ( $\eta < 0$ ). Partial differentiation shows that  $\partial D / \partial t > 0$ ,  $\partial D / \partial \eta > 0$ , and  $\partial D / \partial p' > 0$ . That is, with  $\eta$  and  $p'$  held constant,  $D$  increases as  $t(p' | p')$  increases; the seller's price stickiness increases with the number of marginal customers he has. With  $t(p' | p')$  and  $p'$  held constant,  $D$  increases with  $\eta$ ; price stickiness increases as demand becomes

less elastic. With  $t(p' | p')$  and  $\eta$  held constant,  $D$  increases with  $p'$ ; for prices strictly less than the monopoly price, price stickiness increases with price.

## 7. Discussion of the Results

The equilibria described in Sections 3, 4 and 5 should be thought of as stationary states, each reached as the endpoint of some disequilibrium adjustment process. The formal modelling of this adjustment process will be the subject of future research. Disequilibrium will require both sellers and buyers to make decisions on the basis of imperfect estimates of the distribution of selling prices, and then to update these estimates each period after receiving new information; moreover, the true distribution will change unpredictably over time.<sup>7</sup>

The result that the initially identical firms earn different equilibrium profits, some positive and some zero, contradicts the usual presumption that competition drives profits to zero. There are, however, two interpretations of the result which may be consistent with the zero-profit presumption. First, in the disequilibrium adjustments not modelled here, there would be competition for customers, with each seller seeking to establish implicit contracts with high-search-cost customers. Suppose it costs more (in advertising costs, for example) to attract higher-search-cost customers than to attract lower-search-cost customers. Then the positive profits in equilibrium can be interpreted as returns to investments in building customer relationships; it is possible that the present value of each firm's profits is zero when the transition to equilibrium is taken into account. Second, even if there are positive profits generated by a seller's stock of customers, they will be capitalized in the value of the firm. Only the first owner of the firm earns a rent; if the firm is ever sold, subsequent owners earn zero profit. (In valuing a firm, accountants count the firm's "goodwill", one component of which is customer relationships.)

The model investigated the properties of a time-invariant equilibrium. The assumption that there are many firms is crucial to the existence of a time-invariant equilibrium, as the model of Rosenthal (1982) suggests. In Rosenthal's model, two sellers compete to sell to buyers who purchase the commodity repeatedly. Buyers purchase from the same seller as in the last period unless that seller has raised his price, in which case his former customers costlessly observe the other seller's price and then purchase from the cheaper of the two sellers. (Unlike the present model's buyers, Rosenthal's buyers do not optimize.) As long as the two sellers charge different prices, they adopt a randomized strategy, with the high-priced seller randomly lowering his price each period and the low-priced seller randomly raising his price. Either this situation persists forever, or eventually the system attains a steady state at which either one seller has captured the entire market or both sellers charge the same price. Thus the equilibrium of an oligopolistic repeated-purchase market is different from the equilibrium of the market with many sellers analyzed in this paper.

The sources of information in this model are restrictive: buyers can learn about prices only by searching. In real world markets, there are many sources of price information, such as word of mouth or advertising. The qualitative conclusions of the model seem, however, to be robust to the addition of new sources of information. The crucial element is the asymmetric effect of price cuts and price rises due to customers being able to economize on search costs by remaining with one seller. As long as the new information sources do not remove this asymmetry, the conclusions of the model will remain. For example, suppose each period a few buyers search even if their sellers did not raise their prices. This adds a new

source of demand elasticity, but it is symmetric: if a seller raises his price he gains fewer of these searchers; if he lowers his price he gains more of them. The asymmetry caused by the reactions of the seller's own customers is still present.

Adding to the model the possibility that new entrants advertise would eliminate the inability of entrants to attract customers in equilibrium. The model suggests, however, that entry is a much more difficult and costly process than is usually believed, because an entrant has to break the ongoing relationships which have rationally been established between buyers and sellers. (The model thus provides a rationale for the assumption, commonly made in the literature on entry deterrence, that incumbent firms have a cost advantage over potential entrants.) An equilibrium in a model in which entrants can advertise would not be an equal-profit equilibrium, but would be like the equilibrium described above, with some firms earning positive profits. Firms would enter until the marginal profits from customers gained equalled the marginal cost of advertising; with a positive marginal cost of advertising, this process would stop before all positive profits were eliminated.

Brand names serve as a source of information. The above analysis assumes the commodity is homogeneous, but it can easily be reinterpreted as a model of quality search if the quality of the commodity can be measured on a single-dimensional scale. Define commodities by end use: for example, the relevant commodity is not one liter of detergent but a certain number of dishes washed; not one razor blade but a certain number of shaves. The price of such a commodity is not learnt simply by observing price tags: price per unit depends on quality when quantity units are appropriately defined. The



process of search involves sampling such commodities. The model of this paper suggests that brand names serve the socially useful purpose of economizing on consumers' search costs by facilitating repeated (habitual) purchasing: in equilibrium, buyers know the quality of a particular brand without having again to incur search costs. The model thus rationalizes brand loyalty.

#### 8. Conclusion

Three questions were posed in the introduction.

The first question was about the meaning of perfect competition in a world of costly information. In the model of this paper, if all buyers have strictly positive search costs, no matter how small, the only equilibrium has all sellers charging the monopoly price. All sellers charge the perfectly competitive price if and only if all buyers have zero search costs and therefore are always perfectly informed. The informational requirements for perfect competition are thus very stringent in this model. The model moreover has the unusual result that free entry of firms into the industry does not ensure profits are driven down to zero; instead, if there is dispersion, some of the identical sellers earn positive profit while others earn zero profit in equilibrium. This occurs because buyers rationally decide not to search further after finding an acceptable price; thus new entrants can attract no customers once equilibrium is reached, even though some buyers are paying prices above marginal cost.

The second question posed was about the minimal conditions for equilibrium price dispersion. There is price dispersion in this model with repeated purchasing and constant-returns-to-scale production if a significant fraction of the buyers have strictly positive search costs and a significant

fraction have zero search costs. In equilibrium, only buyers with zero search costs buy from stores with price equal to marginal cost. Unlike in the model of Salop and Stiglitz (1977), for example, there are no buyers for whom search is costly who by chance buy from the lowest price sellers. The zero-search-cost buyers do, however, convey an externality on the positive-search-cost buyers in that, if they did not exist, then all stores would charge the monopoly price. The source of the price dispersion described in this paper is new to the literature: it is the multiplicity of equilibrium prices inherent in the kinked-demand model.

The third question was about price inflexibility. The model shows that the existence of the kink in the demand function does not result in prices being inflexible in the face of small cost or demand fluctuations. This inflexibility usually arises in models with kinked demand because, with the marginal revenue function discontinuous, marginal cost lies between the bounds of marginal revenue. In the present model, however, in equilibrium some or all sellers charge prices such that marginal cost equals one of the extreme values of marginal revenue; these prices must change whenever marginal cost changes. In particular, if there is no price dispersion and all sellers charge the monopoly price, then prices are completely flexible. If there is a significant fraction of buyers with zero search costs, so that there is price dispersion, then a significant number of consumers buy at a price equal to marginal cost. In this case the low-priced sellers exhibit complete price flexibility in response to price changes. Since there are multiple equilibria for the higher-priced sellers (for them, marginal cost lies strictly between the bounds of marginal revenue), these sellers' prices may be inflexible in the face of small cost changes.

APPENDIX

In this Appendix the properties of the functions  $\theta$  and  $T$  are derived.

Proof that  $\theta$  is a continuous and strictly increasing function of  $p^r$ :

$$\theta(p^r) = \frac{1}{1-d} \int_{p_\ell}^{p^r} [V(x, \cdot) - V(p^r, \cdot)] dF(x)$$

To show  $\theta$  is strictly increasing with respect to  $p^r$ , consider any  $\epsilon > 0$ .

$$\theta(p^r + \epsilon) = \int_{p_\ell}^{p^r} (V(x, \cdot) - V(p^r + \epsilon, \cdot)) dF(x) + \lim_{p \rightarrow p^r+} \int_{p}^{p^r + \epsilon} (V(x, \cdot) - V(p^r + \epsilon, \cdot)) dF(x) \quad (A1)$$

$V(p^r + \epsilon, \cdot) < V(p^r, \cdot)$  for all  $p^r + \epsilon \leq \bar{p}$ , so

$$\theta(p^r + \epsilon) > \theta(p^r) + \lim_{p \rightarrow p^r+} \int_{p}^{p^r + \epsilon} (V(x, \cdot) - V(p^r + \epsilon, \cdot)) dF(x) > \theta(p^r). \quad (A2)$$

This establishes that  $\theta$  is a strictly increasing function of  $p^r$ .

If  $F$  contains no mass points,  $\theta$  is clearly continuous. To establish

the continuity of  $\theta$  when  $F$  has a mass point at  $p = p^r$ , consider

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \theta(p^r - \epsilon) &= \lim_{\epsilon \rightarrow 0} \int_{p_\ell}^{p^r - \epsilon} (V(x, \cdot) - V(p^r - \epsilon, \cdot)) dF(x) \\ &= \lim_{\epsilon \rightarrow 0} \int_{p_\ell}^{p^r} (V(x, \cdot) - V(p^r - \epsilon, \cdot)) dF(x) - \lim_{\epsilon \rightarrow 0} \int_{p^r - \epsilon}^{p^r} (V(x, \cdot) - V(p^r - \epsilon, \cdot)) dF(x) \\ &= \theta(p^r) - (V(p^r, \cdot) - V(p^r, \cdot)) \lim_{\epsilon \rightarrow 0} (F(p^r) - F(p^r - \epsilon)) \\ &= \theta(p^r) \end{aligned} \quad (A3)$$

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \theta(p^r + \epsilon) &= \lim_{\epsilon \rightarrow 0} \int_{p_\ell}^{p^r} (V(x, \cdot) - V(p^r + \epsilon, \cdot)) dF(x) + \lim_{\epsilon \rightarrow 0} \lim_{p \rightarrow p^r+} \int_{p}^{p^r + \epsilon} (V(x, \cdot) - V(p^r + \epsilon, \cdot)) dF(x) \\ &= \theta(p^r) \end{aligned} \quad (A4)$$

Hence  $\theta$  is continuous at  $p = p^r$ .

Derivation of  $T(p|p')$ :

The derivation proceeds by first discovering the probability  $A(p'|p^r)$  that a buyer with a reservation price  $p^r$  buys at a price no larger than  $p'$ .

$$\begin{aligned}
 A(p'|p^r) &= \sum_{j=1}^{\infty} \Pr(\{p_j \leq p'\} \cap \{\text{stop on the } j^{\text{th}} \text{ observation}\}) \\
 &= \Pr(\{p_1 \leq p'\} \cap \{p_1 \leq p^r\}) \\
 &\quad + \sum_{j=2}^{\infty} \Pr(\{p_j \leq p'\} \cap \{p_j \leq p^r\} \cap (\bigcap_{i=1}^{j-1} \{p_i > p^r\})) \\
 &= F(\min\{p', p^r\}) + \sum_{j=2}^{\infty} F(\min\{p', p^r\})(1 - F(p^r))^{j-1} \\
 &= \frac{F(\min\{p', p^r\})}{F(p^r)}. \tag{A5}
 \end{aligned}$$

The gpdf of  $A(p'|p^r)$  is

$$a(p'|p^r) = \frac{f(\min\{p', p^r\})}{F(p^r)} \tag{A6}$$

The gpdf of  $T(p^r|p')$  is, by Bayes's Theorem,

$$t(p^r|p') = \frac{a(p'|p^r)h(p^r)}{\int_p^{\infty} a(p'|p^r)h(p^r)dp^r} \tag{A7}$$

$$\begin{aligned}
 \text{so } T(p^r|p') &= \int_{p'}^{p^r} \frac{\frac{f(\min\{p', x\})}{F(x)} h(x)}{\int_{p'}^{\infty} \frac{f(\min\{p', x\})}{F(x)} h(x) dx} \\
 &= \int_{p'}^{p^r} \frac{\frac{h(x)}{F(x)} dx}{\int_{p'}^{\infty} \frac{h(x)}{F(x)} dx} \tag{A8}
 \end{aligned}$$

REFERENCES

- Bertrand, J., Review of Recherches sur les Principes Mathématiques de la Théorie des Richesses by Augustin Cournot. Journal des Savants (September 1883): 499-508.
- Braverman, Avishay, "Consumer Search and Alternative Market Equilibria." Review of Economic Studies 47 (April 1980): 487-502.
- Burdett, Kenneth and Kenneth Judd, "Equilibrium Price Dispersion." Social Systems Research Institute, University of Wisconsin, Madison (July 1979).
- Carlson, John A. and R. Preston McAfee, "Discrete Equilibrium Price Dispersion." Journal of Political Economy, to appear.
- Diamond, Peter A., "Aggregate Demand Management in Search Equilibrium." Journal of Political Economy 90 (October 1982): 881-894.
- Green, Edward J., "Noncooperative Price Taking in Large Dynamic Markets." Journal of Economic Theory 22 (April 1980): 155-182.
- Kawasaki, Seiichi, John McMillan, and Klaus F. Zimmermann, "Disequilibrium Dynamics: An Empirical Study." American Economic Review 72 (December 1982): 992-1004.
- Kohn, M. G. and S. Shavell, "The Theory of Search." Journal of Economic Theory 9 (October 1974): 93-123.
- Manning, R., and P. Morgan, "Search and Consumer Theory." Review of Economic Studies 49 (April 1982): 203-216.
- McCall, J., "Economics of Information and Job Search." Quarterly Journal of Economics 84 (February 1970): 113-126.
- Negishi, T., Microeconomic Foundations of Keynesian Macroeconomics, Amsterdam: North-Holland, 1979.

- Okun, Arthur M., Prices and Quantities: A Macroeconomic Analysis, Washington, D.C.: Brookings, 1981.
- Rosenthal, Robert W., "A Dynamic Model of Duopoly with Customer Loyalties," Journal of Economic Theory 27 (June 1982): 69-76.
- Rothschild, M., "Models of Market Organization with Imperfect Information: A Survey." Journal of Political Economy 81 (November/December 1974): 1283-1308.
- Rothschild, M., "Searching for the Lowest Price when the Distribution of Prices in Unknown." Journal of Political Economy 82 (July/August 1974): 689-711.
- Salop, Steven and Joseph Stiglitz, "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion." Review of Economic Studies (October 1977): 493-510.
- Smith, Adam, An Inquiry into the Nature and Causes of the Wealth of Nations, edited by Edwin Cannan (Chicago: University of Chicago Press, 1976).
- Stigler, George J., "The Economics of Information." Journal of Political Economy 69 (June 1961): 213-225.
- Stiglitz, J. E., "Equilibrium in Product Markets with Imperfect Information." American Economic Review: Papers and Proceedings 69 (May 1979): 339-345.
- Von Zur Muehlen, Peter, "Monopolistic Competition and Sequential Search." Journal of Economic Dynamics and Control 2 (August 1980): 257-281.

FOOTNOTES

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<sup>1</sup>For empirical evidence on the varying degrees of price flexibility across industries, see Kawasaki, McMillan, and Zimmermann (1982).

<sup>2</sup>With a continuum of sellers, the set of prices charged may be an open set, so that  $p_\ell$ , the infimum of  $\mathfrak{P}$ , may not actually be charged by any seller. This would not affect any of the results of the paper.

<sup>3</sup>In the model, each seller maximizes only current profits. Since the model describes only stationary states, this is equivalent to long-run profit maximization. Because of the smallness of the sellers, there is no possibility for any seller to use a reaction-function strategy which would make its price depend upon another seller's price in previous periods: compare with Green (1980).

<sup>4</sup>Stigler (1961) discussed the effects of varying the correlation of a particular seller's prices in successive time periods. The foregoing argument shows that, in the present model's equilibrium, this correlation will be perfect.

<sup>5</sup>It might be asked why a buyer, having found a seller with a price higher than average, returns next period rather than searching again. The answer is that searching incurs costs, while returning to the same seller does not. If search costs are high enough, it will pay to return even to a high-price seller. Search costs are to be interpreted as, for example, the costs of locating sellers, and not the transportation costs of going to a store.

<sup>6</sup>The assumption that some buyers have a marginal search cost of zero might be interpreted as meaning that their search activity is literally costless. An alternative interpretation is that the market contains some intermediate agents (not modelled) whose function is to search exhaustively and sell complete price information to buyers for a fee. If this fee is less than a buyer's expected search cost, this buyer will purchase the information and act like a zero-marginal-search-cost buyer.

<sup>7</sup>A partial step towards solving the buyers' side of this disequilibrium problem is provided by the analyses of search behavior with an unknown price distribution of Kohn and Shavell (1974) and Rothschild (1974).