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The Case of Butter**

By

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Price Dynamics in a Vertical Sector: The Case of Butter

by

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and

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Abstract: We develop a reduced-form model of price transmission in a vertical sector, allowing for refined asymmetric, contemporaneous and lagged, own and cross price effects. The model is used to analyze wholesale-retail price dynamics in the US butter market. The analysis provides strong evidence of asymmetric price transmissions. It documents the complex nature of nonlinear price dynamics in a vertical sector and its implications for the distribution of future prices. It finds evidence that the asymmetric response to shocks is stronger in the short run for retail prices, and in the longer run for wholesale prices.

Key Words: price transmission, asymmetry, nonlinear dynamics, butter.

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Price Dynamics in a Vertical Sector: The Case of Butter

1. Introduction

The issue of price transmission in a vertical sector has been the subject of much research. A common issue is that retail prices do not respond very quickly to changes in market conditions. Under fluctuating market conditions, this raises questions about the efficiency of vertical markets. Examples include situations where retail prices remain “sticky” in the face of large decreases in farm or wholesale prices (e.g., Borenstein et al.; Peltzman; Miller and Hayenga). Peltzman finds strong evidence that in many markets retail prices tend to rise faster than they fall, both in the short term and in the longer term.

This has stimulated research on the possible cause of asymmetric price adjustments. Two potential explanations have been explored: imperfect competition and adjustment costs. A traditional explanation under oligopoly is a kinked-demand schedule that generates sticky prices. More generally, barriers to entry can create asymmetric economic adjustments (see Tirole for an overview). Many other sources of asymmetry have been explored. In general, in the presence of adjustment cost, firms and consumers may not respond to small or transitory price changes until the benefits of changing strategies outweigh the cost. Consider, for example, the unequal cost of maintaining high versus low inventory, where the higher cost of experiencing a stockout can generate asymmetric price adjustments (e.g., Reagan and Weitzman). Also, consumers may not respond quickly to price changes in the presence of search costs. This can allow retailers to boost profits by increasing their prices fast as wholesale prices rise, and lowering them slowly when wholesale prices fall. In addition, menu costs can prevent firms from changing prices rapidly in response to small and transitory market changes (e.g., Blinder;

Blinder et al.). Finally, sunk investment costs can create irreversibility in firms' strategies (e.g., Dixit and Pindyck). Thus, there are many reasons why price transmission may be asymmetric in a vertical sector. Peltzman's analysis suggests that current theories fail to explain the prevalence of price asymmetry. His empirical evidence covering many markets shows no correlation between price asymmetry and inventory cost, menu cost or imperfect competition. This raises significant challenges to our theory of markets. It also stresses the need for a better understanding of the empirical regularities found in price transmissions.

The objective of this paper is to develop a dynamic reduced form model of asymmetric price transmission in a vertical sector. The analysis expands on previous models of dynamic price transmission by allowing asymmetry for both contemporaneous and lagged, own and cross price effects. The model is applied to wholesale-retail price dynamics in the US butter market. As illustrated in Figure 1, butter prices have exhibited large fluctuations over the last 10 years. This makes the butter market an interesting case study of dynamic price adjustments in a vertical sector. Following Peltzman, in the absence of a clear theory of asymmetric price adjustments, the analysis is unrepentantly descriptive. The empirical results provide strong evidence of asymmetric price transmissions in the US butter market. They also document the complex nature of nonlinear price dynamics in a vertical sector. They show how asymmetric price responses affect the distribution of future prices. By stressing the effects on skewness of the price distribution, they point out the limitations of previous models of price dynamics that relied solely on autocovariance (or spectral density in the frequency domain, as done by Miller and Hayenga). One of the main findings is that the asymmetry in responses to

shocks is more pronounced in the short run for retail prices, and in the longer run for wholesale prices.

2. A Model of Price Dynamics

Consider a vertical sector involving m markets in a vertical sector. Let $\mathbf{y}_t = (y_{1t}, y_{2t}, \dots, y_{mt})'$ be an $(m \times 1)$ vector of market prices at time t . Assume that the price vector \mathbf{y}_t has a dynamic reduced-form representation given by the vector autoregression (VAR) model¹

$$\mathbf{y}_t = \boldsymbol{\alpha} + \sum_{k=1}^K \mathbf{A}_k \mathbf{y}_{t-k} + \mathbf{e}_t, \quad (1)$$

where $\boldsymbol{\alpha}$ is an $(m \times 1)$ vector, \mathbf{A}_k is an $(m \times m)$ matrix, $k = 1, \dots, K$, and \mathbf{e}_t is an $(m \times 1)$ error term independently and normally distributed with mean zero and variance $\boldsymbol{\Omega}$. This can be alternatively written in terms of the error-correction model (ECM)

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha} + \mathbf{B}_0 \mathbf{y}_{t-1} + \sum_{k=1}^{K-1} \mathbf{B}_k \Delta \mathbf{y}_{t-k} + \mathbf{e}_t, \quad (2)$$

where $\Delta \mathbf{y}_t = \mathbf{y}_t - \mathbf{y}_{t-1}$, $\mathbf{B}_0 = -[\mathbf{I}_K - \mathbf{A}_1 - \mathbf{A}_2 - \dots - \mathbf{A}_K]$, and $\mathbf{B}_k = -[\mathbf{A}_{k+1} + \mathbf{A}_{k+2} + \dots + \mathbf{A}_K]$, $k = 1, 2, \dots, K-1$.

Equation (2) means that $\Delta \mathbf{y}_t$ is stationary if and only if $[\mathbf{B}_0 \mathbf{y}_{t-1} + \sum_{k=1}^{K-1} \mathbf{B}_k \Delta \mathbf{y}_{t-k}]$ is stationary. Obviously, \mathbf{y}_t being stationary is sufficient for $\Delta \mathbf{y}_t$ to be stationary. In addition, if \mathbf{y}_t is not stationary (e.g., in the presence of units roots), then a stationary $\Delta \mathbf{y}_t$ implies that $[\mathbf{B}_0 \mathbf{y}_{t-1}]$ must be stationary. Such a process is cointegrated, and \mathbf{B}_0 identifies stationary linear combinations of the non-stationary variables $(y_{1t}, \dots, y_{mt})'$. In this case, the matrix \mathbf{B}_0 is singular and can be written as $\mathbf{B}_0 \equiv \boldsymbol{\beta} \boldsymbol{\gamma}$, where $\boldsymbol{\beta}$ is an $(m \times c)$ matrix, $\boldsymbol{\gamma}$ is a $(c \times m)$ matrix of c cointegration vectors, with $c = \text{rank}(\mathbf{B}_0)$. In the error-correction model (2), the vector $\mathbf{z}_t \equiv [\boldsymbol{\gamma} \mathbf{y}_{t-1}]$ is stationary, reflecting long-term relationships among prices,

and $\mathbf{B}_0 \mathbf{y}_{t-1} \equiv \boldsymbol{\beta} \mathbf{z}_t$ (see Hamilton, p. 580). The general specification includes as a special case the situation where $\mathbf{B}_0 \equiv -[\mathbf{I}_K - \mathbf{A}_1 - \mathbf{A}_2 - \dots - \mathbf{A}_K] = \mathbf{0}$ and (2) implies that price dynamics can be properly analyzed using a VAR in differences. However, when $\text{rank}(\mathbf{B}_0) \geq 1$, equation (2) shows that a VAR in differences is an inappropriate representation of price dynamics.

The linear specification (1) or (2) can be extended in a number of directions. First, the intercept $\boldsymbol{\alpha}$ can change over time in at least two ways: 1/ it can have a time trend (reflecting inflation, technical progress, or other long term changes); and 2/ it can involve seasonal effects. This corresponds to $\boldsymbol{\alpha} = \mathbf{a}_0 + \mathbf{a}_1 t + \sum_{s=1}^{S-1} \boldsymbol{\alpha}_s D_{ts}$, where D_{ts} is a dummy variable for the s -th season: D_{ts} equals 1 if t is in the s -th season and zero otherwise, $s = 1, \dots, S$. Then, $(\mathbf{a}_0 + \mathbf{a}_1 t)$ is the intercept at time t in the S -th season, and \mathbf{a}_1 measures the change in intercept between two successive periods.

Second, we consider the case where the dynamics in (1) or (2) vary between regimes. For simplicity we focus on the case of binary regimes denoted by the dummy variables R . Let $R_{it} = 1$ if y_{it} is in regime 1 at time t , and $R_{it} = 0$ if y_{it} is in regime 0 at time t , $i = 1, \dots, m$. In equation (2), let $\mathbf{B}_k =$

$$\begin{bmatrix} \mathbf{B}_{k11}^1 R_{1,t-k} + \mathbf{B}_{k11}^0 (1 - R_{1,t-k}) & \dots & \mathbf{B}_{k1m}^1 R_{m,t-k} + \mathbf{B}_{k1m}^0 (1 - R_{m,t-k}) \\ \vdots & \ddots & \vdots \\ \mathbf{B}_{kml}^1 R_{1,t-k} + \mathbf{B}_{kml}^0 (1 - R_{1,t-k}) & \dots & \mathbf{B}_{kmm}^1 R_{mt} + \mathbf{B}_{kmm}^0 (1 - R_{m,t-k}) \end{bmatrix}, k = 1, \dots, K-1. \text{ This}$$

means that the impact of $\Delta y_{j,t-k}$ on Δy_{it} varies across regimes as $\partial \Delta y_{it} / \partial \Delta y_{j,t-k} = \mathbf{B}_{kij}^1 R_{j,t-k} + \mathbf{B}_{kij}^0 (1 - R_{j,t-k})$, which equals \mathbf{B}_{kij}^1 when $y_{j,t-k}$ is in regime 1 but \mathbf{B}_{kij}^0 when in regime 0. As a result, at time t , equation (2) becomes²

$$\begin{aligned} \Delta y_{it} = & \alpha_{i0} + \alpha_{i1} t + \sum_{s=1}^{S-1} \alpha_{is} D_{ts} + \sum_{j=1}^m B_{0ij} y_{j,t-1} \\ & + \sum_{k=1}^{K-1} \sum_{j=1}^m [B_{kij}^1 R_{j,t-k} + B_{kij}^0 (1-R_{j,t-k})B_k] \Delta y_{t-k} + \epsilon_{it}, \end{aligned} \quad (3)$$

$i = 1, \dots, m$. Equation (3) provides a framework to investigate whether price dynamics vary across regimes. Indeed, prices would exhibit the same dynamics under both regimes if $B_{kij}^1 = B_{kij}^0$ for all (k, i, j) . Alternatively, finding that $B_{kij}^1 \neq B_{kij}^0$ for some (k, j, i) would be sufficient to conclude that price dynamics vary across regimes.³

Next, consider the Cholesky decomposition of the variance of ϵ_t : $\Omega \equiv S S'$, where

$$S = \begin{bmatrix} s_{11} & 0 & \cdots & 0 \\ s_{21} & s_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mm} \end{bmatrix} \text{ is a lower triangular matrix satisfying } s_{ii} > 0, i = 1, \dots, m. \text{ It}$$

means that equation (2) can be alternatively written as

$$S^{-1} \Delta y_t = S^{-1} \alpha + S^{-1} B_0 y_{t-1} + \sum_{k=1}^{K-1} S^{-1} B_k \Delta y_{t-k} + \epsilon_t, \quad (2')$$

where $\epsilon_t = S^{-1} \epsilon_t$ is normally distributed with mean zero and variance I_m . Note that the off-diagonal elements of S capture the contemporaneous effects across dependent variables.

For example, the covariance between y_{1t} and y_{2t} is $\text{Cov}(y_{1t}, y_{2t}) = s_{11} s_{21}$, and the

contemporaneous impact of a shock in y_{2t} on y_{1t} is $\partial y_{1t} / \partial y_{2t} = s_{21} / s_{11}$. Also, the

contemporaneous cross-price effects vanish if $s_{ij} = 0$ for all $i > j$. Thus, the presence of

contemporaneous cross-price effects can be confirmed by rejection of the null hypothesis:

$s_{ij} = 0$ for all $i > j$. In addition, if we are interested in exploring whether such

contemporaneous effects are situation-specific, we can consider the more general

specification: $s_{ij} = \sigma_{ij0} + \sigma_{ij} z_t$, where z_t is a vector of predetermined variables at time t , $i \geq$

j . In this context, constant contemporaneous effects across dependent variables implies

that $\sigma_{ij} = 0$ for all $i > j$. Alternatively, finding that $\sigma_{ij} \neq 0$ for some $i > j$ would be

sufficient to conclude that some contemporaneous cross-price effects vary over time. Econometrically, this corresponds to a situation of heteroscedasticity where the covariance matrix $\mathbf{\Omega} \equiv \mathbf{S S}'$ is time-varying. This provides a framework to analyze how contemporaneous cross-price effects vary with market conditions.

In summary, the model exhibits three types of price transmission: contemporaneous cross price effects (captured by the specification for s_{ij}); lagged effects (captured by \mathbf{B}_k , $k = 1, \dots, K$); and long term effects (captured by \mathbf{B}_0). The model is novel in the flexibility with which it captures these different dynamic price relationships.

As discussed in the introduction, much recent research has focused on whether price dynamics respond symmetrically to price increases versus price decreases. The first area of flexibility, then, corresponds to $R_{it} = 1$ if $\Delta y_{it} > 0$ and $R_{it} = 0$ if $\Delta y_{it} \leq 0$. In this context, equation (3) extends previous specifications of asymmetric price response found in the literature.⁴ The \mathbf{B}_k^0 's and \mathbf{B}_k^1 's capture asymmetric response to price shocks after k lags, $k = 1, \dots, K$. This extends Wolfram's specification, which restricts the \mathbf{B}_k^i 's to be the same for all k . By allowing the \mathbf{B}_k^i 's to vary, equation (3) allows for dynamic asymmetry to vary between the short run and the intermediate run (e.g., as investigated by Peltzman). Second, under cointegration, $[\mathbf{B}_0 y_{t-1}]$ is the "error correction term" which captures deviations from long-term relationships among prices. While equation (3) reduces to the Miller-Hayenga specification when $\mathbf{B}_0 = 0$, the Miller-Hayenga specification of asymmetric price response becomes inappropriate when $\mathbf{B}_0 \neq 0$ (e.g., under cointegration). Third, the specification $s_{ij} = \sigma_{ij0} + \sigma_{ij} z_t$ expands on both the Miller-Hayenga and the Peltzman specifications. It allows for situation-specific contemporaneous cross-price effects. The Miller-Hayenga specification implicitly assumes constant s_{ij} 's, thus restricting contemporaneous cross-price effects to be

symmetric and constant (with $\sigma_{ij} = 0$ for all $i \geq j$). The Peltzman specification (Peltzman's equation (2) on p. 476) corresponds to equation (2') above with $y_1 =$ "output price" and $y_2 =$ "input price". It allows for asymmetric contemporaneous effects from "input price" to "output price", but implicitly assumes symmetric and constant contemporaneous effects from "output price" to "input price". The specification $s_{ij} = \sigma_{ij0} + \sigma_{ij} z_t$ is more flexible and allows for more complex contemporaneous cross-price effects (see below).

Finally, as suggested by equations (1) and (2), one must choose between estimating the model "in levels" (equation (1)) or "in differences" (equation (2)). Both approaches can generate consistent parameter estimates. Below, we focus on the specification "in differences" for two reasons. First, the estimation of models "in differences" can perform better in small samples (Hamilton, p. 652). Second, hypothesis testing is easier "in differences" as test statistics exhibit more standard distributions (e.g., the case of Granger causality; see Toda and Phillips). Thus, the analysis presented below focuses on the estimation of equation (3). Equation (3) can be estimated by maximum likelihood, which under a correct specification generates consistent and asymptotically efficient parameter estimates.

3. Application to the US Butter Sector

We apply model (3) to price dynamics in the vertical sector for US butter. The analysis focuses on the dynamics of two prices ($m = 2$): the wholesale and retail prices of butter. The analysis uses monthly data from the period January 1980 to August 2001. The wholesale price is the Chicago Mercantile Exchange AA butter cash price, and the retail price for butter is from the Bureau of Labor Statistics.

First, some diagnostic tests were conducted on each price series. The augmented Dickey-Fuller (ADF) test for a unit root was implemented for each price separately. This was done based on a model with 5 lags in price differences (as suggested by the Schwartz criterion). ADF testing of the null of a unit root yielded t-values of -0.635 for retail prices and -1.10 for wholesale prices. At the 5 percent significance level, the ADF critical value is -3.43. Thus, we failed to reject the null hypotheses of unit roots. This provides evidence that both prices are non-stationary.

Next, we investigated the nature of price dynamics in the butter market. For this purpose, we relied on the specification given in equation (3). For the i -th price at time $t-k$, we defined two market regimes: $R_{i,t-k} = 0$ (regime 0) when $\Delta y_{i,t-k} \leq 0$, and $R_{i,t-k} = 1$ (regime 1) when $\Delta y_{i,t-k} > 0$. This provided a framework to investigate whether price dynamics differ for price increases versus price decreases, including both own price and cross price effects. In addition, we wanted to analyze whether contemporaneous price relationships change with market conditions. With $m = 2$, let $y_1 \equiv y_r$ represent the retail price, and $y_2 \equiv y_w$ represent the wholesale price. We allow the covariance between y_{rt} and y_{wt} to vary with market conditions and consider the specification $s_{21} = \sigma_0 + \sigma_r E_t(\Delta y_{rt}) + \sigma_w E_t(\Delta y_{wt})$, where s_{21} is the off-diagonal element in the Cholesky decomposition of the variance of e_t .⁵ From (3), the expected price change for y_{it} is $E_t(\Delta y_{it}) = a_{i0} + a_{i1} t + \sum_{s=1}^{S-1} \alpha_{is} D_{ts} + \sum_{j=1}^m B_{0ij} y_{j,t-1} + \sum_{k=1}^{K-1} \sum_{j=1}^m [B_{kij}^1 R_{j,t-k} + B_{kij}^0 (1-R_{j,t-k})B_k] \Delta y_{t-k}$. When $\sigma_r \neq 0$ and/or $\sigma_w \neq 0$, this specification allows market conditions to affect the contemporaneous cross price effects between y_r and y_w . For example, finding that $\sigma_r > 0$ ($\sigma_w > 0$) would mean that an expected rise in retail price (wholesale price) would increase the contemporaneous covariance between retail and wholesale prices. Note that, unlike the Peltzman

specification, this allows retail market conditions to affect the contemporaneous relationships between retail and wholesale prices.

Applied to US butter prices, this model specification (3) was estimated using the maximum likelihood method. Based on the Schwartz criterion, the number of lags was chosen to be $K = 6$. The resulting econometric estimates are presented in Table 1. Many of the estimates are found to be significant. In general, the coefficients (α_{is}) of the monthly seasonal dummies D_{st} show more evidence of seasonality in wholesale prices than in retail prices. Also, the time trend effects differ: the trend coefficient a_{11} is negative and significant for wholesale price, while it is positive but insignificant for retail price. This reflects that the marketing margin ($y_r - y_w$) has increased over time during the sample period. Finally, most of the coefficients on lagged prices are significant, indicating the presence of significant dynamic adjustments in the US butter market.

The nature of the dynamic relationships between y_r and y_w was investigated. First, we implemented a Johansen cointegration test for model (3). The null hypothesis of a cointegration relation between y_r and y_w was investigated using a likelihood ratio test of the rank of the \mathbf{B}_0 matrix. Testing the null hypothesis that $\text{rank}(\mathbf{B}_0) = 0$ versus the alternative $\text{rank}(\mathbf{B}_0) = 1$, the Johansen test statistic was 94.19, which is significant at the 5 percent level. This, in conjunction with the results to the Augmented Dickey Fuller test, provides evidence that wholesale and retail butter prices are cointegrated, i.e. that they exhibit long-term relationships. On its own, it also suggests that a VAR in differences (e.g., as used by Miller and Hayenga) would be misspecified.

Second, we test for Granger causality among prices. The null hypothesis of no causality between prices y_i and y_j requires $B_{kij}^1 = 0$ and $B_{kij}^0 = 0$ for $k = 1, \dots, K-1$, and $B_{0ij} = 0$. Under some regularity conditions, the associated likelihood ratio test has a chi-

square distribution under the null hypothesis (Toda and Phillips). For $i \neq j$, the Granger causality test statistic is 114.57 for the effects of lagged retail on wholesale prices, and 132.40 for the effects of lagged wholesale on retail prices. At the 5 percent significance level and with 6 degrees of freedom, the critical value is 1.64. Thus, we strongly reject the null hypothesis of no causality and find strong evidence of lagged cross effects among butter prices. If $i = j$, the test investigates the presence of lagged own price effects. The associated test statistics are 136.53 for retail prices and 116.17 for wholesale prices. At the 5 percent significance level and with 6 degrees of freedom, we therefore strongly reject the null hypotheses of no own lagged effects. This provides evidence of significant dynamic adjustments in both wholesale and retail prices.

Third, we evaluate the symmetry of lagged price effects. In the context of equation (3), the symmetry of dynamic effects of price j on price i corresponds to the null hypothesis $B_{kij}^1 = B_{kij}^0$, $k = 1, \dots, K-1$. Using a likelihood ratio test, the associated test statistics are 66.87 for $(i, j) = (r, r)$, 28.09 for $(i, j) = (r, w)$, 96.02 for $(i, j) = (w, w)$, and 107.89 for $(i, j) = (w, r)$. Based on a chi square distribution with 5 degrees of freedom, the critical value is 1.15 at the 5 percent significance level. Thus, we strongly reject the symmetry of dynamic adjustments for all prices (i, j) . In other words, we find strong evidence that both own price and cross price dynamics exhibit asymmetric adjustments across regimes. The associated non-linearity in price dynamics will be explored in details below.

Fourth, we investigate the presence of contemporaneous effects between prices. This is captured by the Cholesky term $s_{21} = \sigma_0 + \sigma_r E_t(\Delta y_{rt}) + \sigma_w E_t(\Delta y_{wt})$. The null hypothesis that $\sigma_0 = \sigma_r = \sigma_w = 0$ implies a zero correlation between y_r and y_w and thus zero contemporaneous effects between retail and wholesale prices. A likelihood ratio test

of this hypothesis yielded a test statistic of 243.29. Based a chi square distribution with 3 degrees of freedom, we strongly reject the null hypothesis. This provides evidence of significant contemporaneous cross price effects between the two butter prices.

Fifth, we explore the nature of contemporaneous cross price effects. The estimates reported in Table 1 give $s_{21} = 0.0358 + 0.7894 E_t(\Delta y_{rt}) - 1.4368 E_t(\Delta y_{wt})$. As discussed above, the coefficients of s_{21} are jointly significant. In a long run equilibrium situation where $E_t(\Delta y_{rt}) = E_t(\Delta y_{wt}) = 0$, it follows that $s_{21} = 0.0358$, which is positive and significant. This means that y_{rt} and y_{wt} are positively correlated and that any shock in one price has a positive contemporaneous effect on the other. The coefficient on $E_t(\Delta y_{rt})$ is positive and significant, implying that an expected change in retail price has a positive effect on the covariance between y_{rt} and y_{wt} . The coefficient of $E_t(\Delta y_{wt})$ is negative and significant, showing that an expected change in wholesale price has a negative effect on the covariance between y_{rt} and y_{wt} . This provides statistical evidence that the contemporaneous effects of one price on the other are sensitive to market pressure. In particular, it shows that the contemporaneous linkages between retail and wholesale prices become weaker (stronger) when the wholesale (retail) price is expected to increase. This is another form of asymmetry between retail and wholesale butter prices.

Finally, to evaluate explanatory power, predicted prices were obtained from the estimated model and compared with actual prices during the sample period. The results are presented in Figure 1. The model has high explanatory power and provides a good fit to the butter price data, with R-squares of 0.984 for retail prices and 0.886 for wholesale prices.

4. Implications

The empirical results show strong evidence of asymmetry in price effects and dynamics in the US butter market. This asymmetry means that price dynamics are nonlinear in two ways: 1/ contemporaneous cross-price effects vary with market conditions; and 2/ price dynamics vary across regimes between situations of price increases and price decreases. These nonlinearities mean that, in general, the forward path of prices depends on initial conditions (Potter). As a result, the dynamic price response to exogenous shocks is typically situation specific. To evaluate the nature of dynamic adjustments in the US butter market, dynamic stochastic simulations of the estimated model were performed. The nonlinear dynamics imply that there is no simple way of summarizing price effects (since the results always depend on initial conditions). Below, we report some selected simulation results that illustrate the dynamic implications of the estimated model.

The stochastic simulations were performed as follows. A random number generator was used to generate pseudo-random draws for the error terms $\boldsymbol{\varepsilon}_t = (\varepsilon_{rt}, \varepsilon_{wt})'$ distributed $N(0, \mathbf{I}_2)$. For given initial conditions (say at time τ), these error terms were used to simulate forward the estimated model (3) with $\mathbf{e}_{\tau+i} = \mathbf{S}_{\tau+i} \boldsymbol{\varepsilon}_{\tau+i}$, $i = 0, 1, 2, \dots$, where $\boldsymbol{\Omega}_t \equiv \mathbf{S}_t \mathbf{S}_t'$. Repeated dynamic simulation generated a distribution of prices $\mathbf{y}_{\tau+i}$ at time $\tau+i$, $i = 0, 1, 2, \dots$. This distribution simulates the distribution of predicted prices at time $\tau+i$, based on the information available at time τ . In addition, for given pseudo-random draws for the $\boldsymbol{\varepsilon}_t$'s, the dynamic simulation can be repeated after shocking the system at time τ . Comparison of the paths of the simulated series with and without the shock provides a basis for measuring numerically the effects of the shock on the

dynamics of prices and their distribution. It measures the dynamic impulse response to the initial shock, which can shed light on the nature of price dynamics. We consider two kinds of shock: a shock in retail price at time τ , and a shock in wholesale price at time τ . The former is represented by an exogenous change in ε_{rt} , and the latter by an exogenous change in ε_{wt} .

In general, under nonlinear dynamics, the impulse response depends not just on the initial conditions, but also on the nature and magnitude of the shock (Potter). To evaluate the effects of asymmetric price adjustments, we distinguish between positive and negative shocks to prices.

The distribution of impulse responses to 40% shocks in wholesale price in December 1998 is presented in Figure 2.⁶ Figure 2 shows the evolution of the 10th, 25th, 50th, 75th, and 90th percentiles of the distribution over the 12-month period following the shock. In general, a positive (negative) shock in wholesale price has a positive (negative) impact on retail price, with effects that decay slowly over time. Figure 2 illustrates the asymmetric effects generated by a positive shock versus a negative shock. Indeed, it shows how the distribution of the impulse response can vary: compared to a negative shock, a positive wholesale shock generates greater short-term variability in wholesale price, but lower short-term variability in retail price. Also, Figure 2 suggests that the nonlinear dynamics generate a skewed distribution of retail price responses to a wholesale price shock (see below).

Similarly, Figure 3 presents the distribution of impulse response to 10% shocks in retail price on December 1998.⁷ Again, a positive (negative) shock in retail price has a positive (negative) impact on wholesale price, with effects that decay slowly over time. Here, the differences between a positive and a negative shock are not apparent: Figure 3

shows similar patterns of impulse response whether the retail price initially rises or falls. However, it does indicate the presence of skewness in the distribution of the price response. In addition, the initial shock in retail price (0.21 \$/lb) has a magnified contemporaneous impact on wholesale price (0.40 \$/lb). This large cross-price effect is due to a high s_{21} estimate generating a large covariance between y_r and y_w .

To show that the results presented in Figures 2 and 3 can be sensitive to initial conditions, we present the impulse response to a 10% retail price shock on September 1995 (see Figure 4). Figure 4 illustrates the non-stationarity of the model: a positive (negative) retail shock tends to increase (decrease) retail and wholesale prices both in the short run and in the long run. The absence of decay over time is in sharp contrast with Figure 3. Yet the only differences between Figures 3 and 4 are the initial conditions (December 1998 versus September 1995). This indicates that stationarity conditions can become “local” in nonlinear models, making price forecasts much more complex. Both Figure 3 and Figure 4 show that, in response to a retail shock, price variability tends to be larger for wholesale prices than retail prices. This reflects, in part, the fact that the variance of e_w is larger than the variance of e_r . Finally, from Figure 4, the initial shock in retail price (0.21) has a smaller short-term impact (compared to Figure 3) on wholesale price (0.15). This is because s_{21} is time varying: it is smaller in September 1995 than in December 1998.

The implications of nonlinear dynamics for the asymmetry of impulse response to positive versus negative shocks are investigated further. Table 2 reports formal testing of the null hypothesis of symmetry (the distribution of impulse responses at a point in time is symmetric for a price increase versus an equivalent price decrease). This is done using a chi-square Pearson test. The results are presented for different initial conditions (shock

date), for different shock sizes and at three time intervals (the 2nd, 6th and 12th months of the simulation). First, Table 2 makes it clear that the magnitude of the shock has a large impact on the presence of asymmetry. The evidence of asymmetry is very weak in the case of a small shock (e.g., 1% shock), but becomes strong with increases in the size of the shock. This reflects in large part the piece-wise linearity in model (3): it may take large changes to switch from one regime to another. As a result, the model can still exhibit “linear properties” locally, i.e. in the neighborhood of some path. The non-linearities become apparent only globally, when path changes are large enough to induce regime switching.

Second, the evidence of asymmetry in wholesale price response tends to be weak in the short run but become stronger in the longer run (e.g., August 96). With the exception of retail price shock in September 1995, this applies in response to either a wholesale price shock or a retail price shock (see Table 2). It suggests that the wholesale market exhibits symmetric short-term price adjustments, but asymmetric long-term price adjustments. To the extent that asymmetry is motivated by adjustment costs, this indicates the presence of significant long-term adjustment costs in the butter wholesale industry. This includes adjustment costs in investment and capital formation in butter manufacturing.

Third, in stark contrast to our results on asymmetry in wholesale responses, Table 2 shows that asymmetry in retail price responses tends to be stronger in the short run (after 2 months) but declines in the longer run (12 months). This holds in response to either a wholesale price shock or a retail price shock (see Table 2). It indicates that the retail market exhibits significant asymmetric short-term price adjustments, and that such asymmetry becomes weaker in the longer run. Also, Table 2 shows that the evidence of

asymmetry is in general stronger for retail price responses (compared to wholesale price responses). Again, to the extent that asymmetry is motivated by adjustment costs, this indicates the presence of significant short-term adjustment costs in the butter retail sector. This includes adjustment costs for consumers (e.g., search cost) as well as retailers.

Finally, we evaluate the skewness of the distribution of impulse response. Table 3 presents relative skewness obtained from the simulated effects of shocks in September 1995. It also reports tests of the null hypothesis of zero skewness (corresponding to a symmetric distribution of an impulse response around its mean). This is done using the Bera-Jarque test. The evidence against the null hypothesis is weaker when considering the effect of a positive wholesale shock on the retail price. However, the statistical evidence of skewness is rather strong in all other cases, and is found to be stronger in the longer term. The importance of skewness points out that mean-variance representations cannot provide sufficient statistics for the distribution of future prices. This shows the limitations of previous analyses of price dynamics based solely on autocovariance (or spectral density in the frequency domain, as used by Miller and Hayenga). Table 3 also shows that positive shocks tend to generate positive skewness for own price shocks and negative skewness for cross price ones (with opposite effects obtained under negative shocks). This means that an unanticipated shock in price y_{it} increases the relative probability mass in the tail of the distribution of prices $y_{it'}$ in the direction of the initial shock, for $t' > t$. And it decreases the relative probability mass in the tail of the distribution of prices $y_{jt'}$ in the direction of the shock for $j \neq i$, $t' > t$. This illustrates how non-linear dynamics and asymmetric adjustments affect the distribution of future prices in a marketing channel.

5. Concluding remarks

This paper developed a model of asymmetric price transmission in a vertical sector, allowing for refined asymmetry for both contemporaneous and lagged own and cross price effects. Applied to wholesale-retail price dynamics in the US butter market, the model provides strong evidence of asymmetric price transmissions. The asymmetry generates nonlinear dynamics in price adjustments in a vertical sector. We document the complex nature of price dynamics in the butter market. First, the effects of market shocks depend on initial conditions. For example, the impact of a change in retail price on wholesale price is found to vary significantly with market conditions (see Figures 1 and 3). Also, the evidence of asymmetry grows with the size of the shock. Second, we show how asymmetric price responses affect the distribution of prices. We find strong evidence of skewness in the response to large price shocks. For example, an unanticipated increase in wholesale price tends to create positive skewness in the distribution of future wholesale price, but negative skewness in the distribution of future retail price. This highlights the limitations of previous analyses of price dynamics that relied only on the autocovariance (or spectral density in the frequency domain). Third, for retail price, the asymmetric response is stronger in the short run but declines in the longer run. This is consistent with the presence of consumer search costs and/or menu costs facing retailers. Fourth, in contrast with retail price, the evidence of asymmetry in wholesale price response is weak in the short run but stronger in the longer run. This is consistent with the presence of sunk costs in investment and capital formation in the butter sector.

The analysis has focused on vertical price adjustments in the butter sector. It can be extended in several directions. First, it would be useful to investigate whether our empirical findings hold for other sectors. Second, there may be more complex forms of

nonlinear dynamics that are relevant in vertical price adjustments. Finally, following Peltzman, our empirical findings suggest significant challenges for improving our conceptual understanding of dynamic market adjustments. These are good topics for further research.

Table 1: Maximum likelihood estimate of the parameters

Parameter	Estimate	Std. Error	Parameter	Estimate	Std. Error
a_{w0}	-0.1562**	0.0574	a_{r0}	-0.0934**	0.0217
a_{w1}	-0.0003*	0.0002	a_{r1}	0.0001	0.0001
α_{w1}	0.0849**	0.0249	α_{r1}	0.0804**	0.0104
α_{w2}	0.0648**	0.0275	α_{r2}	0.0162	0.0128
α_{w3}	0.0952**	0.0373	α_{r3}	0.0292**	0.0115
α_{w4}	0.0571*	0.0328	α_{r4}	0.0098	0.0205
α_{w5}	0.0770**	0.0261	α_{r5}	0.0451**	0.0115
α_{w6}	0.0939**	0.0327	α_{r6}	0.0067	0.0107
α_{w7}	0.0712**	0.0288	α_{r7}	0.0016	0.0132
α_{w8}	0.0943**	0.0359	α_{r8}	0.0081	0.0110
α_{w9}	0.0760**	0.0351	α_{r9}	-0.0122	0.0111
α_{w10}	0.0527**	0.0218	α_{r10}	0.0008	0.0069
α_{w11}	0.0812**	0.0354	α_{r11}	-0.0329**	0.0101
B_{1ww}^1	0.2064**	0.0841	B_{1rw}^1	0.5834**	0.0613
B_{1ww}^0	0.2635**	0.0631	B_{1rw}^0	0.5712**	0.0495
B_{2ww}^1	0.2013**	0.0753	B_{2rw}^1	0.2708**	0.0570
B_{2ww}^0	0.0726	0.0914	B_{2rw}^0	0.3770**	0.0420
B_{3ww}^1	0.4104**	0.1135	B_{3rw}^1	0.5031**	0.0591
B_{3ww}^0	0.0342	0.0507	B_{3rw}^0	0.2338**	0.0762
B_{4ww}^1	-0.4495**	0.1914	B_{4rw}^1	0.1509**	0.0507
B_{4ww}^0	0.0850	0.0619	B_{4rw}^0	0.2230**	0.0715
B_{5ww}^1	-0.1296*	0.0742	B_{5rw}^1	0.0332	0.0645
B_{5ww}^0	0.4816**	0.1519	B_{5rw}^0	0.2997**	0.0439
B_{1wr}^1	0.0848	0.0894	B_{1rr}^1	0.0180	0.0744
B_{1wr}^0	-0.6344**	0.2052	B_{1rr}^0	-0.6602**	0.0746
B_{2wr}^1	-0.5860**	0.1989	B_{2rr}^1	-0.3567**	0.0630
B_{2wr}^0	0.3828**	0.1674	B_{2rr}^0	-0.0215	0.0807
B_{3wr}^1	0.0206	0.0960	B_{3rr}^1	-0.3580**	0.0675
B_{3wr}^0	-0.3387**	0.1367	B_{3rr}^0	-0.3502**	0.0920
B_{4wr}^1	0.2694**	0.0912	B_{4rr}^1	0.2885**	0.0855
B_{4wr}^0	-0.1458	0.0990	B_{4rr}^0	-0.4615**	0.0778
B_{5wr}^1	-0.1013	0.0777	B_{5rr}^1	-0.0574	0.0638
B_{5wr}^0	-0.1119	0.0961	B_{5rr}^0	0.1480*	0.0892
B_{0ww}	-0.1049*	0.0593	B_{0rw}	0.0429	0.0335
B_{0wr}	0.1253**	0.0547	B_{0rr}	0.0037	0.0251
s_{11}	0.0458**	0.0028	σ_0	0.0344*	0.0180
s_{22}	0.0501**	0.0030	σ_w	-2.4823**	0.9699
			σ_r	0.8692**	0.3920

Log Likelihood = 824.5952. Number of Observations = 254.

** means |t|-value greater than 2; * means |t|-value greater than 1.6

Table 2: Testing the symmetry of impulse price response to a price increase versus a price decrease

P-Values for the Null Hypothesis that Wholesale Shocks Produce Symmetric Wholesale Price Responses						
Shock Date	Time	1% shock	5% shock	10% shock	20% shock	40% shock
Jan-82	2	0.9935	0.4231	0.0003	0.0000	0.0000
	6	0.9267	0.5930	0.1704	0.0022	0.0002
	12	0.9912	0.2861	0.0007	0.0000	0.0000
Sep-95	2	0.9998	0.9966	0.9688	0.8281	0.3330
	6	0.9979	0.9843	0.6468	0.0021	0.0001
	12	0.9968	0.6265	0.0463	0.0000	0.0000
Aug-96	2	1.0000	1.0000	1.0000	1.0000	0.9980
	6	0.9997	0.9965	0.9432	0.9392	0.7191
	12	0.9989	0.8897	0.8629	0.7049	0.2581
Dec-98	2	0.9960	0.8172	0.4611	0.0001	0.0000
	6	0.9981	0.9715	0.8441	0.0946	0.0028
	12	0.9808	0.1176	0.0002	0.0000	0.0000
P-Values for the Null Hypothesis that Wholesale Shocks Produce Symmetric Retail Price Responses						
Shock Date	Time	1% shock	5% shock	10% shock	20% shock	40% shock
Jan-82	2	0.9398	0.0025	0.0000	0.0000	0.0000
	6	0.9975	0.7555	0.0154	0.0000	0.0000
	12	0.5994	0.9613	0.2638	0.0000	0.0000
Sep-95	2	0.9682	0.1342	0.0001	0.0000	0.0000
	6	0.8702	0.0150	0.0000	0.0000	0.0000
	12	0.9854	0.5873	0.0351	0.0000	0.0000
Aug-96	2	0.9996	0.4583	0.0362	0.0000	0.0000
	6	0.9995	0.9426	0.7110	0.1203	0.0000
	12	0.9939	0.8864	0.2188	0.0854	0.0087
Dec-98	2	0.9602	0.0060	0.0000	0.0000	0.0000
	6	0.9276	0.0008	0.0000	0.0000	0.0000
	12	0.9477	0.1138	0.0003	0.0000	0.0000

P-Values for the Null Hypothesis that Retail Shocks Produce Symmetric Wholesale Price Responses				
Shock Date	Time	1% shock	5% shock	10% shock
Jan-82	2	1.0000	1.0000	1.0000
	6	0.6376	0.0003	0.0000
	12	0.5114	0.7442	0.0520
Sep-95	2	1.0000	1.0000	1.0000
	6	0.9988	0.7743	0.2477
	12	0.9985	0.8215	0.8254
Aug-96	2	1.0000	1.0000	1.0000
	6	0.9985	0.3943	0.0471
	12	0.9975	0.5421	0.0074
Dec-98	2	1.0000	1.0000	1.0000
	6	0.9997	0.8877	0.0733
	12	0.9911	0.4985	0.0001
P-Values for the Null Hypothesis that Retail Shocks Produce Symmetric Retail Price Responses				
Shock Date	Time	1% shock	5% shock	10% shock
Jan-82	2	0.0000	0.0000	0.0000
	6	0.0000	0.0000	0.0000
	12	0.4657	0.0000	0.0000
Sep-95	2	0.0000	0.0000	0.0000
	6	0.9236	0.0042	0.0000
	12	0.9886	0.9113	0.4137
Aug-96	2	0.0000	0.0000	0.0000
	6	0.8575	0.0014	0.0000
	12	0.8755	0.0000	0.0000
Dec-98	2	0.0000	0.0000	0.0000
	6	0.9899	0.2603	0.0000
	12	0.9960	0.4684	0.0009

Table 3: Skewness of the distribution of impulse response (September 1995 shocks)

	A Positive Wholesale Shock				A Positive Retail Shock			
Responding Price:	Wholesale		Retail		Wholesale		Retail	
Month	Relative Skewness	P-Value	Relative Skewness	P-Value	Relative Skewness	P-Value	Relative Skewness	P-Value
1	0.999103	0.0000	-1.0019	0.0000	0.999909	0.0000	0.999585	0.0000
2	0.018136	0.7322	-0.12991	0.0935	-0.10688	0.1677	-1.0005	0.0000
3	0.85994	0.0000	-0.26077	0.0008	-0.87527	0.0000	-0.10676	0.1681
4	0.956083	0.0000	-0.16268	0.0357	-1.19661	0.0000	0.232713	0.0027
5	1.512311	0.0000	-0.04885	0.5283	-1.38537	0.0000	0.580954	0.0000
6	1.688903	0.0000	-0.14151	0.0677	-1.83915	0.0000	0.816	0.0000
7	1.769619	0.0000	-0.06146	0.4275	-1.5596	0.0000	0.830869	0.0000
8	2.525748	0.0000	-0.11229	0.1472	-1.40426	0.0000	0.664662	0.0000
9	4.288709	0.0000	-0.11962	0.1225	-1.44261	0.0000	0.951117	0.0000
10	6.150138	0.0000	-0.2188	0.0047	-1.65875	0.0000	0.650639	0.0000
11	5.770587	0.0000	-0.21556	0.0054	-1.63818	0.0000	0.868161	0.0000
12	2.491966	0.0000	-0.35109	0.0000	-2.31332	0.0000	1.074575	0.0000
	A Negative Wholesale Shock				A Negative Retail Shock			
Responding Price:	Wholesale		Retail		Wholesale		Retail	
Month	Relative Skewness	P-Value	Relative Skewness	P-Value	Relative Skewness	P-Value	Relative Skewness	P-Value
1	-0.9987	0.0000	1.001285	0.0000	-0.99916	0.0000	-0.99955	0.0000
2	-0.01632	0.8331	0.125657	0.1048	0.106875	0.1677	1.000401	0.0000
3	0.040325	0.6027	-0.22315	0.0040	0.875086	0.0000	0.106875	0.1677
4	-0.16857	0.0295	-0.21401	0.0057	1.191057	0.0000	-0.26156	0.0007
5	-0.60019	0.0000	0.244514	0.0016	1.417848	0.0000	-0.55379	0.0000
6	-0.6913	0.0000	0.472841	0.0000	1.924504	0.0000	-0.8149	0.0000
7	-0.78773	0.0000	0.450066	0.0000	1.545587	0.0000	-0.83036	0.0000
8	-1.0791	0.0000	0.61254	0.0000	1.638787	0.0000	-0.81439	0.0000
9	-2.19873	0.0000	0.711232	0.0000	1.507778	0.0000	-1.14421	0.0000
10	-3.28016	0.0000	0.751143	0.0000	1.738341	0.0000	-0.73588	0.0000
11	-2.62252	0.0000	0.745978	0.0000	2.019929	0.0000	-0.93841	0.0000
12	-0.30226	0.0001	0.796072	0.0000	2.609708	0.0000	-1.15015	0.0000

Figure 1: Actual and predicted butter prices: January 1980-August 2001.

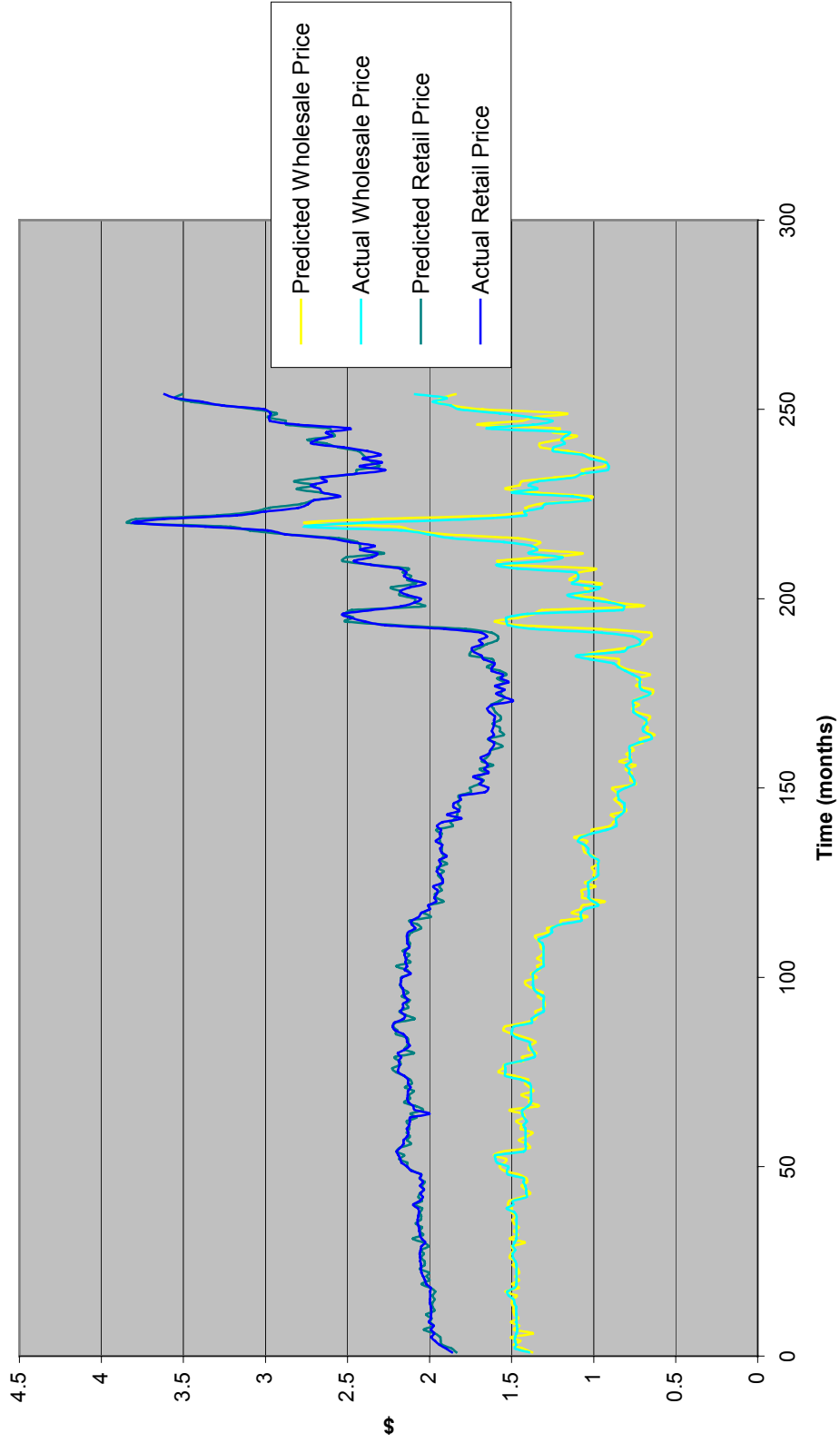


Figure 2: Impulse Responses to 40% Wholesale Price Shocks in December 1998: 10th, 25th, 50th, 75th and 90th Percentiles

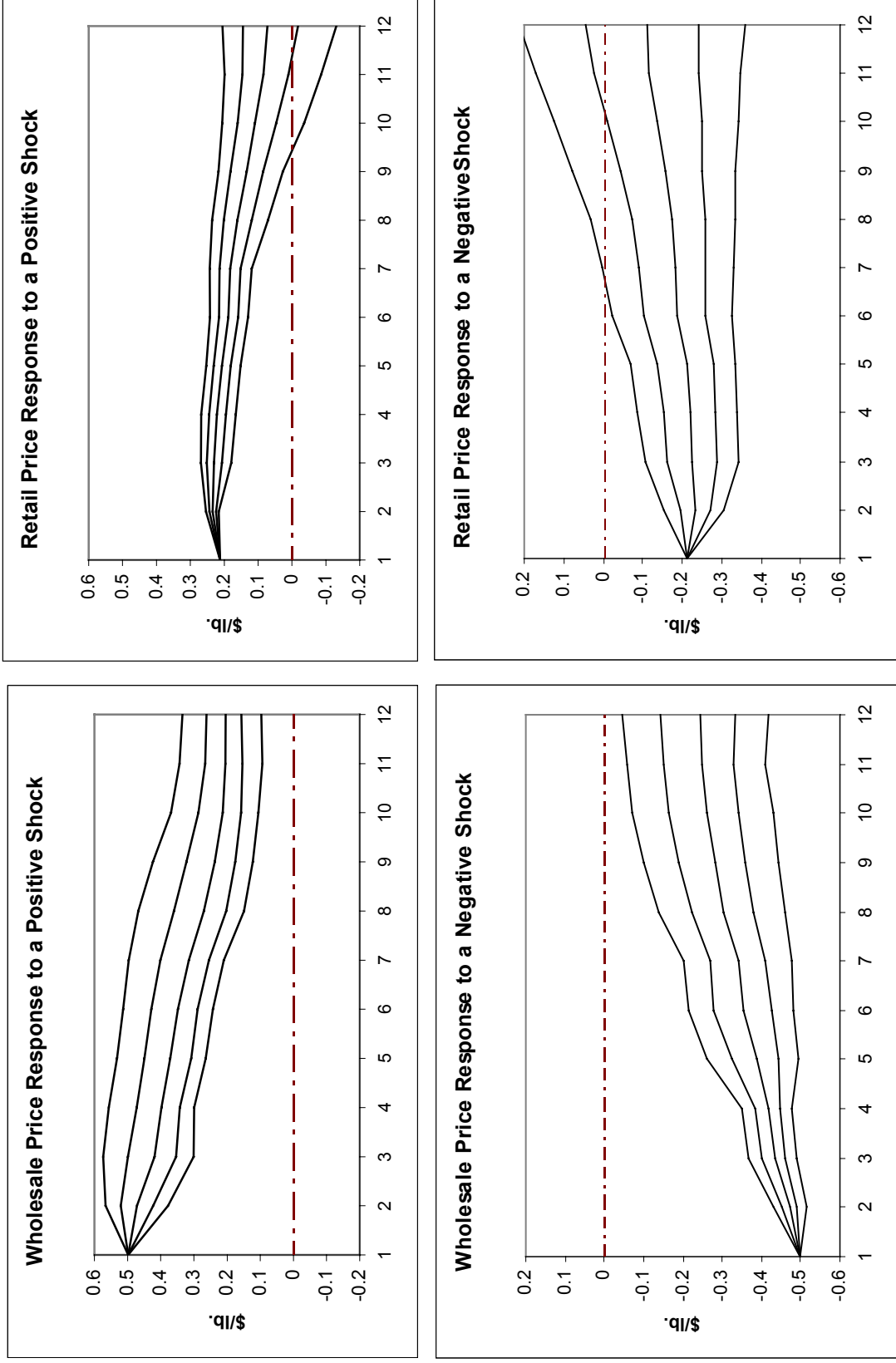


Figure 3: Impulse Responses to 10% Retail Shocks in December 1998: 10th, 25th, 50th, 75th and 90th Percentiles

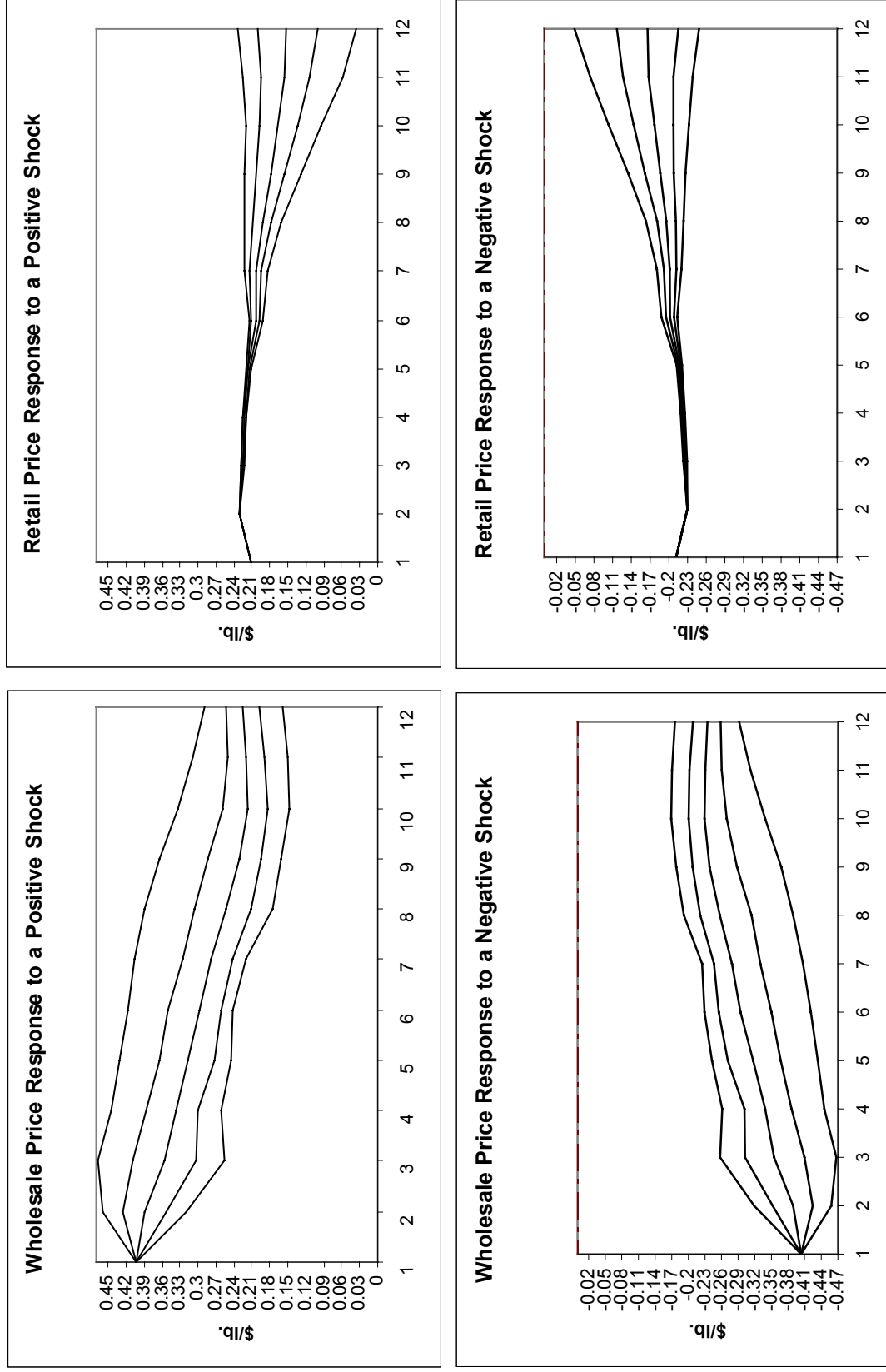
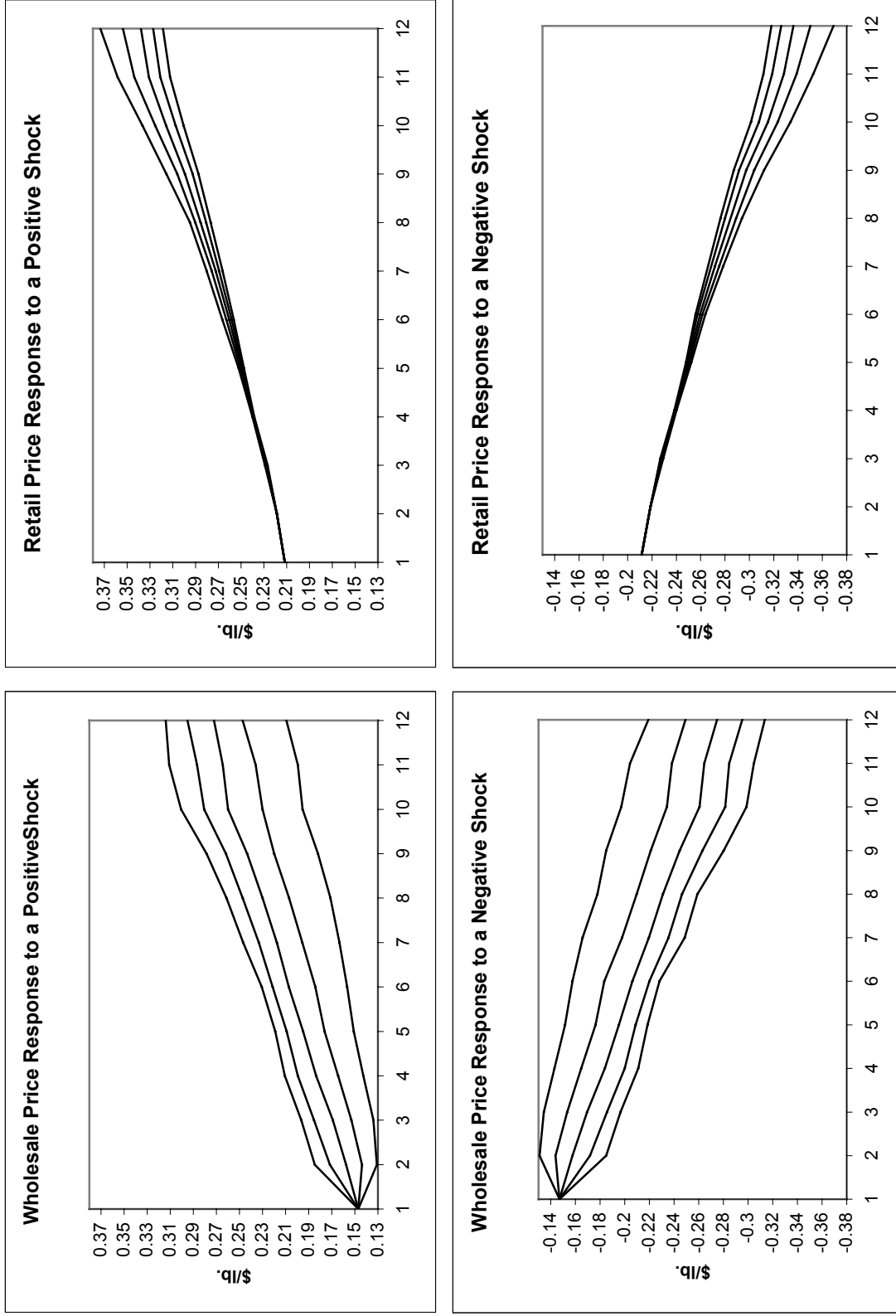


Figure 4: Impulse Responses to 10% Retail Price Shocks in September 1995: 10th, 25th, 50th, 75th and 90th Percentiles



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Footnotes

¹ See Zellner and Palm for a discussion of the linkages between a structural model of price determination and the time series representation (1).

² Note that equation (3) can be equivalently expressed in “levels” as

$$y_{it} = a_{i0} + a_{i1} t + \sum_{s=1}^{S-1} \alpha_{is} D_{ts} + \sum_{k=1}^K \sum_{j=1}^m [A_{kij}^1 R_{j,t-k} + A_{kij}^0 (1-R_{j,t-k})B_k] y_{t-k} + e_{it},$$

$i, j = 1, \dots, m$, where the A's satisfy $\sum_{k=1}^K A_{kij}^1 = \sum_{k=1}^K A_{kij}^0$, for $i, j = 1, \dots, m$.

³ Equation (3) restricts the B_{0ij} 's to be the same across regimes. It assumes that cointegration relationships among the dependent variables are not regime specific. This will prove convenient in the implementation of the Johansen test for cointegration (see below).

⁴ More general forms of asymmetry can treat the regime switching as endogenous. This includes threshold autoregression (TAR; see Hansen, and Koop and Potter), or Markov chains with regime switching (e.g., Hamilton, chapter 22).

⁵ Allowing the s_{ij} 's to become time-varying means that the model specification changes with the ordering of the prices. To evaluate this issue, we also estimated the same model with $y_1 = y_w$ and $y_2 = y_r$. This resulted in a lower log-likelihood value of the sample.

⁶ The choice of 40 percent was made to reflect some of the larger shocks to wholesale butter price observed during the sample period.

⁷ A 10 percent shock reflects some of the larger shocks to retail butter price observed during the sample period.