Price Leadership in a Homogeneous Product Market

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Abstract

Existing studies of asymmetric duopoly show that price leadership by a lower cost firm is beneficial for both firms if cost difference between firms is large (dominant leadership). We reexamine Ono's (1978) pioneering work on price leadership. Ono assumes that the follower undercuts the leader's price and that the leader meets residual demand. We endogenize the follower's price. We find that, in contrast to the existing studies, mutually beneficial price leadership by the higher (lower) cost firm may arise (cannot arise). We also find that price leadership by the higher cost firm is mutually beneficial when the cost difference between firms is small.

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1 Introduction

We reexamine price leadership in homogenous goods markets with increasing marginal costs. We consider an asymmetric duopoly and analyze whether a lower cost firm or a higher cost firm takes price leadership. Existing works indicates that the lower cost firm takes the price leadership (dominant price leadership). We find that in contrast to the existing works, both firms may prefer price leadership by the higher, but not the lower, cost firm. We also find that payoff dominant price leadership arises when cost difference between firms is small.

Price leadership has attracted the attention of both economic and legal (especially anti-trust) researchers and has been intensively discussed. In his pioneering work, Ono (1978) investigates an asymmetric duopoly. He formulates a following model of price leadership with increasing marginal costs. The leader chooses its price. The follower sets a slightly lower price than the leader's (hence undercutting the price) and chooses its output, and the leader meets the residual demand. He compares more efficient firm's and less efficient firm's leadership. He shows that if cost difference between firms is large, both firms prefer leadership by the lower cost firm. He concludes that the lower cost firm takes price leadership (Dominant leadership).[1]

Subsequent researchers have developed other models and obtained similar results. Denekere and Kovenock (1982) and Furth and Kovenock (1993) investigate price leadership under capacity constraints. They show that a firm with more capacity becomes the leader. Denekere et al. (1992) demonstrate that the stronger firm which has the larger segment of loyal consumers becomes the leader. van Damme and Hurkens (2004) and Amir and Stepanova (2006) investigate a model with product differentiation. As does Ono, they show that the lower (higher) cost firm prefers being the leader (follower) if cost difference is large. They also show that this type of price leadership is risk-dominant in the observable delay game. Ishibashi (2008) shows that dominant firm's price leadership stabilizes collusion.

In Ono's original model of homogenous goods duopoly, he assumes, rather than derives, that the

[1] For the oligopoly case, see Ono (1982). For applications of this model, see Itoh and Ono (1982) and Ono (1984).

follower always undercuts the leader's price. In other words, he endogenizes the leader's price only. We formulate a model where both the follower and the leader can fully choose their prices. We find that the follower never undercuts the leader's price in equilibrium when its cost is lower than the leader's, and that it may undercuts when its cost is higher than the leader's; thus endogenizing the price of the follower matters. Endogenizing the follower's price substantially affects the results on price leadership, too. Unlike in existing works of price leadership, in our model mutually beneficial price leadership by the lower cost firm cannot arise, whereas mutually beneficial price leadership by the higher cost firm does arise when the cost difference between firms is **small**. We thus provide a game theoretic model of non-dominant firm price leadership.[2]

Our result also gives a new insight for the analysis under capacity constraints. We can regard the model with capacity constraint is a special model with increasing marginal costs. Since in this paper we assume that cost function is concave and continuously differentiable, the model with capacity constraint is not a special case of our analysis. However, we can construct a concave and continuously differentiable cost functions which is arbitrarily close to the discontinuous cost functions discussed by the capacity constraint models. Thus, we can discuss whether or not the result of capacity constraint model is on knife-edge.

Regarding the endogenous follower's cost in a homogeneous product market, Dastidar (2004) has already established one important contribution for this point. He investigates a symmetric Stackelberg duopoly (both firms have the same cost function) and shows that the follower always takes this price strategy rather than price undercutting strategy in equilibrium. In this paper we allow asymmetry (cost difference) between two firms and finds that his result holds unless the follower is highly inefficient than the leader.

The paper is organized as follows. We describe the model in Section 2, and analyze the equilibrium in Section 3. We present our results and provide examples in Section 4. Finally, we conclude in Section 4.

[2] Dominant firm price leadership as well as non-dominant firm price leadership are widely observed. See, among others, Konishi (2001), Markham (1951), Scherer and Ross (1990), Tasnadi (2004), and Viscusi et al (2005).

2 The Model

Two firms produce homogenous products the market demand for which is $D(p)$ (quantity as a function of price). We suppose $D(p)$ satisfies two assumptions.

Assumption 1: *There exists $\bar{P} > 0$ such that $D(p) = 0$ if and only if $p \geq \bar{P}$.*

Assumption 2: *$D(p)$ is strictly decreasing, twice continuously differentiable, and concave on $[0, \bar{P}]$.*

Firm i's cost is $C_i(y_i)$ $(i = 1, 2.)$. Firm i's payoff is $\pi_i = p_i y_i - C_i(y_i)$, where p_i is firm i's price. Let $MC_i(y_i)$ denote the marginal cost of firm i. We assume that $MC_i(y_i)$ is continuous and strictly increasing. Let $S_i(p)$ denote the supply of firm i when it is a price taker. It is given by $S_i(p) = MC_i^{-1}$ (the inverse function of MC_i). We make the following two assumptions on S_i.

Assumption 3: $S_1(0) = S_2(0) = 0$.

Assumption 4: *$S_1 - S_2$ is non-decreasing in p.*

Assumptions 3–4 imply $MC_1(y) \leq MC_2(y)$ for all $y \in \mathbb{R}_+$ (i.e., firm 1 is more efficient than firm 2 or both firms are equally efficient). A typical example of cost functions satisfying these assumptions is $C_i = \alpha_i y_i^n$ where $\alpha_1 \leq \alpha_2$ and $n \geq 2$.

We formulate a perfect information game. First, firm l $(\in \{1, 2\})$ sets its price $p_l \in [0, \bar{P}]$. Second, after observing the leader's price, firm f $(\in \{1, 2\} \setminus \{l\})$ sets its price $p_f \in [0, \bar{P}]$. Amounts of the supplies are determined by the following efficient rationing: If $p_l < p_f$, then $y_l = \min\{S_l(p_l), D(p_l)\}$ and $y_f = \min\{S_f(p_f), \max(0, D(p_f) - y_l)\}$. If $p_l \geq p_f$, then $y_l = \min\{S_l(p_l), \max(0, D(p_l) - y_f)\}$ and $y_f = \min\{S_f(p_f), D(p_f)\}$.[3]

We consider the two Stackelberg games. One is $l = 1$ (the leadership by the more efficient firm) and the other is $l = 2$ (the leadership by the less efficient firm). In what follows, we sometimes use two subscripts at the same time to distinguish both the efficiency and the timing. First subscripts and second subscripts denote the timing and the efficiency respectively. For example, p_{l1} stands

[3] We explain the reason of the asymmetric treatment between the leader and the follower when $p_f = p_l$ in footnote 4.

for the more efficient firm's price when it is the leader.

We explain the story of the former case ($l = 1$). The leader, firm 1, sets its price p_{l1}. One possible strategy of the follower (firm 2) is undercutting firm 1's price (choosing $p_{f2} \leq p_{l1}$). If firm 2 undercuts firm 1's price, it supplies $y_{f2} = \min\{S_2(p_{f2}), D(p_{f2})\}$, and firm 1 obtains the residual demand, $\max\{0, D(p_{l1}) - y_{f2}\}$. That is, the leader supplies $y_{l1} = \min\{S_1(p_{l1}), \max(0, D(p_{l1}) - y_{f2})\}$. Ono (1978) assumes that the follower always undercuts firm1's price (i.e., sets $p_{f2} = p_{l1}$) and produces $y_{f2} = \min\{S_2(p_{l1}), D(p_{l1})\}$.[4] Another plausible strategy of firm 2, which is neglected by Ono, is setting $p_{f2} > p_{l1}$. Then firm 2 rather than firm 1, obtains the residual demand. In this case $y_{l1} = \min\{S_1(p_{l1}), D(p_{l1})\}$ and $y_{f2} = \min\{S_2(p_{f2}) \max(D(p_{f2}) - y_{l1}, 0)\}$. Our model incorporate this as a possible strategy for the follower, i.e., we endogenize the follower's price as well as the leader's price. The latter case ($l = 2$) is similar.

3 Equilibrium

3.1 The follower's pricing

We solve the game by backward induction. First, we consider the behavior of the follower.

Let p_i^M denote the monopoly price by firm i. If $p_l \geq p_f^M$ (the follower's monopoly price), firm f sets $p_f = p_f^M$ and obtains the whole demand. Suppose that $p_l < p_f^M$. Firm f chooses (i) setting $p_f = p_l$ and $y_f = \min\{D(p_l), S_f\}$ (undercutting) or (ii) setting $p_f > p_l$ and obtaining the residual demand (non-undercutting).

If firm f adopts (i) (i.e., firm f undercuts p_l), its profit is:

$$\pi_f^U(p_l) = \int_0^{y_f^U(p_l)} (p_l - MC_f(q))dq, \tag{1}$$

where $y_f^U(p_l) := \min\{S_f(p_l), D(p_l)\}$. Assumptions 1 and 2 (concavity of the demand function) guarantees that setting $p_f < p_l$ never becomes optimal unless $p_l > p_f^M$.

[4] Strictly speaking, the follower undercuts the leader's price by setting a slightly below that of the leader. Following Ono (1978) we describe the situation where the follower undercuts the leader's price by $p_l = p_f$, not $p_l = p_f - \varepsilon$.

Next we consider the case where the follower adopts (ii). Define

$$p_f^{NU}(p_l) := \operatorname*{argsup}_{p>p_l} \pi_f(p;p_l) \qquad \pi_f^{NU}(p_l) := \sup_{p>p_l} \pi_f(p;p_l), \tag{2}$$

where $\pi_f(p_f;p_l)$ is the follower's profit when it sets p_f and the opponent sets p_l. Assumptions 1 and 2 guarantee that p_f^{NU} is uniquely determined. If firm f adopts (ii), firm l supplies $y_l = \min\{S_l(p_l), D(p_l)\}$.

Since π_f^{NU} is decreasing in p_l and π_f^U is increasing in p_l, $\pi_f^{NU} - \pi_f^U > 0$ when $p_l = 0$, and $\pi_f^{NU} - \pi_f^U < 0$ when $p_l = p_f^M$, there exists a threshold value $\tilde{p}_l \in (0, p_f^M)$ such that firm f does not undercut if and only if $p_l \le \tilde{p}_l$. The threshold value $\tilde{p}_l$ is derived from $\pi_f^{NU}(\tilde{p}_l) = \pi_f^U(\tilde{p}_l)$. We present two supplementary results on $\tilde{p}_l$, which we use in the proofs of our main results.

Lemma 1: $S_l(\tilde{p}_l) < D(\tilde{p}_l)$.

Proof: Suppose otherwise. When the follower does not undercut $\tilde{p}_l$, the residual demand for the follower is zero, so the profit of the follower is zero. If the follower undercuts it, it obtains strictly positive profit. These contradict to the definition of $\tilde{p}_l$. ■

Lemma 2: $S_l(\tilde{p}_l) + S_f(\tilde{p}_l) > D(\tilde{p}_l)$.

Proof: Suppose otherwise. If $S_l(\tilde{p}_l) + S_f(\tilde{p}_l) < D(\tilde{p}_l)$, it is obvious that the follower has an incentive to set $p_f > \tilde{p}_l$ when $p_l = \tilde{p}_l$. Moreover, even if $S_l(\tilde{p}_l) + S_f(\tilde{p}_l) = D(\tilde{p}_l)$ the same holds since $(\partial/\partial p_f)\pi_f(\tilde{p}_l;\tilde{p}_l) = S_f(\tilde{p}_l) + D'[\tilde{p}_l - MC(S_f)] = S_f(\tilde{p}_l) > 0$. That is, $S_l(\tilde{p}_l) + S_f(\tilde{p}_l) \le D(\tilde{p}_l)$ contradicts the definition of $\tilde{p}_l$. ■

3.2 The leader's pricing

In this subsection, we discuss the optimal pricing of the leader. If the leader sets $p_l > \tilde{p}_l$, the follower undercuts it and the leader obtains residual demand $\max(D(p_l) - S_f(p_l), 0)$ which is smaller than $S_l(p_l)$ by Lemma 2. Define

$$p_l^U := \operatorname*{argmax}_{p_l} \pi_l(p_l;p_l) \qquad \pi_l^U := \max_{p_l} \pi_l(p_l;p_l), \tag{3}$$

where $\pi_l(p_l; p_f)$ is the leader's profit when it sets p_l and the opponent sets p_f, and

$$\pi_l^{NU} := p_l S_l(\tilde{p}_l) - C_l(S_l(\tilde{p}_l)). \tag{4}$$

In words, π_l^U is the leader's maximum profit provided that the follower undercuts any price p_l, and π_l^{NU} is the leader's profit when it sets $\tilde{p}_l$ and the follower does not undercut it. Note that when $p_l = \tilde{p}_l < p_f$, firm l's output is $y_l^{NU}(\tilde{p}_l) = \min\{S_l(\tilde{p}_l), D(\tilde{p}_l)\} = S_l(\tilde{p}_l)$. (Lemma 1).

If $\pi_l^{NU} > \pi_l^U$, the leader sets $\tilde{p}_l$ and the follower does not undercut it, and it is the unique subgame perfect equilibrium outcome. If $\pi_l^{NU} < \pi_l^U$, the leader sets p_l^U and the follower undercuts it, and it is the unique SPE outcome. If $\pi_l^{NU} = \pi_l^U$, both are the SPE outcomes.

4 Results

First, we investigate equilibrium prices. Proposition 1 states that the less efficient leader chooses its price to deter price cutting by the follower.[5] This implies Ono's (1978) assumption that the follower **always** undercuts the leader's price is generally invalid when both firms can choose prices.

Proposition 1: *If firm 1 (the more efficient firm) is the follower, it never undercuts firm 2's price* p_{l2} *in equilibrium i.e.,* $p_{l2}^E = \tilde{p}_{l2}$.

Proof: See Appendix.

Intuition behind Proposition 1 is as follows. Given p_l, when the follower obtains the residual demand, it can charge any price higher than p_l. On the other hand, when the leader obtains the residual demand, its price is p_l and cannot change it. In this sense, the follower has a stronger incentive for obtaining residual demand than the leader.

Suppose that Proposition 1 fails to hold. The less efficient leader (firm 2) prefers setting $p_2 = p_{l2}^U > \tilde{p}_{l2}$ and being undercutted to setting $p_2 = \tilde{p}_{l2}$ and not being undercutted. In other

[5] Dastidar (2004) has already shown that this result holds when both firms have the same cost function. His result is a special case of our Proposition 1.

words, firm 2 prefers obtaining residual demand rather than setting $p_2 = \tilde{p}_{l2}$ and producing $y_2 = S_2(\tilde{p}_{l2})$.

Under that supposition, let us consider the leader's incentive when $p_{l2} = \tilde{p}_{l2}$. As is discussed above, the follower has stronger incentive to obtain residual demand than the leader. In addition the follower (firm 1) has the cost advantage $S_1 \geq S_2$. Thus, the residual demand of firm 1 ($D - S_2$) is larger than that of firm 2 ($D - S_1$). Furthermore, the loss of profit from the decrease of supply ($\int (p - MC(q))dq$) is smaller for firm 1 than for firm 2. Because of these cost advantages, firm 1 has a larger incentive for obtaining the residual demand. Combining these cost advantage effects and the follower's effect above, the more efficient follower (firm 1) must strictly prefers setting $p_{f1} = p^U_{l2}$ to undercutting $\tilde{p}_{l2}$. (Recall our supposition that firm 2 prefers to set p^U_{l2} and obtain residual demand.) However, it contradicts to the definition of $\tilde{p}_{l2}$, i.e., the follower must be indifferent between undercutting and non-undercutting when $p_{l2} = \tilde{p}_{l2}$. This implies it is impossible that the less efficient leader (firm 2) prefers setting $p_{l2} = p^U_{l2} > \tilde{p}_{l2}$ and being undercutted to setting $p_2 = \tilde{p}_{l2}$ and not being undercutted.

Next, we compare firms' profits under the more efficient firm's leadership with those under the less efficient firm's. If $\pi^E_{l1} \leq \pi^E_{f1}$ (the more efficient firm prefers to follow) and $\pi^E_{l2} \geq \pi^E_{f2}$ (the less efficient firm prefers to lead), both firms prefer the less efficient firm to lead (dominance of less efficient firm's leadership). On the contrary, if $\pi^E_{l1} \geq \pi^E_{f1}$ and $\pi^E_{l2} \leq \pi^E_{f2}$ both firms prefer the more efficient firm to lead (dominance of more efficient firm's leadership). In Ono (1978) and other studies of price leadership, both firms prefer the more efficient firm to lead if the cost difference between the two is large, whereas less efficient firm's leadership is never mutually beneficial. In contrast to existing studies of price leadership, Proposition 2(i) states that dominance of more efficient firm's leadership never arises.

Proposition 2: *Suppose that $S_1 > S_2$, i.e., firm 1 is more efficient than firm 2 (not equally efficient). (i) The more efficient firm (firm 1) always strictly prefers following to leading. (ii) If the cost difference between firm 1 and 2 is sufficiently small, both firms strictly prefer the less efficient firm's leadership to the more efficient firm's leadership.*

Proof: See Appendix.

Proposition 2(ii) also contrasts sharply with the results of existing studies. In our model, there is no conflict of interest over the distribution of roles when the cost difference is small. Suppose that the cost difference is small. We show in Proof of Proposition 2(ii) that the leader sets $p_l = \tilde{p}_l$ and the follower dose not undercut it, no matter which firm takes leadership. Consequently, the leader's profit is equal to undercutting profits when it were the follower. The follower does not undercut, but by definition of $\tilde{p}_l$, it is indifferent between undercutting and not undercutting. Since both firms' profits are undercutting one, there is no conflict of interest and both firms prefer the leadership which induces higher p_l^E. Since the higher cost firm sets the higher equilibrium price, both firms prefers the higher cost firm to lead.[6]

Our result also gives a new insight for the model with capacity constraint. Deneckere and Kovenock (1992) investigate a model where both firms face capacity constraint and show that the leadership by the firm with more capacity is dominant to the leadership by the firm with less capacity. In the capacity-constraint model, the marginal cost is constant until the firm meets the capacity constraint and after then the marginal cost becomes infinity. We can regard the model with capacity constraint is a special model of increasing marginal costs. In this model we assume that cost function is concave and continuously differentiable, thus the model with capacity constraint discussed by Deneckere and Kovenock (1992) is not a special case of our analysis. However, we can construct a cost functions satisfying all of our assumptions which is arbitrarily close to the above discontinuous cost functions. Thus, we can say that Deneckere and Kovenock's (1992) results are degenerate while ours are generic.

Finally we present examples. The first is an example of the dominance of less efficient firm's leadership. This example indicates that the mutually beneficial leadership by the less efficient firm is not a measure-zero event but holds for broad range of parameter values.

[6] Although our result is quite different from Ono's, there is an important similarity between our results and his. In both models, the more efficient firm obtains the residual demand. We might be able to interpret the model of Ono is discussing who behaves as a price taker and who obtains the residual demand, not discussing the sequential price choice model.

Example 1: Suppose that $D = 1 - p$, $MC_1(y) = (2/3)y$ and $MC_2(y) = y$. When the less efficient firm is the leader, $p^E_{l2} = \tilde{p}_{l2} = 1/3$ and $(\pi^E_{l2}, \pi^E_{f1}) = (1/18, 1/12)$. When the more efficient firm is the leader, $p^E_{l1} = \tilde{p}_{l1} = (\frac{3}{2} + \sqrt{3})^{-1}$ and $(\pi^E_{f2}, \pi^E_{l1}) = ((1/2)(1.5 + \sqrt{3})^{-2}, (3/4)(1.5 + \sqrt{3})^{-2}) \simeq (0.048, 0.072)$.

The second example shows the possibility that the both firms prefer to follow. That is, the condition of Proposition (ii) that difference of costs is small is not redundant. This example also indicates that price-undercutting takes place in equilibrium when the more efficient firm is the leader.

Example 2: Suppose that $D = 1 - p$, $MC_1(y) = (1/10)y$ and $MC_2(y) = y$. When the less efficient firm is the leader, $p^E_{l2} = \tilde{p}_{l2} = 121/541 \simeq 0.2237$ and $(\pi^E_{l2}, \pi^E_{f1}) \simeq (0.025, 0.144)$. When the more efficient firm is the leader, $p^E_{l1} = 3/11 \neq \tilde{p}_{l1}$ and $(\pi^E_{f2}, \pi^E_{l1}) \simeq (0.037, 0.114)$.

5 Concluding Remarks

This paper investigate whether the more efficient or the less efficient firm takes price leadership. On the contrary to the existing works, we show that the less efficient firm takes the leadership. This might explain the behaviors in Japanese brewery industry in 1970s and 80s, which is considered as a typical example of price leadership in Japan. The largest firm and the most efficient firm, Kirin, seldom took price leadership, and either of two smaller firms, Sapporo and Asahi, often took leadership.[7] Recently, Asahi has established great competitive advantage to Kirin and becomes the largest firm in the industry. Nowadays Kirin often takes price leadership.

We investigate a duopoly model with homogeneous good market, like Ono (1978). Whether or not the result is knife edge (whether a slight product differentiation[8] and/or an increase in the

[7] See Konishi (2001). Another example is energy markets in Tokai area in Japan. Toho which is much smaller than Chubu takes price leadership.

[8] From Amir and Stepanova (2006) we know that a large degree of product differentiation which guarantees the existence of pure strategy equilibria in the simultaneous-move games changes the results completely. What happens under a smaller degree of production is still unknown.

number of firms[9] changes the result) remains for future researches.

[9] In sequential move games it is possible that an increase in the number of firms change the result drastically. See, e.g., Shinkai (2000).

Appendix

Proof of Proposition 1: Suppose otherwise: i.e. there exists $p > \tilde{p}_{l2}$ such that $\pi_{l2}(p;p) \geq \pi_{l2}^{NU}(\tilde{p}_{l2})$. If $y_{f1}^{U}(p) := \min\{S_1(p), D(p)\} = D(p)$, then $y_{l2}(p;p) = 0$, so $\pi_{l2}(p;p) = 0$ and $\pi_{l2}(p;p) > \pi_{l2}^{NU}(\tilde{p}_{l2})$ is never satisfied. Thus, $y_{f1}^{U}(p) = S_1(p)$. We use this for deriving (6).

Since $\pi_l(p_l; p_f)$ is decreasing in p_f so far as $p_f \leq p_l$, it follows that $\pi_{l2}(p;p) - \pi_{l2}^{NU}(\tilde{p}_{l2}) < \pi_{l2}(p;\tilde{p}_{l2}) - \pi_{l2}^{NU}(\tilde{p}_{l2})$. Thus, we have

$$\pi_{l2}(p;\tilde{p}_{l2}) - \pi_{l2}^{NU}(\tilde{p}_{l2}) > 0. \tag{5}$$

Manipulating $\pi_{l2}(p;\tilde{p}_{l2}) - \pi_{l2}^{NU}(\tilde{p}_{l2})$, we have

$$\begin{aligned} \pi_{l2}(p;\tilde{p}_{l2}) - \pi_{l2}^{NU}(\tilde{p}_{l2}) &= p(D(p) - S_1(\tilde{p}_{l2})) - C_2\big(D(p) - S_1(\tilde{p}_{l2})\big) - \Big(p_{l2}S_2(\tilde{p}_{l2}) - C_2(S_2(\tilde{p}_{l2}))\Big) \\ &= (p - p_{l2})(S_2(\tilde{p}_{l2}) - \Delta) - \int\limits_{S_2(\tilde{p}_{l2})-\Delta}^{S_2(\tilde{p}_{l2})} (\tilde{p}_{l2} - MC_2(t))dt, \end{aligned} \tag{6}$$

where $\Delta := S_1(\tilde{p}_{l2}) + S_2(\tilde{p}_{l2}) - D(p)$ which is strictly positive by Lemma 2.

From (5) and (6) we have:

$$(p - p_{l2})(S_2(\tilde{p}_{l2}) - \Delta) - \int\limits_{S_2(\tilde{p}_{l2})-\Delta}^{S_2(\tilde{p}_{l2})} (\tilde{p}_{l2} - MC_2(t))dt > 0. \tag{7}$$

Next let us consider the follower's profits.

$$\pi_{f1}(p;\tilde{p}_{l2}) - \pi_{f1}^{U}(\tilde{p}_{l2}) = (p - \tilde{p}_{l2})(S_1(\tilde{p}_{l2}) - \Delta) - \int\limits_{S_1(\tilde{p}_{l2})-\Delta}^{S_1(\tilde{p}_{l2})} (\tilde{p}_{l2} - MC_1(t))dt. \tag{8}$$

Since $S_1 \geq S_2$, we have

$$(p - p_{l2})(S_1(\tilde{p}_{l2}) - \Delta) \geq (p - p_{l2})(S_2(\tilde{p}_{l2}) - \Delta). \tag{9}$$

By Assumption 4 we have

$$\int\limits_{S_2(\tilde{p}_{l2})-\Delta}^{S_2(\tilde{p}_{l2})} (\tilde{p}_{l2} - MC_2(t))dt \geq \int\limits_{S_1(\tilde{p}_{l2})-\Delta}^{S_1(\tilde{p}_{l2})} (\tilde{p}_{l2} - MC_1(t))dt. \tag{10}$$

The left-hand side in (10) is the left triangular in Figure 1 and the right-hand side in (10) is the right triangular in the same figure. Assumption 4 ensures that the left triangular is larger than the right triangular.

From (7), (9), and (10), we have that (8) is strictly positive. However, this inequality implies that firm 1, the more efficient follower, strictly prefers not undercutting $\tilde{p}_{l2}$ to undercutting it. This contradicts to the definition of $\tilde{p}_{l2}$. ■

We now prove Proposition 2. So as to prove Proposition 2, we present the following two supplementary lemmata.

Lemma 3: $\tilde{p}_{l2} \geq \tilde{p}_{l1}$ *and the strict inequality holds unless* $S_1(\tilde{p}_{l2}) = S_2(\tilde{p}_{l2})$*.*

Proof: In the proof of Proposition 1, we suppose that $\pi_{f2}(p; \tilde{p}_{l2}) - \pi^U_{f2}(\tilde{p}_{l2}) > 0$ for some $p > \tilde{p}_{l2}$ and derive a contradiction. This implies that $\pi^{NU}_{f2}(\tilde{p}_{l2}) \leq \pi^U_{f2}(\tilde{p}_{l2})$. By the definition of $\tilde{p}_l$, we conclude that $\tilde{p}_{l1} \leq \tilde{p}_{l2}$.

When $S_1(\tilde{p}_{l2}) > S_2(\tilde{p}_{l2})$, inequalities (9) and (10) hold with strict inequality. Suppose that $\pi_{f2}(p; \tilde{p}_{l2}) - \pi^U_{f2}(\tilde{p}_{l2}) = 0$ for some $p > \tilde{p}_{l2}$. We can derive a similar contradiction. It implies that $\pi^{NU}_{f2}(\tilde{p}_{l2}) < \pi^U_{f2}(\tilde{p}_{l2})$ and $\tilde{p}_{l1} < \tilde{p}_{l2}$ when $S_1 > S_2$. ■

Lemma 4: $\pi_{f1} \geq \pi_{l1}$ *and* $\pi_{l2} \geq \pi_{f2}$ *(both firms prefer the less efficient firm to lead) if and only if* $p^E_{l2} \geq p^E_{l1}$*.*

Proof: First, consider the firm 2's equilibrium payoff. Consider the case where $l = 2$ (firm 2 is the leader). Proposition 1 implies that $p^E_{l2} = \tilde{p}_{l2}$ and firm 1 (the more efficient firm) does not undercut it. Thus, the leader's profit must be equal to $\pi^U_{f2}(\tilde{p}_{l2})$. Note that $y_{l2} = S_2(\tilde{p}_{l2})$ and it is the same when firm 2 is the follower and it undercuts $p_{l1} = \tilde{p}_{l2}$.

We then consider the case $l = 1$. If $p^E_{l1} > \tilde{p}_{l1}$, firm 2 undercuts the leader's price p_{l1} and its profit is $\pi^U_{f2}(p^E_{l1})$. If $p^E_{l1} = \tilde{p}_{l1}$, firm 2 does not undercuts it. By definition of $\tilde{p}_{l1}$, however, firm 2's profit is equal to $\pi^U_{f2}(\tilde{p}_{l1})$ in the both cases. [10]

Therefore, firm 2's equilibrium profit is always $\pi^U_{f2}(p^E_l)$ no matter whether it is the leader or

[10] We have already shown that $p^E_l \geq \tilde{p}_l$.

the follower. Since $\pi^U_{fi}(p_l)$ is increasing in p_l, the firm 2 prefers to lead if and only if its leadership yields higher equilibrium leader's price ($p^E_{l2} \geq p^E_{l1}$). This implies only if part of Lemma 3.

Second, consider firm 1's equilibrium payoff. We only need to show that it prefers to follow if $p^E_{l2} \geq p^E_{l1}$. When it is the follower, by proposition 1 and the definition of $\tilde{p}_l$, it earns $\pi^U_{f1}(p^E_{l2})$. When it is the leader, its profit is equal to or smaller than $\pi^U_{f1}(p^E_{l1})$ no matter whether firm 2 undercuts p^E_{l1} or not.[11] Since $\pi^U_{fi}(p_l)$ is increasing, firm 1 prefers following to leading if $p^E_{l2} \geq p^E_{l1}$. This implies if part of the statement and completes the proof. ■

Proof of Proposition 2 (i): Suppose that the more efficient firm (firm 1) prefers to lead. Lemma 4 implies $p^E_{l1} \geq p^E_{l2}$.

We first show that $p^E_{l1} = \tilde{p}_{l1}$ under the assumption. Suppose otherwise; i.e., firm 2 undercuts the price p^E_{l1}. Then, when firm 2 is the leader, firm 1 obtains the residual demand given the rival's output $S_2(p^E_{l1})$. When it is the follower and the opponent sets $p_{l2} = \tilde{p}_{l2}$, however, it can sell more than when it is the leader at the same price p^E_{l1}, since $S_2(\tilde{p}_{l1}) < S_2(p^E_{l1})$. That is, firm 1 can earn more profit when it is the follower than when it is the leader: a contradiction to the assumption that it prefers to lead.

Therefore, if firm 1 prefers to lead, $p^E_{l1} = \tilde{p}_{l1} \geq p^E_{l2}$. Proposition 1 and Lemma 2 imply, however, $p^E_{l2} = \tilde{p}_{l2} > \tilde{p}_{l1}$, a contradiction. ■

Proof of Proposition 2 (ii): From Lemma 4, all we have to show is that $p^E_{l2} \geq p^E_{l1}$ when the cost difference between the two is small. From Proposition 1 we have $p^E_{l2} = \tilde{p}_{l2}$. From Lemma 3 we have $\tilde{p}_{l2} > \tilde{p}_{l1}$. Thus, all we have to show is that $p^E_{l1} = \tilde{p}_{l1}$ (or equivalently $\pi^{NU}_{l1} \geq \pi^U_{l1}$) when the cost difference between the two is small. It is clear that $\pi^{NU}_{l1} > \pi^U_{l1}$ if $MC_1 = MC_2$, since $\tilde{p}_{l1} = \tilde{p}_{l2}$ in that case. It is also obvious that, given MC_2, all of $\tilde{p}_{l1}, \pi^{NU}_{l1}$ and π^U_{l1} are continuous in MC_1 in terms of L^1 metric. Thus, we conclude that $\pi^{NU}_{l1} > \pi^U_{l1}$ when MC_1 is sufficiently close to MC_2 in L^1 space. ■

[11] If firm 2 does not undercut p^E_{l1}, $y^E_1 = S_1(p^E_{l1})$ and its resulting profit is equal to $\pi^U_{f1}(p^E_{l1})$. If firm 2 undercuts p^E_{l1}, it obtains only residual demand and $y^E_1 < S_1(p^E_{l1})$. Its resulting profit is smaller than $\pi^U_{f1}(p^E_{l1})$.

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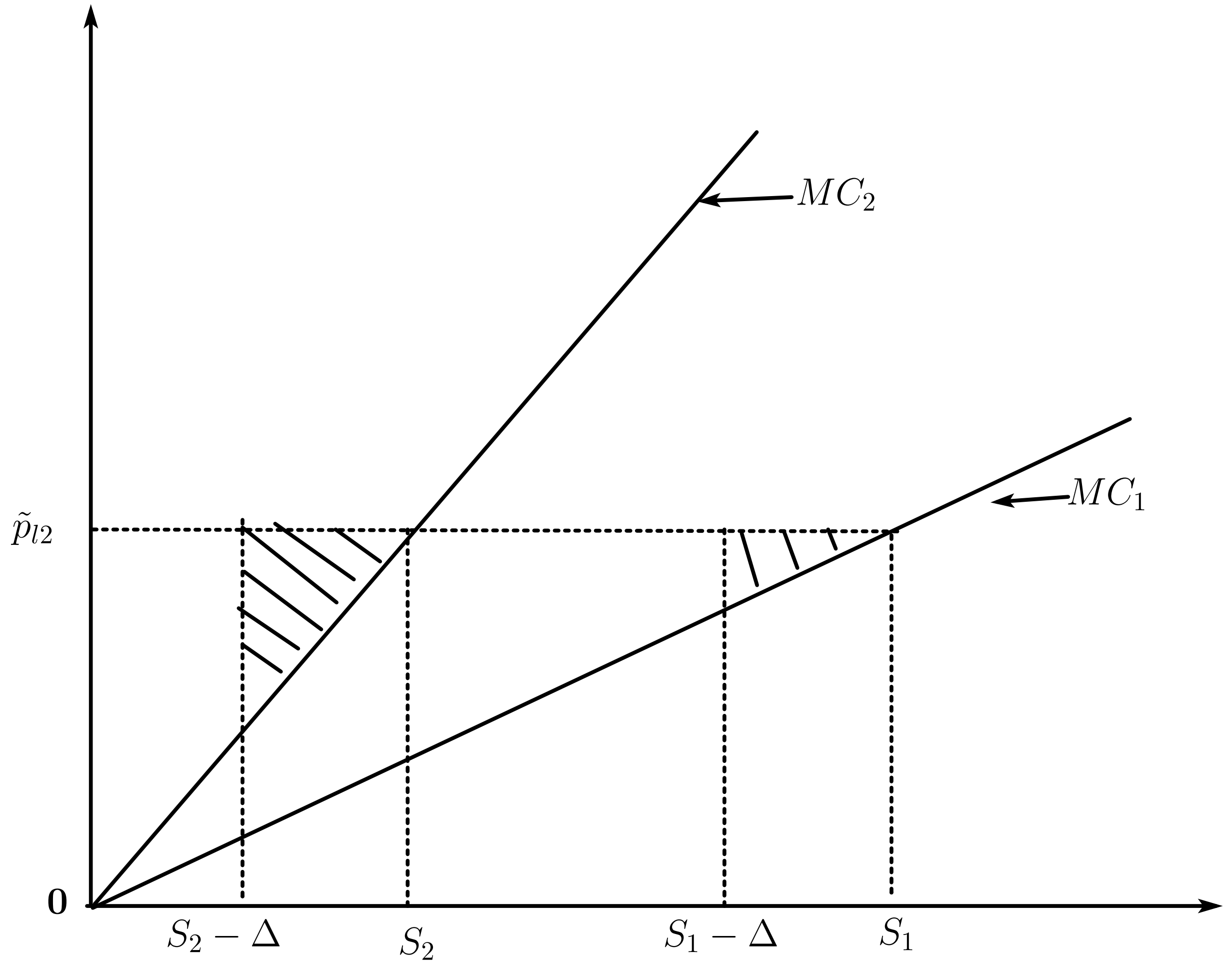

Figure 1