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### Price Regulation in Property-Liability Insurance: A Contingent-Claims Approach

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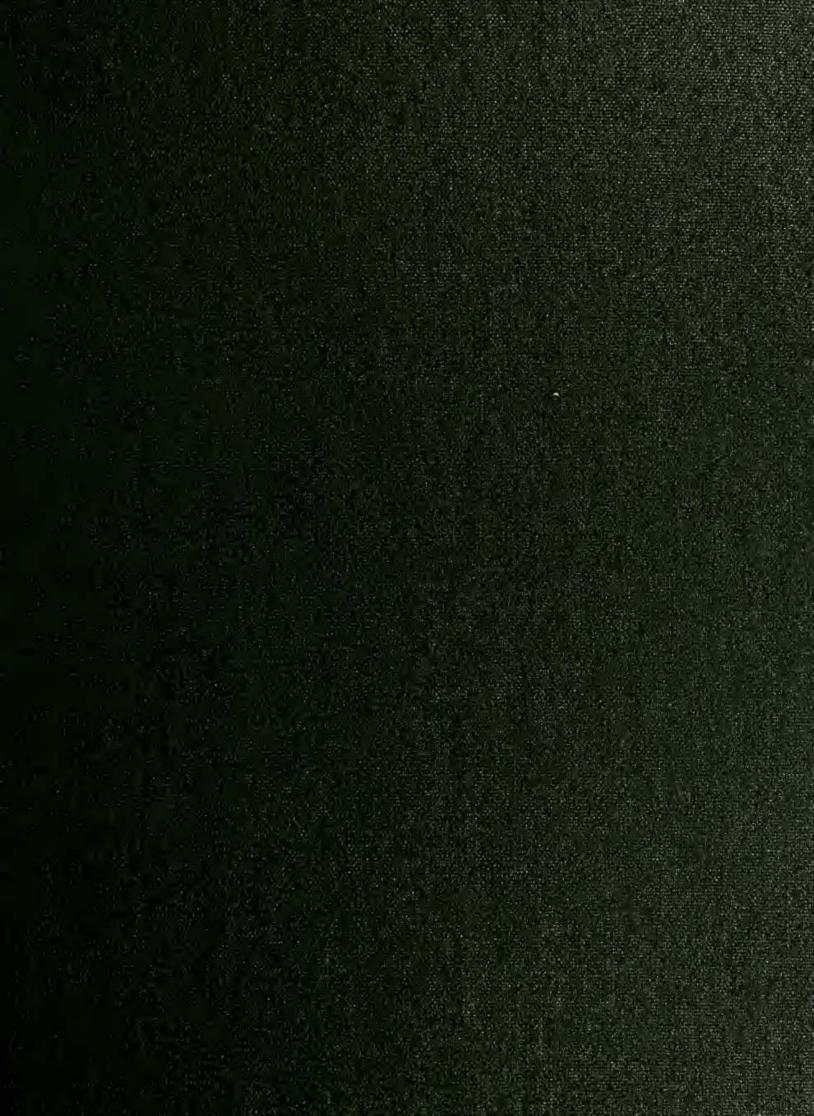
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Price Regulation in Property-Liability Insurance:
A Contingent Claims Approach

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#### ABSTRACT

A Discrete time option pricing model is used to derive the "fair" rate of return for the property liability firm. The rationale for the use of this model is that the financial claims of the policyholders have the characteristics of a European call option written on the firm's asset portfolio. By setting the value of this option equal to the initial surplus, an implicit solution for the fair insurance price may be derived. This approach does not require the direct estimation of risk premiums and thereby offers both analytic and practical advantages over existing methods.



#### I. INTRODUCTION

In recent years, the "fair" rate of return on equity criterion has been used in the regulation of property-liability insurance premiums. As with utility regulation, the "fair" rate of return usually is interpreted as that which would prevail under competitive conditions and in some cases the Sharpe [1964] - Lintner [1965] - Mossin [1966] capital asset pricing model (CAPM) has been used to derive the equilibrium relationship (cf. Hill [1979]; Fairley [1979]). But discontent with this model has led to questioning of its use. In addition to doubt over testability of the CAPM (cf. Roll [1977]), it leaves unexplained some significant pricing anomalies such as the earnings yield and size effects (cf. Reinganum [1981]). Moreover, in applying the CAPM to insurance regulation, estimates of underwriting betas are required in order to determine fair underwriting rates of return. The peculiar difficulties in estimating these underwriting betas are well documented (cf. Fairley [1979]; Hill [1979]; Cummins and Harrington [1985]). The resulting betas are subject to such serious sampling error, or are so unstable, as to be of limited value from a regulatory viewpoint.

In connection with utility regulation, Bower, Bower, and Logue [1984] have recently noted the irony that at the time the CAPM is gaining acceptance by regulators, its preeminent role in the explanation of security returns is being challenged by the arbitrage pricing theory (APT). This paradox applies equally to insurance regulation. Does then the APT offer a more attractive alternative for insurance regulation? An analytic solution for the fair return in an APT

framework has been derived by Kraus and Ross [1982], and attempts at the empirical application of the APT to insurance regulation have already been made by Urritia [1984]. The answer is that it is too early to say. As in the case of the CAPM, doubts have also been raised in the finance literature over the testability of the APT (cf. Shanken [1982]; Dhrymes, Friend and Gultekin [1984]). Furthermore, it is not yet clear that the APT explains the well known pricing anomalies left unanswered by the CAPM. Furthermore, in applying the APT to insurance regulation, the estimation problems associated with calculating underwriting betas remain due to the severe data limitations which have already been noted to exist.

The problems noted with the CAPM, and possibly the APT, imply that the risk premium(s) embodied in the fair rate of return is (are) subject to serious error. Here we suggest an alternative approach that does not require direct calculation of risk premiums. We view the liabilities of the insurer to policyholders (as well as to the government and equityholders) as contingent claims written on the insurer's asset portfolio. If the market value of this asset portfolio is observable, an implicit value for the policyholder's claim may be derived by means of a risk neutral valuation relationship. An explicit specification of the model by which the asset portfolio is valued is not required, nor is it necessary to calculate any further risk premiums to derive the value of the insurance claims. In this way, option pricing techniques may be used to derive the competitive price of the insurance contract and in so doing, to derive the "fair"

rate of return on equity. Since we do not require explicit specification of the asset valuation model, and since the direct calculation of risk premiums for the insurance contracts is not required, we suggest that this approach has both analytic and practical features that make for an attractive alternative to the previous regulatory models.

We will offer an alternative option pricing model which requires restrictions on investor preferences and upon the distributions that underly insurance company investment returns and claim costs. While our model does not yield a closed form solution to the fair rate of return, it can be solved iteratively with a simple algorithm. Indeed, this may be a small price to pay for avoiding the troublesome estimation of risk premiums.

II. BASIC VALUATION RELATIONSHIPS FOR A PROPERTY LIABILITY INSURER

Consider a one period model of the insurance firm in which

investors contribute E paid in equity and policyholders pay premiums

of P. For convenience premiums will be defined net of production and

marketing expenses. Therefore the opening cash flow is

$$(1) Y_0 = E + P$$

The respective claims of the policyholders and the government are discharged at the end of the period, leaving a residual claim for equityholders. Allowing for investment income at a rate  $\mathbf{r}_i$ , we obtain an expression for terminal cash flow  $\mathbf{Y}_1$ 

(2) 
$$Y_1 = (E+kP)(1+r_i)$$

The term k is the funds generating coefficient. This represents an adjustment to compensate for the difference between the period of our model (say one year) and the average delay between receipt of premiums and payment of policyholder claims.

The value  $\mathbf{Y}_1$  is allocated to various claimholders in a set of payoffs having the characteristics of call options.<sup>4</sup> The payoffs to policyholders,  $\mathbf{H}_1$ , and government,  $\mathbf{T}_1$ , are given in the next two equations:

(3) 
$$H_1 = MIN[L, Y_1, 0]$$

(4) 
$$T_1 = MAX[t(\theta(Y_1-Y_0)+P-L),0]$$

where t is the corporate tax rate. The effective rate on the insurer's investment income is considerably less than t in view of its holding of tax exempts, the somewhat lower capital gains rate and the 85 percent shield of dividend income for corporations. The effective tax rate on investments income therefore is denoted  $\theta$ t. Since these claims either directly or indirectly involve the valuation of call options, the appropriate expressions for the values of these claims are given as follows:

(5) 
$$H_0 = V(Y_1) - C(Y_1; L)$$

(6) 
$$T_0 = tC(\theta Y_1; \theta Y_0 - P + L)$$

where:

V( ) = the valuation operator;

The value of the residual claim of the equityholders,  $Q_0$ , is simply the difference between the value of the terminal assets,  $V(Y_1)$ , and the values of the policyholders' and government's claims; i.e.,

$$Q_{0} = V(Y_{1}) - [H_{0} + T_{0}]$$

$$= C(Y_{1}; L) - tC(\theta Y_{1}; \theta Y_{0} - P + L)$$

$$= C_{1} - tC_{2}$$

The regulatory problem may now be couched in straightforward terms. Insurance prices must be set such that a "fair" return is delivered to equityholders. This will be achieved if the current market value of the equity claim  $\mathbf{Q}_0$  is equal to the initial equity investment  $\mathbf{E}_0$ . Noting that  $\mathbf{Y}_1$  is a function of  $\mathbf{P}_0$ , we can state the fair rate of return as that implied by a value of P\* which satisfies the following equation:

(8) 
$$E = C(Y_1(P^*); L) - tC(\theta Y_1(P^*); \theta Y_0 - P + L)$$
$$= C_1 - tC_2$$

Since P is non-stochastic, the selection of a particular value such as  $P_0^*$  merely determines the location parameter of the distribution of  $Y_1$ . Moments of the distribution of  $Y_1$  other than its mean are unaffected by the choice of a specific value for P.

The solution of equation (8) for P\* requires the use of an appropriate option pricing framework, which we present next. Since the payoffs on these call options depend upon the outcomes of the two random variables  $\mathbf{r_i}$  and  $\mathbf{L}$ , our analysis requires the valuation of options with stochastic exercise prices.

# III. AN IMPLICIT SOLUTION FOR THE FAIR RATE OF RETURN The following assumptions are required

- (a) Investment assets held by the insurance firm are competitively priced. Specifically, we assume that the conditions for aggregation are met so that securities are priced as if all investors have the same characteristics as the representative investor.
- (b) The wealth of the representative investor, the rate of return on the insurer's asset portfolio and the aggregate value of the insurer's loss payments are jointly normally distributed.
- (c) The utility function of the representative investor exhibits constant absolute risk aversion.

These assumptions can be used to derive risk neutral valuation relationships in discrete time. Before proceeding, it should be noted that alternative risk neutral valuation relationships can be derived using different sets of assumptions. For example, an alternative discrete time model can be obtained using joint lognormality and constant relative risk aversion. Alternatively, continuous time Black-Scholes type models may be derived by evoking the continuous hedging assumptions. Thus, our solution is illustrative of an entire set of potential option pricing solutions to the fair rate of return on underwriting.

To value the calls  $C(Y_1;L)$  and  $C(\theta Y_1; \theta Y_0 - P_w + L)$ , we must first determine the competitive rate of return on underwriting in the

absence of default risk and tax shield redundancy. Following Hill and Modigliani [1981] and Fairley [1979], the expected rate of return on the insurer's equity, E(r), may be stated as the weighted average of the expected returns on investment  $E(r_i)$  and underwriting  $E(r_i)$ :

(9) 
$$E(r) = [1+k(P/E)](1-\theta t)E(r_i)+(1-t)(P/E)E(r_u)$$

Given the assumptions, the equilibrium return on equity will depend upon the representative investor's absolute risk aversion parameter and the covariance of returns with the wealth of that investor, w.

$$E(r) = r_f + \alpha Cov(r, w)$$

(10)

= 
$$r_f + \alpha[1+k(P/E)](1-\theta t)Cov(r_i, w) + \alpha(1-t)(P/E)Cov(r_i, w)$$

where  $r_f$  is the riskless rate of interest.

Similarly, the equilibrium return on the investment portfolio is

(11) 
$$E(r_i) = r_f + \alpha Cov(r_i, w).$$

An equilibrium condition can be established by equating (9) and (10) which, after substitution of (11) yield the equilibrium return on underwriting

(12) 
$$E(r_u) = -(\frac{1-\theta t}{1-t})kr_F + \frac{E}{P} \frac{\theta t}{1-t} r_F + \alpha Cov(r_u, w)$$

Hill and Modigliani derive a comparable expression using the CAPM, and a similar relationship is derived by Fairley.

To value the call options, we employ the risk neutral valuation framework pioneered by Rubinstein [1976] and extended by Brennan [1979] and Stapleton and Subrahmanyam [1984]. The value of the call option may be written in the form

(13) 
$$C_1 = R_F^{-1} \int_0^\infty Xf'(X) dX$$

where 
$$R_F = 1 + r_f$$
  
 $X = E + (E+kP)r_i + P - L$ 

 $f'(\cdot)$  = the risk neutral density function of  $(\cdot)$  defined by relocating the density function about its certainty equivalent  $E'(\cdot)$ 

 $r_u$  = the rate of underwriting profit, (P-L)/P, in the absence of default risk and tax shield redundancy.

The following properties of X are required

(14) 
$$E'(X) = (E+kP)E'(r_{i}) + E - PE'(r_{u}).$$

$$\sigma_{X} = [(E+kP)^{2}\sigma_{i}^{2} + P^{2}\sigma_{u}^{2} + 2P(E+kP)\sigma_{iu}]^{1/2}$$

However, by the definition of the risk neutral density function, and the expectations given by (11) and (12) we may rewrite E'(X) as follows

$$E'(X) = (E+kP)r_f + E + P[-(\frac{1-\theta t}{1-t})kr_f + \frac{E}{P} \cdot \frac{\theta t}{1-t}r_f]$$

(15) 
$$= E_0 \left[ 1 + \frac{1 - (1 - \theta)t}{1 - t} r_f \right] - kP \left[ \frac{(1 - \theta)t}{1 - t} \right] r_f$$

$$= E_0 \left[ 1 + \frac{r_f}{1 - t} \right] - \left[ E_0 + kP \right] \left[ \frac{1 - \theta t}{1 - t} \right] r_f$$

Since X is normally distributed, equation (13) may be rewritten in terms of the standard normal variate  $z = (X-E'(X))/\sigma_X$ ; hence,

(16) 
$$C_{1} = R_{F}^{-1} \int_{-E'(X)/\sigma_{X}}^{\infty} [E'(X) + \sigma_{X}z] e^{-.5Z^{2}} (2\pi)^{-.5} dz$$

Using the properties of the truncated normal distribution and of the standard normal variate, together with the expressions for E'(X) and  $\sigma_{\chi}$ , the value of the call can be written in the following form.

(17) 
$$C_{1} = R_{F}^{-1} X_{1} N[\frac{X_{1}}{\sigma_{X}}] + R_{F}^{-1} \sigma_{X} n[\frac{X_{1}}{\sigma_{X}}]$$

where 
$$X_1 = E[1 + \frac{r_f}{1-t}] - (E+kP)[\frac{(1-\theta)t}{(1-t)}]r_f$$

 $N(\cdot)$  = the standard normal distribution valued at  $(\cdot)$ 

n(') = the standard normal density valued at (')

The second tax call may be presented as

$$(18) C_2 = R_F^{-1} \int_0^\infty Wf'(W) dW$$

where 
$$W = \partial(E+kP)r_i + P - L$$
  
=  $\partial(E+kP)r_i + Pr_u$ 

Using identical analysis to that shown above, the value of the second call is derived as

(19) 
$$C_2 = R_F^{-1} W_1 N[\frac{W_1}{\sigma_W}] + R_F^{-1} \sigma_W n[\frac{W_1}{\sigma_W}]$$

where 
$$W_1 = E\left[\frac{r_f}{1-t}\right] - [E+kP]\left[\frac{(1-\theta)r_f}{1-t}\right]$$

$$\sigma_{W} = [(E+kP)^{2}\theta^{2}\sigma_{i}^{2} + P^{2}\sigma_{u}^{2} + 2P(E+kP)\theta\sigma_{iu}]^{1/2}$$

Bringing together the valuations (17) and (19) to provide a solution to the fair return requirement (equation (8)) yields

(20) 
$$E = R_f^{-1} \left[ E\left\{ N\left(\frac{X_1}{\sigma_X}\right) + \frac{r_f}{(1-t)} \left[ N\left(\frac{X_1}{\sigma_X}\right) - tN\left(\frac{W_1}{\sigma_W}\right) \right] \right\}$$

$$- \left[ E + kP * \right] \left[ \frac{(1-\theta)t}{1-t} \right] r_f \left[ N\left(\frac{X_1}{\sigma_X}\right) - N\left(\frac{W_1}{\sigma_W}\right) \right]$$

$$+ \sigma_X n\left(\frac{X_1}{\sigma_X}\right) - t\sigma_W n\left(\frac{W_1}{\sigma_W}\right) \right]$$

The implicit solution P\* may be translated into a required rate of underwriting profit  $r_u^{\star}$  by the routine solution of

(21) 
$$r_u^* = \frac{P^* - E(L)}{P^*}$$

Our solution to the fair rate of return  $r_u^*$  depends upon the value of the opening equity, E, the variances of the investment return  $\sigma_1^2$  and underwriting return  $\sigma_u^2$ , the covariance between investment and underwriting returns  $\sigma_{iu}$ , the tax rate t, the tax shield on investment income  $\theta$  and the riskless interest rate  $r_f$ . The virtue of the model

is that it requires no direct calculation of a risk premium for the insurance return. However, we can easily show that the risk premium is implicit in the solution as is a premium for default risk and tax shield redundancy.

We may state the identity

(22) 
$$r_u^* = E(r_u) + [r_u^* - E(r_u)]$$

Recalling the definition of  $E(r_u)$  as the expected underwriting profit with zero probability of default and tax shield redundancy (equations 9-12), we may interpret (22) as the sum of the required return in the absence of default plus a default premium. However decomposing  $E(r_u)$  from equation (12) yields

(23) 
$$\mathbf{r}_{\mathbf{u}}^{*} = \left\{-\left[\frac{1-\theta t}{1-t}\mathbf{k}\right] + \left(\frac{E}{P}\right) \left(\frac{\theta t}{1-t}\right)\right] \mathbf{r}_{\mathbf{f}} + \left\{\alpha \operatorname{Cov}(\mathbf{r}_{\mathbf{u}}, \mathbf{w})\right\} + \left\{\mathbf{r}_{\mathbf{u}}^{*} - \operatorname{E}(\mathbf{r}_{\mathbf{u}})\right\}$$

Thus the required rate of return may be thought of as the sum of (a) the return required in a risk neutral world devoid of the risks of default and tax shield redundancy; (b) the reward for bearing systematic risk in a default-free setting; and (c) a premium which compensates for default risk and the loss of valuable tax shields. This interpretation captures the essence of our model by illustrating that rewards for risk bearing are implicitly provided for even though direct calculation is not required.

IV. OPERATIONAL COMPARISONS WITH ALTERNATIVE REGULATORY MODELS Solution of the fair rate of return requires the following information:

- (i) The opening surplus of the insurer E. This may be calculated by subtracting the value of oustanding policy liabilities from the market value of the insurer's asset portfolio.
- (ii) An estimate of the development of losses to be incurred over the relevant period.
- (iii) Historical data on losses and investment income. These are required to provide estimates of  $\sigma_{iu}$  and  $\sigma_{iu}$ .
- (iv) The riskless interest rate for current bills spanning the regulated period.

All these data are required both for the application of alternatives, such as CAPM and APT, and for the current option pricing model. Data deficiencies will equally affect this and competing models. Of particular concern is the need for a market valuation of the insurer's investment portfolio. While stocks are reported at "end of year" market quotations, bonds are recorded of amortized values. This data limitation faces any serious attempt to estimate the fair return for an insurance contract.

But there are operational differences. CAPM and APT regulatory solutions require explicit calculation of risk premia for the insurance contract. In turn, this calls for calculation of underwriting betas. Underwriting betas may be calculated from market data (at least for the handful of insurance firms with traded equity) or from accounting data. But the estimation problems associated with each are considerable and well documented [cf. Biger and Kahane [1978]; Hill [1979]; Fairley [1979]; Hill and Modigliani [1981]; Myers and Cohn [1981]; Cummins and Harrington [1985]). In contrast, the option model does not require the explicit calculation of risk premia. A risk neutral valuation relationship is defined in relation to the observable value of the asset portfolio. Consequently, we are not

plagued with problems of accepting proxies for the market portfolio a la CAPM. Nor do we encounter problems of identifying the priced factors that beset APT. Nor again are we reduced to using accounting underwriting data to proxy market values. These are significant advantages for the option approach. And as a corollary, it may be noted that the option model does not require forecasts of the return on the market portfolio or other priced indices. Instead, the option approach impounds the expectations of investors as reflected in the current market value of the insurance firm's asset portfolio.

#### V. CONCLUSION

We have developed a contingent claim model for estimating the fair rate of return for the property liability insurance firm. This model offers an alternative regulatory device to the Capital Asset Pricing Model and to Arbitrage Pricing Theory. Such an alternative is considered to be useful in light of the unsettled academic score about whether APT has, or has not, succeeded CAPM as the appropriate asset pricing paradigm. Moreover, the proposed option model offers operational advantages over these alternatives since direct calculation of risk premia are not required. Consequently, the troublesome beta estimates are not needed.

Clearly the option model and its rivals are strictly applicable only in the circumstances that each postulates. Our particular model is developed in discrete time and requires both restrictions on the preference functions of investors and normality. Clearly such restrictions are burdensome. However, option pricing models (and, for our purposes, option pricing models with stochastic striking prices)

can be motivated differently. Alternative preference restrictions have been used together with different distributional assumptions, though these have not been used in regulatory applications to our knowledge. Other possibilities arise from the more familiar continuous time option pricing framework with the continuous hedge assumptions. Our model is intended to represent a menu of such regulatory approaches using option pricing theory.

Finally, our model specifically applies to the property liability insurance firm. The feature of this particular institution that lends it to option pricing application is that its output is a contingent financial claim and that this claim is written on an underlying asset for which a reasonable market value can be provided. With the possible exception of deposit banking, such conditions do not necessarily prevail in other regulated industries.

#### NOTES

The economics which underly the notion of a fair rate of return to underwriting for a multi-line insurance firm are conceptually quite similar to the economics of a multidivisional nonfinancial firm in which each division is assigned its own unique, or divisional cost of capital.

When we speak of risk neutral valuation, we do not mean that investors are risk neutral. Rather, this terminology implies that there is something about the economic structure of the valuation problem that makes it possible to value contingent claims as if such claims and their underlying assets are traded in a risk neutral economy. In the case of the original Black-Scholes [1973] article, the underlying economic structure involves the formation by investors of arbitrage-free riskless hedge portfolios in continuous time. if riskless hedging is not feasible (e.g., as in the case of infrequently traded or non-traded assets), Rubinstein [1976] has shown that if the representative investor exhibits constant proportional risk aversion and the underlying asset price is bivariate lognormally distributed with respect to aggregate wealth, then the resulting valuation relationship between the contingent claim and the underlying asset is also compatible with risk neutral investor preferences. Brennan's [1979] extension of Rubinstein demonstrates, among other things, that a risk neutral valuation relationship also obtains under the assumptions of bivariate normality and constant absolute risk aversion on the part of the representative investor.

Depending upon the type of risk being insured, the time lag between the receipt of the premium and payment of the claim can vary considerably. For example, most casualty insurance lines are characterized by claim delays of less than one year, whereas most liability lines have claim delays of more than one year. Consequently, for every dollar of premiums written, lines of insurance with longer claim delays generate more investable funds than insurance lines with shorter claim delays. Therefore, the "funds-generating coefficient" can be interpreted as the average amount of investable funds per dollar of annual premiums. This type of adjustment is also used in the papers by Hill and Fairley, Biger and Kahane [1978], and Hill and Modigliani [1981].

For the past decade or so, financial economists have applied option pricing theory to the valuation of corporate financial claims. In their seminal article, Black and Scholes [1973] suggest that the equity of a levered firm can be valued as a call option on the terminal value of the firm, with an exercise price equal to the face value of debt. Galai and Masulis [1976] combine Merton's [1973] continuous time CAPM with the Black-Scholes option pricing model in order to value levered equity and investigate the valuation and risk effects of changes in corporate investment policy. Galai [1983] extends the contingent claim formulation of the firm's capital structure to a

valuation of the government's tax claim. Like Galai, we view share-holders as holding opposite positions in two separate call options. Specifically, shareholders hold a long position in a call option on the pre-tax terminal value of the insurer's investment portfolio, and a short position in a call option on the taxable income derived from that portfolio.

 $^5\vartheta$  is a factor of proportionality defined over the interval [0,1]. This parameter is functionally related to the composition of the insurer's investment portfolio. For example, if the investment portfolio is comprised of strictly tax-exempt securities, then  $\theta$  = 0. Conversely, if only fully taxable claims such as corporate bonds and U.S. Treasury securities are chosen, then  $\theta$  = 1.

<sup>6</sup>Fischer (1978) is to be credited for first addressing the pricing of an option with a stochastic exercise price. Stapleton and Subrahmanyam present an alternative derivation which is at odds with Fischer's result because the valuation relationship which results is risk neutral, whereas Fischer's valuation relationship involves expected returns other than the riskless rate of interest. Like Stapleton and Subrahmanyam, the valuation relationships which we derive are risk neutral.

Since we only consider corporate income taxation, the riskless rate of interest is simply the before-tax rate of interest on riskless bonds (e.g., T-bills). However, in the presence of personal and corporate taxes, it is not entirely clear whether the riskless rate of interest is the before-tax rate of interest on riskless bonds or the certainty-equivalent municipal bond rate. If investors are able to "launder" all of their personal taxes a la Miller and Scholes [1978], then  $r_f$  would continue to be defined as the before-tax rate of interest on riskless bonds. However, if investors are not able to launder taxes on investment income, then the certainty-equivalent municipal bond rate is the appropriate rate. For a lucid discussion of these points, see Hamada and Scholes [1985].

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