

Price Rigidity and the Volatility of Vacancies and Unemployment^{*}

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Abstract

The successful matching model developed by Mortensen and Pissarides seems to find its hardest task in explaining the cyclical movements of some key labor market variables such as the vacancy rate and the vacancy-unemployment ratio. Several authors have discussed mechanisms compatible with the matching technology that are able to deliver the kind of correlations observed in the data. In this paper we explore the contribution of price rigidity, within the framework of a full blown SDGE model, to explain the dynamics of these variables. We find that price rigidity greatly improves the model's empirical performance making it capable of reproducing second moments of the data, in particular those related to the vacancy rate and market tightness. Other realistic features of these models such as intertemporal substitution, endogenous match destruction and capital accumulation do not seem to play a relevant role in a flexible prices setting.

Keywords: unemployment, vacancies, business cycle, price rigidities

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1. Introduction

The Mortensen and Pissarides model provides an engaging explanation of the determinants of unemployment dynamics (see Mortensen and Pissarides, 1999, and the refer-

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ences therein). While the model has gained widespread acceptance as a theory of the Natural Rate of unemployment its implications for the dynamics of some key labor market variables at the business cycle frequency are less well accepted. In a widely cited paper, Shimer (2005) argues that the model is incapable of reproducing the volatility of unemployment, vacancies and the vacancy-unemployment (v/u) ratio observed in the data for a reasonable parameter calibration. This is most unfortunate, as the Mortensen and Pissarides model has become the workhorse for introducing unemployment and labor market frictions in a coherent and yet tractable way in dynamic general equilibrium models. Several authors have looked at this issue in more detail and found that the ability of the model to match data moments can be enhanced by enlarging the model in different directions (for example, Mortensen and Nagypál, 2005, Hagedorn and Manovskii, 2005, or Costain and Reiter, 2005). A very promising line of research has emphasized the role of wage rigidity as a means of overcoming the shortcomings of the basic model (see, for example, Shimer, 2004, Hall, 2005a, Gertler and Trigari, 2005, Blanchard and Galí, 2006, Pissarides, 2007, and Bodart, Pierrard and Sneessens, 2005). In particular, Gertler and Trigari (2005) forcefully argue that nominal wage stickiness in the form of a Calvo (1983) adjustment process of the Nash bargaining wage moderates the volatility of real wages making labor market variables more volatile.

In this paper we take an alternative stance and approach the issue in a complementary way. Like Gertler and Trigari (2005) and den Haan, Ramey and Watson (2000), we argue that the model performance at business cycle frequency can be greatly improved by embedding the basic search and matching model in a broader general equilibrium framework, but we stick to the assumption of wage flexibility and explore other mechanisms instead, namely, endogenous separation rates, price rigidity, intertemporal substitution, capital and taxes. These seemingly unrelated features may have different or even offsetting effects on the model's capability to match the data but have, nonetheless, something in common: they all bring the model closer to a state-of-the-art SDGE model and thus provide a richer framework to assess the usefulness of the search and matching structure to explain the data. Besides, each of these mechanisms is relevant on its own. Endogenous separation seems the right choice if we want to give firms an additional margin with which to optimize and adjust employment in the presence of technology shocks. Price rigidity might contribute to smoothing out the response of real wages. Real interest rate fluctuations affect the present value of future surpluses. Capital accumulation is a key component of a model of business cycle fluctuations whose interaction with the labor market cannot be ignored. Finally, distortionary taxes influence the response of investment and the net values of surpluses, thus affecting unemployment and vacancies.

Our main result is that price rigidity is critical for the model to deliver the historical volatility of the vacancy rate and the unemployment-vacancy ratio. We see price rigidity as mechanism akin to that of wage stickiness. Under price stickiness supply shocks generate large swings in the mark-up that greatly amplify fluctuations in the expected surplus of matches and the value of vacancies. Thus the incentive to post new vacancies becomes much more sensitive to variations in productivity than in a flexible price environment.

We also discuss the role of other realistic model features. Among these only endogenous destruction makes a significant contribution to the volatility of labor market rates although taking the model farther away from the data. Endogenous separation moderates (enhances) match destruction following positive (negative) technology shocks, thus reducing the response of vacancy posting. Other additional features also help the model in predicting higher volatility but their qualitative importance is smaller as compared with that of price rigidity.

The rest of the paper is organized as follows. In the second section we outline a general version of the model used in the paper. In the third section we present the empirical evidence and discuss the calibration in detail. Section fourth presents the main results summarized above and the fifth section concludes.

2. The model

There are three types of agents in this economy: firms, workers and the government. Households maximize the discounted present value of expected utility operating in perfect capital markets. They offer labor and store their wealth in bonds and capital. The productive sector is organized in three different levels: (1) firms in the wholesale sector (indexed by j) use labor and capital to produce a homogenous good that is sold in a competitive flexible price market; (2) the homogenous good is bought by firms (indexed by \tilde{j}) and converted, without the use of any other input, into a firm specific variety that is sold in a monopolistically competitive market, in which prices may not be flexible; (3) finally there is a competitive retail aggregator that buys differentiated varieties ($y_{\tilde{j}t}$) and sells a homogeneous final good (y_t) with flexible prices. Thus, the model embeds Mortensen and Pissarides trading technology in the labor market into a fairly general equilibrium model with capital and sticky prices. Therefore, our model extends den Haan, Ramey and Watson (2000) to an economy with sticky prices, and generalizes Walsh (2005) to an economy with capital.

2.1 Households

Households maximize the β discounted present value of the following utility function,

$$\bar{U}_{it}(c_{it}^*, A_i) = U(c_{it}^*) - \chi_{it} A_i \quad (1)$$

where:

$$U_i(c_{it}^*) = \frac{(c_{it}^*)^{1-\sigma}}{1-\sigma} \quad (2)$$

$$c_{it}^* = \frac{c_{it}}{c_{it-1}^h} \quad (3)$$

and h is a parameter that if different from zero indicates the presence of consumption habits, A_i stands for the disutility of working with $\chi_i = 1$ if the worker is employed and $\chi_i = 0$ otherwise. The budget constraint is given by

$$(1 + \tau_t^c) c_{it} + e_{it} + \frac{M_{it}}{P_t} + \frac{B_{it}}{P_t} = \left[\begin{array}{l} \chi_{it} y_{it}^l + (1 - \tau_t^k) r_t k_{it-1} + \\ \frac{M_{it-1}}{P_t} + (1 + i_{t-1}) \frac{B_{it-1}}{P_t} + \int_0^1 \frac{\Omega_{i\tilde{j}t}}{P_t} d\tilde{j} \\ + (1 - \chi_{it}) \tilde{g}_t^u + g_t^s + \frac{M_{it}^s}{P_t} \end{array} \right] \quad (4)$$

where c_{it} stands for real consumption, e_{it} for real investment, M_{it} are money holdings, B_{it} bond holdings, r_t the real return on capital, i_t nominal interest rate, and $\Omega_{i\tilde{j}}$ is the share of profits from the $\tilde{j}th$ monopolistically competitive firm in the intermediate sector, that flows to household i . \tilde{g}^u is the unemployment benefit, g_i^s is a lump sum transfer from the government, k_{it-1} is the stock of capital at the end of period $t - 1$ held by household i , y_{it}^l represents household's disposable real labor income (see the definition below) and M_{it}^s the monetary transfers from the government (in aggregate, $M_t^s = M_t - M_{t-1}$). The model has taxes on capital (τ_t^k) and labor (τ_t^w) incomes, and consumption (τ_t^c).

Money is required to make transactions,

$$P_t (1 + \tau_t^c) c_{it} \leq M_{it-1} + M_{it}^s \quad (5)$$

and households accumulate capital for which they have to pay installation costs ϕ_t and then rent it to firms at the rental cost r_t

$$k_{it} = (1 - \delta) k_{it-1} + \phi_t k_{it-1} \quad (6)$$

where $\phi_t = \phi \left(\frac{e_{it}}{k_{it-1}} \right)$. We further assume that households are homogenous and that they pool their incomes at the end of the period (perfect risk sharing) regardless of

their employment status. This makes the first order conditions symmetric across households:

$$\frac{c_t^{-\sigma}}{c_{t-1}^{1-\sigma}} - E_t \beta h \frac{c_{t+1}^{1-\sigma}}{c_t^{h(1-\sigma)+1}} - \lambda_{1t} (1+\tau^c) - \lambda_{2t} (1+\tau^c) = 0 \quad (7)$$

$$\lambda_{1t} - \lambda_{3t} \phi' = 0 \quad (8)$$

$$\begin{aligned} E_t \beta \lambda_{1t+1} (1 - \tau_{t+1}^k) r_{t+1} - \lambda_{3t} + \\ E_t \beta \lambda_{3t+1} \left[(1 - \delta) + \phi_t - \phi_t' \frac{e_{t+1}}{k_t} \right] = 0 \end{aligned} \quad (9)$$

$$\lambda_{1t} - E_t \beta \lambda_{1t+1} \frac{P_t}{P_{t+1}} - E_t \beta \lambda_{2t+1} \frac{P_t}{P_{t+1}} = 0 \quad (10)$$

$$\lambda_{1t} - E_t \beta \lambda_{1t+1} (1 + i_t) \frac{P_t}{P_{t+1}} = 0 \quad (11)$$

where λ_{1t+1} is the Lagrangian multiplier associated to the budget constraint, λ_{2t+1} is the Lagrangian multiplier associated to the CIA constraint and λ_{3t+1} is the Lagrangian multiplier associated to the law of motion of capital. Expressions (8)-(11) can be rearranged in a more familiar format

$$E_t \lambda_{2t+1} = i_t E_t \lambda_{1t+1} \quad (12)$$

$$\lambda_{1t} \beta^{-1} = (1 + i_t) E_t \left(\lambda_{1t+1} \frac{P_t}{P_{t+1}} \right) \quad (13)$$

$$\frac{\lambda_{3t}}{\lambda_{1t}} = [\phi_t']^{-1} = q_t \quad (14)$$

$$q_t \beta^{-1} = E_t \left\{ \frac{\lambda_{1t+1}}{\lambda_{1t}} \left((1 - \tau_t^k) r_{t+1} + q_{t+1} \left[(1 - \delta) + \phi_t - \phi_t' \frac{e_{t+1}}{k_t} \right] \right) \right\} \quad (15)$$

where we express the ratio of shadow prices as the Tobin's q .

2.2 The competitive retail sector and aggregation

There is a competitive retail aggregator that buys differentiated goods to firms in the intermediate sector and sells a homogeneous final good y_t at price P_t . Each variety y_{jt} is purchased at a price P_{jt} . Profit maximization by the retailer implies

$$\text{Max}_{y_{jt}} \left\{ P_t y_t - \int P_{jt} y_{jt} d\tilde{j} \right\}$$

subject to,

$$y_t = \left[\int y_{jt}^{(1-1/\theta)} d\tilde{j} \right]^{\frac{\theta}{\theta-1}} \quad (16)$$

where $\theta > 1$ is a parameter that can be expressed in terms of the elasticity of substitution between intermediate goods $\varkappa \geq 0$, as $\theta = (1 + \varkappa) / \varkappa$.

The first order condition gives us the following expression for the demand of each variety:

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} y_t \quad (17)$$

Also from the zero profit condition of the aggregator the retailer's price is given by:

$$P_t = \left[\int_0^1 \left(\frac{P_{jt}}{P_t} \right)^{1-\theta} d\tilde{j} \right]^{\frac{1}{1-\theta}} \quad (18)$$

2.3 The monopolistically competitive intermediate sector

The monopolistically competitive intermediate sector is composed of $\tilde{j} = 1, \dots, \tilde{J}$ firms each one buying the production of competitive wholesale firms at a common price P_t^w and selling a differentiated good at a price P_{jt} to the final competitive retailing sector described above.

Variety producers y_{jt} set prices in a staggered fashion. Following Calvo (1983) only some firms set their prices optimally each period. Those firms that do not reset their prices optimally at t adjust them according to a simple indexation rule to catch up with lagged inflation. Thus, each period a proportion ω of firms simply set $P_{jt} = (1 + \pi_{t-1})^\varsigma P_{jt-1}$ (with ς representing the degree of indexation and π_{t-1} the inflation rate in $t-1$). The fraction of firms (of measure $1 - \omega$) that set the optimal price at t seek to maximize the present value of expected profits. Consequently, $1 - \omega$ represents the probability of adjusting prices each period, whereas ω can be interpreted as a measure of price rigidity. Thus, the maximization problem of the representative variety producer

can be written as:

$$\max_{P_{jt}^*} E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \omega^s \left[P_{jt}^* \bar{\pi}_{t+s} y_{jt+s} - P_{t+s} m c_{jt,t+s} y_{jt+s} \right] \quad (19)$$

subject to

$$y_{jt+s} = \left(P_{jt}^* \prod_{s'=1}^s (1 + \pi_{t+s'-1})^\zeta \right)^{-\theta} P_{t+s}^\theta y_{t+s} \quad (20)$$

where P_{jt}^* is the price set by the optimizing firm at time t , $m c_{jt,t+s} = \frac{P_{t+s}^w}{P_{t+s}} = \mu_{t+s}^{-1}$ represents the real marginal cost (inverse mark-up) borne at $t + j$ by the firm that last set its price in period t , P_{t+s}^w the price of the good produced by the wholesale competitive sector, and $\Lambda_{t,t+s}$ is a price kernel which captures the marginal utility of an additional unit of profits accruing to households at $t + s$, i.e.,

$$\frac{E_t \Lambda_{t,t+s}}{E_t \Lambda_{t,t+s-1}} = \frac{E_t (\lambda_{1t+s} / P_{t+s})}{E_t (\lambda_{1t+s-1} / P_{t+s-1})} \quad (21)$$

$$P_{jt}^* = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\beta \omega)^s \Lambda_{t,t+s} \left[\mu_{t+s}^{-1} (P_{t+s})^{\theta+1} y_{t+s} \left(\prod_{s'=1}^s (1 + \pi_{t+s'-1})^\zeta \right)^{-\theta} \right]}{E_t \sum_{s=0}^{\infty} (\beta \omega)^s \Lambda_{t,t+s} \left[(P_{t+s})^\theta y_{t+s} \left(\prod_{s'=1}^s (1 + \pi_{t+s'-1})^\zeta \right)^{1-\theta} \right]} \quad (22)$$

Then, taking into account (18) and that θ is assumed time invariant, the corresponding aggregate price the aggregate price level in the retail price sector is given by,

$$P_t = \left[\omega (P_{t-1} \pi_{t-1}^\zeta)^{1-\theta} + (1 - \omega) (P_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (23)$$

2.4 The competitive wholesale sector

The competitive wholesale sector consists of $j = 1, \dots, J$ firms each one selling a different quantity of a homogeneous good to the same price P_t^w to the monopolistically competitive intermediate sector. Firms in the perfectly competitive wholesale sector carry out the actual production using labor and capital. Each producing firm employs one worker and technology is given by,

$$y_{jt} = z_t a_{jt} k_{jt}^\alpha \quad (24)$$

where k_{jt} is the amount of capital (capital-labor ratio) optimally decided by the firm, z_t is a common aggregate AR(1) shock with root ρ_z and a_{jt} is a firm specific productivity shock. Both shocks have a mean of 1. Nominal income at t is $P_t^w y_{jt}$ but only becomes available in period $t+1$; thus, real income is given by $\frac{P_t^w}{P_{t+1}} y_{jt}$. Present value real income is given by,

$$\left(\frac{1}{1+i_t} \right) \frac{P_t^w}{P_t} y_{jt} = \left(\frac{1}{1+i_t} \right) \frac{z_t a_{jt} k_{jt}^\alpha}{\mu_t} \quad (25)$$

where we have made use of the appropriate discount factor obtained from (11),

$$\beta E_t \left(\frac{\lambda_{1t+1} P_t}{\lambda_{1t} P_{t+1}} \right) = \frac{1}{1+i_t} \equiv \frac{1}{R_t} \quad (26)$$

2.5 Bargaining

Let us normalize the population to 1. Matching and production take place in the whole-sale sector. At the beginning of period t some workers and firms are matched while others are not. In particular, workers start period t either matched (n_t) or unmatched ($1 - n_t$). Some of these matches are destroyed throughout this period while others are created. Unmatched firms and those whose match is severed at that period decide whether or not to post a vacancy. This decision is studied later. Posted vacancies are visited randomly by unemployed workers and all visited vacancies are occupied so that a new match occurs.

In period t not all matches become productive. Before production takes place there is an exogenous probability ρ^x of the match being severed, so only $(1 - \rho^x)n_t$ matches survive this exogenous selection. Surviving matches observe the realization of the random firm specific productivity shock a_{jt} . If a_{jt} is higher than some (endogenous) threshold a'_{jt} then the match becomes a productive firm, otherwise ($a_{jt} < a'_{jt}$) the match is (endogenously) severed with probability

$$\rho_{jt}^n = I(a'_t) = \int_{-\infty}^{a'_{jt}} \varphi(a_{jt}) da_{jt} \quad (27)$$

so the (match specific) survival rate is given by $\rho_{jt}^s = (1 - \rho_{jt}) = (1 - \rho^x) \left(1 - I(a'_{jt}) \right)$ where $\rho_{jt} = \rho^x + (1 - \rho^x)\rho_{jt}^n$ is the proportion of matches that do not survive.

We define by $u_t \equiv (1 - n_t) + \rho_t n_t$ the number of workers that are unemployed during period t . Notice that this variable is neither the beginning nor the end of period unemployment rate but rather the amount of workers that have been unemployed at some point during period t . These unemployed workers are actively looking for va-

cancies that will eventually become productive (if they ever do) in $t + 1$. The number of new matches in period t is ϑ , so that employment evolves according to:

$$n_{t+1} = (1 - \rho_t)n_t + \vartheta \quad (28)$$

The number of matches in period t depends on the amount of vacancies posted and unemployed workers looking for jobs. The mapping from u_t and v_t into the number of matches is given by an aggregate matching function $\vartheta(u_t, v_t)$. The probability of a worker finding a job is given by

$$\rho_t^w = \frac{\vartheta(u_t, v_t)}{u_t} \quad (29)$$

and similarly, the probability of firm with a posted vacancy actually finding a match is

$$\rho_t^f = \frac{\vartheta(u_t, v_t)}{v_t} \quad (30)$$

Let us look at the choices the firm makes throughout this process in more detail. When a vacancy is visited the job offer is accepted with probability $1 - \rho_{jt}$. With probability ρ_{jt} the match is severed. The joint payoff of this match is

$$\left[\left(\frac{1}{1+i_t} \right) \frac{z_t a_{jt} k_{jt}^\alpha}{\mu_t} - r_t k_{jt} \right] - A + x_{jt} \quad (31)$$

where x_t is the expected current value of future payoffs obtained if the relationship continues into the next period. A match continues if the expected payoff (31) compensates for the loss of alternative opportunities available to firms and workers. There are no alternative opportunities for firms and the alternative opportunities for workers is the value if unemployed w_{jt}^u , where $w_{jt}^u = \tilde{g}^u + \overline{w_{jt}^u}$ is the present value of future worker opportunities if unemployed in period t to be defined below and \tilde{g}^u represents unemployment compensation.

The threshold specific shock a'_{jt} below which existing matches do not produce satisfies

$$\left[\frac{z_t a'_{jt} \left(k_{jt}^* \right)^\alpha}{(1+i_t)\mu_t} - r_t k_{jt}^* \right] - A + x_{jt} - w_{jt}^u = 0 \quad (32)$$

that is evaluated at k_{jt}^* , which represents the optimal value of capital had a'_{jt} occurred.

This optimal capital (labor ratio) is given by:

$$k_{jt}^* = \left(\frac{\alpha z_t a'_{jt}}{(1+i_t)\mu_t r_t} \right)^{\frac{1}{1-\alpha}} \quad (33)$$

If production takes place the firm chooses its capital optimally to satisfy,

$$\max_{k_{jt}} \left[\left(\frac{1}{1+i_t} \right) \frac{z_t a_{jt} k_{jt}^\alpha}{\mu_t} - r_t k_{jt} \right] - A + x_{jt} \quad (34)$$

$$\frac{\alpha z_t a_{jt} k_{jt}^{\alpha-1}}{(1+i_t)\mu_t} - r_t = 0 \rightarrow k_{jt}^* = \left(\frac{\alpha z_t a_{jt}}{(1+i_t)\mu_t r_t} \right)^{\frac{1}{1-\alpha}} \quad (35)$$

Define $x_t^u = x_t - w_t^u$ as the expected excess value of a match that continues into period $t + 1$ and s_{jt+1} as the joint surplus of a match at the start of $t + 1$, then for the optimal capital

$$s_{jt+1}^* = \left[\left(\frac{1}{1+i_{t+1}} \right) \frac{z_{t+1} a_{jt+1} (k_{jt+1}^*)^\alpha}{\mu_{t+1}} - r_{t+1} k_{jt+1}^* \right] - A + x_{jt+1}^u \quad (36)$$

The wage is determined as a result of a Nash bargaining process whereby the surplus is split among the worker and the firm according to the relative bargaining power of each side. In particular, a proportion η of the surplus will be received by the worker, who pays $\tau_{t+1}^w \eta s_{jt+1}^*$ in taxes, while the firm receives $1 - \eta$ of the match surplus. Hence total after tax labor income is given by

$$y_{jt+1}^l = (1 - \tau_{t+1}^w) [\eta s_{jt+1}^* + A - x_{jt+1}^u] \quad (37)$$

while the government receives a total of per match:

$$\tau_{t+1}^w (\eta s_{jt+1}^* + A - x_{jt+1}^u) \quad (38)$$

The firm will receive $(1 - \eta) s_{jt+1}^* + r_{t+1} k_{jt+1}^*$ that is used to pay the rental cost of capital and the vacancy posting costs. Total production can be obtained by adding up total rents.

An unemployed worker at t finds a match with probability ρ_t^w . With probability $1 - \rho_t^w(1 - \rho_{t+1})$ the worker either fails to make a match or makes a match that does not produce in $t + 1$. In either case the worker only receives w_{t+1}^u . The expected discounted value net of taxes for an unmatched worker, and hence her relevant opportunity cost of

being matched, is:²

$$w_{jt}^u = \tilde{g}_t^u + \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) \left[\rho_t^w (1 - \rho^x) \int_{a'_{t+1}}^{a_{\max}} (1 - \tau_{t+1}^w) \eta s_{jt+1}^* \varphi(a_i) da_i + w_{jt+1}^u \right] \quad (39)$$

Existing matches produce in $t + 1$ with probability $1 - \rho_{t+1}$. In this case the worker will receive y_{t+1}^l net of taxes. For a worker and firm already matched the joint discounted value of an existing match is $(1 - \eta) s_{jt+1}^* + (1 - \tau_{t+1}^w) \eta s_{jt+1}^* + w_{t+1}^u$, with probability $1 - \rho_{t+1}$, and w_{t+1}^u , with probability ρ_{t+1} . This allows us to write the expected current value of future payoffs of an existing match as:

$$x_{jt} = \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) \left[(1 - \rho^x) \int_{a'_{t+1}}^{a_{\max}} (1 - \eta \tau_{t+1}^w) s_{jt+1}^* \varphi(a_j) da_j + w_{jt+1}^u \right] \quad (40)$$

Therefore:

$$x_{jt}^u \equiv x_{jt} - w_{jt}^u = \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) (1 - \rho^x) [1 - \eta \rho_t^w - \eta \tau_{t+1}^w (1 - \rho_t^w)] \int_{a'_{t+1}}^{a_{\max}} s_{jt+1}^* \varphi(a_j) da_j - \tilde{g}_t^u \quad (41)$$

Unmatched firms or those whose matches terminated may enter the labor market and post a vacancy. Posting a vacancy costs γ per period and the probability of filling a vacancy is ρ_t^f . Free entry ensures that

$$\beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) \rho_t^f (1 - \rho^x) \int_{a'_{t+1}}^{a_{\max}} (1 - \eta) s_{jt+1}^* \varphi(a_t) da = \gamma \quad (42)$$

hence

$$x_{jt}^u = \frac{\gamma [1 - \eta \rho_t^w - \eta \tau_{t+1}^w (1 - \rho_t^w)]}{\rho_t^f (1 - \eta)} - \tilde{g}_t^u \quad (43)$$

Equilibrium in the capital market is determined by the following market-clearing condition:

$$(1 - \rho^x) n_t \int_{a'_t}^{\infty} k_{jt}^* \varphi(a_t) da_t = k_{it-1} \quad (44)$$

where the left hand side indicates the demand for capital to produce in t and the right hand side is the supply of capital available to produce in t derived from (7) to (11).

² Note that recursivity in equation (39) implies a permanent flow of income from \tilde{g}_t^u that should be taken into account in the calibration.

2.6 Aggregation

The economy-wide level of output can be obtained either by looking at production by the monopolistic firms (\tilde{j}) or aggregating across all competitive productive units (j). To clarify the matter, consider the following relationships that hold in our model. The nominal value of total production can be expressed in terms of the different varieties:

$$P_t y_t = \int P_{\tilde{j}t} y_{\tilde{j}t} d\tilde{j} \quad (45)$$

which does not imply total output (y_t) being equal to the integral of varieties produced by monopolistic firms, $\int y_{\tilde{j}t} d\tilde{j}$.

However, turning to the competitive wholesale sector, it is also true that

$$P_t^w y_t = \int P_t^w y_{jt} d_j \quad (46)$$

and thus

$$y_t = \int y_{jt} d_j \quad (47)$$

that implies

$$\int y_{jt} d_j = \left[\int y_{\tilde{j}t}^{(1-1/\theta)} d\tilde{j} \right]^{\frac{\theta}{\theta-1}} \quad (48)$$

Total production therefore can be obtained by aggregating the output from the competitive wholesale firms.

Due to the presence of the match idiosyncratic shock, aggregation requires a double integral, one for all possible realizations of the specific shock and the other for all firms that actually produce. The result of the latter integral gives the number of active matches $(1 - \rho_t)n_t$, whereas the former integral can be interpreted as the average realization of the shock. Therefore aggregate output net of vacancy costs of the wholesale sector is obtained from:

$$y_t = (1 - \rho_t)n_t z_t \int_{a'_t}^{a_{\max}} a_t (k_{jt}^*)^\alpha \frac{\varphi(a_t)}{1 - I(a'_t)} da - \gamma v_t \quad (49)$$

or,

$$y_t = (1 - \rho^x)n_t z_t \left(\frac{\alpha z_t}{(1+i_t)\mu_t r_t} \right)^{\frac{\alpha}{1-\alpha}} \int_{a'_t}^{a_{\max}} a^{\left(\frac{1}{1-\alpha}\right)} \varphi(a_t) da - \gamma v_t \quad (50)$$

where we have considered that the distribution function for a_j is common across firms and independent over time. Aggregation also implies that the average optimal capital

and the average joint surplus of the match at the start of $t + 1$ can be represented as:

$$k_t^* = \int_{a'_t}^{a_{\max}} k_{jt}^* \frac{\varphi(a_t)}{1 - I(a'_t)} da \quad (51)$$

$$s_{t+1}^* = \int_{a'_{t+1}}^{a_{\max}} s_{jt+1}^* \frac{\varphi(a_t)}{1 - I(a'_{t+1})} da \quad (52)$$

Given that the expected excess value of a match is equal for all matches, we can suppress the subindex j and write x_{jt}^u as x_t^u .

2.7 Government

Tax revenues are defined as:

$$t_t = \tau_t^c c_t + \tau_t^k r_t k_{t-1} + \tau_t^w (1 - \rho_t) n_t (\eta s_t^* + A - x_t^u) \quad (53)$$

The budget constraint in real terms for the government is defined by:

$$\frac{M_t}{P_t} + \frac{B_t}{P_t} = (1 + i_{t-1}) \frac{B_{t-1}}{P_t} = g_t^c + g_t^s + g^u u_t + \frac{M_{t-1}}{P_t} + \frac{M_t^s}{P_t} - t_t \quad (54)$$

where g_t^c represents public consumption. Define $b_t = \frac{B_t}{P_t}$ and $\pi_t = \frac{P_t}{P_{t-1}}$. Given the definition in aggregate for M_t^s is reduced to:

$$b_t - (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} = g_t^c + g_t^s + g^u u_t - t_t \quad (55)$$

To close the model it is necessary to specify both a fiscal rule and a monetary rule. The fiscal rule avoids explosive path of public debt (Leeper, 1991, Woodford, 1996, Leith and Wren-Lewis, 2000, or Andrés and Doménech, 2006) and reflects the adjustment of public expenditure φ to deviation from a debt objective:

$$g_t^\varphi = g_{t-1}^\varphi + \psi_1^\varphi \left[\overline{\left(\frac{b}{y}\right)} - \left(\frac{b_t}{y_t}\right) \right] + \psi_2^\varphi \left[\left(\frac{b_{t-1}}{y_{t-1}}\right) - \left(\frac{b_t}{y_t}\right) \right] \quad (56)$$

where φ stands for superscript c, s . In the same vein, to rule out non-stationary paths of inflation we also assume that the nominal interest rate is set as a function of the output gap and the deviation of inflation with respect to a target inflation rate $\bar{\pi}$:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [\rho_\pi (\pi_t - \bar{\pi}_t) + \rho_y (y_t - \bar{y}) + \bar{i}] \quad (57)$$

3. Calibration

The quantitative implications of the model are derived from the simulation of a numerical solution of the steady state as well as of the log-linearized system (see Appendixes 1 to 3). Parameter values are chosen so that the baseline solution replicates the steady state U.S. economy. The calibrated parameters and exogenous variables appear in Table 1 and the implied steady state in Table 2. The calibration strategy begins by solving for the separation rate $\bar{\rho}$, the rate of unemployed workers looking for a job \bar{u} , the vacancy rate \bar{v} , the specific productivity threshold \bar{a}' , and ν_0 , the scale parameter in the matching function, using the steady-state equations (see Appendix 2). To obtain these five unknown variables we need to choose the steady-state values of some endogenous variables. Thus the employment rate, \bar{n} , has been set to the sample average, 0.9433 and the mean quarterly separation rate is approximately 0.09 (as in Hall, 2005). Consistent with these values the average rate of workers looking for a job within each quarter is $\bar{u} = 0.142$ and the condition $\bar{\rho}\bar{n} = \bar{u}\bar{\rho}^w$ implies a value of $\bar{\rho}^w$ equal to 0.6. This value of $\bar{\rho}^w$ is consistent with our definition of the unemployment rate \bar{u} and corresponds to a value of 1.479 of the quarterly job-finding rate consistent with the average US unemployment rate, slightly higher than the value of 1.35 estimated by Shimer (2005). Also from the steady-state condition $\bar{\rho}^f\bar{v} = \bar{\rho}^w\bar{u}$ and using data from JOLTS in which the average 2001:1-2004:3 ratio $\bar{v}/(1 - \bar{n})$ equals 0.58, we obtain $\bar{v} = 0.033$ and $\bar{\rho}^f = 2.58$, which implies that a vacancy is open on average for 5 weeks. We assume that $\rho^x = 0.072$ that implies that the exogenous separation rate is 80 per cent of the total separation rate, a value between the one assumed by den Haan, Ramey and Watson (2000) but smaller than the one used by Hall (2005b) who suggests that the total separation rate is almost completely acyclical. Finally, we assume that $\{a_t\}$ follows a log normal distribution with standard deviation of 0.10, the same as den Haan, Ramey and Watson (2000). We set the share of the match surplus that the worker receives (η) equal to $2/3$, between 0.5 (Walsh, 2005) and 0.72 (Shimer, 2005), and the elasticity of matching with respect vacancies, ν , to 0.4. With these numbers equations (2.1) to (2.5) imply that $\bar{a}' = 0.8133$ and $\nu_0 = 1.075$.

The preference parameters are set to conventional values. In particular we take the following parameters from Walsh (2005): the discount rate ($\beta = 0.989$), the risk aversion ($\sigma = 2$), the elasticity of demand of differentiated goods ($\theta = 11$) and habits ($h = 0.78$). The elasticity of demand for the differentiated retail goods implies a steady state mark-up $\bar{\mu}$ value of 1.1:

$$\bar{\mu} = \frac{\theta}{\theta - 1} \quad (58)$$

The elasticity of output to private capital (α) is set to 0.4 and we consider a standard

value for the depreciation rate (δ) of 0.02. Capital adjustment costs are assumed to satisfy the following properties: $\phi^{-1}(\delta) = \delta$ and $\phi'(\frac{\bar{e}}{\bar{k}}) = 1$. Therefore, in steady state equation (2.9) implies $\bar{q} = 1$, which allows equations (2.19) and (2.8) to be rewritten as:

$$\bar{e} = \delta \bar{k} \quad (59)$$

$$1 = \beta \left(1 - \bar{\tau}^k\right) \bar{r} + \beta (1 - \delta) \quad (60)$$

so the rental cost of capital is obtained given by

$$\bar{r} = \frac{1 - \beta (1 - \delta)}{\beta (1 - \bar{\tau}^k)} \quad (61)$$

Capital adjustment costs ($\Phi = \phi''(\bar{e}/\bar{k})$) are equal to -0.25 as in Bernanke, Gertler and Gilchrist (1999). Since the discount factor (β) is 0.989, following Christiano and Eichenbaum (1992), equation (2.7) implies a steady-state value of \bar{i}

$$\bar{i} = \frac{\bar{\pi}}{\beta} - 1 \quad (62)$$

The values of \bar{a}' , \bar{i} , \bar{r} and $\bar{\mu}$ can be plugged in equation (2.13) and (2.11) to obtain the steady-state value for the optimal individual capital demand

$$\bar{k}^* = \left(\frac{\alpha \bar{a}'}{(1 + \bar{i}) \bar{\mu} \bar{r}} \right)^{\frac{1}{1-\alpha}} \quad (63)$$

and optimal average capital

$$\bar{k}^* = \frac{1}{(1 - I(\bar{a}'))} \left(\frac{\alpha}{(1 + \bar{i}) \bar{\mu} \bar{r}} \right)^{\frac{1}{1-\alpha}} \int_{\bar{a}'}^{a_{\max}} a^{\frac{1}{1-\alpha}} \varphi(a) da \quad (64)$$

whereas steady-state aggregate capital stock is calculated from (2.12) as

$$(1 - \bar{\rho}) \bar{n} \bar{k}^* = \bar{k} \quad (65)$$

Government consumption (\bar{g}^c/\bar{y}) and government investment (\bar{g}^p/\bar{y}) are set to historical average values. Capital and consumption tax rates have been taken from Boscá, García and Taguas (2005), whereas τ^w has been calibrated to obtain a debt to GDP ratio

equal to 2 at quarterly frequency. For simplicity, unemployment benefits are assumed to be equal to the replacement rate times the average labor income:

$$\bar{g}^u = \frac{\bar{y}^l}{\bar{r}\bar{r}} \quad (66)$$

where $\bar{r}\bar{r} = 0.26$, taken from the average value from 1960 to 1995 in Blanchard and Wolfers (2000). Then, using the approximation (66), equations (2.14), (2.15), (2.16), (2.23) can be solved simultaneously for the four unknowns A , \bar{x}^u , \bar{s}^* , \bar{y}^l . Once we have the value of A the steady state equation (2.17) allows us to obtain the cost of vacancies γ . We calibrate transfers \bar{g}^s assuming that total transfers are 15.5 per cent of GDP, that is

$$\frac{g^u \bar{u} + \bar{g}^s}{\bar{y}} = \frac{\bar{r}\bar{r} \frac{\bar{y}^l}{\bar{n}} \bar{u} + \bar{g}^s}{\bar{y}} = 0.155 \quad (67)$$

and hence:

$$\frac{\bar{g}^s}{\bar{y}} = 0.155 - \bar{r}\bar{r} \frac{\bar{y}^l}{\bar{y}\bar{n}} \bar{u} \quad (68)$$

Given the steady state value for \bar{n} , \bar{k}^* , $\bar{\rho}$, $\bar{\mu}$, \bar{r} , \bar{i} , \bar{v} and the parameters γ and α , expression (2.18) gives the steady-state value of output net of vacancy costs \bar{y} . Since the steady-state investment is given by equation (59), the aggregate resource constraint (2.20) allows to obtain private consumption \bar{c} , making it possible to solve for $\bar{\lambda}_1$ in expression (2.21) and \bar{m} in expression (2.22). Finally, \bar{t} and \bar{b} can be solved recursively in equations (2.24) and (2.25).

Some relevant parameters cannot be obtained from the steady-state relationships. Thus, we adopt a value of 0.7 for ω (the share of firms that do not set their prices optimally), close to empirical estimates of the average duration of price stickiness (Gali and Gertler, 1999, Sbordone, 2002), whereas for inflation indexation we take an intermediate value ($\varsigma = 0.5$). As regards the fiscal policy, we assume that only transfers respond to debt deviations from the target so that the dynamics of the all others variables are unaffected. This implies that ψ_1^s is the only parameter of the fiscal rule initially set different to zero. The parameters in the interest rule are standard in the literature: $\rho_i = 0.75$, $\rho_\pi = 1.50$ and $\rho_y = 0$. Finally the standard deviation of productivity shocks (σ_z) and its autocorrelation parameter (ρ_z) are calibrated to reproduce the average historical volatility and autocorrelation of the US output gap.

The model with transitory supply shocks (that is, shocks in z_t) has been simulated 1000 times, with 260 observations in each simulation. We take the last 160 quarters

TABLE 1 – PARAMETER VALUES

ν_0	1.075	γ	0.500	ω	0.700
ρ^x	0.072	h	0.780	ς	0.500
β	0.989	\bar{g}^c/\bar{y}	0.150	Φ	-0.25
δ	0.020	\bar{g}^s/\bar{y}	0.141	ρ_i	0.750
θ	11	\bar{g}^p/\bar{y}	0.035	ρ_π	1.500
$\alpha = \nu$	0.400	τ^w	0.345	ρ_y	0.000
\bar{r}	0.260	τ^k	0.350	σ_a	0.100
σ	2.000	τ^c	0.100	σ_z	1.600
A	1.524	η	0.666	ρ_z	0.402

TABLE 2 – STEADY STATE

$\bar{\rho}$	0.090	\bar{r}	0.048	$\bar{\lambda}$	0.078
\bar{u}	0.141	\bar{q}	1.000	\bar{m}/\bar{y}	0.731
\bar{v}	0.033	$\bar{\mu}$	1.100	\bar{x}^u/\bar{y}	0.017
\bar{a}'	0.813	\bar{k}^*/\bar{y}	8.793	\bar{s}^*/\bar{y}	0.193
\bar{n}	0.943	\bar{k}/\bar{y}	7.548	\bar{b}/\bar{y}	2.000
$\bar{\rho}^f$	2.581	\bar{y}	3.344	\bar{k}^*/\bar{y}	6.104
$\bar{\rho}^w$	0.600	\bar{e}/\bar{y}	0.151	\bar{y}^l/\bar{y}	0.319
\bar{i}	0.011	\bar{c}/\bar{y}	0.664	$\bar{\pi}$	1.000

and compute the averages over the 1000 simulations of the standard deviation of each variable (x) relative to that of output (σ_x/σ_y , except for GDP which is just σ_y), the first-order autocorrelation (ρ_x) and the contemporaneous correlation with output (ρ_{xy}) of each variable.

These moments are compared with basic labor markets facts of the US business cycles from 1951:1 to 2005:3. The data source is basically the same as in Shimer (2005). We use FRED Economic Data from the Federal Reserve Bank of St. Louis for unemployment, the help wanted index (for vacancies) and civilian employment. As the frequency of these data is monthly, we compact the data set by taking quarterly averages. The real quarterly GDP (billions of chained 2000 dollars) is obtained from the Bureau of Economic Analysis of the Department of Commerce. We take logs of these quarterly variables and obtain their cyclical components using the Hodrick-Prescott filter with a smoothing parameter equal to 1600.³

³ We have checked that we obtain the same results as in Shimer (2005) if the analysed period

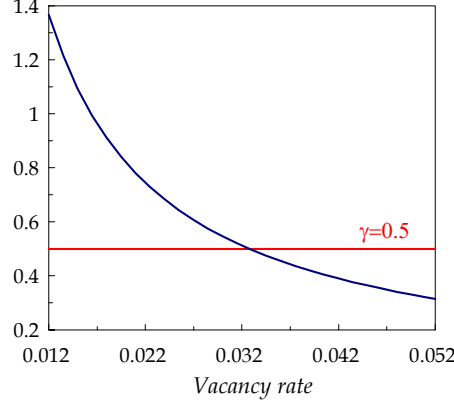


Figure 1: Free entry condition.

4. Results

The results discussed in this section can be explained with the help of two critical expressions in the model: the free entry condition for posting vacancies, equation (42), and the related definition of the surplus, equation (36). Figure 1 represents the free entry condition as a negative function of vacancies, holding constant the rest of the implied variables. Vacancies enter this expression through the probability of filling a vacancy $\rho_t^f = \vartheta\left(\frac{u_t}{v_t}, 1\right)$, whereas changes in other variables shift the curve thus affecting the equilibrium or the impact response and volatility of the vacancy rate. For instance, for a given number of vacancies, an increase in unemployment shifts the curve upwards increasing the number of posted vacancies. The volatility of the vacancy rate depends on the interaction of all these variables in general equilibrium.

Expressions (42) and (36) contain the main parameters that determine the volatility of labor market variables and that have been the subject of much discussion in this literature. The value of non-market activities A and \tilde{g}^u (inside x_{jt+1}^u) on the one hand, and the bargaining power of workers η , on the other hand, are the key parameters in the calibration discussion for Hagedorn and Manovskii (2005) and Costain and Reiter (2005). More specifically, the expression (42) can be rewritten in terms of the survival rate $(1-\rho^x) \left(1-I\left(a'_{jt}\right)\right)$ as:

$$\beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) \rho_t^f (1-\rho^x) \left(1-I\left(a'_{jt}\right)\right) \int_{a'_{t+1}}^{a_{\max}} (1-\eta) s_{jt+1}^* \frac{\varphi(a)}{\left(1-I\left(a'_{jt}\right)\right)} da = \gamma \quad (69)$$

dates from 1951:1 to 2003(4) and the smoothing parameter is 100000.

We can get a glimpse of the main mechanisms behind the volatility of labor market variables with the help of equations (69) and (36). A positive shock to aggregate productivity (z_t) increases the surplus and shifts the free entry condition upwards in Figure 1, increasing the optimal vacancy rate. If the change in vacancy posting is small, so is the volatility of the vacancy rate. Some authors have proposed alternative models of wage determination as a means of increasing the proportion of the observed volatility of labor market variables that the model is able to explain, while the importance of the price formation mechanism has gone quite unnoticed. Gertler and Trigari (2005) have looked at the role of wage rigidity, whereas Costain and Reiter (2005) have allowed for countercyclical movements in η . With flexible prices the mark-up $\mu_t = \frac{P_t}{P_t^w}$ barely responds to technology shocks, while with some degree of price stickiness the mark-up increases sharply on impact (due to a fall in P_t^w not compensated by a fall in P_t) and adjusts thereafter. Thus, price inertia induces an expected fall in the markup that gives an additional impulse to the surplus at $t + 1$ and hence to the optimal vacancy rate.

Endogenous destruction also matters through the effect of a'_{t+1} in equation (69). A decrease in a'_{jt} , as a consequence of a positive shock in productivity, affects the survival rate as well as the average surplus measured by the integral in the above expression. Furthermore, the volatility of vacancies will depend on how much the general equilibrium real interest rate $\frac{\lambda_{1t+1}}{\lambda_{1t}}$ varies after a positive productivity shock. Capital, in turn, enters (36), reducing surplus in levels and therefore making the free entry condition more sensitive to shocks. Taxes affect both the net surplus as well as the dynamics of investment and vacancy posting. We show the effects of these mechanisms in detail in the fourth appendix.

The simulation results of the general model in the previous sections appear in the last column of Table 3, as well as the empirical evidence for the United States (first column) and the results for the simplest version of our model, which is comparable to Shimer's (2005). The last row displays the steady-state values of some relevant variables related to the calibration of each model: the ratio of the surplus to the output ($\frac{\bar{s}}{\bar{y}}$), the net flow surplus enjoyed by an employed worker ($\frac{\eta \bar{s}^*}{A - \bar{x}^u}$), the worker's bargaining power (η), and the worker's value of non-market activities (A). The replacement rate rr is held constant at 0.26 across all experiments.

The model in column (2) is a particular case of the model described in Section 2 that assumes perfect competition in the goods market and price flexibility, with neither capital nor government so that consumption smoothing is not possible and in which job destruction is completely exogenous. Hereafter we refer to this specification as the Shimer's model. In column (2) we present the results of this model using Shimer's calibration for vacancy posting cost ($\gamma = 0.213$), the rate of discount ($1/\beta = 1.012$), utility from leisure

($A = 0.4$), the separation rate ($\rho = 0.1$), worker's bargaining power ($\eta = 0.72$, also equal to the matching elasticity with respect to u) and the scale parameter in the matching function ($\nu_0 = 1.355$); we also set the variance and autocorrelation of technology shocks (σ_z and ρ_z) at the values needed to reproduce second GDP moments. The results in column (2) corroborate Shimer's results: the basic search and matching model generates relative volatilities of unemployment and vacancies which are respectively 20 and 7.5 times smaller than those observed in the data.

Shimer's calibration applied to model in column (2) leads to some unrealistic steady-state values. Both the implicit flow arrival rate of job offers ($\bar{\rho}^w = 1.34$) and the employment rate ($\bar{n} = 1.03$) are far away from our benchmark calibration. Also, as Costain and Reiter (2005) point out, there is a relatively large match surplus calibrated in the Shimer's model. Thus, in column (3) we use an alternative calibration for the same basic model. In particular, we choose a set of parameters so that the steady-state values are compatible with those corresponding to the general model. This means the same $\bar{\rho}^w$, \bar{n} , $\bar{\rho}^f$, \bar{u} and \bar{v} that we have in the benchmark model in column (5). Also the value of A is set so that the basic model reproduces the surplus/GDP ratio of the benchmark model, as reflected at the bottom of the table.

The results in column (3) contain a clear message: the poor performance of the Shimer's model was, to a certain extent, driven by a calibration that does not reproduce the main observed first moments in general equilibrium. This also confirms previous findings in the literature (as those, for example, of Costain and Reiter, 2005, and Hagedorn and Manovskii, 2005) that point out that the size of the match surplus is critical to increase volatilities. This is indeed the case for the unemployment rate but also, although to a lesser extent for the vacancy rate and the probability of finding a job.

However, the main point in our paper is to appraise the incidence of price rigidities on the volatility of vacancies. To that end we compare volatilities across models that share some key features. First, to make sure that we control for the amount of variability in our simulated variables, we calibrate all models to replicate the observed standard deviation and autocorrelation of the GDP in the U.S. Second, all our models imply the same-steady state value for the key parameters and ratios in the process of wage bargaining.

Column (4) presents the results of our general model described above assuming price flexibility. This model incorporates a number of mechanisms with respect to the basic model in column (3): endogenous job destruction, intertemporal substitution, habits, capital and taxes. The detailed analysis of the impact of each of these mechanisms on the relevant volatilities is left to Appendix 4. The joint effect of all these channels is a reduction to a half of the volatility of vacancies whereas the volatility of unemployment

TABLE 3 – MAIN RESULTS

		US	Basic model Shimer	Basic model (recali- brated)	Benchmark model (flexible prices)	Benchmark model (sticky prices)
		(1)	(2)	(3)	(4)	(5)
\hat{y}_t	σ_y	1.58	1.58	1.58	1.58	1.58
	ρ_y	0.84	0.84	0.84	0.84	0.84
$\ln u_t$	σ_u/σ_y	7.83	0.41	7.94	8.08	8.71
	ρ_u	0.87	0.70	0.80	0.84	0.83
	$\sigma_{u,y}$	-0.84	-0.83	-0.99	-0.99	-0.91
$\ln v_t$	σ_v/σ_y	8.85	1.18	5.29	2.57	9.60
	ρ_v	0.91	0.69	0.30	0.15	0.29
	$\sigma_{v,y}$	0.90	0.97	0.66	0.47	0.56
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	1.49	12.34	9.55	14.26
	ρ_{vu}	0.90	0.83	0.62	0.71	0.68
	$\sigma_{vu,y}$	0.89	0.99	0.92	0.97	0.93
ρ^w	σ_{ρ^w}/σ_y	4.86	0.42	3.84	2.93	4.36
	ρ_{ρ^w}	0.91	0.83	0.62	0.70	0.68
	$\sigma_{\rho^w,y}$		0.99	0.92	0.97	0.94
$\frac{\bar{s}^*}{\bar{y}}$			0.67	0.19	0.19	0.19
$\frac{\eta \bar{s}^*}{A - \bar{x}^u}$			1.15	0.13	0.29	0.29
η			0.72	0.67	0.67	0.67
A			0.40	0.91	1.52	1.52

remains basically unaltered. As a result, the market tightness becomes less volatile.

To facilitate a fair assessment of the role of price stickiness in column (5) we augment the model with price stickiness ($\omega = 0.7$) and indexation ($\varsigma = 0.5$) and calibrate it to fit the volatility of output and to maintain the main steady-state labor market ratios: $\frac{\bar{s}^*}{\bar{y}}$, $\frac{\eta \bar{s}^*}{A - \bar{x}^u}$, η , A . The direct consequence of allowing for price rigidity is a sharp increase in the volatilities of all labor market variables that affects specially the vacancy rate⁴. The greatest change affects the volatility of vacancies that is almost four times higher than

⁴ There are few differences in the volatility of other business cycle variables between our general model with and without price rigidity. For instance, the absolute standard deviation of consumption, investment and inflation are respectively 1.24, 5.72 and 0.67 in the model with price stickiness of column (5), whereas these figures turns to be 1.31, 5.45 and 0.69 in the model with flexible prices of column (4).

the one obtained in the flex-price model. Unlike the flexible price model, the benchmark model with sticky prices almost replicates the volatility of unemployment, vacancies, and market tightness observed in the data. Notice also that the ratio $\frac{\eta \bar{s}^*}{A - \bar{x}^u}$ increases in the benchmark model with respect to the basic recalibrated model. The small surplus gain of being employed is one of the main critique of obtaining a high volatility performance using a particular calibration strategy⁵ that does not seem to apply to our results.

To clarify the economics of the contribution of price rigidity to the increase in volatilities we can make use of the entry condition. Substituting out the first order conditions of the households into (42) we obtain:

$$E_t \left(\frac{P_{t+1}}{P_t} \frac{1}{1 + i_t} \right) \rho_t^f (1 - \rho^x) \int_{a'_{t+1}}^{a_{\max}} (1 - \eta) s_{jt+1}^* \varphi(a) da = \gamma \quad (70)$$

After a positive technology shock the left hand side of (70) shifts upwards, thus increasing the amount of vacancies posted in period t in Figure 1. Apart from the real interest rate, two components of this equation are influenced by the degree of price stickiness in the model. First, the mark-up ($\mu_t = P_t/P_t^w$) increases on impact, due to the downward rigidity of P_t . Once the downward adjustment of prices is underway, μ_{t+1} falls. The cyclical response of the mark-up is more intense the stronger the degree of price rigidity and hence the response of s_{t+1}^* is also more pronounced. Second, the sharp increase in μ_t pushes the optimal threshold value a'_{jt} up in (32) and, as a consequence, endogenous destruction rises and unemployment increases. More unemployment reduces labor market tightness increasing the probability (in relative terms) of filling a vacancy ρ_t^f . These two effects reinforce each other and induce an upward shift on the left hand side of (70) that is larger the higher the degree of price stickiness. Thus the volatilities of vacancies and unemployment increase substantially as prices become more rigid. All these effects are reflected both in Figure 2 that displays the IR functions for the benchmark model with price rigidity and Figure 3 that does the same for the benchmark model with flexible prices.

The channel just described hinges crucially on the dynamics of the technology shock. When this shock is very persistent the downward movement of μ_{t+1} after a positive innovation at t is dampened by an upward reaction following the positive realization of z_{t+1} . Models with high price inertia require low values of ρ_z to match the volatility of GDP. Thus, to isolate the role of price stickiness we have repeated our analysis in models with low and high shock persistence. In both cases the volatility of vacancies increases significantly with price stickiness although this increase is more pronounced in

⁵ Mortensen and Nagypál (2005) estimates in 2.8 per cent this flow surplus in the Hagedorn and Monovskii (2005) calibration, ten times smaller than in our benchmark model.

TABLE 4 - THE IMPORTANCE OF PRICE RIGIDITY

Taxes		No						
		No				Yes		
Capital		No		Yes		Yes		
		No	Yes	No	Yes	No	Yes	
Habits		No		Yes		Yes		
		No	Yes	No	Yes	No	Yes	
Price rigidity		US	(2)	(3)	(4)	(5)	(6)	(7)
\widehat{y}_t	σ_y	1.58	1.58	1.58	1.58	1.58	1.58	1.58
	ρ_y	0.84	0.93	0.93	0.93	0.93	0.84	0.84
$\ln u_t$	σ_u/σ_y	7.83	11.94	10.12	12.61	11.69	8.10	8.00
	ρ_u	0.87	0.93	0.92	0.90	0.87	0.85	0.87
	$\sigma_{u,y}$	-0.84	-0.99	-0.94	-0.98	-0.96	-0.99	-0.95
$\ln v_t$	σ_v/σ_y	8.85	2.87	5.54	3.76	18.52	2.41	6.59
	ρ_v	0.91	0.44	0.50	0.53	0.14	0.15	0.31
	$\sigma_{v,y}$	0.90	0.56	0.57	0.37	0.30	0.45	0.56
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	13.76	12.72	14.87	22.86	9.40	11.72
	ρ_{vu}	0.90	0.87	0.99	0.84	0.53	0.72	0.72
	$\sigma_{vu,y}$	0.89	0.98	0.91	0.92	0.74	0.97	0.96
ρ^w	σ_{ρ^w}/σ_y	4.86	4.17	3.91	4.48	6.61	2.88	3.61
	ρ_{ρ^w}	0.91	0.87	0.91	0.84	0.57	0.72	0.72
	$\sigma_{\rho^w,y}$		0.98	0.99	0.93	0.76	0.98	0.96
$\frac{\bar{s}^*}{\bar{y}}$			0.19	0.19	0.19	0.19	0.19	0.19
$\frac{\eta \bar{s}^*}{A - \bar{x}^u}$			0.15	0.15	0.15	0.15	0.29	0.29
η			0.67	0.67	0.67	0.67	0.67	0.67
A			0.66	0.66	0.66	0.66	2.13	2.13

models in which shocks to productivity are less persistent.

Finally, to gauge the sensitivity of our previous results, in Table 4 we show the effects of price stickiness in three alternative settings: a model with no taxes, no capital and no habits in columns (2) and (3); a model with no taxes, no capital but with habits in consumption in columns (4) and (5); and a model of no taxes with capital and habits in columns (6) and (7).⁶ The sensitivity analysis in Table 4 confirms our main result:

⁶ The model without capital cannot reproduce the observed persistence of the output, even when the common productivity shock is assumed to be white noise. This is because the autocorrelation induced by the law of motion of the employment is very high and firms can not substitute away from employment when they can not use capital, so the simulated persistence of the output chosen in columns (2) to (5) is the maximum of the minimum simulated autocorrelation coefficient reachable by each of the models with no capital.

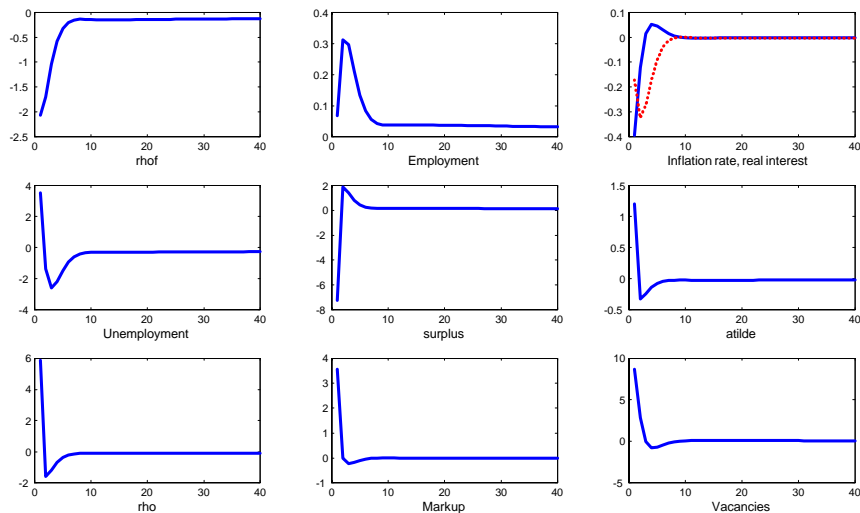


Figure 2: IR for sticky prices model

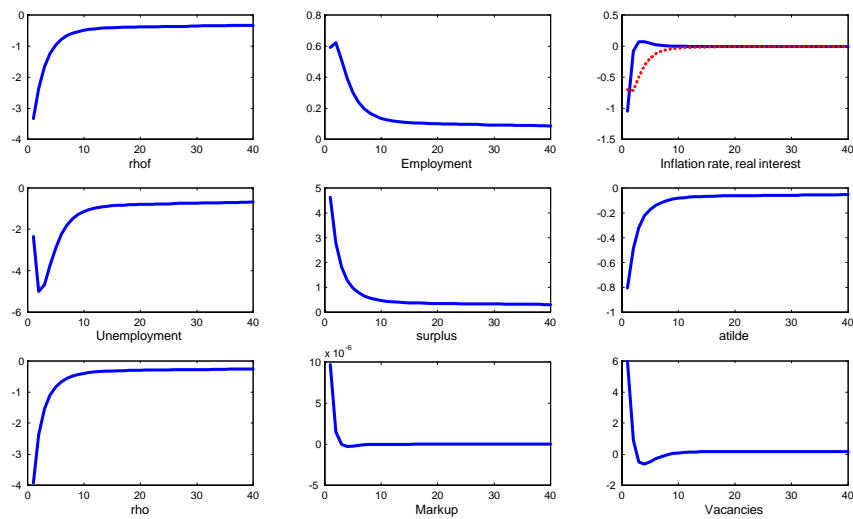


Figure 3: IR for flexible prices model

regardless of other model features, price stickiness always induces a small change in the volatility of unemployment but shoots the volatility of vacancies very much.

5. Concluding Remarks

In the standard search and matching model, the level of unemployment hinges upon the number of vacancies posted, which in turn depends on the determinants of the free-entry condition. This condition relates the cost of vacancy posting with the probability of a vacancy being filled as well as with the expected surplus of the vacancy and the discount rate. These three components are model-specific and vary to make vacancy posting more or less responsive to a total factor productivity shock. Shimer (2005) looked at the business cycle implications of search and matching frictions and showed that in fact the volatilities of vacancies and unemployment (as well as the vacancy to unemployment ratio) predicted by the basic model are far lower than those observed in US data.

In this paper we have proposed a more general neo-keynesian dynamic general equilibrium model in which the empirical predictions match the empirical evidence remarkably well. In particular the model predicts a relative (to output) volatility of vacancies, unemployment and the v/u ratio that matches those observed in the data almost perfectly. The model also does well in explaining autocorrelations and cross correlations among variables, although the implied persistence of vacancies is somewhat low, a results that can be improved with nominal wage rigidities as in Gertler and Trigari (2005).

The main result of the paper is that price stickiness turns out to be of paramount importance to increase labor market variability in line with that observed in the data. This is particularly the case for the vacancy rate and the unemployment/vacancy ratio. Price rigidity has a direct effect on all components of the free entry condition and has proved to be very significant in quantitative terms. In this sense, we see our results as akin to those emphasizing the importance of wage stickiness as a way of improving the empirical performance of matching models. The combination of wage and price stickiness seems a natural extension aimed at both further improving the model and also assessing the relative importance of different sources of nominal inertia for the purpose at hand. However, compared with the relevance of price rigidities, adding endogenous destruction, intertemporal substitution, habits, capital and taxes do not contribute very much to explain the cyclical performance in the labor market.

A final comment on calibration is pertinent here. Our empirical analysis has been ushered in by a thorough calibration exercise based on a careful analysis of the existing literature on the issue, as well as of the basic steady-state variables for the US economy. The main result in our paper, namely the importance of price rigidity to explain labor

market volatilities, is robust to reasonable changes in calibration values. However, we have also corroborated that some predictions of the basic Mortensen and Pissarides model might be sensitive to the choice of some key parameter values. This leads us to believe that more research is needed on this matter and in particular a deep econometric analysis is called for to obtain a better empirical counterpart of some of the parameters used in this literature. This is next on the research agenda.

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Appendix 1: Equilibrium

The dynamic equilibrium is defined by the following equations:

$$y_t = \frac{(1 - \rho_t)n_t(1 + i_t)\mu_t r_t}{\alpha} k_t^* - \gamma v_t \quad (1.1)$$

$$c_t + e_t + g_t^c = y_t \quad (1.2)$$

$$\frac{c_t^{-\sigma}}{c_{t-1}^{h(1-\sigma)}} - E_t \beta h \frac{c_{t+1}^{1-\sigma}}{c_t^{h(1-\sigma)+1}} - \lambda_{1t}(1 + \tau^c) - \lambda_{2t}(1 + \tau^c) = 0 \quad (1.3)$$

$$E_t \lambda_{2t+1} = i_t E_t \lambda_{1t+1} \quad (1.4)$$

$$\lambda_{1t} \beta^{-1} = (1 + i_t) E_t \left(\lambda_{1t+1} \frac{P_t}{P_{t+1}} \right) \quad (1.5)$$

$$P_t (1 + \tau_t^c) c_t = M_t \quad (1.6)$$

$$k_t = (1 - \delta) k_{t-1} + \phi \left(\frac{e_t}{k_{t-1}} \right) k_{t-1} \quad (1.7)$$

$$\left[\phi' \left\{ \frac{e_t}{k_{t-1}} \right\} \right]^{-1} = q_t \quad (1.8)$$

$$q_t \beta^{-1} = E_t \left[\frac{\lambda_{1t+1}}{\lambda_{1t}} \left(q_{t+1} \left[(1 - \delta) + \phi \left\{ \frac{e_{t+1}}{k_t} \right\} - \phi' \left\{ \frac{e_{t+1}}{k_t} \right\} \frac{e_{t+1}}{k_t} \right] \right) \right] \quad (1.9)$$

$$P_t^* = \left(\frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[\mu_{t+s}^{-1} (P_{t+s})^{\theta+1} c_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} \left[(P_{t+s})^{\theta} c_{t+s} \right]} \quad (1.10)$$

$$P_t^{1-\theta} = (1 - \omega) P_t^{*1-\theta} + \omega P_{t-1}^{1-\theta} \quad (1.11)$$

$$k_{jt}^* = \left(\frac{\alpha z_t a_{jt}}{(1+i_t) \mu_t r_t} \right)^{\frac{1}{1-\alpha}} \quad (1.12)$$

$$\left[\frac{z_t \tilde{a}_t (k_{jt}^*)^\alpha}{(1+i_t) \mu_t} - r_t k_{jt}^* \right] - A + x_t^u = 0 \quad (1.13)$$

$$\rho_t^n = \int_{-\infty}^{\tilde{a}_t} \varphi(a_t) da \quad (1.14)$$

$$\rho_t = \rho^x + (1 - \rho^x) \rho_t^n \quad (1.15)$$

$$\rho_t^s = 1 - \rho_t \quad (1.16)$$

$$s_{jt+1}^* = \left[\left(\frac{1}{1+i_{t+1}} \right) \frac{z_{t+1} a_{jt+1} (k_{jt+1}^*)^\alpha}{\mu_{t+1}} - r_{t+1} k_{jt+1}^* \right] - A + x_{t+1}^u \quad (1.17)$$

$$s_{t+1}^* = \frac{1-\alpha}{\alpha} r_{t+1} k_{t+1}^* - A + x_{t+1}^u \quad (1.18)$$

$$x_t^u = \beta E_t \left(\frac{\lambda_{1t+1}}{\lambda_{1t}} \right) (1 - \rho_{t+1}) [1 - \eta \rho_t^w - \eta \tau_{t+1}^w (1 - \rho_t^w)] s_{t+1}^* - \tilde{g}^u \quad (1.19)$$

$$x_t^u = \frac{\gamma [1 - \eta \rho_t^w - \eta \tau_{t+1}^w (1 - \rho_t^w)]}{\rho_t^f (1 - \eta)} - \tilde{g}^u \quad (1.20)$$

$$y_t^l = (1 - \rho_t) n_t (1 - \tau_t^w) [\eta s_t^* + A - x_t^u] \quad (1.21)$$

$$u_t = 1 - (1 - \rho_t) n_t \quad (1.22)$$

$$\rho_t^w = \frac{\vartheta(u_t, v_t)}{u_t} \quad (1.23)$$

$$\rho_t^f = \frac{\vartheta(u_t, v_t)}{v_t} \quad (1.24)$$

$$n_{t+1} = (1 - \rho_t)n_t + \vartheta(u_t, v_t) \quad (1.25)$$

$$(1 - \rho_t) n_t k_t^* = k_{t-1} \quad (1.26)$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) [\rho_\pi (\pi_t - \bar{\pi}_t) + \rho_y (\hat{y}_t) + \bar{i}] \quad (1.27)$$

$$t_t = \tau_t^c c_t + \tau_t^k r_t k_{t-1} + \tau_t^w (1 - \rho_t) n_t (\eta s_t^* + A - x_t^u) \quad (1.28)$$

$$b_t - (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} = g_t^c + g_t^s + g^u u_t - t_t \quad (1.29)$$

$$g_t^\varphi = g_{t-1}^\varphi + \psi_1^\varphi \left[\left(\frac{b}{y} \right) - \left(\frac{b_t}{y_t} \right) \right] + \psi_2^\varphi \left[\left(\frac{b_{t-1}}{y_{t-1}} \right) - \left(\frac{b_t}{y_t} \right) \right] \quad (1.30)$$

$$\frac{E_t \Lambda_{t,t+s}}{E_t \Lambda_{t,t+s-1}} = \frac{E_t (\lambda_{1t+s} / P_{t+s})}{E_t (\lambda_{1t+s-1} / P_{t+s-1})} \quad (1.31)$$

$$k_t^* = \int_{a'_t}^{a_{\max}} k_{jt}^* \frac{\varphi(a)}{1 - \Phi(a')} da = \left(\frac{\alpha z_t}{(1 + i_t) \mu_t r_t} \right)^{\frac{1}{1-\alpha}} \int_{a'_t}^{a_{\max}} \frac{a_t^{\frac{1}{1-\alpha}} \varphi(a)}{1 - I(a'_t)} da \quad (32)$$

$$k_t^* = \left(\frac{\alpha z_t a_t'}{(1+i_t) \mu_t r_t} \right)^{\frac{1}{1-\alpha}} \quad (1.33)$$

$$\pi_t = \frac{P_{t+1}}{P_t} \quad (1.34)$$

Endogenous variables: $c_t, e_t, y_t, \lambda_{1t}, i_t, r_t, v_t, u_t, a_t', n_t, k_{jt}^*, \pi_t, M_t, P_t, q_t, P_t^*, \Lambda_t, \mu_t, x_t^u, \rho_t^n, \rho_t, \rho_t^w, \rho_t^f, \rho_t^s, t_t, b_t, g_t^\varphi, k_t, y_t^l, k_t^*, k_t^{l*}, s_{jt+1}^*, s_{t+1}^*$
 (33 equations=33 variables)

Appendix 2: The steady-state model

From (1.22):

$$\bar{u} = 1 - (1 - \bar{\rho}) \bar{n} \quad (2.1)$$

From (1.25):

$$\bar{\rho} \bar{n} = \vartheta(\bar{u}, \bar{v}) \equiv \nu_0 \bar{v}^\nu \bar{u}^{1-\nu} \quad (2.2)$$

From (1.23):

$$\bar{\rho}^w = \frac{\vartheta(\bar{u}, \bar{v})}{\bar{u}} \quad (2.3)$$

From (1.24):

$$\bar{\rho}^f = \frac{\vartheta(\bar{u}, \bar{v})}{\bar{v}} \quad (2.4)$$

From (1.14) and (1.15):

$$\bar{\rho} = \rho^x + (1 - \rho^x) I(a') \quad (2.5)$$

From (1.16):

$$\bar{\rho}^s = 1 - \bar{\rho} \quad (2.6)$$

From (1.5):

$$\beta = \frac{\bar{\pi}}{1 + \bar{i}} \quad (2.7)$$

From (1.9):

$$\bar{q}\beta^{-1} = \left((1 - \bar{\tau}^k) \bar{r} + \bar{q} \left[(1 - \delta) + \phi \left\{ \frac{\bar{e}}{\bar{k}} \right\} - \phi' \left\{ \frac{\bar{e}}{\bar{k}} \right\} \frac{\bar{e}}{\bar{k}} \right] \right) \quad (2.8)$$

From (1.8):

$$\left[\phi' \left\{ \frac{\bar{e}}{\bar{k}} \right\} \right]^{-1} = \bar{q} \quad (2.9)$$

From (1.10):

$$\left(\frac{\theta}{\theta - 1} \right) = \bar{\mu} \quad (2.10)$$

From (1.32):

$$\bar{k}^* = \frac{1}{(1 - I(\bar{a}'))} \left(\frac{\alpha}{(1 + \bar{i}) \bar{\mu} \bar{r}} \right)^{\frac{1}{1-\alpha}} \int_{\bar{a}'}^{a_{\max}} a^{\frac{1}{1-\alpha}} \varphi(a) da \quad (2.11)$$

From (1.26):

$$(1 - \rho) \bar{n} \bar{k}^* = \bar{k} \quad (2.12)$$

From (1.33):

$$\bar{k}^* = \left(\frac{\alpha \bar{a}'}{(1 + \bar{i}) \bar{\mu} \bar{r}} \right)^{\frac{1}{1-\alpha}} \quad (2.13)$$

From (1.19)⁷:

$$\bar{x}^u = \beta (1 - \bar{\rho}) [1 - \eta \bar{\rho}^w - \eta \bar{\tau}^w (1 - \bar{\rho}^w)] \bar{s}^* - \left(1 - \beta^2 [1 - \rho^w (1 - \rho^x)]^2\right) g^u \quad (2.14)$$

From (1.13):

$$\bar{x}^u = A - \left[\frac{\bar{a}' (\bar{k}^{f*})^\alpha}{(1 + \bar{i}) \bar{\mu}} - \bar{r} \bar{k}^{f*} \right] \quad (2.15)$$

From (1.18):

$$\bar{s}^* = \frac{1 - \alpha}{\alpha} \bar{r} \bar{k}^{f*} - A + \bar{x}^u \quad (2.16)$$

From (1.20):

$$A - \left(\frac{\bar{a}' \bar{k}^{f*\alpha}}{\bar{\mu} (1 + \bar{i})} - \bar{r} \bar{k}^{f*} \right) = \frac{\gamma [1 - \bar{\rho}^w \eta - \bar{\tau}^w \eta (1 - \bar{\rho}^w)]}{(1 - \eta) \bar{\rho}^f} - \left(1 - \beta^2 [1 - \rho^w (1 - \rho^x)]^2\right) g^u \quad (17)$$

From (1.1):

$$\bar{y} = \frac{(1 - \bar{\rho}) \bar{n} (1 + \bar{i}) \bar{\mu} \bar{r}}{\alpha} \bar{k}^{f*} - \gamma \bar{v} \quad (2.18)$$

From (1.7):

$$\frac{\bar{e}}{\bar{k}} = \phi^{-1}(\delta) \quad (2.19)$$

⁷ From 39 can be obtained the steady-state expected present value of income coming from \tilde{g}^u as:

$$\left[1 + \beta (1 - \rho^w (1 - \rho^x)) + \beta^2 (1 - \rho^w (1 - \rho^x))^2 + \beta^3 (1 - \rho^w (1 - \rho^x))^3 \dots \right] \tilde{g}^u$$

We wish to calibrate \tilde{g}^u so that the observed unemployment benefits (g^u) is received only during two consecutive periods:

$$[1 + \beta (1 - \rho^w (1 - \rho^x))] g^u = \left[\frac{1}{1 - \beta (1 - \rho^w (1 - \rho^x))} \right] \tilde{g}^u$$

Therefore

$$\tilde{g}^u = \left(1 - [\beta (1 - \rho^w (1 - \rho^x))]^2 \right) g^u$$

From (1.2):

$$\bar{c} + \bar{e} + \bar{g}^c = \bar{y} \quad (2.20)$$

From (1.3) and (1.4):

$$(1 + \bar{\tau}^c) (1 + \bar{i}) \bar{\lambda}_1 = (1 - \beta h) \frac{\bar{c}^{\sigma(h-1)}}{\bar{c}^h} \quad (2.21)$$

From (1.6):

$$(1 + \bar{\tau}^c) \bar{c} = \frac{\bar{M}}{\bar{P}} \quad (2.22)$$

From (1.21):

$$\bar{y}^l = (1 - \bar{\rho}) \bar{n} (1 - \bar{\tau}^w) [\eta \bar{s}^* + A - \bar{x}^u] \quad (2.23)$$

From (1.28):

$$\bar{t} = \bar{\tau}^c \bar{c} + \bar{\tau}^k \bar{r} \bar{k} + \bar{\tau}^w (1 - \bar{\rho}) \bar{n} (\eta \bar{s}^* + A - \bar{x}^u) \quad (2.24)$$

From (1.29):

$$\bar{g}^c + \bar{g}^s + g^u \bar{u} + \bar{i} \bar{b} = \bar{t} \quad (2.25)$$

Exogenous variables: $\bar{\pi}$ and $\bar{\tau}^c, \bar{\tau}^k, \bar{\tau}^w, \bar{g}^c, \bar{g}^s, \bar{g}^u$. Endogenous: $\bar{c}, \bar{e}, \bar{y}, \bar{\lambda}_1, \bar{i}, \bar{r}, \bar{v}, \bar{u}, a', \bar{n}, \bar{m}, \bar{q}, \bar{\mu}, \bar{x}^u, \bar{\rho}, \bar{s}^*, \bar{\rho}^w, \bar{\rho}^f, \bar{y}^l, \bar{t}, \bar{b}, \bar{k}, \bar{k}'^*, \bar{k}^*, \bar{\rho}^s$ (25 endogenous=25 equations)

Appendix 3: Log-linearized model

Let \hat{x} be the variable that tell us how much x differs from its steady-state value.

From (1.13):

$$\begin{aligned} \hat{a}_t = & \left(\frac{\bar{i}}{1 + \bar{i}} \right) \hat{i}_t - \hat{z}_t + \hat{\mu}_t - \left(\alpha - \frac{\bar{r} \bar{k}'^*}{\bar{r} \bar{k}'^* + A - \bar{x}^u} \right) \hat{k}'_t \\ & + \left(\frac{\bar{r} \bar{k}'^*}{\bar{r} \bar{k}'^* + A - \bar{x}^u} \right) \hat{r}_t - \left(\frac{\bar{x}^u}{\bar{r} \bar{k}'^* + A - \bar{x}^u} \right) \hat{x}_t^u \end{aligned} \quad (3.1)$$

From (1.14):

$$\hat{\rho}_t^n = \frac{\varphi(\bar{a}') \bar{a}'}{I(\bar{a}')} \hat{a}'_t \quad (3.2)$$

From (1.15):

$$\hat{\rho}_t = \left[\frac{(1 - \rho^x) \bar{\rho}^n}{\bar{\rho}} \right] \hat{\rho}_t^n \quad (3.3)$$

From (1.16):

$$\hat{\rho}_t^s = \frac{-\bar{\rho}}{1 - \bar{\rho}} \hat{\rho}_t \quad (3.4)$$

From (1.25):

$$\hat{n}_{t+1} = (1 - \bar{\rho}) \hat{n}_t - \bar{\rho} \hat{\rho}_t + \bar{\rho}^w \frac{\bar{v}^\nu}{\bar{u}^\nu + \bar{v}^\nu} \frac{\bar{u}}{\bar{n}} \hat{u}_t + \bar{\rho}^f \frac{\bar{u}^\nu}{\bar{u}^\nu + \bar{v}^\nu} \frac{\bar{v}}{\bar{n}} \hat{v}_t \quad (3.5)$$

From (1.22):

$$\hat{u}_t = - (1 - \bar{\rho}) \frac{\bar{n}}{\bar{u}} \hat{n}_t + \bar{\rho} \frac{\bar{n}}{\bar{u}} \hat{\rho}_t \quad (3.6)$$

From (1.24):

$$\hat{\rho}_t^f = \frac{\bar{v}^\nu}{\bar{u}^\nu + \bar{v}^\nu} (\hat{u}_t - \hat{v}_t) \quad (3.7)$$

From (1.23):

$$\hat{\rho}_t^w = \frac{\bar{u}^\nu}{\bar{u}^\nu + \bar{v}^\nu} (\hat{v}_t - \hat{u}_t) \quad (3.8)$$

From (1.20):

$$\hat{\rho}_t^f = - \frac{\eta \bar{\rho}^w (1 - \bar{\tau}^w)}{1 - \eta \bar{\rho}^w (1 - \bar{\tau}^w) - \eta \bar{\tau}^w} \hat{\rho}_t^w - \frac{\bar{x}_t^u}{\bar{x}_t^u + \bar{g}^u} \hat{x}_t^u \quad (3.9)$$

From (1.1):

$$\hat{y}_t = \left(\frac{\bar{y} + \gamma \bar{v}}{\bar{y}} \right) \left[\hat{n}_t - \left(\frac{\bar{\rho}}{1 - \bar{\rho}} \right) \hat{\rho}_t + \left(\frac{\bar{i}}{1 + \bar{i}} \right) \hat{i}_t + \hat{\mu}_t + \hat{r}_t + \hat{k}_t^* \right] - \frac{\gamma \bar{v}}{\bar{y}} \hat{v}_t \quad (3.10)$$

From (1.19):

$$\begin{aligned} \hat{x}_t^u &= \left(\frac{\bar{x}^u + \bar{g}^u}{\bar{x}^u} \right) \left(E_t \hat{\lambda}_{t+1} - \hat{\lambda}_t + E_t \hat{s}_{t+1}^* \right) - \\ &\quad \left(\frac{\beta s \eta \bar{\rho}^w}{\bar{x}^u} \right) (1 - \bar{\tau}^w) (1 - \bar{\rho}) \hat{\rho}_t^w - \left(\frac{\beta s \bar{\rho}}{\bar{x}^u} \right) (1 - \eta \bar{\rho}^w - \eta \bar{\tau}^w (1 - \bar{\rho}^w)) E_t \hat{\rho}_{t+1}^w \end{aligned} \quad (3.11)$$

From (1.18):

$$\widehat{s}_t^* = \left(\frac{1-\alpha}{\alpha} \right) \frac{\bar{r}\bar{k}^*}{\bar{s}^*} \left(\widehat{k}_t^* + \widehat{r}_t \right) + \frac{\bar{x}^u}{\bar{s}^*} \widehat{x}_t^u \quad (3.12)$$

From (1.2):

$$\widehat{y}_t = \frac{\bar{c}}{\bar{y}} \widehat{c}_t + \frac{\bar{e}}{\bar{y}} \widehat{e}_t + \frac{\bar{g}}{\bar{y}} \widehat{g}_t^c \quad (3.13)$$

From (1.5):

$$\widehat{\lambda}_{1t} = \frac{\bar{i}}{(1+\bar{i})} \widehat{i}_t + E_t \left(\widehat{\lambda}_{1t+1} - \widehat{\pi}_{t+1} \right) \quad (3.14)$$

From (1.6):

$$\widehat{M}_t = \widehat{P}_t + \widehat{c}_t \quad (3.15)$$

From (1.7):

$$\widehat{k}_t = \left(1 - \frac{\bar{e}}{\bar{k}} \right) \widehat{k}_{t-1} + \frac{\bar{e}}{\bar{k}} \widehat{e}_t \quad (3.16)$$

From (1.8):

$$\widehat{q}_t = \phi'' \frac{\bar{e}}{\bar{k}} \left(\widehat{k}_{t-1} - \widehat{e}_t \right) \quad (3.17)$$

From (1.9):

$$\begin{aligned} \widehat{q}_t &= E_t \left(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} \right) + \beta \bar{r} \left(1 - \bar{\tau}^k \right) E_t \widehat{r}_{t+1} + \\ &\beta \left(1 - \frac{\bar{e}}{\bar{k}} \right) E_t \widehat{q}_{t+1} - \beta \left(\frac{\bar{e}}{\bar{k}} \right)^2 \phi'' E_t \left(\widehat{e}_{t+1} - \widehat{k}_t \right) \end{aligned} \quad (3.18)$$

From (1.11):

$$E_t \widehat{P}_{t+1}^* = \frac{1}{(1-\omega)} E_t \left(\widehat{P}_{t+1} - \widehat{P}_t \right) + \widehat{P}_t \quad (3.19)$$

From (1.27):

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + (1-\rho_i) \rho_\pi \bar{\pi} \widehat{\pi}_t + (1-\rho_i) \rho_y \bar{y} \widehat{y}_t \quad (3.20)$$

Fom (1.10):

$$\widehat{P}_t^* = \beta \omega E_t \widehat{P}_{t+1}^* + (1-\beta\omega) \left(\widehat{P}_t - \widehat{\mu}_t \right) \quad (3.21)$$

From (1.31):

$$E_t \widehat{\Lambda}_{t+1} = \widehat{\Lambda}_t + E_t \left(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} \right) - E_t \left(\widehat{P}_{t+1} - \widehat{P}_t \right) \quad (3.22)$$

From (1.3) and (1.4):

$$\begin{aligned} \widehat{\lambda}_{1t} = & \frac{\beta h(1+h(1-\sigma)) - \sigma}{1-\beta h} \widehat{c}_t - \frac{h(1-\sigma)}{1-\beta h} \widehat{c}_{t-1} \\ & - \frac{\beta h(1-\sigma)}{1-\beta h} E_t \widehat{c}_{t+1} - \frac{\bar{i}}{1+\bar{i}} \widehat{i}_{t-1} \end{aligned} \quad (3.23)$$

From (1.21):

$$\begin{aligned} \widehat{y}_t^l = & \widehat{n}_t - \left(\frac{\bar{\rho}}{1-\bar{\rho}} \right) \widehat{\rho}_t \\ & + \frac{\eta(1-\rho)\bar{n}(1-\bar{\tau}^w)}{\bar{y}^l} s_t^* - \frac{(1-\rho)\bar{n}(1-\bar{\tau}^w)\bar{x}^u}{\bar{y}^l} \widehat{x}_t^u \end{aligned} \quad (3.24)$$

From (1.26):

$$\widehat{k}_{t-1} = \widehat{n}_t - \frac{\bar{\rho}}{1-\bar{\rho}} \widehat{\rho}_t + \widehat{k}_t^* \quad (3.25)$$

New Phillips curve:

$$\widehat{\pi}_t = \frac{\beta}{1+\varsigma\beta} E_t \widehat{\pi}_{t+1} - \frac{(1-\beta\omega)(1-\omega)}{\omega(1+\varsigma\beta)} \widehat{\mu}_t + \frac{\varsigma}{1+\varsigma\beta} \widehat{\pi}_{t-1} \quad (3.26)$$

From (1.28):

$$\begin{aligned} \widehat{t}_t = & \frac{\bar{\tau}^c \bar{c}}{\bar{t}} \widehat{c}_t + \frac{\bar{\tau}^k \bar{r} \bar{k}}{\bar{t}} \left(\widehat{k}_{t-1} + \widehat{r}_r \right) + \frac{\bar{\tau}^w \bar{n} (1-\bar{\rho})}{\bar{t}} \left(\eta \bar{s}^* + A - \bar{x}^u \right) \left(\widehat{n}_t - \frac{\bar{\rho}}{1-\bar{\rho}} \widehat{\rho}_t \right) \\ & + \frac{\bar{\tau}^w \bar{n} (1-\bar{\rho}) \eta \bar{s}^*}{\bar{t}} \widehat{s}_t^* - \frac{\bar{\tau}^w \bar{n} (1-\bar{\rho}) \bar{x}^u}{\bar{t}} \widehat{x}_t^u \end{aligned} \quad (3.27)$$

From (1.30):

$$\bar{g}^\varphi \widehat{g}_t^\varphi = \bar{g}^\varphi \widehat{g}_{t-1}^\varphi + \left(\frac{\bar{b}}{\bar{y}} \right) (\psi_1^\varphi + \psi_2^\varphi) \left(\widehat{y}_t - \widehat{b}_t \right) + \psi_2^\varphi \left(\frac{\bar{b}}{\bar{y}} \right) \left(\widehat{b}_{t-1} - \widehat{y}_{t-1} \right) \quad (3.28)$$

From (1.29):

$$\widehat{tt}_t = \bar{g}^c \widehat{g}_t^c + \bar{g}^s \widehat{g}_t^s + g^u \bar{u} \widehat{u}_t + \frac{\bar{b}}{\bar{\pi}} \widehat{ii}_{t-1} - \frac{\bar{b}}{\bar{\pi}} (1+\bar{i}) \widehat{\pi}_t + \frac{\bar{b}}{\bar{\pi}} (1+\bar{i}) \widehat{b}_{t-1} - \bar{b} \widehat{b}_t \quad (3.29)$$

From (1.32):

$$\widehat{k}_t^* = \left(\frac{1}{1-\alpha} \right) \left(\widehat{z}_t - \frac{\bar{i}}{1+\bar{i}} \widehat{i}_t - \widehat{\mu}_t - \widehat{r}_t \right) - \Psi(\bar{a}') \widehat{a}'_t \quad (3.30)$$

where:

$$\Psi(\bar{a}') = \bar{a}' \varphi(\bar{a}') \left[\frac{1}{1-I(\bar{a}')} - \frac{(\bar{a}')^{\left(\frac{1}{1-\alpha}\right)}}{\int_{\bar{a}'}^{a_{\max}} (a)^{\left(\frac{1}{1-\alpha}\right)} \varphi(a) da} \right] \quad (3.31)$$

From (1.33):

$$\widehat{k}'_t^* = \left(\frac{1}{1-\alpha} \right) \left(\widehat{z}_t + \widehat{a}'_t - \frac{\bar{i}}{1+\bar{i}} \widehat{i}_t - \widehat{\mu}_t - \widehat{r}_t \right) \quad (3.32)$$

Appendix 4: Endogenous job destruction, intertemporal substitution, habits, capital and taxes

There are many differences between our benchmark model and the basic model making it difficult to gauge the contribution of the different components of the model in explaining the improvement in empirical performance. This appendix is devoted to exploring these mechanisms in detail, by taking each of them at a time from the basic to the more general specification in a setting without price rigidity. Given the complexity of the model and the lack of an analytical solution this can be only done by relying on numerical simulations and analyzing the sensitivity of the results in each particular case.

Table A4.1 contains the results for six different models. Given that the simulated persistence of the output in some models without capital is always higher than the observed one, we have re-calibrated the corresponding coefficient of the productivity shock in all the models to match an autocorrelation of 0.93 for the output. This is higher than the observed one, but as the aim of the exercise is to study how cyclical properties of the labor market change as we enrich the model, we have preferred to maintain constant this moment to facilitate comparability across models. However, it is important to note that this strategy means that the persistence and volatility of the common productivity shock is now different across models, thus creating an additional margin affecting the results.

The main message from Table A4.1 is that adding other mechanisms but price rigidity does not contribute to raise the volatility of vacancies. Quite the opposite, some of them seems to work in the wrong direction. Thus, column (2) corresponds to a model without price rigidity, endogenous job destruction, intertemporal substitution, habits, capital or taxes. This is equivalent to our basic model in Table 3 although, as has been

said, the results do not coincide because the calibrated persistence of the output is different⁸. In column (3) we introduce endogenous destruction (that amounts to 1.8 per cent in steady state, representing 20 per cent of the total quarterly separation rate). Compared with the results in column (2) this model predicts a lower volatility in vacancies and unemployment. We next in column (4) embed the matching mechanism in a dynamic model in which agents make their intertemporal decisions operating through a perfect financial market. As we can see, this model does a worse job of fitting the relative volatility of u (increasing it) and v (lowering it). The presence of habits ($h = 0.78$) in column (5) seems to improve the performance of the model related with the volatility of vacancies, but pushes up further the volatility of unemployment. Column (6) introduces capital that leads to a sharp fall in the volatility of vacancies and unemployment, making the relative standard deviation of unemployment closer to the observed one, but widening the gap between the empirical and the simulated volatility of vacancies. Finally, in column (7) taxes are considered, without adding too much in terms of volatilities in a model of flexible prices.

Table A4.2 shows how the results would change for the case in which the productivity shock has the same volatility and persistence. Qualitatively the message learnt from changing the model in the flexible prices case is the same: enrichment of the model do not add too much to explain the cyclical performance in the labor market, although in this case the gap between the observed and simulated volatilities for unemployment and vacancies widens as a consequence of intertemporal substitution.

⁸ As commented before, the higher the persistence of the productivity shock, the lower the volatility of vacancies.

TABLE A4.1 VOLATILITIES ACROSS MODELS
 Same persistence and volatility in output

			No					
			No	Yes				
			No		Yes			
			No			Yes		
			No				Yes	
			No					Yes
US			(2)	(3)	(4)	(5)	(6)	(7)
\hat{y}_t	σ_y	1.58	1.58	1.58	1.58	1.58	1.58	1.58
	ρ_y	0.84	0.93	0.93	0.93	0.93	0.93	0.93
$\ln u_t$	σ_u/σ_y	7.83	9.09	8.47	11.94	12.61	8.53	8.25
	ρ_u	0.87	0.90	0.91	0.93	0.90	0.93	0.93
	$\sigma_{u,y}$	-0.84	-0.99	-0.99	-0.99	-0.98	-0.99	-0.99
$\ln v_t$	σ_v/σ_y	8.85	4.70	3.60	2.87	3.76	1.89	1.96
	ρ_v	0.91	0.53	0.48	0.44	0.53	0.32	0.33
	$\sigma_{v,y}$	0.90	0.71	0.67	0.56	0.37	0.53	0.54
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	13.09	11.37	13.76	14.87	9.65	9.46
	ρ_{vu}	0.90	0.81	0.82	0.87	0.84	0.87	0.86
	$\sigma_{vu,y}$	0.89	0.94	0.95	0.98	0.92	0.98	0.98
ρ^w	σ_{ρ^w}/σ_y	4.86	4.05	3.52	4.17	4.48	2.92	2.87
	ρ_{ρ^w}	0.91	0.81	0.82	0.87	0.84	0.86	0.86
	$\sigma_{\rho^w,y}$		0.94	0.95	0.98	0.93	0.98	0.98
$\frac{\bar{s}^*}{\bar{y}}$			0.19	0.19	0.19	0.19	0.19	0.19
$\frac{\eta \bar{s}^*}{A - \bar{x}^u}$			0.13	0.13	0.15	0.15	0.29	0.29
η			0.67	0.67	0.67	0.67	0.67	0.67
A			0.91	0.95	0.66	0.66	2.13	1.52

TABLE A4.2 VOLATILITIES ACROSS MODELS
Same persistence and volatility in the shock

			No					
			No	Yes				
			No		Yes			
			No			Yes		
			No				Yes	
			No					Yes
US			(2)	(3)	(4)	(5)	(6)	(7)
\hat{y}_t	σ_y	1.58	1.58	3.17	4.44	5.28	1.62	1.64
	ρ_y	0.84	0.93	0.92	0.98	0.97	0.91	0.92
$\ln u_t$	σ_u/σ_y	7.83	9.09	9.28	18.65	20.28	8.49	8.29
	ρ_u	0.87	0.90	0.89	0.92	0.82	0.91	0.91
	$\sigma_{u,y}$	-0.84	-0.99	-0.97	-0.81	-0.69	-0.99	-0.99
$\ln v_t$	σ_v/σ_y	8.85	4.70	3.69	1.94	2.52	2.05	2.09
	ρ_v	0.91	0.53	0.46	0.88	0.78	0.27	0.29
	$\sigma_{v,y}$	0.90	0.71	0.66	0.76	0.55	0.50	0.52
$\ln \frac{v_t}{u_t}$	σ_{vu}/σ_y	16.33	13.09	12.15	19.86	21.47	9.67	9.55
	ρ_{vu}	0.90	0.81	0.81	0.92	0.83	0.83	0.83
	$\sigma_{vu,y}$	0.89	0.94	0.94	0.83	0.71	0.98	0.99
ρ^w	σ_{ρ^w}/σ_y	4.86	4.05	3.51	3.93	3.98	2.93	2.90
	ρ_{ρ^w}	0.91	0.81	0.81	0.98	0.95	0.83	0.83
	$\sigma_{\rho^w,y}$		0.94	0.95	0.99	0.97	0.98	0.98
$\frac{\bar{s}^*}{\bar{y}}$			0.19	0.19	0.19	0.19	0.19	0.19
$\frac{\eta \bar{s}^*}{A - \bar{x}^u}$			0.13	0.13	0.15	0.15	0.29	0.29
η			0.67	0.67	0.67	0.67	0.67	0.67
A			0.91	0.95	0.66	0.66	2.13	1.52