Price-Taker Bidding Strategy Under Price Uncertainty

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***Abstract*—This paper provides a framework to obtain the optimal bidding strategy of a price-taker producer. An appropriate forecasting tool is used to estimate the probability density functions of next-day hourly market-clearing prices. This probabilistic information is used to formulate a self-scheduling profit maximization problem that is solved taking advantage of its particular structure. The solution of this problem allows deriving a simple yet informed bidding rule. Results from a realistic case study are discussed in detail.**

***Index Terms*—Bidding strategy, forecasting, MILP, pool-based electricity market, price-taker producer, self-scheduling.**

Nomenclature

The notation used throughout the paper is stated below.

a_t	Constant used to obtain the upper bound of the confidence interval for hour t.
b_t	Constant used to obtain the lower bound of the confidence interval for hour t.
B	Random variable describing the total profit of the thermal generator (all hours) in \$.
B^{avg}	Average value of the total profit of the thermal generator (all hours) in \$.
B_t	Random variable describing the profit of the thermal generator at hour t in \$.
B_t^{avg}	Average value of the profit of the thermal generator at hour t in \$.
c_t	Operating cost function of the thermal generator at hour t in \$/h. It is precisely described in the Appendix.
c_t^*	Optimal value of the operating cost function of the thermal generator at hour t in \$/h.
n	Number of scenarios.
p_t	Power produced by the thermal generator at hour t in MW.
p_t^*	Optimal power produced by the thermal generator at hour t in MW.
$\bar{P}$	Maximum power output of the thermal generator in MW.
R	Covariance matrix of random variables $\lambda_1, \ldots, \lambda_T$.
s	Scenario index.
t	Hour index.
T	Time span in hours.
$\gamma_{\alpha\%}$	α-percentage point of the $N(0,1)$ distribution.
λ_t	Random variable describing the market-clearing price at hour t in \$/MWh.
λ_t^{avg}	Average value of random variable λ_t in \$/MWh.
λ_t^{est}	Estimate of the market-clearing price at hour t in \$/MWh.
λ_t^{true}	True market-clearing price at hour t in \$/MWh.
λ_{ts}	Market-clearing price at hour t and scenario s in \$/MWh.
Π	Feasible operating region of the thermal generator. It is precisely described in the Appendix.
ω_s	Probability of scenario s.
σ_t^{est}	Estimate of the standard deviation of random variable λ_t in \$/MWh.
$E_{\lambda_1,\ldots,\lambda_T}\{\cdot\}$	Expected value operator with respect to random variables $\lambda_1, \ldots, \lambda_T$.
$E_{\lambda_t}\{\cdot\}$	Expected value operator with respect to random variable λ_t.
$\text{Var}_{B_t}\{\cdot\}$	Variance operator with respect to random variable B_t.
$\text{Var}_{B_1,\ldots,B_T}\{\cdot\}$	Variance operator with respect to random variables $B_1, \ldots, B_T$.

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I. Introduction

THIS paper addresses the bidding problem faced by a thermal price-taker producer in a pool-based electric energy market.

It is assumed that price uncertainty is high and that an appropriate forecasting tool is available to forecast next-day hourly prices and to estimate their associated probability density functions.

This paper provides a simple yet informed bidding rule that allows a price-taker producer to obtain optimal bidding decisions.

The analysis consists of three steps:

1) An appropriate price-forecasting tool is used to estimate the probability density functions corresponding to next-day hourly energy prices.
2) A self-scheduling problem is formulated using the probabilistic price information derived in 1). This problem is efficiently solved taking advantage of its singular structure, which is described in Section III.
3) The solution of the problem formulated and solved in 2) allows deriving a simple yet informed bidding rule for the price-taker.

The framework for the analysis above is a pool-based electricity market [1]–[3]. It is assumed that the market is cleared one day in advance on an hourly basis, and that producers and consumers submit hourly bidding curves consisting of blocks of energy and their corresponding prices. It is considered that, in each hour, every generator must bid all its available power in one or several power blocks at increasing prices of its choice [1]. No particular assumption is made on the market structure and its participants. The market-clearing price is used to pay any accepted production bid, and it is also the price paid by any accepted demand bid.

The analysis in this paper is restricted to considering a price-taker producer, i.e., a producer with no capability of altering market-clearing prices. In such situation, the profit maximization problem faced by the producer decomposes into independent subproblems. Each of these subproblems corresponds to the profit maximization of each generator owned by the producer [4]. Therefore, for the sake of simplicity, a single generator is considered henceforth. The problem faced by a price-maker producer, i.e., a producer with capability of altering market-clearing prices is outside the scope of this paper.

This paper extends the model reported in [4], providing a probabilistic framework for the treatment of uncertain market-clearing prices, and deriving a simple yet informed bidding rule.

Quite a few bidding methods addressing the strategic bidding problem have been published so far [5]. Pioneering paper [6] solves the optimal bidding problem for a single time period using dynamic programming. In [7], an analytical formulation for building the optimal bidding strategy in the former pool-based electricity market of England & Wales was developed under the assumption of a perfectly competitive market. However, this assumption does not seem reasonable for most electricity markets. In [8], a bidding strategy was proposed for the situation of two buyers competing for a single block of energy. Huse *et al.* [9] proposed a simple bidding strategy based on heuristics without taking into consideration the effect of intertemporal constraints. In [10], a simple bidding model is derived after estimating the probability of winning below and on the margin. In [11], a bidding strategy is developed to maximize the profit obtained by a supplier by adjusting its submitted operational parameters such as the declared minimum power output.

In [12]–[15], game theory is applied to find an equilibrium state (Nash equilibrium) of the bidding game, corresponding to the optimal bidding strategies achieved by the participants. Contreras *et al.* [16] propose an iterative Cournot model to find the optimal bidding policy of a generating company. In [17], Nash equilibrium is applied under the framework of bilateral based electricity markets. These methods are more suitable for analyzing strategic behavior rather than for proposing a tool to develop bidding strategies.

Under the framework of multi-round auctions, several bidding strategies are proposed [18]–[20]. In [21], [22], evolutionary and artificial intelligence techniques such as genetic algorithms, genetic programming and finite state automata are used to develop adaptive and evolutionary bidding strategies. Unfortunately, iterative auctions are not implemented in most electricity markets.

Other methods such as ordinal optimization [23], Lagrangian relaxation [24], stochastic optimization [25], and Markov decision process [26] have also been applied to solve the optimal bidding strategy problem. In [23], an ordinal optimization based bidding strategy is used for seeking good enough bids with high probabilities. An innovative model and a Lagrangian relaxation-based method are presented in [24] to solve the bidding and self-scheduling problem. In [25], the optimal bidding problem is modeled as a stochastic optimization problem taking into account the effect of competitors through Monte Carlo simulation. In [26], this problem is represented as a multiple stage probabilistic decision-making problem and a Markov decision process was applied to calculate bidding decisions.

The remainder of this paper is organized as follows. Section II provides the proposed modeling framework to deal with price uncertainty. In Section III, the price-taker self-scheduling problem under price uncertainty is formulated, analyzed and solved. Section IV provides a probabilistic description of the profit achieved by the price-taker. Section V presents the simple yet informed bidding rule derived from the solution of the self-scheduling problem. In Section VI, the results of a realistic case study are analyzed in detail. Section VII presents some relevant conclusions. Finally, in the Appendix, a mathematical linear description of the cost function and the feasible operating region of a generator is provided.

II. Price Uncertainty Modeling

A model for the uncertainty in the hourly market-clearing prices is proposed in this section. This model provides a probabilistic characterization of these prices that has a definitive impact on the self-scheduling problem presented in Section III. Moreover, the model also provides a probabilistic characterization of the profit for the price-taker, which is described in Section IV.

The model is based on the probability density functions of forecast prices. Several techniques to forecast electricity prices can be found in the technical literature. For instance, jump diffusion/mean reversion models have been proposed in [27]. Neural networks are used to predict prices in the England & Wales pool [28], in California [29], and in the Victorian market [30]. Techniques based on Fourier and Hartley transforms have been studied in [31]. Recently, in [32], two models based on time series analysis have been proposed. These models, which produce accurate predictions, relate actual prices to demands and past prices.

All the aforementioned forecasting procedures assume that the market-clearing price at hour $t(t=1,\ldots,T)$ is a random variable denominated λ_t, which has to be forecast. It should be noted that random variables $\lambda_t(t=1,\ldots,T)$ depend on the actual price values of the time series that is used for forecasting. From a statistical point of view, they are random variables "conditioned" to the actual price values of the time series used for forecasting. This time series spans from an arbitrary origin up to hour 24 of the day preceding the one whose prices have to be forecast. Under the above assumption, and using a time series forecasting procedure [33], the expected value of random variable λ_t is the actual price prediction at hour t, λ_t^{est}, that is

$$\lambda_t^{\text{est}} = \lambda_t^{\text{avg}} = E_{\lambda_t}\left\{\lambda_t\right\}. \tag{1}$$

This is a key fact that is used in Section III.

The estimate of the standard deviation of the random variable λ_t is readily available from the forecasting procedure, and it is denominated σ_t^{est}. Moreover, it can be shown that the distribution of random variable λ_t is approximately Lognormal [32], i.e.,

$$\lambda_t \equiv \text{Lognormal}\left(\lambda_t^{\text{est}}, \sigma_t^{\text{est}}\right). \tag{2}$$

Upper and lower bounds of the confidence interval are computed respectively as

$$\begin{aligned}&\lambda_t^{\text{est}} + a_t\sigma_t^{\text{est}}\\ &\lambda_t^{\text{est}} - b_t\sigma_t^{\text{est}}.\end{aligned} \tag{3}$$

It should be noted that parameters a_t and b_t are obtained directly from the forecasting procedure and depend on the considered level of confidence to be guaranteed, e.g., 99% or 95% [33].

They are computed so as to cover 99% or 95% of the total area under the Lognormal distribution. Formulae to compute parameters a_t and b_t are provided below [34]:

$$a_t = \frac{\exp\left(\frac{1}{2}\left(\sigma_t^{\text{est}}\right)^2 + \lambda_t^{\text{est}} + \gamma_{\alpha\%}\sigma_t^{\text{est}}\right) - \lambda_t^{\text{est}}}{\sigma_t^{\text{est}}} \tag{4}$$

$$b_t = \frac{\lambda_t^{\text{est}} - \exp\left(\frac{1}{2}\left(\sigma_t^{\text{est}}\right)^2 + \lambda_t^{\text{est}} - \gamma_{\alpha\%}\sigma_t^{\text{est}}\right)}{\sigma_t^{\text{est}}} \tag{5}$$

where $\gamma_{\alpha\%}$ depends on the desired level of confidence.

For a level of confidence of 99%, $\gamma_{99\%}$ is obtained from Probability $(N(0,1) > \gamma_{99\%}) = 0.01/2$, which results in $\gamma_{99\%} = 2.5758$. Analogously, $\gamma_{95\%} = 1.9600$.

III. Self-Scheduling

Under price uncertainty, the profit maximization problem of a price-taker generator can be formulated as

$$\begin{aligned}&\underset{p_t}{\text{maximize}}\ E_{\lambda_1,\ldots,\lambda_T}\left\{\sum_{t=1}^{T}\lambda_t p_t\right\} - \sum_{t=1}^{T} c_t\\ &\text{subject to } p_t \in \Pi.\end{aligned} \tag{6}$$

The objective function of the problem above is the expected value of profit for selling energy, i.e., expected revenues minus incurred operating costs (as described in the Appendix). Note that 1 hour time intervals are considered. The only constraint of this problem states that the generator must operate within its feasible operating region (power output limits, ramp-rate constraints, and minimum up and down time constraints). This feasible operating region is also precisely described in the Appendix.

It should be noted that the random variable describing market-clearing prices only affects the objective function, and particularly, the term corresponding to revenues.

Formulation (6) suggests a reformulation to allow a scenario-based solution approach, i.e.,

$$\begin{aligned}&\underset{p_t}{\text{maximize}}\ \sum_{s=1}^{n}\left(\omega_s\sum_{t=1}^{T}\lambda_{ts}p_t\right) - \sum_{t=1}^{T} c_t\\ &\text{subject to } p_t \in \Pi\end{aligned} \tag{7}$$

where $\sum_{s=1}^{n}\omega_s = 1$.

Note, however, that this scenario formulation leads to an intractable problem. For instance, considering 24 hours and 3 price values per hour results in 3^{24} scenarios which is a number higher than 2.8×10^{11}, and this number constitutes an excessive number of scenarios. An alternative approach is therefore needed, and it is developed in what follows.

Using basic probability theory [35], expectation and summation operators can be swapped in (6), resulting in

$$\begin{aligned}&\underset{p_t}{\text{maximize}}\ \sum_{t=1}^{T} E_{\lambda_t}\left\{\lambda_t\right\}p_t - \sum_{t=1}^{T} c_t\\ &\text{subject to } p_t \in \Pi.\end{aligned} \tag{8}$$

And using the expected value λ_t^{est} defined in (1), problem (8) becomes

$$\begin{aligned}&\underset{p_t}{\text{maximize}}\sum_{t=1}^{T}\left(\lambda_t^{\text{est}}p_t - c_t\right)\\ &\text{subject to } p_t \in \Pi.\end{aligned} \tag{9}$$

This problem is mixed-integer and linear (see the Appendix) and its size is moderate [4]. It can be easily solved using a standard branch and cut solver such as CPLEX under GAMS [36].

The solution of problem (9) provides the best possible production decision under price uncertainty: p_t^*, $t = 1, \ldots, T$. Note that all information available on prices (probability density functions) is used to reach the above optimal production decision. Note, also, that no additional information is available before the bidding procedure is carried out and the market cleared. Therefore, no additional information alters the optimal production decision.

The next step is to establish bidding rules to ensure that the generator gets allocated its optimal self-scheduled production, i.e., p_t^*, $t = 1, \ldots, T$. This is done in Section V.

IV. Profit

The profit achieved by the generator at hour t, B_t, is also a random variable. Using basic statistics theory [35], if the actual price is within the confidence interval, the average value of B_t is computed as

$$B_t^{\text{avg}} = \lambda_t^{\text{est}}p_t^* - c_t^* \tag{10}$$

and its variance is estimated as

$$\text{Var}_{B_t}(B_t) = \left(\sigma_t^{\text{est}}\right)^2\left(p_t^*\right)^2. \tag{11}$$

The profit achieved by the generator during the 24 hours of the day, B, is also a random variable. Its average value is easily computed as [35]

$$B^{\text{avg}} = \sum_{t=1}^{T}\lambda_t^{\text{est}}p_t^* - c_t^*. \tag{12}$$

The variance of B can be estimated as follows

$$\begin{aligned}\mathrm{Var}_{B_1,\ldots,B_T}(B) =& \mathrm{Var}_{B_1,\ldots,B_T}\left(\sum_{t=1}^{T} B_t\right) \\ =& [\,p_1^* \quad \ldots \quad p_T^*\,]\,R \begin{bmatrix} p_1^* \\ \vdots \\ p_T^* \end{bmatrix}. \end{aligned} \qquad (13)$$

The diagonal elements of the $T \times T$ covariance matrix R are the estimates of the variances of the random variables λ_t, i.e., $(\sigma_t^{\mathrm{est}})^2$, $t = 1, \ldots, T$. The off-diagonal elements are the covariances between each couple of random variables λ_t. That is, element $R_{i,j} (i \neq j)$ is the covariance of random variables λ_i and λ_j. The computation of matrix R is somehow involved but it is precisely described in [37].

V. Bidding Strategy

Using the results obtained in Section III, the proposed bidding strategy is stated below. The generator should submit to the market operator a bidding curve for each hour of the market horizon. Each one of these hourly bidding curves consists of a set of blocks of power and their corresponding increasing prices. For example, a 42-MW unit in hour 21 may bid powers 10, 20 and 12 MW at prices 20, 25, and 30 \$/MWh. It should be noted that a convex bidding curve is required, i.e., prices have to be associated with the power blocks bid.

The bidding rule formulated below to determine the hourly bidding curve of the generator only requires up to two blocks of power and their corresponding prices. If, as a result of the market rules, each hourly bidding curve should have a number of blocks larger than two, the rule below can be modified straightforwardly.

In the rule below, recall that σ_t^{est} is the estimate of the standard deviation of the probability density function describing the market-clearing price at hour t. Additionally, it is assumed that parameters a_t and b_t have been obtained for a level of confidence of 99%.

The bidding curve for hour t is formulated as a function of the optimal self-scheduled production in that hour, p_t^*. Three cases are possible and are analyzed below.

Case 1) If $p_t^* = 0$, the bidding curve consists of a single block of power $\bar{P}$ at price $\lambda_t^{\mathrm{est}} + a_t\sigma_t^{\mathrm{est}}$. See Fig. 1(a).

It should be noted that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is 0, which is the optimal self-scheduled power for this case.

Case 2) If p_t^* is such that $0 < p_t^* < \bar{P}$, the bidding curve consists of two blocks of power and their corresponding prices. These two blocks of powers are p_t^*, and $\bar{P} - p_t^*$, and their prices are $\lambda_t^{\mathrm{est}} - b_t\sigma_t^{\mathrm{est}}$ and $\lambda_t^{\mathrm{est}} + a_t\sigma_t^{\mathrm{est}}$, respectively. See Fig. 1(b).

Note that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is p_t^*, which is the optimal self-scheduled power for this case.

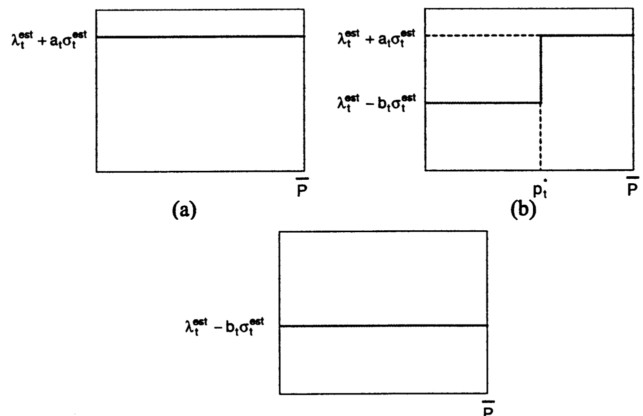

Fig. 1. Bid curves for the proposed bidding strategy.

TABLE I
VARIABLE COST

	Cost (\$/MWh)	Power (MW)
Block 1	25.84	130
Block 2	26.52	148
Block 3	27.83	166
Block 4	29.00	184
Block 5	30.62	202
Block 6	34.97	220
Block 7	33.14	238
Block 8	36.37	256
Block 9	39.00	274
Block 10	41.27	294

Case 3) If $p_t^* = \bar{P}$, the bidding curve consists of a single block of power $\bar{P}$ at price $\lambda_t^{\mathrm{est}} - b_t\sigma_t^{\mathrm{est}}$. See Fig. 1(c). It should be noted that this bidding curve guarantees with a level of confidence of 99% that the power accepted in this situation is $\bar{P}$, which is the optimal self-scheduled power for this case.

If the required level of confidence has to be larger than 99%, parameters a_t and b_t should be computed accordingly. However, note that a low profile bidding behavior is convenient in many markets, and this requires bidding prices not far away from the actual market-clearing prices.

VI. Case Study

This section provides a comparison in terms of profit and power schedule obtained by a generator under price uncertainty and under perfect knowledge of true energy prices.

Data for the considered power unit as well as price values are given below. The characteristics of the unit are based on the data of [38]. The shut-down cost is considered constant and equal to \$56 and the fixed cost is \$700. The start-up cost is considered constant and equal to \$1038. Reference [4] provides a model to consider the exponential variation of the start-up cost that can be easily integrated in the framework provided in this paper.

A ten-block nonconvex variable cost is considered and given in Table I. A nonconvex cost has been selected to illustrate the capability of the proposed formulation to handle this type of costs.

TABLE II
FEASIBLE OPERATING REGION

$\underline{P}$ [MW]	$\overline{P}$ [MW]	SU [MW/h]	SD [MW/h]	RU [MW/h]	RD [MW/h]	UT [h]	DT [h]
112	294	170	160	60	50	4	4

TABLE III
ENERGY PRICE DATA ($/MWh)

t	λ_t^{true}	λ_t^{est}	Lower bound	Upper bound	σ_t^{est}
1	28.52	33.30	27.22	40.75	2.61
2	25.23	26.52	21.63	32.51	2.10
3	23.45	22.15	18.04	27.20	1.76
4	22.93	23.10	18.81	28.36	1.84
5	22.79	22.59	18.40	27.74	1.80
6	22.93	23.15	18.85	28.43	1.84
7	23.29	24.64	20.07	30.26	1.96
8	25.46	24.75	20.15	30.39	1.97
9	22.93	25.50	20.76	31.31	2.03
10	28.28	27.58	22.46	33.86	2.20
11	31.08	31.59	25.73	38.79	2.52
12	39.93	35.59	28.99	43.70	2.84
13	40.24	41.05	33.43	50.40	3.27
14	40.55	41.60	33.88	51.08	3.32
15	39.93	38.98	31.74	47.86	3.11
16	39.93	39.73	32.36	48.79	3.17
17	40.75	42.02	34.22	51.59	3.35
18	41.56	42.09	34.28	51.68	3.36
19	40.75	40.74	33.18	50.02	3.25
20	39.74	38.80	31.60	47.64	3.09
21	39.54	39.63	32.27	48.66	3.16
22	42.77	46.14	37.58	56.66	3.68
23	40.55	39.03	31.79	47.93	3.11
24	29.23	33.67	27.42	41.35	2.68

Table II shows the limits that constrain the feasible operating region, Π, of the generator: minimum power output, maximum capacity, start-up ramp rate limit, shut-down ramp rate limit, ramp-up rate limit, ramp-down rate limit, minimum up time, and minimum down time.

Finally, in the hour before the market horizon the unit has been running for 11 hours and producing 170 MW.

It is assumed that the power output of the thermal unit is constant throughout each hour. However, a linear variation of the power output during each hour can be modeled as stated in [39]. For clarity, this model is not considered in this paper.

Price data are provided in Table III. The second column corresponds to the actual prices obtained in the electricity market of mainland Spain on Wednesday August 29th, 2001 [1]. The third column shows the estimates of the energy prices using the forecasting method proposed in [32] with a level of confidence equal to 99%. Lower and upper bounds of the estimate of each price, as well as an estimate of its standard deviation are also shown in this table. Fig. 2 depicts the actual energy prices, the forecast energy prices and their bounds.

Firstly, the self-scheduling problem is solved with the forecast prices. The hourly power output can be found in Table IV. For this production schedule, the profit that the generator would have obtained can be computed using the true prices (settlement procedure). In this case, the actual settlement profit, computed using optimal self-scheduled powers and true prices is

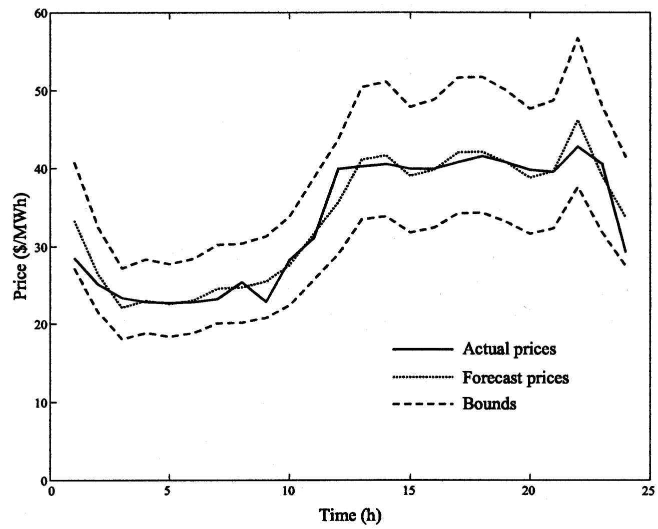

Fig. 2. Price forecast versus actual prices.

U.S. $27 207.70. Note that the optimal self-scheduled powers have been obtained with forecast prices.

In order to assess the actual loss in profit due to price uncertainty, it is assumed that the generator is a perfect fortune-teller, i.e., it has perfect knowledge of the true prices in advance. Note, however, that this information is not available in the real world. If the self-scheduling problem is solved with the true price profile, the settlement profit is equal to U.S. $27 268.95, which represents the maximum profit the generator can make. Note that there is only a 0.22% difference between the maximum attainable profit and the actual profit obtained with the forecast schedule. This slight difference shows that both, the forecasting technique and the bidding strategy, are efficient. Power output in each hour is shown in Table IV for the two price profiles. As it can be noted, both schedules meet the technical constraints presented in Table II. Moreover, the production schedules obtained with both price profiles are different (hours 14, 15, 17, 20, 23, and 24) implying that several blocks of energy are bid at different prices in one case versus the other; however, the commitment status is identical for both cases.

Finally, Table V presents the structure of the hourly bids that the generator should submit to the market operator. For the sake of simplicity, it is assumed that the generator bids one or two blocks in every hour, as proposed in Section V. In hours of scheduled power between 0 and maximum power output (hours 1, 11–13, 15, 16, 19–21, 23, and 24), the actual scheduled energy is bid at a price smaller than the true one (the lower bound of the confidence interval) and the remaining power at a price greater than the true one (the upper bound of the confidence interval). In those hours of scheduled power equal to maximum power output (hours 14, 17, 18, and 22) price bids are smaller than the corresponding true prices. On the other hand, in those hours where the scheduled power is equal to 0 (hours 2–10) the corresponding price bid is greater than the true price. Note that with the above bidding strategy the desired schedule does become the actual one.

The model has been implemented on a SGI R12000, 400 MHz based processor with 500 MB of RAM using CPLEX 7.5 under

TABLE IV
SELF-SCHEDULE IN MW

t	With forecast prices	With true prices
1	160.0	160.0
2	0.0	0.0
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	0.0	0.0
7	0.0	0.0
8	0.0	0.0
9	0.0	0.0
10	0.0	0.0
11	170.0	170.0
12	230.0	230.0
13	274.0	274.0
14	294.0	274.0
15	256.0	274.0
16	274.0	274.0
17	294.0	274.0
18	294.0	294.0
19	274.0	274.0
20	256.0	274.0
21	274.0	274.0
22	294.0	294.5
23	256.0	252.0
24	206.0	202.0

TABLE V
BIDDING STRATEGY

t	Power bid (MW)		Price bid ($/MWh)		λ_t^{true}
	Block 1	Block 2	Block 1	Block 2	
1	160	134	27.22	40.75	28.52
2	294	-	32.51	-	25.23
3	294	-	27.20	-	23.45
4	294	-	28.36	-	22.93
5	294	-	27.74	-	22.79
6	294	-	28.43	-	22.93
7	294	-	30.26	-	23.29
8	294	-	30.39	-	25.46
9	294	-	31.31	-	22.93
10	294	-	33.86	-	28.28
11	170	124	25.73	38.79	31.08
12	230	64	28.99	43.70	39.93
13	274	20	33.43	50.40	40.24
14	294	-	33.88	-	40.55
15	256	38	31.74	47.86	39.93
16	274	20	32.36	48.79	39.93
17	294	-	34.22	-	40.75
18	294	-	34.28	-	41.56
19	274	20	33.18	50.02	40.75
20	256	38	31.60	47.64	39.74
21	274	20	32.27	48.66	39.54
22	294	-	37.58	-	42.77
23	256	38	31.79	47.93	40.55
24	206	88	27.42	41.35	29.23

GAMS [36]. The optimal solutions to both cases (with forecast and with actual energy prices) were achieved in 2.0 seconds of computing time.

VII. CONCLUSIONS

This paper provides a bidding rule that allows a price-taker producer to achieve, under price uncertainty, its optimal self-schedule. An appropriate probability description of hourly market-clearing prices is provided. It is used to formulate and solve an expected maximum profit self-scheduling problem. The solution of this problem allows determining a simple yet informed bidding rule to achieve the actual optimal self-schedule.

The analysis of multiple case studies allows concluding that the rule developed is effective in achieving the optimal (and feasible) schedule of the price-taker producer.

APPENDIX

The operating cost and the set of operating constraints, Π, are presented in this Appendix.

The nonlinear and nonconvex operating cost, c_t, can be formulated as

$$c_t = Cz(t) + Av(t) + d(p_t) + Sy(t) \quad \forall\, t = 1, \ldots, T \quad \text{(A1)}$$

where C is the shut-down cost [$/h], $z(t)$ is a 0/1 variable which is equal to 1 if the unit is shut-down at the beginning of hour t, A is the fixed cost [$/h], $v(t)$ is a 0/1 variable which is equal to 1 if the unit is on-line at hour t, $d(p_t)$ expresses the variable production cost at hour t [$/h], which is a nonlinear function of the power output at that hour. Finally, $y(t)$ is a 0/1 variable which is equal to 1 if the unit is started-up at the beginning of hour t, and S is the start-up cost [$/h].

A mixed-integer linear formulation of the nonconvex and nondifferentiable variable production cost is provided in this Appendix and can also be found in [4]

$$\tilde{d}(t) = \sum_{\ell=1}^{\text{NL}} \left[F_\ell \delta_\ell(t) + t_\ell(t) \sum_{m=1}^{\ell-1} F_m \left(T_m - T_{m-1}\right) \right] \quad \forall\, t = 1, \ldots, T \quad \text{(A2)}$$

$$p_t = \sum_{\ell=1}^{\text{NL}} \left[\delta_\ell(t) + t_\ell(t) T_{\ell-1}\right] \quad \forall\, t = 1, \ldots, T \quad \text{(A3)}$$

$$\sum_{\ell=1}^{\text{NL}} t_\ell(t) = 1 \quad \forall\, t = 1, \ldots, T \quad \text{(A4)}$$

$$\delta_\ell(t) \le \left(T_\ell - T_{\ell-1}\right) t_\ell(t) \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \quad \text{(A5)}$$

$$\delta_\ell(t) \ge 0 \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \quad \text{(A6)}$$

$$t_\ell(t) \in \{0, 1\} \quad \forall\, \ell = 1, \ldots, NL,\ \forall\, t = 1, \ldots, T \quad \text{(A7)}$$

where $\tilde{d}(t)$ is the piecewise linear variable cost at hour t [$/h] which replaces the nonlinear variable cost, $d(\cdot)$, in (A1), F_ℓ is the slope of block ℓ of the variable cost [$/MWh], NL is the number of blocks of the variable cost, $\delta_\ell(t)$ is the power produced in the block ℓ at hour t [MW], $t_\ell(t)$ is a 0/1 variable which is equal to 1 if block ℓ determines the power at hour t, and T_ℓ is the upper limit of block ℓ [MW].

Finally, the following set of linear constraints formulates the feasible operating region, Π, comprising power limits, ramp rate limits and minimum up and down time constraints [4]

$$p_t \ge \underline{P} v(t) \quad \forall\, t = 1, \ldots, T \quad \text{(A8)}$$

$$p_t \le \bar{P}\left[v(t) - z(t+1)\right] + z(t+1)\text{SD} \quad \forall\, t = 1, \ldots, T \quad \text{(A9)}$$

$$p_t \le p_{t-1} + \text{RU} v(t-1) + \text{SU} y(t) \quad \forall\, t = 1, \ldots, T \quad \text{(A10)}$$

$$p_{t-1} - p_t \leq \mathrm{RD}v(t) + \mathrm{SD}z(t) \quad \forall\, t = 1, \ldots, T \tag{A11}$$

$$\sum_{t=1}^{G} [1 - v(t)] = 0 \tag{A12}$$

$$\sum_{j=t}^{t+\mathrm{UT}-1} v(j) \geq \mathrm{UT}y(t) \quad \forall\, t = G+1, \ldots, T - \mathrm{UT} + 1 \tag{A13}$$

$$\sum_{j=t}^{T} [v(j) - y(t)] \geq 0 \quad \forall\, t = T - \mathrm{UT} + 2, \ldots, T \tag{A14}$$

$$\sum_{t=1}^{F} v(t) = 0 \tag{A15}$$

$$\sum_{j=t}^{t+\mathrm{DT}-1} [1 - v(j)] \geq \mathrm{DT}z(t) \quad \forall\, t = F+1, \ldots, T - \mathrm{DT} + 1 \tag{A16}$$

$$\sum_{j=t}^{T} [1 - v(j) - z(t)] \geq 0 \quad \forall\, t = T - \mathrm{DT} + 2, \ldots, T \tag{A17}$$

$$y(t) - z(t) = v(t) - v(t-1) \quad \forall\, t = 1, \ldots, T \tag{A18}$$

$$y(t) + z(t) \leq 1 \quad \forall\, t = 1, \ldots, T \tag{A19}$$

$$z(t) \in \{0, 1\} \quad \forall\, t = 1, \ldots, T \tag{A20}$$

where

F $\quad \mathrm{Min}\, \{T, [\mathrm{DT} - s(0)]\, [1 - v(0)]\}$;

G $\quad \mathrm{Min}\, \left[T, \left(\mathrm{UT} - U^0\right) v(0)\right]$.

In the above formulation, $\underline{P}$ is the minimum power output [MW], SD is the shut-down ramp rate limit [MW/h], RU is the ramp-up rate limit [MW/h], SU is the start-up ramp rate limit [MW/h], RD is the ramp-down rate limit [MW/h], G is the number of intervals the unit must be initially on-line due to the minimum up time constraint [h], UT is the minimum up time [h], F is the number of intervals the unit must be initially off-line due to the minimum down time constraint [h], DT is the minimum down time [h], $s(0)$ is the number of periods the unit has been off-line at the beginning of the market horizon (end of hour 0) [h], and U^0 is the number of periods the unit has been on-line at the beginning of the market horizon (end of hour 0) [h].

Constraints (A8) and (A9) set the limits on the power output. Ramp rate limits (ramp-up, start-up, ramp-down and shut-down) are imposed by constraints (A10) and (A11). Constraints (A12)–(A14) and (A15)–(A17) enforce the minimum up and down time constraints respectively. Constraints (A18) and (A19) preserve the logic of the variables representing running, start-up, and shut-down status changes [40]. Finally, variables $z(t)$ are stated as binary in constraints (A20).

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