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Price versus Quantities versus Indexed Quantities

— FAERE Working Paper version —

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Abstract

We develop a stochastic model to rank different policies (tax, fixed cap and relative cap) according to their expected total social costs. Three types of uncertainties are taken into account: uncertainty about abatement costs, business-as-usual (BAU) emissions and future economic output (the two latter being correlated). Two parameters: the ratio of slopes of marginal benefits and marginal costs, and the above-mentioned correlation, are crucial to determine which instrument is preferred.

When marginal benefits are relatively flatter than marginal costs, prices are preferred over fixed caps (Weitzman's result). When the former correlation is higher than a parameter- dependent threshold, relative caps are preferred to fixed caps. An intermediate condition is found to compare the tax instrument and the relative cap.

The model is then empirically tested for seven different regions (China, the United States, Europe, India, Russia, Brazil and Japan). We find that tax is preferred to caps (absolute or relative) in all cases, and that relative caps are preferred to fixed caps in the US and emerging countries (except Brazil where it is ambiguous), whereas fixed cap are preferred to relative cap in Europe and Japan.

Key-words

Instrument; Price; Quantity; Intensity Target; Regulation; post-Kyoto; Uncertainty; Climate Policy

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1. Introduction

At the Durban UNFCCC conference of the parties in 2011, the international community agreed on limiting the mean temperature increase to 2 degrees Celsius with respect to pre-industrial level. While there is an agreement on this long-term goal, there is much debate on the design of the policy mechanisms to achieve this objective. International negotiations are at a standstill, and the potential successor of the top-down Kyoto protocol is likely to be a minimal agreement, leaving room to countries to implement their own climate policies in a bottom-up architecture (Rayner, 2010).

For cost-effectiveness reasons, economists advocate emissions pricing for abatement, in the form of a tax (price instrument) or tradable permits (quantity instrument), rather than command-and-control regulation. Following the seminal paper of Weitzman (1974), a considerable literature has developed, comparing these two instruments or hybrid systems in terms of administrative costs, political acceptance and the way they address uncertainties, among others (Goulder and Schein, 2013).

For climate change mitigation, this Weitzman argument favors the carbon tax, since there is an important inertia of accumulation of carbon emissions in the atmosphere, so the marginal abatement cost curve is steeper than the marginal benefit curve (Hoel and Karp, 2002; Newell and Pizer, 2003). However, tradable permits have been the most frequent chosen option among the few carbon pricing pioneers such as the European Union in 2005, New Zealand in 2008 or California in 2012. At the federal level of the United States, the prevailing view has been that a tax was a political taboo (Newell and Pizer, 2008; Webster et al., 2010). However, this option may now attract decision makers as a part of a broader tax reform (Goulder and Schein, 2013).

In a cap-and-trade system, the level of emissions is fixed, and the price of the allowance varies and is therefore uncertain. This may induce two potential problems (Conte Grand, 2013) linked to "twin uncertainties" (Kim and Baumert, 2002). In case of higher than expected economic activity, the soaring of abatement costs constitutes an "economic risk". Conversely, during an economic recession, emissions may be greater in the presence of a cap-and-trade scheme than without any policy, which represents an "environmental risk". Then, if there is a demand for allowances in other regions or other sectors, "hot air" is generated leading to unjustified rent such as in the Former Soviet Union within the context of the Kyoto protocol. Otherwise, a surplus of allowances makes the allowance price plummet like in the European Union Emissions Trading System (EU ETS). As the uncertainty of abatement costs federates opposition to climate policies and so bear political costs, decision makers have given more importance to the economic risk. However, the severe crisis of the EU ETS threatening its very existence made a case for the second problem and attracted the attention of policy makers.

To cope with these issues, some authors have proposed a price cap, also known as "safety valve" (Jacoby and Ellerman, 2004; Webster et al., 2010) or to replace a fixed cap by an "intensity target", where the cap of emissions is indexed on the gross domestic product. Argentina proposed such a cap in 1999 (Barros and Grand, 2002) and after its opposition to sign the Kyoto protocol, the Bush administration announced in February 2002 a decrease in emissions intensity by 2012 of 18% compared to 2002[1]. While advocating for intensity-based caps at the international level, Chinese officials opted for absolute caps in their pilot ETS projects (Ecofys, 2013; Wang, 2013).

The main case for intensity targets is to foster the participation of developing countries (Baumert et al., 1999; Frankell, 1999) which fear that climate policies would stifle their economic development. At the company level, an indexed regulation would act as a production subsidy and thereby generate an inefficient allocation of resources (Dudek and Golub, 2003; Fischer, 2003; Holland, 2012). This argument is justified for policies like the phasing out of lead in gasoline, when the index is taken into account in firms' decisions. However as noted in Sue Wing et al. (2006), a nationwide intensity target would not have the same effect on firms production processes, as GDP is not considered in firms' decisions and economic growth is pursued by countries for its own sake.

As categorized by Marschinski and Lecocq (2006), there are two strands of literature comparing the effectiveness of the different economic instruments under uncertainty. The first one is derived from Weitzman (1974) "price versus quantities" article and is reasoning in terms of *welfare*, weighting environmental benefits against abatement costs. As mentioned, in this framework, authors advocate for the superiority of the carbon tax (Newell and Pizer, 2003). Quirion (2005) extended this approach to indexed regulation with a stochastic analytical model featuring uncertainty in business-as-usual emissions and in the slope of the marginal abatement curve. He found that intensity targets ranked better than price instrument when uncertainty in abatement costs was high. Newell and Pizer (2008) reexamined this problem with an additive uncertainty in abatement cost and a correlation between shocks in abatement costs and economic activity (Quirion (2005) had a multiplicative uncertainty and an implicit perfect correlation). They found that the ranking of policies depended on parameters describing the first and second moment of the index and the ex post optimal quantity level.

The second strand of literature studies the impact of the different policies on uncertainties of different policy variables, mainly *uncertainty in abatement*. Sue Wing et al. (2006) established a formula to rank intensity targets over absolute targets (if the correlation coefficient of shocks on GDP and business-as-usual (BAU) emissions is greater than a parameter-dependent threshold, intensity targets are preferred). An equivalent condition was obtained by Marchinski and Lecocq (2006) in a more generalized version and in Marschinski and Edenhofer (2010). Jotzo and Pezzey (2007) obtained a different condition due to a different

[1]The majority of analysts noted that this objective was not far from a business as usual scenario, and the US indeed decreased their emission intensity by more than 30% during this decade.

framework[2], but the general idea was identical: the more emissions and GDP are linked the better intensity targets compared to absolute targets.

The purpose of this paper is to extend Newell and Pizer's model so as to unify these two strands of literature. For this we consider three types of uncertainty: uncertainty in abatement costs, BAU emissions and future economic output. Only the latter two uncertainties are correlated. In this more general context, we confirm Weitzman's result that prices are preferred to quantities when marginal benefits are relatively flat compared to marginal costs, and that intensity targets are preferred to absolute targets when the correlation of uncertainty in emissions and economic output is greater than a parameter-dependent threshold. An intermediary condition is obtained to compare a tax and intensity cap. Further, we are able to compare our results to the literature which minimizes the variance of abatement costs, including Sue Wing et al. (2006), and show that reasoning in terms of abatement costs only and setting environmental benefits aside introduces a bias supporting relative caps, but that this bias is small in the case of climate change mitigation.

We then estimate the model for seven different regions (China, the United States, Europe, India, Russia, Brazil and Japan) using past GDP and emissions data, and International Energy Outlook forecasting. There is a high uncertainty in the value of model parameters which are sensitive to the estimation method. However some relatively robust findings can be drawn. First, the price instrument is preferred to absolute or relative caps. Second, relative caps are preferred to fixed caps in the US and emerging countries (except Brazil where it is ambiguous), whereas fixed cap are preferred to relative cap in Europe and Japan.

The rest of the article is structured as follows. Section 2 explains the model which is used for the price versus quantities problem in Section 3. Next, section 4 studies intensity targets. Empirical estimation of the model is given in section 5. Proofs are in the Appendix.

2. The model

The total environmental costs (TEC) depend on the level of *emissions*:

$$TEC_e = b_0 + b_1 e + \frac{1}{2} b_2 e^2 \tag{1}$$

while the total abatement costs (TAC) are a function of *abatement* $\tilde{e}^b - e$ where $\tilde{e}^b$ are *business-as-usual* emissions i.e. emissions without a price on emissions (parameters with a tilde are random variables).

$$TAC_e = \tilde{c_1}(\tilde{e}^b - e) + \frac{1}{2} c_2 (\tilde{e}^b - e)^2 \tag{2}$$

[2] In their model, a share *alpha* of the emissions has a perfect correlation with GDP, and the rest has no correlation, which is different than an imperfect correlation *rho* for all the emissions.

Abatement costs are uncertain through a random additive shock in marginal cost: $\tilde{c_1} = c_1 + \tilde{\varepsilon_c}$, with $\tilde{\varepsilon_c}$, random variable of zero mean and standard deviation σ_c. Business as usual emissions are uncertain in a similar way, $\tilde{e}^b = e_0^b + \tilde{\varepsilon_e}$, with $\tilde{\varepsilon_e}$, random variable of zero mean and standard deviation σ_e.

We do not model uncertainties in marginal environmental costs as in this framework they do not influence the instruments ranking (they would matter only if correlated to abatement costs (Stavins, 1996), and there are no obvious arguments for suspecting such a correlation in the case of greenhouse gases).

Quadratic functions are a simplification to facilitate the tractability of the model and could be considered as approximations of real functions near arbitrary points (Weitzman, 1974). Uncertainties are modeled in an additive way for tractability and to be comparable to the post-Weitzman literature.

The total social costs are then:

$$\begin{aligned} TSC_e &= TAC_e + TEC_e \\ &= b_0 + b_1 e + \frac{1}{2} b_2 e^2 + \tilde{c_1}(\tilde{e}^b - e) + \frac{1}{2} c_2 (\tilde{e}^b - e)^2 \end{aligned}$$

Developping and sorting the different terms, we have:

$$\begin{aligned} TSC_e = b_0 &+ (c_1 + \tilde{\varepsilon_c})(e_0^b + \tilde{\varepsilon_e}) + \frac{1}{2} c_2 (e_0^b + \tilde{\varepsilon_e})^2 \\ &- [c_1 - b_1 + \tilde{\varepsilon_c} + c_2(e_0^b + \tilde{\varepsilon_e})]e \\ &+ \frac{1}{2}(b_2 + c_2)e^2 \end{aligned}$$

For each policy instrument, we look for the policy variable (which could be the carbon price, the fixed cap or relative cap) that minimizes the *expected* total social costs. Then we calculate the differences of expected total social costs for pair-wise comparison of policies, rank the policies according to key parameters and represent the best policy in a two-dimension diagram.

3. Price versus Quantities revisited

3.1. Quantity instrument (Q)

The *expected* total social costs depending on emission level e are

$$E(TSC_e) = b_0 + c_1 e_0^b + \frac{1}{2} c_2 ((e_0^b)^2 + \sigma_e^2) - [c_1 - b_1 + c_2 e_0^b]e + \frac{1}{2}(b_2 + c_2)e^2 \quad (3)$$

It is a quadratic function of e with a positive coefficient for e^2 so it has a unique minimum.

The optimal cap minimizing expected total social costs is:

$$e^* = \frac{c_2 e_0^b + c_1 - b_1}{b_2 + c_2} = \frac{e_0^b}{1 + \frac{b_2}{c_2}} [1 + \frac{c_1 - b_1}{c_2 e_0^b}] \quad (4)$$

(the second formulation using two adimensional parameters $\frac{b_2}{c_2}$ and $\frac{c_1 - b_1}{c_2 e_0^b}$ is useful in the rest of the paper[3])

Proposition 1. *Using the quantity instrument Q, the cap set is identical to the cap set if there were no uncertainty.*

Proof. e^* does not depend on parameters σ_c or σ_e. □

This is the "uncertainty equivalence" of Weitzman (1974). The cap is biding ($e^* < e_0^b$) and strictly positive ($e^* > 0$) if and only if $(c_1 - b_1) < b_2 e_0^b$ and $(b_1 - c_1) < c_2 e_0^b$. We suppose that it is the case.

The relationship between emissions and carbon price (allowance price in case of a quantity instrument or value of the tax in case of a price instrument) is as follows: the carbon price equals the marginal abatement cost at the emissions level:

$$p(e) = MAC(e) = -\frac{\partial TAC}{\partial e} = c_1 + c_2(e_0^b - e) + c_2\tilde{\varepsilon_e} + \tilde{\varepsilon_c} \tag{5}$$

In the case of a quantity instrument, emissions are equal to the (ex ante) optimal emissions level e^* and then:

$$p(e^*) = c_1 + c_2\frac{b_2 e_0^b - c_1 + b_1}{b_2 + c_2} + \tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e} \tag{6}$$

In case of a quantity instrument, emissions are capped but the allowance price is not known in advance. Once the cap is fixed, a positive cost shock or a positive BAU emissions shock increase the allowance price necessary to achieve it because these shocks induce a larger than expected abatement. Would they be known in advance (optimal policy, see further), the cap would have been set higher to trade off reduced abatement costs and increased environmental costs.

3.2. Price instrument (P)

Replacing e by p thanks to equation (5) in TSC_e and minimizing expected total social costs leads to (see detailed proof in AppendixB):

$$p^* = b_1 + b_2 e^* \tag{7}$$

The corresponding emissions are

$$e(p^*) = \frac{c_2}{b_2 + c_2}e_0^b + \frac{c_1 - b_1}{b_2 + c_2} + \frac{\tilde{\varepsilon_c}}{c_2} + \tilde{\varepsilon_e} \tag{8}$$

We have the relationships:

[3]Specifically, instead of 10 dimensional parameters: b_1, b_2 c_1, c_2, e_0^b, x_0^b, σ_c, σ_e, σ_x, σ_{ex} (see further for the two latter ones), all the results can be expressed with 6 adimensional parameters: $\frac{b_2}{c_2}$, $\frac{c_1 - b_1}{c_2 e_0^b}$, ν_c $(=\frac{\sigma_c}{c_2 e_0^b})$, ν_e $(=\frac{\sigma_e}{e_0^b})$, ν_x and ρ_{ex}

$$\begin{cases} e(p^*) = e^* + \dfrac{\tilde{\varepsilon_c}}{c_2} + \tilde{\varepsilon_e} \\ p(e^*) = p^* + \tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e} \end{cases}$$

Proposition 2. *Without uncertainty on abatement costs and future BAU emissions, the price instrument P and the quantity instrument Q are equivalent.*

Proof. If $\tilde{\varepsilon_c} = 0$ and $\tilde{\varepsilon_e} = 0$, then $e(p^*) = e^*$ and $p(e^*) = p^*$. □

3.3. Optimal policy (O)

We can define the ex post optimal policy for both the quantity and the price instrument as in Newell and Pizer (2008). The optimal cap is

$$\begin{aligned} e_{opt} &= \frac{c_1 - b_1 + \tilde{\varepsilon_c} + c_2(e_0^b + \tilde{\varepsilon_e})}{b_2 + c_2} \\ &= e^* + \frac{c_2}{b_2 + c_2}(\tilde{\varepsilon_e} + \frac{\tilde{\varepsilon_c}}{c_2}) \\ &= e(p^*) - \frac{b_2}{b_2 + c_2}(\tilde{\varepsilon_e} + \frac{\tilde{\varepsilon_c}}{c_2}) \end{aligned}$$

and the optimal price is

$$\begin{aligned} p_{opt} &= c_1 - \frac{c_2}{b_2 + c_2}(c_1 - b_1) + \frac{b_2 c_2}{b_2 + c_2} e_0^b + \frac{b_2}{b_2 + c_2}(\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e}) \\ &= p^* + \frac{b_2}{b_2 + c_2}(\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e}) \\ &= p(e^*) - \frac{c_2}{b_2 + c_2}(\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e}) \end{aligned}$$

Then if $\tilde{\varepsilon_c}$ and $\tilde{\varepsilon_e}$ are positive:

$$\begin{cases} p^* < p_{opt} < p(e^*) \\ e^* < e_{opt} < e(p^*) \end{cases}$$

And vice-versa if they are negative.

3.4. Comparison of P, Q and O

To compare instruments we compare the minimum expected total social costs[4].

[4]We have $E(TSC_e)_{|e=e^*} = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) - \dfrac{1}{2}(b_2 + c_2)(e^*)^2$, $E(TSC_p)_{|p=p^*} = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) - \dfrac{1}{2}(b_2 + c_2)(e^*)^2 + \dfrac{c_2 - b_2}{2c_2^2}(\sigma_c^2 + c_2^2\sigma_e^2)$ and $E(TSC_{opt}) = c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) - \dfrac{1}{2}(b_2 + c_2)(e^*)^2 - \dfrac{1}{2(b_2 + c_2)}(\sigma_c^2 + c_2^2\sigma_e^2)$

$$\begin{aligned}\Delta_{P-Q} &= E(TSC_p)_{|p=p^*} - E(TSC_e)_{|e=e^*} \\ &= -\frac{c_2 - b_2}{2c_2^2}(\sigma_c^2 + c_2^2\sigma_e^2)\end{aligned} \tag{9}$$

Proposition 3. *(Weitzman's result). The price instrument P performs better than the quantity instrument Q if marginal environmental costs are flatter than marginal abatement costs: $P \succ Q \Leftrightarrow b_2 < c_2$.*

Proof. As we reason in terms of social costs (and not in terms of benefits or welfare), a policy is preferred to another when expected total social costs are lower (so when the difference of expected total social costs are negative, e.g $P \succ Q \Leftrightarrow \Delta_{P-Q} < 0$). □

This is the classical Weitzman criterion (Weitzman, 1974). The magnitude of the difference in expected total social costs between the two policies depends on uncertainty in incurred costs ($(\sigma_c^2 + c_2^2\sigma_e^2)$): the bigger it is and the larger is the difference. Uncertainty in incurred costs come from both uncertainty in marginal abatement costs (σ_c) and uncertainty of future baseline emissions (σ_e) which determine the abatement needed to meet the target (Marschinski and Edenhofer, 2010). Uncertainty in BAU emissions and uncertainty in abatement costs are commensurate, the former being "converted" into the latter through the marginal abatement cost curve. The steeper is the curve (c_2 high), the greater is the impact of uncertainty in BAU emissions into uncertainty in incurred costs.

The criterion to prefer tax or trading remains the same compared to Newell and Pizer (2008), but the difference in expected total social costs are larger because of the additional term $c_2^2\sigma_e^2$. Numerical estimations (see part 5.2) lead to the conclusion that in terms of incurred costs uncertainty, BAU emissions uncertainty matters more than structural abatement costs uncertainty ($c_2\sigma_e > \sigma_c$). Pezzey and Jotzo (2012) find similar observation in their multi-party numerical model incorporating these two uncertainties: the inclusion of BAU emissions uncertainty increases the tax versus trading advantage by a factor 40. This point gives a justification for the introduction of BAU emissions uncertainty compared to Newell and Pizer (2008) model[5].

We can compare P and Q to the ex post optimal policy O:

$$\begin{aligned}\Delta_{O-Q} &= E(TSC_{opt}) - E(TSC_e)_{|e=e^*} \\ &= -\frac{1}{2(b_2 + c_2)}(\sigma_c^2 + c_2^2\sigma_e^2)\end{aligned}$$

[5] Actually in the earlier working paper version (Newell and Pizer, 2006), emissions uncertainties are introduced, but only in the empirical part to estimate σ_c. The comparison between P and R does not take uncertainty in future BAU emissions into account.

$$\Delta_{O-P} = E(TSC_{opt}) - E(TSC_p)_{|p=e^*}$$
$$= -\frac{(\frac{c_2}{b_2})^2}{2(b_2+c_2)}(\sigma_c^2 + c_2^2\sigma_e^2)$$

Proposition 4. *The bigger uncertainties on abatement costs and/or BAU emissions, the larger the advantage of O over P and Q.*

Proof. Because of the factor $(\sigma_c^2 + c_2^2\sigma_e^2)$, the greater σ_c or σ_e, the bigger the difference in total expected social costs. □

4. Relative cap

4.1. Relative cap (R)

In this policy (called relative cap, indexed regulation or intensity target in the literature), the cap is set proportionally to the future economic output $\tilde{x}$ (in the case of nationwide climate policies, $\tilde{x}$ is the gross domestic product) through a ratio r: $e = r\tilde{x}$. The future economic output is uncertain, where $\tilde{x} = x_0^b + \tilde{\varepsilon_x}$, with ε_x random variable of zero mean and standard deviation σ_x.

Uncertainty in economic output is correlated to uncertainty in BAU emissions: $cov(\varepsilon_e, \varepsilon_x) = \sigma_{ex} = \rho_{ex}\sigma_e\sigma_x = \rho_{ex}\nu_e\nu_x e_0^b x_0^b$, noting the two adimensional parameters $\nu_e = \frac{\sigma_e}{e_0^b}$ and $\nu_x = \frac{\sigma_x}{x_0^b}$ (and in the rest of the paper we note $\nu_c = \frac{\sigma_c}{c_2 e_0^b}$). Indeed, uncertainty in BAU emissions depends on future economic output but not only, in particular the type of economic development (predominance of services or industry) and the energy mix matter as well (Kim and Baumert, 2002). We suppose that $\rho_{ex} > 0$ (although theoretical formulations would also work for $\rho_{ex} \leq 0$).

Replacing e by $r(x_0^b + \tilde{\varepsilon_x})$ in TSC_e, taking the mean and minimizing with respect to r leads to the optimal rate of intensity target (see proof in AppendixC):

$$r^* = \frac{e^*}{x_0^b(1+\nu_x^2)}[1 + \frac{\rho_{ex}\nu_e\nu_x}{1 + \frac{c_1 - b_1}{c_2 e_0^b}}] \tag{10}$$

Contrary to the fixed cap, the relative cap is not equivalent to the one chosen if there was no uncertainty (which would be $r^* = \frac{e^*}{x_0^b}$). The relative cap takes into account uncertainties about BAU emissions, future economic output and their correlation in order to minimize expected total social costs.

Proposition 5. *The more prediction errors of future economic activity and future BAU emissions are correlated, (i) the better the R instrument performs (ii) the less stringent the* ex ante *relative cap r^* is.*

Proof. The more prediction errors of future economic activity and future BAU emissions are correlated means the bigger ρ_{ex}. (i) $E(TSC_r)$ is increasing with ρ_{ex} (see AppendixC) (ii) r^* is also increasing with ρ_{ex}. □

Indeed, the larger ρ_{ex}, the more future economic output is a good proxy for future BAU emissions, and then it is not necessary to set a more stringent cap to hedge against unexpectedly high future BAU emissions.

4.2. Relative or absolute cap?

Proposition 6. *When targets are set at their optimal level ($e = e^*$ and $r = r^*$), the relative cap R performs better than the absolute cap Q when the correlation between uncertainty in future economic output and uncertainty in future BAU emissions (ρ_{ex}) is higher than a parameter- dependent threshold ρ_{ex}^{min}.*

$R \succ Q \Leftrightarrow \rho_{ex} > \rho_{ex}^{min}$ *with*

$$\rho_{ex}^{min} = \frac{1}{1+\sqrt{1+\nu_x^2}}\frac{\nu_x}{\nu_e}[1+\frac{c_1-b_1}{c_2 e_0^b}] \tag{11}$$

Proof. See AppendixD □

As in Quirion (2005), the comparison between absolute and relative caps does not depend on uncertainty in abatement costs. In this framework it depends only on uncertainty in future BAU emissions, uncertainty in future economic output, and their correlation. A strong correlation between the two or a high uncertainty in BAU emissions favors relative caps, while a high uncertainty in future economic output favors fixed caps[6].

This condition applies only if caps (relative or absolute) are set at their optimal level. If we compare expected total social costs of the two policies for any "comparable target " (like in Marschinski and Edenhofer (2010), e^{target} and r^{target} such as $\frac{e^{target}}{e_0^b} = \frac{r^{target}}{\frac{e_0^b}{x_0^b}}$ or $e^{target} = r^{target}x_0^b$), i.e. the targets are the same in expectation, we have the following proposition.

Proposition 7. *When targets are not set at their optimal level but are "comparable", e.g. $e^{target} = r^{target}x_0^b$, the relative cap $\hat{R}$ performs better than the absolute cap $\hat{Q}$ when the correlation between prediction errors of future economic activity and future BAU emissions (ρ_{ex}) is higher than a parameter dependent threshold. Contrary to the threshold for optimal targets, this threshold depends on the stringency of the cap, and on the ratio of slopes of marginal benefits and marginal costs.*

$$\hat{R} \succ \hat{Q} \Leftrightarrow \rho_{ex} > \frac{1}{2}\frac{e^{target}}{e_0^b}\frac{\nu_x}{\nu_e}(1+\frac{b_2}{c_2}) \tag{12}$$

[6]A derivative calculus shows that $\frac{\nu_x}{1+\sqrt{1+\nu_x^2}}$ always increases when ν_x increases.

Proof. See AppendixD □

We use the notations $\hat{R}$ and $\hat{Q}$ instead of R and Q to symbolize suboptimality[7] ($R \succeq \hat{R}$ and $Q \succ \hat{Q}$ with equivalence if and only if $r^{target} = r^*$ and $e^{target} = e^*$ respectively).

Relative caps are favored over absolute caps when: (i) uncertainty in future economic output is low (ii) uncertainty in future BAU emissions is large (ii) the correlation between the two is high (iii) the target is stringent compared to BAU emissions (iv) the ratio of slopes of marginal environmental and marginal abatement costs is low.

The last point is a specificity of this model. Indeed the previous condition can be rewritten as follows:

$$\tilde{R} \succ \tilde{Q} \Leftrightarrow \frac{1}{2}\frac{e^{target}}{e_0^b} < \rho_{ex}\frac{\nu_e}{\nu_x}\frac{1}{1+\dfrac{b_2}{c_2}} \tag{13}$$

and

$$\tilde{R} \succ \tilde{Q} \Leftrightarrow \rho_{ex} > \frac{1}{2}\frac{r^{target}}{\dfrac{e_0^b}{x_0^b}}\frac{\nu_x}{\nu_e}(1+\frac{b_2}{c_2}) \tag{14}$$

These formulas can be compared, converting the notations, to those of Sue Wing et al. (2006) formula (18) p11, $\frac{1}{2}\frac{e^{target}}{e_0^b} < \rho_{ex}\frac{\nu_e}{\nu_x}$, and Marschinski and Edenhofer (2010) formula (4) p5050 or Marschinski and Lecocq (2006) proposition 1 for "pure" intensity target $\rho_{ex} > \frac{1}{2}\frac{r^{target}}{\frac{e_0^b}{x_0^b}}\frac{\nu_x}{\nu_e}$.

In these papers the studied criteria is the variance of abatement (and therefore abatement costs) while environmental costs are left aside. These conditions differ from (13) and (14) only by a factor $1+\frac{b_2}{c_2}$ supporting relative caps. Indeed, the reduction of the variance of environmental costs provided by the fixed target is accounted for in our model but not in an "abatement costs only" framework.

So taking into account abatement costs only and setting aside environmental benefits introduces a bias favoring relative caps over fixed cap. The larger the

[7] If $e^{target} = e^*$, the "comparable" relative target is $r^{target} = \frac{e^*}{x_0^b}$ (which is different than r^*, so suboptimal). The former condition can be rewritten with ρ_{ex}^{min} as $\hat{R} \succ \hat{Q} \Leftrightarrow \rho_{ex} > \frac{1+\sqrt{1+\nu_x^2}}{2}\frac{e^{target}}{e^*}\rho_{ex}^{min}$, so we have $\hat{R} \succ Q \Leftrightarrow \rho_{ex} > \frac{1+\sqrt{1+\nu_x^2}}{2}\rho_{ex}^{min}$ which is a harder condition to meet than $\rho_{ex} > \rho_{ex}^{min}$, especially if uncertainty about future economic output is large.

ratio $\frac{b_2}{c_2}$ (the same underlying the price versus quantities preference[8]), the larger the bias. So far numerical estimates in the literature have suggested that $\frac{b_2}{c_2} \ll 1$ (Pizer, 2005), so that this bias would be very low.

4.3. Prices vs relative caps

We now compare P and R.

Proposition 8. *The price instrument P is preferable to the relative cap R when correlation between uncertainty in future economic output and uncertainty in future BAU emissions (ρ_{ex}) is lower than a parameter dependent threshold, depending among other on the ratio of slopes of marginal environmental and marginal abatement costs.*

$$P \succ R \Leftrightarrow \rho_{ex} < \frac{\rho_{ex}^{min}}{2}[\sqrt{(\alpha_x+1)^2 + 4\beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)} + 1 - \alpha_x] \tag{15}$$

with

$$\alpha_x = \frac{\sqrt{1+\nu_x^2}+1}{\sqrt{1+\nu_x^2}-1}(>1) \tag{16}$$

and

$$\beta_{x,c,e} = \frac{1+\nu_x^2}{(\sqrt{1+\nu_x^2}-1)^2}(1+\frac{\nu_c^2}{\nu_e^2})\frac{\nu_e^2}{(1+\frac{c_1-b_1}{c_2 e_0^b})^2} \tag{17}$$

Proof. See AppendixE □

4.4. Diagram

Instead of the 10 dimensional parameters b_1, b_2, c_1, c_2, e_0^b, x_0^b, ν_c, σ_e, σ_x and σ_{ex}; we have shown that all the results can be expressed as a function of 6 adimensional parameters: $\frac{b_2}{c_2}$, ρ_{ex}, $\frac{c_1-b_1}{c_2 e_0^b}$, ν_c, ν_e, and ν_x. The first one, $\frac{b_2}{c_2}$, could be called the Weitzman criterion (ratio of slopes of marginal costs and abatement). If it is lower than one, P is preferred to Q. The second one is the correlation of future economic output and future BAU emissions, which has a crucial role in the efficiency of the R instrument. The third one, $\frac{c_1-b_1}{c_2 e_0^b}$, is more difficult to figure out, but can be considered as negligible compared to one[9], so

[8]If this ratio is greater than 1, quantity instrument is preferred

[9]Indeed, if we consider that marginal abatement costs near total abatement (or zero emissions) ($c_2 e_0^b$) are (i) much greater than marginal abatement costs near BAU emissions ($c_1 \ll c_2 e_0^b$) (ii) much larger than marginal environmental costs near zero emissions ($b_1 \ll c_2 e_0^b$), then we have (iii) $\frac{c_1-b_1}{c_2 e_0^b} \ll 1$

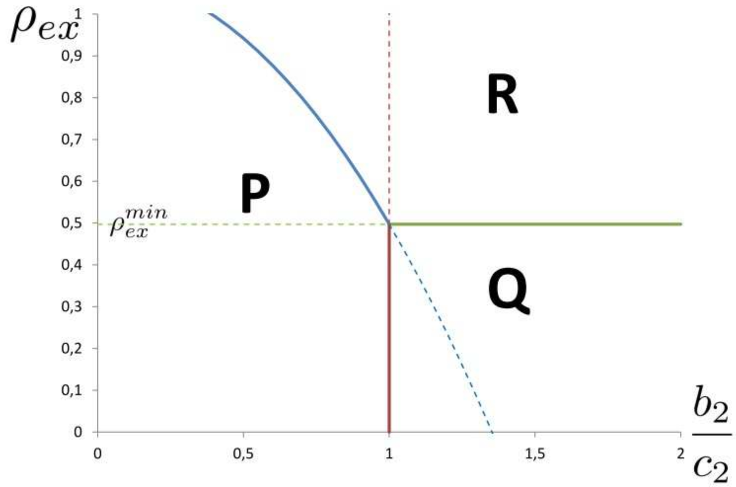
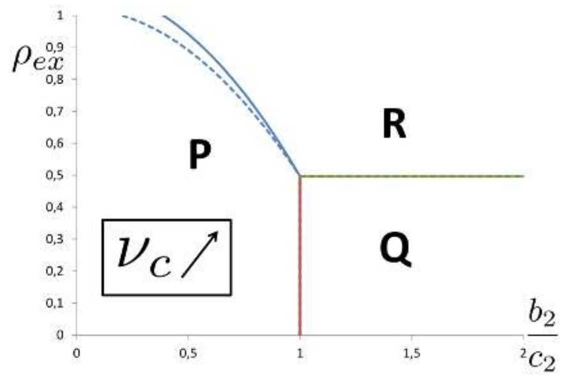


Figure 1: Diagram of optimal instrument (ν_c=3%, ν_e=15% and ν_x=15%)

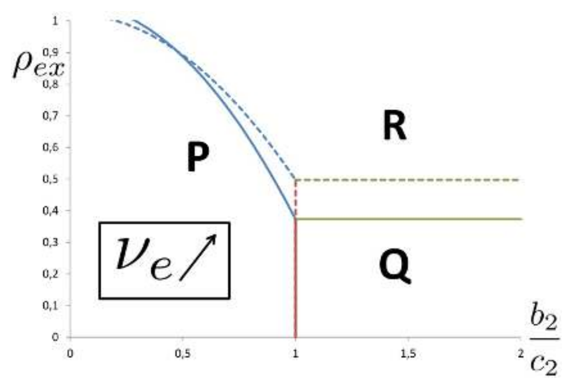
it has no influence on the results (because it is the term $(1+\dfrac{c_1-b_1}{c_2e_0^b})$ which is involved in formulas). Finally, the other ones, ν_c, ν_e and ν_x, correspond to normalized values of the magnitude of uncertainty in future abatement costs, future BAU emissions, and future economic output.

Except for the P-Q comparison ($P \succ Q \Leftrightarrow \dfrac{b_2}{c_2} < 1$), more than two parameters are involved in the R-Q and R-P comparisons, so representing the optimal policy in a 2-axis diagram is not straightforward. As $\rho_{ex}^{min} = \dfrac{1}{1+\sqrt{1+\nu_x^2}}\dfrac{\nu_x}{\nu_e}[1+\dfrac{c_1-b_1}{c_2e_0^b}]$ does not depend on the ratio $\dfrac{b_2}{c_2}$, we represent the optimal policy in the diagram ($\dfrac{b_2}{c_2}$,ρ_{ex}), other parameters being fixed. Then, in a second time, we see how change in the magnitude of uncertainty in abatement, future economic output, and BAU emissions (ν_c, ν_x and ν_e) modify the diagram. We do not consider a change in the parameter $\dfrac{c_1-b_1}{c_2e_0^b}$ because of its little influence. Simplification of formulas is given in AppendixA with this approximation, and in the empirical section this approximation are also undertaken (saving the estimation of b_1, c_1 and b_2).

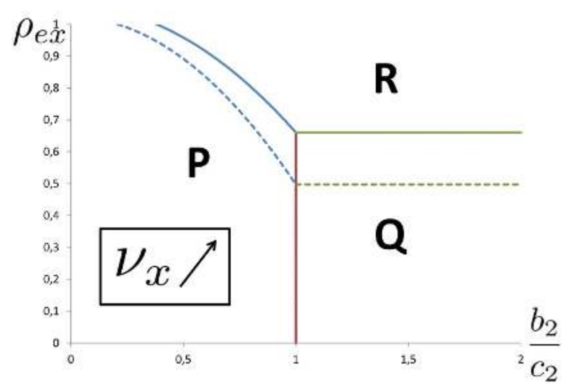
Results are presented in Figure 1. The comparison of P and Q on one hand, and R and Q on the other hand, are direct: $P \succ Q \Leftrightarrow \dfrac{b_2}{c_2} < 1$ and $R \succ Q \Leftrightarrow \rho_{ex} > \rho_{ex}^{min}$ respectively. An intermediate condition is found to compare P and R thanks to the previous section. The frontier between R and P is a decreasing function of $\dfrac{b_2}{c_2}$ which is equal to ρ_{ex}^{min} for $b_2 = c_2$, and to 1



(a) Increase in uncertainty in future abatement costs (ν_c switches from 3% to 6%)



(b) Increase in uncertainty in future BAU emissions (ν_e switches from 15% to 20%)



(c) Increase in uncertainty in future economic output (ν_x switches from 15% to 20%)

Figure 2: Change in the diagram of optimal instrument with change in uncertainty in future (i)Abatement costs (ν_c) (ii)Emissions (ν_e) (iii)Economic output (ν_x). The solid lines indicate the frontiers after the increase in uncertainty, the dashed lines the frontiers before.

for a value of $\frac{b_2}{c_2}$ between 0 and 1[10]: for low values of $\frac{b_2}{c_2}$, R cannot beat the price instrument even if the correlation of BAU emissions and future economic output is perfect.

Figure 2 shows how the diagram evolves with changes in ν_c, ν_e and ν_x. Beyond single example for visualization in the diagram, proofs are given in AppendixF. As the $P-Q$ comparison only depends on the $\frac{b_2}{c_2}$ ratio and not on ν_c, ν_e and ν_x, we only dicuss changes in the $Q-R$ and $P-R$ frontiers. Uncertainty in abatement costs (ν_c), BAU emissions (ν_e) and economic output (ν_x) have the following effects on the optimal policy diagram:

- An increase of uncertainty in abatement costs (ν_c) has no effect on the $Q-R$ comparison, but supports P over R, especially when the ratio $\frac{b_2}{c_2}$ is low.
- For the $R-Q$ frontier: an increase of uncertainty in BAU emissions (ν_e) favors R over Q, and on the contrary an increase of uncertainty in economic output (ν_x) favors R over Q.
- For the $R-P$ frontier, things are clear when the ratio $\frac{b_2}{c_2}$ is relatively high (closer to 1 than 0): an increase of uncertainty in BAU emissions (ν_e) supports R over P, and on the contrary an increase of uncertainty in economic output favors (ν_x) P over R. The situation can be opposite when $\frac{b_2}{c_2}$ decreases, but it depends on the specific values of ν_c, ν_e and ν_x.

5. Empirical application

Contrary to the Kyoto Protocol, the architecture of post-Kyoto international agreements is likely to be bottom-up: most likely, countries or groups of countries will decide a target to achieve and will be free to decide the instrument to achieve this target (for the instruments that concern us in this article: price instrument, absolute or relative caps). In terms of timing, these targets are likely to apply after 2020. Based on the EU ETS, a period of five years before rules can be changed seems reasonable.

[10]With the expression (F.7) in AppendixF (and the simplification), when $\frac{b_2}{c_2} = 0$, the $P-R$ frontier is at $\frac{1}{\nu_e\nu_x}[\sqrt{(1+\nu_x^2)(1+\nu_e^2+\nu_c^2)}-1]$. A study of the function $f(x,y) = \frac{1}{xy}[\sqrt{(1+x^2)(1+y^2)}-1]$ shows that it is always superior to one for x an y values between 0 and 1. As the frontier is a decreasing function with a value > 1 in 0, and a value < 1 in 1, it reaches 1 for a value of $\frac{b_2}{c_2}$ comprised between 0 and 1.

Therefore, we empirically test our model for the period 2020-2025 and for seven countries[11] which represented two thirds of world CO_2 emissions in 2010: China, the United States, Europe[12], India, Russia[13], Brazil and Japan.

5.1. Estimation of parameters ν_e, ν_x and ρ_{ex}

Both future GDP and BAU emissions are highly uncertain and cannot be predicted with accuracy, especially over the medium term. Further, if there is a consensus that emissions and economic output are not independent, there is much debate around the nature of their interaction. The Environmental Kuznets Curve (Grossman and Krueger, 1995) implies an inverted U-shaped relationship between pollution and economic development, but its validity, especially for carbon emissions, has been more and more criticized (Stern, 2004).

However in our case it is not the relationship between absolute values of BAU emissions and GDP that is relevant, but the *correlation of their forecasting errors*. We then need to estimate forecasting errors of emissions and GDP under a common framework to estimate ν_e, ν_x, and their correlation ρ_{ex}.

A first option would be to gather estimates of GDP and BAU emissions for 2020-2025 in different models, consider the mean as the reference value, and compute variances in results, like in Marschinski and Lecocq (2006). On the contrary, we choose to estimate these parameters by confronting past forecasting results on real data (historical forecasting error). An evident advantage of this method is that (if we consider that measurement errors are negligible) the reference values, x_0^b and e_0^b, are exact values. However it implies an implicit hypothesis of temporal stability, that is forecasting errors for a certain country computed in the 2000's are a good approximation of forecasting errors for 2020-2025[14].

Few institutions forecast both GDP and CO_2 emissions for fossil fuel combustion on a yearly basis. It is the case of the US Energy Administration with its International Energy Outlook (IEO). We compute forecasting errors[15] of GDP and emissions forecast of year 2005 (IEO 1999 to 2003) and year 2010 (IEO 2004 to 2008), and resulting values of ν_e, ν_x and ρ_{ex}. Results are reported in Table 1.

Another way to forecast emissions and GDP is to simply extend trends. We compute forecasting values of GDP[16] and carbon emissions[17] in year X (between 2000 and 2010) by using the mean average growth of period [X-9,X-5] to year

[11] We do not consider that the World as a whole is a relevant jurisdiction in terms of climate policy in a post-Kyoto world.

[12] For the Trend estimation, EU 27, and for the IEO estimation, Western Europe or OECD Europe (for IEO versions after 2005).

[13] Former Soviet Union for IEO versions before 2004

[14] If this hypothesis seems too bold; the obtained results can still be considered as an ex post analysis.

[15] The "real" value being those reported in IEO 2013 to avoid break in time series.

[16] GDP data comes from the IMF (http://www.econstats.com/weo/V001.htm) in constant prices in national currency

[17] Carbon emissions come from the World Bank database (World development indicators)

X-5. Forecasting errors are then obtained by comparing these estimates to real values. Results are also reported in Table 1.

Table 1: Empirical estimation of parameters ν_x, ν_e and ρ_{ex}

	ν_x		ν_e		ρ_{ex}	
Region	IEO	Trend	IEO	Trend	IEO	Trend
China	22%	14%	30%	33%	0,95	0,38
US	8%	6%	10%	7%	0,78	0,55
Europe	17%	14%	4%	8%	-0,74	0,01
India	17%	10%	17%	13%	0,71	0,83
Russia	29%	32%	23%	16%	0,90	0,96
Brazil	24%	8%	7%	19%	0,78	0,63
Japan	23%	5%	5%	9%	0,36	0,02

Year 2005 and year 2010 are very different in terms of forecasting: the economic crisis after 2008 was certainly not anticipated. We consider that it is a good thing for our numerical analysis: ν_x, ν_e and ρ_{ex}, which are mean and correlation of random variables, are better estimated when a wide range of possibilities is considered. However, knowing if considering estimations based on "common" or crisis year (2005 or 2010) only would totally change the results is a legitimate question. Separated results of estimations based on "common" or crisis year are given in see AppendixG. In the rest of the paper we make particular attention to the robustness of the mentioned results.

Despite its simplicity, the Trend forecast performs better at predicting GDP[18] (except for Russia). However the IEO forecasts are more accurate for emissions (except for the US, Russia and India). For the IEO forecasts, the errors are larger in GDP predictions than in emissions, whereas they are in the same order of magnitude for the Trend forecasts. For Europe however, as half of emissions have been covered by the Emission Trading Scheme since 2005, emissions cannot technically be considered as business-as-usual.

Table 2: Summary findings of ν_x, ν_e and ρ_{ex}

	ν_x	ν_e	ρ_{ex}
Western Countries	12%	7%	0,16
BRIC Countries	20%	20%	0,77

Individual values remain quite sensitive to the method employed. In order to give stylized facts, we split the countries into two groups: BRIC countries (Brazil, Russia, India and China) and Western countries (Europe, the United

[18]Though as IEO forecasts are in dollars (whereas the Trend forecasts are in national currencies), it adds the uncertainty of currency valuation

States and Japan), and make a double averaging, by group of countries and by forecasting method (IEO and Trend). Results are visible in Table 2.

Three observations can be made (these observations are still valid for separated estimations based on “common” or crisis year):

- Forecasting errors are greater in magnitude for BRIC countries both in GDP and in emissions: $\nu_x(BRIC) > \nu_x(Western)$ and $\nu_e(BRIC) > \nu_e(Western)$.
- Forecasting errors of emissions and GDP are in the same order of magnitude for BRIC countries, whereas GDP are comparatively more uncertain than emissions for Western countries: $\nu_x(BRIC) \simeq \nu_e(BRIC)$ and $\nu_x(Western) > \nu_e(Western)$.
- Forecasting errors are much more correlated for BRIC countries than for Western countries: $\rho_{ex}(BRIC) > \rho_{ex}(Western)$

These three observations favor fixed caps for Western countries and relative caps for BRIC countries. To confirm these findings, we reason from now on for individual regions/countries.

Based on 4.2, using the approximation $|c_1 - b_1| \ll c_2 e_0^b$ (see part AppendixA), when targets are set at their optimal level, relative caps are preferred to fixed caps, when the following ratio

$$K = (1 + \sqrt{1 + \nu_x^2})\rho_{ex}\frac{\nu_e}{\nu_x} \tag{18}$$

is superior to one. If we want to compare policies when they are not at their optimal level (but comparable i.e. $e^{target} = r^{target}x_0^b$), the relevant ratio is:

$$\hat{K} = \frac{e_0^b}{e^{target}} 2\rho_{ex}\frac{\nu_e}{\nu_x} \tag{19}$$

When reasoning in terms of *uncertainty of abatement*, like in Sue Wing et al. (2006) (and Marschinski and Lecocq (2006) and Marschinski and Edenhofer (2010) which are equivalent), $\tilde{R} \succ \hat{Q} \Leftrightarrow \hat{K} > 1$. But if we reason in terms of *welfare*, $\hat{R} \succ \hat{Q} \Leftrightarrow \hat{K} > 1 + \dfrac{b_2}{c_2}$ (see part 4.2).

However, because (i) the ratio $\dfrac{b_2}{c_2}$ is significantly lower than one (ii) uncertainties regarding the key parameters are high; these differences (optimality/non optimality and abatement/welfare) can be neglected in the light of this numerical analysis when comparing R and Q[19].

[19]Indeed we have: $\dfrac{\hat{K}}{1+\dfrac{b_2}{c_2}} = K \times \dfrac{e_0^b}{e^{target}}\dfrac{2}{1+\sqrt{1+\nu_x^2}}\dfrac{1}{1+\dfrac{b_2}{c_2}}$. For respective values of

In order to give results as robust as possible in the light of the various estimations of (ν_x, ν_e and ρ_{ex}); we compute two ratios for each country, picking whichever value of IEO or Trend supporting fixed caps (ν_x high, ν_e and ρ_{ex} low) or relative caps (ν_x low, ν_e and ρ_{ex} high). Results are reported on Table 3.

Table 3: Fixed or Relative Caps?

K	min	max	Instrument
China	1.0	4.7	R
US	0.9	2.5	R
Europe	-0.4	0.0	Q
India	1.1	2.7	R
Russia	1.0	1.6	R
Brazil	0.3	3.5	Ambiguous
Japan	0.0	1.2	Q

Fixed caps are preferred in Europe (where they have been in place since 2005) and in Japan, while in other countries, relative caps are favored (except in Brazil where it is ambiguous).

5.2. Estimation of ν_c

We do not consider differentiated abatement costs per country but base our estimates on global abatement simulated by CGE models, because there is no consistent multi-model ensemble of estimates for our seven countries/regions . Abatement costs are derived from the Energy Modeling Forum 21 (Weyant, 2006). For 18 different models, we gather: (i) the percentage of emissions reduction in 2025 for the fossil fuel/cement sector (compared to the reference case) in the CO_2 only scenario (found in Table 5 of Weyant (2006)), noted A, and the carbon permit price (in G \$ 2000/tCO_2eq (found in Table 15 of (Weyant, 2006) converting carbon into carbon dioxide), noted P. Estimates are reported in Figure 3.

In the model we have (see equation (5)):

$$MAC = \tilde{c_1} + c_2(e_0^b - \tilde{e}) = \tilde{c_1} + c_2 e_0^b \tilde{A} \tag{20}$$

In these CGE models, the price of the carbon permit equals the marginal cost of abatement, so we perform the linear regression:

$$P_i = \lambda_0 + \lambda_1 A_i + \varepsilon_i (= \tilde{c_1} + c_2 e_0^b A_i) \tag{21}$$

10% and 30% for ν_x, $\dfrac{2}{1+\sqrt{1+\nu_x^2}}$ equals 0.998 or 0.978. So because of the high uncertainties regarding the other parameters we can consider that $\dfrac{\hat{K}}{1+\dfrac{b_2}{c_2}} \approx K \approx \hat{K}$.

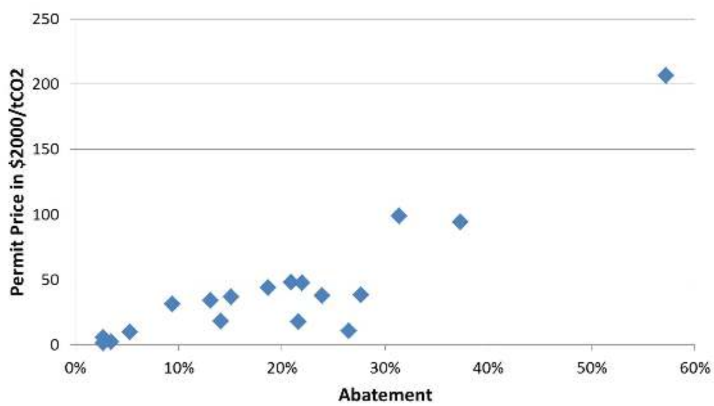


Figure 3: Abatement costs in EMF 21. Each point represent a different model.

The linear regression (regression coefficient $R^2 = 0.78$ gives an intercept of -18 (standard deviation 10) and a coefficient of 315 (standard deviation 41), i.e. at the global level, each additional percent reduction in CO_2 emissions raises marginal costs by 3.15\$/tCO_2. Therefore we assess $\sigma_c = 10$ \$ tCO_2, $c_2 e_0^b$=315\$ tCO_2, and then $\nu_c = \dfrac{\sigma_c}{c_2 e_0^b}$=3%.

Is it reasonable to use this estimation based on a global level for all the different regions? As in the models, the less costly mitigation options are used first, an absolute emission reduction at the global level is then less costly than at the regional level, so c_2 is likely to be larger for smaller geographical entities. However, e_0^b is also smaller, so the effect on the product $c_2 e_0^b$ is undetermined. Further, errors in the assessment of abatement costs can compensate at the global level, therefore it is likely that σ_c is higher for smaller geographical entities. Altogether, we consider that ν_c=3% is a low bound estimation of abatement costs uncertainty, and consider a double of this estimate (6%) as well in the following.

We are now able to compare values of ν_c and ν_e for different configurations (see Table 4). We compute the ratio $(\dfrac{\sigma_c}{c_2 \sigma_e})^2 = (\dfrac{\nu_c}{\nu_e})^2$ (as it appears several time in part 3) for a high (31.5%) and low (6%) value of ν_e[20] and a high (6%) and low (3%) value of ν_c. We recall that if this ratio is lower than one, it means that in terms of incurred costs uncertainty, BAU emissions uncertainty matters more than structural abatement costs uncertainty.

As for all configurations, $\nu_e \geq \nu_c$, it was straightforward that this ratio was lower or equal to one (and so that BAU emissions uncertainty had the biggest impact on incurred cost uncertainty). In case of important emissions

[20] In Table 1, we take the mean of the IEO and Trend forecasts, and then choose the lowest and highest value for ν_e (for Europe and China respectively).

Table 4: Which among structural abatement costs uncertainty and BAU emissions uncertainty matters most in term of incurred costs? Value of the ratio $(\frac{\sigma_c}{c_2\sigma_e})^2 = (\frac{\nu_c}{\nu_e})^2$.

	ν_e low	ν_e high
ν_c low	0.25	0.01
ν_c high	1	0.04

uncertainty (such as in BRIC countries), we have $(\frac{\nu_c}{\nu_e})^2 \ll 1$, and so abatement costs uncertainty is negligible in incurred costs uncertainty.

5.3. Around the ratio $\frac{b_2}{c_2}$

To compare tax and intensity target, we consider the inverse problem. For high and low values of ν_x, ν_e, ρ_{ex} and ν_c, we compute the limit value of the $\frac{b_2}{c_2}$ ratio which makes P and R equivalent[21].

If $\frac{b_2}{c_2} < (\frac{b_2}{c_2})^{lim}$, $P \succ R$ (we can also see indirectly that when $Q \succ R$, $(\frac{b_2}{c_2})^{lim} > 1$). Results are reported in Table 6. The lowest value of $(\frac{b_2}{c_2})^{lim}$ is for Russia at 0.34. How this value compares to plausible estimations of $\frac{b_2}{c_2}$?

Table 5: Price or Relative Caps?

$(\frac{b_2}{c_2})^{lim}$	min	max
China	0.62	0.98
US	0.66	1.09
Europe	1.87	4.43
India	0.60	0.91
Russia	0.34	1.05
Brazil	0.71	2.96
Japan	0.96	4.65

The damages caused by carbon emissions are much more uncertain than abatement costs. The most recent IPCC report (Field et al., 2014) p.19 states that "estimates of the incremental economic impact of emitting carbon dioxide lie between a few dollars and several hundreds of dollars per tonne of carbon

[21] As in the rest of the empirical part, we use the simplification $|c_1 - b_1| \ll c_2 e_0^b$

(3.668 tCO2) (robust evidence, medium agreement). Estimates vary strongly with the assumed damage function and discount rate".

Table 6 summarizes values of the ratio $\frac{b_2}{c_2}$ used in the literature. It seems that it has increased over the past years, but that it remains significantly lower than 1 (and than 0.1).

Table 6: Estimations of the ratio $\frac{b_2}{c_2}$ used in the literature

Reference	Value
Newell and Pizer (2003)	5×10^{-6}
Newell and Pizer (2006)	from 2×10^{-6} to 5×10^{-5} depending on c_2
Newell and Pizer (2008)	$\ll 1$ so that only R and Q are compared
Jotzo and Pezzey (2007)	b_2 chosen "to be a small constant times" c_2

Uncertainty is then very large for $\frac{b_2}{c_2}$, but it is still more likely that this ratio is less than one (and also likely than it is less than 0.1). Therefore, we can be confident that P is the best instrument, superior to R and Q in our 2020-2025 policy framework.

6. Conclusion

Our model, including three types of uncertainty (on abatement costs, BAU emissions and future economic output, the latter two being correlated), allowed to unify Newell and Pizer (2008) and Sue Wing et al. (2006) frameworks. Two parameters proved to be crucial to rank the three considered policies (tax, fixed cap and relative cap). The first one, related to the Weitzman (1974) literature, is the ratio of slopes of marginal benefit and marginal costs. The lower it is, the more the price instrument is preferred. The second one, related to the Sue Wing et al. (2006) literature, is the correlation between BAU emissions and future economic output. The bigger it is, the more the relative instrument is preferred.

The model allowed displaying the optimal policy in a diagram with each one of these two parameters as an axis, and seeing how this diagram changed when the magnitude of the three considered types of uncertainty changed. Further, we showed that reasoning in terms of abatement costs only and setting environmental benefits aside introduces a bias favoring relative caps, but that this bias was likely to be small in the case of climate change mitigation.

Testing empirically the model led us to the following points. First, as we considered a short-time period policy (2020-2025) and as greenhouse gases are a stock pollutant (so marginal benefits are relatively flatter than marginal costs), tax dominates cap, whether the cap is absolute (Weitzman criterion) or relative to economic output (our own contribution). As mentioned, the tax is often seen as difficult to implement in practice for political reasons, so the policy choice is often narrowed to picking between absolute and intensity targets. Our model allowed us to compare these two policies, whether or not caps were put at their optimum level, which is virtually impossible in practice. Empirical estimation revealed that the situation differed among countries. Emerging countries (except for Brazil where it is ambiguous) and the United States would be better off with intensity targets, whereas fixed caps would be preferable for Europe and Japan.

Admittedly, our model remains simple in its assumptions and other aspects (reviewed in Kim and Baumert (2002), Marschinski and Edenhofer (2010) and Goulder and Schein (2013)) have to be considered when determining the choice of an instrument, such as acceptability by population or industry, administrative costs, issues with reporting and verification or carbon price volatility, among others.

As recent evidence in climate science suggests (Matthews et al., 2009), maintaining the global increase in temperature below two degrees would be closely linked to not exceeding one trillion ton of *cumulative carbon emissions* at the global level, regardless of the emission trajectory. This notion of "carbon budget" (half being already consumed, and a quarter since 1990) would make a case for a quantity instrument at a global level and in a theoretically infinite temporal horizon. However the implementation of such a "first best" policy would necessitate a temporal and geographical burden sharing of the cap, as well as strong enforcement at the international level, which currently seems far from achievable by international negotiations on climate change.

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AppendixA. Simplification

The simplification is:

$$|c_1 - b_1| \ll c_2 e_0^b \tag{A.1}$$

In Newell and Pizer (2008), they justify $c_1 = b_1$ by reasoning around the social optimum.

It implies:

$$e^* = \frac{1}{1 + \frac{b_2}{c_2}} e_0^b \tag{A.2}$$

$$p^* = c_2 - \frac{b_2 c_2}{b_2 + c_2} e_0^b \tag{A.3}$$

$$r^* = \frac{e_0^b}{x_0^b(1+\nu_x^2)(1+\frac{b_2}{c2})}(1+\rho_{ex}\nu_x\nu_e) = \frac{e_0^b}{x_0^b(1+\frac{b_2}{c2})}(1+\rho_{ex}\nu_x\nu_e) \tag{A.4}$$

with $\nu_x^2 \ll 1$.

$$\rho_{ex}^{min} = \frac{1}{1+\sqrt{1+\nu_x^2}} \frac{\nu_x}{\nu_e} = \frac{1}{2}\frac{\nu_x}{\nu_e} \tag{A.5}$$

with $\nu_x^2 \ll 1$.

$$\alpha_x = \frac{\sqrt{1+\nu_x^2}+1}{\sqrt{1+\nu_x^2}-1} = \frac{4}{\nu_x^2} \tag{A.6}$$

with $\nu_x^2 \ll 1$.

$$\beta_{x,c,e} = \frac{1+\nu_x^2}{(\sqrt{1+\nu_x^2}-1)^2}(\nu_c^2+\nu_e^2) = \frac{4}{\nu_x^4}(\nu_c^2+\nu_e^2) \tag{A.7}$$

with $\nu_x^2 \ll 1$.

AppendixB. Optimal price instrument proof

Remplacing directly e by p leads to a too complicated formula. First we rewrite TSC_e with e^*:

$$TSC_e = b_0 + (c_1 + \tilde{\varepsilon_c})(e_0^b + \tilde{\varepsilon_e}) + \frac{1}{2}c_2(e_0^b + \tilde{\varepsilon_e})^2$$
$$- \frac{1}{2}(b_2 + c_2)(e^*)^2 - (\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e})e$$
$$+ \frac{1}{2}(b_2 + c_2)(e - e^*)^2 \tag{B.1}$$

(5) is equivalent to $e = \frac{1}{c_2}[(b_2 + c_2)e^* + b_1 - p + \tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e}]$.
Then we have:

$$TSC_p = b_0 + (c_1 + \tilde{\varepsilon_c})(e_0^b + \tilde{\varepsilon_e}) + \frac{1}{2}c_2(e_0^b + \tilde{\varepsilon_e})^2$$
$$- \frac{1}{2}(b_2 + c_2)(e^*)^2 - \frac{1}{c_2}(\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e})[(b_2 + c_2)e^* - (b_1 - p + \tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e})]$$
$$+ \frac{1}{2c_2^2}(b_2 + c_2)(b_2e^* + b_1 - p + \tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e})^2 \tag{B.2}$$

The regulator chooses p^* in order to minimize expected total social costs:

$$E(TSC_p) = b_0 + c_1e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2)$$
$$- \frac{1}{2}(b_2 + c_2)(e^*)^2 - \frac{c_2 - b_2}{2c_2^2}(\sigma_c^2 + c_2^2\sigma_e^2)$$
$$+ \frac{1}{2c_2^2}(b_2 + c_2)(b_2e^* + b_1 - p)^2 \tag{B.3}$$

We find:

$$p^* = b_1 + b_2e^* \tag{B.4}$$

AppendixC. Optimal relative cap instrument proof

Replacing e by $r(x_0^b + \tilde{\varepsilon_x})$ in TSC_e (written with e*) gives:

$$TSC_r = b_0 + (c_1 + \tilde{\varepsilon_c})(e_0^b + \tilde{\varepsilon_e}) + \frac{1}{2}c_2(e_0^b + \tilde{\varepsilon_e})^2$$
$$- \frac{1}{2}(b_2 + c_2)(e^*)^2 - (\tilde{\varepsilon_c} + c_2\tilde{\varepsilon_e})(x_0^b + \tilde{\varepsilon_x})r$$
$$+ \frac{1}{2}(b_2 + c_2)(r(x_0^b + \tilde{\varepsilon_x}) - e^*)^2 \tag{C.1}$$

The regulator chooses r^* so as to minimize expected total social costs:

$$
\begin{aligned}
E(TSC_r) = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) \\
- (b_2 + c_2)(e^* + \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b) r x_0^b \quad \text{(C.2)} \\
+ \frac{1}{2}(b_2 + c_2)(1 + \nu_x^2)(r x_0^b)^2 \quad \text{(C.3)}
\end{aligned}
$$

We find:

$$
r^* x_0^b = \frac{1}{(1 + \nu_x^2)}[e^* + \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b] \quad \text{(C.4)}
$$

Using the formula $(1 + \frac{b_2}{c_2})\frac{e^*}{e_0^b} = 1 + \frac{c_1 - b_1}{c_2 e_0^b}$, we have:

$$
r^* = \frac{e^*}{x_0^b(1 + \nu_x^2)}[1 + \frac{\rho_{ex}\nu_e\nu_x}{1 + \frac{c_1 - b_1}{c_2 e_0^b}}] \quad \text{(C.5)}
$$

AppendixD. Absolute or Relative caps proof

$$
\begin{aligned}
E(TSC_r)_{|r=r^*} = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) \\
+ \frac{1}{2}\frac{b_2 + c_2}{1 + \nu_x^2}(e^* + \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b)^2
\end{aligned}
$$

As

$$
\begin{aligned}
E(TSC_e) = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) - \frac{1}{2}(b_2 + c_2)(e^*)^2 \\
+ \frac{1}{2}(b_2 + c_2)(e - e^*)^2
\end{aligned}
$$

and

$$
E(TSC_e)_{|e=e^*} = b_0 + c_1 e_0^b + \frac{1}{2}c_2((e_0^b)^2 + \sigma_e^2) - \frac{1}{2}(b_2 + c_2)(e^*)^2 \quad \text{(D.1)}
$$

We have:

$$
\begin{aligned}
\Delta_{R-Q} &= E(TSC_r)_{|r=r^*} - E(TSC_e)_{|e=e^*} \\
&= \frac{1}{2}\frac{b_2 + c_2}{1 + \nu_x^2}[(\sqrt{1 + \nu_x^2}e^*)^2 - (e^* + \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b)^2] \\
&= \frac{1}{2}\frac{b_2 + c_2}{1 + \nu_x^2}[(\sqrt{1 + \nu_x^2} - 1)e^* - \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b][(\sqrt{1 + \nu_x^2} + 1)e^* + \frac{c_2}{b_2 + c_2}\rho_{ex}\nu_e\nu_x e_0^b] \\
&= \frac{1}{2}\frac{(\nu_e\nu_x e_0^b)^2}{1 + \nu_x^2}\frac{c_2^2}{b_2 + c_2}[\frac{\sqrt{1 + \nu_x^2} - 1}{\nu_x}\frac{1}{\nu_e}(1 + \frac{b_2}{c2})\frac{e^*}{e_0^b} - \rho_{ex}] \\
&\times [\frac{\sqrt{1 + \nu_x^2} + 1}{\nu_x}\frac{1}{\nu_e}(1 + \frac{b_2}{c2})\frac{e^*}{e_0^b} + \rho_{ex}] \quad \text{(D.2)}
\end{aligned}
$$

Noting

$$\rho_{ex}^{min} = \frac{\sqrt{1+\nu_x^2}-1}{\nu_x}\frac{1}{\nu_e}(1+\frac{b_2}{c2})\frac{e^*}{e_0^b} = \frac{1}{1+\sqrt{1+\nu_x^2}}\frac{\nu_x}{\nu_e}(1+\frac{b_2}{c_2})\frac{e^*}{e_0^b} \quad \text{(D.3)}$$

We have

$$\Delta_{R-Q} = \frac{1}{2}\frac{(\nu_e\nu_x e_0^b)^2}{1+\nu_x^2}\frac{c_2^2}{b_2+c_2}[\rho_{ex}^{min}-\rho_{ex}][\frac{\sqrt{1+\nu_x^2}+1}{\sqrt{1+\nu_x^2}-1}\rho_{ex}^{min}+\rho_{ex}] \quad \text{(D.4)}$$

As the second factor is always positive (for $\rho_{ex} > 0$), we conclude to

$$R \succ Q \Leftrightarrow \rho_{ex} > \rho_{ex}^{min} \quad \text{(D.5)}$$

With $e^{target} = r^{target}x_0^b$,

$$\begin{aligned} ETSC_r - ETSC_e &= -c_2\rho_{ex}\nu_e\nu_x e_0^b e^{target} + \frac{1}{2}(b_2+c_2)\nu_x^2(e^{target})^2 \\ &= c_2\nu_e\nu_x e_0^b e^{target}[\frac{1}{2}(1+\frac{b_2}{c_2})\frac{\nu_x}{\nu_e}\frac{e^{target}}{e_0^b} - \rho_{ex}] \end{aligned}$$

Then

$$\hat{R} \succ \hat{Q} \Leftrightarrow \rho_{ex} > \frac{1}{2}\frac{e^{target}}{e_0^b}\frac{\nu_x}{\nu_e}(1+\frac{b_2}{c_2}) \quad \text{(D.6)}$$

AppendixE. Prices versus relative caps proof

Noting $\alpha_x = \frac{\sqrt{1+\nu_x^2}+1}{\sqrt{1+\nu_x^2}-1}$ we have (see equation (D.4)):

$$\begin{aligned} \Delta_{R-Q} &= \frac{1}{2}\frac{\nu_x^2\sigma_e^2}{1+\nu_x^2}\frac{c_2^2}{b_2+c_2}[\rho_{ex}^{min}-\rho_{ex}][\alpha_x\rho_{ex}^{min}+\rho_{ex}] \\ &= \frac{1}{2}\frac{\nu_x^2\sigma_e^2}{1+\nu_x^2}\frac{c_2^2}{b_2+c_2}[\alpha_x(\rho_{ex}^{min})^2+(1-\alpha_x)\rho_{ex}^{min}\rho_{ex}-(\rho_{ex})^2] \end{aligned}$$

We first rewrite Δ_{Q-P} (equation (9)) in a similar way:

$$\begin{aligned} \Delta_{Q-P} &= \frac{c_2-b_2}{2c_2^2}(\sigma_c^2+c_2^2\sigma_e^2) \\ &= \frac{1}{2}\frac{\nu_x^2\sigma_e^2}{1+\nu_x^2}\frac{c_2^2(\rho_{ex}^{min})^2}{b_2+c_2} \times \frac{(c_2-b_2)}{c_2^2}(\sigma_c^2+c_2^2\sigma_e^2) \\ &\times \frac{1+\nu_x^2}{\nu_x^2\sigma_e^2}\frac{b_2+c_2}{c_2^2} \times \frac{\nu_x^2\sigma_e^2}{(\sqrt{1+\nu_x^2}-1)^2(e_0^b+\frac{c_1-b_1}{c_2})^2} \\ &= \frac{1}{2}\frac{\nu_x^2\sigma_e^2}{1+\nu_x^2}\frac{c_2^2}{b_2+c_2} \times \beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)(\rho_{ex}^{min})^2 \end{aligned} \quad \text{(E.1)}$$

with

$$\beta_{x,c,e} = \frac{1+\nu_x^2}{(\sqrt{1+\nu_x^2}-1)^2}(1+\frac{\sigma_c^2}{c_2^2\sigma_e^2})\frac{\sigma_e^2}{(e_0^b+\frac{c_1-b_1}{c_2})^2} \tag{E.2}$$

Then, as $\Delta_{R-P} = \Delta_{R-Q} + \Delta_{Q-P}$,

$$\Delta_{R-P} = \frac{1}{2}\frac{\nu_x^2\sigma_e^2}{1+\nu_x^2}\frac{c_2^2}{b_2+c_2}[(\beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)+\alpha_x)(\rho_{ex}^{min})^2+(1-\alpha_x)\rho_{ex}^{min}\rho_{ex}-(\rho_{ex})^2] \tag{E.3}$$

The right term is a degree 2 polynom in ρ_{ex} with a discriminant:

$$\begin{aligned} D &= (\rho_{ex}^{min})^2[(1-\alpha_x)^2 + 4((1-(\frac{b_2}{c_2})^2)\beta_{x,c,e}+\alpha_x)] \\ &= (\rho_{ex}^{min})^2[(1+\alpha_x)^2 + 4\beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)] \end{aligned} \tag{E.4}$$

$D > 0$ when $b_2 < c_2$ (which is a relevant domain because when $b_2 > c_2$, P is dominated by Q so it cannot be the optimal instrument).

The roots are:

$$\rho_{12} = \frac{\rho_{ex}^{min}}{2}[\pm\sqrt{(\alpha_x+1)^2 - 4\beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)} - (\alpha_x-1)] \tag{E.5}$$

One root is always negative because $\alpha_x > 1$.

Therefore we conclude that:

$$P \succ R \Leftrightarrow \rho_{ex} < \frac{\rho_{ex}^{min}}{2}[\sqrt{(\alpha_x+1)^2 + 4\beta_{x,c,e}(1-(\frac{b_2}{c_2})^2)} + 1 - \alpha_x] \tag{E.6}$$

We note $(\frac{b_2}{c_2})^{lim}$ the value of the ratio $\frac{b_2}{c_2}$ for which R and P are equivalent. Δ_{R-P} is a decreasing function of $\frac{b_2}{c_2}$. So when $\frac{b_2}{c_2} < (\frac{b_2}{c_2})^{lim}$, $P \succ R$ and conversely. With the approximation $|c_1 - b_1| \ll c_2 e_0^b$ (see AppendixA), $\rho_{ex}^{min} = \frac{1}{1+\sqrt{1+\nu_x^2}}\frac{\nu_x}{\nu_e}$ is independent of $\frac{b_2}{c_2}$.

Then (E.3) brings the equation:

$$(\beta_{x,c,e}(1-((\frac{b_2}{c_2})^{lim})^2)+\alpha_x)(\rho_{ex}^{min})^2 + (1-\alpha_x)\rho_{ex}^{min}\rho_{ex} - (\rho_{ex})^2 = 0 \tag{E.7}$$

which is equivalent to

$$\beta_{x,c,e}(1-((\frac{b_2}{c_2})^{lim})^2)+\alpha_x = (\frac{\rho_{ex}}{\rho_{ex}^{min}})^2 + (\alpha_x-1)\frac{\rho_{ex}}{\rho_{ex}^{min}} \tag{E.8}$$

or

$$((\frac{b_2}{c_2})^{lim})^2 = 1 + \frac{1}{\beta_{x,c,e}}[\alpha_x - (\frac{\rho_{ex}}{\rho_{ex}^{min}})^2 + (1-\alpha_x)\frac{\rho_{ex}}{\rho_{ex}^{min}}] \tag{E.9}$$

If $\rho_{ex} = \rho_{ex}^{min}$, which means that R and Q are equivalent, then $(\frac{b_2}{c_2})^{lim} = 1$. Under this configuration of parameters, R, P and Q are equivalent. If $\rho_{ex} < \rho_{ex}^{min}$, which means that $Q \succ R$, then $(\frac{b_2}{c_2})^{lim} > 1$[22]. If $\rho_{ex} < \rho_{ex}^{min}$, which means that $R \succ Q$, then $(\frac{b_2}{c_2})^{lim} < 1$.

AppendixF. Diagram changes proof

With the simplification $|c_1 - b_1| \ll c_2 e_0^b$ we have:

$$\rho_{ex}^{min} \simeq \frac{1}{1+\sqrt{1+\nu_x^2}}\frac{\nu_x}{\nu_e}$$

Because $\frac{\nu_x}{1+\sqrt{1+\nu_x^2}}$ is an increasing function of ν_x, ρ_{ex}^{min} increases (respectively decreases) when ν_x (respectively ν_e) increases.

The $P-R$ frontier is given by the function of $\xi = \frac{b_2}{c_2}$:

$$f(\xi) = \frac{\rho_{ex}^{min}}{2}[\sqrt{(\alpha+1)^2 + 4\beta(1-\xi^2)} + 1 - \alpha] \tag{F.1}$$

with

$$\alpha = \frac{\sqrt{1+\nu_x^2}+1}{\sqrt{1+\nu_x^2}-1}(> 1) \tag{F.2}$$

and

$$\beta = \frac{1+\nu_x^2}{(\sqrt{1+\nu_x^2}-1)^2}(\nu_c^2 + \nu_e^2) \tag{F.3}$$

To ease calculus we need to express the formulas with different notations. We note

$$u_x = \sqrt{1+\nu_x^2}(> 1) \tag{F.4}$$

[22]The factor multiplying $\frac{1}{\beta_{x,c,e}}$ is a polynom of degree 2 in $\frac{\rho_{ex}}{\rho_{ex}^{min}}$ with roots 1 and $-\alpha_x < 0$, therefore is positive for $\frac{\rho_{ex}}{\rho_{ex}^{min}}$ between 0 and 1.

Then

$$\alpha = \frac{u_x + 1}{u_x - 1} \tag{F.5}$$

and

$$\beta = \frac{u_x^2}{(u_x - 1)^2}(\nu_e^2 + \nu_c^2) \tag{F.6}$$

$f(\xi)$ can be simplified to (by factorizing by $\frac{2}{u_x - 1}$):

$$f(\xi) = \frac{1}{\nu_x \nu_e}[u_x \sqrt{A_{\xi,c,e}} - 1] \tag{F.7}$$

with

$$A_{\xi,c,e} = 1 + (\nu_c^2 + \nu_e^2)(1 - \xi^2) \tag{F.8}$$

$A_{\xi,c,e}$ is a decreasing function of ξ: when $\frac{b_2}{c_2}$ varies from 1 to 0, $A_{\xi,c,e}$ varies from 1 to $1+\nu_c^2+\nu_e^2$. To see a change in the $P-R$ frontier (for example with an increase of ν_x), we compute $\frac{\partial f(\xi)}{\partial \nu_x}$. For a given ξ, if it is positive, it means that the frontier goes up (so the zone where P dominates expands at the expense of the zone where R dominates).

We have

$$\frac{\partial A_{\xi,c,e}}{\partial \nu_x} = 0 \tag{F.9}$$

$$\frac{\partial A_{\xi,c,e}}{\partial \nu_c} > 0 \tag{F.10}$$

$$\frac{\partial A_{\xi,c,e}}{\partial \nu_e} = \frac{2\nu_e}{\nu_e^2 + \nu_c^2}(A_{\xi,c,e} - 1) \tag{F.11}$$

$$\frac{\partial u_x}{\partial \nu_x} = \frac{\nu_x}{\sqrt{1 + \nu_x^2}} \tag{F.12}$$

After these preliminary steps, it is relatively straightforward to obtain:

$$\frac{\partial f(\xi)}{\partial \nu_c} = \frac{(...)}{(...)} \times \frac{\partial A_{\xi,c,e}}{\partial \nu_c} > 0 \tag{F.13}$$

$$\frac{\partial f(\xi)}{\partial \nu_x} = \frac{1}{(...)}[\sqrt{1+\nu_x^2} - \sqrt{A_{\xi,c,e}}] \tag{F.14}$$

$$\frac{\partial f(\xi)}{\partial \nu_e} = \frac{(...)}{(...)}[\sqrt{A_{\xi,c,e}} - \frac{1+\nu_x^2}{\nu_c^2+\nu_e^2}(\nu_c^2 A_{\xi,c,e} + \nu_e^2)] \tag{F.15}$$

We recall that $A_{\xi,c,e} = 1 + (\nu_c^2 + \nu_e^2)(1-\xi^2)$ is a decreasing function of ξ: when $\frac{b_2}{c_2}$ varies from 1 to 0, $A_{\xi,c,e}$ varies from 1 to $1+\nu_c^2+\nu_e^2$. Therefore:

- An increase of ν_c always favors P over R ($\frac{\partial f(\xi)}{\partial \nu_c} > 0$)
- When $\frac{b_2}{c_2}$ is close to one (ξ close to 0 or $A_{\xi,c,e}$ close to 1); an increase of ν_x supports P over R ($\frac{\partial f(\xi)}{\partial \nu_x} > 0$, but an increase of ν_e favors R over P ($\frac{\partial f(\xi)}{\partial \nu_e} < 0$)
- When $\frac{b_2}{c_2}$ is close to zero, it is possible that the situation is opposite (an increase of ν_x supports R over P and an increase of ν_e favors P over R), but in each case it depends on the value of parameters ν_x, ν_c and ν_e. More specifically:
- For an increase of ν_x, it is the case if $\nu_x^2 < (\nu_c^2+\nu_e^2)$ (however the situation is policy relevant if $f(\xi) < 1$ (because $\rho_{ex} < 1$) so if $\nu_x < \nu_e$). More precisely the situation is opposite whenever ξ^2 is lower than $1 - \frac{\nu_x^2}{\nu_c^2+\nu_e^2}$. This situation does not happen in Figure 2.
- For an increase of ν_x, it is the case if $\sqrt{1+\nu_c^2+\nu_e^2} > (1+\nu_c^2)(1+\nu_x^2)$. The situation happens in Figure 2.

AppendixG. Estimations of parameters ν_e, ν_x and ρ_{ex} with IEO forecasts for "common" and crisis year

Estimations are given in Tables G.7 and G.8 for year 2005 (no crisis) and 2010 (crisis). General patterns are:

- ν_x is much higher for 2010 (crisis) in Western countries, but in the same order of magnitude for BRIC countries.
- ν_e is higher for 2010 (crisis) for both Western and BRIC countries
- ρ_{ex} is higher for 2010 (crisis) for both Western (especially Japan and the US) and BRIC countries

Table G.7: Estimations of parameters ν_e, ν_x and ρ_{ex} with IEO forecasts (Year 2005)

Region	ν_x	ν_e	ρ_{ex}
China	20%	33%	0.95
US	6%	3%	-0.24
Europe	2%	3%	-0.73
India	14%	8%	0.28
Russia	9%	8%	0.23
Brazil	31%	7%	0.78
Japan	11%	5%	-0.89

Table G.8: Estimations of parameters ν_e, ν_x and ρ_{ex} with IEO forecasts (Year 2010)

Region	ν_x	ν_e	ρ_{ex}
China	23%	28%	0.97
US	10%	14%	0.96
Europe	24%	5%	-0.85
India	20%	22%	0.86
Russia	40%	32%	0.94
Brazil	14%	6%	0.84
Japan	31%	5%	0.83