# Pricing and Persuasive Advertising in a Differentiated Market

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## Abstract

We examine how competitive firms' pricing and persuasive advertising strategies as well as profits are affected by changes in consumer preferences, unit production costs, and advertising efficiencies. We highlight three interesting findings from our analyses. First, when firms make endogenous advertising decisions in addition to pricing, a seemingly favorable exogenous shock to one firm alone may lead to a lower equilibrium profit for that firm and a higher profit for its competitor. Second, firms' advertising depends on the degree of product differentiation and firms' optimal advertising responses to an exogenous change are opposite—one firm increases advertising whereas the other reduces it. The firm receiving a favorable shock in product valuation is likely to complement it with additional advertising, especially when advertising is inefficient or expensive. Third, an exogenous shock may reduce the price dispersion in the industry even though it increases the separation between the firms' valuations or perceived qualities. Our results are robust to alternative game structures, heterogeneity in consumer response or exposure to advertising, nonlinearity of consumer taste preferences, and alternative functional forms for advertising costs.



***Key Words***: competitive strategy, persuasive advertising, pricing, product differentiation, competition, game theory

# 1. Introduction

When Motorola Inc. introduced its Droid smartphone in late 2009 to compete with Apple Inc.'s iPhone, consumers generally gave very positive reviews to the Droid's key features such as its high resolution touch screen, multitasking capabilities, open Android platform, and so on.[1] Such positive third-party reviews and recommendations increase consumers' willingness to buy Motorola's Droid. In light of such favorable changes in the marketplace, how should Motorola adjust its marketing strategies? For example, should Motorola lower its advertising expenditure since these favorable reviews are similar to free advertising? Does such a seemingly favorable shock necessarily lead to higher profit for Motorola? In this paper, we will examine how competitive firms' pricing and persuasive advertising strategies as well as profits are affected by changes in consumer preferences, unit production costs, and advertising efficiencies. Many exogenous factors can lead to such changes. For example, the consumer's valuation for a company's product may be influenced by product reviews or third-party endorsements, price shocks in complementary products, or even actions by non-profit organizations or governments. Similarly, a firm's production cost may change due to exogenous factors such as shocks in labor costs, other input prices, and government subsidies or tax regulations. Note that oftentimes even a seemingly common exogenous shock may affect different firms to a different degree; for example, a 10% drop in metal prices may reduce the unit cost for an SUV much more than that for a compact car. We develop a game-theoretic framework to study the effects of both symmetric and asymmetric shocks in the market.

We contribute to the stream of literature on competitive strategy and economics that study Bertrand traps and supertraps. The term "Bertrand trap" refers to the situation in which, because of price competition as in the Hotelling model, firms' equilibrium profits remain constant despite seemingly positive exogenous changes such as reduced costs (Hermalin 1993). Note that, if firms do not adjust their prices to such exogenous changes, then all firms will make higher profits. Or, if such a positive exogenous change happens to only one firm, then at equilibrium that firm will become better off (and other firms worse off). The underlying cause for the Bertrand trap is that, if all firms' marginal costs are reduced by the *same* amount, no firm will be able to profit from such a favorable change in the industry because each firm has an *equal* incentive to reduce prices, hence benefiting only the consumers. Cabral and Villas-Boas (2005) show that, in the presence of intra-firm product interactions, there may even be "supertraps" for multi-product firms—a seemingly positive shock in the industry (e.g., an increased degree of economies of scope or demand synergies) may lead to *lower* equilibrium profits. Cabral and Villas-Boas provide a unifying framework to understand a number of papers in the economics, marketing,

[1] See, for example, http://www.wirelessweek.com/Reviews/2009/11/Review--Droid-vs--the-iPhone-3GS/ and http://reviews.cnet.com/motorola-droid-review/, last accessed in June 2010.

and strategy literature. For an excellent review and discussion of such literature in relation to Bertrand supertraps, please refer to Cabral and Villas-Boas (2005). Our research extends even beyond supertraps; we show that when firms make endogenous advertising (or fixed cost) decisions in addition to pricing, a seemingly favorable exogenous shock (e.g., reduced marginal cost of production) to *one* firm alone may lead to a *lower* equilibrium profit for *that firm* and a *higher* equilibrium profit for *its competitor*. This result does not rely on any ex ante asymmetry between the firms, but rather the insight lies in firms' endogenous competitive responses in advertising. This result provides firms with a warning to "be careful what you wish for" in such a competitive landscape.

We focus on examining competitive strategies of persuasive advertising rather than informative advertising (e.g., Amaldoss and He 2010, Dukes 2004, Grossman and Shapiro 1984, Iyer et al. 2005). Our research contributes to the stream of literature on persuasive advertising and product differentiation. Bloch and Manceau (1999) study persuasive advertising in a Hotelling model of product differentiation and assume that advertising will shift the distribution of consumer tastes towards the advertised product. They show that there exist some mathematical distributions of consumers before and after advertising such that advertising may lead to a decrease in the price of the advertised product. Their model assumes that only one firm can advertise; as the authors also point out, their model does not allow them to analyze the firm's choice of advertising levels or study any strategic effects of advertising by both firms. Another related paper by von der Fehr and Stevik (1998) shows that, when persuasive advertising increases the consumer's willingness to pay, both firms will advertise at equilibrium and the amount of advertising does not depend on the degree of product differentiation.[2] Their models do not consider any asymmetry between the firms' product valuations, nor between their marginal costs or their costs of advertising. Their analysis is entirely focused on symmetric equilibrium. In contrast, we explicitly allow for asymmetry between competing firms. We obtain qualitatively different results and the insights cannot be obtained from von der Fehr and Stevik (1998). For example, we show that, generally, firms' advertising depends on the degree of product differentiation (i.e., the strength of consumers' taste preferences), and that firms' optimal advertising responses to an exogenous change in the degree of product differentiation are opposite—one firm increases advertising whereas the other reduces it. As the degree of product differentiation becomes larger, the firm with a relatively low variable margin tends to increase advertising whereas its competitor tends to reduce advertising.

[2] The authors also study two alternative models of advertising (also by symmetric firms); one assumes that advertising changes the ideal product variety and the other assumes that advertising increases perceived product differences. To provide direct comparison in marketing implications, we focus on their case of advertising raising the consumer's willingness to pay since this is consistent with our model.

We highlight a few additional findings. First, although it seems intuitive that the firm with an exogenous shock that increases consumers' willingness to pay for its product should reduce its advertising especially when advertising is very costly, this may be incorrect. In a competitive market, a firm will complement an exogenous increase in its product valuation with additional advertising especially when advertising is costly or inefficient. This is because the firm's competitor will find it inefficient to increase advertising to compensate for the shock leading to a higher marginal benefit for the firm's advertising. Second, if an exogenous shock increases only one firm's advertising efficiency, that firm may make a lower equilibrium profit whereas its competitor's profit increases. The intuition also comes from the fact that, if advertising is efficient, the competitor will respond to the firm's "favorable" shock in efficiency with a large increase in advertising. Such a response reduces the firm's marginal benefit for advertising resulting in decreased equilibrium advertising and price yielding to a lower profit, whereas the competitor can take advantage of the firm's reduced incentive to advertise and make a higher profit. Third, the larger the shock that improves a firm's valuation or unit cost and the more advertising efficient that firm is, the more likely the total advertising expenditure in the industry will increase. Further, the exogenous shock may reduce the price dispersion in the industry even though it increases the separation between the firms' valuations or perceived qualities, and vice versa. We offer some testable hypotheses for empirical researchers and hope our research motivates empirical studies to systematically test our theoretical findings.

In addition, we show that our main results and intuition are robust to alternative game structure, heterogeneity in consumer responses or exposure to advertising, nonlinearity of horizontal preferences, and alternative functional forms for advertising costs. Instead of the persuasive advertising and product differentiation setting, our model can also be reinterpreted in other settings such as high-tech product markets, where products comprise hardware and software components. Our analysis implies, for example, that if the software component of a firm's product can increase the consumer's valuations very efficiently, then a seemingly favorable shock to the consumers' valuation or the production cost for the hardware for one firm's product will reduce that firm's profit and benefit its competitor.

The rest of this paper is organized as follows. In Section 2, we present our model framework and solve for the subgame perfect Nash equilibrium. In Section 3, we analyze the impacts of exogenous shocks on firms' advertising and pricing strategies as well as their profits. In Section 4, we show that our results are robust to alternative game structures, heterogeneity in consumer responses or exposure to advertising, nonlinearity of horizontal preferences, and alternative functional forms for advertising costs. We conclude the paper in Section 5.

## 2. Model

Two firms each offer one durable product. Each consumer will buy at most one product. The firms' products are both horizontally and vertically differentiated. Firm *i* offers product *i* of quality $v_i$ and of marginal cost $c_i$. Without loss of generality, we assume that firm 1's product is located at zero and firm 2's product at one. We use $x$ to represent the consumer's horizontal or taste preference. Consumers are heterogeneous with respect to their horizontal preferences and are uniformly distributed on the line segment between zero and one: $x \sim uniform[0, 1]$. The consumer's disutility from non-exactly matched taste preference is $td_i$, where $d_i$ is the distance between the consumer's location (*x*) and product $i$'s location, and *t* represents the strength of consumers' taste preferences. We assume for now that consumers' taste/horizontal preference is linear, but will show later that nonlinear preferences such as quadratic transportation cost do not change our results. Without loss of generality, we normalize the total number of consumers to one.

Each firm *i* maximizes its own profit by choosing two decision variables—the level of advertising ($a_i$) and price ($p_i$). In the first stage of the game, the firms simultaneously decide their respective advertising levels, and in the second stage they simultaneously decide their prices. Lastly, consumers make purchase decisions.[3] We model the advertising and pricing decisions sequentially since in reality fixed-cost decisions such as advertising tend to be made earlier whereas prices may be much more easily changed. As in Adams and Yellen (1977), we assume that firm *i*'s advertising can increase consumers' willingness to pay for its product by some amount, which we denote as $a_i$. This assumption is also consistent with the persuasive view of advertising—advertising alters consumers' preferences and creates brand loyalty—and the complementary view of advertising—advertising (when properly carried out) provides additional utility to consumers such as creating a feeling of greater social prestige (Bagwell 2007). We use a general convex function $g_i(a_i)$ to denote firm *i*'s fixed cost required for its advertising level $a_i$.

In summary, given firms' advertising ($a_i$) and price ($p_i$) decisions, a consumer of type $x$ will derive a net utility of $U_i = V_i + a_i - td_i - p_i$ from product $i$, where $d_i = x$ for $i = 1$ and $d_i = 1 - x$ for $i = 2$.[4] Without loss of generality, we assume that consumers' outside option has a utility of zero; consumers will thus buy the product that yields a higher non-negative utility.

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[3] We will show later that our main results remain qualitatively the same even for static games (i.e., when both firms simultaneously select both advertising levels and prices).

[4] Though the *level* of advertising is added linearly to the consumer's utility, we actually model these factors in a nonlinear way (through the nonlinear cost functions associated with advertising), e.g., there is a diminishing return

Note that, if the product valuations ($V_i + a_i$) are too low, both firms will be localized monopolies and the market will not be covered. Further, if the difference in the two products' valuations is too large, one firm will profitably squeeze the other out of the market and become a monopoly. We focus on the more interesting case of a competitive market, and will implicitly assume that the product valuations are not too low and that the difference in the two products' valuations is not too large. This assumption is equivalent to assuming that the strength (*t*) of consumers' horizontal preference is not too small and not too large. We will be more explicit about this assumption later in this paper. We now use standard backwards induction to solve for the pure-strategy subgame perfect Nash equilibrium.

### 2.1. Competitive Pricing Decisions

Let $x_{in}$ be the consumer who is indifferent between the two products (i.e., $U_1 = U_2$): $V_1 - tx_{in} - p_1 + a_1 = V_2 - t(1 - x_{in}) - p_2 + a_2$. We easily find that $x_{in} = \frac{V_1+a_1-V_2-a_2+t-p_1+p_2}{2t}$.

Consumers with $x \le x_{in}$ prefer to buy product 1 and those with $x > x_{in}$ prefer product 2. Firms' market shares ($m_i$) are $x_{in}$ and $1 - x_{in}$, respectively; their profits are as follows.

$$\Pi_1(p_1, p_2, a_1, a_2) = (p_1 - c_1)x_{in} - g_1(a_1) \quad (1)$$

$$\Pi_2(p_1, p_2, a_1, a_2) = (p_2 - c_2)(1 - x_{in}) - g_2(a_2) \quad (2)$$

In the second stage, firms simultaneously choose prices given their advertising levels to maximize their respective profits. Firms' optimal (equilibrium) prices are obtained by simultaneously solving the first-order conditions: $\frac{\partial \Pi_1}{\partial p_1} = 0$ and $\frac{\partial \Pi_2}{\partial p_2} = 0$.

$$p_1^*(a_1, a_2) = \frac{V_1+a_1-V_2-a_2+3t+2c_1+c_2}{3} \quad (3)$$

$$p_2^*(a_1, a_2) = \frac{V_2+a_2-V_1-a_1+3t+c_1+2c_2}{3} \quad (4)$$

We now digress to discuss our implicit assumption of a competitive (rather than monopoly) equilibrium in which both firms have a positive market share. For any firm *i* to profitably sell its product, its price must cover its marginal cost: $p_i^*(a_1, a_2) > c_i$, $i \in \{1, 2\}$; otherwise, the firm will prefer not selling and have a zero market share. The necessary condition for a competitive pricing equilibrium straightforwardly simplifies to

$$\left|V_i + a_i - V_j - a_j - c_i + c_j\right| < 3t, \quad \text{(A1)}$$

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of advertising. An alternative and equivalent formulation is to use the advertising expenditure ($s_i$) as the firm's decision variable rather than $a_i$; the consumer's utility will then be nonlinear in $s_i$: $U_i = V_i + g^{-1}(s_i) - td_i - p_i$.

where $i, j \in \{1, 2\}$ and $i \neq j$.[5] Note that $V_i + a_i - c_i$ is firm *i*'s maximum feasible profit margin—the highest willing-to-pay consumer's valuation subtracted by firm *i*'s marginal cost— and that this essentially represents how competitive firm *i* can be. Thus, the condition (A1) is quite intuitive; it simply states that the difference in firms' feasible profit margins must be below a threshold; otherwise, there may not be a pure-strategy pricing equilibrium, or the more competitive firm will effectively be a monopoly.[6]

Substituting (3) and (4) into (1) and (2), we simplify the firms' profits to

$$\Pi_1^*(a_1, a_2) = \frac{(3t+V_1+a_1-V_2-a_2-c_1+c_2)^2}{18t} - g_1(a_1), \tag{5}$$

$$\Pi_2^*(a_1, a_2) = \frac{(3t-V_1-a_1+V_2+a_2+c_1-c_2)^2}{18t} - g_2(a_2). \tag{6}$$

Firms' market shares are given by $m_1 = x_{in}^* = \frac{3t+V_1+a_1-V_2-a_2-c_1+c_2}{6t}$ and $m_2 = 1 - m_1 = \frac{3t+V_2+a_2-V_1-a_1+c_1-c_2}{6t}$, respectively.

## 2.2. Competitive Advertising Decisions

We now analyze the first stage of the game, in which firms simultaneously choose their advertising levels $(a_i)$ to maximize their profits (5) and (6), respectively. To facilitate closed-form analytical solutions, we assume that firms' advertising costs are quadratic (e.g., Tirole 1988, Bagwell 2007): $g_i(a_i) = k_i {a_i}^2$, $i \in \{1, 2\}$. For the existence of a competitive equilibrium, advertising cannot be too "effective" at increasing consumers' valuations (i.e., $k_i$ cannot be too close to zero). In particular, we assume $k_i > \frac{1}{18t}$, $i \in \{1, 2\}$, which implies that firms' profits (5) and (6) are single-peaked, quadratic functions in terms of their advertising levels.

The first-order conditions are

$$\frac{\partial \Pi_1^*(a_1, a_2)}{\partial a_1} = \frac{3t+V_1+a_1-V_2-a_2-c_1+c_2}{9t} - 2k_1 a_1 = 0, \tag{7}$$

$$\frac{\partial \Pi_2^*(a_1, a_2)}{\partial a_2} = \frac{3t-V_1-a_1+V_2+a_2+c_1-c_2}{9t} - 2k_2 a_2 = 0. \tag{8}$$

[5] In this paper, when the subscripts *i* and *j* appear in the same expression or context, it is always assumed that $i \neq j$.

[6] Condition (A1) implies that the "transportation cost" (*t*) is not too small, i.e., the two firms are not too closely located. D'Aspremont et al. (1979) demonstrate that if two firms are closely located, there is no pure strategy equilibrium in the price subgame (with zero marginal costs). In our model, since firms are asymmetric with positive marginal costs, there are two possibilities when condition (A1) is violated—non-existence of a pure strategy pricing equilibrium or a monopoly price equilibrium (with the more competitive firm effectively setting a monopoly price and the other firm pricing at its marginal cost but getting zero market share).

We can obtain the equilibrium advertising levels by simultaneously solving (7) and (8). Boundary solutions (i.e., $a_i^* = 0$ for either firm) imply that the firm with zero advertising will get a *zero* market share at the pricing stage, i.e., the market becomes a monopoly at equilibrium. The existence of interior solutions (rather than boundary solutions) comes from our implicit assumption that firms' abilities to advertise are not drastically different—a necessary condition for a *competitive* (rather than *monopoly*) equilibrium outcome. This is consistent with assumption (A1); if firms' advertising efficiencies are too drastically different, the efficient firm will be able to profitably squeeze out its advertising-inefficient competitor by running a high enough level of advertising. The condition for competitive equilibrium—that both firms are comparably efficient at advertising—gives us two parameter regions of interest:

(i) inefficient-advertising market: $k_i > k_i^{(c)}$ for $i \in \{1, 2\}$,

(ii) efficient-advertising market: $\frac{1}{18t} < k_i < k_i^{(c)}$ for $i \in \{1, 2\}$, where for expositional conciseness, we have defined two constant expressions:

$k_i^{(c)} \equiv \frac{1}{3[3t - V_i + V_j + c_i - c_j]}$ for $i, j \in \{1, 2\}$.

Note that "efficient-advertising" and "inefficient-advertising" do not refer to whether one firm's advertising efficiency is higher or lower than that of the other firm, but rather depend on their efficiencies relative to some thresholds. For example, even if both firms have the same efficiency, we still have efficient-advertising and inefficient-advertising markets depending on whether the efficiency falls below or above the thresholds $k_i^{(c)}$.

LEMMA 1. *At equilibrium, both firms choose positive levels of advertising and make positive profits. The equilibrium outcome is given by*

$$a_i^* = \frac{3k_j(3t + V_i - V_j - c_i + c_j) - 1}{3(18k_i k_j t - k_i - k_j)} \quad (9)$$

$$p_i^* = \frac{2k_i t[3k_j(3t + V_i - V_j - c_i + c_j) - 1]}{18k_i k_j t - k_i - k_j} + c_i \quad (10)$$

$$\Pi_i^* = \frac{k_i(18k_i t - 1)[3k_j(3t + V_i - V_j - c_i + c_j) - 1]^2}{9(18k_i k_j t - k_i - k_j)^2} \quad (11).$$

## 3. Results

Lemma 1 shows that when firms have comparable advertising efficiencies, both will advertise and earn a positive profit in the subgame perfect Nash equilibrium. This is consistent with von der Fehr and Stevik (1998), which shows that when persuasive advertising increases the consumer's willingness to pay, competitive firms will advertise at symmetric equilibrium. These authors do not consider any asymmetry between the firms' product valuations, nor between their marginal costs or their costs of advertising. Their analysis is entirely focused on symmetric equilibrium. In contrast, we explicitly model the asymmetry between the two competing firms. Therefore, as shown in our next few propositions, our results are qualitatively different and the insights cannot be obtained from von der Fehr and Stevik (1998).

PROPOSITION 1. Effect of *t* on Firms' Advertising Levels

*As consumers' taste preference becomes stronger (i.e., as t increases), one firm will increase advertising whereas the other will reduce its advertising. The firm with the lower potential variable-margin $V_i - c_i$ is more likely to increase advertising as t increases.*

Proposition 1 directly contrasts von der Fehr and Stevik (1998), which shows that, if advertising increases the consumer's willingness to pay, the degree of horizontal product differentiation will not affect firms' equilibrium advertising levels. We show that generally, *t*, the strength of consumers' taste preferences (i.e., the degree of product differentiation) affects firms' advertising decisions. A larger *t* has two effects. First, it reduces consumers' willingness to pay because their horizontal preferences are not exactly matched. Second, a larger *t* implies that firms' products are more differentiated and hence the price competition between the two firms tends to be alleviated, a fact that is manifested in equations (3) and (4). Since a larger *t* tends to lower both the consumer's product valuations and the price competition between the firms, one might intuitively expect that both firms should have incentives to increase the level of persuasive advertising to raise consumers' valuations. Proposition 1 shows a surprising result—firms' optimal advertising responses to an increase in *t* are qualitatively different: one firm will increase advertising whereas the other will reduce it. As *t* becomes larger, the firm with a relatively low variable margin tends to increase advertising whereas its competitor tends to reduce advertising. For example, if both firms are equally effective at advertising ($k_i = k_j$), the firm with a lower variable margin (defined by $V_i - c_i$) will have a higher marginal incentive to increase advertising when *t* becomes larger. As a result, the firm with a higher variable margin will find it optimal to reduce its advertising rather than to engage in an advertising war.

Now we examine how exogenous changes in consumers' product valuations ($V_i$) or firms' production costs ($c_i$) may influence their advertising and pricing strategies. Many exogenous factors can lead to changes in consumers' product valuations or firms' production costs. For example, the consumer's valuation or willingness to pay for a company's product may be influenced by product reviews or changing trends in consumer preferences, price shocks in complements, or even actions by third parties such as non-profit organizations or governments. Similarly, a firm's production cost may also change due to many exogenous factors such as changes in labor costs, input prices, and government subsidies or tax law/regulations. Let us take electric cars as an example. Tax credits to consumers who purchase such environmentally-friendly cars will reduce costs of ownership and hence increase consumers' willingness to pay for such vehicles. If environmental agencies run public campaigns promoting "green" causes and praising eco-friendly firms, consumers may increase their valuations for green brands/products such as electric cars (relative to conventional gasoline cars) because by using such products consumers may project a desirable self-image in public. Of course, if gasoline prices rise, then consumers will also have an increased valuation for an electric car because of its lowered relative cost of operation. Similarly, for firms' production costs, exogenous factors in the supply chain may increase (or decrease) one firm's cost more than its competitor's. Even when unit labor costs or input prices change by the same amount for different firms, the effect of such changes on the unit production cost may differ across firms. This is because firms may use different amounts of input or labor for each unit of their product; for example, each electric car requires fewer metal parts (e.g., no engine needed) than a conventional car, hence an increase in metal prices will raise the unit cost of a conventional car much more than that of an electric car.

It is not clear how firms should optimally respond, in terms of their advertising and pricing strategies, to these exogenous changes that influence their production costs or the consumer's product valuations. If external factors have raised a firm's product valuation to consumers or reduced the firm's cost relative to its competitor, should it increase its advertising or price? Should the firm's competitor increase advertising or reduce its price to compensate for its exogenously lowered competitiveness?

**Table 1 Key Comparative Statics Regarding Product Valuations and Production Cost**

| Parameter Regions | Advertising | Price | Profit |
|---|---|---|---|
| Efficient Advertising:<br>$\frac{1}{18t} < k_i < k_i^{(c)}$ for $i \in \{1,2\}$ | $\frac{\partial a_i^*}{\partial V_i} < 0,\ \frac{\partial a_i^*}{\partial c_i} > 0$<br>$\frac{\partial a_j^*}{\partial V_i} > 0,\ \frac{\partial a_j^*}{\partial c_i} < 0$ | $\frac{\partial p_i^*}{\partial V_i} < 0,\ \frac{\partial p_i^*}{\partial c_i} > 0$<br>$\frac{\partial p_j^*}{\partial V_i} > 0,\ \frac{\partial p_j^*}{\partial c_i} < 0$ | $\frac{\partial \Pi_i^*}{\partial V_i} < 0,\ \frac{\partial \Pi_i^*}{\partial c_i} > 0$<br>$\frac{\partial \Pi_j^*}{\partial V_i} > 0,\ \frac{\partial \Pi_j^*}{\partial c_i} < 0$ |
| Inefficient Advertising:<br>$k_i > k_i^{(c)}$ for $i \in \{1,2\}$ | $\frac{\partial a_i^*}{\partial V_i} > 0,\ \frac{\partial a_i^*}{\partial c_i} < 0$<br>$\frac{\partial a_j^*}{\partial V_i} < 0,\ \frac{\partial a_j^*}{\partial c_i} > 0$ | $\frac{\partial p_i^*}{\partial V_i} > 0,\ \frac{\partial p_i^*}{\partial c_i} > 0$<br>$\frac{\partial p_j^*}{\partial V_i} < 0,\ \frac{\partial p_j^*}{\partial c_i} <, > 0$ | $\frac{\partial \Pi_i^*}{\partial V_i} > 0,\ \frac{\partial \Pi_i^*}{\partial c_i} < 0$<br>$\frac{\partial \Pi_j^*}{\partial V_i} < 0,\ \frac{\partial \Pi_j^*}{\partial c_i} > 0$ |

Table 1 shows the comparative statics in the parameter regions of interest. Note that the effects on firms' advertising and profits of an exogenous change in consumers' valuations or firms' unit costs depend only on the relative change for the two firms. For example, if exogenous factors increase both firms' valuations (or unit costs) by the same amount, neither firm will change its advertising; mathematically, $\frac{\partial a_i^*}{\partial V_i} = \frac{\partial a_i^*}{\partial (V_i - V_j)}$. Thus, for brevity, we will list the comparative statics with respect to $V_i$ and $c_i$ rather than also for the relative changes. We start with Proposition 2, which sheds light on how exogenous changes in consumers' valuation and firms' production costs may affect their advertising strategies.

PROPOSITION 2. *Firms' advertising responses to an exogenous relative change in consumers' valuations or firms' unit production costs are opposite—one firm increases advertising whereas the other decreases it. Interestingly, when advertising is costly or inefficient (i.e., $k_i > k_i^{(c)}$ for $i \in \{1, 2\}$), the firm with an exogenous increase in its product valuation or a decrease in its unit cost will increase its advertising rather than decrease it.*

Note that an exogenous increase in consumers' valuation for a firm's product has a similar effect to the firm's advertising, which also increases consumers' willingness to pay for its product. One may thus intuit that the firm with an exogenous shock that increases its product valuation will reduce its advertising especially when advertising is very costly or inefficient (i.e., when $k_i$ is large). Our analysis shows that this naïve intuition turns out to be incorrect. If advertising is inefficient, the firm will in fact consider advertising as a complement to the exogenous increase in its product valuation. That is, the firm will find it optimal to increase its advertising rather than decrease it. The intuition comes from the competitive effect. Let firm 1 be the firm with the exogenous increase in product valuation. When advertising is costly, firm 2 (firm 1's competitor) will find it inefficient to increase its advertising to compensate for the unfavorable exogenous change in product valuation; its best response is mainly to reduce its price to make its product offer more attractive. In fact, at equilibrium, firm 2 will marginally reduce its advertising level. This competitive advertising response makes firm 1's marginal benefit of advertising higher than its marginal cost; thus, firm 1 will increase rather than decrease its advertising (and raise its price as well). The intuition is similar with respect to an exogenous change in unit production cost. That is, in a competitive market, a firm complements an exogenous increase in its product valuation with additional advertising, especially when advertising is costly and inefficient.

The flip side of the story is that when advertising is efficient at raising consumers' willingness to pay (i.e., $k_1$ and $k_2$ are small), firm 1, which has an exogenous increase in consumers' valuation (or a decrease in unit production cost) relative to its competitor (firm 2) will see a lowered marginal benefit of advertising and reduce its advertising because, if advertising is efficient, firm 2 will significantly increase its advertising expenditure to compensate for the exogenous change.

We now examine how an exogenous shock in consumers' valuations and firms' unit costs affects the firms' profits. Clearly, a monopolist firm will benefit from an exogenous increase in consumers' valuation or an exogenous decrease in production cost. This is because the monopolist can, for example, raise its price to take advantage of the consumer's increased valuation, or enjoy a higher margin (even without a price increase) due to the exogenously reduced cost. In a competitive market, will a firm necessarily benefit and its competitor hurt from an exogenous shock that increases the firm's product valuation or decreases the firm's unit cost relative to its competitor? One may intuitively expect so. However, we show that, interestingly, the opposite may be true when firms' decisions on persuasive advertising are endogenously determined.

PROPOSITION 3. *An exogenous shock that increases a firm's product valuation or decreases its unit cost relative to its competitor, though costless to the firm, will reduce the firm's profit and increase its competitor's profit if advertising is efficient (i.e.,* $\frac{1}{18t} < k_i < k_i^{(c)}$ *for* $i \in \{1, 2\}$*).*

Our analysis shows that a seemingly favorable exogenous shock to a firm's product valuation or unit cost may actually hurt the firm's profitability and benefit its competitor. This result is very counter-intuitive, so we will closely study the underlying cause. Suppose that an exogenous shock has increased consumers' valuation for firm 1's product by an amount $\epsilon > 0$ relative to firm 2's product.[7] This shock clearly seems favorable to firm 1—if both firms do not change their advertising and pricing decisions, firm 1 will be better off and firm 2 worse off. Let us examine how this exogenous shock will alter firms' advertising and pricing strategies. If advertising is very efficient, firm 2 will find it effective to simply boost its advertising to compensate for the exogenous shock; its optimal advertising response is an advertising increase of $\Delta a_2^* = \frac{k_1 \epsilon}{k_1 + k_2 - 18 k_1 k_2 t} > 0$. As a result of the $\epsilon$ shock and firm 2's response, firm 1's marginal advertising benefit will drop and its optimal response turns out to be a small reduction in advertising (to avoid an advertising war): $\Delta a_1^* = \frac{-k_2 \epsilon}{k_1 + k_2 - 18 k_1 k_2 t} < 0$. Without considering firm 2's competitive response (i.e., holding firm 2's actions fixed), firm 1 will be better off because it can increase its price and still keep the same market share, or keep the same price and increase its market share even without having to adjust its advertising. However, because firm 2 will respond to the shock with a relatively large increase in advertising, firm 1 will actually *reduce* its price at equilibrium: $\Delta p_1^* = \frac{-6 k_1 k_2 t \epsilon}{k_1 + k_2 - 18 k_1 k_2 t} < 0$, even though firm 2 will *increase* its price: $\Delta p_2^* = \frac{6 k_1 k_2 t \epsilon}{k_1 + k_2 - 18 k_1 k_2 t} > 0$. This is because, as one can easily show, firm 2's price increase is much smaller than its increase in advertising: $\Delta p_2^* <$

[7] The intuition from an exogenous shock in unit production cost is similar.

$\Delta a_2^*$. The effect on firm 1's market share is straightforwardly computed as $\Delta x_{in} = -\frac{3k_1k_2\epsilon}{k_1+k_2-18k_1k_2t} < 0$. In summary, if advertising is efficient, the seemingly favorable shock to firm 1 can trigger a large advertising increase by its competitor, whose market share and profit will both increase despite its slightly increased price. As a result, firm 1 will actually become worse off as it optimally reduces both its advertising and price to reach the new equilibrium.

One natural question may arise from the above result. In an efficient-advertising market, a firm appears to have an incentive to intentionally lower its product value or engage in cost-increasing actions (e.g., to an extreme, burning some money for each unit sold or disabling certain value features). Will a firm necessarily do that to improve its profit? No. First, such a cost-increasing (or value-destroying) action must be credible, observable, and pre-committable before the firm's advertising decisions. Otherwise, a firm will always be better off by not carrying out the action in the pricing subgame or after sales, i.e., a strategy with a cost-increasing, wasteful action *not* committed and observed before firms' advertising decisions is not credible and fails subgame perfection. Second, in theory, an observable, pre-committable, value-destroying or cost-increasing action can be taken by both firms. If both do it to the same extent, the equilibrium outcome will remain unaffected. Or, it will be done to a degree that violates the advertising-efficiency constraint and hence the incentive for such actions reverses. Third, in practice, firms face uncertainties in many things. For example, firms may not know yet, at the time of manufacturing the products and incurring the costs, whether advertising will be efficient (i.e., which parameter region the advertising efficiencies will fall into). Hence, at that stage of decision making, it is in a firm's best interest to take an action that creates the highest competitiveness in terms of value-cost margin. In addition, if firms do not observe each other's value/cost decisions before making advertising decisions, then both firms will find it optimal to choose the most margin-efficient strategy.

Proposition 3 contrasts with the Bertrand supertraps result by Cabral and Villas-Boas (2005). They show that, in the presence of intra-firm product interactions, a seemingly positive shock in the industry (e.g., an increased degree of economies of scope or demand synergies) may lead to lower equilibrium profits for all (multi-product) firms. In their setting, if the positive shock happens to only one firm rather than to all firms in the industry, that firm will become better off and other firms worse off. Our result extends beyond Bertrand supertraps: when firms make endogenous advertising decisions in addition to pricing, a seemingly favorable exogenous shock (e.g., increased product valuation or reduced marginal cost of production) to *one* firm alone may lead to a *lower* equilibrium profit for that firm and a *higher* equilibrium profit for its competitor. It is important to note that this result does not rely on the ex ante asymmetry between the firms. As explained before, the insight lies in firms' endogenous (fixed-cost) advertising decisions.

In practice, a firm's advertising efficiency may also be (exogenously) improved by, for example, favorable third-party reviews or endorsement/recommendations, which can make a firm's advertising more convincing or persuasive. We consider this seemingly positive, exogenous shock in a firm's advertising efficiency and show that such a "favorable" shock to a firm's advertising efficiency may actually reduce the firm's equilibrium profit and increase the competitor's profit. Let us examine this key intuition specifically in a market with ex ante symmetric firms. The first part of Proposition 4 shows that ex ante symmetric firms both become worse off if both firms' advertising efficiencies increase equally (i.e., $k_1 = k_2$ and both decrease by the same amount). One can easily show that when the symmetric firms both improve their advertising efficiencies, both firms will increase their advertising expenditure equally, and neither can profitably raise their prices because of competition. The second part of Proposition 4 shows that if the exogenous shock increases only one firm's advertising efficiency, that firm may make a lower equilibrium profit. Similarly, the intuition comes from the fact that when advertising is efficient, the competitor will respond to the firm's "favorable" shock in efficiency with a large increase in advertising. Such a response reduces the firm's marginal benefit for advertising resulting in decreased equilibrium advertising and price, which leads to a lower firm profit whereas the competitor can take advantage of the firm's reduced incentive to advertise and hence make a higher profit.

PROPOSITION 4. *Suppose that firms are ex ante symmetric (i.e., $V_1 = V_2$, $c_1 = c_2$, $k_1 = k_2$). (a) When the exogenous shock improves both firms' advertising efficiencies equally, both firms become worse off. (b) When advertising is efficient, an exogenous increase in only one firm's advertising efficiency will reduce that firm's equilibrium profit and increase its competitor's profit.*

We now examine how shocks to firms' relative product valuations or unit production costs affect the total advertising expenditure and the price dispersion in the industry. The total advertising expenditure in the industry is given by $T \equiv \sum g_i(a_i^*) = \sum k_i \, {a_i^*}^2$. We know from equation (9) that firms' advertising decisions are affected by an exogenous shock in the consumer's baseline product valuations or unit production costs only to the extent that the two firms' valuations or unit costs are changed by a different amount. A relative increase in a firm's production valuation has the same effect on advertising and profit as a relative decrease in the firm's unit production cost. Without loss of generality, let firm *i* be the firm that has an exogenous increase in its product valuation or a decrease in its unit production cost relative to its competitor (*j*). One may also think of $V_i - V_j$ as the relative separation between firms' baseline (perceived) quality levels. If the exogenous shock increases the separation between consumers' valuations (or perceived qualities), will it necessarily increase the price difference between the two firms? Will the exogenous shock lead to an increase or a decrease in the advertising spending in the industry as a whole?

The following propositions shed light on how exogenous shocks in consumer valuations or firm unit costs affect the industry's advertising expenditure and price dispersion.

PROPOSITION 5. *The larger the exogenous shock is in the relative valuation or unit production cost between the two firms and the more advertising-efficient firm i is (i.e., smaller $k_i$, or smaller $k_i - k_j$), the more likely the total advertising expenditure in the industry will increase.*

PROPOSITION 6. *An exogenous shock that increases (decreases) the separation between consumer's valuations for the firms' products may reduce (increase) the difference between firms' equilibrium prices.*

To explain the intuition behind Proposition 5, let us consider a specific case of an efficient-advertising market. Since advertising is not efficient in raising consumers' willingness to pay, when the exogenous shock increases firm *i*'s relative valuation, the competitor, firm *j*, will see an increased marginal benefit from advertising and significantly increase its advertising to compensate for its reduced competitiveness. Firm *j*'s advertising response and the exogenous shock reduce firm *i*'s marginal benefit for advertising, and it actually finds it optimal to reduce its advertising level but only slightly. The larger the shock, the more likely firm *j*'s increase in advertising expenditure will be more than firm *i*'s reduction in advertising. Similarly, the more efficient firm *i* is, the less it will reduce its advertising spending after the shock, and hence the more likely firm *j*'s increase in advertising expenditure will be more than firm *i*'s reduction in advertising leading to an increase in advertising spending in the industry. In contrast, in an inefficient-advertising market, firm *j* finds it very expensive to use advertising to compensate for the unfavorable, exogenous shock, and firm *i* will have an increased marginal benefit for advertising and will raise its spending to further its advantage in consumer valuation. Firm *i*'s advertising response and the exogenous shock actually reduce firm *j*'s marginal benefit for advertising leading to a small decrease in firm *j*'s advertising. The larger the shock and the more efficient firm *i*, the more likely firm *i*'s increase in advertising expenditure will be more than firm *j*'s reduction in advertising resulting in an increase in advertising spending in the industry.

One may intuit that, if the exogenous shock increases the difference in consumers' valuations between the firms' products, the price difference between the firms will increase as well and vice versa. Proposition 6 shows that this may not necessarily be the case. The exogenous shock may have opposite effects on firms' valuation separation and their price dispersion. The main insight here also comes from the competitive advertising responses to the exogenous shock. In essence, when the shock increases the separation in the firms' product valuations, the difference in firms' marginal incentives to advertise may help counter that increased separation and can, under many circumstances, yield higher rather than lower competitive pressure on prices leading to smaller price dispersion.

# 4. Model Extensions and Reinterpretation

## 4.1. Alternative Static Game

In this section, we consider whether our results depend on the sequential nature of the game. Thus far we have assumed that firms make their decisions in the following sequence: advertising, and then price. Will our results remain robust if firms make both their advertising and price decisions together simultaneously rather than sequentially (i.e., each firm makes both advertising and pricing decisions jointly without observing the competing firm's advertising choice)? This setting makes the game a static one.

Solving for the location of the consumer who is indifferent between the two products (i.e., $U_i = U_j$), we obtain each firm's market share: $m_i = \frac{V_i+a_i-V_j-a_j+t-p_i+p_j}{2t}$. Firm $i$'s profit function is thus expressed as $\Pi_i(p_i, p_j, a_i, a_j) = [p_i - c_i]\, m_i - k_i a_i^2$ . We simultaneously solve the four first-order conditions for profit maximization: $\frac{\partial \Pi_i}{\partial p_i} = 0$, $\frac{\partial \Pi_i}{\partial a_i} = 0$, $i$ = 1, 2. The final equilibrium outcome is as follows.

$$a_i^* = \frac{2k_j(3t+V_i-V_j-c_i+c_j)-1}{2(12k_ik_jt-k_i-k_j)}$$

$$p_i^* = \frac{2k_it[2k_j(3t+V_i-V_j-c_i+c_j)-1]}{12k_ik_jt-k_i-k_j} + c_i$$

$$\Pi_i^* = \frac{k_i(8k_it-1)[2k_j(3t+V_i-V_j-c_i+c_j)-1]^2}{4(12k_ik_jt-k_i-k_j)^2}$$

We observe that the simultaneous (static) equilibrium outcome has similar functional forms to the sequential-move (dynamic) equilibrium outcome (12)-(14) except that a few numeric coefficients are different. In fact, all our results from the dynamic game stay qualitatively the same for the static game; only the parameter regions (of inefficient versus efficient advertising) are different. The new advertising efficiency cutoffs are $k_i^{(c)} \equiv \frac{1}{2(3t-V_i+V_j+c_i-c_j)}$. The efficient advertising market is defined by $\frac{1}{8t} < k_i < k_i^{(c)}$ for $i \in \{1, 2\}$ while the inefficient advertising market is defined by $k_i > k_i^{(c)}$ for $i \in \{1, 2\}$. Thus, we have shown that our main findings are quite robust to the dynamic/static assumptions of the game.

## 4.2. Heterogeneous Consumer Responses or Exposure to Advertising

We have so far assumed that advertising influences consumers uniformly—changing all consumers' willingness to pay by the same amount. It is possible that consumers may have different levels of knowledge about the product, perceive advertising differently, or have different exposures to advertising. That is, it is possible that advertising may change some consumers' valuations more than others. However, we expect that our main results will remain qualitatively the same even if we introduce some heterogeneity in consumers' characteristics other than their taste preferences, which we have modeled. We will rely on future research to find any additional insights that other types of consumer heterogeneity may bring forth. We will study here one type of consumer heterogeneity with respect to advertising; in particular, there are two types of consumers, one of which is less affected by or exposed to advertising than the other. Without loss of generality, we assume that one type of consumer is unaffected or unexposed to advertising.

Let $\beta$ be the fraction of consumers who are exposed and respond to the persuasive advertising, and $1-\beta$ be the fraction of consumers who are unaffected by persuasive advertising. Assuming the market is covered at equilibrium, we obtain

$$\Pi_i(p_i, p_j; a_i, a_j) = \beta(p_i - c_i)\frac{t+V_i+a_i-V_j-a_j-p_i+p_j}{2t} + (1-\beta)(p_i - c_i)\frac{t+V_i-V_j-p_i+p_j}{2t} - k_i a_i^2.$$

Backward induction from the pricing stage to the advertising stage leads to the following equilibrium:

$$p_i^* = \frac{V_i - V_j + a_i^* - a_j^* + 3t + 2c_i + c_j}{3}$$

$$a_i^* = \frac{\beta[3k_j(3t+V_i-V_j-c_i+c_j)-\beta^2]}{3[18k_ik_jt-\beta^2(k_i+k_j)]}$$

$$\Pi_i^* = \frac{k_i(18k_it-\beta^2)[3k_j(3t+V_i-V_j-c_i+c_j)-\beta^2]^2}{9[18k_ik_jt-\beta^2(k_i+k_j)]^2}.$$

From the equilibrium outcome, we obtain several results. As we intuit, the larger the fraction ($\beta$) of consumers whose valuations are influenced by advertising, the more advertising firms do. Clearly, one can easily prove that as $\beta$ increases, symmetric firms (i.e., $V_1 = V_2$, $c_1 = c_2$, $k_1 = k_2$) are both worse off. The intuition here is similar to that for part (a) of Proposition 4, because essentially firms' advertising efficiency increases as $\beta$ increases.

### 4.3. Nonlinear Horizontal Preference and Alternative Advertising Cost Functions

We now examine whether a nonlinear horizontal preference or alternative advertising cost functions will change our main results. First, instead of the linear taste preference as assumed in the core model, we consider a quadratic preference, which is also commonly used in the literature. That is, given firms' advertising ($a_i$) and price ($p_i$) decisions, a consumer of type $x$ will derive a net utility of $U_i = V_i + a_i - td_i^2 - p_i$ from product $i$, where $d_i = x$ for $i = 1$ and $d_i = 1 - x$ for $i = 2$, and $x \sim uniform[0,1]$. With the earlier assumption that the market is competitive (i.e., the market is covered at equilibrium rather than being either a monopoly or localized monopolies), it is straightforward to show that the indifferent consumer $x_{in}$ is the same as found in section 2.1. Thus, firms' profit functions are the same as those found before. Thus, clearly, all our analysis and main results remain the same even if consumers' taste preferences are quadratic rather than linear.[8]

Next we check the robustness of our model to advertising cost functions. For closed-form solutions, our core model assumes the quadratic cost function commonly used in the literature. We now study more convex cost functions. Unfortunately, we cannot obtain a closed-form solution as we did before. However, our numerical analyses show that our main results and intuitions are robust to these alternative cost functions.[9] Table 2 provides the results from an example of such an alternative advertising-cost function: $g_i(a_i) = k_i a_i^3$.

**Table 2 A numerical example of alternative advertising cost functions:** $g_i(a_i) = k_i a_i^3$

| Marginal Costs | Efficient advertising | Inefficient advertising |
|---|---|---|
| $c_1 = 0.5,\ \ c_2 = 0.5$ | Case (i): $k_1 = k_2 = 0.01$<br>$a_1^* = a_2^* = 3.333$<br>$p_1^* = p_2^* = 1.500$<br>$\Pi_1^* = \Pi_2^* = 0.1296$ | Case (ii) : $k_1 = k_2 = 0.02$<br>$a_1^* = a_2^* = 2.357$<br>$p_1^* = p_2^* = 1.500$<br>$\Pi_1^* = \Pi_2^* = 0.2381$ |
| $c_1 = 0.45,\ \ c_2 = 0.5$ | Case (iii): $k_1 = k_2 = 0.01$<br>$a_1^* = 3.080,\ a_2^* = 3.569$<br>$p_1^* = 1.304,\ p_2^* = 1.646$<br>$\Pi_1^* = 0.0723,\ \Pi_2^* = 0.2023$ | Case (iv): $k_1 = k_2 = 0.02$<br>$a_1^* = 2.447,\ a_2^* = 2.263$<br>$p_1^* = 1.528,\ p_2^* = 1.422$<br>$\Pi_1^* = 0.2879,\ \Pi_2^* = 0.1932$ |
| $c_1 = 0.5,\ \ c_2 = 0.5$ | Case (v): $k_1 = 0.0095, k_2 = 0.01$<br>$a_1^* = 3.027,\ a_2^* = 3.677$<br>$p_1^* = 1.283,\ p_2^* = 1.717$<br>$\Pi_1^* = 0.0433,\ \Pi_2^* = 0.2432$ | Case (vi): $k_1 = 0.019, k_2 = 0.02$<br>$a_1^* = 2.537,\ a_2^* = 2.236$<br>$p_1^* = 1.600,\ p_2^* = 1.400$<br>$\Pi_1^* = 0.2953,\ \Pi_2^* = 0.1812$ |

As discussed before, whether advertising is efficient or inefficient does not refer to one firm's efficiency relative to the other's, but rather to both firms' efficiencies relative to some thresholds. In two sets of numerical examples we present in Table 2, both firms have the same efficiency, where $k_1 = k_2 = 0.01$ falls into the efficient-advertising parameter region and $k_1 = k_2 = 0.02$ falls into the inefficient-

[8] Of course, if we are to compute the total consumer surplus or analyze the comparative statics of the total consumer or social surplus under the two types of taste preferences, the results will be different in a quantitative way. That, however, is not the focus of this research and we have excluded such analyses in the paper.

[9] We have numerically studied both $k_i a_i^3$ and $k_i a_i^4$ functions as alternative advertising cost functions. Since both give similar results and show the robustness of our model, we present the results from only one of the two alternatives.

advertising parameter region. These numerical examples demonstrate the effects of an exogenous change in firm 1's unit production cost (keeping other things including firm 2's cost constant). Since an exogenous change in consumers' valuations has a similar effect to an exogenous change in firms' unit production costs, we will not duplicate those examples here. In all these examples, we have used the following common parameter values: $t = 1$, $V_1 = V_2 = 2$.

We observe that our main results all hold for the alternative advertising cost function. First, by comparing cases (i) and (ii), we see that, in the efficient-advertising market, when firm 1's unit production cost decreases 10% from $c_1 = 0.5$ to $c_1 = 0.45$, firm 1 reduces its advertising from 3.333 to 3.080, making a lower profit (from 0.1296 to 0.0723). In this situation, firm 2 increases its advertising from 3.333 to 3.569, making a higher profit (from 0.1296 to 0.2023). Second, by comparing cases (ii) and (iv), we see that, in the inefficient-advertising market, when firm 1's unit cost decreases from $c_1 = 0.5$ to $c_1 = 0.45$, firm 1 increases its advertising from 2.357 to 2.447 making a higher profit (from 0.2381 to 0.2879). In contrast, firm 2 reduces its advertising from 2.357 to 2.263 making a lower profit (from 0.2381 to 0.1932). Third, by comparing cases (ii) and (i), we see that for symmetric firms, when both their advertising efficiencies increase (i.e., both $k_1$ and $k_2$ change from 0.02 to 0.01), both firms become worse off. Lastly, by comparing cases (i) and (v) for the efficient-advertising market, we see that, when only firm 1 receives the seemingly favorable improvement in advertising efficiency (by 5%), its profit will actually become lower and firm 2's profit will be higher. The opposite is true for the inefficient-advertising market (comparing cases (ii) and (vi)). So, in summary, all our main propositions remain qualitatively the same when we use alternative advertising cost functions.

### 4.4. Reinterpretation of the Model

Instead of the persuasive advertising and product differentiation setting, we can reinterpret our model in other settings. For example, in many high-tech product markets, products have hardware and software components. In our model, $V_i$ can be seen as the product quality or valuation of firm *i*'s hardware, which has a marginal production cost $c_i$, whereas $a_i$ can be considered as the quality or valuation of the product's software component, which has negligible or zero marginal cost but a significant fixed cost $g_i(a_i)$, which depends on the quality level of the software. Consider smartphones for example. The smartphones' hardware typically costs around $200 per unit to manufacture, and the hardware update cycles tend to be longer than software update cycles. So, we may expect that firms make some endogenous software decisions holding the current hardware release fixed, which corresponds to our core model setting. That is, given the current hardware quality (e.g., screen size, memory size, camera resolution, etc.), firms may keep releasing new operating system updates or upgrades to improve the quality of the core software and applications. Such software quality decisions correspond to the persuasive advertising decisions ($a_i$) in our original model interpretation. Similarly, we can apply our model to services settings with $V_i$ corresponding to variable-cost component (e.g., customer service or call centers hiring employees to offer some core services) and $a_i$ corresponding to the fixed-cost component (e.g., an online self-service system, which has a negligible marginal cost when more customers use it, but improving the quality of that system requires significant R&D fixed cost investment). Our previous analyses imply, for example, that if the fixed-cost component of the product or service can increase the consumer's valuations very efficiently, then a seemingly favorable shock to the consumers' valuation or the variable cost of production for one firm's product will reduce that firm's profit and benefit its competitor.[10]

## 5. Discussion and Conclusion

We have examined the effects of shocks in consumer preferences, unit production costs, and advertising efficiency on firm marketing strategies and profits. Many exogenous factors can lead to such shocks. For example, the consumer's valuation for a company's product may be influenced by product reviews, price shocks in complements, or even actions by third parties such as non-profit organizations or governments. Similarly, a firm's production cost may change due to many exogenous factors such as shocks in labor costs, input prices, and government subsidies or tax regulations. Our analyses show several interesting findings. First, the strength of consumers' taste preferences—*t*, which also represents the degree of product differentiation—generally affects firms' advertising decisions. Firms' optimal advertising responses to an increase in the degree of product differentiation are qualitatively different: one firm will increase advertising whereas the other will reduce it. As *t* becomes larger, the firm with a relatively low variable margin tends to increase advertising whereas its competitor tends to reduce advertising. This result directly contrasts von der Fehr and Stevik (1998), which shows that, if advertising increases the consumer's willingness to pay, the degree of product differentiation will not affect firms' equilibrium advertising levels—a result of their restrictive focus on symmetric firms and outcomes.

Second, we show that the naïve intuition that the firm with an exogenous shock that increases its product valuation (or reduces its unit cost) will reduce its advertising, especially when advertising is very costly (i.e., when $k_i$ is large), may be incorrect. In a competitive market, a firm will complement an exogenous increase in its product valuation with additional advertising especially when advertising is costly and inefficient. This is because the firm's competitor will find it inefficient to increase advertising

[10] We can also add persuasive advertising decisions to the reinterpreted model, i.e., firms make both software quality decisions and persuasive advertising decisions. The main results and intuitions will remain qualitatively the same.

to compensate for the unfavorable exogenous shock leading to a higher marginal benefit for the firm's advertising.

Third, we find that even though a monopolist will benefit from an exogenous increase in consumers' valuation or a decrease in production cost, in a competitive market, such seemingly favorable shocks to one firm's product valuation or unit cost may actually hurt that firm's profitability and benefit its competitor. This result extends beyond Bertrand supertraps (Cabral and Villas-Boas 2005), which shows that seemingly favorable shocks to *all* firms may lead to lower profits for *all* firms though if the shock happens to only one firm, that firm will benefit. We show that when firms make endogenous advertising decisions in addition to pricing, a seemingly favorable exogenous shock to *one* firm alone may lead to a *lower* equilibrium profit for that firm and a *higher* equilibrium profit for its competitor. This result does not rely on the ex ante asymmetry between the firms, but rather the insight lies in firms' endogenous (fixed-cost) competitive responses in advertising.

Fourth, if the exogenous shock increases only one firm's advertising efficiency, that firm may make a lower equilibrium profit whereas its competitor's profit increases. The intuition again comes from the fact that, if advertising is efficient, the competitor will respond to the firm's "favorable" shock in efficiency with a large increase in advertising. Such a response reduces the firm's marginal benefit for advertising resulting in decreased equilibrium advertising and price, which leads to a lower firm profit whereas the competitor can take advantage of the firm's reduced incentive to advertise and hence make a higher profit.

Fifth, the larger the shock that improves a firm's valuation or unit cost and the more advertising efficient that firm is, the more likely the total advertising spending in the industry will increase. Further, the exogenous shock may reduce the price dispersion in the industry though it increases the separation between the firms' valuations or perceived qualities, and vice versa.

In addition, we have shown that our main results and intuition are robust to alternative game structure, heterogeneity in consumer responses or exposure to advertising, nonlinearity of horizontal preferences, and alternative functional forms for advertising costs. Instead of the persuasive advertising and product differentiation setting, our model can also be reinterpreted in other settings such as high-tech product markets, where products comprise hardware and software components. Our analysis implies, for example, that if the (fixed-cost) component of the product can increase the consumer's valuation very efficiently, then a seemingly favorable shock to the consumers' valuation or the hardware production cost for one firm's product will reduce that firm's profit and benefit its competitor. In short, one lesson to the firm is to "be careful what you wish for" in such a competitive landscape.

In future research, one may study the competitive advertising and pricing strategies of multi-product firms. A firm with multiple product offerings may have different strategic incentives due to intra-firm product interactions when responding to changes in the competitive landscape. Such analysis may yield new insights beyond the current paper and Cabral and Villas-Boas (2005). Our research also offers some testable hypotheses for empirical researchers. For example, we find that competitive firms have qualitatively different advertising responses to such exogenous shocks in relative valuations or costs, i.e., one firm will increase advertising whereas the other will decrease its advertising. We hope our research motivates empirical studies to systematically test our theoretical findings. In addition, it may be interesting to examine the strategic behaviors of differentiated firms in distribution channel settings.

# Appendix

PROOF OF LEMMA 1.

The competitive equilibrium outcome is obtained by solving simultaneously solving (7) and (8) for interior solutions ($a_i^* > 0$ for *i* = 1, 2). Straightforward algebraic manipulation leads to equilibrium advertising levels given by (9), from which we obtain (10) and (11). We next show the following lemma.

*Lemma A.* $18k_ik_jt - k_i - k_j > 0$ if $k_i > k_i^{(c)}$ and $k_j > k_j^{(c)}$;

$$18k_ik_jt - k_i - k_j < 0 \text{ if } \frac{1}{18t} < k_i < k_i^{(c)} \text{ and } \frac{1}{18t} < k_j < k_j^{(c)}.$$

Proof: Define a function: $Z(k_i, k_j) \equiv 18k_ik_jt - k_i - k_j$. $Z$ is an increasing function in both $k_i$ and $k_j$ since $\frac{\partial Z}{\partial k_i} = 18k_jt - 1 > 0$ and $\frac{\partial Z}{\partial k_j} = 18k_it - 1 > 0$. Note that at $k_i = k_i^{(c)}$ and $k_j = k_j^{(c)}$, $Z(k_i, k_j) = 0$. Thus, $Z(k_i, k_j) = \begin{cases} > 0, \text{ if } k_i > k_i^{(c)} \text{ and } k_j > k_j^{(c)}; \\ < 0, \text{ if } \frac{1}{18t} < k_i < k_i^{(c)} \text{ and } \frac{1}{18t} < k_j < k_j^{(c)}. \end{cases}$ QED.

With Lemma A, it is obvious that $a_i^* > 0$ and $\Pi_i^* > 0$ for *i* = 1, 2 in both the efficient and the inefficient advertising markets (the two parameter regions of interest). □

PROOF OF PROPOSITION 1.

$$\frac{\partial a_i^*}{\partial t} = \frac{3k_j[k_i - k_j - 6k_ik_j(V_i - V_j - c_i + c_j)]}{(18k_ik_jt - k_i - k_j)^2}$$

Switching the labels *i* and *j*, we can easily obtain

$$\frac{\partial a_j^*}{\partial t} = -\frac{3k_i[k_i - k_j - 6k_ik_j(V_i - V_j - c_i + c_j)]}{(18k_ik_jt - k_i - k_j)^2}.$$

Clearly, the effect of *t* on advertising is in opposite directions for the two firms.

Thus, $\frac{\partial a_i^*}{\partial t} > 0$ and $\frac{\partial a_j^*}{\partial t} < 0$ if $k_i - k_j - 6k_ik_j[k_i - k_j - 6k_ik_j(V_i - V_j - c_i + c_j)] > 0$; $\frac{\partial a_i^*}{\partial t} < 0$ and $\frac{\partial a_j^*}{\partial t} > 0$ if $k_i - k_j - 6k_ik_j[k_i - k_j - 6k_ik_j(V_i - V_j - c_i + c_j)] < 0$.

We observe that the firm with the lower potential variable-margin $V_i - c_i$ is more likely to increase advertising as *t* increases. □

PROOF OF PROPOSITION 2.

Using Lemma A, we can easily show that when $k_i > k_i^{(c)}$ and $k_j > k_j^{(c)}$,

$$\frac{\partial a_i^*}{\partial V_i} = \frac{k_j}{18k_ik_jt-k_i-k_j} > 0,\ \frac{\partial p_i^*}{\partial V_i} = \frac{6k_ik_jt}{18k_ik_jt-k_i-k_j} > 0,$$

$$\frac{\partial a_j^*}{\partial V_i} = \frac{-k_i}{18k_ik_jt-k_i-k_j} < 0,\ \frac{\partial p_j^*}{\partial V_i} = \frac{-6k_ik_jt}{18k_ik_jt-k_i-k_j} < 0.$$

$$\frac{\partial a_i^*}{\partial c_i} = \frac{-k_j}{18k_ik_jt-k_i-k_j} < 0,\ \frac{\partial p_i^*}{\partial c_i} = \frac{6k_ik_jt}{18k_ik_jt-k_i-k_j} > 0,$$

$$\frac{\partial a_j^*}{\partial c_i} = \frac{k_i}{18k_ik_jt-k_i-k_j} > 0,\ \frac{\partial p_j^*}{\partial c_i} = \frac{12k_ik_jt-k_i-k_j}{18k_ik_jt-k_i-k_j},$$ which can be negative or positive.

Proposition 2 follows immediately from the above comparative statics. Note that the proofs for the other results in Table 1 are similar and hence, for brevity, excluded from the paper. □

PROOF OF PROPOSITION 3.

$$\frac{\partial \Pi_i^*}{\partial V_i} = \frac{2k_ik_j(18k_it-1)\left[3k_j\left(3t+V_i-V_j-c_i+c_j\right)-1\right]}{3(18k_ik_jt-k_i-k_j)^2}$$

$$\frac{\partial \Pi_j^*}{\partial V_i} = \frac{-2k_ik_j\left(18k_jt-1\right)\left[3k_i\left(3t-V_i+V_j+c_i-c_j\right)-1\right]}{3(18k_ik_jt-k_i-k_j)^2}$$

When advertising is efficient (i.e., $\frac{1}{18t} < k_i < k_i^{(c)}$ for $i$ =1, 2), we obtain $\frac{\partial \Pi_i^*}{\partial V_i} < 0$ and $\frac{\partial \Pi_j^*}{\partial V_i} > 0$.

Similarly, when advertising is efficient, we obtain $\frac{\partial \Pi_i^*}{\partial c_i} > 0$ and $\frac{\partial \Pi_j^*}{\partial c_i} < 0$. □

PROOF OF PROPOSITION 4.

To prove part (b), substituting $V_1 = V_2$, $c_1 = c_2$ into equation (11), we get $\Pi_i^* = \frac{k_i(18k_it-1)\left[9k_jt-1\right]^2}{9(18k_ik_jt-k_i-k_j)^2}$ and hence $\frac{\partial \Pi_i^*}{\partial k_i} = -\frac{\left(1-9k_jt\right)^2\left(18k_ik_jt+k_i-k_j\right)}{9\left(18k_ik_jt-k_i-k_j\right)^3}$. When firms have (ex ante) the same advertising efficiencies, i.e., $k_i = k_j = k$, we get $\frac{\partial \Pi_i^*}{\partial k_i} = \frac{t}{4k(1-9kt)} > 0$ if advertising is efficient (i.e. if $\frac{1}{18t} < k < k_i^{(c)} = \frac{1}{9t}$). Similarly, we can show when firms are ex ante symmetric, $\frac{\partial \Pi_j^*}{\partial k_i} = \frac{1-18kt}{36k^2(1-9kt)} > 0$ if advertising is efficient (i.e., if $\frac{1}{18t} < k < k_i^{(c)} = \frac{1}{9t}$). Thus, when advertising is efficient, an increase in a firm's advertising efficiency (i.e., an incremental exogenous decrease in one firm's $k_i$) will reduce the firm's equilibrium profit and increase its competitor's equilibrium profit.

To prove part (a), we substitute $V_1 = V_2$, $c_1 = c_2$, and $k_1 = k_2 = k$ into (11) and obtain $\Pi_i^* = \frac{18kt-1}{36k}$. Note that $\frac{\partial \Pi_i^*}{\partial k} = \frac{1}{36k^2} > 0$. That is, when the two (ex ante symmetric) firms' advertising efficiencies improve by the same amount (i.e., $k_1$ decreases by the same amount), both firms will become worse off. □

PROOF OF PROPOSITION 5.

Note that $\frac{\partial a_i^*}{\partial(V_i - V_j)} = \frac{\partial a_i^*}{\partial(c_i - c_j)} = \frac{\partial a_i^*}{\partial c_i} = \frac{\partial a_i^*}{\partial V_i}$, so we will use a relative change in product valuation or unit production cost and an absolute change. For example, if exogenous factors increase both firms' valuations (or unit costs) by the same amount, neither firm will change its advertising; mathematically, $\frac{\partial a_i^*}{\partial V_i} = \frac{\partial a_i^*}{\partial(V_i - V_j)}$. With equation (12), firm *i*'s equilibrium advertising expenditure is given by

$$g_i(a_i^*) = k_i a_i^{*2} = \frac{k_i\left[3k_j\left(3t + V_i - V_j - c_i + c_j\right) - 1\right]^2}{9\left(18k_ik_jt - k_i - k_j\right)^2}.$$

$$\frac{\partial g_i}{\partial(V_i - V_j)} = \frac{2k_ik_j\left[3k_j\left(3t + V_i - V_j - c_i + c_j\right) - 1\right]}{3\left(18k_ik_jt - k_i - k_j\right)^2}$$

$$\frac{\partial g_j}{\partial(V_i - V_j)} = \frac{-2k_ik_j\left[3k_i\left(3t + V_j - V_i - c_j + c_i\right) - 1\right]}{3\left(18k_ik_jt - k_i - k_j\right)^2}$$

The total industry advertising expenditure is $T = g_i(a_i^*) + g_j\left(a_j^*\right)$.

Thus, $\frac{\partial T}{\partial(V_i - V_j)} = \frac{\partial g_i}{\partial(V_i - V_j)} + \frac{\partial g_j}{\partial(V_i - V_j)}$

$$= \frac{2k_ik_j\left[3k_j\left(3t + V_i - V_j - c_i + c_j\right) - 1\right]}{3\left(18k_ik_jt - k_i - k_j\right)^2} + \frac{-2k_ik_j\left[3k_i\left(3t + V_j - V_i - c_j + c_i\right) - 1\right]}{3\left(18k_ik_jt - k_i - k_j\right)^2}$$

$$= \frac{2k_ik_j\left[3t(k_j - k_i) + (V_i - V_j - c_i + c_j)(k_i + k_j)\right]}{\left(18k_ik_jt - k_i - k_j\right)^2}.$$

Thus, $\frac{\partial T}{\partial(V_i - V_j)} > 0$ if $V_i - V_j + c_j - c_i > \frac{3t(k_i - k_j)}{k_i + k_j}$, a condition that is more likely to hold for a larger exogenous shock and a smaller $k_i$. That is, the larger the exogenous shock—which increases firm *i*'s relative valuation ($V_i - V_j$) or reduces its relative unit production cost ($c_i - c_j$)—and the more advertising-efficient firm *i* is, the more likely the industry advertising expenditure will increase. □

PROOF OF PROPOSITION 6.

Note that it is implicitly assumed that the exogenous shock is incremental and not drastic (i.e., it will not create any crossover in price or product valuations). Without loss of generality, let firm *i* be the firm that has an exogenous increase in its product valuation relative to its competitor (*j*). From equation (10), it is straightforward to show that $\frac{\partial(p_i^* - p_j^*)}{\partial(V_i - V_j)} = \frac{6k_ik_jt}{18k_ik_jt - k_i - k_j}$. We examine two cases.

Case 1: $V_i > V_j$

Since firm *i* received a positive shock in valuation relative to firm *j*, the separation between consumers' valuations between firms' products has *increased*. If $p_i^* > p_j^*$, $\frac{\partial|p_i^* - p_j^*|}{\partial|V_i - V_j|} = \frac{\partial(p_i^* - p_j^*)}{\partial(V_i - V_j)} = \frac{6k_ik_jt}{18k_ik_jt - k_i - k_j} < 0$ for an *efficient*-advertising market (with $\frac{1}{18t} < k_i < k_i^{(c)}$ and $\frac{1}{18t} < k_j < k_j^{(c)}$), i.e., the price difference between the products will *decrease* as a result of the shock. Similarly, if $p_i^* \le p_j^*$, $\frac{\partial |p_i^* - p_j^*|}{\partial |V_i - V_j|} = \frac{\partial (p_j^* - p_i^*)}{\partial (V_i - V_j)} = \frac{-6k_ik_jt}{18k_ik_jt - k_i - k_j} < 0$ for an *inefficient*-advertising market (with $k_i > k_i^{(c)}$ and $k_j > k_j^{(c)}$), i.e., the price difference between the products will *decrease* as a result of the shock.

Case 2: $V_i \le V_j$

Since firm *i* received a positive shock in valuation relative to firm *j*, the separation between consumers' valuations between firms' products has *decreased*. If $p_i^* > p_j^*$ , $\frac{\partial |p_i^* - p_j^*|}{\partial |V_i - V_j|} = \frac{\partial (p_i^* - p_j^*)}{\partial (V_j - V_i)} = \frac{-6k_ik_jt}{18k_ik_jt - k_i - k_j} < 0$ for an *efficient*-advertising market (with $\frac{1}{18t} < k_i < k_i^{(c)}$ and $\frac{1}{18t} < k_j < k_j^{(c)}$), i.e., the price difference between the products will *increase* as a result of the shock. Similarly, if $p_i^* \le p_j^*$, $\frac{\partial |p_i^* - p_j^*|}{\partial |V_i - V_j|} = \frac{\partial (p_j^* - p_i^*)}{\partial (V_j - V_i)} = \frac{6k_ik_jt}{18k_ik_jt - k_i - k_j} < 0$ for an *inefficient*-advertising market (with $k_i > k_i^{(c)}$ and $k_j > k_j^{(c)}$), i.e., the price difference between the products will *increase* as a result of the shock.

In conclusion, we have shown that under the conditions discussed above, an exogenous shock can *reduce* (increase) the equilibrium price difference between firms' products even though it *increases* (reduces) the separation between firms' valuations. □

PROOF OF PROPOSITION 7.

$$\frac{\partial}{\partial t}\left(\frac{\partial a_i^*}{\partial \beta_i}\right) = \frac{\partial}{\partial t}\left(\frac{k_jV_i}{18k_ik_jt - k_i - k_j}\right) = \frac{-18k_ik_j{}^2V_i}{(18k_ik_jt - k_i - k_j)^2} < 0.$$

$$\frac{\partial}{\partial t}\left(\frac{\partial a_i^*}{\partial \beta_j}\right) = \frac{\partial}{\partial k_j}\left(\frac{-k_jV_j}{18k_ik_jt - k_i - k_j}\right) = \frac{18k_ik_j{}^2V_j}{(18k_ik_jt - k_i - k_j)^2} > 0.$$

From Table 1, we know

$$\frac{\partial a_i^*}{\partial \beta_i} = \begin{cases} > 0, \text{ if } k_i > k_i^{(c)} \text{ and } k_j > k_j^{(c)}; \\ < 0, \text{ if } \frac{1}{18t} < k_i < k_i^{(c)} \text{ and } \frac{1}{18t} < k_j < k_j^{(c)}; \end{cases}$$

$$\frac{\partial a_i^*}{\partial \beta_j} = \begin{cases} > 0, \text{ if } \frac{1}{18t} < k_i < k_i^{(c)} \text{and } \frac{1}{18t} < k_j < k_j^{(c)}; \\ < 0, \text{ if } k_i > k_i^{(c)} \text{and } k_j > k_j^{(c)}; \end{cases}$$

Thus, if $\frac{1}{18t} < k_i < k_i^{(c)}$ and $\frac{1}{18t} < k_j < k_j^{(c)}$, then $\frac{\partial}{\partial t}\left(\left|\frac{\partial a_i^*}{\partial \beta_i}\right|\right) > 0$ and $\frac{\partial}{\partial t}\left(\left|\frac{\partial a_i^*}{\partial \beta_j}\right|\right) > 0$; if $k_i > k_i^{(c)}$ and $k_j > k_j^{(c)}$, then $\frac{\partial}{\partial t}\left(\left|\frac{\partial a_i^*}{\partial \beta_i}\right|\right) < 0$ and $\frac{\partial}{\partial t}\left(\left|\frac{\partial a_i^*}{\partial \beta_j}\right|\right) < 0$. That is, as *t* increases, the effect of consumer reviews on firms' advertising levels becomes stronger (weaker) if advertising is efficient (inefficient). □

## References

Adams, W. J., J. Yellen. 1977. What makes advertising profitable? *The Economic Journal* **87**(347) 427-449.

Amaldoss W., C. He. 2010. Product variety, informative advertising and price competition. *Journal of Marketing Research* **47** (1) 146-156.

Bagwell, K. 2007. The economic analysis of advertising. M. Armstrong, R. Porter, eds. *Handbook of Industrial Organization*, Vol. 3, Chapter 28. North-Holland, Amsterdam.

Bloch, F., D. Manceau. 1999. Persuasive advertising in Hotelling's model of product differentiation. *International Journal of Industrial Organization* **17**(4) 557-574.

Cabral, L., M. Villas-Boas. 2005. Bertrand supertraps. *Management Science* **51**(4) 599–613.

D'Aspremont, C., J. J. Gabszewicz, J.-F. Thisse. 1979. On Hotelling's "Stability in competition." *Econometrica* **47**(5) 1145–1150.

eMarketer. "What are UK Women Doing Online?" *The eMarketer Daily Newsletter* 7 April 2009.

Grossman, G., C. Shapiro. 1984. Informative advertising with differentiated products. *Review of Economic Studies* **51**(1) 63 – 81.

Hermalin, B. 1993. Notes on Microeconomics. Mimeo, University of California, Berkeley, CA.

Iyer, G., D. Soberman, and J.M. Villas-Boas. 2005. The targeting of advertising. *Marketing Science* **24**(3) 461-476.

Tirole, J. 1988. *The Theory of Industrial Organization*. The MIT Press, Cambridge, MA.

von der Fehr, N-H., K. Stevik. 1998. Persuasive advertising and product differentiation. *Southern Economic Journal* **65**(1) 113-126.