

# Pricing and Resource Allocation via Game Theory for a Small-Cell Video Caching System

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**Abstract**—Evidence indicates that downloading on-demand videos accounts for a dramatic increase in data traffic over cellular networks. Caching popular videos in the storage of small-cell base stations (SBS), namely, small-cell caching, is an efficient technology for reducing the transmission latency while mitigating the redundant transmissions of popular videos over back-haul channels. In this paper, we consider a commercialized small-cell caching system consisting of a network service provider (NSP), several video retailers (VRs), and mobile users (MUs). The NSP leases its SBSs to the VRs for the purpose of making profits, and the VRs, after storing popular videos in the rented SBSs, can provide faster local video transmissions to the MUs, thereby gaining more profits. We conceive this system within the framework of Stackelberg game by treating the SBSs as specific types of resources. We first model the MUs and SBSs as two independent Poisson point processes, and develop, via stochastic geometry theory, the probability of the specific event that an MU obtains the video of its choice directly from the memory of an SBS. Then, based on the probability derived, we formulate a Stackelberg game to jointly maximize the average profit of both the NSP and the VRs. In addition, we investigate the Stackelberg equilibrium by solving a non-convex optimization problem. With the aid of this game theoretic framework, we shed light on the relationship between four important factors: the optimal pricing of leasing an SBS, the SBSs allocation among the VRs, the storage size of the SBSs, and the popularity distribution of the VRs. Monte Carlo simulations show that our stochastic geometry-based analytical results closely match the empirical ones. Numerical results are also provided for quantifying the proposed game-theoretic framework by showing its efficiency on pricing and resource allocation.

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**Index Terms**—Small-cell caching, cellular networks, stochastic geometry, Stackelberg game.

## I. INTRODUCTION

WIRELESS data traffic is expected to increase exponentially in the next few years driven by a staggering proliferation of mobile users (MU) and their bandwidth-hungry mobile applications. There is evidence that streaming of on-demand videos by the MUs is the major reason for boosting the tele-traffic over cellular networks [1]. According to the prediction of mobile data traffic by Cisco, mobile video streaming will account for 72% of the overall mobile data traffic by 2019. The on-demand video downloading involves repeated wireless transmission of videos that are requested multiple times by different users in a completely asynchronous manner, which is different from the transmission style of live video streaming.

Often, there are numerous repetitive requests of popular videos from the MUs, such as online blockbusters, leading to redundant video transmissions. The redundancy of data transmissions can be reduced by locally storing popular videos, known as caching, into the storage of intermediate network nodes, effectively forming a local caching system [1], [2]. The local caching brings video content closer to the MUs and alleviates redundant data transmissions via redirecting the downloading requests to the intermediate nodes.

Generally, wireless data caching consists of two stages: data placement and data delivery [3]. In the data placement stage, popular videos are cached into local storages during off-peak periods, while during the data delivery stage, videos requested are delivered from the local caching system to the MUs. Recent works advanced the caching solutions of both device-to-device (D2D) networks and wireless sensor networks [4]–[6]. Specifically, in [4] a caching scheme was proposed for a D2D based cellular network relaying on the MUs' caching of popular video content. In this scheme, the D2D cluster size was optimized for reducing the downloading delay. In [5] and [6], the authors proposed novel caching schemes for wireless sensor networks, where the protocol model of [7] was adopted.

Since small-cell embedded architectures will dominate in future cellular networks, known as heterogeneous networks (HetNet) [8]–[13], caching relying on small-cell base stations (SBS), namely, small-cell caching, constitutes a promising solution for HetNets. The advantages brought about by small-cell caching are threefold. Firstly, popular videos are placed closer to the MUs when they are cached in SBSs, hence

78 reducing the transmission latency. Secondly, redundant trans-  
 79 missions over SBSs' back-haul channels, which are usually  
 80 expensive [14], can be mitigated. Thirdly, the majority of video  
 81 traffic is offloaded from macro-cell base stations to SBSs.

82 In [15], a small-cell caching scheme, named  
 83 'Femtocaching', is proposed for a cellular network having  
 84 embedded SBSs, where the data placement at the SBSs is  
 85 optimized in a centralized manner for the sake of reducing  
 86 the transmission delay imposed. However, [15] considers an  
 87 idealized system, where neither the interference nor the impact  
 88 of wireless channels is taken into account. The associations  
 89 between the MUs and the SBSs are pre-determined without  
 90 considering the specific channel conditions encountered.  
 91 In [16], small-cell caching is investigated in the context of  
 92 stochastic networks. The average performance is quantified  
 93 with the aid of stochastic geometry [17], [18], where the  
 94 distribution of network nodes is modeled by Poisson point  
 95 process (PPP). However, the caching strategy of [16] assumes  
 96 that the SBSs cache the same content, hence leading to a  
 97 sub-optimal solution.

98 As detailed above, current research on wireless caching  
 99 mainly considers the data placement issue optimized for reduc-  
 100 ing the downloading delay. However, the entire caching system  
 101 design involves numerous issues apart from data placement.  
 102 From a commercial perspective, it will be more interesting  
 103 to consider the topics of pricing for video streaming, the  
 104 rental of local storage, and so on. A commercialized caching  
 105 system may consist of video retailers (VR), network service  
 106 providers (NSP) and MUs. The VRs, e.g., Youtube, purchase  
 107 copyrights from video producers and publish the videos on  
 108 their web-sites. The NSPs are typically operators of cellular  
 109 networks, who are in charge of network facilities, such as  
 110 macro-cell base stations and SBSs.

111 In such a commercial small-cell caching system, the VRs'  
 112 revenue is acquired from providing video streaming for  
 113 the MUs. As the central servers of the VRs, which store  
 114 the popular videos, are usually located in the backbone net-  
 115 works and far away from the MUs, an efficient solution is  
 116 to locally cache these videos, thereby gaining more profits  
 117 from providing faster local transmissions. In turn, these local  
 118 caching demands raised by the VRs offer the NSPs profit-  
 119 able opportunities from leasing their SBSs. Additionally, the  
 120 NSPs can save considerable costs due to reduced redundant  
 121 video transmissions over SBSs' back-haul channels. In this  
 122 sense, both the VRs and NSPs are the beneficiaries of the  
 123 local caching system. However, each entity is selfish and  
 124 wishes to maximize its own benefit, raising a competition  
 125 and optimization problem among these entities, which can be  
 126 effectively solved within the framework of game theory.

127 We note that game theory has been successfully applied  
 128 to wireless communications for solving resource allocation  
 129 problems. In [19], the authors propose a dynamic spectrum  
 130 leasing mechanism via power control games. In [20],  
 131 a price-based power allocation scheme is proposed for spec-  
 132 trum sharing in Femto-cell networks based on Stackelberg  
 133 game. Game theoretical power control strategies for maxi-  
 134 mizing the utility in spectrum sharing networks are studied  
 135 in [21] and [22].

In this paper, we propose a commercial small-cell caching  
 system consisting of an NSP, multiple VRs and MUs. We opti-  
 mize such a system within the framework of Stackelberg game  
 by viewing the SBSs as a specific type of resources for the  
 purpose of video caching. Generally speaking, Stackelberg  
 game is a strategic game that consists of a leader and several  
 followers competing with each other for certain resources [23].  
 The leader moves first and the followers move subsequently.  
 Correspondingly, in our game theoretic caching system, we  
 consider the NSP to be the leader and the VRs as the followers.  
 The NSP sets the price of leasing an SBS, while the VRs  
 compete with each other for renting a fraction of the SBSs.

To the best of the authors' knowledge, our work is the first  
 of its kind that optimizes a caching system with the aid of  
 game theory. Compared to many other game theory based  
 resource allocation schemes, where the power, bandwidth  
 and time slots are treated as the resources, our work has  
 a totally different profit model, established based on our  
 coverage derivations. In particular, our contributions are as  
 follows.

- 1) By following the stochastic geometry framework  
 of [17] and [18], we model the MUs and SBSs in  
 the network as two different ties of a Poisson point  
 process (PPP) [24]. Under this network model, we define  
 the concept of a successful video downloading event  
 when an MU obtains the requested video directly from  
 the storage of an SBS. Then we quantify the probability  
 of this event based on stochastic geometry theory.
- 2) Based on the probability derived, we develop a profit  
 model of our caching system and formulate the profits  
 gained by the NSP and the VRs from SBSs leasing and  
 renting.
- 3) A Stackelberg game is proposed for jointly maximizing  
 the average profit of the NSP and the VRs. Given this  
 game theoretic framework, we investigate a non-uniform  
 pricing scheme, where the price charged to different VRs  
 varies.
- 4) Then we investigate the Stackelberg equilibrium of this  
 scheme via solving a non-convex optimization problem.  
 It is interesting to observe that the optimal solution is  
 related both to the storage size of each SBS and to the  
 popularity distribution of the VRs.
- 5) Furthermore, we consider an uniform pricing scheme.  
 We find that although the uniform pricing scheme is  
 inferior to the non-uniform one in terms of maximizing  
 the NSP's profit, it is capable of reducing more back-  
 haul costs compared with the latter and achieves the  
 maximum sum profit of the NSP and the VRs.

The rest of this paper is organized as follows. We describe  
 the system model in Section II and establish the related profit  
 model in Section III. We then formulate Stackelberg game for  
 our small-cell caching system in Section IV. In Section V,  
 we investigate Stackelberg equilibrium for the non-uniform  
 pricing scheme by solving a non-convex optimization prob-  
 lem, while in Section VI, we further consider the uniform  
 pricing scheme. Our simulations and numerical results are  
 detailed in Section VII, while our conclusions are provided  
 in Section VIII.

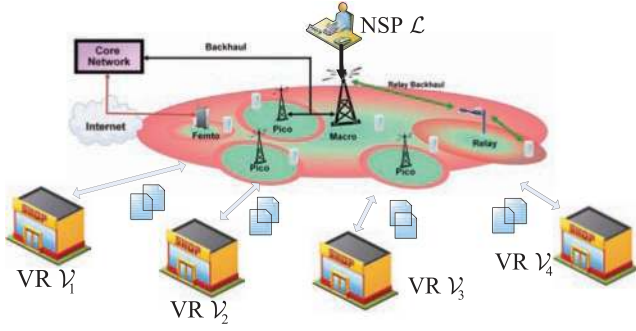


Fig. 1. An example of the small-cell caching system with four VRs.

## II. SYSTEM MODEL

We consider a commercial small-cell caching system consisting of an NSP,  $V$  VRs, and a number of MUs. Let us denote by  $\mathcal{L}$  the NSP, by  $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_V\}$  the set of the VRs, and by  $\mathcal{M}$  one of the MUs. Fig. 1 shows an example of our caching system relying on four VRs. In such a system, the VRs wish to rent the SBSs from  $\mathcal{L}$  for placing their videos. Both the NSP and each VR aim for maximizing their profits.

There are three stages in our system. In the first stage, the VRs purchase the copyrights of popular videos from video producers and publish them on their web-sites. In the second stage, the VRs negotiate with the NSP on the rent of SBSs for caching these popular videos. In the third stage, the MUs connect to the SBSs for downloading the desired videos. We will particularly focus our attention on the second and third stages within this game theoretic framework.

### A. Network Model

Let us consider a small-cell based caching network composed of the MUs and the SBSs owned by  $\mathcal{L}$ , where each SBS is deployed with a fixed transmit power  $P$  and the storage of  $Q$  video files. Let us assume that the SBSs transmit over the channels that are orthogonal to those of the macro-cell base stations, and thus there is no interference incurred by the macro-cell base stations. Also, assume that these SBSs are spatially distributed according to a homogeneous PPP (HPPP)  $\Phi$  of intensity  $\lambda$ . Here, the intensity  $\lambda$  represents the number of the SBSs per unit area. Furthermore, we model the distribution of the MUs as an independent HPPP  $\Psi$  of intensity  $\zeta$ .

The wireless down-link channels spanning from the SBSs to the MUs are independent and identically distributed (*i.i.d.*), and modeled as the combination of path-loss and Rayleigh fading. Without loss of generality, we carry out our analysis for a typical MU located at the origin. The path-loss between an SBS located at  $x$  and the typical MU is denoted by  $\|x\|^{-\alpha}$ , where  $\alpha$  is the path-loss exponent. The channel power of the Rayleigh fading between them is denoted by  $h_x$ , where  $h_x \sim \exp(1)$ . The noise at an MU is Gaussian distributed with a variance  $\sigma^2$ .

We consider the steady-state of a saturated network, where all the SBSs keep on transmitting data in the entire frequency band allocated. This modeling approach for saturated networks characterizes the worst-case scenario of the real systems, which has been adopted by numerous studies on PPP analysis,

such as [18]. Hence, the received signal-to-interference-plus-noise ratio (SINR) at the typical MU from an SBS located at  $x$  can be expressed as

$$\rho(x) = \frac{Ph_x\|x\|^{-\alpha}}{\sum_{x' \in \Phi \setminus x} Ph_{x'}\|x'\|^{-\alpha} + \sigma^2}. \quad (1)$$

The typical MU is considered to be “covered” by an SBS located at  $x$  as long as  $\rho(x)$  is no lower than a pre-set SINR threshold  $\delta$ , i.e.,

$$\rho(x) \geq \delta. \quad (2)$$

Generally, an MU can be covered by multiple SBSs. Note that the SINR threshold  $\delta$  defines the highest delay of downloading a video file. Since the quality and code rate of a video clip have been specified within the video file, the download delay will be the major factor predetermining the QoS perceived by the mobile users. Therefore, we focus our attention on the coverage and SINR in the following derivations.

### B. Popularity and Preferences

We now model the popularity distribution, i.e., the distribution of request probabilities, among the popular videos to be cached. Let us denote by  $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_N\}$  the file set consisting of  $N$  video files, where each video file contains an individual movie or video clip that is frequently requested by MUs. The popularity distribution of  $\mathcal{F}$  is represented by a vector  $\mathbf{t} = [t_1, t_2, \dots, t_N]$ . That is, the MUs make independent requests of the  $n$ -th video  $\mathcal{F}_n$ ,  $n = 1, \dots, N$ , with the probability of  $t_n$ . Generally,  $\mathbf{t}$  can be modeled by the Zipf distribution [25] as

$$t_n = \frac{1/n^\beta}{\sum_{j=1}^N 1/j^\beta}, \quad \forall n, \quad (3)$$

where the exponent  $\beta$  is a positive value, characterizing the video popularity. A higher  $\beta$  corresponds to a higher content reuse, where the most popular files account for the majority of download requests. From Eq. (3), the file with a smaller  $n$  corresponds to a higher popularity.

Note that each SBS can cache at most  $Q$  video files, and usually  $Q$  is no higher than the number of videos in  $\mathcal{F}$ , i.e., we have  $Q \leq N$ . Without loss of generality, we assume that  $N/Q$  is an integer. The  $N$  files in  $\mathcal{F}$  are divided into  $F = N/Q$  file groups (FG), with each FG containing  $Q$  video files. The  $n$ -th video,  $\forall n \in \{(f-1)Q + 1, \dots, fQ\}$ , is included in the  $f$ -th FG,  $f = 1, \dots, F$ . Denote by  $\mathcal{G}_f$  the  $f$ -th FG, and by  $p_f$  the probability of the MUs’ requesting a file in  $\mathcal{G}_f$ , and we have

$$p_f = \sum_{n=(f-1)Q+1}^{fQ} t_n, \quad \forall f. \quad (4)$$

File caching is then carried out on the basis of FGs, where each SBS caches one of the  $F$  FGs.

At the same time, the MUs have unbalanced preferences with regard to the  $V$  VRs, i.e., some VRs are more popular than others. For example, the majority of the MUs may tend to access Youtube for video streaming. The preference distribution among the VRs is denoted by  $\mathbf{q} = [q_1, q_2, \dots, q_V]$ ,

where  $q_v$ ,  $v = 1, \dots, V$ , represents the probability that the MUs prefer to download videos from  $\mathcal{V}_v$ . The preference distribution  $\mathbf{q}$  can also be modeled by the Zipf distribution. Hence, we have

$$q_v = \frac{1/v^\gamma}{\sum_{j=1}^V 1/j^\gamma}, \quad \forall v, \quad (5)$$

where  $\gamma$  is a positive value, characterizing the preference of the VRs. A higher  $\gamma$  corresponds to a higher probability of accessing the most popular VRs.

### C. Video Placement and Download

Next, we introduce the small-cell caching system with its detailed parameters. In the first stage, each VR purchases the  $N$  popular videos in  $\mathcal{F}$  from the producers and publishes these videos on its web-site. In the second stage, upon obtaining these videos, the VRs negotiate with the NSP  $\mathcal{L}$  for renting its SBSs. As  $\mathcal{L}$  leases its SBSs to multiple VRs, we denote by  $\boldsymbol{\tau} = [\tau_1, \tau_2, \dots, \tau_V]$  the fraction vector, where  $\tau_v$  represents the fraction of the SBSs that are assigned to  $\mathcal{V}_v$ ,  $\forall v$ . We assume that the SBSs rented by each VR are uniformly distributed. Hence, the SBSs that are allocated to  $\mathcal{V}_v$  can be modeled as a ‘‘thinned’’ HPPP  $\Phi_v$  with intensity  $\tau_v \lambda$ .

The data placements of the second stage commence during network off-peak time after the VRs obtain access to the SBSs. During the placements, each SBS will be allocated with one of the  $F$  FGs. Generally, we assume that the VRs do not have the *a priori* information regarding the popularity distribution of  $\mathcal{F}$ . This is because the popularity of videos is changing periodically, and can only be obtained statistically after these videos quit the market. It is clear that each VR may have more or less some statistical information on the popularity distribution of videos based on the MUs’ downloading history. However, this information will be biased due to limited sampling. In this case, the VRs will uniformly assign the  $F$  FGs to the SBSs with equal probability of  $\frac{1}{F}$  for simplicity. We are interested in investigating the uniform assignment of video files for drawing a bottom line of the system performance. As the FGs are randomly assigned, the SBSs in  $\Phi_v$  that cache the FG  $\mathcal{G}_f$  can be further modeled as a ‘‘more thinned’’ HPPP  $\Phi_{v,f}$  with an intensity of  $\frac{1}{F} \tau_v \lambda$ .

In the third stage, the MUs start to download videos. When an MU  $\mathcal{M}$  requires a video of  $\mathcal{G}_f$  from  $\mathcal{V}_v$ , it searches the SBSs in  $\Phi_{v,f}$  and tries to connect to the nearest SBS that covers  $\mathcal{M}$ . Provided that such an SBS exists, the MU  $\mathcal{M}$  will obtain this video directly from this SBS, and we thereby define this event by  $\mathcal{E}_{v,f}$ . By contrast, if such an SBS does not exist,  $\mathcal{M}$  will be redirected to the central servers of  $\mathcal{V}_v$  for downloading the requested file. Since the servers of  $\mathcal{V}_v$  are located at the backbone network, this redirection of the demand will trigger a transmission via the back-haul channels of the NSP  $\mathcal{L}$ , hence leading to an extra cost.

## III. PROFIT MODELING

We now focus on modeling the profit of the NSP and the VRs obtained from the small-cell caching system. The average profit is developed based on stochastically geometrical

distributions of the network nodes in terms of per unit area times unit period ( $/UAP$ ), e.g.,  $/month \cdot km^2$ .

### A. Average Profit of the NSP

For the NSP  $\mathcal{L}$ , the revenue gained from the caching system consists of two parts: 1) the income gleaned from leasing SBSs to the VRs and 2) the cost reduction due to reduced usage of the SBSs’ back-haul channels. First, the leasing income/ $UAP$  of  $\mathcal{L}$  can be calculated as

$$S^{RT} = \sum_{j=1}^V \tau_j \lambda s_j, \quad (6)$$

where  $s_j$  is the price per unit period charged to  $\mathcal{V}_j$  for renting an SBS. Then we formulate the saved cost/ $UAP$  due to reduced back-haul channel transmissions. When an MU demands a video in  $\mathcal{G}_f$  from  $\mathcal{V}_v$ , we derive the probability  $\Pr(\mathcal{E}_{v,f})$  as follows.

*Theorem 1:* The probability of the event  $\mathcal{E}_{v,f}$ ,  $\forall v, f$ , can be expressed as

$$\Pr(\mathcal{E}_{v,f}) = \frac{\tau_v}{C(\delta, \alpha)(F - \tau_v) + A(\delta, \alpha)\tau_v + \tau_v}, \quad (7)$$

where we have  $A(\delta, \alpha) \triangleq \frac{2\delta}{\alpha-2} {}_2F_1(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta)$  and  $C(\delta, \alpha) \triangleq \frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B(\frac{2}{\alpha}, 1 - \frac{2}{\alpha})$ . Furthermore,  ${}_2F_1(\cdot)$  in the function  $A(\delta, \alpha)$  is the hypergeometric function, while the Beta function in  $C(\delta, \alpha)$  is formulated as  $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$ .

*Proof:* Please refer to Appendix A. ■

*Remark 1:* From *Theorem 1*, it is interesting to observe that the probability  $\Pr(\mathcal{E}_{v,f})$  is independent of both the transmit power  $P$  and the intensity  $\lambda$  of the SBSs. Furthermore, since  $Q$  is inversely proportional to  $F$ , we can enhance  $\Pr(\mathcal{E}_{v,f})$  by increasing the storage size  $Q$ .

We assume that there are on average  $K$  video requests from each MU within unit period, and that the average back-haul cost for a video transmission is  $s^{bh}$ . Based on  $\Pr(\mathcal{E}_{v,f})$  in Eq. (7), we obtain the cost reduction/ $UAP$  for the back-haul channels of  $\mathcal{L}$  as

$$S^{BH} = \sum_{j_1=1}^F \sum_{j_2=1}^V p_{j_1} q_{j_2} \zeta K \Pr(\mathcal{E}_{j_2, j_1}) s^{bh}. \quad (8)$$

By combining the above two items, the overall profit/ $UAP$  for  $\mathcal{L}$  can be expressed as

$$S^{NSP} = S^{RT} + S^{BH}. \quad (9)$$

### B. Average Profit of the VRs

Note that the MUs can download the videos either from the memories of the SBSs directly or from the servers of the VRs at backbone networks via back-haul channels. In the first case, the MUs will be levied by the VRs an extra amount of money in addition to the videos’ prices because of the higher-rate local streaming, namely, local downloading surcharge (LDS). We assume that the LDS of each video is set as  $s^{ld}$ . Then the revenue/ $UAP$  for a VR  $\mathcal{V}_v$  gained from the LDS can be calculated as

$$S_v^{LD} = \sum_{j=1}^F p_j q_v \zeta K \Pr(\mathcal{E}_{v,j}) s^{ld}. \quad (10)$$

387 Additionally,  $\mathcal{V}_v$  pays for renting the SBSs from  $\mathcal{L}$ . The related  
388 cost/*UAP* can be written as

$$389 \quad S_v^{RT} = \tau_v \lambda s_v. \quad (11)$$

390 Upon combining the two items, the profit/*UAP* for  $\mathcal{V}_v$ ,  $\forall v$ ,  
391 can be expressed as

$$392 \quad S_v^{VR} = S_v^{LD} - S_v^{RT}. \quad (12)$$

#### 393 IV. PROBLEM FORMULATION

394 In this section, we first present the Stackelberg game for-  
395 mulation for our price-based SBS allocation scheme. Then the  
396 equilibrium of the proposed game is investigated.

##### 397 A. Stackelberg Game Formulation

398 Again, Stackelberg game is a strategic game that consists of  
399 a leader and several followers competing with each other for  
400 certain resources [23]. The leader moves first and the followers  
401 move subsequently. In our small-cell caching system, we  
402 model the NSP  $\mathcal{L}$  as the leader, and the  $V$  VRs as the followers.  
403 The NSP imposes a price vector  $\mathbf{s} = [s_1, s_2, \dots, s_V]$  for  
404 the lease of its SBSs, where  $s_v$ ,  $\forall v$ , has been defined in the  
405 previous section as the price per unit period charged on  $\mathcal{V}_v$   
406 for renting an SBS. After the price vector  $\mathbf{s}$  is set, the VRs  
407 update the fraction  $\tau_v$ ,  $\forall v$ , that they tend to rent from  $\mathcal{L}$ .

408 1) *Optimization Formulation of the Leader*: Observe from  
409 the above game model that the NSP's objective is to maximize  
410 its profit  $S^{NSP}$  formulated in Eq. (9). Note that for  $\forall v$ , the  
411 fraction  $\tau_v$  is a function of the price  $s_v$  under the Stackelberg  
412 game formulation. This means that the fraction of the SBSs  
413 that each VR is willing to rent depends on the specific price  
414 charged to them for renting an SBS. Consequently, the NSP  
415 has to find the optimal price vector  $\mathbf{s}$  for maximizing its profit.  
416 This optimization problem can be summarized as follows.

417 *Problem 1*: The optimization problem of maximizing  $\mathcal{L}$ 's  
418 profit can be formulated as

$$419 \quad \begin{aligned} & \max_{\mathbf{s} \geq \mathbf{0}} S^{NSP}(\mathbf{s}, \boldsymbol{\tau}), \\ & \text{s.t.} \quad \sum_{j=1}^V \tau_j \leq 1. \end{aligned} \quad (13)$$

421 2) *Optimization Formulation of the Followers*: The profit  
422 gained by the VR  $\mathcal{V}_v$  in Eq. (12) can be further written as

$$423 \quad \begin{aligned} S_v^{VR}(\tau_v, s_v) &= \sum_{j=1}^F p_j q_v \zeta K \Pr(\mathcal{E}_{v,j}) s^{ld} - \tau_v \lambda s_v \\ 424 &= \sum_{j=1}^F \frac{p_j q_v \zeta K s^{ld} \tau_v}{(A(\delta, \alpha) - C(\delta, \alpha) + 1) \tau_v + C(\delta, \alpha) F} \\ 425 &\quad - \lambda s_v \tau_v. \end{aligned} \quad (14)$$

426 We can see from Eq. (14) that once the price  $s_v$  is fixed, the  
427 profit of  $\mathcal{V}_v$  depends on  $\tau_v$ , i.e., the fraction of SBSs that  
428 are rented by  $\mathcal{V}_v$ . If  $\mathcal{V}_v$  increases the fraction  $\tau_v$ , it will gain  
429 more revenue by levying surcharges from more MUs, while  
430 at the same time,  $\mathcal{V}_v$  will have to pay for renting more SBSs.

Therefore,  $\tau_v$  has to be optimized for maximizing the profit  
of  $\mathcal{V}_v$ . This optimization can be formulated as follows.

*Problem 2*: The optimization problem of maximizing  $\mathcal{V}_v$ 's  
profit can be written as

$$435 \quad \max_{\tau_v \geq 0} S_v^{VR}(\tau_v, s_v). \quad (15)$$

*Problem 1* and *Problem 2* together form a Stackelberg  
game. The objective of this game is to find the Stackelberg  
Equilibrium (SE) points from which neither the leader (NSP)  
nor the followers (VRs) have incentives to deviate. In the  
following, we investigate the SE points for the proposed game.

##### 441 B. Stackelberg Equilibrium

For our Stackelberg game, the SE is defined as follows.

*Definition 1*: Let  $\mathbf{s}^* \triangleq [s_1^*, s_2^*, \dots, s_V^*]$  be a solution for  
*Problem 1*, and  $\boldsymbol{\tau}_v^*$  be a solution for *Problem 2*,  $\forall v$ . Define  
 $\boldsymbol{\tau}^* \triangleq [\tau_1^*, \tau_2^*, \dots, \tau_V^*]$ . Then the point  $(\mathbf{s}^*, \boldsymbol{\tau}^*)$  is an SE for  
the proposed Stackelberg game if for any  $(\mathbf{s}, \boldsymbol{\tau})$  with  $\mathbf{s} \geq \mathbf{0}$   
and  $\boldsymbol{\tau} \geq \mathbf{0}$ , the following conditions are satisfied:

$$448 \quad \begin{aligned} S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*) &\geq S^{NSP}(\mathbf{s}, \boldsymbol{\tau}^*), \\ S_v^{VR}(\tau_v^*, \tau_v^*) &\geq S_v^{VR}(s_v^*, \tau_v), \quad \forall v. \end{aligned} \quad (16)$$

449 Generally speaking, the SE of a Stackelberg game can be  
450 obtained by finding its perfect Nash Equilibrium (NE). In our  
451 proposed game, we can see that the VRs strictly compete  
452 in a non-cooperative fashion. Therefore, a non-cooperative  
453 subgame on controlling the fractions of rented SBSs is for-  
454 mulated at the VRs' side. For a non-cooperative game, the  
455 NE is defined as the operating points at which no players can  
456 improve utility by changing its strategy unilaterally. At the  
457 NSP's side, since there is only one player, the best response  
458 of the NSP is to solve *Problem 1*. To achieve this, we need to  
459 first find the best response functions of the followers, based  
460 on which, we solve the best response function for the leader.

461 Therefore, in our game, we first solve *Problem 2* given a  
462 price vector  $\mathbf{s}$ . Then with the obtained best response function  
463  $\boldsymbol{\tau}^*$  of the VRs, we solve *Problem 1* for the optimal price  $\mathbf{s}^*$ . In  
464 the following, we will have an in-depth investigation on this  
465 game theoretic optimization. 466

#### 467 V. GAME THEORETIC OPTIMIZATION

468 In this section, we will solve the optimization problem in  
469 our game under the non-uniform pricing scheme, where the  
470 NSP  $\mathcal{L}$  charges the VRs with different prices  $s_1, \dots, s_V$  for  
471 renting an SBS. In this scheme, we first solve *Problem 2* at  
472 the VRs, and rewrite Eq. (14) as

$$473 \quad S_v^{VR}(\tau_v, s_v) = \frac{\Gamma_v s^{ld} \tau_v}{\Theta \tau_v + \Lambda} - \lambda s_v \tau_v. \quad (17)$$

474 where  $\Gamma_v \triangleq \sum_{j=1}^F p_j q_v \zeta K$ ,  $\Theta \triangleq A(\delta, \alpha) - C(\delta, \alpha) + 1$ , and  
475  $\Lambda \triangleq C(\delta, \alpha) F$ . We observe that Eq. (17) is a concave function  
476 over the variable  $\tau_v$ . Thus, we can obtain the optimal solution  
477 by solving the Karush-Kuhn-Tucker (KKT) conditions, and we  
478 have the following lemma. 479

479 *Lemma 1:* For a given price  $s_v$ , the optimal solution of  
480 *Problem 2* is

$$481 \tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{ld}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_v}} - \frac{\Lambda}{\Theta} \right)^+, \quad (18)$$

482 where  $(\cdot)^+ \triangleq \max(\cdot, 0)$ .

483 *Proof:* The optimal solution  $\tau_v^*$  of  $\mathcal{V}_v$  can be obtained by  
484 deriving  $S_v^{VR}$  with respect to  $\tau_v$  and solving  $\frac{dS_v^{VR}}{d\tau_v} = 0$  under  
485 the constraint that  $\tau_v \geq 0$ . ■

486 We can see from *Lemma 1* that if the price  $s_v$  is set too  
487 high, i.e.,  $s_v \geq \frac{\Gamma_v s^{ld}}{\Lambda \lambda}$ , the VR  $\mathcal{V}_v$  will opt out for renting any  
488 SBS from  $\mathcal{L}$  due the high price charged. Consequently, the  
489 VR  $\mathcal{V}_v$  will not participate in the game.

490 In the following derivations, we assume that the LDS on  
491 each video  $s^{ld}$  is set by the VRs to be the cost of a video trans-  
492 mission via back-haul channels  $s^{bh}$ . The rational behind this  
493 assumption is as follows. Since a local downloading reduce a  
494 back-haul transmission, this saved back-haul transmission can  
495 be potentially utilized to provide extra services (equivalent to  
496 the value of  $s^{bh}$ ) for the MUs. In addition, the MUs enjoy the  
497 benefit from faster local video transmissions. In light of this,  
498 it is reasonable to assume that the MUs are willing to accept  
499 the price  $s^{bh}$  for a local video transmission.

500 Substituting the optimal  $\tau_v^*$  of Eq. (18) into Eq. (9) and  
501 carry out some further manipulations, we arrive at

$$502 S^{NSP} = \sum_{j=1}^V \lambda s_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ \\ 503 + \frac{\sum_{i=1}^F p_i q_j \zeta K s^{bh} \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+}{\Theta \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ + \Lambda} \\ 504 = \sum_{j=1}^V \frac{\zeta_j}{\Theta} \left( -\Lambda \lambda s_j + \left( \sqrt{s^{bh}} - \frac{s^{bh}}{\sqrt{s^{bh}}} \right) \sqrt{\Gamma_j \Lambda \lambda s_j} + \Gamma_j s^{bh} \right) \\ 505 = \sum_{j=1}^V \frac{\zeta_j}{\Theta} \left( -\Lambda \lambda s_j + \Gamma_j s^{bh} \right), \quad (19)$$

506 where  $\zeta_j$  is the indicator function, with  $\zeta_j = 1$  if  $s_j < \frac{\Gamma_j s^{bh}}{\Lambda \lambda}$   
507 and  $\zeta_j = 0$  otherwise. Upon defining the binary vector  $\boldsymbol{\xi} \triangleq$   
508  $[\zeta_1, \zeta_2, \dots, \zeta_V]$ , we can rewrite *Problem 1* as follows.

509 *Problem 3:* Given the optimal solutions  $\tau_v^*$ ,  $\forall v$ , gleaned  
510 from the followers, we can rewrite *Problem 1* as

$$511 \min_{\boldsymbol{\xi}, s \geq \mathbf{0}} \sum_{j=1}^V \zeta_j \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right), \\ 512 \text{s.t. } \sum_{j=1}^V \zeta_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \quad (20)$$

513 Observe from Eq. (20) that *Problem 3* is non-convex due  
514 to  $\boldsymbol{\xi}$ . However, for a given  $\boldsymbol{\xi}$ , this problem can be solved by  
515 satisfying the KKT conditions. In the following, we commence  
516 with the assumption that  $\boldsymbol{\xi} = \mathbf{1}$ , i.e.,  $\zeta_v = 1, \forall v$ , and then we  
517 extend this result to the general case.

518 *A. Special Case:*  $\zeta_v = 1, \forall v$

519 In this case, all the VRs are participating in the game, and  
520 we have the following optimization problem.

521 *Problem 4:* Assuming  $\zeta_v = 1, \forall v$ , we rewrite *Problem 3* as

$$522 \min_{s \geq \mathbf{0}} \sum_{j=1}^V s_j, \\ 523 \text{s.t. } \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} \leq (V \Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda s^{bh}}}. \quad (21)$$

524 The optimal solution of *Problem 4* is derived and given in  
525 the following lemma.

526 *Lemma 2:* The optimal solution to *Problem 4* can be  
527 derived as  $\hat{\mathbf{s}} \triangleq [\hat{s}_1, \dots, \hat{s}_V]$ , where

$$528 \hat{s}_v = \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda (V \Lambda + \Theta)^2}, \quad \forall v. \quad (22)$$

529 *Proof:* Please refer to Appendix B. ■

530 Note that the solution given in *Lemma 2* is found under  
531 the assumption that  $\zeta_v = 1, \forall v$ . That is,  $\hat{s}_v$  given in Eq. (22)  
532 should ensure that  $\tau_v^* > 0, \forall v$ , in Eq. (18), i.e.,

$$533 \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda (V \Lambda + \Theta)^2} < \frac{\Gamma_v s^{bh}}{\Lambda \lambda}. \quad (23)$$

534 Given the definitions of  $\Gamma_v, \Lambda$ , and  $\Theta$ , it is interesting to find  
535 that the inequality (23) can be finally converted to a constraint  
536 on the storage size  $Q$  of each SBS, which is formulated as

$$537 Q > \max \left\{ \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \forall v \right\}. \quad (24)$$

538 The constraint imposed on  $Q$  can be expressed in a concise  
539 manner in the following theorem.

540 *Theorem 2:* To make sure that  $\hat{s}_v$  in Eq. (22) does become  
541 the optimal solution of *Problem 4* when  $\zeta_v = 1, \forall v$ , the  
542 sufficient and necessary condition to be satisfied is

$$543 Q > Q_{min} \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \quad (25)$$

544 where  $q_v$  is the minimum value in  $\mathbf{q}$  according to Eq. (5).

545 *Proof:* Please refer to Appendix C. ■

546 *Remark 2:* Observe from Eq. (25) that since  $\frac{q_j}{q_v}$  increases  
547 exponentially with  $\gamma$  according to Eq. (5), the value of  $Q_{min}$   
548 ensuring  $\zeta_v = 1, \forall v$ , will increase exponentially with  $\gamma/3$ .

549 Note that we have  $Q \leq N$ . In the case that  $Q_{min}$  in Eq. (25)  
550 is larger than  $N$  for a high VR popularity exponent  $\gamma$ , some  
551 VRs with the least popularity will be excluded from the game.

552 *B. Further Discussion on Q*

553 We define a series of variables  $U_v, \forall v$ , as follows:

$$554 U_v \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^v \sqrt[3]{\frac{q_j}{q_v}} - v \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \quad (26)$$

555 and formulate the following lemma.

*Lemma 3:*  $U_v$  is a strictly monotonically-increasing function of  $v$ , i.e., we have  $U_V > U_{V-1} > \dots > U_1$ .

*Proof:* Please refer to Appendix D. ■

For the special case of the previous subsection, the optimal solution for  $\zeta_v = 1, \forall v$ , is found under the condition that the storage size obeys  $Q > U_V$ . In other words,  $Q$  should be large enough such that every VR can participate in the game. However, when  $Q$  reduces, some VRs have to leave the game as a result of the increased competition. Then we have the following lemma.

*Lemma 4:* When  $U_v < Q \leq U_{v+1}$ , the NSP can only retain at most the  $v$  VRs of  $\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_v$  in the game for achieving its optimal solution.

*Proof:* Please refer to Appendix E. ■

From *Lemma 4*, when we have  $U_v < Q \leq U_{v+1}$ , and given that there are  $u$  VRs,  $u \leq v$ , in the game, we can have an optimal solution for  $s$ .

*Problem 5:* When  $U_v < Q \leq U_{v+1}$  is satisfied, and given that there are  $u, u \leq v$ , VRs in the game, we can formulate the following optimization problem as

$$\begin{aligned} \min_{s \geq \mathbf{0}} \quad & \sum_{j=1}^u s_j, \\ \text{s.t.} \quad & \sum_{j=1}^u \sqrt{\frac{\Gamma_j}{s_j}} \leq (u\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda_S^{bh}}}. \end{aligned} \quad (27)$$

Similar to the solution of *Problem 4*, we arrive at the optimal solution for the above problem as  $\hat{s}_u \triangleq [\hat{s}_{1,u}, \dots, \hat{s}_{i,u}, \dots, \hat{s}_{v,u}]$ , where

$$\hat{s}_{i,u} = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^u \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_i}}{\lambda(u\Lambda + \Theta)^2}, & i = 1, \dots, u, \\ \infty, & i = u+1, \dots, v. \end{cases} \quad (28)$$

### C. General Case

Let us now focus our attention on the general solution of the original optimization problem, i.e., of *Problem 3*. Without loss of generality, we consider the case of  $U_v < Q \leq U_{v+1}$ . Then *Problem 3* is equivalent to the following problem.

*Problem 6:* When  $U_v < Q \leq U_{v+1}$ , there are at most  $v$  VRs in the game. Then *Problem 3* can be converted to

$$\begin{aligned} \min_{\xi, s \geq \mathbf{0}} \quad & \sum_{j=1}^v \xi_j \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right), \\ \text{s.t.} \quad & \sum_{j=1}^v \xi_j \left( \sqrt{\frac{\Gamma_j \Lambda_S^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \end{aligned} \quad (29)$$

The problem in Eq. (29) is again non-convex due to the uncertainty of  $\xi_u, u = 1, \dots, v$ . We have to consider the cases, where there are  $u, \forall u$ , most popular VRs in the game. We observe that for a given  $u$ , *Problem 6* converts to *Problem 5*. Therefore, to solve *Problem 6*, we first solve *Problem 5* with a given  $u$  and obtain  $\hat{s}_u$  according to Eq. (28).

TABLE I  
THE CENTRALIZED ALGORITHM AT THE NSP FOR OBTAINING THE OPTIMAL SOLUTION  $s^*$

#### Algorithm 1 :

**Input:** Storage size  $Q$ , number of videos  $N$ , VRs' preference distribution  $\mathbf{q}$ , channel exponent  $\alpha$ , and pre-set threshold  $\delta$ .

**Output:** Optimal pricing vector  $s^*$ .

**Steps:**

- 1: Based on  $N, \mathbf{q}, \alpha$ , and  $\delta$ , the NSP calculates  $U_v, \forall v$ , according to Eq. (26);
- 2: By comparing  $Q$  to  $U_v$ , the NSP obtains the value of the integer  $T$  in Eq. (33);
- 3: Calculate  $S_u, u = 1, 2, \dots, T$ , according to Eq. (33);
- 4: Compare among  $S_1, \dots, S_T$  for finding the index  $\hat{u}$  of the minimum  $S_{\hat{u}}$ ;
- 5: Based on  $\hat{u}, N, \mathbf{q}, \alpha$ , and  $\delta$ , the NSP obtains the optimal solution  $s^*$  according to Eq. (31).

Then we choose the optimal solution, denoted by  $s_v^*$ , among  $\hat{s}_1, \dots, \hat{s}_v$  as the solution to *Problem 6*, which is formulated as

$$\begin{aligned} s_v^* &= \arg \min_{\hat{s}_u} \left\{ \min \left( \sum_{j=1}^u \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right) \right), u = 1, \dots, v \right\}. \end{aligned} \quad (30)$$

Based on the above discussions, we can see that the optimal solution  $s^*$  of *Problem 3* is a piece-wise function of  $Q$ , i.e.,  $s^* = s_v^*$  when  $U_v < Q \leq U_{v+1}$ . Now, we formulate the solution  $s^* = [s_1^*, \dots, s_V^*]$  to *Problem 3* in a general manner as follows.

$$s_v^* = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^{\hat{u}} \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda(\hat{u}\Lambda + \Theta)^2}, & v = 1, \dots, \hat{u}, \\ \infty, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (31)$$

where regarding  $\hat{u}$ , we have

$$\hat{u} = \arg \min_u \{S_u : u = 1, 2, \dots, T\}, \quad (32)$$

with  $S_u$  formulated as

$$\begin{aligned} S_u &= \sum_{j_1=1}^u \left( \frac{\Lambda^2 s^{bh} \left( \sum_{j_2=1}^u \sqrt[3]{\Gamma_{j_2}} \right)^2 \sqrt[3]{\Gamma_{j_1}}}{(u\Lambda + \Theta)^2} - \Gamma_{j_1} s^{bh} \right), \\ T &= \begin{cases} 1, & U_1 < Q \leq U_2, \\ \dots, & \\ v, & U_v < Q \leq U_{v+1}, \\ \dots, & \\ V, & U_V < Q. \end{cases} \end{aligned} \quad (33)$$

To gain a better understanding of the optimal solution in Eq. (31), we propose a centralized algorithm at  $\mathcal{L}$  in Table I for obtaining  $s^*$ .

*Remark 3:* The optimal solution  $s^*$  in Eq. (31), combined with the solution of  $\tau^*$  given by Eq. (18) in *Lemma 1*, constitutes the SE for the Stackelberg game.

Furthermore, by substituting the optimal  $\mathbf{s}^*$  into the expression of  $S^{NSP}$  in Eq. (19), we get

$$S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*) = \frac{1}{\Theta} \sum_{j_1=1}^{\hat{u}} \left( \Gamma_{j_1} s^{bh} - \frac{\Lambda^2 s^{bh} \left( \sum_{j_2=1}^{\hat{u}} \sqrt[3]{\Gamma_{j_2}} \right)^2 \sqrt[3]{\Gamma_{j_1}}}{(\hat{u}\Lambda + \Theta)^2} \right). \quad (34)$$

*Remark 4:* Since we have  $\Gamma_v \propto q_v$ ,  $\forall v$ , and  $q_v$  increases exponentially with the VR preference parameter  $\gamma$  according to Eq. (5),  $S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*)$  also increases exponentially with  $\gamma$ .

## VI. DISCUSSIONS OF OTHER SCHEMES

Let us now consider two other schemes, namely, an uniform pricing scheme and a global optimization scheme.

### A. Uniform Pricing Scheme

In contrast to the non-uniform pricing scheme of the previous section, the uniform pricing scheme deliberately imposes the same price on the VRs in the game. We denote the fixed price by  $s$ . In this case, similar to *Lemma 1, Problem 2* can be solved by

$$\tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s}} - \frac{\Lambda}{\Theta} \right)^+. \quad (35)$$

We first focus our attention on the special case of  $\zeta_v = 1$ ,  $\forall v$ . Then *Problem 4* can be converted to that of minimizing  $s$  subject to the constraint  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s}} \leq (V\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda s^{bh}}}$ . We then obtain the optimal  $\hat{s}$  for this special case as

$$\hat{s} = \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt{\Gamma_j} \right)^2}{\lambda (V\Lambda + \Theta)^2}. \quad (36)$$

To guarantee that all the VRs are capable of participating in the game, i.e.,  $\zeta_v = 1$ ,  $\forall v$ , with the optimal price  $\hat{s}$ , we let  $\hat{s} < \frac{\Gamma_v s^{bh}}{\Lambda \lambda}$ . Then we have the following constraint on the storage  $Q$  as

$$Q > Q'_{min} \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (37)$$

We can see that we require a larger storage size  $Q$  in Eq. (37) than that in Eq. (25) under the non-uniform pricing scheme to accommodate all the VRs, since we have  $\sum_{j=1}^V \sqrt{\frac{q_j}{q_v}} > \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}}$ . Following *Remark 2*, we conclude that  $Q'_{min}$  of the uniform pricing scheme will increase exponentially with  $\gamma/2$ .

Then based on this special case, the optimal  $\mathbf{s}^* = [s_1^*, \dots, s_V^*]$  in the uniform pricing scheme can be readily obtained by following a similar method to that in the previous section. That is,

$$s_v^* = \begin{cases} \frac{\Lambda s^{bh} \left( \sum_{j=1}^{\hat{u}} \sqrt{\Gamma_j} \right)^2}{\lambda (\hat{u}\Lambda + \Theta)^2}, & v = 1, \dots, \hat{u}, \\ \infty, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (38)$$

where regarding  $\hat{u}$ , we have

$$\hat{u} = \arg \min_u \{S_u : u = 1, 2, \dots, T\}, \quad (39)$$

with

$$S_u = \frac{u\Lambda^2 s^{bh} \left( \sum_{j=1}^u \sqrt{\Gamma_j} \right)^2}{(u\Lambda + \Theta)^2} - \sum_{j=1}^u \Gamma_j s^{bh},$$

$$T = \begin{cases} 1, & \bar{U}_1 < Q \leq \bar{U}_2, \\ \dots, \\ v, & \bar{U}_v < Q \leq \bar{U}_{v+1}, \\ \dots, \\ V, & \bar{U}_V < Q. \end{cases} \quad (40)$$

Note that  $\bar{U}_v$  in Eq. (40) is defined as

$$\bar{U}_v \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^v \sqrt{\frac{q_j}{q_v}} - v \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (41)$$

It is clear that the uniform pricing scheme is inferior to the non-uniform pricing scheme in terms of maximizing  $S^{NSP}$ . However, we will show in the following problem that the uniform pricing scheme offers the optimal solution to maximizing the back-haul cost reduction  $S^{BH}$  at the NSP in conjunction with  $\tau_v^*$ ,  $\forall v$ , from the followers.

*Problem 7:* With the aid of the optimal solutions  $\tau_v^*$ ,  $\forall v$ , from the followers, the maximization on  $S^{BH}$  is achieved by solving the following problem:

$$\begin{aligned} \min_{\xi, s \geq 0} & \sum_{j=1}^V \xi_j \left( \sqrt{s^{bh}} \sqrt{\Gamma_j \Lambda \lambda} \sqrt{s_j} - \Gamma_j s^{bh} \right), \\ \text{s.t.} & \sum_{j=1}^V \xi_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \end{aligned} \quad (42)$$

The optimal solution to *Problem 7* can be readily shown to be  $\mathbf{s}^*$  given in Eq. (38). This proof follows the similar procedure of the optimization method presented in the previous section. Thus it is skipped for brevity. In this sense, the uniform pricing scheme is superior to the non-uniform pricing scheme in terms of reducing more cost on back-haul channel transmissions.

### B. Global Optimization Scheme

In the global optimization scheme, we are interested in the sum profit of the NSP and VRs, which can be expressed as

$$\begin{aligned} S^{GLB} &= S^{NSP} + \sum_{j=1}^V S_j^{VR} \\ &= \sum_{j_1=1}^V \sum_{j_2=1}^F \frac{2p_{j_2} q_{j_1} \zeta K s^{bh} \tau_{j_1}}{(A(\delta, \alpha) - C(\delta, \alpha) + 1) \tau_{j_1} + C(\delta, \alpha) F} \\ &= 2S^{BH}. \end{aligned} \quad (43)$$

Observe from Eq. (43), we can see that the sum profit  $S^{GLB}$  is twice the back-haul cost reduction  $S^{BH}$ , where the vector  $\boldsymbol{\tau}$  is the only variable of this maximization problem.



694 *Problem 8*: The optimization of the sum profit  $S^{GLB}$  can  
695 be formulated as

$$696 \quad \max_{\tau \geq 0} \sum_{j_1=1}^V \frac{\tau_{j_1} \sum_{j_2=1}^F p_{j_2} q_{j_1} \zeta K s^{bh}}{(A(\delta, \alpha) - C(\delta, \alpha) + 1)\tau_{j_1} + C(\delta, \alpha)F},$$

$$697 \quad \text{s.t.} \quad \sum_{j=1}^V \tau_j \leq 1. \quad (44)$$

698 *Problem 8* is a typical water-filling optimization problem.  
699 By relying on the classic Lagrangian multiplier, we arrive at  
700 the optimal solution as

$$701 \quad \hat{\tau}_v = \left( \frac{\frac{\sqrt{q_v}}{\eta} - C(\delta, \alpha)F}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \right)^+, \quad \forall v, \quad (45)$$

702 where we have  $\eta = \frac{\sum_{j=1}^{\bar{v}} \sqrt{q_j}}{\bar{v}C(\delta, \alpha)F + A(\delta, \alpha) - C(\delta, \alpha) + 1}$ , and  $\bar{v}$  satisfies  
703 the constraint of  $\hat{\tau}_v > 0$ .

### 704 C. Comparisons

705 Let us now compare the optimal SBS allocation variable  $\tau_v$   
706 in the context of the above two schemes. First, we investigate  
707  $\tau_v^*$  in the uniform pricing scheme. By substituting Eq. (38)  
708 into Eq. (35), we have

$$709 \quad \tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_v^*}} - \frac{\Lambda}{\Theta} \right)^+$$

$$710 \quad = \begin{cases} \frac{\frac{\sqrt{q_v}}{\eta'} - C(\delta, \alpha)F}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, & v = 1, \dots, \hat{u} \\ 0, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (46)$$

711 where  $\eta' = \frac{\sum_{j=1}^{\hat{u}} \sqrt{q_j}}{\hat{u}C(\delta, \alpha)F + A(\delta, \alpha) - C(\delta, \alpha) + 1}$ , and  $\hat{u}$  ensures  $\tau_v^* > 0$ .

712 Then, comparing  $\tau_v^*$  given in Eq. (46) to the optimal  
713 solution  $\hat{\tau}$  of the global optimization scheme given by Eq. (45),  
714 we can see that these two solutions are the same. In other  
715 words, the uniform pricing scheme in fact represents the global  
716 optimization scheme in terms of maximizing the sum profit  
717  $S^{GLB}$  and maximizing the back-haul cost reduction  $S^{BH}$ .

## 718 VII. NUMERICAL RESULTS

719 In this section, we provide both numerical as well as  
720 Monte-Carlo simulation results for evaluating the performance  
721 of the proposed schemes. The physical layer parameters of  
722 our simulations, such as the path-loss exponent  $\alpha$ , transmit  
723 power  $P$  of the SBSs and the noise power  $\sigma^2$  are similar to  
724 those of the 3GPP standards. The unit of noise power and  
725 transmit power is Watt, while the SBS and MU intensities are  
726 expressed in terms of the numbers of the nodes per square  
727 kilometer.

728 Explicitly, we set the path-loss exponent to  $\alpha = 4$ , the  
729 SBS transmit power to  $P = 2$  Watt, the noise power to  
730  $\sigma^2 = 10^{-10}$  Watt, and the pre-set SINR threshold to  $\delta = 0.01$ .  
731 For the file caching system, we set the number of files in  
732  $\mathcal{F}$  to  $N = 500$  and set the number of VRs to  $V = 15$ .  
733 For the network deployments, we set the intensity of the

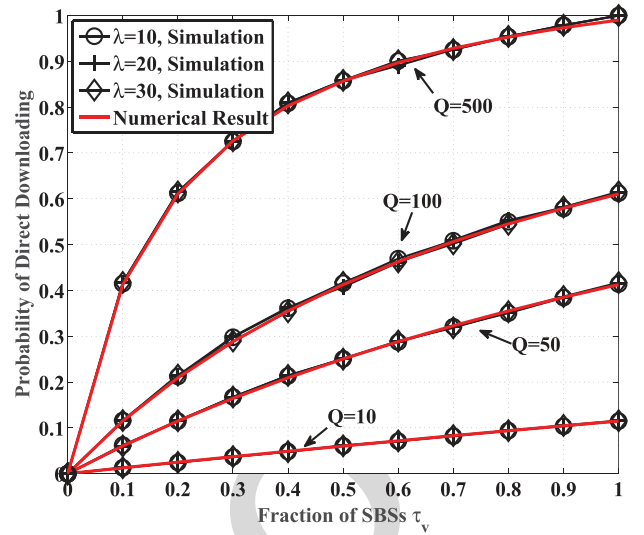


Fig. 2. Comparisons between the simulations and analytical results on  $\Pr(\mathcal{E}_{v,f})$ . We consider four kinds of storage size  $Q$  in each SBS, i.e.,  $Q = 10, 50, 100, 500$ , and three kinds of SBS intensity, i.e.,  $\lambda = 10, 20, 30$ .

MUs to  $\zeta = 50/km^2$ , and investigate three cases of the SBS  
734 deployments as  $\lambda = 10/km^2, 20/km^2$  and  $30/km^2$ . 735

736 For the pricing system, the profit/ $UAP$  is considered to  
737 be the profit gained per month within an area of one square  
738 kilometer, i.e.,  $/month \cdot km^2$ . We note that the profits gained  
739 by the NSP and by the VRs are proportional to the cost  $s^{bh}$  of  
740 back-haul channels for transmitting a video. Hence, without  
741 loss of generality, we set  $s^{bh} = 1$  for simplicity. Additionally,  
742 we set  $K = 10/month$ , which is the average number of video  
743 requests from an MU per month.

744 We first verify our derivation of  $\Pr(\mathcal{E}_{v,f})$  by comparing the  
745 analytical results of *Theorem 1* to the Monte-Carlo simulation  
746 results. Upon verifying  $\Pr(\mathcal{E}_{v,f})$ , we will investigate the  
747 optimization results within the framework of the proposed  
748 Stackelberg game by providing numerical results.

### 749 A. Performance Evaluation on $\Pr(\mathcal{E}_{v,f})$

750 For the Monte-Carlo simulations of this subsection, all the  
751 average performances are evaluated over a thousand network  
752 scenarios, where the distributions of the SBSs and the MUs  
753 change from case to case according to the PPPs characterized by  
754  $\Phi$  and  $\Psi$ , respectively.

755 Note that  $\Pr(\mathcal{E}_{v,f})$  in *Theorem 1* is the probability that an  
756 MU can obtain its requested video directly from the memory  
757 of an SBS rented by  $\mathcal{V}_v$ . We can see from the expression of  
758  $\Pr(\mathcal{E}_{v,f})$  in Eq. (7) that it is a function of the fraction  $\tau_v$   
759 of the SBSs that are rented by  $\mathcal{V}_v$ . Although  $\tau_v$  should be  
760 optimized according to the price charged by the NSP, here  
761 we investigate a variety of  $\tau_v$  values, varying from 0 to 1, to  
762 verify the derivation of  $\Pr(\mathcal{E}_{v,f})$ .

763 Fig. 2 shows our comparisons between the simulations and  
764 analytical results on  $\Pr(\mathcal{E}_{v,f})$ . We consider four different  
765 storage sizes  $Q$  in each SBS by setting  $Q = 10, 50, 100, 500$ .  
766 Correspondingly, we have four values for the number of file  
767 groups, i.e.,  $F = 50, 10, 5, 1$ . Furthermore, we consider the  
768 SBS intensities of  $\lambda = 10, 20, 30$ . From Fig. 2, we can

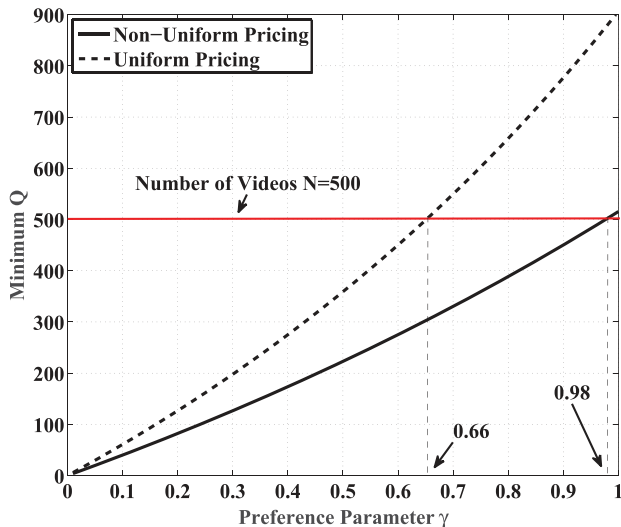


Fig. 3. The minimum number of  $Q$  that allows all the VRs to participate in the game under different preference parameter  $\gamma$ . In the case that the minimum  $Q$  is larger than  $N$ , it means that some VRs will be inevitably excluded from the game.

769 see that the simulations results closely match the analytical  
 770 results derived in *Theorem 1*. Our simulations show that the  
 771 intensity  $\lambda$  does not affect  $\Pr(\mathcal{E}_{v,f})$ , which is consistent with  
 772 our analytical results. Furthermore, a larger  $Q$  leads to a higher  
 773 value of  $\Pr(\mathcal{E}_{v,f})$ . Hence, enlarging the storage size is helpful  
 774 for achieving a higher probability of direct downloading.

### 775 B. Impact of the VR Preference Parameter $\gamma$

776 The preference distribution  $\mathbf{q}$  of the VRs defined in Eq. (5)  
 777 is an important factor in predetermining the system perfor-  
 778 mance. Indeed, we can see from Eq. (5) that this distribution  
 779 depends on the parameter  $\gamma$ . Generally, we have  $0 < \gamma \leq 1$ ,  
 780 with a larger  $\gamma$  representing a more uneven popularity among  
 781 the VRs. First, we find the minimum  $Q$  that can keep all  
 782 the VRs in the game. This minimum  $Q$  for the non-uniform  
 783 pricing scheme (NUPS) is given by Eq. (25), while the  
 784 minimum  $Q$  for the uniform-pricing scheme (UPS) is given by  
 785 Eq. (37). From the two equations, this minimum  $Q$  increases  
 786 exponentially with  $\gamma/3$  in the NUPS, while it also increases  
 787 exponentially with a higher exponent of  $\gamma/2$  in the UPS.  
 788 Fig. 3 shows this minimum  $Q$  for different values of the  
 789 VR preference parameter  $\gamma$ .

790 We can see that the UPS needs a larger  $Q$  than the NUPS  
 791 for keeping all the VRs. This gap increases rapidly with the  
 792 growth of  $\gamma$ . For example, for  $\gamma = 0.3$ , the uniform pricing  
 793 scheme requires almost 80 more storages, while for  $\gamma = 0.6$ ,  
 794 it needs 200 more. We can also observe in Fig. 3 that for  
 795  $\gamma > 0.66$  in the UPS and for  $\gamma > 0.98$  in the NUPS,  
 796 the minimum  $Q$  becomes larger than the overall number of  
 797 videos  $N$ . In both cases, since we have  $Q \leq N$  ( $Q > N$   
 798 results in the same performance as  $Q = N$ ), some unpopular  
 799 VRs will be excluded from the game.

800 Next, we study the number of VR participants that stay in  
 801 the game for the two schemes upon increasing  $\gamma$ . We can see  
 802 from Fig. 4 that the number of VR participants keeps going  
 803 down upon increasing  $\gamma$  in the both schemes. The NUPS

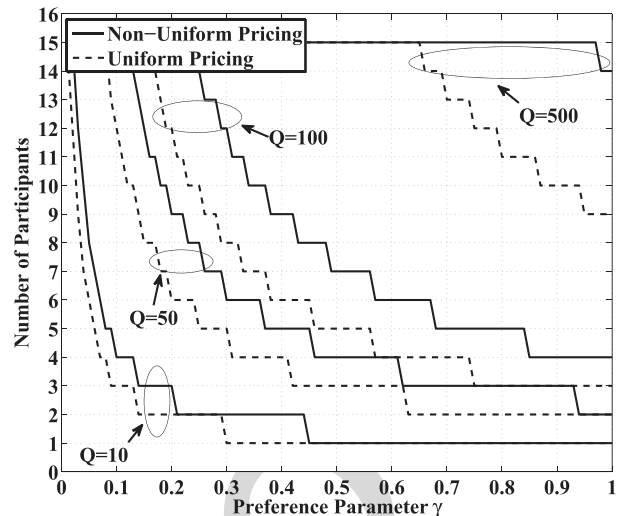


Fig. 4. Number of participants, i.e., the VRs that are in the game, vs. the preference parameter  $\gamma$ , under the two schemes. We also consider four different values of the storage size  $Q$ , i.e., 10, 50, 100, 500.

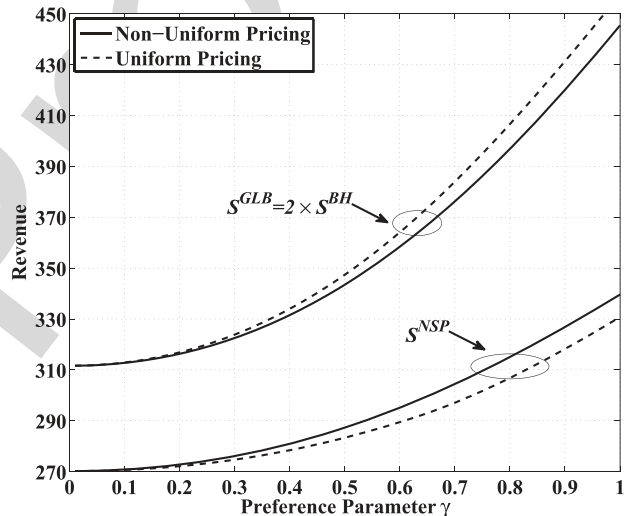


Fig. 5. Various revenues, including  $S^{NSP}$  and  $S^{GLB}$ , vs. the preference parameter  $\gamma$ , under the two schemes.

804 always keeps more VRs in the game than the UPS under  
 805 the same  $\gamma$ . At the same time, by considering  $Q =$   
 806 10, 50, 100, 500, it is shown that for a given  $\gamma$ , a higher  $Q$   
 807 will keep more VRs in the game.

808 Fig. 5 shows two kinds of revenues gained by the two  
 809 schemes for a given storage of  $Q = 500$ , namely, the global  
 810 profit  $S^{GLB}$  defined in Eq. (43) and the profit of the NSP  
 811  $S^{NSP}$  defined in Eq. (9). Recall that we have  $S^{GLB} = 2S^{BH}$   
 812 according to Eq. (43). We can see that the revenues of both  
 813 schemes increase exponentially upon increasing  $\gamma$ , as stated  
 814 in *Remark 4*. As our analytical result shows, the profit  $S^{NSP}$   
 815 gained by the NUPS is optimal and thus it is higher than  
 816 that gained by the UPS, while the UPS maximizes both  
 817  $S^{GLB}$  and  $S^{BH}$ . Fig. 5 verifies the accuracy of our derivations.

### 818 C. Impact of the Storage Size $Q$

819 Since  $\gamma$  is a network parameter that is relatively fixed,  
 820 the NSP can adapt the storage size  $Q$  for controlling

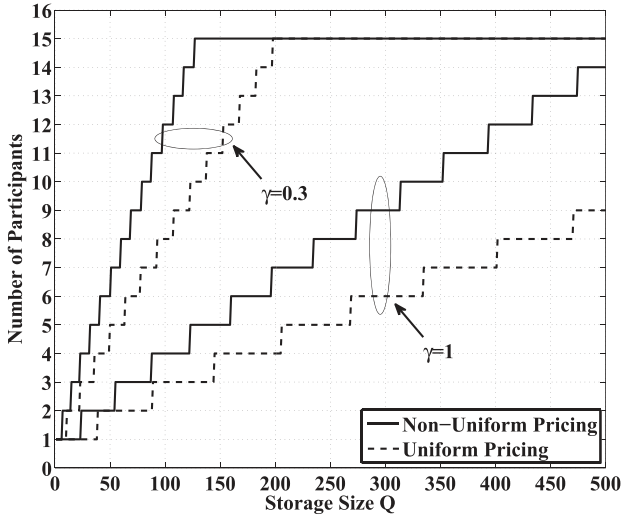


Fig. 6. Number of participants vs. the storage size  $Q$ , under the two schemes. We also consider two different values of  $\gamma$ , i.e.,  $\gamma = 0.3, 1$ .

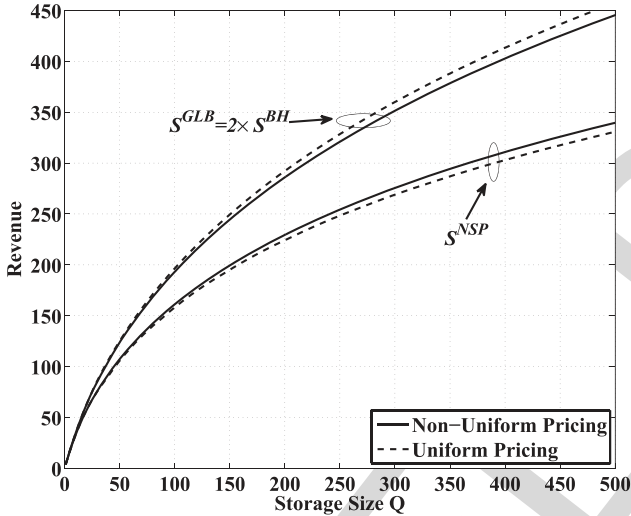


Fig. 7. Various revenues, including  $S^{NSP}$  and  $S^{GLB}$ , vs. the storage size  $Q$ , under the two schemes.

its performance. In this subsection, we investigate the performance as a function of  $Q$ . Fig. 6 shows the number of participants in the game versus  $Q$ , where  $\gamma = 0.3$  and 1 are considered. It is shown that for a larger  $Q$ , more VRs are able to participate in the game. Again, the NUPS outperforms the UPS owing to its capability of accommodating more VRs for a given  $Q$ . By comparing the scenarios of  $\gamma = 0.3$  and 1, we find that for  $\gamma = 0.3$ , a given increase of  $Q$  can accommodate more VRs in the game than  $\gamma = 1$ .

Fig. 7 shows both  $S^{NSP}$  and  $S^{GLB}$  versus  $Q$  for the two schemes for a given  $\gamma = 1$ . We can see that the revenues of both schemes increase with the growth of  $Q$ . It is shown that the profit  $S^{NSP}$  gained by the NUPS is higher than the one gained by the UPS, while the UPS outperforms the NUPS in terms of both  $S^{GLB}$  and  $S^{BH}$ .

#### D. Individual VR Performance

In this subsection, we investigate the performance of each individual VR, including the price charged to them for renting

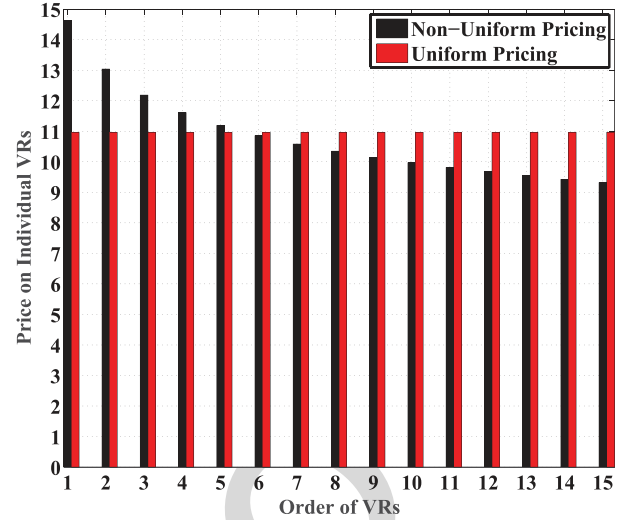


Fig. 8. Price charged on each VR for renting an SBS per month.

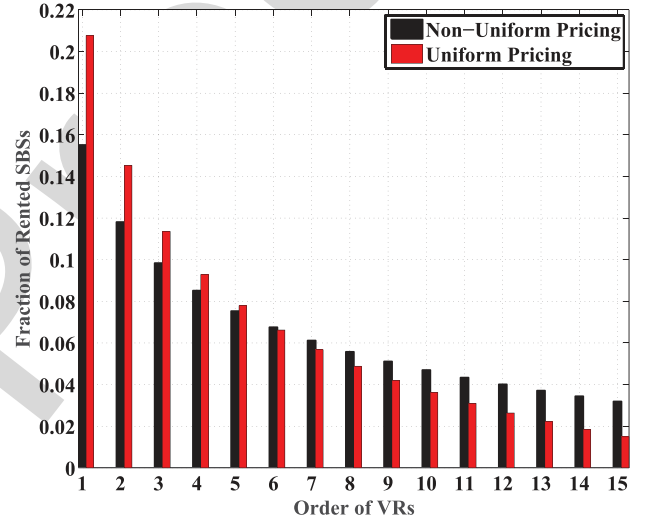


Fig. 9. The fraction of SBSs that are rented by each VR.

an SBS per month, and the fractions of the SBSs they rent from the NSP. We fix  $\gamma = 0.5$  and choose a large storage size of  $Q = 500$  for ensuring that all the VRs can be included. Fig. 8 shows the price charged to each VR for renting an SBS. The VRs are arranged according to their popularity order, ranging from  $\mathcal{V}_1$  to  $\mathcal{V}_{15}$ , with  $\mathcal{V}_1$  having the highest popularity and  $\mathcal{V}_{15}$  the lowest one. We can see from the figure that in the NUPS, the price for renting an SBS is higher for the VRs having a higher popularity than those with a lower popularity. By contrast, in the UPS, this price is fixed for all the VRs. Fig. 9 shows the specific fraction of the rented SBSs at each VR. In both schemes, the VRs associated with a high popularity tend to rent more SBSs. The UPS in fact represents an instance of the water-filling algorithm. Furthermore, the UPS seems more aggressive than the NUPS, since the less popular VRs of the UPS are more difficult to rent an SBS, and thus these VRs are likely to be excluded from the game with a higher probability.

#### VIII. CONCLUSIONS

In this paper, we considered a commercial small-cell caching system consisting of an NSP and multiple VRs, where

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860 the NSP leases its SBSs to the VRs for gaining profits and for  
 861 reducing the costs of back-haul channel transmissions, while  
 862 the VRs, after storing popular videos to the rented SBSs, can  
 863 provide faster transmissions to the MUs, hence gaining more  
 864 profits. We proposed a Stackelberg game theoretic framework  
 865 by viewing the SBSs as a type of resources. We first modeled  
 866 the MUs and SBSs using two independent PPPs with the aid of  
 867 stochastic geometry, and developed the probability expression  
 868 of direct downloading. Then, based on the probability derived,  
 869 we formulated a Stackelberg game for maximizing the average  
 870 profit of the NSP as well as individual VRs. Next, we investi-  
 871 gate the Stackelberg equilibrium by solving the associated non-  
 872 convex optimization problem. We considered a non-uniform  
 873 pricing scheme and an uniform pricing scheme. In the former  
 874 scheme, the prices charged to each VR for renting an SBS  
 875 are different, while the latter imposes the same price for  
 876 each VR. We proved that the non-uniform pricing scheme  
 877 can effectively maximize the profit of the NSP, while the  
 878 uniform one maximizes the sum profit of the NSP and the VRs.  
 879 Furthermore, we derived a relationship between the optimal  
 880 pricing of renting an SBS, the fraction of SBSs rented by each  
 881 VR, the storage size of each SBS and the popularity of the  
 882 VRs. We verified by Monte-Carlo simulations that the direct  
 883 downloading probability under our PPP model is consistent  
 884 with our derived results. Then we provided several numerical  
 885 results for showing that the proposed schemes are effective in  
 886 both pricing and SBSs allocation.

887 APPENDIX A  
 888 PROOF OF THEOREM 1

889 Recall that the SBSs allocated to the VR  $\mathcal{V}_v$  and cache  $\mathcal{G}_f$   
 890 are modeled as a “thinned” HPPP  $\Phi_{v,f}$  having the intensity  
 891 of  $\frac{1}{F}\tau_v\lambda$ . We consider a typical MU  $\mathcal{M}$  who wishes to connect  
 892 to the nearest SBS  $\mathcal{B}$  in  $\Phi_{v,f}$ . The event  $\mathcal{E}_{v,f}$  represents that  
 893 this SBS can support  $\mathcal{M}$  with an SINR no lower than  $\delta$ , and  
 894 thus  $\mathcal{M}$  can obtain the desired file from the cache of  $\mathcal{B}$ .

895 We carry out the analysis on  $\Pr(\mathcal{E}_{v,f})$  for the typical MU  
 896  $\mathcal{M}$  located at the origin. Since the network is interference  
 897 dominant, we neglect the noise in the following. We denote by  
 898  $z$  the distance between  $\mathcal{M}$  and  $\mathcal{B}$ , by  $x_Z$  the location of  $\mathcal{B}$ , and  
 899 by  $\rho(x_Z)$  the received SINR at  $\mathcal{M}$  from  $\mathcal{B}$ . Then the average  
 900 probability that  $\mathcal{M}$  can download the desired video from  $\mathcal{B}$  is

$$\begin{aligned}
 & \Pr(\rho(x_Z) \geq \delta) \\
 &= \int_0^\infty \Pr\left(\frac{h_{x_Z} z^{-\alpha}}{\sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}} \geq \delta \middle| z\right) f_Z(z) dz \\
 &= \int_0^\infty \Pr\left(h_{x_Z} \geq \frac{\delta \left(\sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}\right)}{z^{-\alpha}} \middle| z\right) \\
 & \quad 2\pi \frac{1}{F} \tau_v \lambda z \exp\left(-\pi \frac{1}{F} \tau_v \lambda z^2\right) dz \\
 &= \int_0^\infty \mathbb{E}_I(\exp(-z^\alpha \delta I)) 2\pi \frac{1}{F} \tau_v \lambda z \exp\left(-\pi \frac{1}{F} \tau_v \lambda z^2\right) dz,
 \end{aligned} \tag{47}$$

907 where we have  $I \triangleq \sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}$ , and the PDF of  $z$ , i.e.,  
 908  $f_Z(z)$ , is derived by the null probability of the HPPP  $\Phi_{v,f}$   
 909 with the intensity of  $\frac{1}{F}\tau_v\lambda$ . More specifically in  $\Phi_{v,f}$ , since  
 910 the number of the SBSs  $k$  in an area of  $A$  follows the Poisson  
 911 distribution, the probability of the event that there is no SBS  
 912 in the area with the radius of  $z$  can be calculated as [17]

$$\Pr(k = 0 \mid A = \pi z^2) = e^{-A \frac{1}{F} \tau_v \lambda} \frac{(A \frac{1}{F} \tau_v \lambda)^k}{k!} = e^{-\pi z^2 \frac{1}{F} \tau_v \lambda}. \tag{48}$$

915 By using the above expression, we arrive at  $f_Z(z) =$   
 916  $2\pi \frac{1}{F} \tau_v \lambda z \exp(-\pi \frac{1}{F} \tau_v \lambda z^2)$ . Note that the interference  $I$  con-  
 917 sists of  $I_1$  and  $I_2$ , where  $I_1$  emanates from the SBSs in  $\Phi$   
 918 excluding  $\Phi_{v,f}$ , while  $I_2$  is from the SBSs in  $\Phi_{v,f}$  excluding  
 919  $\mathcal{B}$ . The SBSs contributing to  $I_1$ , denoted by  $\Phi_{v,f}^c$ , have the  
 920 intensity of  $(1 - \frac{1}{F}\tau_v)\lambda$ , while those contributing to  $I_2$  have  
 921 the intensity of  $\frac{1}{F}\tau_v\lambda$ .

922 Correspondingly, the calculation of  $\mathbb{E}_I(\exp(-z^\alpha \delta I))$  will  
 923 be split into the product of two expectations over  $I_1$  and  $I_2$ .  
 924 The expectation over  $I_1$  is calculated as

$$\begin{aligned}
 & \mathbb{E}_{I_1}(\exp(-z^\alpha \delta I_1)) \\
 & \stackrel{(a)}{=} \mathbb{E}_{\Phi_{v,f}^c} \left( \prod_{x \in \Phi_{v,f}^c} \int_0^\infty \exp(-z^\alpha \delta h_x \|x\|^{-\alpha}) \exp(-h_x) dh_x \right) \\
 & \stackrel{(b)}{=} \exp\left(-\left(1 - \frac{1}{F}\tau_v\right)\lambda \int_{\mathbb{R}^2} \left(1 - \frac{1}{1 + z^\alpha \delta \|x_k\|^{-\alpha}}\right) dx_k\right) \\
 & = \exp\left(-2\pi \left(1 - \frac{1}{F}\tau_v\right)\lambda \frac{1}{\alpha} z^2 \delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right)\right), \\
 & = \exp\left(-\pi \left(1 - \frac{1}{F}\tau_v\right)\lambda C(\delta, \alpha) z^2\right),
 \end{aligned} \tag{49}$$

930 where (a) is based on the independence of chan-  
 931 nel fading, while (b) follows from  $\mathbb{E}\left(\prod_x u(x)\right) =$   
 932  $\exp(-\lambda \int_{\mathbb{R}^2} (1 - u(x)) dx)$ , where  $x \in \Phi$  and  $\Phi$  is an PPP in  
 933  $\mathbb{R}^2$  with the intensity  $\lambda$  [24], and  $C(\delta, \alpha)$  has been defined as  
 934  $\frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right)$ .

935 The expectation over  $I_2$  has to take into account  $z$  as the  
 936 distance from the nearest interfering SBS. Then we have

$$\begin{aligned}
 & \mathbb{E}_{I_2}(\exp(-z^\alpha \delta I_2)) \\
 & = \exp\left(-\frac{1}{F}\tau_v \lambda 2\pi \int_z^\infty \left(1 - \frac{1}{1 + z^\alpha \delta r^{-\alpha}}\right) r dr\right) \\
 & \stackrel{(a)}{=} \exp\left(-\frac{1}{F}\tau_v \lambda \pi \delta^{\frac{2}{\alpha}} z^2 \frac{2}{\alpha} \int_{\delta^{-1}}^\infty \frac{\kappa^{\frac{2}{\alpha}-1}}{1 + \kappa} dx\right) \\
 & \stackrel{(b)}{=} \exp\left(-\frac{1}{F}\tau_v \lambda \pi \delta z^2 \frac{2}{\alpha - 2} {}_2F_1\left(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta\right)\right),
 \end{aligned} \tag{50}$$

942 where (a) defines  $\kappa \triangleq \delta^{-1} z^{-\alpha} r^\alpha$ , and  ${}_2F_1(\cdot)$   
 943 in (b) is the hypergeometric function. As we  
 944 defined  $A(\delta, \alpha) = \frac{2\delta}{\alpha - 2} {}_2F_1\left(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta\right)$ , by

945 substituting (49) and (50) into (47), we have

$$\begin{aligned}
946 & \Pr(\rho(x_Z) \geq \delta) \\
947 &= \int_0^\infty \exp\left(-\pi\left(1 - \frac{1}{F}\tau_v\right)\lambda C(\delta, \alpha)z^2\right) \\
948 & \exp\left(-\pi\frac{1}{F}\tau_v\lambda z^2 A(\delta, \alpha)\right) 2\pi\frac{1}{F}\tau_v\lambda z \exp\left(-\pi\frac{1}{F}\tau_v\lambda z^2\right) dz \\
949 &= \frac{\frac{1}{F}\tau_v}{C(\delta, \alpha)\left(1 - \frac{1}{F}\tau_v\right) + A(\delta, \alpha)\frac{1}{F}\tau_v + \frac{1}{F}\tau_v}. \quad (51)
\end{aligned}$$

950 This completes the proof.  $\blacksquare$

#### 951 APPENDIX B 952 PROOF OF LEMMA 2

953 By applying Lagrangian multipliers to the objective func-  
954 tion, we have

$$\begin{aligned}
955 & L(\mathbf{s}, \mu, \mathbf{v}) \\
956 &= \sum_{j=1}^V s_j + \mu \left( \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \right) - \sum_{j=1}^V v_j s_j, \\
957 & \quad (52)
\end{aligned}$$

958 where  $\mu$  and  $v_j$  are non-negative multipliers associated with  
959 the constraints  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \leq 0$  and  $s_j \geq 0$ ,  
960 respectively. Then the KKT conditions can be written as

$$\begin{aligned}
961 & \frac{\partial L(\mathbf{s}, \mu, \mathbf{v})}{\partial s_j} = 0, \quad \forall j = 1, \dots, V, \\
962 & \mu \left( \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \right) = 0, \text{ and } v_j s_j = 0, \quad \forall j. \\
963 & \quad (53)
\end{aligned}$$

964 From the first line of Eq. (53), we have

$$965 s_j = \sqrt[3]{\frac{\mu^2 \Gamma_j}{4(1 - v_j)^2}}. \quad (54)$$

966 Obviously, we have  $s_j \neq 0, \forall j$ , otherwise the constraint  
967  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \leq 0$  cannot be satisfied.  
968 Thus, we have  $v_j = 0, \forall j$ . Furthermore, we have  $\mu \neq 0$   
969 according to Eq. (54) since  $s_j$  is non-zero. This means that  
970  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} = 0$ .

971 By substituting Eq. (54) into this constraint, we have

$$972 \sqrt[3]{\mu} = \frac{\sqrt{\Lambda_S^{bh}} \sum_{j=1}^V \sqrt[3]{2\Gamma_j}}{\sqrt{\lambda}(V\Lambda + \Theta)}. \quad (55)$$

973 Then it follows that

$$974 s_j = \frac{\Lambda_S^{bh} \left( \sum_{v=1}^V \sqrt[3]{\Gamma_v} \right)^2 \sqrt[3]{\Gamma_j}}{\lambda(V\Lambda + \Theta)^2}. \quad (56)$$

975 This completes the proof.  $\blacksquare$

#### APPENDIX C PROOF OF THEOREM 2

976 As discussed in Eq. (23) and Eq. (24), we have proved that  
977  
978  $Q > \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}$  is a sufficient condition for the  
979 optimal solution in Eq. (22). In other words, as long as  $Q$  is  
980 satisfied, we have the conclusion that the solution in Eq. (22)  
981 is optimal and  $\xi_v = 1, \forall v$ .  
982

983 Next, we prove the necessary aspect. Without loss of  
984 generality, we assume that

$$\begin{aligned}
985 & \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{V-1} \sqrt[3]{\frac{q_j}{q_{V-1}}} - V + 1 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} < Q \\
986 & \leq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (57)
\end{aligned}$$

987 This leads to  $s_V \geq \frac{\Gamma_V s^{bh}}{\Lambda \lambda}$ , and the VR  $\mathcal{V}_V$  will be excluded  
988 from the game. In this case, we have  $\xi_j = 1, j = 1, \dots, V-1$ ,  
989 and *Problem 4* will be rewritten as follows.

990 *Problem 9:* We rewrite *Problem 4* as

$$\begin{aligned}
991 & \min_{\mathbf{s} \geq \mathbf{0}} \sum_{j=1}^{V-1} s_j, \\
992 & \text{s.t. } \sum_{j=1}^{V-1} \sqrt{\frac{\Gamma_j}{s_j}} \leq ((V-1)\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}}. \quad (58)
\end{aligned}$$

993 Similar to the proof of *Lemma 2*, and combined with the  
994 constraint of  $Q$  in Eq. (57), the optimal solution of *Problem 9*  
995 is given by

$$\begin{aligned}
996 & \hat{s}_v = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^{V-1} \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda((V-1)\Lambda + \Theta)^2}, & v = 1, \dots, V-1, \\ \infty, & v = V. \end{cases} \\
997 & \quad (59)
\end{aligned}$$

998 We can see that the optimal solution given in Eq. (59)  
999 contradicts to the optimal solution of *Problem 4* given in  
1000 Eq. (22). Hence,  $Q > \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}$  is a necessary  
1001 condition for finding the optimal solution in Eq. (22). This  
1002 completes the proof.  $\blacksquare$

#### APPENDIX D PROOF OF LEMMA 3

1003 Consider  $v_1, v_2 = 1, \dots, V$  and  $v_1 = v_2 + 1$ . Then we  
1004 prove that  $U_{v_1} > U_{v_2}$ . We have

$$\begin{aligned}
1005 & U_{v_1} = \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_1} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_1 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
1006 & = \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_2 + \sum_{j=v_2+1}^{v_1} \sqrt[3]{\frac{q_j}{q_{v_1}}} - (v_1 - v_2) \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
1007 & = \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_2 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
1008 & = \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_2}}} - v_2 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
1009 & \stackrel{(a)}{>} \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_2}}} - v_2 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} = U_{v_2}, \quad (60) \\
1010 &
\end{aligned}$$

1011 where (a) comes from the fact that  $q_{v_1} < q_{v_2}$ . This completes  
1012 the proof. ■

1013 APPENDIX E  
1014 PROOF OF LEMMA 4

1015 It is plausible that if  $\mathcal{L}$  can only keep at most  $v$  VRs, it has  
1016 to retain the  $v$  most popular VRs to maximize its profit. Let  
1017 us now prove that if  $\mathcal{L}$  keeps  $(v+w)$  VRs,  $w = 1, \dots, V-v$ ,  
1018 in the game, it cannot achieve the optimal solution for  
1019  $U_v < Q \leq U_{v+1}$ .

1020 *Problem 10:* In the case that  $\mathcal{L}$  keeps  $(v+w)$  VRs, we have  
1021 the optimization problem of

$$1022 \quad \min_{s_j \geq 0} \sum_{j=1}^{v+w} s_j,$$

$$1023 \quad \text{s.t.} \quad \sum_{j=1}^{v+w} \sqrt{\frac{\Gamma_j}{s_j}} \leq ((v+w)\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda_S b h}}. \quad (61)$$

1024 Similar to the proof of *Theorem 2*, we obtain that  $Q >$   
1025  $\frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v+w} \sqrt{\frac{q_j}{q_{v+w}}} - (v+w) \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} = U_{v+w}$  is the necessary con-  
1026 dition for the  $(v+w)$  VRs to participate in the game. This  
1027 contradicts to the premise  $U_v < Q \leq U_{v+1}$ , since we have  
1028  $Q > U_{v+1}$  according to *Lemma 3*. Let us now consider  
1029 the cases of  $w' = 0, -1, \dots, 1-v$ . To ensure there are  
1030  $(v+w')$  VRs in the game,  $Q$  has to satisfy the condition  
1031 that  $Q > U_{v+w'}$ . Since  $Q > U_v \geq U_{v+w'}$ , this implies that  
1032 given  $(v+w')$  VRs in the game, the NSP can achieve an  
1033 optimal solution. This completes the proof. ■

1034 REFERENCES

1035 [1] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire,  
1036 "Femtocaching and device-to-device collaboration: A new architecture  
1037 for wireless video distribution," *IEEE Commun. Mag.*, vol. 51, no. 4,  
1038 pp. 142–149, Apr. 2013.

1039 [2] X. Wang, M. Chen, T. Taleb, A. Ksentini, and V. C. M. Leung, "Cache  
1040 in the air: Exploiting content caching and delivery techniques for 5G  
1041 systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 131–139, Feb. 2014.

1042 [3] M. A. Maddah-Ali and U. Niesen, "Decentralized coded caching attains  
1043 order-optimal memory-rate tradeoff," in *Proc. 51st Annu. Allerton Conf.*  
1044 *Commun., Control, Comput. (Allerton)*, Oct. 2013, pp. 421–427.

1045 [4] N. Golrezaei, P. Mansourifard, A. F. Molisch, and A. G. Dimakis, "Base-  
1046 station assisted device-to-device communications for high-throughput  
1047 wireless video networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7,  
1048 pp. 3665–3676, Jul. 2014.

1049 [5] M. Ji, G. Caire, and A. F. Molisch. (May 2013). "Wireless device-to-  
1050 device caching networks: Basic principles and system performance."  
1051 [Online]. Available: <http://arxiv.org/abs/1305.5216>

1052 [6] M. Ji, G. Caire, and A. F. Molisch, "Optimal throughput-outage trade-  
1053 off in wireless one-hop caching networks," in *Proc. IEEE Int. Symp. Inf.*  
1054 *Theory (ISIT)*, Jul. 2013, pp. 1461–1465.

1055 [7] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE*  
1056 *Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

1057 [8] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski,  
1058 "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*,  
1059 vol. 52, no. 2, pp. 74–80, Feb. 2014.

1060 [9] A. Damnjanovic *et al.*, "A survey on 3GPP heterogeneous networks,"  
1061 *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, Jun. 2011.

1062 [10] J. Akhtman and L. Hanzo, "Heterogeneous networking: An enabling  
1063 paradigm for ubiquitous wireless communications," *Proc. IEEE*, vol. 98,  
1064 no. 2, pp. 135–138, Feb. 2010.

1065 [11] S. Bayat, R. H. Y. Louie, Z. Han, B. Vucetic, and Y. Li, "Distributed  
1066 user association and femtocell allocation in heterogeneous wireless  
1067 networks," *IEEE Trans. Commun.*, vol. 62, no. 8, pp. 3027–3043,  
1068 Aug. 2014.

1069 [12] M. Mirahmadi, A. Al-Dweik, and A. Shami, "Interference modeling  
1070 and performance evaluation of heterogeneous cellular networks," *IEEE*  
1071 *Trans. Commun.*, vol. 62, no. 6, pp. 2132–2144, Jun. 2014.

1072 [13] A. K. Gupta, H. S. Dhillon, S. Vishwanath, and J. G. Andrews, "Down-  
1073 link multi-antenna heterogeneous cellular network with load balancing,"  
1074 *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4052–4067, Nov. 2014.

1075 [14] M. Liebsch, S. Schmid, and J. Awano, "Reducing backhaul costs for  
1076 mobile content delivery—An analytical study," in *Proc. IEEE Int. Conf.*  
1077 *Commun. (ICC)*, Jun. 2012, pp. 2895–2900.

1078 [15] K. Shanmugam, N. Golrezaei, A. G. Dimakis, A. F. Molisch, and  
1079 G. Caire, "FemtoCaching: Wireless content delivery through distrib-  
1080 uted caching helpers," *IEEE Trans. Inf. Theory*, vol. 59, no. 12,  
1081 pp. 8402–8413, Dec. 2013.

1082 [16] E. Baştuğ, M. Bennis, and M. Debbah, "Cache-enabled small cell  
1083 networks: Modeling and tradeoffs," in *Proc. 11th Int. Symp. Wireless*  
1084 *Commun. Syst. (ISWCS)*, Aug. 2014, pp. 649–653.

1085 [17] D. Stoyan, W. S. Kendall, and M. Mecke, *Stochastic Geometry and Its*  
1086 *Applications*. 2nd ed. New York, NY, USA: Wiley, 2003.

1087 [18] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and  
1088 M. Franceschetti, "Stochastic geometry and random graphs for the  
1089 analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*,  
1090 vol. 27, no. 7, pp. 1029–1046, Sep. 2009.

1091 [19] G. Vazquez-Vilar, C. Mosquera, and S. K. Jayaweera, "Primary user  
1092 enters the game: Performance of dynamic spectrum leasing in cogni-  
1093 tive radio networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12,  
1094 pp. 3625–3629, Dec. 2010.

1095 [20] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for  
1096 spectrum-sharing femtocell networks: A Stackelberg game approach,"  
1097 *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 538–549, Apr. 2012.

1098 [21] D. Niyato and E. Hossain, "Competitive spectrum sharing in cognitive  
1099 radio networks: A dynamic game approach," *IEEE Trans. Wireless*  
1100 *Commun.*, vol. 7, no. 7, pp. 2651–2660, Jul. 2008.

1101 [22] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and  
1102 multiple-buyer spectrum trading in cognitive radio networks: A game-  
1103 theoretic modeling approach," *IEEE Trans. Mobile Comput.*, vol. 8,  
1104 no. 8, pp. 1009–1022, Aug. 2009.

1105 [23] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA, USA:  
1106 MIT Press, 1993.

1107 [24] D. J. Daley and D. Vere-Jones, *An Introduction to the Theory of Point*  
1108 *Processes: Elementary Theory and Methods*, vol. 1. Springer, 1996. AQ:4

1109 [25] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "iTube, You  
1110 Tube, everybody tubes: Analyzing the world's largest user generated  
1111 content video system," in *Proc. 7th ACM SIGCOMM Conf. Internet*  
1112 *Meas.*, 2007, pp. 1–14. AQ:5



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IEEE PROOF



# Pricing and Resource Allocation via Game Theory for a Small-Cell Video Caching System

Jun Li, *Member, IEEE*, He Chen, *Member, IEEE*, Youjia Chen, Zihuai Lin, *Senior Member, IEEE*, Branka Vucetic, *Fellow, IEEE*, and Lajos Hanzo, *Fellow, IEEE*

**Abstract**—Evidence indicates that downloading on-demand videos accounts for a dramatic increase in data traffic over cellular networks. Caching popular videos in the storage of small-cell base stations (SBS), namely, small-cell caching, is an efficient technology for reducing the transmission latency while mitigating the redundant transmissions of popular videos over back-haul channels. In this paper, we consider a commercialized small-cell caching system consisting of a network service provider (NSP), several video retailers (VRs), and mobile users (MUs). The NSP leases its SBSs to the VRs for the purpose of making profits, and the VRs, after storing popular videos in the rented SBSs, can provide faster local video transmissions to the MUs, thereby gaining more profits. We conceive this system within the framework of Stackelberg game by treating the SBSs as specific types of resources. We first model the MUs and SBSs as two independent Poisson point processes, and develop, via stochastic geometry theory, the probability of the specific event that an MU obtains the video of its choice directly from the memory of an SBS. Then, based on the probability derived, we formulate a Stackelberg game to jointly maximize the average profit of both the NSP and the VRs. In addition, we investigate the Stackelberg equilibrium by solving a non-convex optimization problem. With the aid of this game theoretic framework, we shed light on the relationship between four important factors: the optimal pricing of leasing an SBS, the SBSs allocation among the VRs, the storage size of the SBSs, and the popularity distribution of the VRs. Monte Carlo simulations show that our stochastic geometry-based analytical results closely match the empirical ones. Numerical results are also provided for quantifying the proposed game-theoretic framework by showing its efficiency on pricing and resource allocation.

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Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

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**Index Terms**—Small-cell caching, cellular networks, stochastic geometry, Stackelberg game.

## I. INTRODUCTION

WIRELESS data traffic is expected to increase exponentially in the next few years driven by a staggering proliferation of mobile users (MU) and their bandwidth-hungry mobile applications. There is evidence that streaming of on-demand videos by the MUs is the major reason for boosting the tele-traffic over cellular networks [1]. According to the prediction of mobile data traffic by Cisco, mobile video streaming will account for 72% of the overall mobile data traffic by 2019. The on-demand video downloading involves repeated wireless transmission of videos that are requested multiple times by different users in a completely asynchronous manner, which is different from the transmission style of live video streaming.

Often, there are numerous repetitive requests of popular videos from the MUs, such as online blockbusters, leading to redundant video transmissions. The redundancy of data transmissions can be reduced by locally storing popular videos, known as caching, into the storage of intermediate network nodes, effectively forming a local caching system [1], [2]. The local caching brings video content closer to the MUs and alleviates redundant data transmissions via redirecting the downloading requests to the intermediate nodes.

Generally, wireless data caching consists of two stages: data placement and data delivery [3]. In the data placement stage, popular videos are cached into local storages during off-peak periods, while during the data delivery stage, videos requested are delivered from the local caching system to the MUs. Recent works advanced the caching solutions of both device-to-device (D2D) networks and wireless sensor networks [4]–[6]. Specifically, in [4] a caching scheme was proposed for a D2D based cellular network relaying on the MUs' caching of popular video content. In this scheme, the D2D cluster size was optimized for reducing the downloading delay. In [5] and [6], the authors proposed novel caching schemes for wireless sensor networks, where the protocol model of [7] was adopted.

Since small-cell embedded architectures will dominate in future cellular networks, known as heterogeneous networks (HetNet) [8]–[13], caching relying on small-cell base stations (SBS), namely, small-cell caching, constitutes a promising solution for HetNets. The advantages brought about by small-cell caching are threefold. Firstly, popular videos are placed closer to the MUs when they are cached in SBSs, hence

78 reducing the transmission latency. Secondly, redundant trans-  
 79 missions over SBSs' back-haul channels, which are usually  
 80 expensive [14], can be mitigated. Thirdly, the majority of video  
 81 traffic is offloaded from macro-cell base stations to SBSs.

82 In [15], a small-cell caching scheme, named  
 83 'Femtocaching', is proposed for a cellular network having  
 84 embedded SBSs, where the data placement at the SBSs is  
 85 optimized in a centralized manner for the sake of reducing  
 86 the transmission delay imposed. However, [15] considers an  
 87 idealized system, where neither the interference nor the impact  
 88 of wireless channels is taken into account. The associations  
 89 between the MUs and the SBSs are pre-determined without  
 90 considering the specific channel conditions encountered.  
 91 In [16], small-cell caching is investigated in the context of  
 92 stochastic networks. The average performance is quantified  
 93 with the aid of stochastic geometry [17], [18], where the  
 94 distribution of network nodes is modeled by Poisson point  
 95 process (PPP). However, the caching strategy of [16] assumes  
 96 that the SBSs cache the same content, hence leading to a  
 97 sub-optimal solution.

98 As detailed above, current research on wireless caching  
 99 mainly considers the data placement issue optimized for reduc-  
 100 ing the downloading delay. However, the entire caching system  
 101 design involves numerous issues apart from data placement.  
 102 From a commercial perspective, it will be more interesting  
 103 to consider the topics of pricing for video streaming, the  
 104 rental of local storage, and so on. A commercialized caching  
 105 system may consist of video retailers (VR), network service  
 106 providers (NSP) and MUs. The VRs, e.g., Youtube, purchase  
 107 copyrights from video producers and publish the videos on  
 108 their web-sites. The NSPs are typically operators of cellular  
 109 networks, who are in charge of network facilities, such as  
 110 macro-cell base stations and SBSs.

111 In such a commercial small-cell caching system, the VRs'  
 112 revenue is acquired from providing video streaming for  
 113 the MUs. As the central servers of the VRs, which store  
 114 the popular videos, are usually located in the backbone net-  
 115 works and far away from the MUs, an efficient solution is  
 116 to locally cache these videos, thereby gaining more profits  
 117 from providing faster local transmissions. In turn, these local  
 118 caching demands raised by the VRs offer the NSPs profit-  
 119 able opportunities from leasing their SBSs. Additionally, the  
 120 NSPs can save considerable costs due to reduced redundant  
 121 video transmissions over SBSs' back-haul channels. In this  
 122 sense, both the VRs and NSPs are the beneficiaries of the  
 123 local caching system. However, each entity is selfish and  
 124 wishes to maximize its own benefit, raising a competition  
 125 and optimization problem among these entities, which can be  
 126 effectively solved within the framework of game theory.

127 We note that game theory has been successfully applied  
 128 to wireless communications for solving resource allocation  
 129 problems. In [19], the authors propose a dynamic spectrum  
 130 leasing mechanism via power control games. In [20],  
 131 a price-based power allocation scheme is proposed for spec-  
 132 trum sharing in Femto-cell networks based on Stackelberg  
 133 game. Game theoretical power control strategies for maxi-  
 134 mizing the utility in spectrum sharing networks are studied  
 135 in [21] and [22].

In this paper, we propose a commercial small-cell caching  
 system consisting of an NSP, multiple VRs and MUs. We opti-  
 mize such a system within the framework of Stackelberg game  
 by viewing the SBSs as a specific type of resources for the  
 purpose of video caching. Generally speaking, Stackelberg  
 game is a strategic game that consists of a leader and several  
 followers competing with each other for certain resources [23].  
 The leader moves first and the followers move subsequently.  
 Correspondingly, in our game theoretic caching system, we  
 consider the NSP to be the leader and the VRs as the followers.  
 The NSP sets the price of leasing an SBS, while the VRs  
 compete with each other for renting a fraction of the SBSs.

To the best of the authors' knowledge, our work is the first  
 of its kind that optimizes a caching system with the aid of  
 game theory. Compared to many other game theory based  
 resource allocation schemes, where the power, bandwidth  
 and time slots are treated as the resources, our work has  
 a totally different profit model, established based on our  
 coverage derivations. In particular, our contributions are as  
 follows.

- 1) By following the stochastic geometry framework  
 of [17] and [18], we model the MUs and SBSs in  
 the network as two different ties of a Poisson point  
 process (PPP) [24]. Under this network model, we define  
 the concept of a successful video downloading event  
 when an MU obtains the requested video directly from  
 the storage of an SBS. Then we quantify the probability  
 of this event based on stochastic geometry theory.
- 2) Based on the probability derived, we develop a profit  
 model of our caching system and formulate the profits  
 gained by the NSP and the VRs from SBSs leasing and  
 renting.
- 3) A Stackelberg game is proposed for jointly maximizing  
 the average profit of the NSP and the VRs. Given this  
 game theoretic framework, we investigate a non-uniform  
 pricing scheme, where the price charged to different VRs  
 varies.
- 4) Then we investigate the Stackelberg equilibrium of this  
 scheme via solving a non-convex optimization problem.  
 It is interesting to observe that the optimal solution is  
 related both to the storage size of each SBS and to the  
 popularity distribution of the VRs.
- 5) Furthermore, we consider an uniform pricing scheme.  
 We find that although the uniform pricing scheme is  
 inferior to the non-uniform one in terms of maximizing  
 the NSP's profit, it is capable of reducing more back-  
 haul costs compared with the latter and achieves the  
 maximum sum profit of the NSP and the VRs.

The rest of this paper is organized as follows. We describe  
 the system model in Section II and establish the related profit  
 model in Section III. We then formulate Stackelberg game for  
 our small-cell caching system in Section IV. In Section V,  
 we investigate Stackelberg equilibrium for the non-uniform  
 pricing scheme by solving a non-convex optimization prob-  
 lem, while in Section VI, we further consider the uniform  
 pricing scheme. Our simulations and numerical results are  
 detailed in Section VII, while our conclusions are provided  
 in Section VIII.

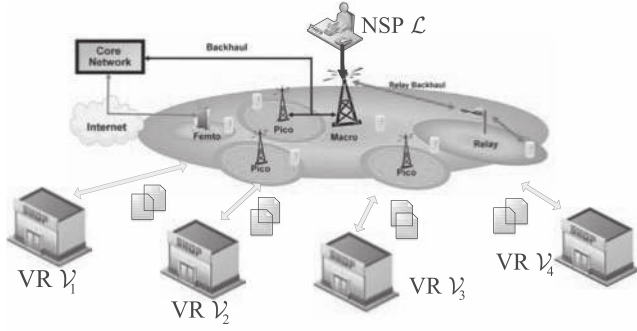


Fig. 1. An example of the small-cell caching system with four VRs.

## II. SYSTEM MODEL

We consider a commercial small-cell caching system consisting of an NSP,  $V$  VRs, and a number of MUs. Let us denote by  $\mathcal{L}$  the NSP, by  $\mathcal{V} = \{\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_V\}$  the set of the VRs, and by  $\mathcal{M}$  one of the MUs. Fig. 1 shows an example of our caching system relying on four VRs. In such a system, the VRs wish to rent the SBSs from  $\mathcal{L}$  for placing their videos. Both the NSP and each VR aim for maximizing their profits.

There are three stages in our system. In the first stage, the VRs purchase the copyrights of popular videos from video producers and publish them on their web-sites. In the second stage, the VRs negotiate with the NSP on the rent of SBSs for caching these popular videos. In the third stage, the MUs connect to the SBSs for downloading the desired videos. We will particularly focus our attention on the second and third stages within this game theoretic framework.

### A. Network Model

Let us consider a small-cell based caching network composed of the MUs and the SBSs owned by  $\mathcal{L}$ , where each SBS is deployed with a fixed transmit power  $P$  and the storage of  $Q$  video files. Let us assume that the SBSs transmit over the channels that are orthogonal to those of the macro-cell base stations, and thus there is no interference incurred by the macro-cell base stations. Also, assume that these SBSs are spatially distributed according to a homogeneous PPP (HPPP)  $\Phi$  of intensity  $\lambda$ . Here, the intensity  $\lambda$  represents the number of the SBSs per unit area. Furthermore, we model the distribution of the MUs as an independent HPPP  $\Psi$  of intensity  $\zeta$ .

The wireless down-link channels spanning from the SBSs to the MUs are independent and identically distributed (*i.i.d.*), and modeled as the combination of path-loss and Rayleigh fading. Without loss of generality, we carry out our analysis for a typical MU located at the origin. The path-loss between an SBS located at  $x$  and the typical MU is denoted by  $\|x\|^{-\alpha}$ , where  $\alpha$  is the path-loss exponent. The channel power of the Rayleigh fading between them is denoted by  $h_x$ , where  $h_x \sim \exp(1)$ . The noise at an MU is Gaussian distributed with a variance  $\sigma^2$ .

We consider the steady-state of a saturated network, where all the SBSs keep on transmitting data in the entire frequency band allocated. This modeling approach for saturated networks characterizes the worst-case scenario of the real systems, which has been adopted by numerous studies on PPP analysis,

such as [18]. Hence, the received signal-to-interference-plus-noise ratio (SINR) at the typical MU from an SBS located at  $x$  can be expressed as

$$\rho(x) = \frac{Ph_x\|x\|^{-\alpha}}{\sum_{x' \in \Phi \setminus x} Ph_{x'}\|x'\|^{-\alpha} + \sigma^2}. \quad (1)$$

The typical MU is considered to be “covered” by an SBS located at  $x$  as long as  $\rho(x)$  is no lower than a pre-set SINR threshold  $\delta$ , i.e.,

$$\rho(x) \geq \delta. \quad (2)$$

Generally, an MU can be covered by multiple SBSs. Note that the SINR threshold  $\delta$  defines the highest delay of downloading a video file. Since the quality and code rate of a video clip have been specified within the video file, the download delay will be the major factor predetermining the QoS perceived by the mobile users. Therefore, we focus our attention on the coverage and SINR in the following derivations.

### B. Popularity and Preferences

We now model the popularity distribution, i.e., the distribution of request probabilities, among the popular videos to be cached. Let us denote by  $\mathcal{F} = \{\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_N\}$  the file set consisting of  $N$  video files, where each video file contains an individual movie or video clip that is frequently requested by MUs. The popularity distribution of  $\mathcal{F}$  is represented by a vector  $\mathbf{t} = [t_1, t_2, \dots, t_N]$ . That is, the MUs make independent requests of the  $n$ -th video  $\mathcal{F}_n$ ,  $n = 1, \dots, N$ , with the probability of  $t_n$ . Generally,  $\mathbf{t}$  can be modeled by the Zipf distribution [25] as

$$t_n = \frac{1/n^\beta}{\sum_{j=1}^N 1/j^\beta}, \quad \forall n, \quad (3)$$

where the exponent  $\beta$  is a positive value, characterizing the video popularity. A higher  $\beta$  corresponds to a higher content reuse, where the most popular files account for the majority of download requests. From Eq. (3), the file with a smaller  $n$  corresponds to a higher popularity.

Note that each SBS can cache at most  $Q$  video files, and usually  $Q$  is no higher than the number of videos in  $\mathcal{F}$ , i.e., we have  $Q \leq N$ . Without loss of generality, we assume that  $N/Q$  is an integer. The  $N$  files in  $\mathcal{F}$  are divided into  $F = N/Q$  file groups (FG), with each FG containing  $Q$  video files. The  $n$ -th video,  $\forall n \in \{(f-1)Q + 1, \dots, fQ\}$ , is included in the  $f$ -th FG,  $f = 1, \dots, F$ . Denote by  $\mathcal{G}_f$  the  $f$ -th FG, and by  $p_f$  the probability of the MUs’ requesting a file in  $\mathcal{G}_f$ , and we have

$$p_f = \sum_{n=(f-1)Q+1}^{fQ} t_n, \quad \forall f. \quad (4)$$

File caching is then carried out on the basis of FGs, where each SBS caches one of the  $F$  FGs.

At the same time, the MUs have unbalanced preferences with regard to the  $V$  VRs, i.e., some VRs are more popular than others. For example, the majority of the MUs may tend to access Youtube for video streaming. The preference distribution among the VRs is denoted by  $\mathbf{q} = [q_1, q_2, \dots, q_V]$ ,

where  $q_v$ ,  $v = 1, \dots, V$ , represents the probability that MUs prefer to download videos from  $\mathcal{V}_v$ . The preference distribution  $\mathbf{q}$  can also be modeled by the Zipf distribution. Hence, we have

$$q_v = \frac{1/v^\gamma}{\sum_{j=1}^V 1/j^\gamma}, \quad \forall v, \quad (5)$$

where  $\gamma$  is a positive value, characterizing the preference of the VRs. A higher  $\gamma$  corresponds to a higher probability of accessing the most popular VRs.

### C. Video Placement and Download

Next, we introduce the small-cell caching system with its detailed parameters. In the first stage, each VR purchases the  $N$  popular videos in  $\mathcal{F}$  from the producers and publishes these videos on its web-site. In the second stage, upon obtaining these videos, the VRs negotiate with the NSP  $\mathcal{L}$  for renting its SBSs. As  $\mathcal{L}$  leases its SBSs to multiple VRs, we denote by  $\boldsymbol{\tau} = [\tau_1, \tau_2, \dots, \tau_V]$  the fraction vector, where  $\tau_v$  represents the fraction of the SBSs that are assigned to  $\mathcal{V}_v$ ,  $\forall v$ . We assume that the SBSs rented by each VR are uniformly distributed. Hence, the SBSs that are allocated to  $\mathcal{V}_v$  can be modeled as a ‘‘thinned’’ HPPP  $\Phi_v$  with intensity  $\tau_v \lambda$ .

The data placements of the second stage commence during network off-peak time after the VRs obtain access to the SBSs. During the placements, each SBS will be allocated with one of the  $F$  FGs. Generally, we assume that the VRs do not have the *a priori* information regarding the popularity distribution of  $\mathcal{F}$ . This is because the popularity of videos is changing periodically, and can only be obtained statistically after these videos quit the market. It is clear that each VR may have more or less some statistical information on the popularity distribution of videos based on the MUs’ downloading history. However, this information will be biased due to limited sampling. In this case, the VRs will uniformly assign the  $F$  FGs to the SBSs with equal probability of  $\frac{1}{F}$  for simplicity. We are interested in investigating the uniform assignment of video files for drawing a bottom line of the system performance. As the FGs are randomly assigned, the SBSs in  $\Phi_v$  that cache the FG  $\mathcal{G}_f$  can be further modeled as a ‘‘more thinned’’ HPPP  $\Phi_{v,f}$  with an intensity of  $\frac{1}{F} \tau_v \lambda$ .

In the third stage, the MUs start to download videos. When an MU  $\mathcal{M}$  requires a video of  $\mathcal{G}_f$  from  $\mathcal{V}_v$ , it searches the SBSs in  $\Phi_{v,f}$  and tries to connect to the nearest SBS that covers  $\mathcal{M}$ . Provided that such an SBS exists, the MU  $\mathcal{M}$  will obtain this video directly from this SBS, and we thereby define this event by  $\mathcal{E}_{v,f}$ . By contrast, if such an SBS does not exist,  $\mathcal{M}$  will be redirected to the central servers of  $\mathcal{V}_v$  for downloading the requested file. Since the servers of  $\mathcal{V}_v$  are located at the backbone network, this redirection of the demand will trigger a transmission via the back-haul channels of the NSP  $\mathcal{L}$ , hence leading to an extra cost.

## III. PROFIT MODELING

We now focus on modeling the profit of the NSP and the VRs obtained from the small-cell caching system. The average profit is developed based on stochastically geometrical

distributions of the network nodes in terms of per unit area times unit period ( $/UAP$ ), e.g.,  $/month \cdot km^2$ .

### A. Average Profit of the NSP

For the NSP  $\mathcal{L}$ , the revenue gained from the caching system consists of two parts: 1) the income gleaned from leasing SBSs to the VRs and 2) the cost reduction due to reduced usage of the SBSs’ back-haul channels. First, the leasing income/ $UAP$  of  $\mathcal{L}$  can be calculated as

$$S^{RT} = \sum_{j=1}^V \tau_j \lambda s_j, \quad (6)$$

where  $s_j$  is the price per unit period charged to  $\mathcal{V}_j$  for renting an SBS. Then we formulate the saved cost/ $UAP$  due to reduced back-haul channel transmissions. When an MU demands a video in  $\mathcal{G}_f$  from  $\mathcal{V}_v$ , we derive the probability  $\Pr(\mathcal{E}_{v,f})$  as follows.

*Theorem 1:* The probability of the event  $\mathcal{E}_{v,f}$ ,  $\forall v, f$ , can be expressed as

$$\Pr(\mathcal{E}_{v,f}) = \frac{\tau_v}{C(\delta, \alpha)(F - \tau_v) + A(\delta, \alpha)\tau_v + \tau_v}, \quad (7)$$

where we have  $A(\delta, \alpha) \triangleq \frac{2\delta}{\alpha-2} {}_2F_1(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta)$  and  $C(\delta, \alpha) \triangleq \frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B(\frac{2}{\alpha}, 1 - \frac{2}{\alpha})$ . Furthermore,  ${}_2F_1(\cdot)$  in the function  $A(\delta, \alpha)$  is the hypergeometric function, while the Beta function in  $C(\delta, \alpha)$  is formulated as  $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$ .

*Proof:* Please refer to Appendix A. ■

*Remark 1:* From *Theorem 1*, it is interesting to observe that the probability  $\Pr(\mathcal{E}_{v,f})$  is independent of both the transmit power  $P$  and the intensity  $\lambda$  of the SBSs. Furthermore, since  $Q$  is inversely proportional to  $F$ , we can enhance  $\Pr(\mathcal{E}_{v,f})$  by increasing the storage size  $Q$ .

We assume that there are on average  $K$  video requests from each MU within unit period, and that the average back-haul cost for a video transmission is  $s^{bh}$ . Based on  $\Pr(\mathcal{E}_{v,f})$  in Eq. (7), we obtain the cost reduction/ $UAP$  for the back-haul channels of  $\mathcal{L}$  as

$$S^{BH} = \sum_{j_1=1}^F \sum_{j_2=1}^V p_{j_1} q_{j_2} \zeta K \Pr(\mathcal{E}_{j_2, j_1}) s^{bh}. \quad (8)$$

By combining the above two items, the overall profit/ $UAP$  for  $\mathcal{L}$  can be expressed as

$$S^{NSP} = S^{RT} + S^{BH}. \quad (9)$$

### B. Average Profit of the VRs

Note that the MUs can download the videos either from the memories of the SBSs directly or from the servers of the VRs at backbone networks via back-haul channels. In the first case, the MUs will be levied by the VRs an extra amount of money in addition to the videos’ prices because of the higher-rate local streaming, namely, local downloading surcharge (LDS). We assume that the LDS of each video is set as  $s^{ld}$ . Then the revenue/ $UAP$  for a VR  $\mathcal{V}_v$  gained from the LDS can be calculated as

$$S_v^{LD} = \sum_{j=1}^F p_j q_v \zeta K \Pr(\mathcal{E}_{v,j}) s^{ld}. \quad (10)$$

387 Additionally,  $\mathcal{V}_v$  pays for renting the SBSs from  $\mathcal{L}$ . The related  
388 cost/*UAP* can be written as

$$389 \quad S_v^{RT} = \tau_v \lambda s_v. \quad (11)$$

390 Upon combining the two items, the profit/*UAP* for  $\mathcal{V}_v$ ,  $\forall v$ ,  
391 can be expressed as

$$392 \quad S_v^{VR} = S_v^{LD} - S_v^{RT}. \quad (12)$$

#### 393 IV. PROBLEM FORMULATION

394 In this section, we first present the Stackelberg game for-  
395 mulation for our price-based SBS allocation scheme. Then the  
396 equilibrium of the proposed game is investigated.

##### 397 A. Stackelberg Game Formulation

398 Again, Stackelberg game is a strategic game that consists of  
399 a leader and several followers competing with each other for  
400 certain resources [23]. The leader moves first and the followers  
401 move subsequently. In our small-cell caching system, we  
402 model the NSP  $\mathcal{L}$  as the leader, and the  $V$  VRs as the followers.  
403 The NSP imposes a price vector  $\mathbf{s} = [s_1, s_2, \dots, s_V]$  for  
404 the lease of its SBSs, where  $s_v$ ,  $\forall v$ , has been defined in the  
405 previous section as the price per unit period charged on  $\mathcal{V}_v$   
406 for renting an SBS. After the price vector  $\mathbf{s}$  is set, the VRs  
407 update the fraction  $\tau_v$ ,  $\forall v$ , that they tend to rent from  $\mathcal{L}$ .

408 1) *Optimization Formulation of the Leader*: Observe from  
409 the above game model that the NSP's objective is to maximize  
410 its profit  $S^{NSP}$  formulated in Eq. (9). Note that for  $\forall v$ , the  
411 fraction  $\tau_v$  is a function of the price  $s_v$  under the Stackelberg  
412 game formulation. This means that the fraction of the SBSs  
413 that each VR is willing to rent depends on the specific price  
414 charged to them for renting an SBS. Consequently, the NSP  
415 has to find the optimal price vector  $\mathbf{s}$  for maximizing its profit.  
416 This optimization problem can be summarized as follows.

417 *Problem 1*: The optimization problem of maximizing  $\mathcal{L}$ 's  
418 profit can be formulated as

$$419 \quad \begin{aligned} & \max_{\mathbf{s} \geq \mathbf{0}} S^{NSP}(\mathbf{s}, \boldsymbol{\tau}), \\ & \text{s.t. } \sum_{j=1}^V \tau_j \leq 1. \end{aligned} \quad (13)$$

421 2) *Optimization Formulation of the Followers*: The profit  
422 gained by the VR  $\mathcal{V}_v$  in Eq. (12) can be further written as

$$423 \quad \begin{aligned} S_v^{VR}(\tau_v, s_v) &= \sum_{j=1}^F p_j q_v \zeta K \Pr(\mathcal{E}_{v,j}) s^{ld} - \tau_v \lambda s_v \\ 424 &= \sum_{j=1}^F \frac{p_j q_v \zeta K s^{ld} \tau_v}{(A(\delta, \alpha) - C(\delta, \alpha) + 1) \tau_v + C(\delta, \alpha) F} \\ 425 &\quad - \lambda s_v \tau_v. \end{aligned} \quad (14)$$

426 We can see from Eq. (14) that once the price  $s_v$  is fixed, the  
427 profit of  $\mathcal{V}_v$  depends on  $\tau_v$ , i.e., the fraction of SBSs that  
428 are rented by  $\mathcal{V}_v$ . If  $\mathcal{V}_v$  increases the fraction  $\tau_v$ , it will gain  
429 more revenue by levying surcharges from more MUs, while  
430 at the same time,  $\mathcal{V}_v$  will have to pay for renting more SBSs.

Therefore,  $\tau_v$  has to be optimized for maximizing the profit  
of  $\mathcal{V}_v$ . This optimization can be formulated as follows.

*Problem 2*: The optimization problem of maximizing  $\mathcal{V}_v$ 's  
profit can be written as

$$435 \quad \max_{\tau_v \geq 0} S_v^{VR}(\tau_v, s_v). \quad (15)$$

*Problem 1* and *Problem 2* together form a Stackelberg  
game. The objective of this game is to find the Stackelberg  
Equilibrium (SE) points from which neither the leader (NSP)  
nor the followers (VRs) have incentives to deviate. In the  
following, we investigate the SE points for the proposed game.

##### 441 B. Stackelberg Equilibrium

For our Stackelberg game, the SE is defined as follows.

*Definition 1*: Let  $\mathbf{s}^* \triangleq [s_1^*, s_2^*, \dots, s_V^*]$  be a solution for  
*Problem 1*, and  $\boldsymbol{\tau}_v^*$  be a solution for *Problem 2*,  $\forall v$ . Define  
 $\boldsymbol{\tau}^* \triangleq [\tau_1^*, \tau_2^*, \dots, \tau_V^*]$ . Then the point  $(\mathbf{s}^*, \boldsymbol{\tau}^*)$  is an SE for  
the proposed Stackelberg game if for any  $(\mathbf{s}, \boldsymbol{\tau})$  with  $\mathbf{s} \geq \mathbf{0}$   
and  $\boldsymbol{\tau} \geq \mathbf{0}$ , the following conditions are satisfied:

$$448 \quad \begin{aligned} S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*) &\geq S^{NSP}(\mathbf{s}, \boldsymbol{\tau}^*), \\ S_v^{VR}(\tau_v^*, \tau_v^*) &\geq S_v^{VR}(s_v^*, \tau_v), \quad \forall v. \end{aligned} \quad (16)$$

449 Generally speaking, the SE of a Stackelberg game can be  
450 obtained by finding its perfect Nash Equilibrium (NE). In our  
451 proposed game, we can see that the VRs strictly compete  
452 in a non-cooperative fashion. Therefore, a non-cooperative  
453 subgame on controlling the fractions of rented SBSs is for-  
454 mulated at the VRs' side. For a non-cooperative game, the  
455 NE is defined as the operating points at which no players can  
456 improve utility by changing its strategy unilaterally. At the  
457 NSP's side, since there is only one player, the best response  
458 of the NSP is to solve *Problem 1*. To achieve this, we need to  
459 first find the best response functions of the followers, based  
460 on which, we solve the best response function for the leader.

461 Therefore, in our game, we first solve *Problem 2* given a  
462 price vector  $\mathbf{s}$ . Then with the obtained best response function  
463  $\boldsymbol{\tau}^*$  of the VRs, we solve *Problem 1* for the optimal price  $\mathbf{s}^*$ . In  
464 the following, we will have an in-depth investigation on this  
465 game theoretic optimization.

#### 467 V. GAME THEORETIC OPTIMIZATION

468 In this section, we will solve the optimization problem in  
469 our game under the non-uniform pricing scheme, where the  
470 NSP  $\mathcal{L}$  charges the VRs with different prices  $s_1, \dots, s_V$  for  
471 renting an SBS. In this scheme, we first solve *Problem 2* at  
472 the VRs, and rewrite Eq. (14) as

$$473 \quad S_v^{VR}(\tau_v, s_v) = \frac{\Gamma_v s^{ld} \tau_v}{\Theta \tau_v + \Lambda} - \lambda s_v \tau_v. \quad (17)$$

474 where  $\Gamma_v \triangleq \sum_{j=1}^F p_j q_v \zeta K$ ,  $\Theta \triangleq A(\delta, \alpha) - C(\delta, \alpha) + 1$ , and  
475  $\Lambda \triangleq C(\delta, \alpha) F$ . We observe that Eq. (17) is a concave function  
476 over the variable  $\tau_v$ . Thus, we can obtain the optimal solution  
477 by solving the Karush-Kuhn-Tucker (KKT) conditions, and we  
478 have the following lemma.

479 *Lemma 1:* For a given price  $s_v$ , the optimal solution of  
480 *Problem 2* is

$$481 \tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{ld}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_v}} - \frac{\Lambda}{\Theta} \right)^+, \quad (18)$$

482 where  $(\cdot)^+ \triangleq \max(\cdot, 0)$ .

483 *Proof:* The optimal solution  $\tau_v^*$  of  $\mathcal{V}_v$  can be obtained by  
484 deriving  $S_v^{VR}$  with respect to  $\tau_v$  and solving  $\frac{dS_v^{VR}}{d\tau_v} = 0$  under  
485 the constraint that  $\tau_v \geq 0$ . ■

486 We can see from *Lemma 1* that if the price  $s_v$  is set too  
487 high, i.e.,  $s_v \geq \frac{\Gamma_v s^{ld}}{\Lambda \lambda}$ , the VR  $\mathcal{V}_v$  will opt out for renting any  
488 SBS from  $\mathcal{L}$  due the high price charged. Consequently, the  
489 VR  $\mathcal{V}_v$  will not participate in the game.

490 In the following derivations, we assume that the LDS on  
491 each video  $s^{ld}$  is set by the VRs to be the cost of a video trans-  
492 mission via back-haul channels  $s^{bh}$ . The rational behind this  
493 assumption is as follows. Since a local downloading reduce a  
494 back-haul transmission, this saved back-haul transmission can  
495 be potentially utilized to provide extra services (equivalent to  
496 the value of  $s^{bh}$ ) for the MUs. In addition, the MUs enjoy the  
497 benefit from faster local video transmissions. In light of this,  
498 it is reasonable to assume that the MUs are willing to accept  
499 the price  $s^{bh}$  for a local video transmission.

500 Substituting the optimal  $\tau_v^*$  of Eq. (18) into Eq. (9) and  
501 carry out some further manipulations, we arrive at

$$502 S^{NSP} = \sum_{j=1}^V \lambda s_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ \\ 503 + \frac{\sum_{i=1}^F p_i q_j \zeta K s^{bh} \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+}{\Theta \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ + \Lambda} \\ 504 = \sum_{j=1}^V \frac{\zeta_j}{\Theta} \left( -\Lambda \lambda s_j + \left( \sqrt{s^{bh}} - \frac{s^{bh}}{\sqrt{s^{bh}}} \right) \sqrt{\Gamma_j \Lambda \lambda s_j} + \Gamma_j s^{bh} \right) \\ 505 = \sum_{j=1}^V \frac{\zeta_j}{\Theta} \left( -\Lambda \lambda s_j + \Gamma_j s^{bh} \right), \quad (19)$$

506 where  $\zeta_j$  is the indicator function, with  $\zeta_j = 1$  if  $s_j < \frac{\Gamma_j s^{bh}}{\Lambda \lambda}$   
507 and  $\zeta_j = 0$  otherwise. Upon defining the binary vector  $\boldsymbol{\xi} \triangleq$   
508  $[\zeta_1, \zeta_2, \dots, \zeta_V]$ , we can rewrite *Problem 1* as follows.

509 *Problem 3:* Given the optimal solutions  $\tau_v^*$ ,  $\forall v$ , gleaned  
510 from the followers, we can rewrite *Problem 1* as

$$511 \min_{\boldsymbol{\xi}, s \geq \mathbf{0}} \sum_{j=1}^V \zeta_j \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right), \\ 512 \text{s.t.} \sum_{j=1}^V \zeta_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \quad (20)$$

513 Observe from Eq. (20) that *Problem 3* is non-convex due  
514 to  $\boldsymbol{\xi}$ . However, for a given  $\boldsymbol{\xi}$ , this problem can be solved by  
515 satisfying the KKT conditions. In the following, we commence  
516 with the assumption that  $\boldsymbol{\xi} = \mathbf{1}$ , i.e.,  $\zeta_v = 1, \forall v$ , and then we  
517 extend this result to the general case.

518 *A. Special Case:*  $\zeta_v = 1, \forall v$

519 In this case, all the VRs are participating in the game, and  
520 we have the following optimization problem.

521 *Problem 4:* Assuming  $\zeta_v = 1, \forall v$ , we rewrite *Problem 3* as

$$522 \min_{s \geq \mathbf{0}} \sum_{j=1}^V s_j, \\ 523 \text{s.t.} \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} \leq (V \Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda s^{bh}}}. \quad (21)$$

524 The optimal solution of *Problem 4* is derived and given in  
525 the following lemma.

526 *Lemma 2:* The optimal solution to *Problem 4* can be  
527 derived as  $\hat{\mathbf{s}} \triangleq [\hat{s}_1, \dots, \hat{s}_V]$ , where

$$528 \hat{s}_v = \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda (V \Lambda + \Theta)^2}, \quad \forall v. \quad (22)$$

529 *Proof:* Please refer to Appendix B. ■

530 Note that the solution given in *Lemma 2* is found under  
531 the assumption that  $\zeta_v = 1, \forall v$ . That is,  $\hat{s}_v$  given in Eq. (22)  
532 should ensure that  $\tau_v^* > 0, \forall v$ , in Eq. (18), i.e.,

$$533 \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda (V \Lambda + \Theta)^2} < \frac{\Gamma_v s^{bh}}{\Lambda \lambda}. \quad (23)$$

534 Given the definitions of  $\Gamma_v, \Lambda$ , and  $\Theta$ , it is interesting to find  
535 that the inequality (23) can be finally converted to a constraint  
536 on the storage size  $Q$  of each SBS, which is formulated as

$$537 Q > \max \left\{ \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \forall v \right\}. \quad (24)$$

538 The constraint imposed on  $Q$  can be expressed in a concise  
539 manner in the following theorem.

540 *Theorem 2:* To make sure that  $\hat{s}_v$  in Eq. (22) does become  
541 the optimal solution of *Problem 4* when  $\zeta_v = 1, \forall v$ , the  
542 sufficient and necessary condition to be satisfied is

$$543 Q > Q_{min} \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \quad (25)$$

544 where  $q_v$  is the minimum value in  $\mathbf{q}$  according to Eq. (5).

545 *Proof:* Please refer to Appendix C. ■

546 *Remark 2:* Observe from Eq. (25) that since  $\frac{q_j}{q_v}$  increases  
547 exponentially with  $\gamma$  according to Eq. (5), the value of  $Q_{min}$   
548 ensuring  $\zeta_v = 1, \forall v$ , will increase exponentially with  $\gamma/3$ .

549 Note that we have  $Q \leq N$ . In the case that  $Q_{min}$  in Eq. (25)  
550 is larger than  $N$  for a high VR popularity exponent  $\gamma$ , some  
551 VRs with the least popularity will be excluded from the game.

552 *B. Further Discussion on Q*

553 We define a series of variables  $U_v, \forall v$ , as follows:

$$554 U_v \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^v \sqrt[3]{\frac{q_j}{q_v}} - v \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, \quad (26)$$

555 and formulate the following lemma.

*Lemma 3:*  $U_v$  is a strictly monotonically-increasing function of  $v$ , i.e., we have  $U_V > U_{V-1} > \dots > U_1$ .

*Proof:* Please refer to Appendix D. ■

For the special case of the previous subsection, the optimal solution for  $\zeta_v = 1, \forall v$ , is found under the condition that the storage size obeys  $Q > U_V$ . In other words,  $Q$  should be large enough such that every VR can participate in the game. However, when  $Q$  reduces, some VRs have to leave the game as a result of the increased competition. Then we have the following lemma.

*Lemma 4:* When  $U_v < Q \leq U_{v+1}$ , the NSP can only retain at most the  $v$  VRs of  $\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_v$  in the game for achieving its optimal solution.

*Proof:* Please refer to Appendix E. ■

From *Lemma 4*, when we have  $U_v < Q \leq U_{v+1}$ , and given that there are  $u$  VRs,  $u \leq v$ , in the game, we can have an optimal solution for  $s$ .

*Problem 5:* When  $U_v < Q \leq U_{v+1}$  is satisfied, and given that there are  $u, u \leq v$ , VRs in the game, we can formulate the following optimization problem as

$$\begin{aligned} \min_{\mathbf{s} \geq \mathbf{0}} \quad & \sum_{j=1}^u s_j, \\ \text{s.t.} \quad & \sum_{j=1}^u \sqrt{\frac{\Gamma_j}{s_j}} \leq (u\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda_S^{bh}}}. \end{aligned} \quad (27)$$

Similar to the solution of *Problem 4*, we arrive at the optimal solution for the above problem as  $\hat{\mathbf{s}}_u \triangleq [\hat{s}_{1,u}, \dots, \hat{s}_{i,u}, \dots, \hat{s}_{v,u}]$ , where

$$\hat{s}_{i,u} = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^u \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_i}}{\lambda(u\Lambda + \Theta)^2}, & i = 1, \dots, u, \\ \infty, & i = u + 1, \dots, v. \end{cases} \quad (28)$$

### C. General Case

Let us now focus our attention on the general solution of the original optimization problem, i.e., of *Problem 3*. Without loss of generality, we consider the case of  $U_v < Q \leq U_{v+1}$ . Then *Problem 3* is equivalent to the following problem.

*Problem 6:* When  $U_v < Q \leq U_{v+1}$ , there are at most  $v$  VRs in the game. Then *Problem 3* can be converted to

$$\begin{aligned} \min_{\xi, \mathbf{s} \geq \mathbf{0}} \quad & \sum_{j=1}^v \zeta_j \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right), \\ \text{s.t.} \quad & \sum_{j=1}^v \zeta_j \left( \sqrt{\frac{\Gamma_j \Lambda_S^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \end{aligned} \quad (29)$$

The problem in Eq. (29) is again non-convex due to the uncertainty of  $\zeta_u, u = 1, \dots, v$ . We have to consider the cases, where there are  $u, \forall u$ , most popular VRs in the game. We observe that for a given  $u$ , *Problem 6* converts to *Problem 5*. Therefore, to solve *Problem 6*, we first solve *Problem 5* with a given  $u$  and obtain  $\hat{\mathbf{s}}_u$  according to Eq. (28).

TABLE I  
THE CENTRALIZED ALGORITHM AT THE NSP FOR  
OBTAINING THE OPTIMAL SOLUTION  $\mathbf{s}^*$

#### Algorithm 1 :

**Input:** Storage size  $Q$ , number of videos  $N$ , VRs' preference distribution  $\mathbf{q}$ , channel exponent  $\alpha$ , and pre-set threshold  $\delta$ .

**Output:** Optimal pricing vector  $\mathbf{s}^*$ .

**Steps:**

- 1: Based on  $N, \mathbf{q}, \alpha$ , and  $\delta$ , the NSP calculates  $U_v, \forall v$ , according to Eq. (26);
- 2: By comparing  $Q$  to  $U_v$ , the NSP obtains the value of the integer  $T$  in Eq. (33);
- 3: Calculate  $S_u, u = 1, 2, \dots, T$ , according to Eq. (33);
- 4: Compare among  $S_1, \dots, S_T$  for finding the index  $\hat{u}$  of the minimum  $S_{\hat{u}}$ ;
- 5: Based on  $\hat{u}, N, \mathbf{q}, \alpha$ , and  $\delta$ , the NSP obtains the optimal solution  $\mathbf{s}^*$  according to Eq. (31).

Then we choose the optimal solution, denoted by  $\mathbf{s}_v^*$ , among  $\hat{\mathbf{s}}_1, \dots, \hat{\mathbf{s}}_v$  as the solution to *Problem 6*, which is formulated as

$$\begin{aligned} \mathbf{s}_v^* &= \arg \min_{\hat{\mathbf{s}}_u} \left\{ \min \left( \sum_{j=1}^u \left( \Lambda \lambda s_j - \Gamma_j s^{bh} \right) \right), u = 1, \dots, v \right\}. \end{aligned} \quad (30)$$

Based on the above discussions, we can see that the optimal solution  $\mathbf{s}^*$  of *Problem 3* is a piece-wise function of  $Q$ , i.e.,  $\mathbf{s}^* = \mathbf{s}_v^*$  when  $U_v < Q \leq U_{v+1}$ . Now, we formulate the solution  $\mathbf{s}^* = [s_1^*, \dots, s_V^*]$  to *Problem 3* in a general manner as follows.

$$\mathbf{s}_v^* = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^{\hat{u}} \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda(\hat{u}\Lambda + \Theta)^2}, & v = 1, \dots, \hat{u}, \\ \infty, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (31)$$

where regarding  $\hat{u}$ , we have

$$\hat{u} = \arg \min_u \{S_u : u = 1, 2, \dots, T\}, \quad (32)$$

with  $S_u$  formulated as

$$\begin{aligned} S_u &= \sum_{j_1=1}^u \left( \frac{\Lambda^2 s^{bh} \left( \sum_{j_2=1}^u \sqrt[3]{\Gamma_{j_2}} \right)^2 \sqrt[3]{\Gamma_{j_1}}}{(u\Lambda + \Theta)^2} - \Gamma_{j_1} s^{bh} \right), \\ T &= \begin{cases} 1, & U_1 < Q \leq U_2, \\ \dots, & \\ v, & U_v < Q \leq U_{v+1}, \\ \dots, & \\ V, & U_V < Q. \end{cases} \end{aligned} \quad (33)$$

To gain a better understanding of the optimal solution in Eq. (31), we propose a centralized algorithm at  $\mathcal{L}$  in Table I for obtaining  $\mathbf{s}^*$ .

*Remark 3:* The optimal solution  $\mathbf{s}^*$  in Eq. (31), combined with the solution of  $\boldsymbol{\tau}^*$  given by Eq. (18) in *Lemma 1*, constitutes the SE for the Stackelberg game.

Furthermore, by substituting the optimal  $\mathbf{s}^*$  into the expression of  $S^{NSP}$  in Eq. (19), we get

$$S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*) = \frac{1}{\Theta} \sum_{j=1}^{\hat{u}} \left( \Gamma_{j_1} s^{bh} - \frac{\Lambda^2 s^{bh} \left( \sum_{j_2=1}^{\hat{u}} \sqrt[3]{\Gamma_{j_2}} \right)^2 \sqrt[3]{\Gamma_{j_1}}}{(\hat{u}\Lambda + \Theta)^2} \right). \quad (34)$$

*Remark 4:* Since we have  $\Gamma_v \propto q_v$ ,  $\forall v$ , and  $q_v$  increases exponentially with the VR preference parameter  $\gamma$  according to Eq. (5),  $S^{NSP}(\mathbf{s}^*, \boldsymbol{\tau}^*)$  also increases exponentially with  $\gamma$ .

## VI. DISCUSSIONS OF OTHER SCHEMES

Let us now consider two other schemes, namely, an uniform pricing scheme and a global optimization scheme.

### A. Uniform Pricing Scheme

In contrast to the non-uniform pricing scheme of the previous section, the uniform pricing scheme deliberately imposes the same price on the VRs in the game. We denote the fixed price by  $s$ . In this case, similar to *Lemma 1, Problem 2* can be solved by

$$\tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s}} - \frac{\Lambda}{\Theta} \right)^+. \quad (35)$$

We first focus our attention on the special case of  $\zeta_v = 1$ ,  $\forall v$ . Then *Problem 4* can be converted to that of minimizing  $s$  subject to the constraint  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s}} \leq (V\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda s^{bh}}}$ . We then obtain the optimal  $\hat{s}$  for this special case as

$$\hat{s} = \frac{\Lambda s^{bh} \left( \sum_{j=1}^V \sqrt{\Gamma_j} \right)^2}{\lambda(V\Lambda + \Theta)^2}. \quad (36)$$

To guarantee that all the VRs are capable of participating in the game, i.e.,  $\zeta_v = 1$ ,  $\forall v$ , with the optimal price  $\hat{s}$ , we let  $\hat{s} < \frac{\Gamma_v s^{bh}}{\Lambda \lambda}$ . Then we have the following constraint on the storage  $Q$  as

$$Q > Q'_{min} \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt{\frac{q_j}{q_v}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (37)$$

We can see that we require a larger storage size  $Q$  in Eq. (37) than that in Eq. (25) under the non-uniform pricing scheme to accommodate all the VRs, since we have  $\sum_{j=1}^V \sqrt{\frac{q_j}{q_v}} > \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_v}}$ . Following *Remark 2*, we conclude that  $Q'_{min}$  of the uniform pricing scheme will increase exponentially with  $\gamma/2$ .

Then based on this special case, the optimal  $\mathbf{s}^* = [s_1^*, \dots, s_V^*]$  in the uniform pricing scheme can be readily obtained by following a similar method to that in the previous section. That is,

$$s_v^* = \begin{cases} \frac{\Lambda s^{bh} \left( \sum_{j=1}^{\hat{u}} \sqrt{\Gamma_j} \right)^2}{\lambda(\hat{u}\Lambda + \Theta)^2}, & v = 1, \dots, \hat{u}, \\ \infty, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (38)$$

where regarding  $\hat{u}$ , we have

$$\hat{u} = \arg \min_u \{S_u : u = 1, 2, \dots, T\}, \quad (39)$$

with

$$S_u = \frac{u\Lambda^2 s^{bh} \left( \sum_{j=1}^u \sqrt{\Gamma_j} \right)^2}{(u\Lambda + \Theta)^2} - \sum_{j=1}^u \Gamma_j s^{bh},$$

$$T = \begin{cases} 1, & \bar{U}_1 < Q \leq \bar{U}_2, \\ \dots, & \\ v, & \bar{U}_v < Q \leq \bar{U}_{v+1}, \\ \dots, & \\ V, & \bar{U}_V < Q. \end{cases} \quad (40)$$

Note that  $\bar{U}_v$  in Eq. (40) is defined as

$$\bar{U}_v \triangleq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^v \sqrt{\frac{q_j}{q_v}} - v \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \quad (41)$$

It is clear that the uniform pricing scheme is inferior to the non-uniform pricing scheme in terms of maximizing  $S^{NSP}$ . However, we will show in the following problem that the uniform pricing scheme offers the optimal solution to maximizing the back-haul cost reduction  $S^{BH}$  at the NSP in conjunction with  $\tau_v^*$ ,  $\forall v$ , from the followers.

*Problem 7:* With the aid of the optimal solutions  $\tau_v^*$ ,  $\forall v$ , from the followers, the maximization on  $S^{BH}$  is achieved by solving the following problem:

$$\begin{aligned} \min_{\xi, s \geq 0} & \sum_{j=1}^V \xi_j \left( \sqrt{s^{bh}} \sqrt{\Gamma_j \Lambda \lambda} \sqrt{s_j} - \Gamma_j s^{bh} \right), \\ \text{s.t.} & \sum_{j=1}^V \xi_j \left( \sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\lambda s_j}} - \Lambda \right) \leq \Theta. \end{aligned} \quad (42)$$

The optimal solution to *Problem 7* can be readily shown to be  $\mathbf{s}^*$  given in Eq. (38). This proof follows the similar procedure of the optimization method presented in the previous section. Thus it is skipped for brevity. In this sense, the uniform pricing scheme is superior to the non-uniform pricing scheme in terms of reducing more cost on back-haul channel transmissions.

### B. Global Optimization Scheme

In the global optimization scheme, we are interested in the sum profit of the NSP and VRs, which can be expressed as

$$\begin{aligned} S^{GLB} &= S^{NSP} + \sum_{j=1}^V S_j^{VR} \\ &= \sum_{j_1=1}^V \sum_{j_2=1}^F \frac{2p_{j_2} q_{j_1} \zeta K s^{bh} \tau_{j_1}}{(A(\delta, \alpha) - C(\delta, \alpha) + 1) \tau_{j_1} + C(\delta, \alpha) F} \\ &= 2S^{BH}. \end{aligned} \quad (43)$$

Observe from Eq. (43), we can see that the sum profit  $S^{GLB}$  is twice the back-haul cost reduction  $S^{BH}$ , where the vector  $\boldsymbol{\tau}$  is the only variable of this maximization problem.



694 *Problem 8*: The optimization of the sum profit  $S^{GLB}$  can  
695 be formulated as

$$696 \quad \max_{\tau \geq 0} \sum_{j_1=1}^V \frac{\tau_{j_1} \sum_{j_2=1}^F p_{j_2} q_{j_1} \zeta K s^{bh}}{(A(\delta, \alpha) - C(\delta, \alpha) + 1)\tau_{j_1} + C(\delta, \alpha)F},$$

$$697 \quad \text{s.t.} \quad \sum_{j=1}^V \tau_j \leq 1. \quad (44)$$

698 *Problem 8* is a typical water-filling optimization problem.  
699 By relying on the classic Lagrangian multiplier, we arrive at  
700 the optimal solution as

$$701 \quad \hat{\tau}_v = \left( \frac{\frac{\sqrt{q_v}}{\eta} - C(\delta, \alpha)F}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \right)^+, \quad \forall v, \quad (45)$$

702 where we have  $\eta = \frac{\sum_{j=1}^{\bar{v}} \sqrt{q_j}}{\bar{v}C(\delta, \alpha)F + A(\delta, \alpha) - C(\delta, \alpha) + 1}$ , and  $\bar{v}$  satisfies  
703 the constraint of  $\hat{\tau}_v > 0$ .

### 704 C. Comparisons

705 Let us now compare the optimal SBS allocation variable  $\tau_v$   
706 in the context of the above two schemes. First, we investigate  
707  $\tau_v^*$  in the uniform pricing scheme. By substituting Eq. (38)  
708 into Eq. (35), we have

$$709 \quad \tau_v^* = \left( \sqrt{\frac{\Gamma_v \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_v^*}} - \frac{\Lambda}{\Theta} \right)^+$$

$$710 \quad = \begin{cases} \frac{\frac{\sqrt{q_v}}{\eta'} - C(\delta, \alpha)F}{A(\delta, \alpha) - C(\delta, \alpha) + 1}, & v = 1, \dots, \hat{u} \\ 0, & v = \hat{u} + 1, \dots, V, \end{cases} \quad (46)$$

711 where  $\eta' = \frac{\sum_{j=1}^{\hat{u}} \sqrt{q_j}}{\hat{u}C(\delta, \alpha)F + A(\delta, \alpha) - C(\delta, \alpha) + 1}$ , and  $\hat{u}$  ensures  $\tau_v^* > 0$ .

712 Then, comparing  $\tau_v^*$  given in Eq. (46) to the optimal  
713 solution  $\hat{\tau}$  of the global optimization scheme given by Eq. (45),  
714 we can see that these two solutions are the same. In other  
715 words, the uniform pricing scheme in fact represents the global  
716 optimization scheme in terms of maximizing the sum profit  
717  $S^{GLB}$  and maximizing the back-haul cost reduction  $S^{BH}$ .

## 718 VII. NUMERICAL RESULTS

719 In this section, we provide both numerical as well as  
720 Monte-Carlo simulation results for evaluating the performance  
721 of the proposed schemes. The physical layer parameters of  
722 our simulations, such as the path-loss exponent  $\alpha$ , transmit  
723 power  $P$  of the SBSs and the noise power  $\sigma^2$  are similar to  
724 those of the 3GPP standards. The unit of noise power and  
725 transmit power is Watt, while the SBS and MU intensities are  
726 expressed in terms of the numbers of the nodes per square  
727 kilometer.

728 Explicitly, we set the path-loss exponent to  $\alpha = 4$ , the  
729 SBS transmit power to  $P = 2$  Watt, the noise power to  
730  $\sigma^2 = 10^{-10}$  Watt, and the pre-set SINR threshold to  $\delta = 0.01$ .  
731 For the file caching system, we set the number of files in  
732  $\mathcal{F}$  to  $N = 500$  and set the number of VRs to  $V = 15$ .  
733 For the network deployments, we set the intensity of the

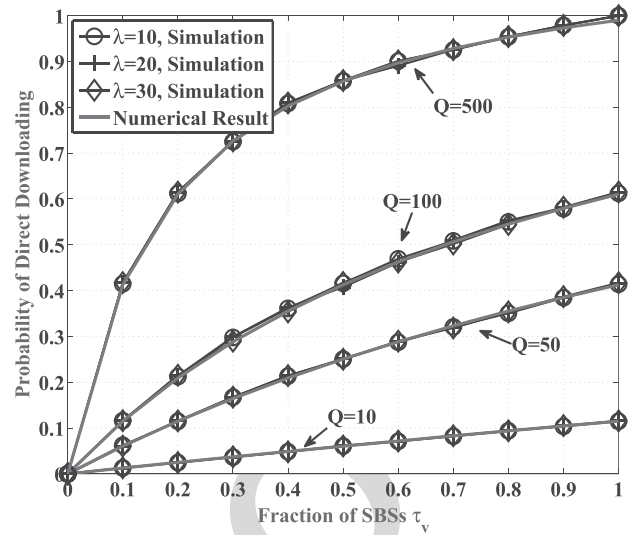


Fig. 2. Comparisons between the simulations and analytical results on  $\Pr(\mathcal{E}_{v,f})$ . We consider four kinds of storage size  $Q$  in each SBS, i.e.,  $Q = 10, 50, 100, 500$ , and three kinds of SBS intensity, i.e.,  $\lambda = 10, 20, 30$ .

MUs to  $\zeta = 50/km^2$ , and investigate three cases of the SBS  
734 deployments as  $\lambda = 10/km^2, 20/km^2$  and  $30/km^2$ . 735

736 For the pricing system, the profit/ $UAP$  is considered to  
737 be the profit gained per month within an area of one square  
738 kilometer, i.e.,  $/month \cdot km^2$ . We note that the profits gained  
739 by the NSP and by the VRs are proportional to the cost  $s^{bh}$  of  
740 back-haul channels for transmitting a video. Hence, without  
741 loss of generality, we set  $s^{bh} = 1$  for simplicity. Additionally,  
742 we set  $K = 10/month$ , which is the average number of video  
743 requests from an MU per month.

744 We first verify our derivation of  $\Pr(\mathcal{E}_{v,f})$  by comparing the  
745 analytical results of *Theorem 1* to the Monte-Carlo simulation  
746 results. Upon verifying  $\Pr(\mathcal{E}_{v,f})$ , we will investigate the  
747 optimization results within the framework of the proposed  
748 Stackelberg game by providing numerical results.

### 749 A. Performance Evaluation on $\Pr(\mathcal{E}_{v,f})$

750 For the Monte-Carlo simulations of this subsection, all the  
751 average performances are evaluated over a thousand network  
752 scenarios, where the distributions of the SBSs and the MUs  
753 change from case to case according to the PPPs characterized by  
754  $\Phi$  and  $\Psi$ , respectively.

755 Note that  $\Pr(\mathcal{E}_{v,f})$  in *Theorem 1* is the probability that an  
756 MU can obtain its requested video directly from the memory  
757 of an SBS rented by  $\mathcal{V}_v$ . We can see from the expression of  
758  $\Pr(\mathcal{E}_{v,f})$  in Eq. (7) that it is a function of the fraction  $\tau_v$   
759 of the SBSs that are rented by  $\mathcal{V}_v$ . Although  $\tau_v$  should be  
760 optimized according to the price charged by the NSP, here  
761 we investigate a variety of  $\tau_v$  values, varying from 0 to 1, to  
762 verify the derivation of  $\Pr(\mathcal{E}_{v,f})$ .

763 Fig. 2 shows our comparisons between the simulations and  
764 analytical results on  $\Pr(\mathcal{E}_{v,f})$ . We consider four different  
765 storage sizes  $Q$  in each SBS by setting  $Q = 10, 50, 100, 500$ .  
766 Correspondingly, we have four values for the number of file  
767 groups, i.e.,  $F = 50, 10, 5, 1$ . Furthermore, we consider the  
768 SBS intensities of  $\lambda = 10, 20, 30$ . From Fig. 2, we can

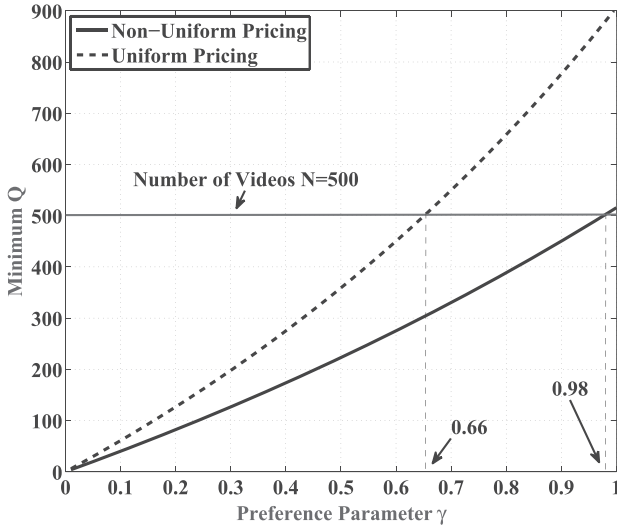


Fig. 3. The minimum number of  $Q$  that allows all the VRs to participate in the game under different preference parameter  $\gamma$ . In the case that the minimum  $Q$  is larger than  $N$ , it means that some VRs will be inevitable excluded from the game.

769 see that the simulations results closely match the analytical  
 770 results derived in *Theorem 1*. Our simulations show that the  
 771 intensity  $\lambda$  does not affect  $\Pr(\mathcal{E}_{v,f})$ , which is consistent with  
 772 our analytical results. Furthermore, a larger  $Q$  leads to a higher  
 773 value of  $\Pr(\mathcal{E}_{v,f})$ . Hence, enlarging the storage size is helpful  
 774 for achieving a higher probability of direct downloading.

### 775 B. Impact of the VR Preference Parameter $\gamma$

776 The preference distribution  $\mathbf{q}$  of the VRs defined in Eq. (5)  
 777 is an important factor in predetermining the system perfor-  
 778 mance. Indeed, we can see from Eq. (5) that this distribution  
 779 depends on the parameter  $\gamma$ . Generally, we have  $0 < \gamma \leq 1$ ,  
 780 with a larger  $\gamma$  representing a more uneven popularity among  
 781 the VRs. First, we find the minimum  $Q$  that can keep all  
 782 the VRs in the game. This minimum  $Q$  for the non-uniform  
 783 pricing scheme (NUPS) is given by Eq. (25), while the  
 784 minimum  $Q$  for the uniform-pricing scheme (UPS) is given by  
 785 Eq. (37). From the two equations, this minimum  $Q$  increases  
 786 exponentially with  $\gamma/3$  in the NUPS, while it also increases  
 787 exponentially with a higher exponent of  $\gamma/2$  in the UPS.  
 788 Fig. 3 shows this minimum  $Q$  for different values of the  
 789 VR preference parameter  $\gamma$ .

790 We can see that the UPS needs a larger  $Q$  than the NUPS  
 791 for keeping all the VRs. This gap increases rapidly with the  
 792 growth of  $\gamma$ . For example, for  $\gamma = 0.3$ , the uniform pricing  
 793 scheme requires almost 80 more storages, while for  $\gamma = 0.6$ ,  
 794 it needs 200 more. We can also observe in Fig. 3 that for  
 795  $\gamma > 0.66$  in the UPS and for  $\gamma > 0.98$  in the NUPS,  
 796 the minimum  $Q$  becomes larger than the overall number of  
 797 videos  $N$ . In both cases, since we have  $Q \leq N$  ( $Q > N$   
 798 results in the same performance as  $Q = N$ ), some unpopular  
 799 VRs will be excluded from the game.

800 Next, we study the number of VR participants that stay in  
 801 the game for the two schemes upon increasing  $\gamma$ . We can see  
 802 from Fig. 4 that the number of VR participants keeps going  
 803 down upon increasing  $\gamma$  in the both schemes. The NUPS

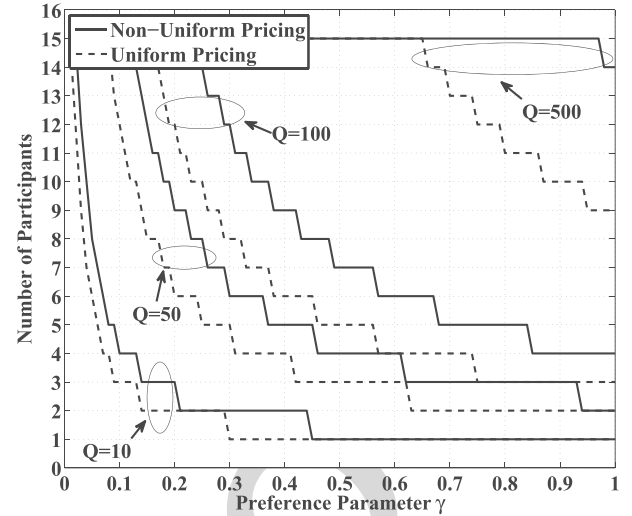


Fig. 4. Number of participants, i.e., the VRs that are in the game, vs. the preference parameter  $\gamma$ , under the two schemes. We also consider four different values of the storage size  $Q$ , i.e., 10, 50, 100, 500.

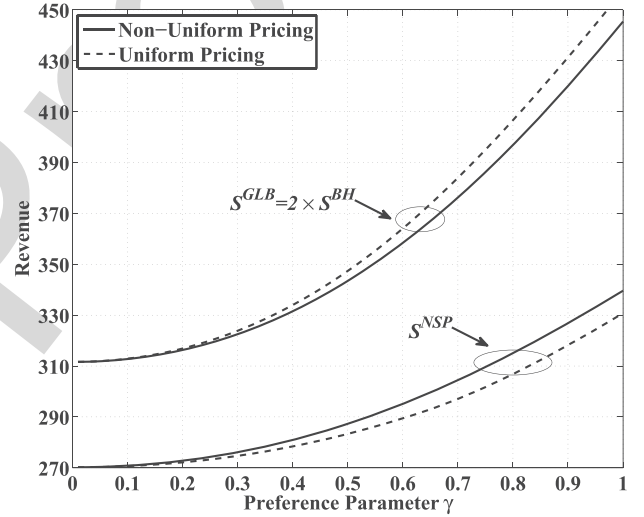


Fig. 5. Various revenues, including  $S^{NSP}$  and  $S^{GLB}$ , vs. the preference parameter  $\gamma$ , under the two schemes.

804 always keeps more VRs in the game than the UPS under  
 805 the same  $\gamma$ . At the same time, by considering  $Q =$   
 806 10, 50, 100, 500, it is shown that for a given  $\gamma$ , a higher  $Q$   
 807 will keep more VRs in the game.

808 Fig. 5 shows two kinds of revenues gained by the two  
 809 schemes for a given storage of  $Q = 500$ , namely, the global  
 810 profit  $S^{GLB}$  defined in Eq. (43) and the profit of the NSP  
 811  $S^{NSP}$  defined in Eq. (9). Recall that we have  $S^{GLB} = 2S^{BH}$   
 812 according to Eq. (43). We can see that the revenues of both  
 813 schemes increase exponentially upon increasing  $\gamma$ , as stated  
 814 in *Remark 4*. As our analytical result shows, the profit  $S^{NSP}$   
 815 gained by the NUPS is optimal and thus it is higher than  
 816 that gained by the UPS, while the UPS maximizes both  
 817  $S^{GLB}$  and  $S^{BH}$ . Fig. 5 verifies the accuracy of our derivations.

### 818 C. Impact of the Storage Size $Q$

819 Since  $\gamma$  is a network parameter that is relatively fixed,  
 820 the NSP can adapt the storage size  $Q$  for controlling

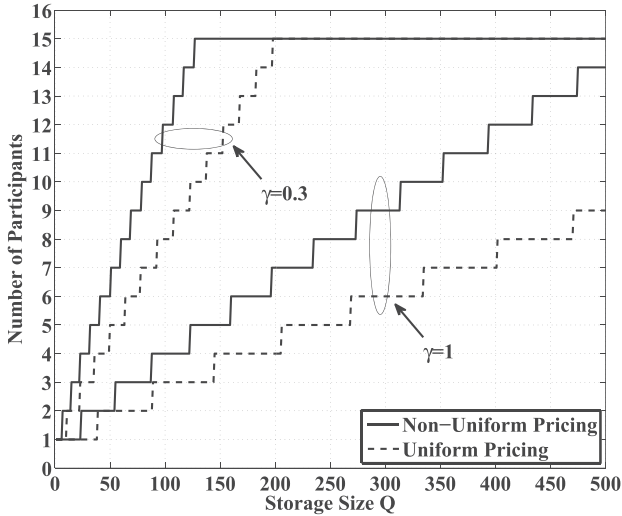


Fig. 6. Number of participants vs. the storage size  $Q$ , under the two schemes. We also consider two different values of  $\gamma$ , i.e.,  $\gamma = 0.3, 1$ .

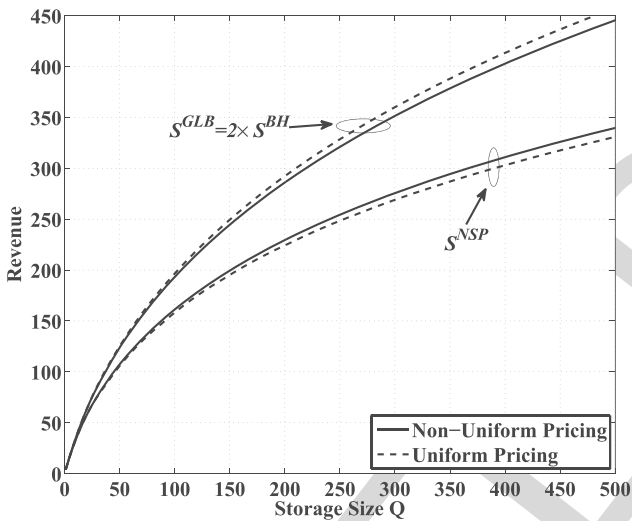


Fig. 7. Various revenues, including  $S^{NSP}$  and  $S^{GLB}$ , vs. the storage size  $Q$ , under the two schemes.

its performance. In this subsection, we investigate the performance as a function of  $Q$ . Fig. 6 shows the number of participants in the game versus  $Q$ , where  $\gamma = 0.3$  and 1 are considered. It is shown that for a larger  $Q$ , more VRs are able to participate in the game. Again, the NUPS outperforms the UPS owing to its capability of accommodating more VRs for a given  $Q$ . By comparing the scenarios of  $\gamma = 0.3$  and 1, we find that for  $\gamma = 0.3$ , a given increase of  $Q$  can accommodate more VRs in the game than  $\gamma = 1$ .

Fig. 7 shows both  $S^{NSP}$  and  $S^{GLB}$  versus  $Q$  for the two schemes for a given  $\gamma = 1$ . We can see that the revenues of both schemes increase with the growth of  $Q$ . It is shown that the profit  $S^{NSP}$  gained by the NUPS is higher than the one gained by the UPS, while the UPS outperforms the NUPS in terms of both  $S^{GLB}$  and  $S^{BH}$ .

#### D. Individual VR Performance

In this subsection, we investigate the performance of each individual VR, including the price charged to them for renting

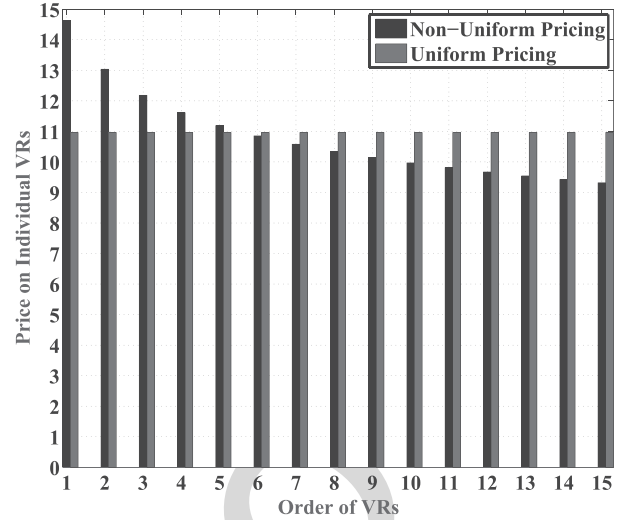


Fig. 8. Price charged on each VR for renting an SBS per month.

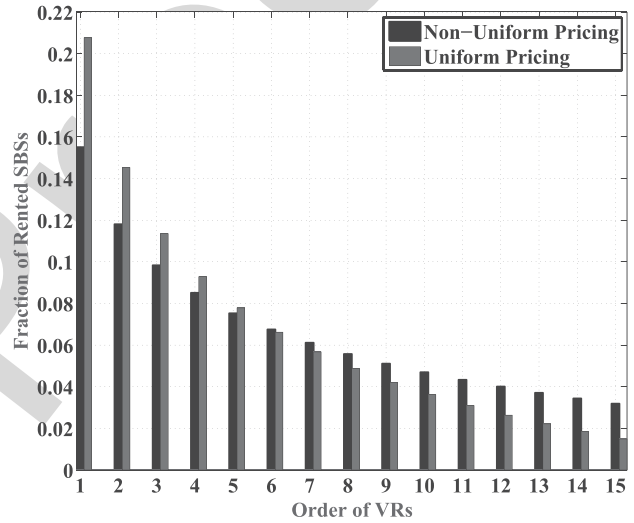


Fig. 9. The fraction of SBSs that are rented by each VR.

an SBS per month, and the fractions of the SBSs they rent from the NSP. We fix  $\gamma = 0.5$  and choose a large storage size of  $Q = 500$  for ensuring that all the VRs can be included. Fig. 8 shows the price charged to each VR for renting an SBS. The VRs are arranged according to their popularity order, ranging from  $\mathcal{V}_1$  to  $\mathcal{V}_{15}$ , with  $\mathcal{V}_1$  having the highest popularity and  $\mathcal{V}_{15}$  the lowest one. We can see from the figure that in the NUPS, the price for renting an SBS is higher for the VRs having a higher popularity than those with a lower popularity. By contrast, in the UPS, this price is fixed for all the VRs. Fig. 9 shows the specific fraction of the rented SBSs at each VR. In both schemes, the VRs associated with a high popularity tend to rent more SBSs. The UPS in fact represents an instance of the water-filling algorithm. Furthermore, the UPS seems more aggressive than the NUPS, since the less popular VRs of the UPS are more difficult to rent an SBS, and thus these VRs are likely to be excluded from the game with a higher probability.

## VIII. CONCLUSIONS

In this paper, we considered a commercial small-cell caching system consisting of an NSP and multiple VRs, where

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860 the NSP leases its SBSs to the VRs for gaining profits and for  
 861 reducing the costs of back-haul channel transmissions, while  
 862 the VRs, after storing popular videos to the rented SBSs, can  
 863 provide faster transmissions to the MUs, hence gaining more  
 864 profits. We proposed a Stackelberg game theoretic framework  
 865 by viewing the SBSs as a type of resources. We first modeled  
 866 the MUs and SBSs using two independent PPPs with the aid of  
 867 stochastic geometry, and developed the probability expression  
 868 of direct downloading. Then, based on the probability derived,  
 869 we formulated a Stackelberg game for maximizing the average  
 870 profit of the NSP as well as individual VRs. Next, we investi-  
 871 gate the Stackelberg equilibrium by solving the associated non-  
 872 convex optimization problem. We considered a non-uniform  
 873 pricing scheme and an uniform pricing scheme. In the former  
 874 scheme, the prices charged to each VR for renting an SBS  
 875 are different, while the latter imposes the same price for  
 876 each VR. We proved that the non-uniform pricing scheme  
 877 can effectively maximize the profit of the NSP, while the  
 878 uniform one maximizes the sum profit of the NSP and the VRs.  
 879 Furthermore, we derived a relationship between the optimal  
 880 pricing of renting an SBS, the fraction of SBSs rented by each  
 881 VR, the storage size of each SBS and the popularity of the  
 882 VRs. We verified by Monte-Carlo simulations that the direct  
 883 downloading probability under our PPP model is consistent  
 884 with our derived results. Then we provided several numerical  
 885 results for showing that the proposed schemes are effective in  
 886 both pricing and SBSs allocation.

887 APPENDIX A  
 888 PROOF OF THEOREM 1

889 Recall that the SBSs allocated to the VR  $\mathcal{V}_v$  and cache  $\mathcal{G}_f$   
 890 are modeled as a “thinned” HPPP  $\Phi_{v,f}$  having the intensity  
 891 of  $\frac{1}{F}\tau_v\lambda$ . We consider a typical MU  $\mathcal{M}$  who wishes to connect  
 892 to the nearest SBS  $\mathcal{B}$  in  $\Phi_{v,f}$ . The event  $\mathcal{E}_{v,f}$  represents that  
 893 this SBS can support  $\mathcal{M}$  with an SINR no lower than  $\delta$ , and  
 894 thus  $\mathcal{M}$  can obtain the desired file from the cache of  $\mathcal{B}$ .

895 We carry out the analysis on  $\Pr(\mathcal{E}_{v,f})$  for the typical MU  
 896  $\mathcal{M}$  located at the origin. Since the network is interference  
 897 dominant, we neglect the noise in the following. We denote by  
 898  $z$  the distance between  $\mathcal{M}$  and  $\mathcal{B}$ , by  $x_Z$  the location of  $\mathcal{B}$ , and  
 899 by  $\rho(x_Z)$  the received SINR at  $\mathcal{M}$  from  $\mathcal{B}$ . Then the average  
 900 probability that  $\mathcal{M}$  can download the desired video from  $\mathcal{B}$  is

$$\begin{aligned}
 & \Pr(\rho(x_Z) \geq \delta) \\
 &= \int_0^\infty \Pr\left(\frac{h_{x_Z} z^{-\alpha}}{\sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}} \geq \delta \middle| z\right) f_Z(z) dz \\
 &= \int_0^\infty \Pr\left(h_{x_Z} \geq \frac{\delta \left(\sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}\right)}{z^{-\alpha}} \middle| z\right) \\
 & \quad 2\pi \frac{1}{F} \tau_v \lambda z \exp\left(-\pi \frac{1}{F} \tau_v \lambda z^2\right) dz \\
 &= \int_0^\infty \mathbb{E}_I(\exp(-z^\alpha \delta I)) 2\pi \frac{1}{F} \tau_v \lambda z \exp\left(-\pi \frac{1}{F} \tau_v \lambda z^2\right) dz,
 \end{aligned} \tag{47}$$

907 where we have  $I \triangleq \sum_{x \in \Phi \setminus \{x_Z\}} h_x \|x\|^{-\alpha}$ , and the PDF of  $z$ , i.e.,  
 908  $f_Z(z)$ , is derived by the null probability of the HPPP  $\Phi_{v,f}$   
 909 with the intensity of  $\frac{1}{F}\tau_v\lambda$ . More specifically in  $\Phi_{v,f}$ , since  
 910 the number of the SBSs  $k$  in an area of  $A$  follows the Poisson  
 911 distribution, the probability of the event that there is no SBS  
 912 in the area with the radius of  $z$  can be calculated as [17]

$$\Pr(k = 0 \mid A = \pi z^2) = e^{-A \frac{1}{F} \tau_v \lambda} \frac{(A \frac{1}{F} \tau_v \lambda)^k}{k!} = e^{-\pi z^2 \frac{1}{F} \tau_v \lambda}. \tag{48}$$

915 By using the above expression, we arrive at  $f_Z(z) =$   
 916  $2\pi \frac{1}{F} \tau_v \lambda z \exp(-\pi \frac{1}{F} \tau_v \lambda z^2)$ . Note that the interference  $I$  con-  
 917 sists of  $I_1$  and  $I_2$ , where  $I_1$  emanates from the SBSs in  $\Phi$   
 918 excluding  $\Phi_{v,f}$ , while  $I_2$  is from the SBSs in  $\Phi_{v,f}$  excluding  
 919  $\mathcal{B}$ . The SBSs contributing to  $I_1$ , denoted by  $\Phi_{v,f}^c$ , have the  
 920 intensity of  $(1 - \frac{1}{F}\tau_v)\lambda$ , while those contributing to  $I_2$  have  
 921 the intensity of  $\frac{1}{F}\tau_v\lambda$ .

922 Correspondingly, the calculation of  $\mathbb{E}_I(\exp(-z^\alpha \delta I))$  will  
 923 be split into the product of two expectations over  $I_1$  and  $I_2$ .  
 924 The expectation over  $I_1$  is calculated as

$$\begin{aligned}
 & \mathbb{E}_{I_1}(\exp(-z^\alpha \delta I_1)) \\
 & \stackrel{(a)}{=} \mathbb{E}_{\Phi_{v,f}^c} \left( \prod_{x \in \Phi_{v,f}^c} \int_0^\infty \exp(-z^\alpha \delta h_x \|x\|^{-\alpha}) \exp(-h_x) dh_x \right) \\
 & \stackrel{(b)}{=} \exp\left(-\left(1 - \frac{1}{F}\tau_v\right)\lambda \int_{\mathbb{R}^2} \left(1 - \frac{1}{1 + z^\alpha \delta \|x_k\|^{-\alpha}}\right) dx_k\right) \\
 & = \exp\left(-2\pi \left(1 - \frac{1}{F}\tau_v\right)\lambda \frac{1}{\alpha} z^2 \delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right)\right), \\
 & = \exp\left(-\pi \left(1 - \frac{1}{F}\tau_v\right)\lambda C(\delta, \alpha) z^2\right),
 \end{aligned} \tag{49}$$

930 where (a) is based on the independence of chan-  
 931 nel fading, while (b) follows from  $\mathbb{E}\left(\prod_x u(x)\right) =$   
 932  $\exp(-\lambda \int_{\mathbb{R}^2} (1 - u(x)) dx)$ , where  $x \in \Phi$  and  $\Phi$  is an PPP in  
 933  $\mathbb{R}^2$  with the intensity  $\lambda$  [24], and  $C(\delta, \alpha)$  has been defined as  
 934  $\frac{2}{\alpha} \delta^{\frac{2}{\alpha}} B\left(\frac{2}{\alpha}, 1 - \frac{2}{\alpha}\right)$ .

935 The expectation over  $I_2$  has to take into account  $z$  as the  
 936 distance from the nearest interfering SBS. Then we have

$$\begin{aligned}
 & \mathbb{E}_{I_2}(\exp(-z^\alpha \delta I_2)) \\
 & = \exp\left(-\frac{1}{F}\tau_v \lambda 2\pi \int_z^\infty \left(1 - \frac{1}{1 + z^\alpha \delta r^{-\alpha}}\right) r dr\right) \\
 & \stackrel{(a)}{=} \exp\left(-\frac{1}{F}\tau_v \lambda \pi \delta^{\frac{2}{\alpha}} z^2 \frac{2}{\alpha} \int_{\delta^{-1}}^\infty \frac{\kappa^{\frac{2}{\alpha}-1}}{1 + \kappa} dx\right) \\
 & \stackrel{(b)}{=} \exp\left(-\frac{1}{F}\tau_v \lambda \pi \delta z^2 \frac{2}{\alpha - 2} {}_2F_1\left(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta\right)\right),
 \end{aligned} \tag{50}$$

942 where (a) defines  $\kappa \triangleq \delta^{-1} z^{-\alpha} r^\alpha$ , and  ${}_2F_1(\cdot)$   
 943 in (b) is the hypergeometric function. As we  
 944 defined  $A(\delta, \alpha) = \frac{2\delta}{\alpha - 2} {}_2F_1\left(1, 1 - \frac{2}{\alpha}; 2 - \frac{2}{\alpha}; -\delta\right)$ , by

945 substituting (49) and (50) into (47), we have

$$\begin{aligned}
 & \Pr(\rho(x_Z) \geq \delta) \\
 &= \int_0^\infty \exp\left(-\pi\left(1 - \frac{1}{F}\tau_v\right)\lambda C(\delta, \alpha)z^2\right) \\
 & \quad \exp\left(-\pi\frac{1}{F}\tau_v\lambda z^2 A(\delta, \alpha)\right) 2\pi\frac{1}{F}\tau_v\lambda z \exp\left(-\pi\frac{1}{F}\tau_v\lambda z^2\right) dz \\
 &= \frac{\frac{1}{F}\tau_v}{C(\delta, \alpha)\left(1 - \frac{1}{F}\tau_v\right) + A(\delta, \alpha)\frac{1}{F}\tau_v + \frac{1}{F}\tau_v}. \tag{51}
 \end{aligned}$$

950 This completes the proof.  $\blacksquare$

## APPENDIX B PROOF OF LEMMA 2

953 By applying Lagrangian multipliers to the objective func-  
954 tion, we have

$$\begin{aligned}
 & L(\mathbf{s}, \mu, \mathbf{v}) \\
 &= \sum_{j=1}^V s_j + \mu \left( \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \right) - \sum_{j=1}^V v_j s_j, \tag{52}
 \end{aligned}$$

958 where  $\mu$  and  $v_j$  are non-negative multipliers associated with  
959 the constraints  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \leq 0$  and  $s_j \geq 0$ ,  
960 respectively. Then the KKT conditions can be written as

$$\begin{aligned}
 & \frac{\partial L(\mathbf{s}, \mu, \mathbf{v})}{\partial s_j} = 0, \quad \forall j = 1, \dots, V, \\
 & \mu \left( \sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \right) = 0, \text{ and } v_j s_j = 0, \quad \forall j. \tag{53}
 \end{aligned}$$

964 From the first line of Eq. (53), we have

$$s_j = \sqrt[3]{\frac{\mu^2 \Gamma_j}{4(1 - v_j)^2}}. \tag{54}$$

966 Obviously, we have  $s_j \neq 0, \forall j$ , otherwise the constraint  
967  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} \leq 0$  cannot be satisfied.  
968 Thus, we have  $v_j = 0, \forall j$ . Furthermore, we have  $\mu \neq 0$   
969 according to Eq. (54) since  $s_j$  is non-zero. This means that  
970  $\sum_{j=1}^V \sqrt{\frac{\Gamma_j}{s_j}} - (V\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}} = 0$ .

971 By substituting Eq. (54) into this constraint, we have

$$\sqrt[3]{\mu} = \frac{\sqrt{\Lambda_S^{bh}} \sum_{j=1}^V \sqrt[3]{2\Gamma_j}}{\sqrt{\lambda}(V\Lambda + \Theta)}. \tag{55}$$

973 Then it follows that

$$s_j = \frac{\Lambda_S^{bh} \left( \sum_{v=1}^V \sqrt[3]{\Gamma_v} \right)^2 \sqrt[3]{\Gamma_j}}{\lambda(V\Lambda + \Theta)^2}. \tag{56}$$

975 This completes the proof.  $\blacksquare$

## APPENDIX C PROOF OF THEOREM 2

976 As discussed in Eq. (23) and Eq. (24), we have proved that  
977  
978  $Q > \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}$  is a sufficient condition for the  
979 optimal solution in Eq. (22). In other words, as long as  $Q$  is  
980 satisfied, we have the conclusion that the solution in Eq. (22)  
981 is optimal and  $\xi_v = 1, \forall v$ .  
982

983 Next, we prove the necessary aspect. Without loss of  
984 generality, we assume that

$$\begin{aligned}
 & \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{V-1} \sqrt[3]{\frac{q_j}{q_{V-1}}} - V + 1 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} < Q \\
 & \leq \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}. \tag{57}
 \end{aligned}$$

987 This leads to  $s_V \geq \frac{\Gamma_V s^{bh}}{\Lambda \lambda}$ , and the VR  $v_V$  will be excluded  
988 from the game. In this case, we have  $\xi_j = 1, j = 1, \dots, V-1$ ,  
989 and *Problem 4* will be rewritten as follows.

990 *Problem 9:* We rewrite *Problem 4* as

$$\begin{aligned}
 & \min_{\mathbf{s} \geq \mathbf{0}} \sum_{j=1}^{V-1} s_j, \\
 & \text{s.t. } \sum_{j=1}^{V-1} \sqrt{\frac{\Gamma_j}{s_j}} \leq ((V-1)\Lambda + \Theta)\sqrt{\frac{\lambda}{\Lambda_S^{bh}}}. \tag{58}
 \end{aligned}$$

993 Similar to the proof of *Lemma 2*, and combined with the  
994 constraint of  $Q$  in Eq. (57), the optimal solution of *Problem 9*  
995 is given by

$$\hat{s}_v = \begin{cases} \frac{\Lambda_S^{bh} \left( \sum_{j=1}^{V-1} \sqrt[3]{\Gamma_j} \right)^2 \sqrt[3]{\Gamma_v}}{\lambda((V-1)\Lambda + \Theta)^2}, & v = 1, \dots, V-1, \\ \infty, & v = V. \end{cases} \tag{59}$$

998 We can see that the optimal solution given in Eq. (59)  
999 contradicts to the optimal solution of *Problem 4* given in  
1000 Eq. (22). Hence,  $Q > \frac{NC(\delta, \alpha) \left( \sum_{j=1}^V \sqrt[3]{\frac{q_j}{q_V}} - V \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1}$  is a necessary  
1001 condition for finding the optimal solution in Eq. (22). This  
1002 completes the proof.  $\blacksquare$

## APPENDIX D PROOF OF LEMMA 3

1005 Consider  $v_1, v_2 = 1, \dots, V$  and  $v_1 = v_2 + 1$ . Then we  
1006 prove that  $U_{v_1} > U_{v_2}$ . We have

$$\begin{aligned}
 U_{v_1} &= \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_1} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_1 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
 &= \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_2 + \sum_{j=v_2+1}^{v_1} \sqrt[3]{\frac{q_j}{q_{v_1}}} - (v_1 - v_2) \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
 &= \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_1}}} - v_2 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} \\
 &\stackrel{(a)}{>} \frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v_2} \sqrt[3]{\frac{q_j}{q_{v_2}}} - v_2 \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} = U_{v_2}, \tag{60}
 \end{aligned}$$

975 This completes the proof.  $\blacksquare$

1011 where (a) comes from the fact that  $q_{v_1} < q_{v_2}$ . This completes  
1012 the proof. ■

#### 1013 APPENDIX E 1014 PROOF OF LEMMA 4

1015 It is plausible that if  $\mathcal{L}$  can only keep at most  $v$  VRs, it has  
1016 to retain the  $v$  most popular VRs to maximize its profit. Let  
1017 us now prove that if  $\mathcal{L}$  keeps  $(v+w)$  VRs,  $w = 1, \dots, V-v$ ,  
1018 in the game, it cannot achieve the optimal solution for  
1019  $U_v < Q \leq U_{v+1}$ .

1020 **Problem 10:** In the case that  $\mathcal{L}$  keeps  $(v+w)$  VRs, we have  
1021 the optimization problem of

$$1022 \min_{s \geq 0} \sum_{j=1}^{v+w} s_j,$$

$$1023 \text{ s.t. } \sum_{j=1}^{v+w} \sqrt{\frac{\Gamma_j}{s_j}} \leq ((v+w)\Lambda + \Theta) \sqrt{\frac{\lambda}{\Lambda_s b h}}. \quad (61)$$

1024 Similar to the proof of *Theorem 2*, we obtain that  $Q >$   
1025  $\frac{NC(\delta, \alpha) \left( \sum_{j=1}^{v+w} \sqrt{\frac{q_j}{q_{v+w}} - (v+w)} \right)}{A(\delta, \alpha) - C(\delta, \alpha) + 1} = U_{v+w}$  is the necessary con-  
1026 dition for the  $(v+w)$  VRs to participate in the game. This  
1027 contradicts to the premise  $U_v < Q \leq U_{v+1}$ , since we have  
1028  $Q > U_{v+1}$  according to *Lemma 3*. Let us now consider  
1029 the cases of  $w' = 0, -1, \dots, 1-v$ . To ensure there are  
1030  $(v+w')$  VRs in the game,  $Q$  has to satisfy the condition  
1031 that  $Q > U_{v+w'}$ . Since  $Q > U_v \geq U_{v+w'}$ , this implies that  
1032 given  $(v+w')$  VRs in the game, the NSP can achieve an  
1033 optimal solution. This completes the proof. ■

#### 1034 REFERENCES

- 1035 [1] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire,  
1036 "Femtocaching and device-to-device collaboration: A new architecture  
1037 for wireless video distribution," *IEEE Commun. Mag.*, vol. 51, no. 4,  
1038 pp. 142–149, Apr. 2013.
- 1039 [2] X. Wang, M. Chen, T. Taleb, A. Ksentini, and V. C. M. Leung, "Cache  
1040 in the air: Exploiting content caching and delivery techniques for 5G  
1041 systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 131–139, Feb. 2014.
- 1042 [3] M. A. Maddah-Ali and U. Niesen, "Decentralized coded caching attains  
1043 order-optimal memory-rate tradeoff," in *Proc. 51st Annu. Allerton Conf.*  
1044 *Commun., Control, Comput. (Allerton)*, Oct. 2013, pp. 421–427.
- 1045 [4] N. Golrezaei, P. Mansourifard, A. F. Molisch, and A. G. Dimakis, "Base-  
1046 station assisted device-to-device communications for high-throughput  
1047 wireless video networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7,  
1048 pp. 3665–3676, Jul. 2014.
- 1049 [5] M. Ji, G. Caire, and A. F. Molisch. (May 2013). "Wireless device-to-  
1050 device caching networks: Basic principles and system performance."  
1051 [Online]. Available: <http://arxiv.org/abs/1305.5216>
- 1052 [6] M. Ji, G. Caire, and A. F. Molisch, "Optimal throughput-outage trade-  
1053 off in wireless one-hop caching networks," in *Proc. IEEE Int. Symp. Inf.*  
1054 *Theory (ISIT)*, Jul. 2013, pp. 1461–1465.
- 1055 [7] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE*  
1056 *Trans. Inf. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.
- 1057 [8] F. Boccardi, R. W. Heath, A. Lozano, T. L. Marzetta, and P. Popovski,  
1058 "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*,  
1059 vol. 52, no. 2, pp. 74–80, Feb. 2014.
- 1060 [9] A. Damnjanovic *et al.*, "A survey on 3GPP heterogeneous networks,"  
1061 *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, Jun. 2011.
- 1062 [10] J. Akhtman and L. Hanzo, "Heterogeneous networking: An enabling  
1063 paradigm for ubiquitous wireless communications," *Proc. IEEE*, vol. 98,  
1064 no. 2, pp. 135–138, Feb. 2010.

- 1065 [11] S. Bayat, R. H. Y. Louie, Z. Han, B. Vucetic, and Y. Li, "Distributed  
1066 user association and femtocell allocation in heterogeneous wireless  
1067 networks," *IEEE Trans. Commun.*, vol. 62, no. 8, pp. 3027–3043,  
1068 Aug. 2014.
- 1069 [12] M. Mirahmadi, A. Al-Dweik, and A. Shami, "Interference modeling  
1070 and performance evaluation of heterogeneous cellular networks," *IEEE*  
1071 *Trans. Commun.*, vol. 62, no. 6, pp. 2132–2144, Jun. 2014.
- 1072 [13] A. K. Gupta, H. S. Dhillon, S. Vishwanath, and J. G. Andrews, "Down-  
1073 link multi-antenna heterogeneous cellular network with load balancing,"  
1074 *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4052–4067, Nov. 2014.
- 1075 [14] M. Liebsch, S. Schmid, and J. Awano, "Reducing backhaul costs for  
1076 mobile content delivery—An analytical study," in *Proc. IEEE Int. Conf.*  
1077 *Commun. (ICC)*, Jun. 2012, pp. 2895–2900.
- 1078 [15] K. Shanmugam, N. Golrezaei, A. G. Dimakis, A. F. Molisch, and  
1079 G. Caire, "FemtoCaching: Wireless content delivery through distrib-  
1080 uted caching helpers," *IEEE Trans. Inf. Theory*, vol. 59, no. 12,  
1081 pp. 8402–8413, Dec. 2013.
- 1082 [16] E. Baştuğ, M. Bennis, and M. Debbah, "Cache-enabled small cell  
1083 networks: Modeling and tradeoffs," in *Proc. 11th Int. Symp. Wireless*  
1084 *Commun. Syst. (ISWCS)*, Aug. 2014, pp. 649–653.
- 1085 [17] D. Stoyan, W. S. Kendall, and M. Mecke, *Stochastic Geometry and Its*  
1086 *Applications*. 2nd ed. New York, NY, USA: Wiley, 2003.
- 1087 [18] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and  
1088 M. Franceschetti, "Stochastic geometry and random graphs for the  
1089 analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*,  
1090 vol. 27, no. 7, pp. 1029–1046, Sep. 2009.
- 1091 [19] G. Vazquez-Vilar, C. Mosquera, and S. K. Jayaweera, "Primary user  
1092 enters the game: Performance of dynamic spectrum leasing in cogni-  
1093 tive radio networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12,  
1094 pp. 3625–3629, Dec. 2010.
- 1095 [20] X. Kang, R. Zhang, and M. Motani, "Price-based resource allocation for  
1096 spectrum-sharing femtocell networks: A Stackelberg game approach,"  
1097 *IEEE J. Sel. Areas Commun.*, vol. 30, no. 3, pp. 538–549, Apr. 2012.
- 1098 [21] D. Niyato and E. Hossain, "Competitive spectrum sharing in cognitive  
1099 radio networks: A dynamic game approach," *IEEE Trans. Wireless*  
1100 *Commun.*, vol. 7, no. 7, pp. 2651–2660, Jul. 2008.
- 1101 [22] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and  
1102 multiple-buyer spectrum trading in cognitive radio networks: A game-  
1103 theoretic modeling approach," *IEEE Trans. Mobile Comput.*, vol. 8,  
1104 no. 8, pp. 1009–1022, Aug. 2009.
- 1105 [23] D. Fudenberg and J. Tirole, *Game Theory*. Cambridge, MA, USA:  
1106 MIT Press, 1993.
- 1107 [24] D. J. Daley and D. Vere-Jones, *An Introduction to the Theory of Point*  
1108 *Processes: Elementary Theory and Methods*, vol. 1. Springer, 1996. AQ:4
- 1109 [25] M. Cha, H. Kwak, P. Rodriguez, Y.-Y. Ahn, and S. Moon, "iTube, You  
1110 Tube, everybody tubes: Analyzing the world's largest user generated  
1111 content video system," in *Proc. 7th ACM SIGCOMM Conf. Internet*  
1112 *Meas.*, 2007, pp. 1–14. AQ:5



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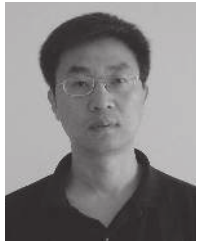
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