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Pricing and warranty decisions in a two-period closed-loop supply chain

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Abstract: For a two-period closed loop supply chain (CLSC) consisting of a manufacturer and a retailer, Stackelberg game analyses are conducted to examine pricing and warranty decisions under two warranty models depending on who offers warranty for new and remanufactured products and the corresponding benchmark models with warranty for new products only. Next, we identify the conditions under which warranty for remanufactured products is offered and investigate how this warranty affects the CLSC operations. Subsequently, comparative studies are carried out to examine equilibrium decisions, profitability and consumer surplus of the CLSC between the two warranty models. Analytical results show that offering warranty for remanufactured products does not affect new product pricing in period 2, but influences pricing of new products in period 1 and remanufactured products in period 2, thereby enhancing remanufacturing, individual and channel profitability, and consumer surplus. Compared to the retailer warranty for remanufactured products, the manufacturer warranty can attain a more equitable profit distribution. If the warranty cost advantage of the manufacturer (retailer) is significant relative to that of the retailer (the manufacturer), the manufacturer (retailer) arises as a natural choice to offer warranty for remanufactured products as this decision enhances both profitability and consumer surplus.

Keywords: Closed loop supply chain (CLSC); Remanufacturing; Warranty; Pricing decision; Stackelberg game

1 Introduction

Remanufacturing is often considered as an ideal choice for recycling and reuse of used products (Esenduran et al., 2016). As a sustainable and green development mode, remanufacturing fosters the formation of a recycling industrial chain and lays the foundation for green and circular development of the economy. Governments at different levels have issued diverse regulatory policies to encourage remanufacturing. For instance, in July 2013, led by the National Development and Reform Commission, China's five ministries jointly issued *the Pilot Program on "Bartering Used for Remanufactured Products"* to promote remanufacturing. In 2015, the United States House of Representatives passed the *Federal Vehicle Repair Cost Savings Act* to require federal agencies to consider using remanufactured parts for vehicle fleet repairs. Since more and more countries recognize the importance of and commit support to remanufacturing, an increasing number of enterprises are entering the remanufacturing arena (such as Kodak, Xerox, Siemens, Lenovo and Caterpillar), resulting in rapid growth in the remanufacturing industry. In 2013, the annual sales of global remanufacturing industry were more than 150 billion U.S. dollars and the United States alone reached more than 100 billion U.S. dollars (Liu et al., 2017). Meanwhile, remanufacturing has also drawn significant attention from academia and has been extensively studied in recent years (Atasu et al., 2009; Ferrer and Swaminathan, 2010; Liu et al., 2019; Ma et al., 2016; Özdemir and Denizel, 2012; Savaskan et al., 2004; Wang et al., 2017).

Although remanufacturing has made significant progress in theory and practice, it still faces many challenges such as consumers' low perceived quality of and willingness-to-pay for remanufactured products (Abbey et al., 2015). Among all factors influencing consumers' willingness-to-pay for remanufactured products, the retail price and consumers' perceived quality are the two most important considerations. Understandably, the lower the retail price and the higher the perceived quality, the greater the demand for remanufactured products (Abbey et al., 2015). However, if the remanufactured product is priced too low, consumers may lose confidence in its quality. Therefore, it is not viable to purely rely on aggressive pricing to stimulate demand for remanufactured products (Esenduran et al., 2016). In many cases, remanufactured and new products have comparable levels of quality, but consumers tend to discount the quality of remanufactured products. If effective methods can be developed to improve consumers' perception on the quality of remanufactured products or appropriate mechanisms can be devised to convey to consumers the right quality signal of remanufactured products, their market demand will be properly stimulated (Abbey et al., 2015). Generally speaking, product warranty is a proven mechanism to signal product quality to consumers. A longer warranty period tends to enhance consumers' confidence in and perception of product quality (Dai et al., 2012). Therefore, offering warranty for remanufactured products is presumably able to strengthen consumers' willingness-to-pay.

Researchers have conducted some studies on warranty policies for remanufactured products (Alqahtani and Gupta, 2017; Liao, 2018; Zhu et al., 2016), and their results indicate that a warranty policy for remanufactured products, such as a refund warranty, can enhance the remanufacturer's profit. In a CLSC where the manufacturer is responsible for remanufacturing, operational decisions are often made in two or multiple periods. For instance, for a two-period CLSC with remanufacturing, the manufacturer prices and provides warranty for new products in the first period and, then, collects used products to produce remanufactured products, prices and provides warranty for both products in the second period. In this context, it would be interesting to know if the results obtained from a single-period model are still applicable to the two-period model. Furthermore, warranty for products can be provided by either the manufacturer or the retailer (Dai et al., 2012; Bian et al., 2019). So, it is a worthy topic to investigate who is a better choice to offer warranty for new and remanufactured products in terms of profitability and consumer surplus in a two-period setting. We attempt to deal with this significant problem by resorting to Stackelberg game analyses. In reality, a warranty policy for new and remanufactured products may contain different clauses such as scope, free replacement, and refund (Alqahtani and Gupta, 2017; Liao, 2018). In this research, we focus on the warranty period of free replacement to investigate the provider's warranty decision and its efficiency.

More specifically, this study first examines the pricing and warranty period decisions under two warranty models where either the manufacturer or the retailer offers warranty for new and remanufactured products and the corresponding benchmark models with warranty for new products only in a two-period CLSC. We then investigate the impact of warranty on the operational decisions and efficiency of the CLSC from the perspectives of individual and channel profitability as well as consumer surplus. Subsequently, we explore who arises as a better warranty provider from the manufacturer's, retailer's, CLSC's and consumer's angle. This study attempts to address the following three questions.

- How does the warranty affect the pricing decisions of new products in both periods and of remanufactured products in the second period?
- Does warranty enhance market demand for remanufactured products and the profit of the warranty provider and the other CLSC member?
- Is the manufacturer or the retailer a better warranty provider in terms of individual and channel profitability as well as consumer surplus?

The remainder of this paper is organized as follows. Section 2 conducts a brief literature review. Section 3 presents the problem description and assumptions. Section 4 establishes two-period CLSC Stackelberg game models with the manufacturer or the retailer being the warranty provider for new and remanufactured products as well as the corresponding benchmark models with warranty for new products only, and derives their equilibrium solutions. In Section 5, we discuss the effect of offering warranty for remanufactured products on the operations of the CLSC, and analyze the warranty efficiency in the two warranty models. Section 6 presents numerical experiment to illustrate the propositions and reports further results for more general cases. Conclusions are drawn in Section 7.

2 Literature Review

This paper draws on and contributes to two streams of literature: pricing decisions of CLSCs and warranty policies for new and second-hand (remanufactured) products.

2.1 Pricing decisions of CLSCs

Extensive research has been conducted on pricing decisions of new, used and remanufactured products in CSLCs under different contexts. For example, some researchers focus on the optimal pricing under certain government regulations (Atasu et al., 2009; Esenduran et al., 2016; Özdemir and Denizel, 2012; Wang et al., 2018; Zhou et al., 2017), some literature investigates these pricing decisions under different collection patterns (Atasu et al., 2013; Govindan et al., 2017; He et al., 2019; Savaskan et al., 2004) and various decision structures (Ferrer and Swaminathan, 2006, 2010; Wang et al., 2017; Xiong et al., 2013). In addition, many articles jointly optimize pricing and other decisions such as product design (Örsdemir et al., 2014; Wu, 2013), remanufacturing authorization (Hong et al., 2017; Oraiopoulos et al., 2012) and warranty policy decisions.

It is a relatively new research topic to consider the pricing decision under warranty for new or remanufactured products. By assuming both linear and nonlinear demand functions for remanufactured products in a single period setting, Yazdian et al. (2014) investigate the remanufacturer's optimal collection price of used products, sale price and warranty period of remanufactured products. Zhu et al. (2016) optimize the warranty length and selling price for remanufactured products under free-replacement and extended warranty services from the remanufacturer's perspective and their empirical analysis indicates that the remanufacturer profits more from the free-replacement warranty than the extended warranty service. Liao (2018) focuses on a partial-refund warranty policy for remanufactured products and develops a newsvendor model to explore the impact of this warranty policy on product pricing decisions, members' profits and consumer surplus. Giri et al. (2018) assume that the warranty period for new products affects the collection quantity of used products and investigate the optimal price of new and remanufactured products and warranty period of new products when the manufacturer refurbishes and remanufactures defective products.

This brief literature review reveals that extant research tends to focus on warranty for either new or remanufactured products in a single period and does not consider pricing decisions with warranty offering to both products by the same warranty provider. In contrast, this paper incorporates warranty for both new and remanufactured products in a two-period CLSC setting and examines the impact of offering warranty on the pricing decisions of both products in different periods.

2.2 Warranty for new and second-hand (remanufactured) products

Researchers have conducted extensive studies on product warranty from different angles. Warranty studies in operational management can be grouped into three streams: Estimation of warranty cost, optimization of the warranty service level and efficiency analysis of warranty policies.

Warranty cost depends on product reliability, rectification and warranty terms, so a bulk of literature considers it from the angles of the product failure rate and warranty policy at the component and product level (Murthy and Djamaludin, 2002; Wang and Xie, 2018). For new products, Polatoglu and Sahin (1998) derive a probability distribution of the warranty cost for pro-rata warranties; Liu et al (2015) develop warranty cost models for series and parallel systems with failure interaction under renewing free-replacement warranty; Park et al. (2016) develop a new warranty cost model for a renewable two-dimensional policy based on failure time and warranty servicing time; Zhang et al. (2018) analyze the expected warranty cost for non-renewing and renewing free replacement policy in a two-series system with stochastically dependent components; Luo and Wu (2019) estimate the warranty cost originated from software, hardware failures or human errors as well as their interactions. As for second-hand products, Chattopadhyay and Murthy (2000) develop probabilistic models to estimate their expected warranty cost under free replacement or pro-rata warranty at the component and system level. Shafiee et al. (2011) estimate the warranty cost based on past age, usage, service strategy and reliability of second-hand products. Alqahtani and Gupta (2017) examine the impact of warranty for remanufactured products from a remanufacturer's perspective and furnish an approach to minimize the warranty cost. The aforesaid literature provides theoretical bases for optimizing warranty policies and many studies build upon these warranty cost models to examine their specific problems (Chen et al., 2012; Dan et al., 2017; Giri et al., 2018; Luo and Wu, 2018; Wei et al., 2015). This paper differs from existing literature by combining the features of cost structure in two papers (Chen et al., 2012; Wei et al., 2015) and takes a more generic form to model the warranty cost in our CLSC.

The warranty service level can be reflected in different aspects, ranging from the length of the warranty period, the comprehensive service level consisting of warranty period and coverage, to refund due to product failure. For new products, Chen et al. (2012, 2017), Chie (2005), Esmaeili et al. (2014), Giri et al. (2018), Park et al. (2016), Taleizadeh et al. (2017), Wei et al. (2015), Xie (2017) and Zhang et al. (2019) explore the optimal warranty period under different specific scenarios. Dan et al. (2017) investigate the optimal warranty service level, added-value service level and their interaction in a cooperative and competitive context. Li et al. (2018) formulate a two-period model to analyze the optimal warranty compensation strategy when the consumer's quality perception and valuation change. For second-hand and remanufactured products, Chattopadhyay and Rahman (2010), Lu and Shang (2019) study the optimal warranty for second-hand products; Yazdian et al. (2014) and Zhu et al. (2016) optimize the warranty period of remanufactured products from the remanufacturer's perspectives. Liao (2018) focuses on a partial-refund warranty policy for remanufactured products. This literature review shows that it is a key concern to determine the warranty period and this has been extensively studied. However, existing research is typically confined to a single period setting and optimizing the warranty period for a single product (either new, second-hand, or remanufactured product). In contrast, this paper establishes a two-period CLSC where a warranty provider (either the manufacturer or the retailer) furnishes warranty for new products in both periods and warranty for remanufactured products in the second period. This new model setup allows us to determine the optimal warranty period of remanufactured products and examine how the operations of the two-period CLSC is affected by offering warranty for both new and remanufactured products.

In the stream of warranty efficiency, Zhou et al. (2009) compare a non-renewing with a renewing warranty policy for repairable products with known market entry and departure under consumer's risk aversion. Their research reveals that the renewing warranty policy is more conductive to the warranty provider. Dai et al. (2012) focus on the impact of product quality and warranty period decisions on supply chain performance under two scenarios where the warranty period is determined by the manufacturer or the supplier. Chari et al. (2016) study the optimal warranty policy of a manufacturer who services products within the warranty period by replacing defective components with new or reconditioned ones. Mo et al. (2017) provide a hybrid warranty policy combining free-replacement warranty with the buyer's investment in prevention management. Zhu et al. (2019) establish a model to determine the product reliability, warranty policy, pricing decisions in the period of regular and promotion sales, and compare four nonrenewable warranty policies to provide managerial insights on warranty policy selection. Existing literature pays more attention to efficiency of different warranty policies, but this research assesses warranty efficiency from a service provider's perspective. More specifically, this paper examines warranty efficiency by allowing the manufacturer or the retailer to offer warranty for both new and remanufactured products in a two-period CLSC and this assessment is conducted under the criteria of profitability and consumer surplus.

Table 1 shows the difference between our paper with existing studies on pricing decision in a CLSC and warranty decision in general supply chain management in a proper literature context. The literature review and Table 1 reveal that the majority of research is confined to either pricing or warranty decision for a single product (new, remanufactured, or second-hand product) and limited attention is dedicated to examining joint warranty and pricing decisions for remanufactured products in a single period (Alqahtani and Gupta, 2017; Liao, 2018; Yazdian et al., 2014; Zhu et al., 2016). On the other hand, the focus of this paper differs from existing studies in that it jointly optimizes pricing and warranty decisions where either the manufacturer or the retailer provides warranty service for both new and remanufactured products in a two-period CLSC setting. We then examine the interaction of warranty and pricing decisions and the implications on the CLSC performance from both profitability and consumer surplus angles, thereby assessing warranty efficiency offered by different parties.

Table 1 Literature positioning of this research and existing studies (*Y*=Yes; *N*=No)

Reference	Multi-period CLSC?	Warranty for new products?	Warranty for remanufactured products?	Warranty period?	The optimal warranty provider?
Alqahtani and Gupta (2017)	*N*	*N*	*Y*	*N*	*N*
Chen et al. (2012)	*N*	*Y*	*N*	*Y*	*N*
Dai et al. (2012)	*N*	*Y*	*N*	*Y*	*Y*
Esmaeili et al. (2014)	*N*	*Y*	*N*	*Y*	*N*

Ferrer and Swaminathan (2010)	*Y*	*N*	*N*	*N*	*N*
Giri et al. (2018)	*N*	*Y*	*N*	*Y*	*N*
He et al. (2019)	*N*	*N*	*N*	*N*	*N*
Liao (2018)	*N*	*N*	*Y*	*N*	*N*
Li et al. (2018)	*Y*	*Y*	*N*	*N*	*N*
Park et al.(2016)	*N*	*Y*	*N*	*Y*	*N*
Savaskan et al. (2004)	*N*	*N*	*N*	*N*	*N*
Taleizadeh et al. (2017)	*N*	*Y*	*N*	*Y*	*N*
Wang et al. (2018)	*Y*	*N*	*N*	*N*	*N*
Wei et al. (2015)	*N*	*Y*	*N*	*Y*	*N*
Xie et al. (2017)	*N*	*Y*	*N*	*Y*	*N*
Yazdian et al. (2016)	*N*	*N*	*Y*	*Y*	*N*
Zhu et al. (2016)	*N*	*N*	*Y*	*Y*	*N*
Zhu et al. (2019)	*N*	*Y*	*N*	*Y*	*N*
Our paper	*Y*	*Y*	*Y*	*Y*	*Y*

3 Problem description and model assumptions

In a two-period CLSC consisting of a manufacturer and a retailer, the manufacturer is modeled as the leader and organizes production in response to market demand for new products in period 1; in period 2, it continues to produce new products, collects and remanufactures used products as a result of new product sales in period 1. The retailer is modeled as the follower and sets the retail prices for new and remanufactured products to sell them to the end market. Generally, only a portion of used products can be collected and remanufactured, so $q_r \le \gamma q_1$, where $0 < \gamma \le 1$ is the collection yield and defined as the fraction of new products made in period 1 that is available for remanufacturing in period 2 (Atasu et al., 2013; Ferrer and Swaminathan, 2006, 2010). Here, q_1 is the sale quantity of new products in period 1, q_r is the demand for remanufactured products in period 2. Conceptually, the profit obtained in period 2 should be discounted into the present value in period 1 (Ferrer and Swaminathan, 2006, 2010). For simplicity and without loss of generality, we ignore this discount factor following extant literature (Ferguson and Toktay, 2010; Hong et al., 2017; Wu, 2013).

Remanufactured products are generally perceived to be of lower quality compared to new products, and consumers tend to demonstrate loss-averse preference. Thus, to promote sales of remanufactured products, the manufacturer or retailer can resort to offering a warranty policy to boost consumers' confidence. Although a warranty policy for products typically contains different items (Alqahtani and Gupta, 2017; Liao, 2018), this research focuses on the warranty period of free replacement. Without loss of generality, the warranty period for new products is the same in the two periods and is denoted by n and that for remanufactured products is denoted by t. If $t > 0$, the manufacturer or retailer provides warranty for remanufactured products. If $t = 0$, the manufacturer or retailer does not provide warranty for remanufactured products. In general, n and t are different and $t \le n$ is commonly observed in practice. For instance, eBay provides a 5-year warranty period for new Big Ball Canister vacuum cleaner but only a 2-year for a remanufactured one. Lenovo provides the same 1-year warranty period for new and refurbished laptops. For simplicity, we normalize n to 1 and assume that $t \le 1$. The results derived from this simplified case can be extended to the more general case when n is set at different values.

Consumers' willingness-to-pay for new products is set as v, which is assumed to be uniformly distributed in the whole market capacity Q in each period. Consumers' willingness-to-pay for remanufactured products in period 2 is set as δv, where $\delta \in [0,1)$ denotes the consumer's value discount for remanufactured products. Given the warranty period n and t for new and remanufactured products, the utilities for consumers to purchase new products are $u_1 = v - p_1 + \varepsilon_n n$ and $u_n = v - p_n + \varepsilon_n n$ in the two periods, respectively; the utility to purchase remanufactured products is $u_r = \delta v - p_r + \varepsilon_r t$ in period 2. Here, $\varepsilon_n > 0$ ($\varepsilon_r > 0$) measures the impact of the warranty period n (t) on the consumers' willingness-to-pay for new (remanufactured) products. Following Liu et al. (2017) and Li and Chen (2018), consumers make purchasing decision in the principle of utility maximization. Assume that Θ_1 and Θ_n denote the set of consumers purchasing new products in period 1 and 2, respectively, and Θ_r represents the set of consumers purchasing remanufactured products in period 2, that is $\Theta_1 = \{v: u_1 \geq 0\}$, $\Theta_n = \{v: u_n \geq \max\{u_r, 0\}\}$ and $\Theta_r = \{v: u_r \geq \max\{u_n, 0\}\}$. Hence, by setting n = 1, demand for new and remanufactured products is derived as $q_1 = Q - p_1 + \varepsilon_n$, $q_n = Q - \frac{p_n - p_r + \varepsilon_r t - \varepsilon_n}{1-\delta}$ and $q_r = \frac{\delta p_n - p_r + \varepsilon_r t - \delta \varepsilon_n}{\delta(1-\delta)}$, respectively. The manufacturer produces as per market demand q_1, q_n and q_r.

We assume that warranty for new and remanufactured products is provided for consumers by either the manufacturer or the retailer and the warranty provider incurs the warranty cost over both periods. In each period, the warranty cost consists of a linear component $c_{nw}^j k$ ($c_{rw}^j k$) and a quadratic portion $\frac{1}{2}\beta_{nw}^j k^2$ ($\frac{1}{2}\beta_{rw}^j k^2$), k=n, t. Parameter c_{nw}^j (c_{rw}^j) is a linear maintenance cost for malfunctions of a new (remanufactured) product in each unit time. Parameter β_{nw}^j (β_{rw}^j) is the quadratic cost coefficient for offering new (remanufactured) product warranty. This warranty cost structure combines the features in two existing articles (Chen et al., 2012; Wei et al., 2015) and takes a more general form. If $c_{nw}^j = 0$ ($c_{rw}^j = 0$), the cost structure is consistent with that in (Wei et al., 2015); if $\beta_{nw}^j = 0$ ($\beta_{rw}^j = 0$), it is the same as that in (Chen et al., 2012). Here, $j \in \{M, R\}$ represents the manufacturer and retailer, respectively. The remanufactured product is assumed to be as good as the new product, and warranty for new and remanufactured products is provided by the same member. As such, we assume that the linear and quadratic warranty cost parameters for a new product are the same as those for a remanufactured product, i.e., $c_{nw}^j = c_{rw}^j = c_w^j$ and $\beta_{nw}^j = \beta_{rw}^j = \beta_w^j$. The unit production costs of new and remanufactured products are c_n and c_r, and $c_r < c_n$. It is noted that a unit collection cost is often assumed in a typical CLSC setting when a used product is recycled for remanufacturing period 2 (Atasu et al., 2013; Ma et al., 2016). Without loss of generality, we assume that c_r here incorporates the collection cost into the unit remanufacturing cost and this assumption does not materially change the main results derived in this paper. To make remanufacturing economically sensible, given consumers' value discount δ, it is assumed that $c_r < \delta(c_n + c_w^j - \varepsilon_n)$. This assumption is similar as that in existing literature (Esenduran et al., 2016).

In this research, we analyze the impact of the warranty policy on pricing decisions and operational efficiency of the CLSC under two warranty models, where either the manufacturer or the retailer offers warranty for new and remanufactured products (denoted by *MM* or *RR* accordingly) over two periods. In addition, to facilitate comparative analyses, we consider two benchmark models where the manufacturer or the retailer offers warranty for new products only (denoted by *MN* or *RN* accordingly). The manufacturer and retailer are rational and profit maximizers. Hereafter, π_j^i denotes the profit of member *j* under model *i*, and π_T^i denotes the channel profit of the CLSC; $i \in \{MM, RR, MN, RN\}$, $j \in \{M, R\}$.

In addition, let *CS* denote consumer surplus obtained from purchasing new and remanufactured products over the two periods , which is calculated as $CS = \frac{1}{2}(q_1^2 + q_n^2 + 2\delta q_n q_r + \delta q_r^2)$ (Yenipazarli, 2016).

Table 2 summarizes the symbols and notation used in this paper. For a more concise presentation of equilibrium solutions, we further introduce $\phi_{j0} = Q - (c_n + c_w^j - \varepsilon_n)$, $\phi_{j1} = \delta(c_n + c_w^j - \varepsilon_n) - c_r$, $\phi_{j2} = \beta_w^j \delta(1-\delta)$, $\phi_{j3} = \varepsilon_r - c_w^j$, $\phi_{j4} = \phi_{j0} + \gamma\phi_{j1}$, $\varphi_0 = 1 + \gamma^2\delta(1-\delta)$, $\phi_{MM} = 4\phi_{M2} - \phi_{M3}^2$, $\phi_{RR} = 4\phi_{R2} - \phi_{R3}^2$, $\varphi_{MM} = 4\beta_w^M + \gamma^2\phi_{MM}$, and $\varphi_{RR} = 2\beta_w^R + \gamma^2\phi_{RR}$.

Table 2. Parameters and decision variables.

Symbol	Definition
c_n / c_r	The unit production cost of a new/remanufactured product
$w_1 / p_1 / q_1$	The unit wholesale price/retail price/sale quantity of new products in period 1
w_n / w_r	The unit wholesale price of a new/remanufactured product in period 2
p_n / p_r	The unit retail price of a new/remanufactured product in period 2
q_n / q_r	Sales quantity of new/remanufactured products in period 2
Q	The whole market capacity in each period
v	Consumer's willingness-to-pay for the new product in each period
δ	Consumer's value discount for the remanufactured product
t	Warranty period for the remanufactured product
γ	The collection yield of used products for remanufacturing
c_w^j / β_w^j	The linear/quadratic warranty cost parameter for remanufactured and new products incurred by the warranty provider *j*, $j \in \{M, R\}$
ε_r	The impact of warranty period t on the consumer's willingness-to-pay for the remanufactured product
ε_n	The impact of warranty period *n* on the consumer's willingness-to-pay for the new product
π_j^i	Profit function of member *j* in model *i*, $i \in \{MM, RR, MN, RN\}$
CS^i	Consumer surplus in model i

4 Model formulation and equilibrium solution

Since we normalize the warranty period for new products to be *n* = 1 and assume the warranty period for remanufactured products $t \le n = 1$, the warranty provider's decision on the warranty period is confined to remanufactured products for the remainder of this research. On the other hand, the linear and quadratic warranty costs for new products in both periods are incorporated into the warranty provider's profit function by using *n* = 1.

4.1 Model *MM*– the manufacturer offers warranty for new and remanufactured products

In model *MM*, the manufacturer provides warranty for new and remanufactured products. The profit functions of the manufacturer and retailer can be formulated as:

$$\pi_M^{MM}(w_1, w_n, w_r, t) = (w_1 - c_n - c_w^M)q_1 + (w_n - c_n - c_w^M)q_n + (w_r - c_r - c_w^M t)q_r - \beta_w^M - \frac{1}{2}\beta_w^M t^2 \quad (1)$$

$$\pi_R^{MM}(p_1, p_n, p_r) = (p_1 - w_1)q_1 + (p_n - w_n)q_n + (p_r - w_r)q_r \quad (2)$$

$$s.t. \quad 0 < q_r \le \gamma q_1$$

The constraint means that the quantity of remanufactured products is constrained by a proportion of new products that are collected and remanufactured after one period of use. Since the manufacturer is the leader and the retailer is the follower, the decision sequence is as follows. Stage 1: the manufacturer decides the wholesale price of new products and, then, the retailer decides its retail price in period 1. Stage 2: the manufacturer decides the wholesale price of new and remanufactured products as well as the warranty period for remanufactured products and, then, the retailer decides the retail price of both products in period 2. According to backward induction, model *MM* has a unique optimal solution if $\phi_{MM} = 4\phi_{M2} - \phi_{M3}^2 > 0$. Given the unit remanufacturing cost c_r, we derive the equilibrium decisions as follows.

If $\delta(Q-\phi_{M0}) - \frac{\gamma\phi_{M0}\phi_{MM}}{4\beta_w^M} \le c_r < \delta(Q-\phi_{M0})$, the manufacturer remanufactures part of available used products (referred to as partial remanufacturing). In this case, the optimal wholesale price and warranty period decisions are $w_1^{MM-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_n^{MM-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_r^{MM-P*} = \frac{\delta Q + c_r}{2} + \frac{\phi_{M1}\phi_{M3}(\varepsilon_r + c_w^M)}{2\phi_{MM}}$, and $t^{MM-P*} = \frac{\phi_{M1}\phi_{M3}}{\phi_{MM}}$; the corresponding optimal retail prices are $p_1^{MM-P*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$, $p_n^{MM-P*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$, and $p_r^{MM-P*} = \frac{\delta(4Q-\phi_{M0})}{4} - \frac{\phi_{M1}(\phi_{M2} - \varepsilon_r\phi_{M3})}{\phi_{MM}}$.

If $0 < c_r < \delta(Q-\phi_{M0}) - \frac{\gamma\phi_{M0}\phi_{MM}}{4\beta_w^M}$, the manufacturer remanufactures all available used products (referred to as complete remanufacturing). In this case, the optimal wholesale price and warranty period decisions are $w_1^{MM-C*} = c_n + c_w^M + \frac{2\beta_w^M\phi_{M4}}{\varphi_{MM}}$, $w_n^{MM-C*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_r^{MM-C*} = c_r + \frac{2\gamma^2(\delta Q - c_r)(2\phi_{M2} + c_w^M\phi_{M3}) + \phi_{M0}(4\beta_w^M\delta(1+\gamma(1-\delta)) - \phi_{M3}(\gamma^2\delta(c_w^M + \varepsilon_r) - 2\gamma c_w^M))}{2\varphi_{MM}}$, and $t^{MM-C*} = \frac{\gamma\phi_{M3}\phi_{M4}}{\varphi_{MM}}$; the corresponding optimal retail prices are $p_1^{MM-C*} = Q + \varepsilon_n - \frac{\beta_w^M\phi_{M4}}{\varphi_{MM}}$, $p_n^{MM-C*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$, and $p_r^{MM-C*} = \delta(Q - \frac{\phi_{M0}}{4}) - \frac{\gamma\phi_{M4}(\phi_{M2} - \varepsilon_r\phi_{M3})}{\varphi_{MM}}$.

The derivation process of the equilibrium solution is furnished in Appendix A-1.

4.2 Model *RR*– the retailer offers warranty for new and remanufactured products

In model *RR*, the retailer offers warranty for new and remanufactured products, and the profit functions of the manufacturer and retailer can be formulated as:

$$\pi_M^{RR}(w_1, w_n, w_r) = (w_1 - c_n)q_1 + (w_n - c_n)q_n + (w_r - c_r)q_r \tag{3}$$

$$\pi_R^{RR}(p_1, p_n, p_r, t) = (p_1 - w_1 - c_w^R)q_1 + (p_n - w_n - c_w^R)q_n + (p_r - w_r - c_w^R t)q_r - \beta_w^R - \frac{1}{2}\beta_w^R t^2 \tag{4}$$

$$s.t. \quad 0 < q_r \le \gamma q_1$$

The constraint is the same as that in model *MM*. Since the manufacturer is the leader and the retailer is the follower, the decision sequence is as follows. Stage 1: the manufacturer decides the wholesale price of new products and, then, the retailer decides its retail price in period 1. Stage 2: the manufacturer decides the wholesale price of new and remanufactured products and, then, the retailer decides the retail price of both products and the warranty period for remanufactured products in period 2.

According to backward induction, model RR has a unique optimal solution if $\phi_{RR}=2\phi_{R2}-\phi_{R3}^2>0$. Given the unit remanufacturing cost c_r, one obtains the equilibrium decisions as follows.

If $\delta(Q-\phi_{R0})-\frac{\gamma\phi_{R0}\phi_{RR}}{2\beta_w^R}\le c_r<\delta(Q-\phi_{R0})$, the manufacturer commits to partial remanufacturing. In this case, the optimal pricing and warranty period decisions are $w_1^{RR-P*}=c_n+\frac{\phi_{R0}}{2}$, $w_n^{RR-P*}=c_n+\frac{\phi_{R0}}{2}$, $w_r^{RR-P*}=\frac{\delta Q+c_r}{2}$, $p_1^{RR-P*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$, $p_n^{RR-P*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$, $p_r^{RR-P*}=\frac{\delta(4Q-\phi_{R0})}{4}-\frac{\phi_{R1}(\phi_{R2}-\varepsilon_r\phi_{R3})}{2\phi_{RR}}$, and $t^{RR-P*}=\frac{\phi_{R1}\phi_{R3}}{2\phi_{RR}}$.

If $0<c_r<\delta(Q-\phi_{R0})-\frac{\gamma\phi_{R0}\phi_{RR}}{2\beta_w^R}$, the manufacturer is engaged in complete remanufacturing, and the equilibrium decisions are obtained as $w_1^{RR-C*}=c_n+\frac{\beta_w^R\phi_{R4}}{\varphi_{RR}}$, $w_n^{RR-C*}=c_n+\frac{\phi_{R0}}{2}$, $w_r^{RR-C*}=\frac{\delta Q+c_r}{2}+\frac{\phi_{R0}}{2\gamma}-\frac{\beta_w^R\phi_{R4}}{\gamma\varphi_{RR}}$, $p_1^{RR-C*}=Q+\varepsilon_n-\frac{\beta_w^R\phi_{R4}}{2\varphi_{RR}}$, $p_n^{RR-C*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$, $p_r^{RR-C*}=\frac{\delta(4Q-\phi_{R0})}{4}-\frac{\gamma\phi_{R4}(\phi_{R2}-\varepsilon_r\phi_{R3})}{2\varphi_{RR}}$, and $t^{RR-C*}=\frac{\gamma\phi_{R3}\phi_{R4}}{2\varphi_{RR}}$.

The derivation process of the equilibrium solution is provided in Appendix A-2.

4.3 Benchmark models

To facilitate the impact analysis of warranty on supply chain operations, two benchmark models are formulated as follows, where the manufacturer and the retailer, respectively, offer warranty for new products only, denoted as MN and RN accordingly.

(1) Model *MN*– the manufacturer offers warranty for new products only

Model MN is the simplified case of model MM where t =0, meaning that the manufacturer provides warranty for new products only. In this model, when $\delta(Q-\phi_{M0})-\gamma\delta(1-\delta)\phi_{M0}\le c_r<\delta(Q-\phi_{M0})$, the manufacturer commits to partial remanufacturing and the equilibrium decisions are $w_1^{MN-P*}=c_n+c_w^M+\frac{\phi_{M0}}{2}$, $w_n^{MN-P*}=c_n+c_w^M+\frac{\phi_{M0}}{2}$, $w_r^{MN-P*}=\frac{\delta Q+c_r}{2}$, $p_1^{MN-P*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}$, $p_n^{MN-P*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}$, and $p_r^{MN-P*}=\delta Q-\frac{\delta Q-c_r}{4}$. When $0<c_r<\delta(Q-\phi_{M0})-\gamma\delta(1-\delta)\phi_{M0}$, the manufacturer commits to complete remanufacturing and the equilibrium decisions are $w_1^{MN-C*}=c_n+c_w^M+\frac{\phi_{M4}}{2\varphi_0}$, $w_n^{MN-C*}=c_n+c_w^M+\frac{\phi_{M0}}{2}$, $w_r^{MN-C*}=c_r+\frac{\delta(\varphi_0\phi_{M0}+\gamma(1-\delta)\phi_{M4})}{2\varphi_0}$, $p_1^{MN-C*}=Q+\varepsilon_n-\frac{\phi_{M4}}{4\varphi_0}$, $p_n^{MN-C*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}$, and $p_r^{MN-C*}=\frac{3Q\delta+c_r}{4}+\frac{\phi_{M1}-\gamma\delta(1-\delta)\phi_{M0}}{4\varphi_0}$.

The derivation process of the equilibrium solution is given in Appendix A-3.

(2) Model *RN*– the retailer offers warranty for new products only

Model RN is the simplified case of model RR where t =0, meaning that the retailer offers warranty for new products only, but no warranty for remanufactured products. In this model, when $\delta(Q-\phi_{R0})-\gamma\delta(1-\delta)\phi_{R0}\le c_r<\delta(Q-\phi_{R0})$, the manufacturer commits to partial remanufacturing; the equilibrium decisions are $w_1^{RN-P*}=c_n+\frac{\phi_{R0}}{2}$, $w_n^{RN-P*}=c_n+\frac{\phi_{R0}}{2}$, $w_r^{RN-P*}=\frac{\delta Q+c_r}{2}$, $p_1^{RN-P*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$, $p_n^{RN-P*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$, and $p_r^{RN-P*}=\delta Q-\frac{\delta Q-c_r}{4}$. When

$0 \le c_r < \delta(Q-\phi_{R0}) - \gamma\delta(1-\delta)\phi_{R0}$, the manufacturer commits to complete remanufacturing; the equilibrium decisions are obtained as $w_1^{RN-C*} = c_n + \frac{\phi_{R4}}{2\varphi_0}$, $w_n^{RN-C*} = c_n + \frac{\phi_{R0}}{2}$, $w_r^{RN-C*} = \frac{\delta Q + c_r}{2} + \frac{\gamma\delta(1-\delta)\phi_{R0} - \phi_{R1}}{2\varphi_0}$, $p_1^{RN-C*} = Q + \varepsilon_n - \frac{\phi_{R4}}{4\varphi_0}$, $p_n^{RN-C*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, and $p_r^{RN-C*} = \delta Q - \frac{\varphi_0\phi_{R0} + \gamma(1-\delta)\phi_{R4}}{4\varphi_0}$.

The derivation process of the equilibrium solution is detailed in Appendix A-4.

5 Main results and comparative analyses

5.1 An impact analysis of warranty on remanufacturing and supply chain operations

Lemma 1. The manufacturer in model *MM* (the retailer in model *RR*) offers warranty for remanufactured products if and only if $\phi_{j3} = \varepsilon_r - c_w^j > 0$, $j \in \{M, R\}$.

The proof of Lemma 1 is obvious by examining the equilibrium warranty period decisions in Section 4.1 and 4.2. Loosely speaking, ϕ_{j3} gauges the net benefit of offering warranty for remanufactured products. The manufacturer in model *MM* and the retailer in model *RR* are willing to offer warranty ($t^{i*} > 0$, $i \in \{MM, RR\}$) if and only if they can benefit from it ($\phi_{j3} > 0$). Otherwise, they will not offer warranty for remanufactured products ($t^{i*} = 0$, $i \in \{MM, RR\}$). If $\phi_{j3} > 0$, keeping other things equal, the larger the ϕ_{j3}, the longer the warranty period. However, there exists an upper limit for the increased warranty period: the existence of a unique optimal solution for model *MM* (*RR*) and the constraint $t^{i*} \le n = 1$ imply that $\phi_{M3} < \frac{1}{2}(\sqrt{\phi_{M1}^2 + 16\phi_{M2}} - \phi_{M1})$ ($\phi_{R3} < \frac{1}{4}\left(\sqrt{\phi_{R1}^2 + 32\phi_{R2}} - \phi_{R1}\right)$) when the manufacturer commits to partial remanufacturing; and $\phi_{M3} < \frac{1}{2\gamma^2}(\sqrt{\phi_{M4}^2 + 16\gamma^2\beta_w^M\varphi_0} - \phi_{M4})$ ($\phi_{R3} < \frac{1}{4\gamma^2}\left(\sqrt{\phi_{R4}^2 + 32\gamma^2\beta_w^R\varphi_0} - \phi_{R4}\right)$) when the manufacturer commits to complete remanufacturing.

Proposition 1. Compared to models *MN* and *RN*, the following results hold:

i) When the manufacturer commits to partial remanufacturing, the wholesale price of remanufactured products is higher in model *MM* than that in model *MN*, but it is the same in models *RR* and *RN*; offering warranty for remanufactured products leads to a higher retail price of remanufactured products, but does not affect the wholesale or retail price of new products in both periods.

ii) When the manufacturer commits to complete remanufacturing, the wholesale price of remanufactured products is higher in model *MM* than that in model *MN*, and it is lower in model *RR* than that in model *RN*; offering warranty for remanufactured products leads to a higher retailer price of remanufactured products, a higher wholesale price and a lower retail price of new products in period 1 , but it has no effect on pricing decisions of new products in period 2.

The proof of Proposition 1 is given in Appendix B.

It is understandable that offering warranty for remanufactured products conveys a right quality signal to customers and increases consumers' utility of purchasing remanufactured products, thereby enhancing the sales of remanufactured products ($q_r^{MM-h*} > q_r^{MN-h*}$ and $q_r^{RR-h*} > q_r^{RN-h*}$, $h \in \{P, C\}$), exacerbating demand cannibalization for new products in period 2

($q_n^{MM-h*} < q_n^{MN-h*}$ and $q_n^{RR-h*} < q_n^{RN-h*}$). To curb profit loss in the new product market and maintain differentiated competition between new and remanufactured products in period 2, the manufacturer (retailer) does not adjust its wholesale price (retail price) of new products, thereby stabilizing the profit margin of new products ($w_n^{MM-h*} = w_n^{MN-h*}$, $w_n^{RR-h*} = w_n^{RN-h*}$, $p_n^{MM-h*} = p_n^{MN-h*}$, and $p_n^{RR-h*} = p_n^{RN-h*}$).

The impact of warranty for remanufactured products on pricing decisions of new products in period 1 and remanufactured products is closely related to the manufacturer's remanufacturing strategy. When the manufacturer commits to partial remanufacturing, offering warranty for remanufactured products has no effect on pricing decisions of new products in period 1. In terms of wholesale pricing of remanufactured products, in model *MM* where the manufacturer offers warranty and incurs warranty cost, it can pass the increased warranty cost onto the retailer by bumping up the wholesale price ($w_r^{MM-P*} > w_r^{MN-P*}$). On the other hand, in model *RR* where the retailer provides warranty and bears warranty cost, as the follower in the CLSC, it has no mechanism to influence the manufacturer's wholesale price of remanufactured products ($w_r^{RR-P*} = w_r^{RN-P*}$). As for the retail price, offering warranty incurs additional cost to the manufacturer (retailer) in the *MM* (*RR*) model. In model *MM*, the manufacturer transfers part of this cost to the retailer by increasing the wholesale price, leading the retailer to raise its retail price ($p_r^{MM-P*} > p_r^{MN-P*}$). In model *RR*, the retailer has to pass this additional warranty cost onto the customers by increasing the retail price of remanufactured products ($p_r^{RR-P*} > p_r^{RN-P*}$).

When the manufacturer remanufactures all available used products, remanufactured products in period 2 and new products in period 1 are complements. In this case, compared to producing new products, remanufacturing has significant cost advantage in the first place and offering warranty for remanufactured products will further enhance consumer's willingness-to-pay and demand for remanufactured products. For the wholesale price of remanufactured products, when the manufacturer offers warranty for remanufactured products in model *MM*, it can raise its wholesale price to share the benefit of enhanced demand for remanufactured products ($w_r^{MM-C*} > w_r^{MN-C*}$). When the retailer offers warranty for remanufactured products in model *RR*, the manufacturer lowers its wholesale price to compensate the retailer for its warranty cost ($w_r^{RR-C*} < w_r^{RN-C*}$). It is understandable that the retailer will charge a higher retail price for remanufactured products to absorb the additional cost when warranty is extended to them by the manufacturer (retailer) in model *MM* (*RR*) ($p_r^{MM-C*} > p_r^{MN-C*}$ and $p_r^{RR-C*} > p_r^{RN-C*}$). Furthermore, the enhanced demand for remanufactured products requires more new products due to the complementary product relationship, allowing the retailer to lower the retail price of new products ($p_1^{MM-C*} < p_1^{MN-C*}$ and $p_1^{RR-C*} < p_1^{RN-C*}$) in period 1, thereby boosting the sale of new products and, eventually, resulting in more available used products for remanufacturing in period 2. As the retailer is closer to the end market and directly benefits from enhanced demand for remanufactured products when warranty is extended to them, the profitability of the retailer increases much more than that of the manufacturer from the remanufactured product channel. To achieve an equitable allocation of the benefit, the manufacturer raises its wholesale price of new products to rake in more profit from this channel ($w_1^{MM-C*} > w_1^{MN-C*}$ and $w_1^{RR-C*} > w_1^{RN-C*}$) in models *MM* and *RR* in period 1. By collaborating with each other, both members achieve higher profitability by balancing their benefit from the new and remanufactured product channels.

Proposition 2. Compared to benchmark models *MN* and *RN*, offering warranty for remanufactured products improves individual and channel profits, and consumer surplus.

The proof of Proposition 2 is given in Appendix C.

Proposition 2 indicates that the combined effect of pricing decisions and resulting quantities from offering warranty for remanufactured products leads to heightened individual and channel profitability as well as consumer surplus in the CLSC in *MM* and *RR* warranty models compared to the corresponding benchmark models *MN* and *RN* offering warranty for new products only ($\pi_j^{MM-h*} > \pi_j^{MN-h*}$, $\pi_j^{RR-h*} > \pi_j^{RN-h*}$, $j \in \{M, R\}$; $\pi_T^{MM-h*} > \pi_T^{MN-h*}$, $\pi_T^{RR-h*} > \pi_T^{RN-h*}$, $CS^{MM-h*} > CS^{MN-h*}$, and $CS^{RR-h*} > CS^{RN-h*}$, $h \in \{P, C\}$). This result remains true regardless of the manufacturer's remanufacturing strategy, indicating that offering warranty for remanufactured products is mutually beneficial for the manufacturer, retailer, and consumers. The implication is that a wise choice is for the manufacturer or the retailer to extend warranty to remanufactured products. This is consistent with business practices in reality: Caterpillar and Lenovo, for example, provide warranty for remanufactured products comparable to that for their new products.

Corollary 1. The following results hold:

i) Compared to model *MN*, the increase in the manufacturer's profit is lower than the retailer's profit enhancement in model *MM*.

ii) Compared to model *RN*, the increase in the manufacturer's profit is higher than the retailer's profit enhancement in model *RR*.

The proof of Corollary 1 is given in Appendix D.

When the manufacturer offers warranty for remanufactured products in model *MM*, the manufacturer directly incurs the warranty cost, but its benefit from enhanced demand for remanufactured products is secondary. On the other hand, the retailer directly benefits from enhanced demand owing to offering warranty for remanufactured products, but its sharing of warranty cost is secondary. As such, it is reasonable that the retailer is better off in grabbing more shares in the enhanced profitability when the manufacturer offers warranty for remanufactured products in model *MM* compared to the benchmark model *MN*.

When the retailer offers warranty for remanufactured products in model *RR*, the retailer directly incurs the warranty cost and also directly enjoys the ensuing benefit of enhanced demand for remanufactured products. In this case, the manufacturer reaps more secondary benefit from enhanced demand for remanufactured products than the secondary expense for offering warranty, leading to a higher share of enhanced profitability for the manufacturer than the retailer.

Furthermore, a closer examination of the two members' profitability in different models reveals that the manufacturer as the leader in this CLSC takes a larger share of the channel profit than the retailer does if the manufacturer's quadratic warranty cost parameters for new and remanufactured products are not too high in models *MN* and *MM*. Corollary 1 implies that the profit inequity is always narrowed down if the manufacturer offers warranty for remanufactured products, but widened if the retailer does.

5.2 Equilibrium solution analysis

In this subsection, we compare equilibrium solutions between the two warranty models *MM* and *RR*. First, we identify the warranty cost parameter boundary conditions under which the manufacturer (retailer) offers a longer warranty period for remanufactured products. For apparent reasons, the manufacturer and retailer usually possess different levels of production technology and market knowledge. This difference tends to be reflected in different linear and quadratic warranty cost parameters ($c_w^M \neq c_w^R$ and $\beta_w^M \neq \beta_w^R$) when the manufacturer (retailer) offers warranty in model *MM* (*RR*). In general, three scenarios may arise: Scenario I ($c_w^M = c_w^R = c$ and $\beta_w^M \neq \beta_w^R$), Scenario II ($c_w^M \neq c_w^R$ and $\beta_w^M = \beta_w^R = \beta$), and Scenario III ($c_w^M \neq c_w^R$ and $\beta_w^M \neq \beta_w^R$). Next, we restrict our discussion to Scenario I only and the analysis for Scenario II can be carried out in a similar fashion. For the more general scenario of $c_w^M \neq c_w^R$ and $\beta_w^M \neq \beta_w^R$, we cannot obtain closed-form solutions to compare warranty periods and warranty efficiency between models *MM* and *RR* and, hence, the relevant analysis is delegated to numerical experiments in Section 6.

Proposition 3. Under scenario I when $c_w^M = c_w^R = c$, if $\overline{\beta}_{M1}^h \leq \beta_w^M < \overline{\beta}_{M2}^h$ ($h \in \{P, C\}$), a longer warranty period for remanufactured products is offered by the manufacturer in model *MM*; if $\beta_w^M = \overline{\beta}_{M2}^h$, models *MM* and *RR* offer the same warranty period; if $\beta_w^M > \overline{\beta}_{M2}^h$, a longer warranty period is offered by the retailer in model *RR*. Here $\beta_w^M \geq \overline{\beta}_{M1}^h$ is the lower bound of the quadratic cost parameter to guarantee that $t^{MM-h*} \leq n = 1$.

The proof of Proposition 3 is given in Appendix E.

As shown in Appendix E, when models *MM* and *RR* have the same linear cost parameter $c_w^M = c_w^R = c$ but different quadratic cost parameters in Scenario I, $\overline{\beta}_{M2}^h$ is a function of β_w^R. If $\overline{\beta}_{M1}^h \leq \beta_w^M < \overline{\beta}_{M2}^h$, the quadratic cost parameter in the manufacturer warranty model *MM* is more efficient compared to that in the retailer warranty model *RR*. In this case, the manufacturer is willing to offer a longer warranty period for remanufactured products in model *MM*. On the other hand, if $\beta_w^M > \overline{\beta}_{M2}^h$, the retailer has a more efficient quadratic cost parameter in model *RR* than the manufacturer does in model *MM*, so the retailer in model *RR* is able to provide a longer warranty period. It is worth noting that this structural insight on warranty period remains valid regardless of the manufacturer's remanufacturing strategy except for a shifted threshold value of $\overline{\beta}_{M2}^h$. It is also confirmed that $\overline{\beta}_{M2}^C$ decreases in collection rate γ, indicating that a higher collection rate makes it easier for the retailer in model *RR* to offer a longer warranty when the manufacturer remanufactures all available used products.

Corollary 2. Under scenario I when $c_w^M = c_w^R = c$, if $\overline{\beta}_{M1}^h \leq \beta_w^M < \overline{\beta}_{M3}^h$, a higher retail price is attained for remanufactured products in model *MM* than *RR*; if $\beta_w^M = \overline{\beta}_{M3}^h$, the retailer sets the same retail price in models *MM* and *RR*; if $\beta_w^M > \overline{\beta}_{M3}^h$, a higher retail price is achieved for remanufactured products in model *RR* than *MM*, where $\overline{\beta}_{M2}^h < \overline{\beta}_{M3}^h$ holds.

The proof of Corollary 2 is discussed in Appendix F.

When $\overline{\beta}_{M1}^h \le \beta_w^M < \overline{\beta}_{M2}^h$, the quadratic cost parameter of the manufacturer warranty model *MM* is more efficient than that in the retailer warranty model *RR*, the manufacturer is willing to provide a longer warranty period for remanufactured products and transfers part of the increased warranty cost to the retailer through the wholesale price, which is eventually passed onto consumers by the retailer at a higher retail price ($p_r^{MM-h*} > p_r^{RR-h*}$). At the other extreme end when $\beta_w^M > \overline{\beta}_{M3}^h > \overline{\beta}_{M2}^h$, the retailer in model *RR* has a significant relative quadratic cost parameter advantage over the manufacturer does in *MM*, it is natural from Proposition 3 that the retailer's cost advantage is translated into a longer warranty period in model *RR*. At the same time, the retailer can also raise its retail price of remanufactured products to maximize its economic benefit ($p_r^{MM-h*} < p_r^{RR-h*}$).

What is more complicated is in the middle range $\overline{\beta}_{M2}^h < \beta_w^M < \overline{\beta}_{M3}^h$, where the retailer in model *RR* has more efficient quadratic cost parameter than the manufacturer does in model *MM*. This cost advantage allows the retailer to offer a longer warranty period for remanufactured products in model *RR* than the manufacturer does in model *MM* as shown in Proposition 3. When the manufacturer offers warranty in model *MM*, even if its warranty period is shorter than that offered by the retailer in *RR*, the higher warranty cost and the double-marginalization nature of cost transfer to the retailer tend to push higher the retail price of remanufactured products in model *MM*. Conversely, if the retailer offers warranty in model *RR*, as it is closer to the market and can directly influence market demand by offering warranty and setting the retail price of remanufactured products, this channel structure allows the retailer to offer a longer warranty period for remanufactured products at a lower retail price in model *RR*. Therefore, the coupling effect of the cost advantage and channel structure leads to $t^{MM-h*} < t^{RR-h*}$ (Proposition 3) and $p_r^{MM-h*} > p_r^{RR-h*}$ when $\overline{\beta}_{M2}^h < \beta_w^M < \overline{\beta}_{M3}^h$. Similarly, the structural insight on the retail price of remanufactured products remains intact regardless of the manufacturer's remanufacturing strategy.

5.3 Analysis of profitability and consumer surplus

We first compare individual profitability between the two warranty models *MM* and *RR*.

Proposition 4. Under scenario I when $c_w^M = c_w^R = c$, the manufacturer achieves higher profitability in model *RR* than *MM*, but the retailer reaches higher profitability in model *MM* than *RR*.

The proof of Proposition 4 is furnished in Appendix G.

When the linear warranty cost parameters are the same in models *MM* and *RR* ($c_w^M = c_w^R = c$), the manufacturer's provision of warranty allows it to charge a higher wholesale price (leading to a higher unit profit margin) of remanufactured products in model *MM* than *RR*. When the manufacturer commits to partial remanufacturing, the unit profit margin for new products stays the same in models *MM* and *RR* in both periods. The quantities of new products in period 1 are the same in both models and the quadratic warranty cost for new products is only occurred in model *MM*, leading to a lower profit contribution from new products in period 1 in model *MM* for the manufacturer. In period 2, if $\beta_w^M < \frac{\beta_w^R}{2}$, the manufacturer warranty in model *MM* has a significant relative quadratic cost advantage; this advantage outweighs the double-marginalization effect of transferring warranty cost down to the retailer and results in a higher demand for remanufactured products but a lower demand for new products in model *MM* than that in model *RR*. Hence, the profit contribution from remanufactured products is higher for the manufacturer in model *MM* than that in model *RR*. On the other hand, the unit profit margin of new products for the manufacturer stays the same in model *MM* and *RR*. This leads to a lower profit contribution from new products for the manufacturer in model *MM* than that in model *RR*. And the reduced profitability from new products in the two periods outweighs the increased profitability from remanufactured products in model *MM* and the net effect is that the manufacturer achieves higher profitability in model *RR*, and this remains true when $\beta_w^M = \frac{\beta_w^R}{2}$. If the manufacturer warranty model *MM* has no significant relative quadratic cost advantage ($\beta_w^M > \frac{\beta_w^R}{2}$), the retailer's offering warranty boosts an even higher demand for remanufactured products and cannibalizes more demand for new products in model *RR*. The heightened demand in this case is more than enough to offset the decreased unit profit margin of remanufactured products for the manufacturer in model *RR*. As such, the profit contribution from remanufactured products in model *RR* is higher than that in model *MM*. This increased profitability from remanufactured products outweighs the change in profitability from new products and the net effect is that the manufacturer achieves higher profitability in model *RR*.

When the manufacturer commits to complete remanufacturing, if the manufacturer warranty model *MM* has a significant relative quadratic cost advantage ($\beta_w^M < \frac{\beta_w^R}{2} < \overline{\beta}_{M2}^C$), a longer warranty period offered in the *MM* model (as per Proposition 3) boosts more demand for remanufactured products (and the complementary new products in period 1) in model *MM* than *RR*. In addition, the manufacturer obtains higher unit profit margins of new products in period 1 and remanufactured products in model *MM* than those in model *RR*. However, the increased demand for remanufactured products produces more severe cannibalization of new products in period 2, leading to a significantly lower profitability from new products in period 2 in model *MM*. The increased profitability from new products in period 1 and remanufactured products is insufficient to recover the reduced profitability from new products in period 2 in model *MM* and the net effect is that the manufacturer achieves higher profitability in model *RR* and this remains true when $\beta_w^M = \frac{\beta_w^R}{2}$. If the manufacturer warranty model *MM* has no significant relative quadratic cost advantage ($\beta_w^M > \frac{\beta_w^R}{2}$), the manufacturer's unit profit margin and sales quantity of new products in period 1 are lower in model *MM* than those in model *RR*. On the other hand, the sale quantity of remanufactured products is lower in model *MM* than *RR* in this case; the impact of the manufacturer's higher unit profit margin of remanufactured products cannot counterbalance that of the lower demand in model *MM* compared to model *RR*, leading to a lower profit contribution from remanufactured products for the manufacturer in model *MM*. It is noted that the manufacturer's profit difference between models *MM* and *RR* from new products in period 2 increases in β_w^M. At the lower end when

$\beta_w^M > \frac{\beta_w^R}{2}$, the manufacturer's profit from new products in period 2 under model *MM* is lower than that under model *RR*. At the higher end when β_w^M is sufficiently large, the manufacturer achieves a higher profit from new products in period 2 in model *MM* than *RR*. However, the higher profit from new products in period 2 is insufficient to recover the reduced profitability from new products in period 1 and remanufactured products for the manufacturer in model *MM* and the net effect is that the manufacturer achieves higher profitability in model *RR* than *MM*. In summary, under scenario I when $c_w^M = c_w^R = c$, the manufacturer achieves higher profitability when the retailer offers warranty in model *RR* than the case when the manufacturer offers warranty in model *MM*.

Following the similar logic, Proposition 4 confirms that the retailer always obtains higher profitability when the manufacturer provides warranty for new and remanufactured products regardless of the manufacturer's remanufacturing strategy. In summary, Proposition 4 demonstrates that, under scenario I when models *MM* and *RR* have the same linear cost parameter ($c_w^M = c_w^R = c$), each member in the CLSC prefers the other to offer warranty for new and remanufactured products, and the CLSC may enter an impasse.

Proposition 5. Under scenario I when $c_w^M = c_w^R = c$, the following results hold.

i) If $\phi_{R3} \le \bar{\phi}_{R3}^h$ and $\beta_w^R > \bar{\beta}_{R2}^h$ ($\phi_{R3} > \bar{\phi}_{R3}^h$), there exist $\bar{\beta}_{M4}^h > \bar{\beta}_{M1}^h$ such that a higher channel profit is attained in model *MM* than *RR* if $\bar{\beta}_{M1}^h < \beta_w^M < \bar{\beta}_{M4}^h$; a higher channel profit is attained in model *RR* than *MM* if $\beta_w^M > \bar{\beta}_{M4}^h$; and an equal channel profit is achieved if $\beta_w^M = \bar{\beta}_{M4}^h$, $h \in \{P, C\}$.

ii) If $\phi_{R3} < \bar{\phi}_{R3}^h$ and $\bar{\beta}_{R1}^h < \beta_w^R < \bar{\beta}_{R2}^h$, a higher channel profit is attained in model *RR* than *MM*, $h \in \{P, C\}$.

The proof of Proposition 5 is furnished in Appendix H.

Proposition 5 shows that the comparison of the channel profit between models *MM* and *RR* depends on the net benefit of offering warranty for remanufactured products (ϕ_{j3}) and quadratic warranty cost parameters regardless of the manufacturer's remanufacturing strategy if $c_w^M = c_w^R = c$. When the net benefit of the retailer offering warranty for remanufactured products is low ($\phi_{R3} \le \bar{\phi}_{R3}^h$) and it has a high quadratic warranty cost parameter ($\beta_w^R > \bar{\beta}_{R2}^h$), or the net benefit of the retailer offering warranty for remanufactured products is high ($\phi_{R3} > \bar{\phi}_{R3}^h$), even if the manufacturer is farther away from the market compared to the retailer, its relative quadratic warranty cost advantage ($\bar{\beta}_{M1}^h < \beta_w^M < \bar{\beta}_{M4}^h$) outweighs the double-marginalization effect , leading to a higher channel profit in model *MM* than that in model *RR*. As β_w^M increases from the lower end, the manufacturer's relative quadratic warranty cost advantage shrinks. When β_w^M increases to $\bar{\beta}_{M4}^h$, the channel profit reaches parity for models *MM* and *RR*. When β_w^M exceeds $\bar{\beta}_{M4}^h$, the retailer has relative quadratic warranty cost advantage, the warranty efficiency for remanufactured products in model *MM* turns lower than that in model *RR* and, then, a higher channel profit is attained in model *RR*. When the net benefit of the retailer offering warranty for remanufactured products is low ($\phi_{R3} < \overline{\phi}_{R3}^h$) but it has an advantageous quadratic warranty cost parameter ($\overline{\beta}_{R1}^h < \beta_w^R < \overline{\beta}_{R2}^h$), the retailer's proximity to the market and direct influence on market demand by offering warranty lead to a higher channel profit in model *RR* than *MM*.

Next, we compare consumer surplus between the two warranty models *MM* and *RR*.

Proposition 6. Under scenario I when $c_w^M = c_w^R = c$, the following results hold.

i) When $\beta_w^R \geq \overline{\beta}_{R3}^h$, there exists $\overline{\beta}_{M5}^h \geq \overline{\beta}_{M1}^h$ such that consumer surplus is higher in model *MM* if $\overline{\beta}_{M1}^h \leq \beta_w^M < \overline{\beta}_{M5}^h$; it is the same for models *MM* and *RR* if $\beta_w^M = \overline{\beta}_{M5}^h$; it is higher in model *RR* if $\beta_w^M > \overline{\beta}_{M5}^h$, $h \in \{P, C\}$.

ii) When $\overline{\beta}_{R1}^h \leq \beta_w^R < \overline{\beta}_{R3}^h$, consumer surplus is higher in model *RR*, $h \in \{P, C\}$.

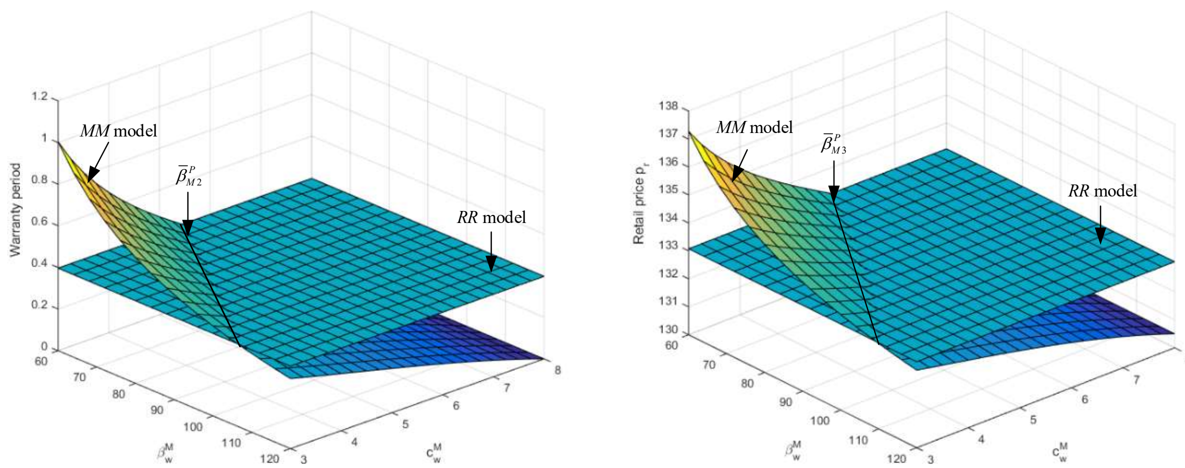
The proof of Proposition 6 is given in Appendix I.

Proposition 6.*i*) shows that consumer surplus is higher in model *MM* than *RR* if the retailer's provision of warranty has a sufficiently large quadratic cost parameter ($\beta_w^R > \overline{\beta}_{R3}^h$) and the manufacturer's provision of warranty has a relative quadratic cost advantage ($\overline{\beta}_{M1}^h < \beta_w^M < \overline{\beta}_{M5}^h$. NB: $\overline{\beta}_{M1}^h$ is the lower bound of β_w^M to ensure that $t^{MM-h*} \leq n = 1$). Under this condition, if the manufacturer commits to partial remanufacturing, demands for new products in period 1 are the same in models *MM* and *RR*, but it is higher in model *MM* than *RR* if the manufacturer commits to complete remanufacturing. On the other hand, regardless of the manufacturer's remanufacturing strategy, in period 2, offering warranty by the manufacturer in model *MM* has a more notable impact on enhancing demand for remanufactured products compared to model *RR*. Although this causes more severe cannibalization of demand for new products in period 2, the increase in the quantity of remanufactured products is more rapid than the decrease in the quantity of new products. Therefore, it is natural that consumer surplus is higher in model *MM* than *RR* regardless of the manufacturer's remanufacturing strategy.

However, when β_w^M increases to $\overline{\beta}_{M5}^h$, the manufacturer's relative quadratic cost advantage of offering warranty disappears and a parity is reached in the quantities of new and remanufactured products, leading to the same consumer surplus in models *MM* and *RR*. When β_w^M further increases beyond $\overline{\beta}_{M5}^h$, the retailer now has a relative quadratic cost advantage of offering warranty. In this case, if the manufacturer commits to partial remanufacturing, demands for new products in period 1 are the same in models *MM* and *RR*, but offering warranty by the retailer in model *RR* has a more significant impact on enhancing demand for remanufactured products, which outweighs the demand cannibalization of new products in period 2, leading to higher consumer surplus in model *RR* than *MM*. If the manufacturer is engaged in complete remanufacturing, offering warranty by the retailer in model *RR* is more effective in enhancing demand for remanufactured products and, subsequently, the complementary new products in period 1. On the other hand, offering warranty by the retailer causes more severe cannibalization of demand for new products in period 2 in model *RR*, but the cannibalization does not outweigh the increases in demands for remanufactured products in period 2 and new products in period 1, resulting in higher consumer surplus in model *RR* than *MM*. Similarly, Proposition 6.*ii*) corresponds to the case that the retailer possesses relative quadratic cost advantage of offering warranty ($\beta_w^R < \overline{\beta}_{R3}^h$) and, hence, higher consumer surplus is achieved when the retailer provides warranty in model *RR*.

6 Numerical experiment

In this section, we present numerical studies to verify the analytical results in Section 5 by comparing equilibrium decisions as well as profits and consumer surplus between different models with a general setting of $c_w^M \neq c_w^R$ and $\beta_w^M \neq \beta_w^R$. Let $Q = 220$, $c_n = 100$, $c_r = 60$, $\delta = 0.7$, $c_w^R = 5$, $\beta_w^R = 80$, $\varepsilon_n = 10$, $\varepsilon_r = 8$, and we obtain the relevant charts by varying the values of parameters β_w^M and c_w^M. Because the manufacturer's remanufacturing strategy has no significant impact on the main results, the numerical studies are confined to the scenario of partial remanufacturing. We first plot the warranty period and retail price of remanufactured products in models *MM* and *RR* in Fig. 1.

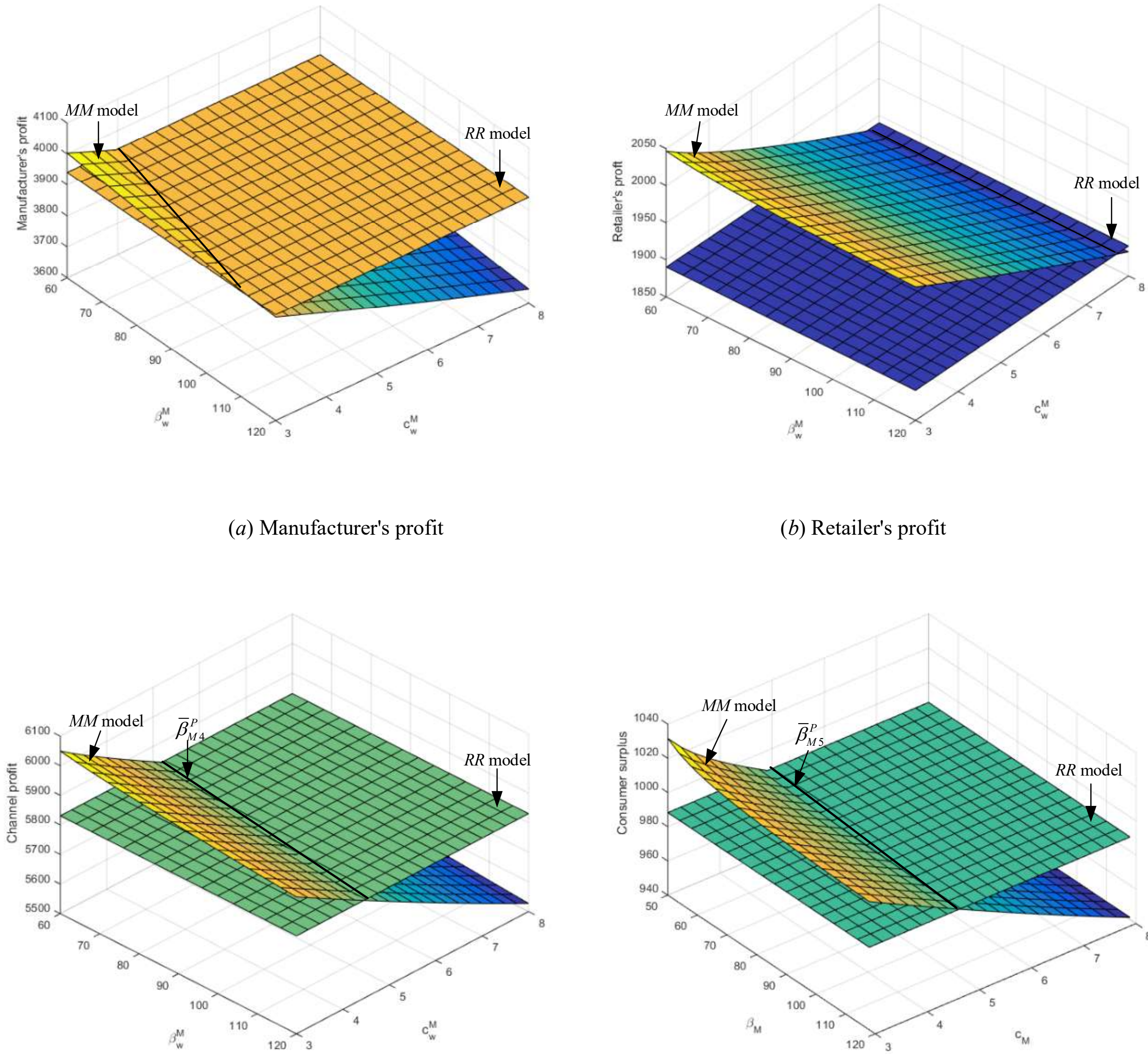


(*a*) Warranty period (*b*) Retail price

Figure 1. The warranty period and retail price of remanufactured products in the two warranty models

It can be seen from Fig. 1 that, when β_w^M and c_w^M are small and the manufacturer warranty has significant cost advantage, the *MM* model has a longer warranty period and higher retail price for remanufactured products than the *RR* model does, and the higher the β_w^M and c_w^M, the shorter (longer) the warranty period and the lower (higher) the retail price for remanufactured products in model *MM* (*RR*). When the warranty cost parameters meet the condition $\frac{(c_w^M-8)(0.7c_w^M+3)}{(c_w^M-8)^2-0.84\beta_w^M} < \frac{c_w^R(0.35c_w^R-1.3)-12}{(c_w^R-8)^2-0.42\beta_w^R}$ ($\frac{8208+c_w^M((0.175c_w^M+134.05)c_w^M-2109.6)-109.62\beta_w^M}{(c_w^M-8)^2-0.84\beta_w^M} < \frac{8304+c_w^R((0.175c_w^R+131.25)c_w^R-2099.2)-54.81\beta_w^R}{(c_w^R-8)^2-0.42\beta_w^R}$), the retailer warranty cost advantage becomes significant and, hence, the warranty period (retail price) of remanufactured products in model *MM* becomes shorter (lower) than that in model *RR*. By fixing $c_w^M = c_w^R = 5$, it is confirmed that, if $\beta_w^M > \overline{\beta}_{M2}^P = -10.714 + \beta_w^R$ ($\beta_w^M > \overline{\beta}_{M3}^P = -13.187 + 1.115\beta_w^R$), the warranty period (retail price) of remanufactured products in model *MM* is shorter (lower) than that in model *RR*. These results verify the conclusions in Proposition 3 and Corollary 2.

Next, under the same settings, we plot the individual and channel profits as well as consumer surplus against the two parameters β_w^M and c_w^M in Fig. 2.



(*a*) Manufacturer's profit (*b*) Retailer's profit

(*c*) Channel profit (*d*) Consumer surplus

Figure 2. Comparisons of individual and channel profits, and consumer surplus between the two warranty models

It is clear from Fig. 2 that, when manufacturer warranty has a significant relative cost advantage at sufficiently small β_w^M and c_w^M, individual and channel profit as well as consumer surplus are all higher in model *MM* than those in model *RR*. However, as β_w^M and c_w^M increase, these metrics in model *MM* decrease and turn lower than those in model *RR*. In terms of individual profit, we find that the manufacturer achieves a higher profit in model *RR* than that in *MM* regardless of β_w^M if $c_w^M \geq c_w^R$, and the retailer attains a higher profit in model *MM* than that in model *RR* regardless of β_w^M if c_w^M is not significantly larger than c_w^R. This result indicates that, if c_w^M is significantly larger than c_w^R, both members are better off in profitability when the retailer offers warranty for new and remanufactured products. In terms of the channel profit, by setting $c_w^M = c_w^R = 5$, one can confirm that $\overline{\beta}_{M4}^P = 74.9396$ exists if $\phi_{R3} < \frac{2}{3}\phi_{R1}$ and $\beta_w^R > \overline{\beta}_{R2}^P = 46.5356$. When $\beta_w^M < \overline{\beta}_{M4}^P = 74.9396$ ($\beta_w^M > \overline{\beta}_{M4}^P = 74.9396$), the channel profit is higher (lower) in model *MM* than that in model *RR*. As for consumer surplus, if $\beta_w^R > \overline{\beta}_{R3}^P = 67.8571$ and $\beta_w^M < \overline{\beta}_{M5}^P = 40$ ($\beta_w^R > \overline{\beta}_{R3}^P = 67.8571$ and $\beta_w^M > \overline{\beta}_{M5}^P = 40$), consumer surplus is higher (lower) in model *MM* than that in model *RR*. These results verify the conclusions in Propositions 4, 5 and 6.

In general, these numerical studies allow us to infer that individual and channel profits as well as consumer surplus in model *MM* are higher than those in model *RR* when both the manufacturer's quadratic and linear warranty cost (β_w^M and c_w^M) are sufficiently efficient compared to those in the retailer warranty model *RR*. Otherwise, they are lower in model *MM* than those in model *RR*.

7 Conclusions

For a two-period CLSC consisting of a manufacturer and a retailer, this study establishes four Stackelberg game models to incorporate the warranty policies for new and remanufactured products. The manufacturer is modeled as the leader and the retailer is the follower. We consider two warranty models where either the manufacturer or the retailer offers warranty for new and remanufactured products and two corresponding benchmark models with warranty for new products only. We derive the optimal wholesale and retail pricing as well as warranty period decisions for the models. By analyzing and comparing equilibrium solutions and the resulting sales quantities, individual and channel profits as well as consumer surplus, the following results are obtained.

Firstly, the conditions are identified to guarantee the offering of warranty for remanufactured products: the manufacturer in model *MM* and the retailer in model *RR* are willing to offer warranty for remanufactured products if and only if they can benefit from it (Lemma 1). Secondly, the remanufacturing strategy (partial or complete remanufacturing) and warranty for remanufactured products do not affect the pricing strategies of new products in the second period, but they influence the pricing of new products in the first period and remanufactured products in the second period (Proposition 1). Thirdly, offering warranty for remanufactured products improves the sales quantity of remanufactured products, thereby enhancing individual and channel profits, and consumer surplus in the CLSC (Proposition 2); meanwhile, it also attains a more equitable channel profit distribution between the two members if the manufacturer provides warranty for remanufactured products (Corollary 1). Finally, by comparing the two warranty models *MM* and *RR*, we find that the member who has relative warranty cost advantage offers a longer warranty period for remanufactured products (Proposition 3). If the manufacturer (retailer) has a significant linear and quadratic warranty cost advantage over the retailer (the manufacturer), the manufacturer (retailer) should be entrusted to offer the warranty for new and remanufactured products, which can lead to higher individual and channel profitability as well as consumer surplus (Fig. 2).

This paper only considers a CLSC consisting of one manufacturer and one retailer where the manufacturer produces new products in two periods and remanufactures used products in the second period, and sells them to the retailer. In practice, remanufacturing may be performed by a third-party remanufacturer, and products may be sold to multiple retailers. Therefore, future research can be extended to consider warranty policies for new and remanufactured products with competition and cooperation among multiple agents.

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Appendices: Supplementary materials

Appendix A: Proof of the derivation process of the equilibrium solutions in the four models

For a more concise presentation of equilibrium solutions, we introduce the following notation, $\phi_{j0}=Q-(c_n+c_w^j-\varepsilon_n)$, $\phi_{j1}=\delta(c_n+c_w^j-\varepsilon_n)-c_r$, $\phi_{j2}=\beta_w^j\delta(1-\delta)$, $\phi_{j3}=\varepsilon_r-c_w^j$, $\phi_{j4}=\phi_{j0}+\gamma\phi_{j1}$, $\varphi_0=1+\gamma^2\delta(1-\delta)$, $\phi_{MM}=4\phi_{M2}-\phi_{M3}^2$, $\phi_{RR}=4\phi_{R2}-\phi_{R3}^2$, $\varphi_{MM}=4\beta_w^M+\gamma^2\phi_{MM}$, and $\varphi_{RR}=2\beta_w^R+\gamma^2\phi_{RR}$, where $j\in\{M,R\}$, representing the manufacturer and retailer, respectively.

Appendix A-1: Equilibrium solution derivation for model *MM*

In model *MM*, the Lagrangian function for the retailer's optimization problem is stated as follows:

$L_R^{MM}(p_1,p_n,p_r)=\pi_R^{MM}+\lambda_1(\gamma q_1-q_r)$, where λ_1 is the Lagrangian multiplier and $\lambda_1(\gamma q_1-q_r)=0$. According to backward induction, given w_1, p_1, w_n, w_r and t, $\frac{\partial^2 L_R^{MM}}{\partial p_n^2}=-\frac{2}{1-\delta}$, $\frac{\partial^2 L_R^{MM}}{\partial p_r^2}=-\frac{2}{\delta(1-\delta)}$, and $\frac{\partial^2 L_R^{MM}}{\partial p_n \partial p_r}=\frac{2}{1-\delta}$, one can confirm that the Lagrangian function $L_R^{MM}(p_1,p_n,p_r)$ is strictly and jointly concave in decision variables p_n and p_r and, hence, has a unique optimal solution. Solving the first-order condition $\frac{\partial L_R^{MM}}{\partial p_n}=0$ and $\frac{\partial L_R^{MM}}{\partial p_r}=0$, we can obtain the optimal response functions for the retailer $p_n^{MM*}(w_1,p_1,w_n,w_r,t)=\frac{Q+\varepsilon_n+w_n}{2}$ and $p_r^{MM*}(w_1,p_1,w_n,w_r,t)=\frac{\delta Q+t\varepsilon_r+w_r+\lambda_1}{2}$.

These response functions are then substituted into the manufacturer's profit function π_M^{MM}. If $\phi_{MM}=4\phi_{M2}-\phi_{M3}^2>0$, the manufacturer's profit function is strictly and jointly concave in decision variables w_n, w_r and *t*. In this case, model *MM* has a unique optimal solution. Solving the first-order condition $\frac{\partial \pi_M^{MM}}{\partial w_n}=0$, $\frac{\partial \pi_M^{MM}}{\partial w_r}=0$, and $\frac{\partial \pi_M^{MM}}{\partial t}=0$, we obtain the manufacturer's optimal solutions $w_r^{MM*}=\frac{\delta Q+c_r}{2}+\frac{\phi_{M1}\phi_{M3}(\varepsilon_r+c_w^M)}{2\phi_{MM}}-\frac{(2\phi_{M2}+c_w^M\phi_{M3})\lambda_1}{\phi_{MM}}$, $w_n^{MM*}=c_n+c_w^M+\frac{\phi_{M0}}{2}$, and $t^{MM*}=\frac{\phi_{M3}(\phi_{M1}-\lambda_1)}{\phi_{MM}}$. Thus, the optimal retail prices of new and remanufactured products are derived as $p_n^{MM*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}$ and $p_r^{MM*}=\frac{\delta}{4}(4Q-\phi_{M0})-\frac{(\phi_{M1}-\lambda_1)(\phi_{M2}-\varepsilon_r\phi_{M3})}{\phi_{MM}}$, respectively.

Substituting p_n^{MM*}, p_r^{MM*}, w_n^{MM*}, w_r^{MM*} and t^{MM*} into L_R^{MM}, since $\frac{d^2 L_R^{MM}}{dp_1^2}=-2$, one can confirm that the Lagrangian function L_R^{MM} is strictly concave in decision variable p_1 and, hence, has a unique optimal solution. Solving the first-order condition $\frac{dL_R^{MM}}{dp_1}=0$, we can obtain the optimal response function for the retailer $p_1^{MM*}(w_1)=\frac{Q+\varepsilon_n+w_1}{2}-\frac{\gamma\lambda_1}{2}$. Then, by substituting p_n^{MM*}, p_r^{MM*}, w_n^{MM*}, w_r^{MM*}, t^{MM*} and p_1^{MM*} into the manufacturer's profit function π_M^{MM}, as $\frac{d^2\pi_M^{MM}}{dw_1^2}=-1$, one can confirm that the manufacturer's profit function π_M^{MM} is strictly concave in decision variable w_1 and, hence, has a unique optimal solution. Solving the first-order condition $\frac{d\pi_M^{MM}}{dw_1}=0$, we can obtain $w_1^{MM*}=c_n+c_w^M+\frac{\phi_{M0}+\gamma\lambda_1}{2}$. Thus, the optimal pricing decisions of new and remanufactured products and the warranty period for remanufactured products are derived as $p_1^{MM*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}-\frac{\gamma\lambda_1}{4}$, $w_n^{MM*}=c_n+c_w^M+\frac{\phi_{M0}}{2}$, $w_r^{MM*}=\frac{\delta Q+c_r}{2}+\frac{\phi_{M1}\phi_{M3}(\varepsilon_r+c_w^M)}{2\phi_{MM}}-\frac{(2\phi_{M2}+c_w^M\phi_{M3})\lambda_1}{\phi_{MM}}$, $t^{MM*}=\frac{\phi_{M3}(\phi_{M1}-\lambda_1)}{\phi_{MM}}$, $p_n^{MM*}=Q+\varepsilon_n-\frac{\phi_{M0}}{4}$ and $p_r^{MM*}=\frac{\delta}{4}(4Q-\phi_{M0})-\frac{(\phi_{M1}-\lambda_1)(\phi_{M2}-\varepsilon_r\phi_{M3})}{\phi_{MM}}$, respectively.

According to the Karush-Kuhn-Tucker condition, there are two possible cases for the optimal solutions corresponding to different Lagrangian coefficient λ_1 as follows.

Case I: When $\lambda_1 = 0$, $\gamma q_1^{MM*} - q_r^{MM*} \geq 0$ (denoted as partial remanufacturing) can be derived based on the Complementary Relaxation Theorem. The manufacturer's optimal solutions are $w_1^{MM-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_n^{MM-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_r^{MM-P*} = \frac{\delta Q + c_r}{2} + \frac{\phi_{M1}\phi_{M3}(\varepsilon_r + c_w^M)}{2\phi_{MM}}$, and $t^{MM-P*} = \frac{\phi_{M1}\phi_{M3}}{\phi_{MM}}$, the retailer's optimal solutions are $p_1^{MM-P*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$, $p_n^{MM-P*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$ and $p_r^{MM-P*} = \frac{\delta(4Q - \phi_{M0})}{4} - \frac{\phi_{M1}(\phi_{M2} - \varepsilon_r \phi_{M3})}{\phi_{MM}}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{MM-P*} = \frac{\phi_{M0}}{4}$, $q_n^{MM-P*} = \frac{\phi_{M0}}{4} - \frac{\delta \beta_w^M \phi_{M1}}{\phi_{MM}}$, $q_r^{MM-P*} = \frac{\beta_w^M \phi_{M1}}{\phi_{MM}}$, $\pi_M^{MM-P*} = \frac{\phi_{M0}^2}{4} + \frac{\beta_w^M \phi_{M1}^2}{2\phi_{MM}} - \beta_w^M$, $\pi_R^{MM-P*} = \frac{\phi_{M0}^2}{8} + \frac{\delta(1-\delta)(\beta_w^M \phi_{M1})^2}{\phi_{MM}^2}$, and $\pi_T^{MM-P*} = \frac{3\phi_{M0}^2}{8} + \frac{\beta_w^M \phi_{M1}^2 (6\phi_{M2} - \phi_{M3}^2)}{2\phi_{MM}^2} - \beta_w^M$, respectively. Then, according to $\gamma q_1^{MM*} - q_r^{MM*} \geq 0$, we obtain the condition for Case I as $\delta(Q - \phi_{M0}) - \frac{\gamma \phi_{M0} \phi_{MM}}{4\beta_w^M} \leq c_r < \delta(Q - \phi_{M0})$.

Case II: When $\lambda_1 > 0$, $\gamma q_1^{MM*} - q_r^{MM*} = 0$ (denoted as complete remanufacturing) can be derived based on the Complementary Relaxation Theorem. It is easy to get $\lambda_1 = \frac{4\phi_{M4}\beta_w^M}{\gamma \varphi_{MM}} - \frac{\phi_{M0}}{\gamma}$ given $\gamma q_1^{MM*} - q_r^{MM*} = 0$. By substituting it into the manufacturer's and retailer's optimal solutions, we obtain $w_1^{MM-C*} = c_n + c_w^M + \frac{2\beta_w^M \phi_{M4}}{\varphi_{MM}}$, $w_n^{MM-C*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_r^{MM-C*} = c_r + \frac{2\gamma^2(\delta Q - c_r)(2\phi_{M2} + c_w^M \phi_{M3}) + \phi_{M0}(4\beta_w^M \delta(1 + \gamma(1-\delta)) - \phi_{M3}(\gamma^2 \delta(c_w^M + \varepsilon_r) - 2\gamma c_w^M))}{2\varphi_{MM}}$, $t^{MM-C*} = \frac{\gamma \phi_{M3} \phi_{M4}}{\varphi_{MM}}$, $p_1^{MM-C*} = Q + \varepsilon_n - \frac{\beta_w^M \phi_{M4}}{\varphi_{MM}}$, $p_n^{MM-C*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$ and $p_r^{MM-C*} = \delta(Q - \frac{\phi_{M0}}{4}) - \frac{\gamma \phi_{M4}(\phi_{M2} - \varepsilon_r \phi_{M3})}{\varphi_{MM}}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{MM-C*} = \frac{\beta_w^M \phi_{M4}}{\varphi_{MM}}$, $q_n^{MM-C*} = \frac{\phi_{M0}}{4} - \frac{\gamma \delta \beta_w^M \phi_{M4}}{\varphi_{MM}}$, $q_r^{MM-C*} = \frac{\gamma \beta_w^M \phi_{M4}}{\varphi_{MM}}$, $\pi_M^{MM-C*} = \frac{1}{8}(\phi_{M0}^2 + \frac{4\beta_w^M {\phi_{M4}}^2}{\varphi_{MM}}) - \beta_w^M$, $\pi_R^{MM-C*} = \frac{\phi_{M0}^2}{16} + \frac{(\beta_w^M \phi_{M4})^2 \varphi_0}{\varphi_{MM}^2}$, and $\pi_T^{MM-C*} = \frac{3\phi_{M0}^2}{16} + \frac{\beta_w^M \phi_{M4}^2 (\varphi_{MM} + 2\beta_w^M \varphi_0)}{2\varphi_{MM}^2} - \beta_w^M$, respectively. Furthermore, $0 < c_r < \delta(Q - \phi_{M0}) - \frac{\gamma \phi_{M0} \phi_{MM}}{4\beta_w^M}$ is derived from $\lambda_1 > 0$, corresponding to the condition for Case II.

Thus, the equilibrium solutions are obtained for model *MM*.

Appendix A-2: Equilibrium solution derivation for model *RR*

In model *RR*, the Lagrangian function for the retailer's optimization problem is given as: $L_R^{RR}(p_1, p_n, p_r, t) = \pi_R^{RR} + \lambda_1(\gamma q_1 - q_r)$, where λ_2 is the Lagrangian multiplier and $\lambda_2(\gamma q_1 - q_r) = 0$. According to backward induction, given w_1, p_1, w_n and w_r, since $\frac{\partial^2 L_R^{RR}}{\partial p_n^2} = -\frac{2}{1-\delta}$, $\frac{\partial^2 L_R^{RR}}{\partial p_r^2} = -\frac{2}{\delta(1-\delta)}$, $\frac{\partial^2 L_R^{RR}}{\partial t^2} = -(\beta_w^R + \frac{2c_w^R \varepsilon_r}{\delta(1-\delta)})$,

$\frac{\partial^2 L_R^{RR}}{\partial p_n \partial t} = -\frac{c_w^R + \varepsilon_r}{1-\delta}$, $\frac{\partial^2 L_R^{RR}}{\partial p_r \partial t} = \frac{c_w^R + \varepsilon_r}{\delta(1-\delta)}$, and $\frac{\partial^2 L_R^{RR}}{\partial p_n \partial p_r} = \frac{2}{1-\delta}$, one confirms that, if $\phi_{RR} = 2\phi_{R2} - \phi_{R3}^2 > 0$, the retailer's Lagrangian function $L_R^{RR}(p_1, p_n, p_r, t)$ is strictly and jointly concave in decision variables p_n, p_r and t, meaning that $L_R^{RR}(p_1, p_n, p_r, t)$ has a unique optimal solution. Solving the first-order conditions $\frac{\partial L_R^{RR}}{\partial p_n} = 0$, $\frac{\partial L_R^{RR}}{\partial p_r} = 0$, and $\frac{\partial L_R^{RR}}{\partial t} = 0$, we can obtain the optimal response functions for the retailer $p_n^{RR*}(w_1, p_1, w_n, w_r) = \frac{Q + \varepsilon_n + w_n + c_w^R}{2}$, $p_r^{RR*}(w_1, p_1, w_n, w_r) = \frac{\delta Q + w_r + \lambda_2}{2} - \frac{(\varepsilon_r^2 - c_w^{R2})(w_r + \lambda_2 - \delta(c_w^R + w_n - \varepsilon_n))}{2\phi_{RR}}$, and $t^{RR*}(w_1, p_1, w_n, w_r) = \frac{\phi_{R3}(\delta(c_w^R + w_n - \varepsilon_n) - \lambda_2)}{\phi_{RR}}$.

These three response functions are then substituted into the manufacturer's profit function π_M^{RR}. If $\phi_{RR} > 0$, $\frac{\partial^2 \pi_M^{RR}}{\partial w_n^2} = -1 - \frac{2\beta_w^R \delta}{\phi_{RR}} < 0$, $\frac{\partial^2 \pi_M^{RR}}{\partial w_r^2} = -\frac{2\beta_w^R}{\phi_{RR}} < 0$, $\frac{\partial^2 \pi_M^{RR}}{\partial w_n \partial w_r} = \frac{\partial^2 \pi_M^{RR}}{\partial w_r \partial w_n} = \frac{2\beta_w^R \delta}{\phi_{RR}} > 0$, then, one can confirm that the manufacturer's profit function π_M^{RR} is strictly and jointly concave in decision variables w_n and w_r. Then, model *RR* has a unique optimal solution. Solving the first-order conditions $\frac{\partial \pi_M^{RR}}{\partial w_n} = 0$ and $\frac{\partial \pi_M^{RR}}{\partial w_r} = 0$, we obtain the manufacturer's optimal solutions $w_n^{RR} = c_n + \frac{\phi_{R0}}{2}$ and $w_r^{RR*} = c_r + \frac{(\delta Q - c_r) - \lambda_2}{2}$. Thus, the optimal retail prices of new and remanufactured products and warranty period decision are derived as $p_n^{RR*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$ and $p_r^{RR*} = \frac{\delta(4Q - \phi_{R0})}{4} - \frac{\phi_{R1}(\phi_{R2} - \varepsilon_r \phi_{R3}) - (2\phi_{R2} - \varepsilon_r(\varepsilon_r - c_w^R))\lambda_2}{2\phi_{RR}}$, and $t^{RR*} = \frac{\phi_{R3}(\phi_{R1} - \lambda_2)}{2\phi_{RR}}$, respectively.

Substituting p_n^{RR*}, p_r^{RR*}, w_n^{RR*}, w_r^{RR*} and t^{RR*} into L_R^{RR}, since $\frac{d^2 L_R^{RR}}{dp_1^2} = -2$, one can confirm that the Lagrangian function L_R^{RR} is strictly concave in decision variable p_1 and, hence, has a unique optimal solution. Solving the first-order condition $\frac{dL_R^{MM}}{dp_1} = 0$, we can obtain the optimal response function for the retailer $p_1^{RR*}(w_1) = Q + \varepsilon_n - \frac{\phi_{R0} + \gamma\lambda_2}{4}$. Then, by substituting p_n^{RR*}, p_r^{RR*}, w_n^{RR*}, w_r^{RR*}, t^{RR*} and p_1^{RR*} into the manufacturer's profit function π_M^{RR}, as $\frac{d^2 \pi_M^{RR}}{dw_1^2} = -1$, one can confirm that the manufacturer's profit function π_M^{RR} is strictly concave in decision variable w_1 and, thus, has a unique optimal solution. Solving the first-order condition $\frac{d\pi_M^{RR}}{dw_1} = 0$, we can obtain $w_1^{RR*} = c_n + \frac{\phi_{R0} + \gamma\lambda_2}{2}$. Thus, the optimal pricing decisions of new and remanufactured products and the warranty period for remanufactured products are derived as $p_1^{RR*} = Q + \varepsilon_n - \frac{\phi_{R0} + \gamma\lambda_2}{4}$, $w_n^{RR} = c_n + \frac{\phi_{R0}}{2}$, $w_r^{RR*} = c_r + \frac{(\delta Q - c_r) - \lambda_2}{2}$, $p_n^{RR*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, $p_r^{RR*} = \frac{\delta(4Q - \phi_{R0})}{4} - \frac{\phi_{R1}(\phi_{R2} - \varepsilon_r \phi_{R3}) - (2\phi_{R2} - \varepsilon_r(\varepsilon_r - c_w^R))\lambda_2}{2\phi_{RR}}$, and $t^{RR*} = \frac{\phi_{R3}(\phi_{R1} - \lambda_2)}{2\phi_{RR}}$, respectively.

According to the Karush-Kuhn-Tucker condition, there exist two possible cases for the optimal solutions corresponding to different Lagrangian coefficient λ_2 as follows.

Case I: When $\lambda_2 = 0$, the manufacturer's optimal solutions are $w_1^{RR-P*} = c_n + \frac{\phi_{R0}}{2}$, $w_n^{RR-P*} = c_n + \frac{\phi_{R0}}{2}$ and $w_r^{RR-P*} = \frac{\delta Q + c_r}{2}$, the retailer's optimal solutions are $p_1^{RR-P*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, $p_n^{RR-P*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, $p_r^{RR-P*} = \frac{\delta(4Q - \phi_{R0})}{4} - \frac{\phi_{R1}(\phi_{R2} - \varepsilon_r \phi_{R3})}{2\phi_{RR}}$, and $t^{RR-P*} = \frac{\phi_{R1}\phi_{R3}}{2\phi_{RR}}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{RR-P*} = \frac{\phi_{R0}}{4}$, $q_n^{RR-P*} = \frac{\phi_{R0}}{4} - \frac{\delta\beta_w^R \phi_{R1}}{2\phi_{RR}}$, $q_r^{RR-P*} = \frac{\beta_w^R \phi_{R1}}{2\phi_{RR}}$, $\pi_M^{RR-P*} = \frac{1}{4}(\phi_{R0}^2 + \frac{\beta_w^R \phi_{R1}^2}{\phi_{RR}})$, $\pi_R^{RR-P*} = \frac{1}{8}(\phi_{R0}^2 + \frac{\beta_w^R \phi_{R1}^2}{\phi_{RR}}) - \beta_w^R$, and $\pi_T^{RR-P*} = \frac{3}{8}(\phi_{R0}^2 + \frac{\beta_w^R \phi_{R1}^2}{\phi_{RR}}) - \beta_w^R$, respectively. Then, $\gamma q_1^{RR*} - q_r^{RR*} \geq 0$ can be obtained from the Complementary Relaxation Theorem. Therefore, we obtain the condition for Case I as $\delta(Q - \phi_{R0}) - \frac{\gamma \phi_{R0}\phi_{RR}}{2\beta_w^R} \leq c_r < \delta(Q - \phi_{R0})$.

Case II: When $\lambda_2 > 0$, $\gamma q_1^{RR*} - q_r^{RR*} = 0$ can be derived based on the Complementary Relaxation Theorem. It is easy to confirm $\lambda_2 = \frac{2\beta_w^R \phi_{R4}}{\gamma \varphi_{RR}} - \frac{\phi_{R0}}{\gamma}$ given $\gamma q_1^{RR*} - q_r^{RR*} = 0$. By substituting it into the manufacturer's and retailer's optimal solutions, we have $w_1^{RR-C*} = c_n + \frac{\beta_w^R \phi_{R4}}{\varphi_{RR}}$, $w_n^{RR-C*} = c_n + \frac{\phi_{R0}}{2}$, $w_r^{RR-C*} = \frac{\delta Q + c_r}{2} + \frac{\phi_{R0}}{2\gamma} - \frac{\beta_w^R \phi_{R4}}{\gamma \varphi_{RR}}$, $p_1^{RR-C*} = Q + \varepsilon_n - \frac{\beta_w^R \phi_{R4}}{2\varphi_{RR}}$, $p_n^{RR-C*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, $p_r^{RR-C*} = \frac{\delta(4Q - \phi_{R0})}{4} - \frac{\gamma \phi_{R4}(\phi_{R2} - \varepsilon_r \phi_{R3})}{2\varphi_{RR}}$, and $t^{RR-C*} = \frac{\gamma \phi_{R3}\phi_{R4}}{2\varphi_{RR}}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{RR-C*} = \frac{\beta_w^R \phi_{R4}}{2\varphi_{RR}}$, $q_n^{RR-C*} = \frac{1}{4}(\phi_{R0} - \frac{2\gamma\delta\beta_w^R \phi_{R4}}{\varphi_{RR}})$, $q_r^{RR-C*} = \frac{\gamma \beta_w^R \phi_{R4}}{2\varphi_{RR}}$, $\pi_M^{RR-C*} = \frac{1}{8}(\phi_{R0}^2 + \frac{2\beta_w^R \phi_{R4}^2}{\varphi_{RR}})$, $\pi_R^{RR-C*} = \frac{1}{16}(\phi_{R0}^2 + \frac{2\beta_w^R \phi_{R4}^2}{\varphi_{RR}}) - \beta_w^R$, and $\pi_T^{RR-C*} = \frac{3}{16}(\phi_{R0}^2 + \frac{2\beta_w^R \phi_{R4}^2}{\varphi_{RR}}) - \beta_w^R$, respectively. Furthermore, $0 < c_r < \delta(Q - \phi_{R0}) - \frac{\gamma \phi_{R0}\phi_{RR}}{2\beta_w^R}$ is derived from $\lambda_2 > 0$, corresponding to the condition for Case II.

Thus, we confirm the equilibrium solutions for model *RR*.

Appendix A-3: Equilibrium solution derivation for model *MN*

In model *MN*, the Lagrangian function for the retailer's optimization problem is furnished as follows: $L_R^{MN}(p_1, p_n, p_r) = \pi_R^{MN} + \lambda_3(\gamma q_1 - q_r)$, where λ_3 is the Lagrangian multiplier and $\lambda_3(\gamma q_1 - q_r) = 0$. According to backward induction, given w_1, p_1, w_n and w_r, since $\frac{\partial^2 L_R^{MN}}{\partial p_n^2} = -\frac{2}{1-\delta}$, $\frac{\partial^2 L_R^{MN}}{\partial p_r^2} = -\frac{2}{\delta(1-\delta)}$, and $\frac{\partial^2 L_R^{MN}}{\partial p_n \partial p_r} = \frac{2}{1-\delta}$, one can confirm that $L_R^{MN}(p_1, p_n, p_r)$ is strictly and jointly concave in decision variables p_n and p_r, hence, has a unique optimal solution. Solving the first-order conditions $\frac{\partial L_R^{MN}}{\partial p_n} = 0$ and $\frac{\partial L_R^{MN}}{\partial p_r} = 0$, we obtain the optimal response functions for the retailer as

$p_n^{MN*}(w_1, p_1, w_n, w_r) = \frac{Q+\varepsilon_n + w_n}{2}$ and $p_r^{MN*}(w_1, p_1, w_n, w_r) = \frac{\delta Q + w_r + \lambda_3}{2}$.

These two response functions are then substituted into the manufacturer's profit function $\pi_M^{MN}(w_1, w_n, w_r)$. Solving the first-order conditions $\frac{\partial \pi_M^{MN}}{\partial w_n} = 0$ and $\frac{\partial \pi_M^{MN}}{\partial w_r} = 0$, we obtain the manufacturer's optimal solutions $w_n^{MN*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$ and $w_r^{MN*} = \frac{\delta Q + c_r - \lambda_3}{2}$. Thus, the optimal retail prices of new and remanufactured products are obtained as $p_n^{MN*} = Q+\varepsilon_n - \frac{\phi_{M0}}{4}$ and $p_r^{MN*} = \frac{c_r + 3\delta Q + \lambda_3}{4}$, respectively. Then, similar to model *MM*, we can obtain $w_1^{MN*} = c_n + c_w^M + \frac{\phi_{M0} + \gamma\lambda_3}{2}$ and $p_1^{MN*} = Q+\varepsilon_n - \frac{\phi_{M0}}{4} - \frac{\gamma\lambda_3}{4}$.

According to the Karush-Kuhn-Tucker condition, two possible cases may arise for the optimal solutions corresponding to different Lagrangian coefficient λ_3 as follows.

Case I: When $\lambda_3 = 0$, the manufacturer's optimal solutions are $w_1^{MN-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_n^{MN-P*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$ and $w_r^{MN-P*} = \frac{\delta Q + c_r}{2}$, the retailer's optimal solutions are $p_1^{MN-P*} = Q+\varepsilon_n - \frac{\phi_{M0}}{4}$, $p_n^{MN-P*} = Q+\varepsilon_n - \frac{\phi_{M0}}{4}$ and $p_r^{MN-P*} = \delta Q - \frac{\delta Q - c_r}{4}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{MN-P*} = \frac{\phi_{M0}}{4}$, $q_n^{MN-P*} = \frac{\phi_{M0} - (\delta Q - c_r)}{4(1-\delta)}$, $q_r^{MN-P*} = \frac{\phi_{M1}}{4\delta(1-\delta)}$, $\pi_M^{MN-P*} = \frac{1}{4}\left(\phi_{M0}^2 + \frac{\phi_{M1}^2}{2\delta(1-\delta)}\right) - \beta_w^M$, $\pi_R^{MN-P*} = \frac{1}{8}\left(\phi_{M0}^2 + \frac{\phi_{M1}^2}{2\delta(1-\delta)}\right)$, and $\pi_T^{MN-P*} = \frac{3}{8}\left(\phi_{M0}^2 + \frac{\phi_{M1}^2}{2\delta(1-\delta)}\right) - \beta_w^M$, respectively. Then, $\gamma q_1^{MN*} - q_r^{MN*} \geq 0$ can be obtained from the Complementary Relaxation Theorem. Therefore, we obtain the condition for Case I as $\delta(Q - \phi_{M0}) - \gamma\delta(1-\delta)\phi_{M0} \leq c_r < \delta(Q - \phi_{M0})$.

Case II: When $\lambda_3 > 0$, $\gamma q_1^{MN*} - q_r^{MN*} = 0$ can be derived based on the Complementary Relaxation Theorem. It is easy to get $\lambda_3 = \frac{\gamma\delta(1-\delta)\phi_{M0} - \phi_{M1}}{\varphi_0}$ given $\gamma q_1^{MN*} - q_r^{MN*} = 0$. By substituting it into the manufacturer's and retailer's optimal solutions, we have $w_1^{MN-C*} = c_n + c_w^M + \frac{\phi_{M4}}{2\varphi_0}$, $w_n^{MN-C*} = c_n + c_w^M + \frac{\phi_{M0}}{2}$, $w_r^{MN-C*} = c_r + \frac{\delta(\varphi_0\phi_{M0} + \gamma(1-\delta)\phi_{M4})}{2\varphi_0}$; the retailer's optimal solutions are $p_1^{MN-C*} = Q + \varepsilon_n - \frac{\phi_{M4}}{4\varphi_0}$, $p_n^{MN-C*} = Q + \varepsilon_n - \frac{\phi_{M0}}{4}$, $p_r^{MN-C*} = \frac{3Q\delta + c_r}{4} + \frac{\phi_{M1} - \gamma\delta(1-\delta)\phi_{M0}}{4\varphi_0}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{MN-C*} = \frac{\phi_{M4}}{4\varphi_0}$, $q_n^{MN-C*} = \frac{Q}{4} - \frac{\gamma\delta\phi_{M0} + (c_n + c_w^M - \varepsilon_n)(1+\gamma^2\delta) - \gamma^2\delta c_r}{4\varphi_0}$, $q_r^{MN-C*} = \frac{\gamma\phi_{M4}}{4\varphi_0}$, $\pi_M^{MN-C*} = \frac{\phi_{M0}^2}{8} + \frac{\phi_{M4}^2}{8\varphi_0} - \beta_w^M$, $\pi_R^{MN-C*} = \frac{\phi_{M0}^2}{16} + \frac{\phi_{M4}^2}{16\varphi_0}$, $\pi_T^{MN-C*} = \frac{3\phi_{M0}^2}{16} + \frac{3\phi_{M4}^2}{16\varphi_0} - \beta_w^M$, respectively. Furthermore, $0 < c_r < \delta(Q - \phi_{M0}) - \gamma\delta(1-\delta)\phi_{M0}$ is derived from $\lambda_3 > 0$, corresponding to the condition for Case II.

Thus, we obtain the equilibrium solutions for model *MN*.

Appendix A-4: Equilibrium solution derivation for model *RN*

In model *RN*, the Lagrangian function for the retailer's optimization problem is given as follows: $L_R^{RN}(p_1, p_n, p_r) = \pi_R^{RN} + \lambda_4(\gamma q_1 - q_r)$, where λ_4 is the Lagrangian multiplier and $\lambda_4(\gamma q_1 - q_r) = 0$. According to backward induction, given w_1, p_1, w_n and w_r, since $\frac{\partial^2 L_R^{RN}}{\partial p_n^2} = -\frac{2}{1-\delta}$, $\frac{\partial^2 L_R^{RN}}{\partial p_r^2} = -\frac{2}{\delta(1-\delta)}$, and $\frac{\partial^2 L_R^{RN}}{\partial p_n \partial p_r} = \frac{2}{1-\delta}$, one can confirm that the Lagrangian function $L_R^{RN}(p_1, p_n, p_r)$ is strictly and jointly concave in decision variables p_n and p_r and, hence, has a unique optimal solution. Solving the first-order conditions $\frac{\partial L_R^{RN}}{\partial p_n} = 0$ and $\frac{\partial L_R^{RN}}{\partial p_r} = 0$, we obtain the optimal response functions for the retailer as $p_n^{RN*}(w_1, p_1, w_n, w_r) = \frac{Q + \varepsilon_n + w_n + c_w^R}{2}$ and $p_r^{RN*}(w_1, p_1, w_n, w_r) = \frac{\delta Q + w_r + \lambda_4}{2}$.

These two functions are then substituted into the manufacturer's profit function π_M^{RN}. Solving the first-order conditions $\frac{\partial \pi_M^{RN}}{\partial w_n} = 0$ and $\frac{\partial \pi_M^{RN}}{\partial w_r} = 0$, we obtain the manufacturer's optimal solutions $w_n^{RN*} = c_n + \frac{\phi_{R0}}{2}$ and $w_r^{RN*} = c_r + \frac{\delta Q - c_r - \lambda_4}{2}$. Thus, the optimal retail prices of new and remanufactured products are derived as $p_n^{RN*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$ and $p_r^{RN*} = \frac{c_r + 3\delta Q + \lambda_4}{4}$, respectively. Then, similar to model *RR*, we can obtain $w_1^{RN} = c_n + \frac{\phi_{R0} + \gamma\lambda_4}{2}$ and $p_1^{RN*} = Q + \varepsilon_n - \frac{\phi_{R0} + \gamma\lambda_4}{4}$.

According to the Karush-Kuhn-Tucker condition, there exist two possible cases for the optimal solutions corresponding to different Lagrangian coefficient λ_4 as follows.

Case I: When $\lambda_4 = 0$, the manufacturer's optimal solutions are $w_1^{RN-P*} = c_n + \frac{\phi_{R0}}{2}$, $w_n^{RN-P*} = c_n + \frac{\phi_{R0}}{2}$ and $w_r^{RN-P*} = \frac{\delta Q + c_r}{2}$, the retailer's optimal solutions are $p_1^{RN-P*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$, $p_n^{RN-P*} = Q + \varepsilon_n - \frac{\phi_{R0}}{4}$ and $p_r^{RN-P*} = \delta Q - \frac{\delta Q - c_r}{4}$. Hence, the optimal quantities, individual and channel profits are derived as $q_1^{RN-P*} = \frac{\phi_{R0}}{4}$, $q_n^{RN-P*} = \frac{\phi_{R0} - \delta Q + c_r}{4(1-\delta)}$, $q_r^{RN-P*} = \frac{\phi_{R1}}{4\delta(1-\delta)}$, $\pi_M^{RN-P*} = \frac{\phi_{R0}^2}{4} + \frac{\phi_{R1}^2}{8\delta(1-\delta)}$, $\pi_R^{RN-P*} = \frac{\phi_{R0}^2}{8} + \frac{\phi_{R1}^2}{16\delta(1-\delta)} - \beta_w^R$, $\pi_T^{RN-P*} = \frac{3\phi_{R0}^2}{8} + \frac{3\phi_{R1}^2}{16\delta(1-\delta)} - \beta_w^R$, respectively. Then, $\gamma q_1^{RN*} - q_r^{RN*} \geq 0$ is confirmed by the Complementary Relaxation Theorem. Therefore, we derive the condition for Case I as $\delta(Q - \phi_{R0}) - \gamma\delta(1-\delta)\phi_{R0} \leq c_r < \delta(Q - \phi_{R0})$.

Case II: When $\lambda_4 > 0$, $\gamma q_1^{RN*} - q_r^{RN*} = 0$ can be verified by the Complementary Relaxation Theorem. It is easy to get $\lambda_4 = \frac{\phi_{R1} - \gamma\delta(1-\delta)\phi_{R0}}{\varphi_0}$ given $\gamma q_1^{RN*} - q_r^{RN*} = 0$. By substituting it into the manufacturer's and retailer's optimal solutions, we have $w_1^{RN-C*}=c_n+\frac{\phi_{R4}}{2\varphi_0}$, $w_n^{RN-C*}=c_n+\frac{\phi_{R0}}{2}$, $w_r^{RN-C*}=\frac{\delta Q+c_r}{2}+\frac{\gamma\delta(1-\delta)\phi_{R0}-\phi_{R1}}{2\varphi_0}$; $p_1^{RN-C*}=Q+\varepsilon_n-\frac{\phi_{R4}}{4\varphi_0}$, $p_n^{RN-C*}=Q+\varepsilon_n-\frac{\phi_{R0}}{4}$ and $p_r^{RN-C*}=\delta Q-\frac{\varphi_0\phi_{R0}+\gamma(1-\delta)\phi_{R4}}{4\varphi_0}$. Hence, the optimal quantities, individual and channel profits are obtained as $q_1^{RN-C*}=\frac{\phi_{R4}}{4\varphi_0}$, $q_n^{RN-C*}=\frac{Q\gamma^2\delta(1-\delta)+(1-\gamma\delta)\phi_{R0}-\gamma^2\delta(c_n+c_w^R-c_r-\varepsilon_n)}{4\varphi_0}$, $q_r^{RN-C*}=\frac{\gamma\phi_{R4}}{4\varphi_0}$, $\pi_M^{RN-C*}=\frac{\phi_{R0}^2}{8}+\frac{\phi_{R4}^2}{8\varphi_0}$, $\pi_R^{RN-C*}=\frac{1}{16}(\phi_{R0}^2+\frac{\phi_{R4}^2}{\varphi_0})-\beta_w^R$, $\pi_T^{RN-C*}=\frac{3}{16}(\phi_{R0}^2+\frac{\phi_{R4}^2}{\varphi_0})-\beta_w^M$, respectively. Furthermore, $0<c_r<\delta(Q-\phi_{R0})-\gamma\delta(1-\delta)\phi_{R0}$ is derived from $\lambda_4>0$, corresponding to the condition for Case II.

Thus, the equilibrium solutions are confirmed for model *RN*.

Appendix B: Proof of Proposition 1

When the manufacturer commits to partial remanufacturing, by subtracting the wholesale and retail prices of remanufactured products under models *MN* and *RN* from those under models *MM* and *RR*, respectively, we get $w_r^{MM-P*}-w_r^{MN-P*}=\frac{\phi_{M1}\phi_{M3}(c_w^M+\varepsilon_r)}{2\phi_{MM}}>0$, $w_r^{RR-P*}-w_r^{RN-P*}=0$; $p_r^{MM-P*}-p_r^{MN-P*}=\frac{\phi_{M1}\phi_{M3}(c_w^M+3\varepsilon_r)}{4\phi_{MM}}>0$, and $p_r^{RR-P*}-p_r^{RN-P*}=\frac{\phi_{R1}\phi_{R3}(c_w^R+\varepsilon_r)}{4\phi_{RR}}>0$. Similarly, for the wholesale and retail prices of new products in the four models, one has $w_1^{MM-P*}-w_1^{MN-P*}=w_1^{RR-P*}-w_1^{RN-P*}=0$, $w_n^{MM-P*}-w_n^{MN-P*}=w_n^{RR-P*}-w_n^{RN-P*}=0$, $p_1^{MM-P*}-p_1^{MN-P*}=p_1^{RR-P*}-p_1^{RN-P*}=0$, and $p_n^{MM-P*}-p_n^{MN-P*}=p_n^{RR-P*}-p_n^{RN-P*}=0$.

When the manufacturer commits to complete remanufacturing, by subtracting the wholesale and retail prices of remanufactured products under models *MN* and *RN* from those under models *MM* and *RR*, respectively, we obtain $w_r^{MM-C*}-w_r^{MN-C*}=\frac{\gamma\phi_{M4}(\gamma^2(1-\delta)\delta(\varepsilon_r^2-c_w^{M2})+2c_w^M\phi_{M3})}{2\varphi_0\varphi_{MM}}>0$, $w_r^{RR-C*}-w_r^{RN-C*}=-\frac{\gamma\phi_{R3}^2\phi_{R4}}{2\varphi_0\varphi_{RR}}<0$; $p_r^{MM-C*}-p_r^{MN-C*}=\frac{\gamma\phi_{M3}\phi_{M4}(4\varepsilon_r+\gamma^2(1-\delta)\delta(c_w^M+3\varepsilon_r))}{4\varphi_0\varphi_{MM}}>0$, and $p_r^{RR-C*}-p_r^{RN-C*}=\frac{\gamma\phi_{R3}\phi_{R4}(2\varepsilon_r+\gamma^2(1-\delta)\delta(\varepsilon_r+c_w^R))}{4\varphi_0\varphi_{RR}}>0$. Similarly, for new products, we have $w_1^{MM-C*}-w_1^{MN-C*}=\frac{\gamma^2\phi_{M3}^2\phi_{M4}}{2\varphi_0\varphi_{MM}}>0$, $w_1^{RR-C*}-w_1^{RR-C*}=\frac{\gamma^2\phi_{R3}^2\phi_{R4}}{2\varphi_0\varphi_{RR}}>0$, $w_n^{MM-C*}-w_n^{MN-C*}=w_n^{RR-C*}-w_n^{RN-C*}=0$; $p_1^{MM-C*}-p_1^{MN-C*}=-\frac{\gamma^2\phi_{M3}^2\phi_{M4}}{4\varphi_0\varphi_{MM}}<0$, $p_1^{RR-C*}-p_1^{RN-C*}=-\frac{\gamma^2\phi_{R3}^2\phi_{R4}}{4\varphi_0\varphi_{RR}}<0$, and $p_n^{MM-C*}-p_n^{MN-C*}=p_n^{RR-C*}-p_n^{RN-C*}=0$.

Proposition 1 is thus proved.

Appendix C: Proof of Proposition 2

When the manufacturer commits to partial remanufacturing, by subtracting the manufacturer's and the retailer's profits under models MN and RN from those under models MM and RR, respectively, we can obtain $\Delta\pi_M^{MM-P*} = \pi_M^{MM-P*} - \pi_M^{MN-P*} = \dfrac{\phi_{M1}^2\phi_{M3}^2}{8\delta(1-\delta)\phi_{MM}} > 0$, $\Delta\pi_R^{MM-P} = \pi_R^{MM-P*} - \pi_R^{MN-P*} = \dfrac{(\phi_{MM}+4\phi_{M2})\phi_{M1}^2\phi_{M3}^2}{16\delta(1-\delta)\phi_{MM}^2} > 0$, $\Delta\pi_M^{RR-P*} = \pi_M^{RR-P*} - \pi_M^{RN-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2}{8\delta(1-\delta)\phi_{RR}} > 0$, and $\Delta\pi_R^{RR-P*} = \pi_R^{RR-P*} - \pi_R^{RN-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2}{16\delta(1-\delta)\phi_{RR}} > 0$. Then, $\Delta\pi_T^{MM-P*} = \Delta\pi_M^{MM-P*} + \Delta\pi_R^{MM-P*} > 0$ and $\Delta\pi_T^{RR-P*} = \Delta\pi_M^{RR-P*} + \Delta\pi_R^{RR-P*} > 0$ can be derived. By subtracting consumer surplus under models MN and RN from that under models MM and RR, respectively, we have $\Delta CS^{MM-P*} = \mathrm{CS}^{MM-P*} - \mathrm{CS}^{MN-P*}$ $= \dfrac{\phi_{M1}^2\phi_{M3}^2(\phi_{MM}+4\phi_{M2})}{32\delta(1-\delta)\phi_{MM}^2} > 0$ and $\Delta CS^{RR-P*} = \mathrm{CS}^{RR-P*} - \mathrm{CS}^{RN-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2(\phi_{RR}+2\phi_{R2})}{32\delta(1-\delta)\phi_{RR}^2} > 0$.

When the manufacturer commits to complete remanufacturing, by subtracting the manufacturer's and the retailer's profits under models MN and RN from those under models MM and RR, respectively, we can obtain $\Delta\pi_M^{MM-C*} = \pi_M^{MM-C*} - \pi_M^{MN-C*} = \dfrac{\gamma^2\phi_{M3}^2\phi_{M4}^2}{8\varphi_0\varphi_{MM}} > 0$, $\Delta\pi_R^{MM-C} = \pi_R^{MM-C*} - \pi_R^{MN-C*} = \dfrac{\gamma^2\phi_{M4}^2(2\varphi_{MM}+\gamma^2\phi_{M3}^2)\phi_{M3}^2}{16\varphi_0\varphi_{MM}^2} > 0$, $\Delta\pi_M^{RR-C*} = \pi_M^{RR-C*} - \pi_M^{RN-C*} = \dfrac{\gamma^2\phi_{R3}^2\phi_{R4}^2}{8\varphi_0\varphi_{RR}} > 0$, $\Delta\pi_R^{RR-C*} = \pi_R^{RR-C*} - \pi_R^{RN-C*} = \dfrac{\gamma^2\phi_{R3}^2\phi_{R4}^2}{16\varphi_0\varphi_{RR}} > 0$. Then, $\Delta\pi_T^{MM-C*} = \Delta\pi_M^{MM-C*} + \Delta\pi_R^{MM-C*} > 0$ and $\Delta\pi_T^{RR-C*} = \Delta\pi_M^{RR-C*} + \Delta\pi_R^{RR-C*} > 0$ can be verified. By subtracting consumer surplus under models MN and RN from that under models MM and RR, respectively, we can obtain $\Delta CS^{MM-C*} = \mathrm{CS}^{MM-C*} - \mathrm{CS}^{MN-C*} = \dfrac{\gamma^2\phi_{M3}^2\phi_{M4}^2(2\varphi_{MM}+\gamma^2\phi_{M3}^2)}{32\varphi_0\varphi_{MM}^2} > 0$ and

$$\Delta CS^{RR-C*} = \mathrm{CS}^{RR-C*} - \mathrm{CS}^{RN-C*} = \frac{\gamma^2\phi_{R3}^2\phi_{R4}^2(2\varphi_{RR}+\gamma^2\phi_{R3}^2)}{32\varphi_0\varphi_{RR}^2} > 0.$$

Proposition 2 is thus proved.

Appendix D: Proof of Corollary 1

When the manufacturer commits to partial remanufacturing, by subtracting the manufacturer's and the retailer's profits under models MN and RN from those under models MM and RR, respectively, we can obtain $\Delta\pi_M^{MM-P*} = \pi_M^{MM-P*} - \pi_M^{MN-P*} = \dfrac{\phi_{M1}^2\phi_{M3}^2}{8\delta(1-\delta)\phi_{MM}}$, $\Delta\pi_R^{MM-P} = \pi_R^{MM-P*} - \pi_R^{MN-P*} = \dfrac{(\phi_{MM}+4\phi_{M2})\phi_{M1}^2\phi_{M3}^2}{16\delta(1-\delta)\phi_{MM}^2}$, $\Delta\pi_M^{RR-P*} = \pi_M^{RR-P*} - \pi_M^{RN-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2}{8\delta(1-\delta)\phi_{RR}}$, and $\Delta\pi_R^{RR-P*} = \pi_R^{RR-P*} - \pi_R^{RN-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2}{16\delta(1-\delta)\phi_{RR}}$. Then, $\Delta\pi_M^{MM-P*} - \Delta\pi_R^{MM-P*} = -\dfrac{\phi_{M1}^2\phi_{M3}^4}{16\delta(1-\delta)\phi_{MM}^2} < 0$; $\Delta\pi_M^{RR-P*} - \Delta\pi_R^{RR-P*} = \dfrac{\phi_{R1}^2\phi_{R3}^2}{16\delta(1-\delta)\phi_{RR}} > 0$.

When the manufacturer commits to complete remanufacturing, by subtracting the manufacturer's and the retailer's profits under models MN and RN from those under models MM and RR, respectively, we have $\Delta\pi_M^{MM-C*} = \pi_M^{MM-C*} - \pi_M^{MN-C*} = \dfrac{\gamma^2\phi_{M3}^2\phi_{M4}^2}{8\varphi_0\varphi_{MM}}$, $\Delta\pi_R^{MM-C} = \pi_R^{MM-C*} - \pi_R^{MN-C*} = \dfrac{(2\varphi_{MM}+\gamma^2\phi_{M3}^2)\gamma^2\phi_{M3}^2\phi_{M4}^2}{16\varphi_0\varphi_{MM}^2}$,

$\Delta\pi_M^{RR-C*}=\pi_M^{RR-C*}-\pi_M^{RN-C*}=\frac{\gamma^2\phi_{R3}^2\phi_{R4}^2}{8\varphi_0\varphi_{RR}}$, and $\Delta\pi_R^{RR-C*}=\pi_R^{RR-C*}-\pi_R^{RN-C*}=\frac{\gamma^2\phi_{R3}^2\phi_{R4}^2}{16\varphi_0\varphi_{RR}}$. Then, $\Delta\pi_M^{MM-C*}-\Delta\pi_R^{MM-C*}=-\frac{\gamma^4\phi_{M4}^2\phi_{M3}^4}{16\varphi_0\varphi_{MM}^2}<0$; $\Delta\pi_M^{RR-C*}-\Delta\pi_R^{RR-C*}=\frac{\gamma^2\phi_{R3}^2\phi_{R4}^2}{16\varphi_0\varphi_{RR}}>0$.

This completes the proof of Corollary 1.

Appendix E: Proof of Proposition 3

It can be seen from $\phi_{MM}=4\delta\beta_w^M(1-\delta)-\phi_{M3}^2>0$ that $\beta_w^M>\frac{\phi_{M3}^2}{4\delta(1-\delta)}=\overline{\beta}_{M0}$ is the condition for model *MM* to have a unique optimal solution. In addition, given the assumption $t\le n=1$, we can derive another boundary of β_w^M, denoted by $\overline{\beta}_{M1}^h$ as shown below.

When the manufacturer commits to partial remanufacturing, by subtracting the optimal warranty period under model *RR* from that under model *MM*, we can obtain $t^{MM-P*}-t^{RR-P*}=\frac{1}{2}\phi_{R1}\phi_{R3}f^P(\beta_w^M)$, where $f^P(\beta_w^M)=\frac{2}{\phi_{MM}}-\frac{1}{\phi_{RR}}$. Solving equation $f^P(\beta_w^M)=0$ for β_w^M, we derive $\overline{\beta}_{M2}^P=\frac{4\phi_{R2}-\phi_{R3}^2}{4\delta(1-\delta)}$. In addition, we can derive $\beta_w^M>\overline{\beta}_{M1}^P=\frac{\phi_{R3}(\phi_{R1}+\phi_{R3})}{4\delta(1-\delta)}>\overline{\beta}_{M0}$ from $t^{MM-P*}\le n=1$. Since $\frac{\partial f^P(\beta_w^M)}{\partial\beta_w^M}<0$ and $\overline{\beta}_{M2}^P>\overline{\beta}_{M1}^P$, the conclusion of Proposition 3 is confirmed under the partial remanufacturing strategy.

When the manufacturer commits to complete remanufacturing, by subtracting the optimal warranty period under model *RR* from that under model *MM*, we have $t^{MM-C*}-t^{RR-C*}=\frac{\gamma\phi_{R3}\phi_{R4}}{2}f^C(\beta_w^M)$, where $f^C(\beta_w^M)=\frac{2}{\varphi_{MM}}-\frac{1}{\varphi_{RR}}$. Solving equation $f^C(\beta_w^M)=0$ for β_w^M, we derive $\overline{\beta}_{M2}^C=\beta_w^R-\frac{\gamma^2\phi_{R3}^2}{4\varphi_0}$. In addition, we can derive $\beta_w^M>\overline{\beta}_{M1}^C=\frac{\gamma^2\phi_{R3}^2+\gamma\phi_{R3}\phi_{M4}}{4\varphi_0}$ from $t^{MM-C*}\le n=1$. Since $\frac{\partial f^C(\beta_w^M)}{\partial\beta_w^M}<0$ and $\overline{\beta}_{M2}^C>\overline{\beta}_{M1}^C$, the conclusion of Proposition 3 is confirmed under the complete remanufacturing strategy as well.

Proposition 3 is thus proved.

Appendix F: Proof of Corollary 2

By subtracting the optimal retail price under model *RR* from that under model *MM*, we obtain $p_r^{MM-P*}-p_r^{RR-P*}=\frac{\phi_{R1}\phi_{R3}}{2\phi_{RR}}H^P(\beta_w^M)$ and $p_r^{MM-C*}-p_r^{RR-C*}=\frac{\gamma\phi_{R3}\phi_{R4}}{2\varphi_{RR}}H^C(\beta_w^M)$, where $H^P(\beta_w^M)=\frac{\phi_{R2}(3\varepsilon_r+c)-2\phi_{M2}(\varepsilon_r+c)-\varepsilon_r\phi_{R3}^2}{\phi_{MM}}$, $H^C(\beta_w^M)=\frac{4(\beta_w^R-\beta_w^M)\varepsilon_r+\gamma^2(\phi_{R2}(3\varepsilon_r+c)-2\phi_{M2}(\varepsilon_r+c)-\varepsilon_r\phi_{R3}^2)}{\varphi_{MM}}$. Solving equations $H^P(\beta_w^M)=0$ and $H^C(\beta_w^M)=0$ for β_w^M, we obtain $\overline{\beta}_{M3}^P=\frac{(3\varepsilon_r+c)\phi_{R2}-\varepsilon_r\phi_{R3}^2}{2\delta(1-\delta)(\varepsilon_r+c)}>\overline{\beta}_{M2}^P$ and $\overline{\beta}_{M3}^C=\frac{\beta_w^R(\phi_{R3}+(3\varepsilon_r+c)\varphi_0)-\gamma^2\varepsilon_r\phi_{R3}^2}{4\varepsilon_r+2\gamma^2\delta(1-\delta)(\varepsilon_r+c)}>\overline{\beta}_{M2}^C$. Hence, Corollary 2 is verified due to $\frac{\partial H^P(\beta_w^M)}{\partial \beta_w^M} < 0$ and $\frac{\partial H^C(\beta_w^M)}{\partial \beta_w^M} < 0$.

This completes the proof of Corollary 2.

Appendix G: Proof of Proposition 4

When the manufacturer commits to partial remanufacturing, by subtracting the manufacturer's profit in model *RR* from that in model *MM*, we obtain $\pi_M^{MM-P*} - \pi_M^{RR-P*} = \frac{F_M^P(\beta_w^M)}{4}$ where $F_M^P(\beta_w^M) = -4\beta_w^M - \frac{\beta_w^R \phi_{R1}^2}{\phi_{RR}} + \frac{2\beta_w^M \phi_{R1}^2}{\phi_{MM}}$. Because $\frac{\partial F_M^P(\beta_w^M)}{\partial \beta_w^M} < 0$, $F_M^P(\overline{\beta}_{M1}^P) < 0$, and $\overline{\beta}_{M1}^P$ is the lower bound to ensure that $t^{MM-P*} \le n = 1$ under the partial remanufacturing strategy, we have $\pi_M^{MM-P*} - \pi_M^{RR-P*} < 0$. By subtracting the retailer's profit in model *RR* from that in model *MM*, we derive $\pi_R^{MM-P*} - \pi_R^{RR-P*} = \frac{F_R^P(\beta_w^M)}{8}$, where $F_R^P(\beta_w^M) = 8\beta_w^R - \frac{\beta_w^R \phi_{R1}^2}{\phi_{RR}} + \frac{8\delta(1-\delta)(\beta_w^M \phi_{R1})^2}{\phi_{MM}^2}$. Given $\beta_w^R > \overline{\beta}_{R1}^P = \frac{2\phi_{R3}^2 + \phi_{R1}\phi_{R3}}{4\delta(1-\delta)}$, $F_R^P(\beta_w^M) > 0$ regardless of β_w^M, where $\overline{\beta}_{R1}^P$ is the lower bound to ensure that $t^{RR-P} \le n = 1$ under the partial remanufacturing strategy and, hence, $\pi_R^{MM-P*} - \pi_R^{RR-P*} > 0$.

When the manufacturer commits to complete remanufacturing, $\pi_M^{MM-C*} - \pi_M^{RR-C*} = \frac{F_M^C(\beta_w^M)}{8}$, where $F_M^C(\beta_w^M) = -8\beta_w^M + \frac{4\beta_w^M \phi_{R4}^2}{\varphi_{MM}} - \frac{2\beta_w^R \phi_{R4}^2}{\varphi_{RR}}$. Since $\frac{\partial F_M^C(\beta_w^M)}{\partial \beta_w^M} < 0$, $F_M^C(\overline{\beta}_{M1}^C) < 0$, and $\overline{\beta}_{M1}^C$ is the lower bound to ensure that $t^{MM-C*} \le n = 1$ under the complete remanufacturing strategy, one has $\pi_M^{MM-C*} - \pi_M^{RR-C*} < 0$. By subtracting the retailer's profit in model *RR* from that in model *MM*, we derive $\pi_R^{MM-C*} - \pi_R^{RR-C*} = \frac{F_R^C(\beta_w^M)}{8}$, where $F_R^C(\beta_w^M) = 8\beta_w^R - \frac{\beta_w^R \phi_{R4}^2}{\varphi_{RR}} + \frac{8\varphi_0(\beta_w^M \phi_{R4})^2}{\varphi_{MM}^2}$. Given $\beta_w^R > \overline{\beta}_{R1}^C = \frac{2\gamma^2\phi_{R3}^2 + \gamma\phi_{R3}\phi_{R4}}{4\varphi_0}$, $F_R^C(\beta_w^M) > 0$ regardless of β_w^M, where $\overline{\beta}_{R1}^C$ is the lower bound to ensure that $t^{RR-C*} \le n = 1$ under the complete remanufacturing strategy and, hence, $\pi_R^{MM-C*} - \pi_R^{RR-C*} > 0$.

Proposition 4 is thus proved.

Appendix H: Proof of Proposition 5

By subtracting the channel profit in model *RR* from that in model *MM* when the manufacturer commits to partial remanufacturing, we derive $\pi_T^{MM-P*} - \pi_T^{RR-P*} = \frac{F_T^P(\beta_w^M, \beta_w^R)}{8}$, where $F_T^P(\beta_w^M, \beta_w^R) = 4\beta_w^R - 4\beta_w^M - \frac{3\beta_w^R \phi_{R1}^2}{\phi_{RR}} + \frac{4\beta_w^M \phi_{R1}^2(\phi_{MM} + 2\phi_{R2})}{\phi_{MM}^2}$. Given $\beta_w^M = \overline{\beta}_{M1}^P$, solving equation $F_T^P(\overline{\beta}_{M1}^P, \beta_w^R) = 0$ for β_w^R yields $\overline{\beta}_{R2}^P$, where $\overline{\beta}_{R2}^P = \frac{\phi_{R3}(\sqrt{52\phi_{R1}^2 + 12\phi_{R1}\phi_{R3} + 9\phi_{R3}^2} - (2\phi_{R1} - 5\phi_{R3}))}{16\delta(1-\delta)}$. If $\phi_{R3} < \frac{2}{3}\phi_{R1}$, $\overline{\beta}_{R2}^P > \overline{\beta}_{R1}^P$; otherwise, $\overline{\beta}_{R2}^P \le \overline{\beta}_{R1}^P$, where $\overline{\beta}_{R1}^P = \frac{\phi_{R3}(\phi_{R1} + 2\phi_{R3})}{4\delta(1-\delta)}$ is the lower bound to ensure that $t^{RR-P*} \le n = 1$ under the partial remanufacturing strategy. Since $\frac{\partial F_T^P(\overline{\beta}_{M1}^P, \beta_w^R)}{\partial \beta_w^R} > 0$ and

$\frac{\partial F_T^P(\beta_w^M,\beta_w^R)}{\partial \beta_w^M}<0$, we have $F_T^P(\overline{\beta}_{M1}^P,\beta_w^R)<0$ when $\phi_{R3}<\frac{2}{3}\phi_{R1}$ and $\overline{\beta}_{R1}^P<\beta_R<\overline{\beta}_{R2}^P$, thus $\pi_T^{MM-P*}-\pi_T^{RR-P*}<0$. When $\phi_{R3}\le\frac{2}{3}\phi_{R1}$ and $\beta_R>\overline{\beta}_{R2}^P$ (or $\phi_{R3}>\frac{2}{3}\phi_{R1}$), $F_T^P(\overline{\beta}_{M1}^P,\beta_w^R)>0$. Given β_w^R, solving equation $F_T^P(\beta_w^M,\beta_w^R)=0$ for β_w^M yields $\overline{\beta}_{M4}^P$, and $\overline{\beta}_{M1}^P<\overline{\beta}_{M4}^P<\beta_w^R$. In this case, if $\overline{\beta}_{M1}^P\le\beta_w^M<\overline{\beta}_{M4}^P$, $F_T^P(\beta_w^M,\beta_w^R)>0$ and, hence, $\pi_T^{MM-P*}-\pi_T^{RR-P*}>0$; if $\beta_w^M=\overline{\beta}_{M4}^P$, $F_T^P(\beta_w^M,\beta_w^R)=0$ and, then, $\pi_T^{MM-P*}-\pi_T^{RR-P*}=0$; if $\beta_w^M>\overline{\beta}_{M4}^P$, $F_T^P(\beta_w^M,\beta_w^R)<0$ and, thus $\pi_T^{MM-P*}-\pi_T^{RR-P*}<0$. Set $\overline{\phi}_{R3}^P=\frac{2}{3}\phi_{R1}$, Proposition 5 is proved when the manufacturer commits to partial remanufacturing.

By subtracting the channel profit in model *RR* from that in model *MM* when the manufacturer commits to complete remanufacturing, we derive $\pi_T^{MM-C*}-\pi_T^{RR-C*}=\frac{F_T^C(\beta_w^M,\beta_w^R)}{16}$, where $F_T^C(\beta_w^M,\beta_w^R)=8\beta_w^R-8\beta_w^M-\frac{6\beta_w^R\phi_{R4}^2}{\varphi_{RR}}+\frac{8\beta_w^M\phi_{R4}^2(6\beta_w^M\varphi_0-\gamma^2\phi_{R3}^2)}{\varphi_{MM}^2}$. Given $\beta_w^M=\overline{\beta}_{M1}^C$, solving equation $F_T^C(\overline{\beta}_{M1}^C,\beta_w^R)=0$ for β_w^R yields $\overline{\beta}_{R2}^C$, where $\overline{\beta}_{R2}^C=\frac{\gamma\phi_{R3}(\sqrt{52\phi_{R4}^2+12\gamma\phi_{R4}\phi_{R3}+9\gamma^2\phi_{R3}^2}-2\phi_{R4}+5\gamma\phi_{R3})}{16\varphi_0}$. If $\phi_{R3}<\frac{2}{3\gamma}\phi_{R4}$, $\overline{\beta}_{R2}^C>\overline{\beta}_{R1}^C$; otherwise, $\overline{\beta}_{R2}^C<\overline{\beta}_{R1}^C$, where $\overline{\beta}_{R1}^C=\frac{\gamma\phi_{R3}(2\gamma\phi_{R3}+\phi_{R4})}{4\varphi_0}$ is the lower bound to ensure that $t^{RR-C*}\le n=1$. Since $\frac{\partial F_T^C(\overline{\beta}_{M1}^C,\beta_w^R)}{\partial \beta_w^R}>0$ and $\frac{\partial F_T^C(\beta_w^M,\beta_w^R)}{\partial \beta_w^M}<0$, we have $F_T^C(\overline{\beta}_{M1}^C,\beta_w^R)<0$ when $\phi_{R3}<\frac{2}{3\gamma}\phi_{R4}$ and $\overline{\beta}_{R1}^C<\beta_R<\overline{\beta}_{R2}^C$, thus $\pi_T^{MM-C*}-\pi_T^{RR-C*}<0$. When $\phi_{R3}\le\frac{2}{3\gamma}\phi_{R4}$ and $\beta_R>\overline{\beta}_{R2}^C$ (or $\phi_{R3}>\frac{2}{3\gamma}\phi_{R4}$), $F_T^C(\overline{\beta}_{M1}^C,\beta_w^R)>0$. Given β_w^R, solving equation $F_T^C(\beta_w^M,\beta_w^R)=0$ for β_w^M yields $\overline{\beta}_{M4}^C$, and $\overline{\beta}_{M1}^C<\overline{\beta}_{M4}^C<\beta_w^R$. In this case, if $\overline{\beta}_{M1}^C\le\beta_w^M<\overline{\beta}_{M4}^C$, $F_T^C(\beta_w^M,\beta_w^R)>0$ and, hence $\pi_T^{MM-C*}-\pi_T^{RR-C*}>0$; if $\beta_w^M=\overline{\beta}_{M4}^C$, $F_T^C(\beta_w^M,\beta_w^R)=0$ and, then, $\pi_T^{MM-C*}-\pi_T^{RR-C*}=0$; if $\beta_w^M>\overline{\beta}_{M4}^C$, $F_T^C(\beta_w^M,\beta_w^R)<0$ and, thus $\pi_T^{MM-C*}-\pi_T^{RR-C*}<0$. Set $\overline{\phi}_{R3}^C=\frac{2}{3\gamma}\phi_{R4}$, Proposition 5 is proved when the manufacturer commits to complete remanufacturing.

This completes the proof of Proposition 5.

Appendix I: Proof of Proposition 6

By subtracting the optimal consumer surplus in model *RR* from that in model *MM*, we have $CS^{MM-P*}-CS^{RR-P*}=\frac{\delta(1-\delta)\phi_{R1}^2}{8}G^P(\beta_w^M)$ when the manufacturer commits to partial remanufacturing, where $G^P(\beta_w^M)=\left(\frac{2\beta_w^M}{\phi_{MM}}\right)^2-\left(\frac{\beta_w^R}{\phi_{RR}}\right)^2$. Since $\frac{\partial G^P(\beta_w^M)}{\partial \beta_w^M}<0$, given β_w^R, solving equation $G^P(\beta_w^M)=0$ for β_w^M yields $\overline{\beta}_{M5}^P=\frac{\beta_w^R}{2}$. If $\beta_w^R>\overline{\beta}_{R3}^P=\frac{\phi_{R3}(\phi_{R1}+\phi_{R3})}{2\delta(1-\delta)}>\overline{\beta}_{R1}^P$, $\overline{\beta}_{M5}^P>\overline{\beta}_{M1}^P$. In this case, if $\overline{\beta}_{M1}^P\le\beta_w^M<\overline{\beta}_{M5}^P$, $G^P(\beta_w^M)>0$ and, hence $CS^{MM-P*}-CS^{RR-P*}>0$; if $\beta_w^M=\overline{\beta}_{M5}^P$, $G^P(\beta_w^M)=0$ and, then, $CS^{MM-P*}-CS^{RR-P*}=0$; if $\beta_w^M>\overline{\beta}_{M5}^P$, $G^P(\beta_w^M)<0$ and, thus

$CS^{MM-P*} - CS^{RR-P*} < 0$. If $\overline{\beta}_{R1}^{P} \le \beta_{w}^{R} < \overline{\beta}_{R3}^{P} = \dfrac{\phi_{R3}(\phi_{R1}+\phi_{R3})}{2\delta(1-\delta)}$, then $\overline{\beta}_{M5}^{P} < \overline{\beta}_{M1}^{P}$, we can infer that $CS^{MM-P*} - CS^{RR-P*} < 0$.

When the manufacturer commits to complete remanufacturing, $CS^{MM-C*} - CS^{RR-C*} = \dfrac{\varphi_0 \phi_{R4}^2}{8} G^C(\beta_w^M)$, where $G^C(\beta_w^M) = \left(\dfrac{2\beta_w^M}{\varphi_{MM}}\right)^2 - \left(\dfrac{\beta_w^R}{\varphi_{RR}}\right)^2$. Since $\dfrac{\partial G^C(\beta_w^M)}{\partial \beta_w^M} < 0$, given β_w^R, solving equation $G^C(\beta_w^M) = 0$, we can obtain the solution $\overline{\beta}_{M5}^{C} = \dfrac{\beta_w^R}{2}$. If $\beta_w^R > \overline{\beta}_{R3}^{C} = \dfrac{\gamma\phi_{R3}(\gamma\phi_{R3}+\phi_{R4})}{2\varphi_0} > \overline{\beta}_{R1}^{C}$, $\overline{\beta}_{M5}^{C} > \overline{\beta}_{M1}^{C}$. In this case, if $\overline{\beta}_{M1}^{C} \le \beta_w^M < \overline{\beta}_{M5}^{C}$, $G^C(\beta_w^M) > 0$ and, then, $CS^{MM-C*} - CS^{RR-C*} > 0$; if $\beta_w^M = \overline{\beta}_{M5}^{C}$, $G^C(\beta_w^M) = 0$ and, then, $CS^{MM-C*} - CS^{RR-C*} = 0$; if $\beta_w^M > \overline{\beta}_{M5}^{C}$, $G^C(\beta_w^M) < 0$ and, then $CS^{MM-C*} - CS^{RR-C*} < 0$. If $\overline{\beta}_{R1}^{C} \le \beta_w^R < \overline{\beta}_{R3}^{C}$, $\overline{\beta}_{M5}^{C} < \overline{\beta}_{M1}^{C}$ and, thus $CS^{MM-C*} - CS^{RR-C*} < 0$.

This completes the proof of Proposition 6.