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Peng Ma
*Nanjing University of Information Science & Technology*

Kevin Li
*University of Windsor*

Zhou-Jing Wang
*Zhejiang University of Finance & Economics*

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# Pricing Decisions in Closed-Loop Supply Chains with Marketing Effort and Fairness Concerns

Peng Ma

School of Economics and Management

Nanjing University of Information Science & Technology

Nanjing 210044, P.R. China

Email: mapeng88@126.com

Kevin W. Li

Odette School of Business, University of Windsor

401 Sunset Ave., Windsor, Ontario, N9B 3P4, Canada

Email: kwli@uwindsor.ca

Zhou-Jing Wang*

School of Information, Zhejiang University of Finance & Economics

Hangzhou, Zhejiang 310018, China.

Email: wangzj@xmu.edu.cn

**Abstract:** We investigate closed-loop supply chains (CLSCs) under four reverse channel structures where a central planner, a manufacturer (M), a retailer (R) or a third party (T), respectively, serves as the collector of used product and demand depends on R's marketing effort. We derive supply chain profitability under both the centralized and decentralized CLSCs and furnish the optimal marketing effort, collection rate and pricing decisions for the supply chain members. We then extend the base models along two directions: the first extension incorporates R's distributional fairness concerns into the M collection model and the second extension considers potential recycle cost advantages by R and T compared to the M collection model.

**Keywords:** Supply chain management; marketing-operations interface; collection rate; pricing decisions; fairness concerns

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* Corresponding author. Tel.: +86 57185043562.

## 1 Introduction

It has been a global trend to recycle end-of-use products and produce remanufactured goods so that manufacturers can reduce their environmental footprint and extract residual values in used products (Agrawal, Atasu, and Ittersum 2015). As the world's largest manufacturer, China recorded a \$2.9 trillion value added in manufacturing in 2014 (Levinson 2017). To meet China's major strategic development needs, its State Council outlined a "Made in China 2025" Program in May 2015 to promote its manufacturing industry. Different priorities such as improving manufacturing innovation, integrating information technology and industry, and enforcing green manufacturing have been identified to help transform China from a manufacturing giant into a real power. In April 2015, Chinese National Development and Reform Commission (NDRC) issued a Circular Economy Promotion Plan for 2015. The document details actions and targets to use resources more efficiently and to better manage resources and waste in industry, agriculture and municipalities[1].

Thanks to increasing environmental consciousness, sustainability concerns, and stringent recycling regulations in recent years, both businesses and academia have been paying more attention to managing closed-loop supply chains (CLSCs). Agrawal, Atasu, and Ittersum. (2015) employ behavioral experiments to study whether and how the presence of remanufactured products and the identity of the remanufacturer influence the perceived value of new products. Their results show that the presence of third-party remanufactured products has a positive effect on the perceived value of the new product.

In practice, the collection rate reflects the collector's effort in collecting used products, so this paper adopts the collection rate as an indicator of the collector's effort. On the other hand, the retailer can increase the sales of products by boosting brand reputation and engaging in promotion and advertising campaigns. These activities signify the retailer's marketing effort. It is understandable that demand depends on the marketing effort.

This paper explores a closed-loop supply chain (CLSC) with a manufacturer (M), a retailer (R) and, possibly, a third-party (T), where the market demand is sensitive to the retail

price and marketing effort. To make the presentation gender neutral, we shall denote M as him, R as her, and T as it. M decides his wholesale price, R chooses her marketing effort and retail price while the collection rate is determined by a central planner, M, R or T in four different models depending on who is responsible for collecting used products. Game theoretic models are established for a centralized CLSC (model I) and decentralized CLSCs under three structures, manufacturer (M) collection (model II), retail (R) collection (model III), and third-party (T) collection (model IV). Key research questions are: (1) What are the optimal pricing decisions, marketing effort and collection rate and profits under different structures? (2) How do the marketing effort coefficient in the demand function and the unit transfer price from M to the collector affect profits and channel strategies? (3) What is the best collection structure in terms of profitability, marketing effort level, and collection rate? (4) How do R's fairness concerns affect the optimal marketing effort, collection rate, and supply chain performance?

This paper follows Savaskan, Bhattacharya, and Van Wassenhove (2004) to consider four collection scenarios: a central planner, M, R or T collects used products. We characterize and compare equilibrium strategies of the four scenarios. In the centralized CLSC, the central planner decides the retail price, marketing effort and collection rate. In the decentralized CLSC, we formulate a Stackelberg game model, where M serves as the leader, setting his optimal wholesale price, R/T as the follower, determining her/its optimal decisions given his wholesale price.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature to put the research in context. The model settings and equilibria are presented in Section 3. Section 4 conducts comparative studies among the four model structures and examines how model parameters affect marketing and collection efforts as well as profitability. Section 5 extends base models along two directions: the first extension incorporates R's distributional fairness concerns into model II and the second extension considers potential recycle cost advantages by R and T compared to the M collection model. Concluding remarks are made in Section 6.

## 2 Related Literature

Many researchers in supply chain management study CLSCs from different angles. Savaskan, Bhattacharya, and Van Wassenhove (2004) investigate four collection models where the collector can be a central planner, a manufacturer, a retailer, or a third party. This research expands their model by considering marketing effort dependent demand and the retailer's fairness concerns as well as collection cost advantage by R and T. Savaskan and Van Wassenhove (2006) examine the interaction between decisions in the forward and reverse logistics channels with competing retailers. Our paper differs by focusing on collection channel selection and optimal pricing decisions where demand depends on both retail price and marketing effort.

## 2.1. Optimal pricing decisions in CLSCs

The first stream of relevant research mainly focuses on remanufacturing problems. Ferrer and Swaminathan (2006) study new and remanufactured products in monopolistic and duopolistic scenarios. Atasu, Sarvary, and Van Wassenhove (2008) investigate remanufacturing as a marketing strategy. The second stream of studies aims to consider different pricing issues in CLSCs. For example, De Giovanni and Zaccour (2014) introduce a two-period CLSC game and compare the equilibrium strategies. Chuang, Wang, and Zhao (2014) establish CLSC models for a high-tech product characterized by a short lifecycle and volatile demand. De Giovanni (2014) examines a CLSC with a manufacturer and a retailer who invest in green advertising to build up goodwill dynamics. Yoo, Kim, and Park (2015) consider pricing and return policies under various supply contracts in a CLSC. Shi et al. (2015) explore the reverse collection channel selection problem under responsibility sharing in a CLSC. Giri and Sharma (2015) consider a closed-loop serial supply chain for a single and multiple manufacturing-remanufacturing cycles by using sequential and global optimizations. Abbey et al. (2015) empirically examine consumer perceptions of remanufactured consumer products in CLSCs. Ma et al. (2016) investigate interactions among a single manufacturer, a single retailer and two recyclers and focus on how cooperative strategies affect CLSC decision-making. Xiong, Zhao, and Zhou (2016) analyze the performance of manufacturer-remanufacturing and supplier-remanufacturing in a decentralized CLSC. Moreover, Yoo and Kim (2016) study the interaction between new and refurbished items in a

three-stage CLSC. Esmaeili, Allameh and Tajvidi (2016) investigate the short- and long-term behaviour of agents in implementing appropriate collection strategies in a two-stage CLSC. Saha, Sarmah and Moon (2016) study a reward-driven policy for acquiring used products earmarked for remanufacturing in a CLSC. Zheng et al. (2017) address the impact of forward channel competition and power structure on a dual-channel CLSC.

Other related works can be found in Amin and Zhang (2012), Chen and Chang (2013), Jena and Sarmah (2014), Das and Dutta (2015), and Han et al. (2017).

**2.2 Optimal pricing decision with fairness concerns**

Most studies focus on distributional fairness concerns in the newsvendor problem or wholesale price contract (Cui, Raju, and Zhang 2007; Caliskan-Demirag, Chen, and Li 2010; Yang et al. 2013; Katok, Olsen, and Pavlov 2014; Du et al. 2014; Wu and Niederhoff 2014). More specifically, Cui, Raju, and Zhang (2007) investigate how fairness may affect channel coordination with linear demand. Then, Caliskan-Demirag, Chen, and Li (2010) provide an extension of Cui, Raju, and Zhang (2007) to explore supply chain coordination with nonlinear demand. Yang et al. (2013) extend Cui, Raju, and Zhang (2007) to study the cooperative advertising problem. Katok, Olsen, and Pavlov (2014) examine the performance of wholesale pricing when the supply chain members' fairness concerns are private information. Du et al. (2014) study the newsvendor problem for a dyadic supply chain in which both the supplier and the retailer have status-seeking preference with fairness concerns. Wu and Niederhoff (2014) investigate the impact of fairness concerns on supply chain performance in a two-party newsvendor setting. Some other papers are concerned with peer-induced fairness (Ho and Su, 2009; Nie and Du, 2016). For instance, Ho and Su (2009) consider peer-induced fairness in games. Nie and Du (2016) explore the impact of distributional and peer-induced fairness on decision-making of a two-stage supply chain with one supplier and two retailers.

Our paper differs from the above literature in two aspects:

(i) Most of the aforesaid studies only consider customer return rate of used products. This research investigates the case that the retailer's marketing effort enhances market demand, and studies the joint effect of marketing effort and return rate on the profitability of the

retailer, the manufacturer and the CLSC. We also examine an extension of incorporating collection cost advantage by R and T compared to the M collection model.

(ii) Existing papers on fairness concerns have analyzed pricing decision and contract coordination issues in classical forward supply chains. In this research, we introduce distributional fairness into the CLSC model and explore its impact on the optimal collection and marketing decisions as well as supply chain performance.

Next, we will address our model assumptions and the four CLSC models.

## 3 Model Settings and Equilibrium Analysis

We consider a demand function depending on the retail price and marketing effort:

$$D(p,e) = a - kp + \gamma e \tag{1}$$

where $a$ measures the market size, $k$ is the price elasticity of demand, $e$ is her marketing effort, $p$ is the retail price. The total marketing effort cost is $\eta e^2/2$, where $\eta$ is the marketing cost coefficient (Mukhopadhyay, Su, and Ghose 2009; Wu, 2013). The parameter $\gamma$ measures the impact of marketing effort on demand. The market demand can be fulfilled in the selling period.

$\tau$ is introduced as a decision variable to gauge the used-product collection rate that reflects the collection effort and signifies the reverse channel performance. The total collection cost is given by $C(\tau)=C_L\tau^2+A\tau D(p,e)$, where $C_L$ is a scale parameter (Savaskan, Bhattacharya, and Van Wassenhove 2004; Chuang, Wang, and Zhao 2014).

For the sake of clarity, the following notations are listed in Table 1 and are used to formulate our CLSC models.

**Table 1.** The Notations in the CLSC Models

| Parameter | |
|---|---|
| $c_m$ | The unit cost of manufacturing a new product |
| $c_r$ | The remanufacturing cost, and without loss of generality, it is assumed that $c_m>c_r$ |
| $\Delta=c_m-c_r$ | The unit cost savings from remanufacturing |
| $A$ | A variable unit cost of collecting and handling one unit of the used product. The fixed payment per unit is less than the savings generated from remanufacturing, i.e., $A<\Delta$ (See Savaskan, Bhattacharya, and Van Wassenhove 2004) |
| $c$ | The average unit production cost, $c=(1-\tau)c_m+\tau c_r=c_m-\Delta\tau$ |
| $C(\tau)$ | Total collection cost |
| $\eta e^2/2$ | The cost of marketing effort, where $\eta$ is the marketing cost coefficient |
| $b$ | The unit transfer price from the manufacturer to the collector for each |

| | collected used product (See Savaskan et al., 2004) |
|---|---|
| $\mu$ | The retailer's equitable payoff parameter |
| $\alpha_0$ | The retailer's disadvantageous inequality parameter |
| Decision variables | |
| $p$ | The retail price of the product |
| $w$ | The wholesale price of the product, $p \geq w$ |
| $\tau$ | The collection rate, i.e., the ratio of products remanufactured from collected used products |
| $e$ | The marketing effort level |
| Superscript/ Subscripts | |
| $(\ )^{i*}$ | Optimal values for different models ($i$=I,II,III,IV,V) |
| $\Pi_i$ | Profit for the supply chain ($i$=SC), the retailer ($i$=R), the manufacturer ($i$=M), and the third-party ($i$=3P) |
| $(\ )^{i**}$ | Optimal values for different models ($i$=III, IV) in section 5.2 |

We will consider the following four cases depending on who collects used products. Firstly, we will investigate the centralized CLSC case where a central planner is responsible for collection (model I) and, then, study the case that M collects used products (model II); next, we consider R as the collector (model III); lastly, T is modeled as the collector (model IV).

### 3.1 The Centralized System (model I)

In the centralized CLSC model, a central planner determines the retail price, marketing and collection efforts. This ideal case is examined here to furnish a benchmark for the other three more realistic decentralized models. The profit of the centralized CLSC can be expressed as follows:

$$\Pi_{SC} = (a - kp + \gamma e)(p - c_m + \Delta\tau) - C_L \tau^2 - A\tau(a - kp + \gamma e) - \frac{\eta}{2}e^2 \qquad (2)$$

After taking the first-order derivative of $\Pi_{SC}$ with respect to $p$, $e$ and $\tau$, we have

$$\frac{\partial \Pi_{SC}}{\partial p} = -k(\Delta\tau + p - c_m) + \gamma e - kp + a + A\tau k = 0 \qquad (3)$$

$$\frac{\partial \Pi_{SC}}{\partial e} = \gamma(\Delta\tau + p - c_m) - A\tau\gamma - \eta e = 0 \qquad (4)$$

$$\frac{\partial \Pi_{SC}}{\partial \tau} = (e\gamma - kp + a)\Delta - 2C_L\tau - A(e\gamma - kp + a) = 0 \qquad (5)$$

For notational convenience, let $F = 2k\eta - \gamma^2, G = a - kc_m, J = \Delta - A.$

**Lemma 1.** If $F > 0$ and $2C_L F - k^2 \eta J^2 > 0$, the profit function $\Pi_{SC}$ is strictly concave in $p$, $e$ and $\tau$.

**Proof.** See Appendix.

Combining Eqs. (3)-(5) yields Proposition 1.

**Proposition 1.** The solutions of the first-order conditions for the centralized case are derived as follows:

$$\tau^{I*} = \frac{Jk\eta G}{2C_L F - k^2 \eta J^2}, e^{I*} = \frac{2\gamma C_L G}{2C_L F - k^2 \eta J^2}, D^{I*} = \frac{2\eta k C_L G}{2C_L F - k^2 \eta J^2},$$

$$p^{I*} = \frac{A^2 a\eta k - 2A\Delta a\eta k + \Delta^2 a\eta k - 2\eta k C_L c_m + 2\gamma^2 C_L c_m - 2a\eta C_L}{-2C_L F + k^2 \eta J^2}, \Pi_{SC}^{I*} = \frac{\eta C_L G^2}{2C_L F - k^2 \eta J^2}.$$

To ensure that all equilibrium values are meaningful, we give following assumption 1.

**Assumption 1**. The scale parameter $C_L$ given in the collection cost function is large enough that $\tau^{I*} \leq 1$. More specifically,

$$C_L \geq \frac{Jk\eta G + k^2 \eta J^2}{2F}.$$

This assumption follows the same line of thinking in Savaskan, Bhattacharya, and Van Wassenhove (2004). As the numerator of $\tau^{I*}$ in Proposition 1 is nonnegative given the basic model setting in Table 1, considering the assumption $2C_L F - k^2 \eta J^2 > 0$ in Lemma 1, it is guaranteed that $0 \leq \tau^{I*} \leq 1$.

### 3.2 The manufacturer collects used products (model II)

In model II, R determines the marketing effort and retail price, and M collects used products from the market. In this case, her profit is expressed as follows:

$$\Pi_R = (p - w)(a - kp + \gamma e) - \frac{\eta}{2} e^2 \tag{6}$$

After taking the first-order derivative of $\Pi_R$ with respect to $p$ and $e$, we can derive that:

$$\frac{\partial \Pi_R}{\partial p} = \gamma e - kp + a - (p - w)k = 0 \tag{7}$$

$$\frac{\partial \Pi_R}{\partial e} = (p - w)\gamma - \eta e = 0 \tag{8}$$

The Hessian matrix of $\Pi_R$ is:

$$H(p,e) = \begin{bmatrix} \dfrac{\partial^2 \Pi_R}{\partial p^2} & \dfrac{\partial^2 \Pi_R}{\partial p \partial e} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial e \partial p} & \dfrac{\partial^2 \Pi_R}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2k & \gamma \\[1mm] \gamma & -\eta \end{bmatrix}$$

Since $\dfrac{\partial^2 \Pi_R}{\partial p^2} = -2k < 0$, if $|H(p,e)| = 2k\eta - \gamma^2 = F > 0$, then the Hessian matrix of

$\Pi_R$ is negative definite. From Eqs. (7) and (8), we derive that

$$e^{II^*}(w,\tau) = \frac{\gamma(-kw+a)}{F} \tag{9}$$

$$p^{II^*}(w,\tau) = \frac{\eta k w - \gamma^2 w + a\eta}{F} \tag{10}$$

His decision problem is to select $w$ and $\tau$ to maximize his profit.

$$\underset{w,\tau}{Max}\,\Pi_M = (w - c_m + \Delta\tau)(a - kp + \gamma e) - C_L \tau^2 - A\tau(a - kp + \gamma e)$$

$$= (w - c_m + \Delta\tau).\frac{\eta k(a - kw)}{F} - C_L \tau^2 - A\tau.\frac{\eta k(a - kw)}{F} \tag{11}$$

After taking the first-order derivative of $\Pi_M$ with respect to $w$ and $\tau$, we obtain

$$\frac{\partial \Pi_M}{\partial w} = \frac{\eta k(Ak\tau - \Delta k\tau - 2kw + kc_m + a)}{F} = 0 \tag{12}$$

$$\frac{\partial \Pi_M}{\partial \tau} = \frac{\Delta \eta k(-kw+a)}{F} - 2C_L\tau - \frac{A\eta k(-kw+a)}{F} = 0 \tag{13}$$

The Hessian matrix of $\Pi_M$ is:

$$H(w,\tau) = \begin{bmatrix} \dfrac{\partial^2 \Pi_M}{\partial w^2} & \dfrac{\partial^2 \Pi_M}{\partial w \partial \tau} \\[2mm] \dfrac{\partial^2 \Pi_M}{\partial \tau \partial w} & \dfrac{\partial^2 \Pi_M}{\partial \tau^2} \end{bmatrix} = \begin{bmatrix} -\dfrac{2\eta k^2}{F} & -\dfrac{Jk^2\eta}{F} \\[2mm] -\dfrac{Jk^2\eta}{F} & -2C_L \end{bmatrix}.$$

As $\dfrac{\partial^2 \Pi_M}{\partial w^2} = \dfrac{-2k^2\eta}{F} < 0$, if $|H(w,\tau)| = \dfrac{k^2\eta}{F^2}[4C_L F - k^2\eta J^2] > 0$, the Hessian matrix of

$\Pi_M$ is negative definite. Combining Eqs. (12) and (13) results in

$$\tau^{II^*} = \frac{\eta k G J}{4C_L F - k^2\eta J^2} \tag{14}$$

$$w^{II^*} = -\frac{a\eta k^2 J^2 - 2C_L(a + kc_m)F}{(4C_L F - k^2\eta J^2)k} \tag{15}$$

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Substituting Eqs. (14) and (15) into Eqs. (9) and (10), we obtain the optimal marketing effort ($e^{II*}$) and optimal retail price ($p^{II*}$). Then, we substitute $e^{II*}$, $p^{II*}$, $\tau^{II*}$ and $w^{II*}$ into Eqs. (6) and (11) and derive the optimal profits of R and M. The following proposition summarizes the equilibrium result.

**Proposition 2.** Under the M collection case, his optimal wholesale price ($w^{II*}$) and used-product collection rate ($\tau^{II*}$), her optimal retail price ($p^{II*}$) and marketing effort ($e^{II*}$), the optimal demand ($D^{II*}$), and the individual and supply chain profits are derived as follows

$$w^{II*} = \frac{2C_L(a+kc_m)F - a\eta k^2 J^2}{(4C_L F - k^2 \eta J^2)k}, \tau^{II*} = \frac{\eta kJG}{4C_L F - k^2 \eta J^2},$$

$$p^{II*} = -\frac{a\eta k^2 J^2 + 2kC_L c_m(\gamma^2 - \eta k) + 2aC_L(\gamma^2 - 3\eta k)}{(4C_L F - k^2 \eta J^2)k}, e^{II*} = \frac{2GC_L\gamma}{4C_L F - k^2 \eta J^2},$$

$$D^{II*} = \frac{2\eta kC_L G}{4C_L F - k^2 \eta J^2}, \Pi_M^{II*} = \frac{\eta C_L G^2}{4C_L F - k^2 \eta J^2}, \Pi_R^{II*} = \frac{2\eta C_L^2 G^2 F}{(4C_L F - k^2 \eta J^2)^2},$$

$$\Pi_{SC}^{II*} = \Pi_M^{II*} + \Pi_R^{II*} = \frac{(6C_L F - k^2 \eta J^2)\eta C_L G^2}{(4C_L F - k^2 \eta J^2)^2}.$$

## 3.3 The Retailer Collects Used Products (model III)

In model III, R is responsible for both marketing effort and collection of used products. Her decision problem is

$$\underset{p,e,\tau}{Max}\Pi_R = (p-w)(a-kp+\gamma e) + b\tau(a-kp+\gamma e) - \frac{\eta}{2}e^2 - C_L\tau^2 - A\tau(a-kp+\gamma e), \quad (16)$$

By taking the first-order derivative of Eq. (16) with respect to $p$, $e$ and $\tau$, we derive

$$\frac{\partial \Pi_R}{\partial p} = \gamma e - kp + a - (p-w)k - b\tau k + A\tau k = 0 \quad (17)$$

$$\frac{\partial \Pi_R}{\partial e} = (p-w)\gamma + b\tau\gamma - \eta e - A\tau\gamma = 0 \quad (18)$$

$$\frac{\partial \Pi_R}{\partial \tau} = b(e\gamma - kp + a) - 2C_L\tau - A(e\gamma - kp + a) = 0 \quad (19)$$

The Hessian matrix of $\Pi_R$ is

$$H(p,e,\tau) = \begin{bmatrix} \dfrac{\partial^2 \Pi_R}{\partial p^2} & \dfrac{\partial^2 \Pi_R}{\partial p \partial e} & \dfrac{\partial^2 \Pi_R}{\partial p \partial \tau} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial e \partial p} & \dfrac{\partial^2 \Pi_R}{\partial e^2} & \dfrac{\partial^2 \Pi_R}{\partial e \partial \tau} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial \tau \partial p} & \dfrac{\partial^2 \Pi_R}{\partial \tau \partial e} & \dfrac{\partial^2 \Pi_R}{\partial \tau^2} \end{bmatrix} = \begin{bmatrix} -2k & \gamma & -(b-A)k \\[2mm] \gamma & -\eta & (b-A)\gamma \\[2mm] -(b-A)k & (b-A)\gamma & -2C_L \end{bmatrix}$$

Since $\dfrac{\partial^2 \Pi_R}{\partial p^2} = -2k < 0$, $\left| H_2(p,e,\tau) \right| = \begin{vmatrix} \dfrac{\partial^2 \Pi_R}{\partial p^2} & \dfrac{\partial^2 \Pi_R}{\partial p \partial e} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial e \partial p} & \dfrac{\partial^2 \Pi_R}{\partial e^2} \end{vmatrix} = F$. For $\left| H_2(p,e,\tau) \right| > 0$ to

hold, we need $F > 0$.

$$\left| H_3(p,e,\tau) \right| = \begin{vmatrix} \dfrac{\partial^2 \Pi_R}{\partial p^2} & \dfrac{\partial^2 \Pi_R}{\partial p \partial e} & \dfrac{\partial^2 \Pi_R}{\partial p \partial \tau} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial e \partial p} & \dfrac{\partial^2 \Pi_R}{\partial e^2} & \dfrac{\partial^2 \Pi_R}{\partial e \partial \tau} \\[2mm] \dfrac{\partial^2 \Pi_R}{\partial \tau \partial p} & \dfrac{\partial^2 \Pi_R}{\partial \tau \partial e} & \dfrac{\partial^2 \Pi_R}{\partial \tau^2} \end{vmatrix} = -2C_L F + k^2 \eta (b-A)^2$$. For $\left| H_3(p,e,\tau) \right| < 0$

to hold, we need $-2C_L F + k^2 \eta (b-A)^2 < 0$. Under this condition, $H(p,e,\tau)$ is negative definite. Combining Eqs. (17)-(19), we derive that

$$\tau^{III*}(w) = \frac{(b-A)k\eta(a-kw)}{2C_L F - k^2 \eta (b-A)^2} \tag{20}$$

$$e^{III*}(w) = \frac{2\gamma C_L (a-kw)}{2C_L F - k^2 \eta (b-A)^2} \tag{21}$$

$$p^{III*}(w) = \frac{-a\eta k (b-A)^2 + 2C_L \left( \eta kw - \gamma^2 w + a\eta \right)}{2C_L F - k^2 \eta (b-A)^2} \tag{22}$$

Then, M's decision problem can be stated as

$$\underset{w}{Max}\, \Pi_M = (w - c_m + \Delta\tau)(a - kp + \gamma e) - b\tau(a - kp + \gamma e) \tag{23}$$

After substituting Eqs. (20)-(22) into Eq. (23), M's problem becomes

$$\underset{w}{Max}\, \Pi_M = \frac{2\left( w - c_m + \dfrac{(\Delta-b)(A-b)k\eta(-kw+a)}{A^2 \eta k^2 - 2Ab\eta k^2 + b^2 \eta k^2 - 4\eta k C_L + 2\gamma^2 C_L} \right) \eta k C_L (a-kw)}{2C_L F - k^2 \eta (b-A)^2} \tag{24}$$

After taking the first-order derivative of Eq. (24) with respect to $w$, we obtain

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$$\frac{\partial \Pi_M}{\partial w} = -\frac{2\eta k C_L}{\left(2C_L F - (A-b)^2 k^2 \eta\right)^2} \{(A-b)\left((c_m - 2w)A - bc_m + 2w\Delta\right)\eta k^3$$

$$+\eta\left(A^2 a - 2A\Delta a + (-4c_m + 8w)C_L - ab(b - 2\Delta)\right)k^2 + 2C_L\left(-2a\eta + \gamma^2(c_m - 2w)\right)k + 2a\gamma^2 C_L\} = 0$$

$$\frac{\partial^2 \Pi_M}{\partial w^2} = -\frac{4\eta k^2 C_L\left(2C_L F - J(b-A)k^2 \eta\right)}{\left(2C_L F - (A-b)^2 k^2 \eta\right)^2}.$$

Under the assumption $2C_L F - J(b-A)k^2\eta > 0$ in Lemma 1, we have $\dfrac{\partial^2 \Pi_M}{\partial w^2} < 0$.

Thus, the objective function in Eq. (24) is concave in $w$. Thus, M's first-order condition characterizes the unique best response,

$$w^{III*} = \frac{1}{2k\left(-2C_L F + k^2\eta J(b-A)\right)}\{A^2\eta k^3 c_m - 2Ab\eta k^3 c_m + b^2\eta k^3 c_m + A^2 a\eta k^2$$

$$-2A\Delta a\eta k^2 + 2\Delta ab\eta k^2 - ab^2\eta k^2 - 4\eta k^2 C_L c_m + 2\gamma^2 k C_L c_m - 4a\eta k C_L + 2a\gamma^2 C_L\}. \tag{25}$$

Substituting Eq. (25) into Eqs. (20)-(22), we obtain the optimal collection rate ($\tau^{III*}$), optimal marketing effort ($e^{III*}$) and optimal retail price ($p^{III*}$). Then, we substitute $\tau^{III*}$, $e^{III*}$ and $p^{III*}$ into Eqs. (16) and (23) and derive the optimal profits of R and M. The following proposition 3 summarizes the optimal solutions in R collection model.

**Proposition 3.** Under the R collection case, M's optimal wholesale price ($w^{III*}$), R's optimal retail price ($p^{III*}$), marketing effort ($e^{III*}$), used-product collection rate ($\tau^{III*}$), the optimal demand ($D^{III*}$) and individual and supply chain profits are as follows

$$w^{III*} = \frac{1}{2k\left(-2C_L F + k^2\eta J(b-A)\right)}\{A^2\eta k^3 c_m - 2Ab\eta k^3 c_m + b^2\eta k^3 c_m + A^2 a\eta k^2$$

$$-2A\Delta a\eta k^2 + 2\Delta ab\eta k^2 - ab^2\eta k^2 - 4\eta k^2 C_L c_m + 2\gamma^2 k C_L c_m - 4a\eta k C_L + 2a\gamma^2 C_L\},$$

$$p^{III*} = \frac{A^2 a\eta k^2 - A\Delta a\eta k^2 - Aab\eta k^2 + \Delta ab\eta k^2 - \eta k^2 C_L c_m + \gamma^2 k C_L c_m - 3a\eta k C_L + a\gamma^2 C_L}{k\left(-2C_L F + k^2\eta J(b-A)\right)},$$

$$e^{III*} = \frac{GC_L\gamma}{2C_L F - k^2\eta J(b-A)}, \quad \tau^{III*} = \frac{1}{2}\cdot\frac{G(b-A)k\eta}{2C_L F - k^2\eta J(b-A)},$$

$$D^{III*} = \frac{G\eta k C_L}{2C_L F - k^2\eta J(b-A)}, \quad \Pi_M^{III*} = \frac{1}{2}\frac{G^2\eta C_L}{2C_L F - k^2\eta J(b-A)},$$

$$\Pi_R^{III*} = \frac{\eta C_L G^2\left(2C_L F - k^2\eta(b-A)^2\right)}{4\left(2C_L F - k^2\eta J(b-A)\right)^2},$$

$$\Pi_{SC}^{III*} = \Pi_M^{III*} + \Pi_R^{III*} = \frac{G^2\eta C_L\left(6C_L F - k^2\eta(b-A)(2\Delta - 3A + b)\right)}{4\left(2C_L F - k^2\eta J(b-A)\right)^2}.$$

**Observation 1**. Because M's profit increases in $b$, the optimal transfer price $b$ should be set at $b = \Delta$ under the R collection case.

**Proof.** See Appendix.

Savaskan et al. (2004) derive the same result when the demand function assumes a simpler form of $D=\varphi-\beta p$. Thus, this observation confirms that our generalized demand function does not affect the value of the optimal transfer price ($b$) when R serves as the collector.

### 3.4 The Third-party (T) Collection Model (model IV)

In reality, many OEM manufacturers prefer to outsource the collection of used products to independent third parties to exploit their specialization in the recycling process (Savaskan, Bhattacharya, and Van Wassenhove 2004). In this model, we assume that M is the Stackelberg leader whereas R and T are the followers. The decision sequence is as follows. M first determines his wholesale price ($w$), T decides its collection rate ($\tau$), and R chooses her retail price ($p$) and marketing effort ($e$). Next, we use backward induction to solve the aforesaid problem. For a given $w$, the two followers' problems are given by

$$\underset{p,e}{Max}\Pi_R = (p-w)(a-kp+\gamma e)-\frac{\eta}{2}e^2 \tag{26}$$

$$\underset{\tau}{Max}\Pi_{3P} = b\tau(a-kp+\gamma e)-C_L\tau^2 - A\tau\left(a-kp+\gamma e\right) \tag{27}$$

Similar to model II, $\Pi_R$ is strictly jointly concave in $p$ and $e$. Based on the R's reaction functions in Eqs. (9) and (10), we have

$$e^{IV*}(w,\tau) = \frac{\gamma\left(-kw+a\right)}{F} \tag{28}$$

$$p^{IV*}(w,\tau) = \frac{\eta kw - \gamma^2 w + a\eta}{F} \tag{29}$$

Substituting Eqs. (28) and (29) into Eq. (27), T's problem is converted to

$$\underset{\tau}{Max}\Pi_{3P} = \frac{(b-A)\tau\eta k\left(-kw+a\right)}{F} - C_L\tau^2 \tag{30}$$

Taking the first- and second-order derivatives of $\Pi_{3P}$ with respect to $\tau$, we have

$\dfrac{\partial \Pi_{3P}}{\partial \tau} = \dfrac{(b-A)\eta k\left(-kw+a\right)}{F} - 2C_L\tau$ , $\dfrac{\partial^2 \Pi_{3P}}{\partial \tau^2} = -2C_L < 0$ . Thus, T's optimal response function is

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$$\tau^{IV*}(w) = \frac{\eta k(-kw+a)(b-A)}{2FC_L} \tag{31}$$

For a given $p$, $e$ and $\tau$, M's problem is given by

$$\underset{w}{Max}\,\Pi_M = (w - c_m + \tau(\Delta - b))(a - kp + \gamma e) \tag{32}$$

By substituting Eqs. (28)-(29) and (31) into Eq. (32), the above problem becomes

$$\underset{w}{Max}\,\Pi_M = \frac{\left(w - c_m + \dfrac{1}{2}\dfrac{\eta k(-kw+a)(-b+A)(\Delta - b)}{-FC_L}\right)\eta k(-kw+a)}{F} \tag{33}$$

Solving the first-order condition of Eq. (33) yields the following equation that the optimal wholesale price must satisfy

$$w^{IV*} = \frac{A\Delta a\eta k^2 - Aab\eta k^2 - \Delta ab\eta k^2 + ab^2\eta k^2 + 2\eta k^2 C_L c_m - \gamma^2 kC_L c_m + 2a\eta kC_L - a\gamma^2 C_L}{k\left(A\Delta\eta k^2 - Ab\eta k^2 - \Delta b\eta k^2 + b^2\eta k^2 + 4\eta kC_L - 2\gamma^2 C_L\right)} \tag{34}$$

Substituting Eq. (34) into (28)-(29) and (31), then, into Eqs. (26), (27) and (33), we derive the following result.

**Proposition 4.** Under the T collection case, M's optimal wholesale price ($w^{IV*}$), R's retail price ($p^{IV*}$) and marketing effort ($e^{IV*}$), T's used-product collection rate ($\tau^{IV*}$), the optimal demand ($D^{IV*}$) and individual and supply chain profits are as follows

$$w^{IV*} = \frac{C_L\left(2\eta k^2 c_m - \gamma^2 kc_m + 2a\eta k - a\gamma^2\right) - ak^2\eta(b-A)(\Delta - b)}{k\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)},$$

$$p^{IV*} = \frac{C_L\left(\eta k^2 c_m - \gamma^2 kc_m + 3a\eta k - a\gamma^2\right) - ak^2\eta(b-A)(\Delta - b)}{k\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)},$$

$$e^{IV*} = \frac{GC_L\gamma}{2C_L F - k^2\eta(b-A)(\Delta - b)}, \tau^{IV*} = \frac{G(b-A)\eta k}{2\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)},$$

$$D^{IV*} = \frac{G\eta kC_L}{2C_L F - k^2\eta(b-A)(\Delta - b)}, \Pi_R^{IV*} = \frac{1}{2}\frac{G^2\eta C_L^2 F}{\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)^2},$$

$$\Pi_{3P}^{IV*} = \frac{1}{4}\frac{C_L(b-A)^2 G^2\eta^2 k^2}{\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)^2}, \Pi_M^{IV*} = \frac{C_L\eta G^2}{2\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)},$$

$$\Pi_{SC}^{IV*} = \frac{C_L\eta G^2\left(6C_L F + k^2\eta\left(A^2 + 2A\Delta - 4Ab - 2\Delta b + 3b^2\right)\right)}{4\left(2C_L F - k^2\eta(b-A)(\Delta - b)\right)^2}.$$

Now, we make the following observation regarding the optimal $b$ value in the T collection model.

**Observation 2**. M's profit function is maximized at $b = \dfrac{A + \Delta}{2}$.

**Proof.** See Appendix.

Savaskan et al. (2004) establish the optimal transfer price ($b$) as a function of $\Delta$ and the variable unit collection cost with their simplified demand function under the T collection model. Observation 2 furnishes a parallel optimal transfer price under our marketing--effort-dependent demand function.

## 4 Comparative Analysis

### 4.1 Comparisons of the Four Closed-loop Supply Chain Models

We shall carry out comparative analyses of the optimal collection rate and marketing effort as well as supply chain performance under the four different CLSC models.

**Proposition 5.** The optimal product return rates are related as $\tau^{IV*} < \tau^{II*} < \tau^{III*} < \tau^{I*}$.

**Proof.** See Appendix.

Proposition 5 confirms an interesting finding in Savaskan, Bhattacharya, and Van Wassenhove (2004) under our generalized demand function: The closer the collector is to the market, the more efficient the collection rate is. More specifically, Proposition 5 indicates that the collection effort level under model I is always the best compared to the other three decentralized cases. This result is natural as all decisions are fully coordinated in this centralized case. For the three decentralized cases, the collection effort level under R collection (model III) is the best, followed by M collection and, lastly, T collection. In the T collection model, the only way that T can influence market demand to enhance its profitability is to invest in the collection rate. On the other hand, in the M collection model, M can not only decide on the collection rate directly, but also strategically set his wholesale price with a second-degree impact on the collection rate. As such, the M collection model realizes a higher collection rate than the T collection model. Along the same line, in the R collection model, R has more tools at her disposal to enhance market demand and her profitability: setting the retail price and investing in the collection and marketing efforts conditional upon M's wholesale price decision. With the second-degree impact on the collection rate by her retail price and marketing effort decisions, the R collection channel is able to achieve higher collection efficiency than the M collection model.

**Proposition 6.** The optimal marketing efforts are related as $e^{IV*} < e^{II*} < e^{III*} < e^{I*}$.

**Proof.** See Appendix.

Proposition 6 establishes a definite relationship for the optimal marketing effort among the centralized (model I), M collection (model II), R collection (model III) and T collection (model IV) cases. It is natural for model I to outperform the three decentralized cases as it is the fully coordinated scenario. The reason that the marketing investment in model III outperforms model II is due to the fact that R directly benefits from her marketing effort in enhancing demand when R is responsible for collecting used products in model III, but this benefit is only secondary when M makes collection effort decision in model II. In the T collection case, the optimal marketing effort level is less than the other three cases due to the fact that both M and T will share the increased sales and profits arising from the R's marketing efforts while only M claims part of the increased sales and profits in model II & III. As such, R is least enthusiastic in exerting marketing efforts under model IV.

Next, we compare the optimal individual and supply chain profits for different models.

**Proposition 7.** If $b=\Delta$ in model III and $b=(A+\Delta)/2$ in model IV, we have

(1) The optimal profits of the supply chain in the four models are related as $\Pi_{SC}^{IV*} < \Pi_{SC}^{II*} < \Pi_{SC}^{III*} < \Pi_{SC}^{I*}$;

(2) For the three decentralized cases, M's profits are related as $\Pi_M^{IV*} < \Pi_M^{II*} < \Pi_M^{III*}$;

(3) For the three decentralized cases, R's profits are related as $\Pi_R^{IV*} < \Pi_R^{II*} < \Pi_R^{III*}$.

**Proof.** See Appendix.

Proposition 7 indicates that the channel profit achieves the best level when the system is fully coordinated (model I). For the three decentralized cases, both individual and channel profits attain the highest when R is the collector (model III), followed by the case when M serves as the collector (model II), and lastly the lowest individual and channel profits are achieved when T is responsible for collection (model IV). This result resonates the conclusion on optimal marketing effort in Proposition 6. A higher marketing effort leads to higher market demand, thereby enhancing profitability. As the optimal marketing effort reaches the highest level in model III, followed by model II and IV in the three decentralized cases, it is understandable that both individual and channel profits follow suit.

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## 4.2 Impact of Model Parameters on Effort Decisions and Profits

Next, we will study the impact of the demand coefficient $\gamma$ and the unit transfer price ($b$) on the collection and marketing efforts, and profits of the CLSC.

### 4.2.1 Impact of the Demand Coefficient $\gamma$

Recall that the demand coefficient $\gamma$ measures how sensitive market demand responds to the marketing effort. The higher the $\gamma$, the more sensitive the demand.

**Proposition 8.** In model I, II, III and IV, the optimal collection rate, the optimal marketing effort, and the channel profit of the CLSC increase in $\gamma$.

**Proof.** See Appendix.

Proposition 8 demonstrates that the optimal collection rate, marketing effort level, and channel profit all increase when the market demand becomes more sensitive to marketing effort for all of the four CLSC cases. This result is sensible: when market demand is more responsive, the central planner (the centralized case) or R (the three decentralized cases) is incentivized to exert more marketing effort. The spillover effect will induce the collector to make a better collection rate, thereby enhancing channel profitability.

### 4.2.2 Effect of the Transfer Price $b$

Now, we examine the impact of the unit transfer price ($b$) on the collection rate and marketing effort, as well as the profits of the CLSC. Since $b$ is the unit transfer price from M to R/T when R/T is the collector of used products, it is irrelevant when the central planner/manufacturer is responsible for collection. Therefore, the discussion below is confined to models III and IV only. Given that the meanings of $b$ and $\Delta$, it is sensible to have $0 \le b \le \Delta$ in the following discussions.

**Proposition 9.** (1) In model III, the collection rate, marketing effort, individual and channel profits increase in $b$.

(2) In model IV, the collection rate increases in $b$;

(3) In model IV, the marketing effort and M's profit increase in $b$ if $0 \le b \le (A+\Delta)/2$, and decrease in $b$ if $(A+\Delta)/2 < b \le \Delta$ .

**Proof.** See Appendix.

Recall that $b$ is the unit transfer price from M to the collector due to the savings from remanufacturing. Proposition 9 (1) shows that the collection rate and marketing effort, as well as individual and channel profits increase in $b$ when R is the collector as long as M controls the unit transfer price to within his optimal level $b = \Delta$ (See Observation 1). This result is

natural as R directly benefits from a higher transfer price, which apparently motivates her to exert more collection and marketing efforts, thereby enhancing her profitability. The spillover effect of R's collection and marketing effort decisions is eventually translated into a higher profit for M and the whole supply chain.

In model IV, T is responsible for collecting used products, it is sensible that the collection rate increases when M transfers more savings to T (a larger $b$). For the meaningful model parameter range ($0 \leq b \leq \Delta$), R's marketing effort and M's profit first increase in $b$ when it is small and, then, decrease in $b$ when it is sufficiently large. Recall that T's collection rate always increases in $b$. When this transfer price increases within a reasonable range ($0 \leq b \leq (A+\Delta)/2$), R has the motivation to exert more marketing effort and M actually attains a higher profit as a result of the heightened collection rate and marketing effort. However, when the transfer price increases beyond the threshold $(A+\Delta)/2$, the imbalanced transfer dampens R's motive for marketing the product and eventually hurts M's profit.

### 4.2.3 Effect of the Marketing Effort Cost Coefficient ($\eta$)

Now, we will study the impact of the marketing effort cost coefficient ($\eta$) on the collection rate and marketing effort, and profits of the CLSC.

**Proposition 10.** In model I, II, III and IV, the optimal collection rate, marketing effort, and individual and channel profits of the CLSC decrease in $\eta$.

Loosely speaking, this proposition is the converse of Proposition 8 and it shows the other side of the coin. As $\eta$ is the quadratic cost coefficient, the marketing cost increases faster at a higher $\eta$. This curbs the central planner (model I) or R (the other three decentralized cases) from investing in marketing effort, resulting in a lower collection rate and driving down individual and channel profitability.

**Proof.** See Appendix.

### 5 Extensions

In this section, we shall consider two extensions for the base model. The first extension entertains the idea that R is concerned with distributional fairness. Model II is taken as an example for this extension where M collects used products with R having distributional fairness concerns. It trivial to extend the same fairness concerns to the other two models, III and IV.

### 5.1 The Manufacturer Collects Used Products with the Retailer Concerning Fairness (Model V)

### 5.1.1 Model Description

In this case, R will maximize a utility function (see Eq. (35) below) that accounts for both her profit and her fairness concerns when R makes her price and marketing effort decisions (Cui, Raju, and Zhang 2007; Caliskan-Demirag, Chen, and Li 2010; Yang et al. 2013; Katok, Olsen, and Pavlov 2014; Nie and Du 2016).

Now, we analyze the case where R cares about profit distributional fairness as reflected in the disadvantageous inequality (i.e., $\mu \Pi_M - \Pi_R \geq 0$). In general, her utility function can be expressed as

$$U_R = \Pi_R(p,w) - \alpha_0(\mu \Pi_M - \Pi_R)^+ \tag{35}$$

where $\Pi_R(p,e) = (p-w)(a-kp+\gamma e) - \dfrac{\eta}{2}e^2$, $\alpha_0$ is her disadvantageous inequality parameter $(0 \leq \alpha_0 \leq 1)$. $\mu > 0$ is her equitable payoff parameter and captures her fairness concerns with the expected profit distribution scheme between herself and M and is exogenous to our model.

We can rewrite R's objective function as follows:

$$\max_{p,e} U_R(p,e) = (p-w)(a-kp+\gamma e) - \frac{\eta}{2}e^2 - \alpha_0\{\mu[(w-c_m+\Delta\tau)(a-kp+\gamma e) - C_L\tau^2$$

$$-A\tau(a-kp+\gamma e)] - [(p-w)(a-kp+\gamma e) - \frac{\eta}{2}e^2]\}$$

$$s.t.\mu[(w-c_m+\Delta\tau)(a-kp+\gamma e) - C_L\tau^2 \tag{36}$$

$$-A\tau(a-kp+\gamma e)] - [(p-w)(a-kp+\gamma e) - \frac{\eta}{2}e^2] \geq 0$$

The profit of the manufacturer is:

$$\max_{w,\tau} \Pi_M = (w-c_m+\Delta\tau)(a-kp+\gamma e) - C_L\tau^2 - A\tau(a-kp+\gamma e). \tag{37}$$

The following Proposition summarizes both parties' optimal solutions.

**Proposition 11.** If there exists the R's disadvantageous inequality aversion, i.e.,

$$\mu(1+\alpha_0)(4C_L(1+\alpha_0+\mu\alpha_0)F - (1+\alpha_0)\eta k^2 J^2) \geq 2C_L(1+\alpha_0+\mu\alpha_0)(1+\alpha_0+3\mu\alpha_0)F,$$

then the optimal used product collection rate ($\tau^{V^*}$), wholesale price ($w^{V^*}$), demand ($D^{V^*}$), marketing effort ($e^{V^*}$), retail price ($p^{V^*}$), the R's profit ($\Pi_R^{V^*}$), and M's profit ($\Pi_M^{V^*}$) are given by

$$\tau^{V^*} = \frac{(1+\alpha_0)JG\eta k}{4C_L(1+\alpha_0+\mu\alpha_0)F - (1+\alpha_0)\eta k^2 J^2},$$

$$w^{V^*} = \frac{(a\eta k^2 J^2 - 2C_L F(a+kc_m))(1+\alpha_0) - 4k\mu C_L\alpha_0 c_m F}{k((k^2\eta J^2 - 4C_L F)(1+\alpha_0) - 4\mu C_L\alpha_0 F)},$$

$$D^{V*} = \frac{2(1+\alpha_0+\mu\alpha_0)G\eta k C_L}{4C_L(1+\alpha_0+\mu\alpha_0)F-(1+\alpha_0)\eta k^2 J^2}, \quad e^{V*} = \frac{2\gamma(1+\alpha_0+\mu\alpha_0)GC_L}{4C_L(1+\alpha_0+\mu\alpha_0)F-(1+\alpha_0)\eta k^2 J^2},$$

$$p^{V*} = \frac{J^2 a\eta k^2(1+\alpha_0)+\left(-2\eta k^2 C_L c_m + 2\gamma^2 k C_L c_m - 6a\eta k C_L + 2a\gamma^2 C_L\right)(1+\alpha_0+\mu\alpha_0)}{k\left(\left(k^2\eta J^2 - 4C_L F\right)(1+\alpha_0)-4\mu C_L\alpha_0 F\right)},$$

$$\Pi_R^{V*} = \frac{2C_L^2\eta(1+\alpha_0+\mu\alpha_0)(1+\alpha_0+3\mu\alpha_0)FG^2}{[4C_L(1+\alpha_0+\mu\alpha_0)F-(1+\alpha_0)\eta k^2 J^2]^2}, \quad \Pi_M^{V*} = \frac{(1+\alpha_0)\eta C_L G^2}{4C_L(1+\alpha_0+\mu\alpha_0)F-(1+\alpha_0)\eta k^2 J^2}.$$

**Proof.** This proof is similar to that of Proposition 2 and is thus omitted here.

Now, we investigate the impact of her disadvantageous inequality parameter $\alpha_0$ on the optimal marketing effort, collection rate as well as individual and channel profits. Then, we derive the following proposition.

**Proposition 12.** (1) The optimal collection rate, marketing effort, and the M's profit all decrease in $\alpha_0$;

(2) If $4C_L F - 3k^2\eta J^2 > 0$, then the R's profit increases in $\alpha_0$.

**Proof.** See Appendix.

Note that $\alpha_0$ measures how sensitive R is concerned with the fairness of profit distributions. In the presence of her disadvantageous inequality ($\mu\Pi_M - \Pi_R \geq 0$), a higher $\alpha_0$ corresponds to a more sensitive retailer. It is natural that R will make less marketing effort when R feels more strongly about the unfair distribution of profit between M and herself. This will drive down market demand and lead to a lower collection rate and profit for M.

For R, as long as $4C_L F - 3k^2\eta J^2 > 0$, which is a slightly stronger condition than that in Lemma 1, her profit will always increase in $\alpha_0$. This is quite understandable as her fairness concerns will force M to give up some profit to appease R.

### 5.1.2 Numerical Studies

Next, numerical studies are carried out to investigate the impact of R's disadvantageous inequality parameter $\alpha_0$ on the marketing effort, collection rate as well as individual and channel profits.

To examine the effect of $\alpha_0$ on the optimal marketing effort and the supply chain profits, we allow $\alpha_0$ to vary in the range of [0, 0.25]. By assuming $a=360$, $A=10$, $C_L=300$, $\eta=4$, $k=5$, $\Delta=15$, $\mu=0.8$, $\gamma=2$ and $c_m=20$, Figs. 1-3 validate Proposition 12. More specifically, Fig. 1 visually depicts a declining marketing effort level when parameter $\alpha_0$ increases. Fig. 2 shows a similar declining trend for M's collection rate when $\alpha_0$ increases. Fig. 3, on the other hand,
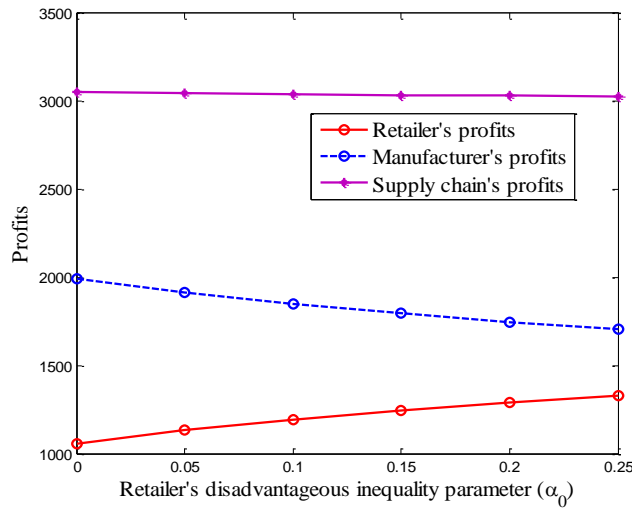
displays how individual and channel profits change with $\alpha_0$. With R's concerns with distributional fairness, R's profit increases, but M's profit drops when $\alpha_0$ increases. This trend helps to close in the gap of R's disadvantageous inequality when R feels more strongly about the distributional fairness. It is worth noting that the system profit suffers slightly in this numerical experiment due to R's non-economic fairness concerns.
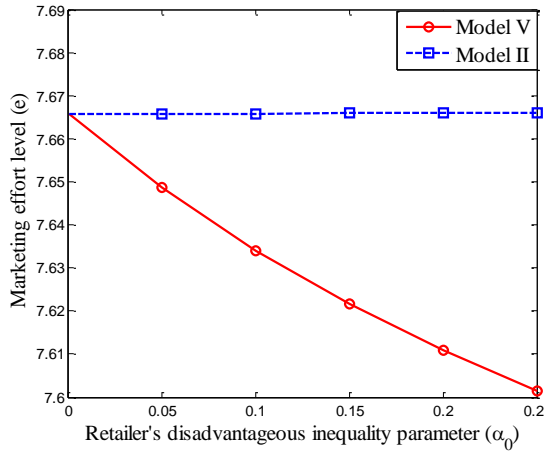


**Figure 1**. The effect of parameter $\alpha_0$ on the optimal marketing effort
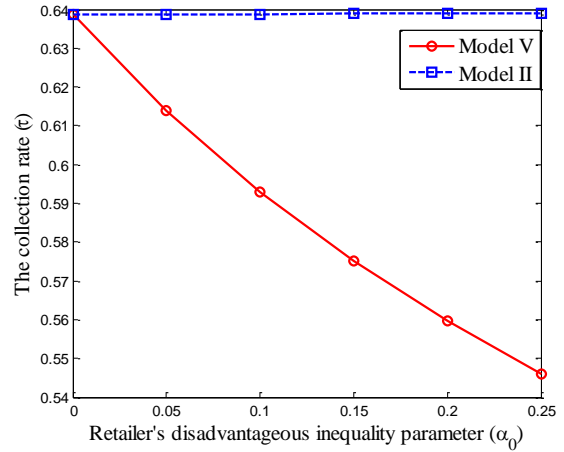


**Figure 2**. The effect of parameter $\alpha_0$ on the optimal collection rate



**Figure 3**. The effect of parameter $\alpha_0$ on individual and channel profits in Model V

Next, we shall consider another extension to examine the impact of different collection efficiencies when M, R, or T serves as the collector of used products.

## 5.2 Three Decentralized Collection Models with Heterogeneous Collection Costs

In this extension, we investigate the case when M, R, and T with different recycle costs, respectively, serve as the collector, where M as the collector is taken as the benchmark with a variable collection cost parameter $A$. As R is closer to the consumer and T tends to be more professional in collecting used products, a parameter $0 \le \lambda \le 1$ is introduced to gauge potential cost advantages with a variable collection cost of $\lambda A$ if R or T serves as the collector.

### 5.2.1 R Collection Model with $\lambda A$ as the Variable Collection Cost Parameter

We can derive the relevant equilibrium values as follows:

$$w^{III**} = \frac{1}{2k\left(-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta\right)}\{A^2\eta k^3\lambda^2 c_m + A^2 a\eta k^2\lambda^2 - 2Ab\eta k^3\lambda c_m$$

$$-2A\Delta a\eta k^2\lambda + b^2\eta k^3 c_m + 2\Delta ab\eta k^2 - ab^2\eta k^2 - 4\eta k^2 C_L c_m + 2\gamma^2 kC_L c_m - 4a\eta kC_L + 2a\gamma^2 C_L\},$$

$$p^{III**} = \frac{A^2 a\eta k^2\lambda^2 - A\Delta a\eta k^2\lambda - Aab\eta k^2\lambda + \Delta ab\eta k^2 - \eta k^2 C_L c_m + \gamma^2 kC_L c_m - 3a\eta kC_L + a\gamma^2 C_L}{k\left(-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta\right)},$$

$$e^{III**} = \frac{GC_L\gamma}{2C_L F - (\Delta - \lambda A)(b - \lambda A)k^2\eta}, D^{III**} = \frac{G\eta kC_L}{2C_L F - (\Delta - \lambda A)(b - \lambda A)k^2\eta},$$

$$\tau^{III**} = \frac{1}{2}\frac{G(A\lambda - b)k\eta}{-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta}, \quad \Pi_M^{III**} = \frac{1}{2}\cdot\frac{G^2\eta C_L}{2C_L F - (\Delta - \lambda A)(b - \lambda A)k^2\eta},$$

$$\Pi_R^{III**} = -\frac{G\eta C_L}{4\left(-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta\right)^2}\{-A^2\eta k^3\lambda^2 c_m + A^2 a\eta k^2\lambda^2 + 2Ab\eta k^3\lambda c_m$$

$$-2Aab\eta k^2\lambda - b^2\eta k^3 c_m + ab^2\eta k^2 + 4\eta k^2 C_L c_m - 2\gamma^2 kC_L c_m - 4a\eta kC_L + 2a\gamma^2 C_L\},$$

$$\Pi_{SC}^{IV**} = -\frac{G\eta C_L}{4\left(-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta\right)^2}\{-3A^2\eta k^3\lambda^2 c_m + 3A^2 a\eta k^2\lambda^2 + 2A\Delta\eta k^3\lambda c_m + 4Ab\eta k^3\lambda c_m - 2A\Delta a\eta k^2\lambda$$

$$-4Aab\eta k^2\lambda - 2\Delta b\eta k^3 c_m - b^2\eta k^3 c_m + 2\Delta ab\eta k^2 + ab^2\eta k^2 + 12\eta k^2 C_L c_m - 6\gamma^2 kC_L c_m - 12a\eta kC_L + 6a\gamma^2 C_L\}.$$

After taking the first-order derivative of $\Pi_M^{III**}$ with respect to $b$, we derive that

$$\frac{\partial \Pi_M^{III**}}{\partial b} = \frac{G^2\eta^2 k^2 C_L(\Delta - A\lambda)}{2\left(-2C_L F + (\Delta - \lambda A)(b - \lambda A)k^2\eta\right)^2} > 0 \quad .$$

Because the manufacturer's profit increases in $b$, the optimal transfer price $b$ should be set at $b=\Delta$ under the R collection case, which is identical to Observation 1.

### 5.2.2 T Collection Model with $\lambda A$ as the Variable Collection Cost Parameter

Similarly, we derive the equilibrium values for this extension model as follows:

$$w^{IV**} = \frac{A\Delta a\eta k^2\lambda - Aab\eta k^2\lambda - \Delta ab\eta k^2 + ab^2\eta k^2 + 2\eta k^2 C_L c_m - \gamma^2 kC_L c_m + 2a\eta kC_L - a\gamma^2 C_L}{k\left(2C_L F - k^2\eta(b - \lambda A)(\Delta - b)\right)},$$

$$p^{IV**} = \frac{A\Delta a\eta k^2\lambda - Aab\eta k^2\lambda - \Delta ab\eta k^2 + ab^2\eta k^2 + \eta k^2 C_L c_m - \gamma^2 k C_L c_m + 3a\eta k C_L - a\gamma^2 C_L}{k\left(2C_L F - k^2\eta(b-\lambda A)(\Delta-b)\right)},$$

$$e^{IV**} = \frac{GC_L\gamma}{2C_L F - k^2\eta(b-\lambda A)(\Delta-b)}, \quad D^{IV**} = \frac{\eta k C_L G}{2C_L F - k^2\eta(b-\lambda A)(\Delta-b)},$$

$$\tau^{IV**} = -\frac{1}{2}\frac{G(A\lambda-b)\eta k}{2C_L F - k^2\eta(b-\lambda A)(\Delta-b)}, \quad \Pi_M^{IV**} = \frac{1}{2}\frac{G^2\eta C_L}{2C_L F - k^2\eta(b-\lambda A)(\Delta-b)},$$

$$\Pi_R^{IV**} = \frac{1}{2}\frac{\eta C_L^2 G^2 F}{\left(2C_L F - k^2\eta(b-\lambda A)(\Delta-b)\right)^2}, \quad \Pi_{3P}^{IV**} = \frac{1}{4}\frac{C_L G^2(A\lambda-b)^2\eta^2 k^2}{\left(2C_L F - k^2\eta(b-\lambda A)(\Delta-b)\right)^2},$$

$$\Pi_{SC}^{IV**} = \frac{G^2\eta C_L\left(A^2\eta k^2\lambda^2 + 2A\Delta\eta k^2\lambda - 4Ab\eta k^2\lambda - 2\Delta b\eta k^2 + 3b^2\eta k^2 + 12\eta k C_L - 6\gamma^2 C_L\right)}{4\left(2C_L F - k^2\eta(b-\lambda A)(\Delta-b)\right)^2}.$$

After taking the first-order derivative of $\Pi_M^{IV**}$ with respect to $b$, we derive that

$$\frac{\partial\Pi_M^{IV**}}{\partial b} = -\frac{1}{2}\frac{G^2 C_L k^2\eta^2\left(-A\lambda - \Delta + 2b\right)}{\left(2C_L F - k^2\eta(b-\lambda A)(\Delta-b)\right)^2} \quad .$$

Solving the equation $\partial\Pi_M^{IV**}/\partial b = 0$ results in $b = \frac{1}{2}A\lambda + \frac{1}{2}\Delta$. This result is consistent with Observation 2 with $A$ being replaced with $\lambda A$.

After substituting $b = \frac{1}{2}A\lambda + \frac{1}{2}\Delta$ into $\Pi_{SC}^{IV**}$, we can derive that

$$\Pi_{SC}^{IV**} = \frac{G^2\eta C_L\left(24C_L F - k^2\eta(\Delta-\lambda A)^2\right)}{\left(8C_L F - k^2\eta(\Delta-\lambda A)^2\right)^2}.$$

Taking the first-order derivative of $\Pi_{SC}^{IV**}$ with respect to $\lambda$, we can derive that

$$\frac{\partial\Pi_{SC}^{IV**}}{\partial\lambda} = \frac{2G^2\eta^2 C_L A k^2(\Delta-A\lambda)\left(40C_L F - (\lambda A-\Delta)^2\eta k^2\right)}{\left(8C_L F - k^2\eta(\Delta-\lambda A)^2\right)^3} < 0.$$

This result shows that the supply chain's profit decrease in $\lambda$.

## 5.2.3 Impact of $\lambda$ on the Supply Chain Profitability for the Three Decentralized Models

Now, by setting $a=360$, $A=10$, $C_L=300$, $\eta=4$, $k=5$, $\Delta=15$, $\gamma=2$, and $c_m=2$, we examine the impact of $\lambda$ on the supply chain profitability. If there does not exist any cost advantage for R or T to collect used products, i.e., $\lambda = 1$, Fig. 4 confirms the conclusion in Proposition 7(1), $\Pi_{SC}^{IV*} < \Pi_{SC}^{II*} < \Pi_{SC}^{III*}$. On the other hand, when the cost advantage of R and T collection models increases (i.e., when $\lambda$ decreases), the supply chain profitability for the R collection model (model III) and T collection model (model IV) increases. When $\lambda$ is sufficiently small ($\lambda$ is less than about 0.8 as shown in Figure 4), the supply chain profitability for the T collection

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model (model IV) surpasses that for the M collection model (model II). Fig. 4 also demonstrates that, in terms of supply chain profitability enhancement, the R collection model (model III) is more sensitive to the cost advantage parameter $\lambda$ than the T collection model (model IV).
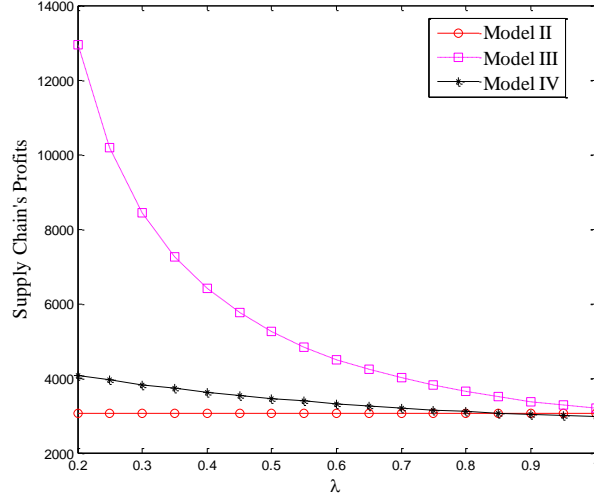


**Figure 4**. Comparison among the Models II, III and IV

## 6 Conclusions

CLSC studies have received increasing attention in recent years. We address the optimal pricing strategies in CLSCs where market demand is influenced by R's marketing effort. In the centralized case, a central planner is responsible for all decisions. In the three decentralized cases where M, R and T are, respectively, modeled as the collector. In terms of decisions, M sets a wholesale price, R takes care of the marketing effort and retail price, and the collector determines the collection rate. Game theoretic models are established to characterize the interactions among the supply chain members. In the decentralized cases where M is not the collector, M is modeled as the Stackelberg leader and R and T are treated as followers.

We derive the optimal pricing strategies for the CLSC members under different collection structures. We then compare the collection rate and marketing effort (Propositions 5 and 6) in the four models, and find that the centralized model achieves the highest collection and marketing efforts, followed by R collection and, then, M collection, and lastly T collection model. We also compare the channel profits in the four models (I, II, III, IV) as well as the profits of R and M in the decentralized cases (models II, III and IV).

We next examine the influence of the demand coefficient ($\gamma$) on optimal decisions and profits of the CLSC in different cases. We find that the collection rate and marketing effort as

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well as individual and channel profits all increase when market demand becomes more sensitive to the marketing effort (a higher $\gamma$) (Proposition 8). Our further study on the impact of the unit transfer price $b$ reveals that the collection rate, marketing effort, and individual and channel profits all increase in $b$ when R serves as the collector (Proposition 9(1)). When T serves as the collector in model IV, its collection rate always increases in $b$. The marketing effort and M's profit increase in the unit transfer price $b$ when it is within a reasonable range, but decrease in $b$ when it is excessive.

Subsequently, we extend model II to consider a fairness-concerned R. On the one hand, the optimal collection rate, marketing effort, and M's profit decrease in R's disadvantageous inequality parameter $\alpha_0$. On the other hand, R's profit increases in $\alpha_0$ under certain conditions, helping to close in the gap between the two supply chain members' profits when R is a stronger proponent of fair distributions (a larger $\alpha_0$). We also extend the base models to accommodate potential collection cost advantages by R and T compared to the M collection model. We find that: (1) The supply chain always attains the highest profit under the R collection model and the T collection model achieves a higher profit than the M collection model when the collection cost advantage is large enough (or a small enough $\lambda$); (2) In terms of supply chain profitability enhancement, the R collection model (model III) is more sensitive to the cost advantage parameter $\lambda$ than the T collection model (model IV).

One worthy future research opportunity is to consider cost sharing of marketing effort and collection rate between M and R/T. As illustrated in the numerical studies, R's fairness concerns lead to a channel efficiency loss. So, another possible direction is to examine the contract design issue in coordinating the CLSC with a fairness-concerned R.

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## Appendix A. Proofs in Sections 3, 4 and 5

**Proof of Lemma 1.** The Hessian matrix of $\Pi_{SC}$ is

$$H(p,e,\tau) = \begin{bmatrix} \dfrac{\partial^2 \Pi_{SC}}{\partial p^2} & \dfrac{\partial^2 \Pi_{SC}}{\partial p \partial e} & \dfrac{\partial^2 \Pi_{SC}}{\partial p \partial \tau} \\ \dfrac{\partial^2 \Pi_{SC}}{\partial e \partial p} & \dfrac{\partial^2 \Pi_{SC}}{\partial e^2} & \dfrac{\partial^2 \Pi_{SC}}{\partial e \partial \tau} \\ \dfrac{\partial^2 \Pi_{SC}}{\partial \tau \partial p} & \dfrac{\partial^2 \Pi_{SC}}{\partial \tau \partial e} & \dfrac{\partial^2 \Pi_{SC}}{\partial \tau^2} \end{bmatrix} = \begin{bmatrix} -2k & \gamma & k(A-\Delta) \\ \gamma & -\eta & \gamma(\Delta-A) \\ k(A-\Delta) & \gamma(\Delta-A) & -2C_L \end{bmatrix}.$$

It is apparent $\dfrac{\partial^2 \Pi_C}{\partial p^2} = -2k < 0$. As $\left| H_2(p,e,\tau) \right| = \begin{vmatrix} \dfrac{\partial^2 \Pi_C}{\partial p^2} & \dfrac{\partial^2 \Pi_C}{\partial p \partial e} \\ \dfrac{\partial^2 \Pi_C}{\partial e \partial p} & \dfrac{\partial^2 \Pi_C}{\partial e^2} \end{vmatrix} = F$, if $F > 0$, we

have $\left| H_2(p,e,\tau) \right| > 0$. Similarly,

$$\left| H_3(p,e,\tau) \right| = \begin{vmatrix} \dfrac{\partial^2 \Pi_C}{\partial p^2} & \dfrac{\partial^2 \Pi_C}{\partial p \partial e} & \dfrac{\partial^2 \Pi_C}{\partial p \partial \tau} \\ \dfrac{\partial^2 \Pi_C}{\partial e \partial p} & \dfrac{\partial^2 \Pi_C}{\partial e^2} & \dfrac{\partial^2 \Pi_C}{\partial e \partial \tau} \\ \dfrac{\partial^2 \Pi_C}{\partial \tau \partial p} & \dfrac{\partial^2 \Pi_C}{\partial \tau \partial e} & \dfrac{\partial^2 \Pi_C}{\partial \tau^2} \end{vmatrix} = -2C_L F + k^2 \eta J^2$$

If $-2C_L F + k^2 \eta J^2 < 0$, we have $\left| H_3(p,e,\tau) \right| < 0$. ∎

**Proof of Observation 1.** After taking the first-order derivative of $\Pi_M^{III*}$ with respect to $b$, we can derive that

$$\frac{\partial \Pi_M^{III*}}{\partial b} = \frac{G^2 \eta^2 k^2 C_L J}{2\left(2C_L F - k^2 \eta J(b-A)\right)^2} > 0.$$

As $\Pi_M^{III*}$ increases in $b$, the optimal transfer price $b$ should be set at its upper bound $\Delta$. ∎

**Proof of Observation 2.** Taking the first-order derivative of $\Pi_M^{IV*}$ with respect to $b$, we have

$$\frac{\partial \Pi_M^{IV*}}{\partial b} = -\frac{C_L k^2 \eta^2 G^2 \left(-A - \Delta + 2b\right)}{2\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^2}.$$ Solving the equation $\partial \Pi_M^{IV*} / \partial b = 0$ results in

its root, i.e., $b^* = \frac{1}{2}A + \frac{1}{2}\Delta$. ∎

**Proof of Proposition 5.** (1) We can show that:

$$\tau^{II*} - \tau^{I*} = \frac{-2C_L F \eta k JG}{\left(4C_L F - k^2 \eta J^2\right)\left(2C_L F - k^2 \eta J^2\right)} < 0, \text{ i.e., } \tau^{II*} < \tau^{I*};$$

(2) If $b=\Delta$, we can find that:

$$\tau^{III*} - \tau^{I*} = -\frac{Jk\eta G}{2\left(2C_L F - k^2 \eta J^2\right)} < 0, \text{ thus } \tau^{III*} < \tau^{I*};$$

(3) If $b = \Delta$, we can prove that

$$\tau^{III*} - \tau^{II*} = \frac{k^2 \eta J^3 \eta k G}{2\left(2C_L F - k^2 \eta J^2\right)\left(4C_L F - k^2 \eta J^2\right)} > 0, \text{ i.e., } \tau^{III*} > \tau^{II*}.$$

(4) Now, we compare the values $\tau^{*IV}$ with $\tau^{*III}$ and $\tau^{*II}$ as follows,

$$\tau^{IV*} - \tau^{III*} = \frac{-k^2 \eta (b-A)^3 G\eta k}{2\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)\left(2C_L F - k^2 \eta (\Delta-A)(b-A)\right)} < 0.$$

Thus, we have $\tau^{IV*} < \tau^{III*}$.

If $b=(A+\Delta)/2$, we can prove that

$$\tau^{IV*} - \tau^{II*} = \frac{-2C_L F J \eta k G}{2\left(2C_L F - \frac{1}{4}k^2 \eta J^2\right)\left(4C_L F - k^2 \eta J^2\right)} < 0. \text{ Thus, we have } \tau^{IV*} < \tau^{II*}.$$

After combining the above results, we complete the proof of Proposition 5. ∎

**Proof of Proposition 6.** In order to compare the optimal marketing effort level in model III with that in model I, we can find that

$$e^{III*} - e^{I*} = \frac{-GC_L\gamma}{2C_LF - k^2\eta J^2} < 0 \text{ if } b=\Delta, \text{ thus } e^{III*} < e^{I*};$$

Comparing the optimal marketing effort level in model III with that in model II, we have

$$e^{III*} - e^{II*} = \frac{k^2\eta J^2 GC_L\gamma}{\left(2C_LF - k^2\eta J^2\right)\left(4C_LF - k^2\eta J^2\right)} > 0 \text{ if } b=\Delta, \text{ thus } e^{III*} > e^{II*}.$$

Now, we compare the values $e^{IV*}$ with $e^{II*}$ and find that:

$$e^{IV*} - e^{II*} = -\frac{k^2\eta GC_L\gamma J^2}{2\left(2C_LF - \frac{1}{4}k^2\eta J^2\right)\left(4C_LF - k^2\eta J^2\right)} < 0 \text{ if } b=(A+\Delta)/2, \text{ thus we have}$$

$$e^{IV*} < e^{II*}.$$

After combining the above results, we complete the proof of Proposition 6.∎

**Proof of Proposition 7.** We can prove that

$$\Pi_{SC}^{I*} - \Pi_{SC}^{II*} = \frac{4C_L^2F^2\eta C_LG^2}{\left(2C_LF - k^2\eta J^2\right)\left(4C_LF - k^2\eta J^2\right)^2} > 0;$$

If $b=\Delta$, we have

$$\Pi_{SC}^{II*} - \Pi_{SC}^{III*} = = \frac{-k^2\eta^2 C_L J^2\left(8C_LF - k^2\eta J^2\right)G^2}{4\left(4C_LF - k^2\eta J^2\right)^2\left(2C_LF - k^2\eta J^2\right)} < 0.$$

If $b=\Delta$, we can prove that

$$\Pi_{SC}^{I*} - \Pi_{SC}^{III*} = \frac{\eta C_LG^2}{2\left(2C_LF - k^2\eta J^2\right)} > 0, \text{ i.e., } \Pi_{SC}^{I*} > \Pi_{SC}^{III*}.$$

If $b=\Delta$, we have

$$\Pi_M^{II*} - \Pi_M^{III*} = \frac{-k^2\eta J^2\eta C_LG^2}{2\left(4C_LF - k^2\eta J^2\right)\left(2C_LF - k^2\eta J^2\right)} < 0,$$

$$\Pi_R^{II*} - \Pi_R^{III*} = \frac{-k^4\eta^2 J^4\eta C_LG^2}{4\left(4C_LF - k^2\eta J^2\right)^2\left(2C_LF - k^2\eta J^2\right)} < 0.$$

Therefore, we have $\Pi_M^{II*} < \Pi_M^{III*}$ and $\Pi_R^{II*} < \Pi_R^{III*}$.

If $b=(A+\Delta)/2$, we have $\Pi_{SC}^{II*} - \Pi_{SC}^{IV*} = \dfrac{\left(12C_L F - 5J^2 k^2 \eta\right) J^2 k^2 \eta^2 F C_L^2 G^2}{8[2C_L F - k^2 \eta \cdot \frac{J^2}{4}]^2 \left(4C_L F - k^2 \eta J^2\right)^2} > 0$,

$$\Pi_R^{II*} - \Pi_R^{IV*} = \frac{[8C_L F - \frac{3}{2}k^2 \eta J^2]G^2 \eta C_L^2 F}{2\left(4C_L F - k^2 \eta J^2\right)\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^2} > 0,$$

$$\Pi_M^{II*} - \Pi_M^{IV*} = \frac{J^2 k^2 \eta^2 C_L G^2}{4\left(4C_L F - k^2 \eta J^2\right)\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)} > 0.$$

After combining the above results, we complete the proof of Proposition 7.■

**Proof of Proposition 8.** (1) Taking the first-order derivative of $\tau$ and $e$ with respect to $\gamma$ for models I, II, III and IV leads to

$$\frac{\partial \tau^{I*}}{\partial \gamma} = \frac{4Jk\eta G\gamma C_L}{\left(A^2 \eta k^2 - 2A\Delta \eta k^2 + \Delta^2 \eta k^2 - 4\eta k C_L + 2\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial \tau^{II*}}{\partial \gamma} = \frac{8\eta k JG\gamma C_L}{\left(A^2 \eta k^2 - 2A\Delta \eta k^2 + \Delta^2 \eta k^2 - 8\eta k C_L + 4\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial \tau^{III*}}{\partial \gamma} = \frac{2G(b-A)k\eta\gamma C_L}{\left(A^2 \eta k^2 - A\Delta \eta k^2 - Ab\eta k^2 + \Delta b\eta k^2 - 4\eta k C_L + 2\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial \tau^{IV*}}{\partial \gamma} = \frac{2G(b-A)k\eta\gamma C_L}{\left(A\Delta \eta k^2 - Ab\eta k^2 - \Delta b\eta k^2 + b^2 \eta k^2 + 4\eta k C_L - 2\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial e^{I*}}{\partial \gamma} = \frac{8\gamma C_L^2 G\gamma}{\left(A^2 \eta k^2 - 2A\Delta \eta k^2 + \Delta^2 \eta k^2 - 4\eta k C_L + 2\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial e^{II*}}{\partial \gamma} = \frac{16G C_L^2 \gamma^2}{\left(A^2 \eta k^2 - 2A\Delta \eta k^2 + \Delta^2 \eta k^2 - 8\eta k C_L + 4\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial e^{III*}}{\partial \gamma} = \frac{G C_L \left(2C_L \left(2k\eta + \gamma^2\right) - k^2 \eta J(b-A)\right)}{\left(2C_L F - k^2 \eta J(b-A)\right)^2} > 0;$$

$$\frac{\partial e^{IV*}}{\partial \gamma} = \frac{G C_L \eta k \left(4C_L - k(b-A)(\Delta-b)\right)}{\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^2} > 0.$$

(ii) Taking the first derivative of $\Pi_{SC}^{I*}$, $\Pi_{SC}^{II*}$, $\Pi_{SC}^{III*}$ and $\Pi_{SC}^{IV*}$ with respect to $\gamma$ results in

$$\frac{\partial \Pi_{SC}^{I*}}{\partial \gamma} = \frac{4\eta C_L^2 G^2 \gamma}{\left(A^2 \eta k^2 - 2A\Delta \eta k^2 + \Delta^2 \eta k^2 - 4\eta k C_L + 2\gamma^2 C_L\right)^2} > 0;$$

$$\frac{\partial \Pi_{SC}^{II*}}{\partial \gamma} = \frac{4\gamma C_L^2 \eta G^2 \left( A^2 \eta k^2 - 2A\Delta\eta k^2 + \Delta^2 \eta k^2 - 24\eta k C_L + 12\gamma^2 C_L \right)}{\left( A^2 \eta k^2 - 2A\Delta\eta k^2 + \Delta^2 \eta k^2 - 8\eta k C_L + 4\gamma^2 C_L \right)^3} > 0;$$

If $b=\Delta$, we have

$$\frac{\partial \Pi_{SC}^{III*}}{\partial \gamma} = \frac{3G^2 \eta C_L^2 \gamma}{\left( 2C_L F - k^2 \eta J^2 \right)^2} > 0;$$

If $b=(A+\Delta)/2$, we have $\quad \dfrac{\partial \Pi_{SC}^{IV*}}{\partial \gamma} = \dfrac{C_L^2 \eta G^2 \gamma [6C_L F + k^2 \eta . \dfrac{J^2}{4}]}{[2C_L F - k^2 \eta . \dfrac{J^2}{4}]^3} > 0$. Combining the above

results proves Proposition 8.∎

**Proof of Proposition 9.**

(1) Recall that $F > 0$, $A < \Delta$, $b \leq \Delta$ and the optimal values of $\tau$, $e$, $\Pi_R$ and $\Pi_M$ in

four models are positive. We can prove that

$$\frac{\partial \tau^{III*}}{\partial b} = \frac{Gk\eta C_L F}{\left( 2C_L F - k^2 \eta J (b-A) \right)^2} > 0; \frac{\partial e^{III*}}{\partial b} = \frac{GC_L \gamma \eta k^2 J}{\left( 2C_L F - k^2 \eta J (b-A) \right)^2} > 0;$$

$$\frac{\partial \Pi_R^{III*}}{\partial b} = -\frac{\eta^2 C_L^2 G^2 k^2 (b-\Delta) F}{\left( 2C_L F - k^2 \eta J (b-A) \right)^3} > 0.$$

After taking the first-order derivative of $\Pi_M^{III*}$ and $\Pi_{SC}^{III*}$ with respect to $b$, we can

derive that

$$\frac{\partial \Pi_M^{III*}}{\partial b} = \frac{\eta^2 k^2 C_L G^2 J}{2\left( 2C_L F - k^2 \eta J (b-A) \right)^2} > 0. \text{ From } \frac{\partial \Pi_R^{III*}}{\partial b} > 0 \text{ and } \frac{\partial \Pi_M^{III*}}{\partial b} > 0, \text{ we}$$

have $\dfrac{\partial \Pi_{SC}^{III*}}{\partial b} > 0$.

(2) We can confirm that

$$\frac{\partial \tau^{IV*}}{\partial b} = \frac{G\eta k \left( 2C_L F - k^2 \eta (b-A)^2 \right)}{2\left( 2C_L F - k^2 \eta (b-A)(\Delta-b) \right)^2} > 0 \quad \text{for all } b.$$

(3) Taking the first-order derivative of $e^{IV*}$ and $\Pi_M^{IV*}$ with respect to $b$, we have

$$\frac{\partial e^{IV*}}{\partial b} = -\frac{GC_L \gamma \eta k^2 (2b - A - \Delta)}{\left( 2C_L F - k^2 \eta (b-A)(\Delta-b) \right)^2}.$$

From Observation 2,

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$$\frac{\partial \Pi_M^{IV*}}{\partial b} = -\frac{C_L k^2 \eta^2 G^2 (2b - A - \Delta)}{2(2C_L F - k^2 \eta (b-A)(\Delta-b))^2}$$ . Solving $\frac{\partial e^{IV*}}{\partial b} = 0$ and $\frac{\partial \Pi_M^{IV*}}{\partial b} = 0$, we

can derive that $b=(A+\Delta)/2$, thus we get $\frac{\partial e^{IV*}}{\partial b} > 0$ and $\frac{\partial \Pi_M^{IV*}}{\partial b} > 0$ if $0 \le b \le \frac{A+\Delta}{2}$, otherwise

$\frac{\partial e^{IV*}}{\partial b} < 0$ and $\frac{\partial \Pi_M^{IV*}}{\partial b} < 0$ if $\frac{A+\Delta}{2} < b \le \Delta$. Combining the above results completes the proof

of Proposition 9.∎

**Proof of Proposition 10.** Recall that $F > 0$, $A < \Delta$, $b \le \Delta$ and the optimal values of $\tau$, $e$,

$\Pi_R$ and $\Pi_M$ in four models are positve. Taking the first derivative of $\tau^{I*}$, $\tau^{II*}$, $\tau^{III*}$ and

$\tau^{IV*}$ with respect to $\eta$, we can obtain that

$$\frac{\partial \tau^{I*}}{\partial \eta} = -\frac{2JkG\gamma^2 C_L}{(2C_L F - k^2 \eta J^2)^2} < 0 \; ; \quad \frac{\partial \tau^{II*}}{\partial \eta} = -\frac{4kJG\gamma^2 C_L}{(4C_L F - k^2 \eta J^2)^2} < 0 \; ;$$

$$\frac{\partial \tau^{III*}}{\partial \eta} = \frac{G(-b+A)k\gamma^2 C_L}{(A^2 \eta k^2 - A\Delta \eta k^2 - Ab\eta k^2 + \Delta b\eta k^2 - 4\eta k C_L + 2\gamma^2 C_L)^2} < 0 \; ;$$

$$\frac{\partial \tau^{IV*}}{\partial \eta} = -\frac{G(b-A)k\gamma^2 C_L}{(2C_L F - k^2 \eta (b-A)(\Delta-b))^2} < 0 \; .$$

(2) Taking the first derivative of $e^{I*}$, $e^{II*}$, $e^{III*}$ and $e^{IV*}$, we can obtain that

$$\frac{\partial e^{I*}}{\partial \eta} = \frac{2\gamma C_L (k^2 J^2 - 4kC_L)G}{(2C_L F - k^2 \eta J^2)^2} < 0 \; ; \quad \frac{\partial e^{II*}}{\partial \eta} = \frac{2C_L \gamma (k^2 J^2 - 8kC_L)G}{(4C_L F - k^2 \eta J^2)^2} < 0 \; ;$$

$$\frac{\partial e^{III*}}{\partial \eta} = \frac{GC_L \gamma (k^2 J(b-A) - 4kC_L)}{(2C_L F - k^2 \eta J(b-A))^2} < 0 \; ; \quad \frac{\partial e^{IV*}}{\partial \eta} = -\frac{GC_L \gamma (4kC_L - k^2 (b-A)(\Delta-b))}{(2C_L F - k^2 \eta (b-A)(\Delta-b))^2} < 0 \; .$$

(3) Taking the first derivative of $\Pi_{SC}^{I*}$, $\Pi_R^{II*}$, $\Pi_M^{II*}$, $\Pi_R^{III*}$, $\Pi_M^{III*}$, $\Pi_{SC}^{III*}$, $\Pi_R^{IV*}$, $\Pi_M^{IV*}$,

$\Pi_{3P}^{IV*}$ and $\Pi_{SC}^{IV*}$, we can obtain:

$$\frac{\partial \Pi_{SC}^{I*}}{\partial \eta} = -\frac{2C_L^2 G^2 \gamma^2}{(2C_L F - k^2 \eta J^2)^2} < 0 \; ;$$

$$\frac{\partial \Pi_R^{II*}}{\partial \eta} = -\frac{2C_L^2 \gamma^2 (4C_L F + k^2 \eta J^2)G^2}{(4C_L F - k^2 \eta J^2)^3} < 0 \; ;$$

$$\frac{\partial \Pi_M^{II*}}{\partial \eta} = -\frac{4C_L^2 \gamma^2 G^2}{[4C_L F - k^2 \eta J^2]^2} < 0 \; ;$$

$$\frac{\partial \Pi_M^{III*}}{\partial \eta} = -\frac{G^2 C_L^2 \gamma^2}{(A^2 \eta k^2 - A\Delta \eta k^2 - Ab\eta k^2 + \Delta b\eta k^2 - 4\eta k C_L + 2\gamma^2 C_L)^2} < 0 \; ;$$

If $b=\Delta$, we have

$$\frac{\partial \Pi_R^{III*}}{\partial \eta} = -\frac{C_L^2 G^2 \gamma^2}{2\left(A^2 \eta k^2 - 2A\Delta\eta k^2 + \Delta^2 \eta k^2 - 4\eta k C_L + 2\gamma^2 C_L\right)^2} < 0 \quad , \qquad \text{thus} \qquad \text{we} \qquad \text{have}$$

$$\frac{\partial \Pi_{SC}^{III*}}{\partial \eta} = \frac{\partial \Pi_R^{III*}}{\partial \eta} + \frac{\partial \Pi_M^{III*}}{\partial \eta} < 0;$$

$$\frac{\partial \Pi_R^{IV*}}{\partial \eta} = -\frac{G^2 C_L^2 \gamma^2 \left(2C_L F + k^2 \eta (b-A)(\Delta-b)\right)}{2\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^3} < 0,$$

$$\frac{\partial \Pi_M^{IV*}}{\partial \eta} = -\frac{G^2 C_L^2 \gamma^2}{\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^2} < 0,$$

$$\frac{\partial \Pi_{3P}^{IV*}}{\partial \eta} = -\frac{C_L^2 G^2 (b-A)^2 \eta k^2 \gamma^2}{\left(2C_L F - k^2 \eta (b-A)(\Delta-b)\right)^3} < 0 \quad , \qquad \text{thus} \qquad \text{we} \qquad \text{have}$$

$$\frac{\partial \Pi_{SC}^{IV*}}{\partial \eta} = \frac{\partial \Pi_R^{IV*}}{\partial \eta} + \frac{\partial \Pi_M^{IV*}}{\partial \eta} + \frac{\partial \Pi_{3P}^{IV*}}{\partial \eta} < 0.$$ Combining the above results completes the proof of

Proposition 10.∎

**Proof of Proposition 12.** Recall that $F > 0$, $A < \Delta$ and the optimal values of $\tau$, $e$, $\Pi_R$ and $\Pi_M$ in model V are positive. Taking the first derivative of $\tau^{V*}$, $e^{V*}$ and $\Pi_R^{V*}$ with respect to $\alpha_0$, we can derive:

$$\frac{\partial \tau^{V*}}{\partial \alpha_0} = -\frac{4JG\eta k \mu C_L F}{\left(4C_L (1+\alpha_0 + \mu\alpha_0)F - (1+\alpha_0)\eta k^2 J^2\right)^2} < 0,$$

$$\frac{\partial e^{V*}}{\partial \alpha_0} = -\frac{2\gamma G C_L \eta k^2 \mu J^2}{\left(4C_L (1+\alpha_0 + \mu\alpha_0)F - (1+\alpha_0)\eta k^2 J^2\right)^2} < 0,$$

$$\frac{\partial \Pi_R^{V*}}{\partial \alpha_0} = -\frac{4C_L^2 \eta FG^2 \mu \left((3\mu\alpha_0 + 2 + 2\alpha_0)\eta k^2 J^2 - 4C_L (1+\alpha_0 + \mu\alpha_0)F\right)}{\left(4C_L (1+\alpha_0 + \mu\alpha_0)F - (1+\alpha_0)\eta k^2 J^2\right)^3}.$$

Let $H(\alpha_0) = 4C_L (1+\alpha_0 + \mu\alpha_0)F - (3\mu\alpha_0 + 2 + 2\alpha_0)\eta k^2 J^2$ , then we can derive that $H(0) = 4C_L F - 2k^2 \eta J^2 > 0$ from Lemma 1 and $H'(\alpha_0) = 4C_L (1+\mu)F - (3\mu+2)\eta k^2 J^2$ . Furthermore, let

$G(\mu) = 4C_L (1+\mu)F - (3\mu+2)\eta k^2 J^2$ , we have $G(0) = 4C_L F - 2k^2 \eta J^2 > 0$ from Lemma 1 and $G'(\mu) = 4C_L F - 3k^2 \eta J^2$. Thus, if $4C_L F - 3k^2 \eta J^2 > 0$, we have $G(\mu) > 0$ for all $\mu$, and then we have $H'(\alpha_0) > 0$. Thus, we have $\frac{\partial \Pi_R^{V*}}{\partial \alpha_0} > 0$ If $4C_L F - 3k^2 \eta J^2 > 0$.

Taking the first derivative of $\Pi_M^{V*}$ with respect to $\alpha_0$, we can derive:

$$\frac{\partial \Pi_M^{V*}}{\partial \alpha_0} = -\frac{4\eta C_L^2 G^2 \mu F}{\left(4C_L\left(\mu\alpha_0 + \alpha_0 + 1\right)F - \left(1 + \alpha_0\right)\eta k^2 J^2\right)^2} < 0 \,,$$

Combining the above results completes the proof of Proposition 12. ∎