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PRICING MORTGAGES: AN INTERPRETATION OF THE MODELS AND RESULTS

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ABSTRACT

Mortgages, like all debt securities, can be viewed as risk-free assets plus or minus contingent claims that can be usefully viewed as options. The most important options are: prepayment, which is a call option giving the borrower the right to buy back the mortgage at par, and default, which is a put option giving the borrower the right to sell the house in exchange for the mortgage. This paper reviews and interprets the large and growing body of literature that applies recent results of option pricing models to mortgages. We also provide a critique of the models and suggest directions for future research.

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Pricing Mortgages: An Interpretation of the Models and Results

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Twenty years ago, nearly all mortgages had long-term fixed rates and most were originated and held for investment by heavily regulated housing-finance institutions granted tax preferences for investing their cheap deposit dollars in mortgages. Today, deposits are more expensive, tax preferences are miniscule, and the regulations are eroding; most mortgages are not held by their originators, and many are widely traded; many mortgages do not have fixed rates; most new investment is in mortgage-backed securities (MBSs) rather than whole loans; and MBSs are now being placed in collateralized mortgage obligations (CMOs) or "stripped". The accurate pricing of various mortgage loans, MBSs, and claims on parts of the MBSs has become a major concern.

It is by now widely recognized that debt securities can be viewed as risk-free assets plus various contingent claims, which are frequently modeled as options. This approach is applicable to mortgages (see Findley and Capozza, 1977, for an early discussion). For instance, prepayment is a call option (i.e., an option to buy back or call the mortgage at par), and default is a put option (i.e., an option to sell or put the house to the lender at a price equal to the value of the mortgage). The application of the formal continuous-time stock and bond option-pricing methodology (Black and Scholes, 1973; Brennan and Schwartz, 1977; Cox, Ingersoll, and Ross (CI&R), 1976/1985ab; and Merton, 1973) has been the centerpiece of most mortgage pricing research (early references are Asay, 1978, and Dunn and McConnell, 1981). In recent years, this high-tech literature has grown geometrically (as exemplified by the October 1984 issue of the Housing Finance Review, the Fall 1985 issue of the AREUEA Journal, and a recent stream of unpublished papers by Kau *et al*, 1985 and 1986ab).[1]

This paper interprets the growing body of formal pricing literature and offers suggestions to make the technology more useful in pricing mortgages. The great insight of the models comes from determining equilibrium prices by imposing zero arbitrage profits; such models give exact, rather than simply qualitative, predictions about mortgage prices. The models combine a carefully chosen portfolio of nonmortgage assets (whose risk and cash flows are identical to those of the mortgage contract being modelled) with mortgages to create a synthetic investment that is instantaneously risk free. That the return on the risk-free portfolio equal the instantaneous risk free rate gives the basic equilibrium condition. In simple cases, specific borrower and lender characteristics are irrelevant, and the number of parameters required for pricing is small. When transaction costs are incorporated, specific characteristics often matter, and pricing is more complex, but presumably more accurate.

Our paper has four sections. The first considers a standard fixed-rate, long-term mortgage assuming no default risk. Such a mortgage (which may be viewed as a pool of individual mortgages) is portrayed as a riskless annuity plus the borrower's right to prepay (to buy the mortgage back at the remaining book value). In Section II, the analysis is extended to other instruments: parts of the fixed-rate mortgage's cash flows (tranches of a collateralized mortgage obligation or strips of mortgage pools) and adjustable-rate mortgages with rate caps and floors. Mortgage default is introduced in Section III, first alone (a nonprepayable mortgage) and then in conjunction with the prepayment option. The analysis is then extended to the valuation of mortgage insurance contracts. The relatively few efforts to price mortgages realistically and to test the models against market data are discussed in Section IV, and an overview of future research opportunities is offered.

The formal option pricing methodology requires some technical apparatus, but it is not our major concern. Rather, we focus on intuitive interpretations to underscore the economic logic underlying the analysis and to assess the simulation experiments that have been undertaken because closed-form solutions to the models do not exist. In this area, comparative static analysis must be based on simulation results and intuition.

I. Default-Free Fixed-Rate Mortgages

A default-free assumable fixed-rate mortgage (FRM) can be viewed as a continuous-payment annuity with a call option giving the borrower (seller) the right to repurchase the annuity at a price equal to its par value at any time before maturity (typically 30 years). Under our assumption of no default risk, the value of a FRM depends on its coupon rate, time to maturity, other details of the contract, and the pattern of market interest rates. Pricing models seek to find and evaluate a function that explains observed prices of mortgages: M(R,t), where R is a vector of interest rates and t is time. We begin by describing the basic model and go on to extend it in a number of directions, including introducing transaction costs. We conclude by discussing techniques for solving the models.

Before turning to specific models, we note four points that apply to all models:

(1) The basic equilibrium condition for any mortgage is that its expected instantaneous yield equal the risk-free rate plus an appropriate risk premium. The models that follow derive this equality, but because this notion of equilibrium is so straightforward, we could start by making it an initial assumption.

(2) The equilibrium condition turns out to be a second-order partial differential equation in R and t and other variables (e.g., house prices if default is possible). This sort of equation will apply to <u>any</u> contingent claim. Hence, an infinite number of functions satisfy the equation. To determine the one function that applies to the mortgage being priced, we incorporate boundary conditions specifying the details of the contract such as the coupon rate, the term of the mortgage, and the value of R at which the mortgage will be called (usually determined via an optimal call strategy).

(3) The solution for M(R,t), not surprisingly, has an expected-present-value interpretation. M is the expected present value of the cash-flows (including call and default) from the mortgage, discounted at the risk-free rate, but where the expected value of R, upon which future cash flows (e.g., because of prepayment) depends, is altered by a risk-adjustment factor.

(4) The models determine mortgage price, not yield. Yield is usually measured as the internal rate of return computed for a given assumption about prepayment (e.g., prepayment in 12 years). Because expected prepayment can vary greatly, depending on the mortgage coupon, whether the mortgage is assumable, etc., conventional yield calculations can be very misleading.

A. The Basic Model

Here we discuss a simple frictionless continuous-time model of mortgage pricing, which draws on Dunn and McConnell (1981), Buser and Hendershott (1984), Brennan and Schwartz (1985), and Kau, et al (1986a). We begin by specifying the state variables and the arbitrage condition and then derive the pricing equation.

1. State Variables

A default-free, fixed-rate mortgage is risky for two reasons. First, interest-rate movements change the value of any fixed income security. Second, mortgages can be refinanced when rates fall. This creates an asymmetry in mortgage payoffs: when rates increase, the value of the security falls, as with any fixed-income security, but when rates fall, the rise in value is limited because of borrowers' refinancing opportunities. Thus interest rates are natural exogenous or "state" variables in the model. Because a mortgage can be outstanding for up to 30 years, all interest rates up to 30 years are potential state variables; which is to say M could depend on a large number of variables. The problem can only be managed if a small number of basic interest rates determine the other rates.

Like most authors, Cox, Ingersoll and Ross (CI&R, 1976/85) being the first, we begin by assuming that all interest rates are driven by a single exogenous rate, the instantaneous short rate, r. Changes in this rate are taken to follow an "Ito process," the evolution of which is governed by the following stochostic differential equation:

(1.1) $$dr = u(r,t)dt + \delta(r,t)dz.$$

In (1.1), $u(r,t)dt$ is the expected change in r over an infinitesimal interval of time of length dt, and $\delta(r,t)dz$ is a disturbance made up of dz, which is normally distributed with zero mean and unit variance, and $\delta(r,t)$, which allows r and t to affect the disturbance. Equation (1.1) is a continuous-time version of a standard difference equation. CI&R, Vasicek (1977), Dothan (1978), and Richard (1978) have shown how (1.1) can be used to determine the entire Treasury yield curve.[2]

A particular specification used in several mortgage pricing studies and analyzed in detail by CI&R is

(1.1') $$dr = k(\theta-r)dt + \sigma r^{\frac{1}{2}}dz,$$

where k and σ are positive constants. In this specification, r tends to revert to its mean level, θ, at rate k; the variance of the disturbance decreases less than proportionately as r falls (so that low interest rates are less volatile) and goes to zero as r goes to zero (so that negative interest rates cannot exist).

2. Perfect Markets Arbitrage Model

Pricing comes from arbitrage in complete markets.[3] We begin by constructing a portfolio of Treasury securities whose cash flows exactly mimic those expected on the mortgage. Thus, the combination of the mortgage and a short position in the Treasury portfolio (the hedge portfolio) absorbs zero wealth and has zero instantaneous risk. Dunn and McConnell, 1981, Buser and Hendershott, 1984, and Brennan and Schwartz, 1977 and 1985, among others, show how this portfolio is derived. Absence of arbitrage profits implies a zero instantaneous return. From this zero return, the basic equilibrium condition is deduced: the instantaneous expected yield on the mortgage must equal the risk-free short rate plus a risk factor (see Brennan and Schwartz, 1985, for a general derivation).

In the case of one state variable,

$$u_m M(r,t) = rM + \lambda(r,t)\delta(r,t)M_r, \tag{1.2}$$

where u_m is the expected yield of the mortgage, r the instantaneous risk-free rate, λ the market price of risk, and M_r the partial derivative of M with respect to r. The risk-adjustment term becomes the product of three terms: the "price" of the risk that r changes, λ, the amount of risk, δ, and the interest-rate sensitivity of the mortgage, M_r. If more state variables exist, more risk-adjustment terms, each with its own λ, come into being.

The arbitrage model does not derive the λ's; their derivation is a general equilibrium problem (see CI&R 1976/85b) that requires knowledge of such market forces as traders' risk aversion. The model does imply that the λ's are objective prices which are the same for all traders. Thus the λ's may be viewed as competitive prices for insurance policies.[4] An important pricing issue is how to estimate the λ's. It turns out that the λ's can be inferred from market prices (see Section B) when the state variables are yields on or prices of traded assets.

The perfect-market arbitrage model assumes no transaction costs are incurred either in shorting the portfolio of Treasuries or in continuously adjusting the portfolio as the expected cash flows from the mortgage change. The existence of such costs implies that arbitrage will keep the mortgage price within a range, the width of which depends on the magnitude of the relevant costs. We focus on the perfect-market arbitrage model as a first approximation to pricing.

3. Pricing Equations

The next step is the derivation of u_m, the expected return on the mortgage. This is a technical step that requires some knowledge of Stochastic Calculus. Particularly heavy use is made of Ito's lemma, which is the stochastic analogue of the chain rule of ordinary calculus.[5] The result, which we simply assert, has a fairly straightforward interpretation. The expected instantaneous return consists of the coupon rate and expected percentage capital gains. The coupon rate is simply the coupon payment, C, divided by M. Expected percentage capital gains come from two sources. The first class of gains occurs if t and r change as expected: we call these "certainty equivalent" gains. These are given by M_t/M (amortization and capital gain from selling at a discount) and $u(r,t)M_r/M$ (expected change in r times the sensitivity of value to interest rates). The second source flows from the stochastic nature of r. Because M is, in general, not linear, random increases in r need not have the same effect on M, in absolute value, as random decreases; thus the certainty equivalent approach of assuming that r changes exactly by u will not reflect expected capital gains.

Figure 1 depicts a hypothetical M(r,t) given t. The function is (tentatively) assumed to be convex to the origin. Suppose the current interest rate is r* and $u(r^*,t) = 0$, i.e., r is as likely to go up as it is to go down. Suppose further that next period's rate will be either $r^*+\Delta$ or $r^*-\Delta$, with equal probability. Because of the convex shape of M, the expected level of M at r*, E[M(r*)], will be greater than M at r*, M[E(r*)]; that is, a capital gain is expected, even though r is not expected to change, because interest-rate declines raise M by more than interest-rate increases lower M (the opposite would be the case if M were concave).[6] Accounting for this "extra" capital gain requires using Ito's lemma. Here we simply assert that expected capital gains from the dispersion of r are given by $\frac{1}{2}\delta^2 M_{rr}/M$; that is, they depend on the volatility of r and the shape of M, disappearing if M_r is linear or r is nonstochastic.[7]

Adding the returns from coupons and capital gains, we have

$$u_m M(r,t) = C + M_t + u(r,t)M_r + \tfrac{1}{2}\delta^2(r,t)M_{rr}. \tag{1.3}$$

Substituting from (1.2),

$$C + M_t + [u(r,t) - \lambda(r,t)\delta(r,t)]M_r + \tfrac{1}{2}\delta^2(r,t)M_{rr} = rM. \tag{1.4}$$

This second-order partial differential equation is the basic equilibrium condition for the one state variable model. An infinite number of functions of r and t satisfy this condition (an infinite combination of coupon and capital gains streams provide a "normal" or equilibrium return). Not surprisingly, we need to incorporate details of the contract to obtain a unique function.

4. A Unique Function

Mathematically we need three boundary conditions to find the right M(r,t), one for t and two for r (1.4 is second order in r). The t boundary is the terminal condition that comes from the amortization schedule of the mortgage. For a fully amortizing mortgage,

$$M(r,T) = 0, \tag{1.5}$$

where T is the time at which the last payment is made. The other two conditions relate to how M is valued when r takes on extreme values. The first of these conditions incorporates the economic intuition that M becomes worthless as r approaches infinity:[8]

$$M(\infty,t) = 0. \tag{1.6}$$

The final condition specifies the interest rate at which the mortgage is called.

But before turning to that, we consider the pricing of a benchmark security, a noncallable mortgage $\bar{M}$, that is equivalent to a portfolio of Treasury securities with constant payout for T years. This is easy to price because the value of $\bar{M}$ is just the present value of the known cash flows discounted at the appropriate rate read off the yield curve determined by (1.1). Hence $\bar{M}(r,t)$ looks like AM in Figure 2: i.e., it has the usual downward sloping concave shape of a fixed-income security (see Brennan and Schwartz, 1977, and CI&R, 1976/85, for a fuller discussion).

The curve for the callable mortgage, M(r,t), lies below $\bar{M}(r,t)$ by an amount equal to the value of the call option. Because the mortgage can be called when M equals PAR, we know that points in the region above the "PAR line" cannot be points on M. For the third boundary condition for a callable mortgage, we use a relationship describing the optimal call strategy for a borrower. Because r is the only exogenous variable upon which decisions can be based, the strategy is characterized by the level of r, r_c, at which the mortgage is called. Rational borrowers (ignoring transaction costs) must choose the call strategy that minimizes the value of M. This strategy maximizes their net worth.[9] Of all the M functions satisfying (1.4), (1.5), and 1.6), the rational borrower chooses the function that has the smallest value subject to touching the PAR line in the Figure. The curve that does this (assuming an interior solution) must be tangent to the PAR line, and the level of r at which it touches is the optimal call rate r_c for a given t. Hence, the final boundary condition is:

(1.7) $$M_r(r_c,t) = 0 \quad \text{at } M = PAR,$$

which gives the minimum M(r,t), represented by the BCM curve in Figure 2.

A few observations on this equilibrium model follow:

1. While $\bar{M}$ is convex throughout, M becomes concave as r approaches r_c. Traders call this feature "negative convexity". It reflects the price of the security anticipating the call option even when r is not especially close to r_c.

2. Whereas volatility tends to produce capital gains on average for noncallable mortgages (see Figure 1), negative convexity implies that volatility produces capital losses on average for mortgages close to being called. Hence, volatility makes callable mortgages less valuable when they are near exercise. This is nothing more than a reflection of the proposition that options increase in value as volatility increases.

3. Again, the solution has an expected-present-value interpretation. In particular, M is the expected present value of future cash flows, discounted at r, with the expected value of dr given by $u - \lambda\delta M_r$ rather than just u (see CI&R, 1976/85b, lemma 4).

4. The coupon need not be constant. For example, the model is capable of pricing graduated-payment mortgages, for which C is a rising function of time, and price-level-adjusted mortgages, by putting the analysis into "real" terms.

B. Extensions

1. Adding State Variables

a. Interest Rates

A logical extension of the model is to increase the number of interest rate variables. Taken literally, the one-rate model above implies a constant rate toward which the short rate reverts for all time. Given the obvious importance of changing inflation on interest rates, this seems like a difficult model to take seriously (although this may not be of empirical significance, see section IV below). The nominal rate could be defined as the sum of the real rate and the expected inflation rate, and these components could be viewed as being governed by separate processes, requiring two state variables (CI&R and Richards, 1978).

In their two state variable model, Brennan and Schwartz (1985) assume different mean reverting processes for the short rate and the rate on a long-term consol bond. Adding a state variable changes the equilibrium condition (1.4), as discussed in the previous section. The expected return on the mortgage now equals the risk-free rate plus adjustments for: (1) the risk of the short rate changing as in (1.4), and (2) the risk of the long rate changing. This leads to a generalization of (1.4). If we let c be the long-term (consol) rate, δ_1^2 and δ_2^2 be the variances of changes in r and c, ρ be their correlation coefficient, u_1 and u_2 be their means and λ_1 and λ_2 be their risk prices, then a second-order Taylor series approximation to equilibrium is given by

$$(1.8)\quad \tfrac{1}{2}M_{rr}\delta_1^2 + M_{rc}\rho\delta_1\delta_2 + \tfrac{1}{2}M_{cc}\delta_2^2 + M_r(u_1 - \lambda_1\delta_1) + M_c(u_2 - \lambda_2\delta_2) + M_t + C = rM.$$

What is interesting in this case is that the values of λ_2 and u_2 can be inferred. Because (1.8) applies to the value of a consol, by substituting the value, 1/c, of a consol paying \$1 into (1.8) and evaluating the derivatives ($M_c = -1/c^2$, $M_{cc} = 2/c^3$, and $M_r = 0$), we can solve for $\lambda_2\delta_2$, and, by substituting the result back into (1.8), produce an expression that contains neither λ_2 nor u_2 (see Brennan and Schwartz, 1985). This insight comes directly from the arbitrage approach and is directly analogous to the result in Black and Scholes (1973) that we do not need to know the mean reverting value of a stock or its risk price to price an option on the stock. The result, which holds for any state variable that is a traded asset, is also useful in the analysis of default (treated in Section IIIA). In the one state model, we could not eliminate u or λ because the instantaneous security is not a traded asset; it does not have a price that we can plug back into (1.8).[10]

b. <u>Default on Fully-Insured FRMs</u>

No default-free, assumable mortgages exist, but GNMAs, which are pools of mortgages guaranteed by the Government National Mortgage Association, a branch of the U.S. government, are a close approximation. GNMAs are assumable by the new homeowner should the house be sold, and they are fully insured. The major difference between a GNMA and a default-free FRM is that the payoff from insurance in the event of default is the par, rather than market, value of the security. Hence, default can still produce a gain or loss for investors. In general, the possibility of defaults on fully-insured callable mortgages raises their value; defaults that might occur when mortgages are above par, and thus would generate capital losses, tend not to occur because the mortgages would already have been prepaid but defaults that occur when mortgages are below par give a windfall gain to the mortgage owners. The possibility of this windfall will keep the mortgages close to par.[11] Whether this effect is quantitatively important is, of course, an empirical question.

Kau et al (1986a) and Rosenberg (1986) model mortgages with both default and prepayment by adding a new state variable, house price, H, to the one-state interest rate model (as explained in Section III). Both papers have a two state variable model in r and H, with an equilibrium condition like (1.8) with H replacing c, to simulate prices and values of the default option. In the case where house price and interest rate variances are "high" relative to recent experience and there is no covariance between r and H, Kau et al (1986a, Table 8) compute that an insured but "defaultable" 80 percent loan-to-value mortgage has a 6 basis point smaller coupon than a similarly priced default-free mortgage.[12] For 95 percent loan-to-value mortgages, the difference is 12 basis points. For moderate variances, the differences would probably be negligible, so we might be comfortable applying the model in Section 4 to GNMAs.

c. Due on Sale

Many mortgages are not fully assumable. Major examples are the pass-through securities guaranteed by the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FHLMC). In general, the automatic repayment when a household moves raises the value of a below-par nonassumable mortgage, and mortgages that would have been above par would have already been called.

Valuing nonassumable mortgages requires a potentially troublesome analysis of household mobility (Cassidy, 1983), including recognition that the potential forced prepayment when mortgages are far below par alters the incentive to move (Hendershott and Hu, 1982). In principle, nonassumability should be handled by introducing new state variables to govern the moving decision.[13]

2. Prepayment Models

a. Prepayment Functions

It is clear (see Foster and Van Order, 1985, and Green and Shoven, 1986) that households do not exercise their options as ruthlessly as the models we have elaborated imply. While prepayments accelerate when interest rates fall, some high-coupon mortgages remain outstanding. Moreover, while prepayments decelerate when interest rates rise, some low-coupon assumable mortgages continue to prepay. For this reason many researchers have added *ad hoc* prepayment functions that allow prepayments for reasons other than hitting the boundary condition.

For instance, Dunn and McConnell (1981) and Brennan and Schwartz (1985) add random prepayments, which they model as Poisson processes, to "rational" prepayments given by the boundary condition (1.7). If p is the probability of a random prepayment, then the expected cash flow (C in (1.4)) is increased by p(M-PAR). Boundary conditions are as before.

Dale-Johnson and Langetieg (1986) and Dietrich *et al* (1983) model FRMs in a similar way. Whereas the Poisson approach assumes that the entire mortgage pool pays off simultaneously, these papers assume gradual prepayment (half in Dietrich *et al*) when call is optimal, which is especially important in the analysis of instruments such as Collateralized Mortgage Obligations that allocate payments sequentially to different classes of security holders. Not surprisingly, these "nonrational" prepayments make the borrowers' call option less valuable and mortgages more valuable.

b. Transaction Costs

Presumably borrowers do not exercise their option ruthlessly because of implicit and explicit transaction costs. For instance, if a prepayment penalty must be paid to the investor, then the model is the same as depicted in Figure 2 except that the tangency at r_c is with a horizontal line that exceeds PAR by the amount of the penalty, PEN. Thus (1.7) becomes

$$M_r(r_c,t) = 0 \quad \text{at } M = PAR + PEN. \tag{1.7'}$$

The mortgage value functions with and without the penalty are drawn in Figure 3. The value function without a prepayment penalty is given by AM; with the penalty the function is given by BM. The level of r at which the mortgage is called falls from r_c^o to r_c' when the penalty is introduced.

Another transaction cost is the cost of taking out a new mortgage. To the borrower this is the same as the prepayment penalty model, a new tangency with a higher exercise price. However, because this "refinancing wedge" is not paid to the investor, the value to the investor is lower than when the cost is a prepayment penalty. The investor's boundary condition is now different from the borrower's. For a refinancing wedge equal to PEN, the borrower determines r_c' by the tangency condition depicted in Figure 3 by the curve BM. Given this, the boundary condition determining value to the investor, i.e., the market price, is that $M(r_c',t)$ equal PAR, as depicted by CM. Not only do we have negative convexity, but a range exists over which falling interest rates lower mortgage value (and thus "duration" is negative). Intuitively, this is because refinancing costs allow the mortgage value to exceed par, but the value must come back to par when interest rates decline sufficiently to make refinancing very likely. Buser and Hendershott (1984) incorporate the refinancing wedge into their simulations, and they do indeed (Table 5, p. 420) obtain the "negative duration" results (find that falling interest rates lower value) in some of their simulations.[14]

C. Solution Techniques

Because no closed-form solution to the model exists, numerical methods are needed to obtain mortgage prices. Taking advantage of the expected present value property discussed above, one might use a random number generating device to generate numerous interest-rate scenarios, calculate present values for each scenario, and average the results. Unfortunately, this Monte Carlo technique won't work because the key element of cash flow, when prepayment occurs, depends on the value of the mortgage, which is not known in the middle of the simulation.

Numerical procedures exist for solving difference-equation approximations to equations like (1.4). These are discussed in Brennan and Schwartz (1977), Kau et al (1986a), and McDonald (1987). Here we discuss a simple technique developed by Bartter and Rendleman (1979). The technique assumes risk neutrality ($\lambda=0$) and uses a simple binomial approximation to the interest-rate process given by (1.1).[15] The approximation is that the change in r can take on two values, up or down, with known probability. This simple scheme is used by Hall (1985) to price callable mortgages.

Figure 4 illustrates how the pricing techniques works in a simple 4-period model, with and without transaction costs. The interest rate is initially 10% and must either increase or decrease by one percentage point with equal probability; the distribution of interest rate levels at the beginning of the four periods is depicted in Figure 4a. Now consider a bond that pays $10 in interest at the end of each period plus $100 in principal at the end of the final period. Figure 4b depicts possible prices at the beginning of each period (with some rounding error). At the beginning of the final period, interest rates will be 13, 11, 9, or 7 percent, and the bonds will be worth (approximately) 97.3 (110/1.13), 99.1 (110/1.11), 100.9 (110/1.09) or 102.8 (110/1.07). Moving backward one period, if rates were at 8 percent (the lower path), then bond holders would get $10 in interest plus a fifty-fifty chance at either a $100.9 bond or a $102.8 bond. Assuming traders are risk neutral, the value of this lottery will be the expected present value: $[10 + \frac{1}{2}(100.9 + 102.8)]/1.08 = 103.6$. Similarly, if rates were at 12 percent (the higher path), the value of the bond at the beginning of period 3 would be about 96.6. Going back to the initial period, the market price would be 100.1. The excess over PAR comes from risk neutrality and the phenomenon depicted in Figure 1.

Suppose that the bond can be called at 100. The pricing is depicted in Figure 4c. At the beginning of the last period we have a simple choice: pay $110 in one period or pay $100 now (call the loan). If the interest rate is 9 or 7 percent, we call $[110/(1+r) > 100]$; if the rate is 13 or 11 percent, we don't call. Moving back a period, we see that if the rate is 8 percent, we pay either the $10 interest and accept a $100 liability at the end of the period (the bond will be worth $100 next period whether r is 9 or 17 percent) or we pay $100 now. Because $110/(1+r) > 100$, we call and pay $100, which is the value of the bond if r is 8 percent in the second period. If r is 12 percent, the value is the same as in the noncallable case.

Working back to the first period, we see that the value at origination is $98.8, so that the value of the call option is $1.2. This approach takes advantage of the expected-present-value interpretation of the model and the fact that we know the value at the end of the term. The model can be extended to as many periods as desirable and advantage can be taken of the property of binomial distributions, that they approach the normal. Thus the simple binomial process can be made into a good approximation to (1.1).

Note that this backward-solution technique incorporates all the information used in the perfect-markets arbitrage model. In particular,

(1) requiring that the beginning of period price equal the expected present value of "payout" at the end of the period, including end-of-period value, is equivalent to requiring that the one period expected return (a coupon plus expected capital gain) equal the risk-free rate, which, with $\lambda = 0$, captures (1.4).

(2) starting at the end automatically captures the terminal condition, (1.5); because the mortgage in our example is nonamortizing, the terminal value is par, not zero.

(3) given the limits on the variation in r, (1.6) is irrelevant; but if we let the number of periods grow, r can become very large and M(r,t) will approach zero.

(4) that the borrower prepays if the par value in less than the value of the mortgage if held another period is another way of stating the optimal prepayment strategy given by (1.7); the borrower chooses the strategy that minimizes M.

Panels 4d and 4e show bond prices with transactions costs. A $2 prepayment penalty increases value (from $98.8 to $99.7) because borrowers do not call when the interest rate declines to 9 percent, and when they do call, lenders receive the $2. A $2 refinancing charge also increases bond value, but by less than prepayment does because the "old" lender doesn't receive the $2 when call occurs.

While our description emphasizes the binomial model, the numerical procedures for solving difference equation approximations use the same principle, that of starting at the end of the term and working backward. With this method, an interest-rate grid is searched over each period. The problem with the Monte Carlo method is also clear here. One can't compute prices in period n without knowing prices in period n+1 because they determine whether call is optimal. Of course, if the call option is not important, as appears to be the case in the pricing of adjustable-rate mortgages with rate caps, then the Monte Carlo method works well.

II. Other Default-Free Claims

Numerous new mortgage instruments are now available and need to be priced. Pools of fixed-rate mortgages are being partitioned, either by unequal division of the interest and principal repayment components of the cashflows ("strips") or by shifting the timing of the mortgage prepayments (CMOs). In addition, large volumes of adjustable-rate mortgages have been issued since 1982, especially in 1984 and 1985. The pricing of these relatively new instruments is analyzed in this Section.

While the tools of Section I can be used to price mortgage derivatives (strips and CMOs) in the manner described below, the perfect-capital-market assumption (zero costs of setting up the hedge portfolio) that underlies these tools suggests that mortgage derivatives should not exist. If costs are incurred in creating the derivatives but no value is added (the parts sum to the whole in perfect markets), then creating them is a negative net present value endeavor. No value is added because the procedures used to set up the GNMA-Treasury hedge portfolio and to derive the pricing function could be used to generate expected cash flows identical to any derivative. However, if setting up the hedge portfolio is expensive because of trading costs, then scale economies might allow investment bankers to create derivatives less expensively than individual investors could. Thus imperfect markets can be used to rationalize the rapid growth ($80 billion in two years) of CMOs.[16] Another rationale for mortgage derivatives are accounting and/or tax advantages (see footnotes 19 and 20). In any event, if derivatives add value (if slick investment bankers aren't just fooling investors), then the pricing model cannot be applied exactly.

The discussion below explains strips and CMOs and describes how the perfect-markets model can be used to price them. In the process, we suggest to whom these new instruments might appeal if markets were imperfect.

A. Stripped Mortgage-Backed Securities[17]

Stripped mortgage-backed securities represent unequal proportions of the cash flows from mortgage pools. For example, a pool could be split into two parts, with each subpool entitled to half of the principal, but one receiving only one-third of the interest and the other two-thirds. Assuming that the underlying pool is valued near par, the first strip will sell at a discount and the second at a premium. Earnings from the parts would respond differently to declines in interest rates that might trigger prepayment, the discount part reaping capital gains and the premium part suffering capital losses.

Pricing strips is a fairly straightforward extension of pricing the underlying mortgage pools. Let us flesh out the previous example, setting underlying pool coupon rate at 9 percent so that the two strips earn 6 and 12 percent, respectively, on half of the principal.[18] The monthly payments based on a 9% coupon are first divided into interest and principal and then subdivided 1/3:2/3 (the interest) or 1/2:1/2 (the principal). Equation (1.4) can be solved for each component, after replacing C with either C/3 or 2C/3. The call condition which determines r_c, (1.8), is unchanged because call is based upon the value of the underlying mortgages, not the individual strip, reaching PAR. With no imperfections, every mortgage in the pool pays off at the same moment; each strip receives half of PAR, so that the boundary condition for each is $M(r_c,t) = PAR/2$.

Figure 5 plots prices for the underlying mortgage pool and the two strips for different values of r, assuming no transaction costs (or imperfections). The value function for the underlying pool is identical to the DEM line in Figure 2. The strips, M_6 and M_{12}, have values equal to half of PAR at call;

M_{12} always lies above M_6 prior to call, reflecting the difference in coupons; and $M_6 + M_{12} = M$ at all r. Note the sharply different responses to interest-rate declines as the interest rate approaches the call rate. As expectations of the below-market 6 percent coupon being called increase, value rises sharply; at the same time the far above-market 12 percent coupon declines in value (the shorter expected life of the interest payments outweighs the increase in present value coming from the lower rate at which the cash flows are discounted).

Figure 6 plots the price curves for a more severe stripping and the underlying mortgage pool. Here an 11 percent coupon is divided 4.95 and 6.05 between the two parts, and the principal is divided 99 and 1 percent. Thus the first part pays a 5 percent coupon (4.95/.99) and the second a 605 percent coupon (6.05/.01). The latter is close to an interest only security (an IO), while the low-coupon strip is not far from a principal only (PO) security. Pure IOs and POs are popular strips. The positive slope of the 605 percent coupon strip over a wide range of r, and the extreme negative slope of the 5 percent coupon strip are startling.[19] The 605 percent coupon could be especially valuable to thrifts who could use it to offset their long positions in mortgages; note that a portfolio split equally between 11 percent mortgages and the 605 strip would appear to be insensitive to changes in interest rates over a wide range. The 5 percent might appeal to pension funds with their long-term fixed dollar liabilities.

B. Collateralized Mortgage Obligations[20]

While CMOs take a wide variety of forms, the trick is to divide amortizing mortgages into different maturities: a short one for investors such as thrift institutions who dislike the "long" repricing period of FRMs, a long-one for investors such as pension funds and life insurance companies who dislike the "short" average life of FRMs, and a catch all in-between category for God knows whom, to account for the remaining cash flows. The classic CMO slices the mortgage pool into four sections or portions (tranches from the french trancher, to cut) by maturity. The shortest three receive interest as they accrue. The last is often an accrual security "Z bond" with no interest paid prior to repayment of the principal; until then the face principal rises at the stated coupon rate for the tranche. The first tranche is repaid entirely prior to any repayment of the other tranches (receives all distributed principal repayments), the second prior to any repayment of the third and fourth, etc.

What can complicate CMO structures is that the interest earned on the underlying mortgage pool need not be allocated directly among the tranche holders. Rather, each tranche usually earns a different coupon rate, such that the pattern of tranche coupons at the time of issue matches that on Treasury securities of comparable maturity. For example, with 2,7,10 and 20 year par-value Treasuries paying 6,7,7.5 and 8 percent, the four tranches might pay 6.5, 7.75, 8.5 and 9.25 percent. If the pool has a coupon above those on all the tranches, there is no problem. But what if the underlying mortgage coupon is 8.5 percent in our example? If the first three tranches prepay immediately, to take an extreme example, only 8.5 percent interest will be available to pay holders of the fourth tranche, although they have been promised 9.25 percent.

The conventional solution to this problem is overcollateralization.[21] The CMO originator might issue only $90 million of CMOs for $100 million of mortgage pools. Thus, when principal payments are made (scheduled or early), not all is used to retire the first tranche. To achieve a triple-A credit rating, a collateralization/payment rule must be followed to ensure that the outstanding CMO tranches are small enough that the cash flows from the remaining collateral prove sufficient to make all promised payments to the remaining tranche holders. The rule is: if any tranche has a coupon greater than that on the underlying mortgage pool, then the outstanding CMO principal must be less than the present value of the mortgage payments (1) assuming no prepayment and (2) using the highest coupon paid on any of the tranches as the discount rate.

In effect, a fifth or "residual" investor in the underlying mortgage pool exists to "finance" the excess collateral. The difference between the cash flow received from the underlying mortgage pool and the CMO payments (interest and principal) goes to the residual holder. Moreover, even if the mortgage isn't overcollateralized (it has a coupon greater than tranche coupons), the tranches are generally paid quarterly or semiannually so that the residual investor gets to use this "float" (and accepts some interest-rate risk). While the residual claim is usually held by the CMO originator, this claim can be sold and can be priced like the other parts of the CMO.

Pricing the tranches is similar to pricing mortgage strips. The cash flows must be carefully identified, taking into account the timing of principal payments as driven by the collateralization requirement (where relevant). Further, the value of all tranches goes to par upon call, which is determined by borrower behavior vis-a-vis the underlying mortgages.

Figure 7 illustrates the price behavior of a three-tranche CMO and its underlying mortgage pool, assuming perfect arbitrage and no transaction costs. To simplify the diagram, all tranches are assumed to place equal claims on the mortgage principal and to have coupons below that on the underlying pool (no overcollateralization). The value responses of the tranches (M_1, M_2 and M_3) to changes in interest rates are markedly different. The short tranche responds little given its short life, while the accruing Z-bond drops off sharply (has a far steeper slope than the underlying mortgage pool).

Again, the perfect-markets pricing model requires that the parts (tranches in this case) sum to the whole. Just as with strips, the pure-arbitrage assumption has to be relaxed for the origination of CMOs to have economic value. Further, with the optimal call model and no mortgage transaction costs (or equal costs for all borrowers), all tranches prepay at the same point in time. This clearly mitigates the point of CMOs (that the first tranche prepays quickly while the last receives no payment for perhaps 10 years or longer). Sluggish prepayment, likely based on varying transaction costs (Section IV), is needed for CMOs to make a lot of sense.

Recent CMOs have departed from the classic form (Roll, 1987). The innovations are a planned-amortization class (PAC), a tranche whose amortization follows a known schedule except under extreme prepayment scenarios, and a floating-rate tranche (almost always with a fairly tight rate cap). Such CMOs greatly complicate the collateralization issue and the pricing of the other tranches and the residual. Analyzing these securities lies beyond the scope of this paper.

C. Adjustable-Rate Mortgages

Another phenomenon of the 1980s is adjustable-rate mortgages (ARMs). Over a third of the loans originated since 1983 have had adjustable rates, and during 1984 and 1985 the percentage exceeded a half. Because pure adjustable-rate mortgages with short adjustment intervals never vary significantly from par, pricing them is trivial, but virtually no pure ARMs exist. Nearly all have life-of-loan rate caps, and most have per-period adjustment caps.

The surge in ARM originations was due to an increased willingness of households to accept interest rate risk, an increased desire of thrift institutions to reduce interest rate risk, and relaxation of most governmental restrictions impeding the issuance of ARMs. When high interest rates sharply reduced the size of loans households could qualify for, ARMs, which had lower initial coupon rates than FRMs owing to both an upward sloping yield curve and the far lower value of call protection given by ARMs (see below), became relatively more valuable to households. Moreover, the desire to match asset and liability repricing periods led thrifts to structure ARMs to address the qualification problem (to offer initial teaser or below-market coupon rates).

Rate-capped ARMs are usefully viewed as loans with freely floating rates combined with both an automatically exercised option to borrow at below-market rates if interest rates rise too fast (adjustment caps) or too far (life-of-loan caps) and an option to refinance if interest rates fall either so fast that rate floors bind or so far that lowering the base for the life-of-loan cap is profitable. The ARM acts like an FRM both when interest rates get very low and when they get very high. When rates are sufficiently low, either the ARM is called or the interest-rate variance gets so low that the probability of binding caps becomes zero. When rates are sufficiently high, the life-of-loan cap binds so tightly that the probability of rates dropping enough for the ARM coupon to decrease approaches zero. Figure 8 plots the value functions for a noncallable mortgage AM, an FRM with the same coupon BM, and an ARM whose coupon plus its life-of-loan cap equals the FRM coupon CM.

Because the rate caps are automatically exercised options, caps can be valued by Monte Carlo methods (Asay, 1984 and Lea, 1985). However, more complex valuation techniques are needed if call (which is an endogenously exercised option) is an important consideration. With call, one has to solve the pricing problem backward, as shown in Figure 4. But with per period adjustment caps, a forward solution is also needed because the previous period coupon rate must be known to determine if the adjustment cap/floor binds. Kau et al (1986b) have solved this problem by introducing a second state variable to keep track of the previous period coupon rate so that backward solution techniques can be used. The importance or value of the call option is thus a significant issue.

Buser, Hendershott and Sanders (1985, pp. 257-58) argue that the call option on a 5 percent life-of-loan rate capped ARM has trivial value. The simulations of Kau et al (1986b, Table I) confirm this. Even in a high rate-volatility environment ($\sigma = 0.05$, see note 11), the call value in a flat interest-rate environment for a near-par ARM with a 5-point life-of-loan cap and no per-period cap (an $\infty/5$ ARM) is 0.005 percent of value with no transactions costs. The same values for ARMs with per-period rate caps/floors of 1 and 2 percentage points along with the 5-point life-of-loan cap (1/5 and 2/5 ARMs) are 1.42 and 0.35 percent. If refinancing costs were added, as in Section IB, the value of call on the 2/5 ARM, the most popular of all ARMs originated, even in a high rate-volatility environment, would probably be negligible.[22] Thus, Monte Carlo methods that ignore the prepayment option are likely sufficient for pricing 2/5 and $\infty/5$ ARMs in a high volatility environment and even the 1/5 ARM in a low volatility environment.

III. Pricing Default Risk

The model of default risk is analogous to that of prepayment risk. Default may be viewed either as a put option that gives the borrower the right to sell the house to the lender at a price equal to the value of the mortgage or a call option that gives the borrower the right to buy back the mortgage in exchange for the house. As in Section I we first analyze a frictionless model, where the lender's only recourse is to take over the house, and work our way up to more complicated models that incorporate other costs. To keep matters simple, we begin by assuming constant (or at least nonstochastic) interest rates, so that we do not have to worry about interest-rate fluctuations and prepayments made in response to them. In the extensions, this assumption is relaxed and transaction costs are introduced. A discussion of mortgage default insurance concludes the section.

A. The Basic Model

The options approach to valuing default in mortgages began with Asay, 1978 (see also Masulis, 1982). Cunningham and Hendershott (1984), Epperson _et. al._ (1985) and Kau _et. al._ (1986a) have studied similar models. The structure of these models parallels that of the prepayment model in Section I.

The basic model has one state variable, house price. We assume that the change in house prices (H) is given by

$$dH = u_H(H,t)dt + \delta(H,t)dz, \tag{3.1}$$

which is interpreted as before. While this function can take on a variety of forms, the most common is

$$dH = hHdt + \sigma Hdz, \tag{3.1'}$$

which implies that the percentage change in H has a constant mean and variance. As in Section I, we view the mortgage as made up of a basic default-free mortgage, $\bar{M}(r,t)$, minus an option, in this case to default on subsequent payments, $D(r,H,t)$. We assume that r is constant and supress it.

The papers cited above proceed by focusing on a portfolio of mortgages and housing constructed to have no instantaneous risk. As was the case in Section I, the zero arbitrage equilibrium condition implies that the expected return on the mortgage must equal the risk free rate plus an adjustment for risk. The analogue to equation (1.2) is

$$u_m M(H,t) = rM + \lambda_H \delta(H,t)M_H, \tag{3.2}$$

where λ_H is the price of house price risk.

As in the case of the two-state variable model in Section I-B (from Brennan and Schwartz, 1977 and 1985), the value of the risk adjustment can be inferred. Indeed in this case the inference is straightforward. We know that the expected return on the house itself, composed of service flow ("dividend") and expected capital gains, is given by

(3.3) $$R_H = sH + u_H(H,t),$$

where s is the per dollar service flow from the house. For reasons identical to those behind (1.2), equilibrium in the market for holding houses requires that:

(3.4) $$R_H = rH + \lambda_H \delta(H,t)$$

Equating (3.3) and (3.4), λ_H can be inferred from [23]

(3.5) $$\lambda_H \delta(H,t) = sH + u_H - rH.$$

The expected yield on the mortgage, for the same reasons developed in Section II for equation (1.3), is given by

(3.6) $$u_m M = C + M_t + u_H(H,t)M_H + \tfrac{1}{2}\delta^2(H,t)M_{HH}.$$

Substituting (3.5) and (3.6) into (3.2) yields

(3.7) $$C + M_t + (r-s)HM_H + \tfrac{1}{2}\delta^2(H,t)M_{HH} = rM,$$

which is the basic equilibrium condition.

Note that both λ_H and u_H, the expected growth rate in house prices, have been eliminated, just as both the risk premium for and expected growth rate of the long rate were eliminated in the Brennan-Schwartz two-state call model.

Unfortunately, the elimination is somewhat less valuable here because the service flow or "rent" from the house is added as a determining variable. This flow is not easily specified.

As before, an infinity of curves satisfies (3.7), and three boundary conditions are needed (one for t and two for H) to determine the optimal one. Again, the terminal condition reflects amortization:

(3.8) $$M(H,T) = 0,$$

as in (1.5). Also, the value of the default option goes to zero as H approaches infinity. Equivalently, the mortgage's value approaches that of the comparable default-free mortgage:

(3.9) $$M(\infty,t) = \bar{M} \quad \text{or} \quad D(\infty,t) = 0.$$

As before, the final or free boundary condition comes from assuming rational exercise of the option. Ignoring transaction costs or recourse that reaches into owner resources beyond the house, the option will be exercised when M = H. Figure 9 depicts the value of the mortgage (at a given t) as a function of H. The value of the default-free mortgage is simply its par value as represented by the horizontal line $\bar{M}$. M lies below that line by the value of the default option, D. M must also be on or below the 45 degree line along which H = M. Finally, M approaches $\bar{M}$ (D goes to zero) as H goes to infinity. The higher is the initial house value (H_o in the figure) and thus the lower the loan-to-value ratio, the lower is D at that time.

As in Section I, an optimal exercise strategy exists. Because the option is exercised along the 45-degree line, the problem is to find the critical H, H_c, at which to put the house to the lender, subject to touching the 45-degree line. Again the borrower chooses an H_c, determining a function M(H,t), that minimizes the value of the mortgage (his liability) for any value of H. The lowest curve that satisfies (3.8) and (3.9) and still touches the 45-degree line is the one that is just tangent to the 45 degree line. Again assuming an interior solution, the condition that nails down the price function is:

$$M_H(H_C,t) = 1 \quad [D_H(H_C,t) = 1] \quad \text{at } M = H. \tag{3.10}$$

This fully specifies a very simple model of the price of a defaultable mortgage. While we analyze default insurance below, here we note that the up-front premium, I, for 100 percent insurance when $\bar{M}$ = PAR is simply D in the model.

Two observations on this model are:

(1) This model of default is entirely equity based; people default if and only if they have no equity. But equity is measured by H-M, not H-$\bar{M}$, the usual measure of equity. The difference is that H-M includes the value of the option to default in the future. Rational households with house price only slightly less than mortgage balance do not default; the gain from default is less than the cost of putting up equity on another house purchase, equity that would be at risk. Households maintain the option to default later on because they have an underpriced option.

(2) The value of M is independent of borrowers' characteristics, unless they affect the equation for house price (e.g., slovenly individuals might raise s). Note, in particular, that "payment burden" has no place in this model. A borrower who can't make the payments sells the house if there is positive equity; if there is negative equity, default occurs in any event. In extended versions with costs of exercising the option (discussed below), personal characteristics are more likely to matter.

B. Extensions

Here we consider two extensions, similar to those in I-B, adding more state variables and transaction costs.

1. Adding State Variables

Asay (1978), Epperson et al (1985), Kau, et al (1986a), Titman and Torous (1986), and Rosenberg (1986) add the short rate to the model. The main reason for the extra variable is that when r changes, the value of the mortgage changes, affecting equity and therefore default. For instance, a decline in H that would otherwise trigger a default might not if r has risen because the borrower would wish to maintain what is now a low-rate mortgage.

We want to specify a pricing function M(r,H,t). Formally, the analysis proceeds along lines entirely analogous to the two state Brennan-Schwartz model in Section I, where the subscript 2 now denotes house price and subscript 1 still denotes the spot rate of interest. Equilibrium requires that:

$$(3.11)\quad \tfrac{1}{2}M_{rr}\delta_1^2 + \tfrac{1}{2}M_{HH}\delta_2^2 + \rho\delta_1\delta_2 M_{rH} + M_r(u_1 - \lambda_1\delta_1) + M_H(r-s) + M_t + C = rM.$$

Note that, as above, we do not need to know the price of house-price risk or the expected growth of house prices, but we cannot eliminate these terms for r.

We assume first that M is a fixed-rate mortgage with no prepayment option. In that case, the five boundary conditions (one for t and two each for r and H) resemble those of the one state variable model with call [equations (1.5), (1.6) and (1.7)] but with default [(3.9) and (3.10)].[24] The difference in boundary conditions is that default, not call, can now be triggered by an interest-rate decline. Hence, we need to replace (1.7) with a condition for default when r changes. The condition is illustrated by the BM curve in Figure 10. When r falls sufficiently, equity will be zero. For reasons entirely analogous to those behind Figure 2 and equation (1.7), the new boundary condition becomes:

$$(3.12)\qquad M_r(r_c^d,t) = 0 \quad \text{at } M = H.$$

where r_c^d is the critical default interest rate. As before, we can view default as a put on a house, or, in this case, a call on the mortgage where the exercise price is the house value. Hence, the similarity between (3.12) and (1.7).

In this expanded model, intuition is complicated because two offsetting effects exist. First, interest-rate variance raises the value of the default option, and thus lowers the value of the mortgage, on the grounds that increasing risk generally raises option values. Second, interest-rate variance increases the value of the mortgage over its convex range because interest-rate volatility raises prices of noncallable fixed-income securities (declines in r raise value by more than increases lower it, as depicted in Figure 1). In the Kau et al (1986a, Table 8) simulations, the second effect overcomes the first except in the case of high values of house price variance or low down payment (in which case D becomes very valuable and very sensitive to changes in r).

These simulations assume no correlation between r and H. One would expect that high negative correlation between r and H (as would be expected when real interest rates change) would make default less valuable because the low house prices that trigger default would be associated with high interest rates, which would make the existing low rate mortgage more valuable and thus reduce the probability of default. Unfortunately, neither Epperson et al (1985) nor Kau et al (1986a) considers simulations with a negative ρ.

2. Both Default and Prepayment for FRMs

Kau et al (1986a), Titman and Torous (1986) and Rosenberg (1986) consider both default and prepayment. These models are similar to that just discussed, but with both optimal prepayment and optimal default.[25] The free boundary condition becomes the minimum of (1.8) and (3.12). Introducing prepayment should make default less valuable because the critical interest rate at which prepayment will trigger r_c, can exceed that at which default will trigger call, r_c^d (PAR, the exercise price for the prepayment call, is less than H, the exercise price for default call). This situation is illustrated by the CM curve lying below the BM curve in Figure 10. This intuition is somewhat confirmed in the Kau et al simulations (their Table 8 comparing contracts III and VI); however, the comparisons with and without prepayment are of mortgages with different coupons, and hence they are not quite comparable.[26]

3. Transaction Costs

Not surprisingly, the frictionless default model does not explain FHA default data very well. Foster and Van Order (1984) estimate an "option-based" model of default and find that even when equity is quite negative, the probability of default is under 10 percent. Nonetheless, they and others (see Campbell and Dietrich, 1983, and Swan's 1982 survey of the literature) find that equity is a major factor in explaining default. This suggests the inclusion of default costs in a way similar to the inclusion of transaction costs in Section I.

In their pricing of mortgage default insurance, Cunningham and Hendershott (1984) consider two types of costs: (1) costs to the borrower (moving costs, lower credit rating, and lender recourse to the borrower's other assets less free rent) and (2) costs to the lender, which they divide into lost interest plus administrative costs and the cost of selling the house. We denote the borrower costs by C_B and the lender costs by $C_L + \alpha H$, where α is the assumed proportional cost of selling the house, and incorporate these costs in Figure 11, which is a modification of Figure 9 and thus is similar in purpose to Figure 3 in which prepayment costs were added to the frictionless prepayment model. To the borrower, the new exercise price is $H + C_B$, and the solution is the same as before, a tangency, but now with $M = H + C_B$. This determines the new, lower critical house price, H_c':

$$(3.10') \qquad M_H(H_c', t) = 1 \quad \text{at } M = H + C_B.$$

In the Figure, the value of the mortgage to the borrower is given by $A\bar{M}$, compared with the curve in the frictionless case, $B\bar{M}$.

Again, a wedge exists between borrower and lender values. As in the prepayment model, the lender's (and hence the market's) value is determined by the borrower's tangency condition. In this case the boundary condition for the lender becomes:

(3.10") $$M(H_c',t) = (1-\alpha)H-C_L.$$

As can be seen from the figure, if C_L and α are zero, then M(H,t) to the lender is always greater if the borrower has transaction costs (curve $C\bar{M}$), but if the lender's costs are large enough, M may be worth less than in the frictionless case (curve $D\bar{M}$).[27]

This model is straightforward, but if we pursue the effects of costs more thoroughly, we are forced to face up to some complications. Consider the cost of selling a house and moving into a new one, and suppose that borrowers move at random intervals. If a borrower is already moving, moving costs do not act as a deterrent to default and the existence of selling costs acts as an incentive. That is, default costs for movers, C_B^M, are significantly less than costs for those who do not have to move, C_B (Cunningham and Hendershott, 1984). Thus, default is more likely for those who have to move. Moreover, default now depends on a personal characteristic.

Foster and Van Order (1984) suggest that we might extend the model by assuming that, for borrowers with positive $H-M+C_B$, default is the intersection of two events: (1) negative equity after selling costs, and (2) the occurrence of some event (more than likely a bad one) that entails a move. A simple way of modelling this is to introduce a new state variable similar to the random prepayment variable in Dunn and McConnell (1981), which has a Poisson distribution and takes on a unit value when people move. This also alters the boundary conditions: when the random prepayment variable is zero, people will only default if $H = M-C_B$; but if the variable is unity, they will default if H

$= M-C^M_B$. Note, however, that the extension really introduces additional state variables (sickness, divorce, unemployment, etc.) and that the model should be expanded to account for optimizing should these events take place.

Finally, we note that households should be expected to default ARMs more frequently than FRMs (and the value of D for ARMs should generally be greater than that for FRMs). An increase in interest rates lowers defaults on FRMs, because existing loans now have below-market rates, but raises defaults on ARMs, because of the payment shock from the rate increases. Defaults on both instruments are largely independent of interest-rate declines because both stay at par (FRMs are called).[28]

C. Mortgage Insurance Contracts

Currently FHA is charging a one-time upfront fee to insure (100%) FHA mortgages. As noted above, in the absence of transaction costs, the "fair" fee should equal the value of the default option in the simple (constant interest rate) model. When interest rates are allowed to vary, when coverage is less than 100 percent (and pays off at PAR rather than market), and especially when transaction costs are introduced, the calculation is altered in significant ways.

The basic partial-differential equation for an insurance contract is essentially the same as (3.11), but with a fee paid to the insurer replacing the coupon payment. The problem is to find the appropriate "upfront" premium, given the fee payment (zero for FHA).[29] The difficulty in pricing insurance is the complicated boundary condition describing the payout at default.

The critical house price at which default occurs is given by (3.10'). At that price, the payoff to the borrower (D) and the cost to the insurer/lender (I) are:

$$D = \bar{M} - (H + C_B) \tag{3.13}$$
$$I = \bar{M} - [(1-\alpha)H - C_L].$$

C_B and C_L are the costs discussed above and α is the fractional cost of selling the house. With 100 percent coverage on the PAR, not the market, value of the mortgage, the insurer pays $I = PAR - [(1-\alpha)H-C_L]$ and the lender gains $PAR - \bar{M}$.[30]

Private mortgage insurance is more complicated in that insurers pay a fraction, β, of the claimable amount (principal balance, delinquent interest, taxes and insurance paid, legal fees and miscellaneous expense). The insurers loss is then $\beta(PAR + C_L)$, and the lender loses $\bar{M} - (1-\alpha)H + (1-\beta)C_L - \beta\ PAR$.[31]

One point of interest is whether a one state (house price) or two state (house price and interest rate) model is needed to value insurance contracts. Asay (1978) shows and Epperson _et al_ (1985) confirm that the interest-rate state variable may not be needed. When the interest-rate variance rises from a moderate to a high level (Table 5, scenario II versus IV), the value of the up-front insurance fee changes by less than one percent.

IV. Realistic Pricing and Testing: Suggestions for Future Research

Early research on pricing mortgages was triggered by an interest in explaining changes in spreads between yields on mortgages traded in the rapidly expanding secondary market and on Treasuries. The GNMA-Treasury spread fell throughout the early and middle 1970s, an observation attributed to increased efficiency of the GNMA market. However, the spread rose in the late 1970s and early 1980s, something which could not be attributed to increased efficiency. At about that time (see Hendershott and Villani, 1981), attention shifted to the call aspect of mortgages; yield spreads could increase because of changes in interest rate volatility or the shape of the yield curve.

Hendershott, Shilling and Villani (1982) regressed the "true" GNMA-Treasury spread (see Hendershott, Shilling and Villani, 1983) on a proxy for the value of the call option (the spread between 20-year utility bonds, with 5-years call protection, and industrial bonds, with 10-years call protection) and the extra taxation of GNMAs at the state and local level. Both variables were statistically significant. Brooks and Quick (1983) regressed conventionally-measured GNMA yields on a set of variables that included interest rate volatility and the slope of the yield curve. While that paper was not intended as a test of option pricing models, it confirmed their propositions that greater variance and a more downward sloping yield curve tend to raise GNMA rates (relative to Treasuries).

This interest in explaining market prices and yields is largely absent from the formal pricing literature; few studies attempt to obtain realistic price estimates and even fewer compare estimates with market prices. Buser, Hendershott, and Sanders (1985) obtain estimates of a five-percent life-of-loan rate cap on ARMs for historically observed term structures and measures of interest-rate volatility, but data on ARMs with this cap are not available for comparison. Cunningham and Hendershott (1984) estimate fair market fees for FHA default insurance for different maturity fixed-rate mortgages over a range of loan-to-value ratios under various assumptions about borrower transaction costs and realistic estimates of FHA losses if default occurred. Because FHA has historically charged a single fee for all mortgages and loan-to-value ratios, market fees could not be contrasted with the estimates.

Only Hall (1985) and Titman and Torous (1986) compare price estimates with market prices. Using a variant of the one-state models in Section I, Hall claims some confirmation that the call option in GNMAs is priced properly, but the data are too shaky to support rigorous inference. Using a variant of the two-state variable models in Section III, Titman and Torous loosely confirm that the default option in commercial mortgages (an area where "ruthless" exercise is more plausible) is priced properly, but uncertainty about the variance of changes in property prices makes testing difficult. Dunn and Singleton (1986) attempt an indirect test of the efficiency of the GNMA market, but they do not test specific option-based pricing models.

That some researchers seem more concerned with whether mortgages can default during or only at the end of the month than with homeowner's obvious reluctance to default when they have negative equity or with the large losses insurers suffer when default occurs illustrates the sharp division of labor that exists in modern economics, which permits one to distain interest in weighing one's explanations against real-world phenomena. The time is ripe for applied econometricians to turn to this field, and they are beginning to do so. In this closing section, we first draw together the useful results of the work to date and then suggest needed extensions of the models.

A. Structuring and Specifying the Pricing Model

Application of the pricing methodology requires specifying state variables and parameters values. These two decisions are discussed in turn.

1. The Needed Number of State Variables

The computational complexities of the backward solving pricing methods are such that minimizing the number of state variables is important. Indeed, to the best of our knowledge, no one has yet worked with a three state variable model. Nevertheless, our analysis suggests that possibly four state variables are needed for pricing: two interest rate variables to model call, house price to model default/insurance, and a fourth variable to model ARMs or CMOs where keeping track of past interest rates (prepayments) is crucial. Fortunately, this list can be pared.

Analysis in process by Buser, Hendershott and Sanders (1987) indicates that only one interest-rate state variable is needed because a one-state interest-rate process can be specified that closely approximates prices from a given two-state process. That is, even if the interest rate process were "known" to be driven by two state variables, a one state process can be used in computing prices for a wide range of parameters. Other economies follow from the simulations of Epperson, Kau and their colleagues (1985, 1986 ab). Because these were discussed earlier, they are just summarized here:

(1) one state variable is sufficient to price fully default-insured mortgages (GNMAs); even though default raises value when the defaulted mortgage is below-par, the impact is trivial,

(2) one state variable is sufficient to price default insurance; even though interest-rate declines could raise the market value of mortgages sufficiently to trigger default, the impact is trivial, and

(3) often a forward pricing (Monte Carlo) methodology is sufficient to price default-free ARMs; only when interest-rate volatility is high and floors of less than two percentage points per adjustment are being analyzed does the call on ARMs have more than a trivial value.

2. Parameterizing the Models

Brown and Dybvig (1986) use the CI&R term-structure model to estimate the parameters of the underlying process [equation (1.1') above] for the short rate. This is relatively easy to accomplish because the CI&R model (with one state variable) yields a closed form solution for the price of a zero-coupon, risk-free security, and the solution is a nonlinear function of three parameters, r, and t. Using nonlinear least squares, these parameters can be estimated on any date with that day's yield curve. From these estimates, the variance parameter, σ in (1.1') can be inferred. The remaining parameters cannot be uniquely determined, but if we know one of them (e.g.,λ), we can determine the other two (θ and k).

Titman and Torous exploit their model and end-of-month yield curve data to estimate end-of-month mortgage prices.[32] A key to using the yield curve data is the observation (in Titman-Torous and Buser, Hendershott, and Sanders, 1985 and 1987) that for a given set of data the answer to the mortgage pricing problem is invariant to the value of λ in the sense that if we pick an arbitrary λ, say λ_o, use λ_o to infer θ and k from the Brown-Dybvig model, and then use that θ, k and λ_o to estimate M, our answer is invariant to the choice of λ_o . In fact, Buser, Hendershott and Sanders (1987) argue that <u>any</u> combination of θ, k and λ which, along with an estimate of σ, reproduces the yield curve will give call values within a percent or two of any other (for either one or two state variable models).

The reason for this can be seen from examining the equilibrium condition (1.4) in the one state variable model in Section I. The expected growth in interest rates (which is determined by θ, k and r) and the price of risk (λ) enter together and additively in the term multiplied by M_r. They do not enter in any other term or in the boundary conditions. Hence, increasing one and decreasing the other in the right proportion won't change the answer. Intuitively, the pricing model arbitrages off the yield curve. Only the shape of the yield curve matters; the specific contributions of expected interest rate changes or risk aversion in forming this shape are irrelevant (Buser, Hendershott and Sanders, 1985 and 1987).

In contrast, no analogue to the term structure exists from which we can deduce the expected split of housing returns between appreciation and rental flow. Thus house prices must be modelled explicitly. Data are a problem in estimating such models because few sources exist of transaction prices on the same houses over time at frequent intervals. Foster and Van Order (1984) use an indirect approach to infer the variance of house prices from a default model. In any event, getting up-dated estimates of the growth rate (or rental flow) and variance of house price changes will be difficult.

B. The Future for Option Pricing Models

Because of the large amounts of money involved in trading mortgages, a competitive market exists in mortgage models. The market is on Wall Street, and it is one with imperfect information in the sense that little is published about the pricing models used by investment bankers. However, from what we can infer from nontechnical publications and private discussions,

(1) an appreciation exists of the importance of the option aspects discussed above. For instance, Askin et al (1987), discussing Drexel-Burnham's model, refer to the Cox, Ross, Rubenstein (1979) approach to option pricing as the basis of this model, and the Salomon Brother's model, apparently uses a Monte Carlo approach to option pricing (Waldman and Gordon, 1986). The output of these models is (see also Jacob and Toevs, 1987) either an option-based price or an option-adjusted yield.

(2) however, the centerpiece of Wall Street research is the recognition that prepayments are very complicated and need to be modelled as more than a boundary condition. For instance, the Salomon Brothers model downplays the rigor of the option model by making prepayment a direct function of interest rates (rather than mortgage price, which the strict model implies is the right variable) and in that way Monte Carlo techniques in conjunction with their prepayment model can be used without the endogeniety problem discussed in I-C above. This approach is also followed at Goldman, Sachs & Co. and, we expect, other Wall Street firms.

For obvious reasons we have nothing to report on the details of the prepayment functions used by investment bankers. Preliminary work by Green and Shoven (1986) and Foster and Van Order (1984, 1985) suggests that more is involved in the sluggishness of exercising the prepayment or default option than can be captured by simply adding transaction costs. In particular, a simple transaction-costs model would imply that mortgage prepayments should be closely clustered, which does not seem to be the case. While an additive transaction costs term of the appropriate magnitude might yield accurate prices for GNMAs (although the market for models does not suggest this possibility is taking seriously), the term would be unlikely to yield accurate prices for the different tranches of CMOs. Even if "average" prepayment for the GNMA pool can be made equivalent to gradual prepayment, with average prepayment the short tranche would prepay too slowly and the long tranche too rapidly.

If the formal pricing models are to be successful in the competitive market for models, far more serious treatment of prepayment will be necessary. We have two alternative suggestions for researchers:

> (1) estimate a probabilistic model of prepayment, along the lines of Foster and Van Order (1985), assuming that the percent of mortgages which prepay is a nonlinear function of the difference between market and/or par values. Then add this function to the coupon, scrapping the boundary condition (1.7) for frictionless call. The nonlinearity of the prepayment function would generate the negative convexity depicted in Figure 2.

> (2) assume transaction costs as in Buser and Hendershott (1984) or Cunningham and Hendershott (1984) but let the costs vary across borrowers. Specifically, the mean and variance of the distribution of transaction costs can be estimated from prepayment data via maximum-likelihood techniques. These models could then be used to price mortgages with different transaction costs, and the value of the pool of mortgages would be the sum of the value of the individual mortgages. This procedure would permit analysis of the effect of changes in transaction costs.

Footnotes

[1] For a short survey of the published mortgage pricing research prior to 1986, see Hendershott (1986). Of course, much research is also being conducted by Wall Street firms (see Fabozzi, 1985 and 1987).

[2] CI&R show how (1.1), (1.1'), or generalizations of them might arise in a general equilibrium model.

[3] By complete we mean that if there are n state variables (e.g., interest rates upon which M depends), then there must be at least n independent securities that depend on these state variables. This is necessary for the hedge portfolio to be created.

[4] The intuition is that the hedge portfolio is perfect insurance and can be created with constant returns to scale. Hence the λ's are the linear prices that would be charged for a competitive insurance contract.

[5] See Malliaris and Brock (1981) for a discussion and some applications.

[6] The reader will probably note the similarity of this issue to the proposition (Jensen's Inequality) that $E[F(X)] \neq F[E(X)]$ if F is nonlinear.

[7] Technically, uncertainty requires keeping some of the second order effects in the Taylor expansion used to approximate M. The "½" in (1.3) comes from this second order term.

[8] In the implementation of pricing models, prices are calculated over a finite interest-rate grid, the highest value of which is far (infinity) short of infinity. A reasonable boundary condition here is that the valuation function is smooth ($M_{rr} = 0$) at extreme r's.

[9] The solutions to (1.4) that satisfy (1.5) and (1.6) can never intersect (in Figure 2). Hence, a trajectory that minimizes the value of M for a single r minimizes value for all r's.

[10] This problem cannot be finessed by manipulating the choice of state variable because the hedge portfolio used to eliminate instantaneous risk must contain the instantaneous T-bill rate.

[11] This possibility can become a probability for high coupon GNMAs on nonperforming rental properties.

[12] Kau et al (1986a) use interest rate uncertainty scale parameters [σ in equation (1.1')] of 0.05 and 0.15; over the last decade this parameter has varied between roughly 0.015 and 0.05. Thus their low parameter value is at the high end of historical experience. For the house price uncertainty scale parameter [σ in (3.1') below], they use the same values; other authors have used values ranging from 0.08 to 0.12. We interpret σ's of 0.05 (interest rate) and 0.15 (house price) as a high variance world.

[13] Cassidy (1983) and Dietrich et al (1983) provide estimates of the value of due on sale; Hendershott (1986) discusses the estimates.

[14] For a fuller discussion of transaction costs, see Dunn and Spatt (1986).

[15] Cox, Ross and Rubinstein (1979) discuss how risk can be handled.

[16] Whether CMOs have lowered mortgage rates to borrowers is a separate empirical question. Brooks and Quick (1983) find some evidence that a dummy variable for CMOs has a negative effect on GNMA rates, but in the best specified equations in the paper, the effect is small (10 to 20 basis points) and of questionable significance.

[17] This discussion draws heavily on Roll (1986).

[18] If current income is more important to some investors than is true economic income, then strips with artificially high coupons have "extra" value.

[19] A highly placed investment banker has suggested that investing in both the IO and the PO could be superior to investing in the underlying mortgage. If interest rates rise, the capital loss on the PO can be taken against current income, while the capital gain on the IO can be allowed to ride. If interest rates fall, the IO can be sold and the PO retained.

[20] This discussion draws heavily on Roll (1987a).

[21] In addition, up to two percent overcollateralization seems to be required in order to establish equity investment by the creator of the CMO. The new REMIC vehicle introduced in the Tax Reform Act of 1986, which can be classified as an asset sale rather than debt issue, removes this requirement.

[22] With an 1½ point up-front fee, the 0.35% drops to 0.11% (Table II).

[23] Solving for s, $s = r + \lambda_H \delta/H - u_H/H$. Some might recognize this as the rental cost equation for owner-occupied housing in the absence of taxes.

[24] Epperson et al (1985) and Kau et al (1986a) modify the default model by arguing that defaults will not happen until a payment is due, so that for a 30-year mortgage there is a string of 360 European options instead of a boundary condition that applies continuously.

[25] Kau et al (1986a) assume that default occurs only at the end of the month, while prepayment can happen anytime. We presume, although no simulations analyze the difference, that whether or not defaults occur at intervals or "continuously" is of little consequence.

[26] In particular, their experiments hold price constant and look at required coupon.

[27] Because the curves come from the same partial differential equation, differing only by boundary conditions, they cannot cross (see footnote 9).

[28] Note that if there are costs of refinancing, ARMs will default less than FRMs when interest rates fall. Hence, it is possible that D will be worth more for FRMs than ARMs when rates are expected to fall.

[29] Insurers receive their income from borrowers as an annual fraction of the remaining balance as long as the contract is in force plus a larger one-time first year fee. The latter varies with the initial loan-to-value, the extent of the coverage (fraction of claimable amount), and the type of mortgage (ARM insurance is more expensive, as was argued above). For over 20% coverage, the continuing fees are 30 basis points for FRMs with loan-to-values of 90% or less, 35 basis points for loan-to-values above 90%, and 40 basis points for "nonfixed-payment loans" (ARMs, short-term balloons, and GPMs). (These prices are from Mortgage Guarantee Insurance Corporation's premium schedule dated May 1, 1985; MGIC's prices were constant for the 25 years prior to 1982 and since then have been increased a number of times.)

[30] See Van Order (1987) for a synthesis and interpretation of the literature on the appropriateness of FHA default insurance fees.

[31] The analysis is further complicated by the possibility that the insurers themselves may default (fail to perform). Thus the lender's potential loss is I.

[32] Note that the Titman-Torous procedure involves an internal inconsistency. The pricing model assumes that k, θ, and λ are constant when making the arbitrage argument, but the parameters are reestimated every month.

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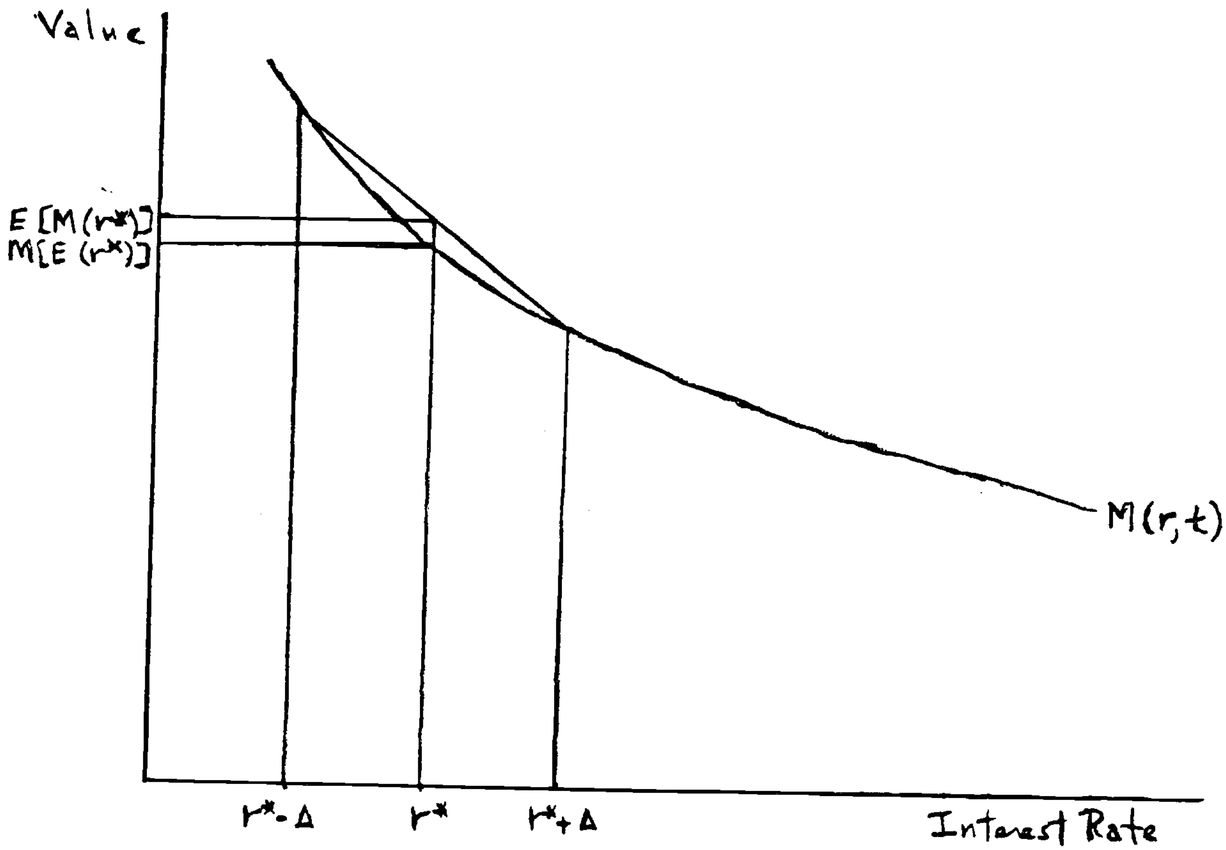


Figure 1: Uncertainty and Expected Returns

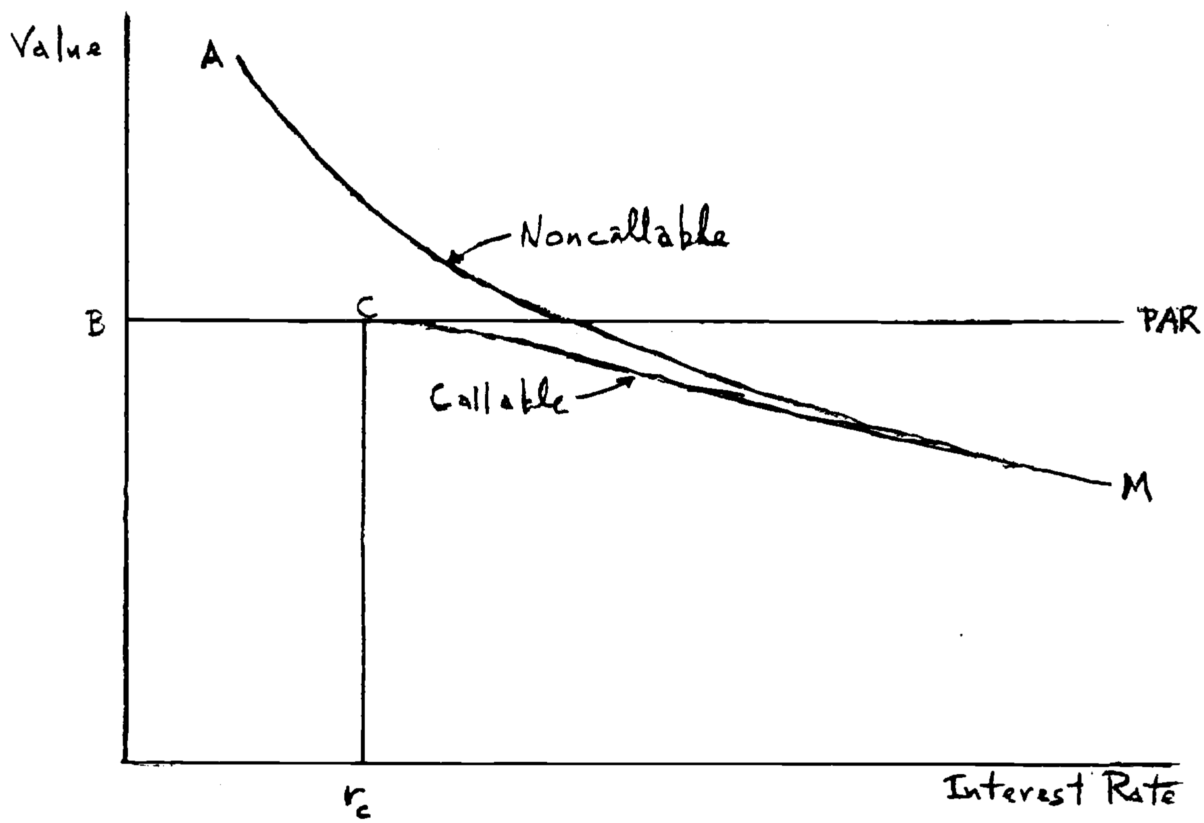


Figure 2: Mortgage Prices with One State Variable and No Transaction Costs

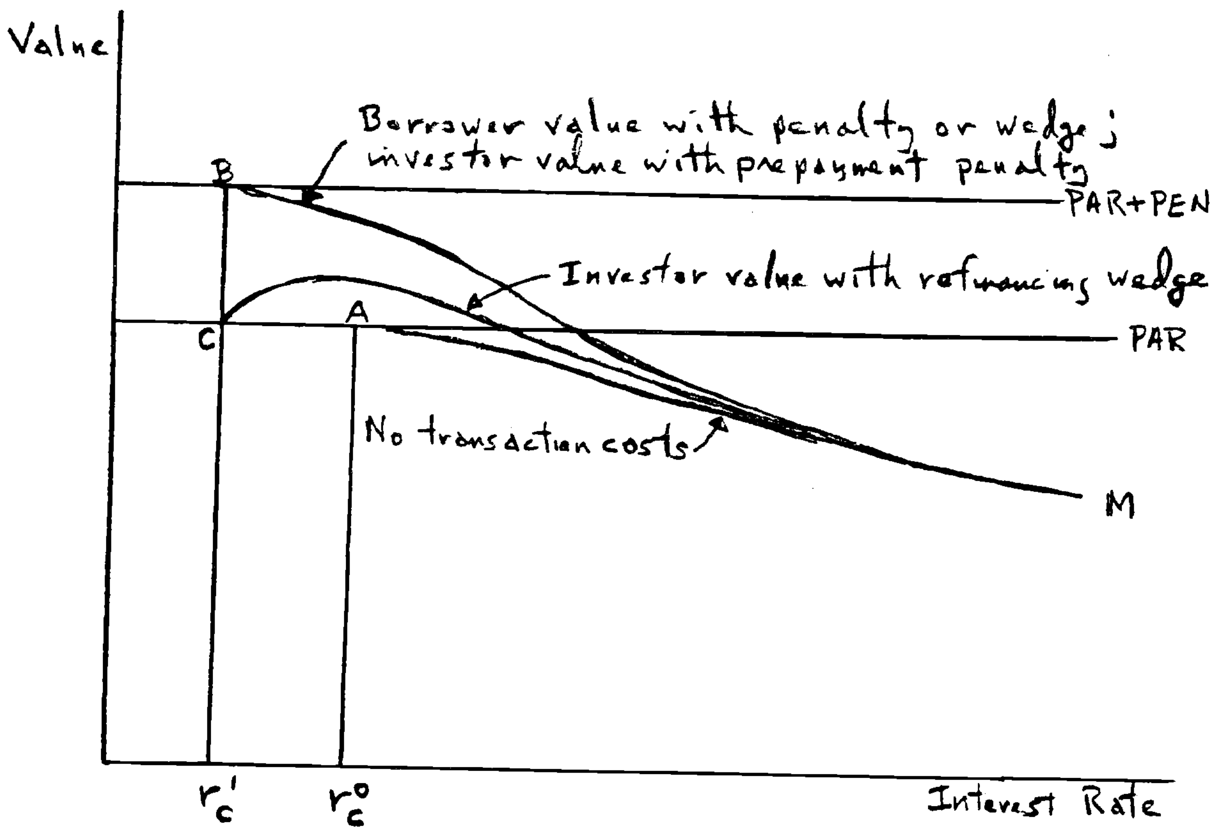
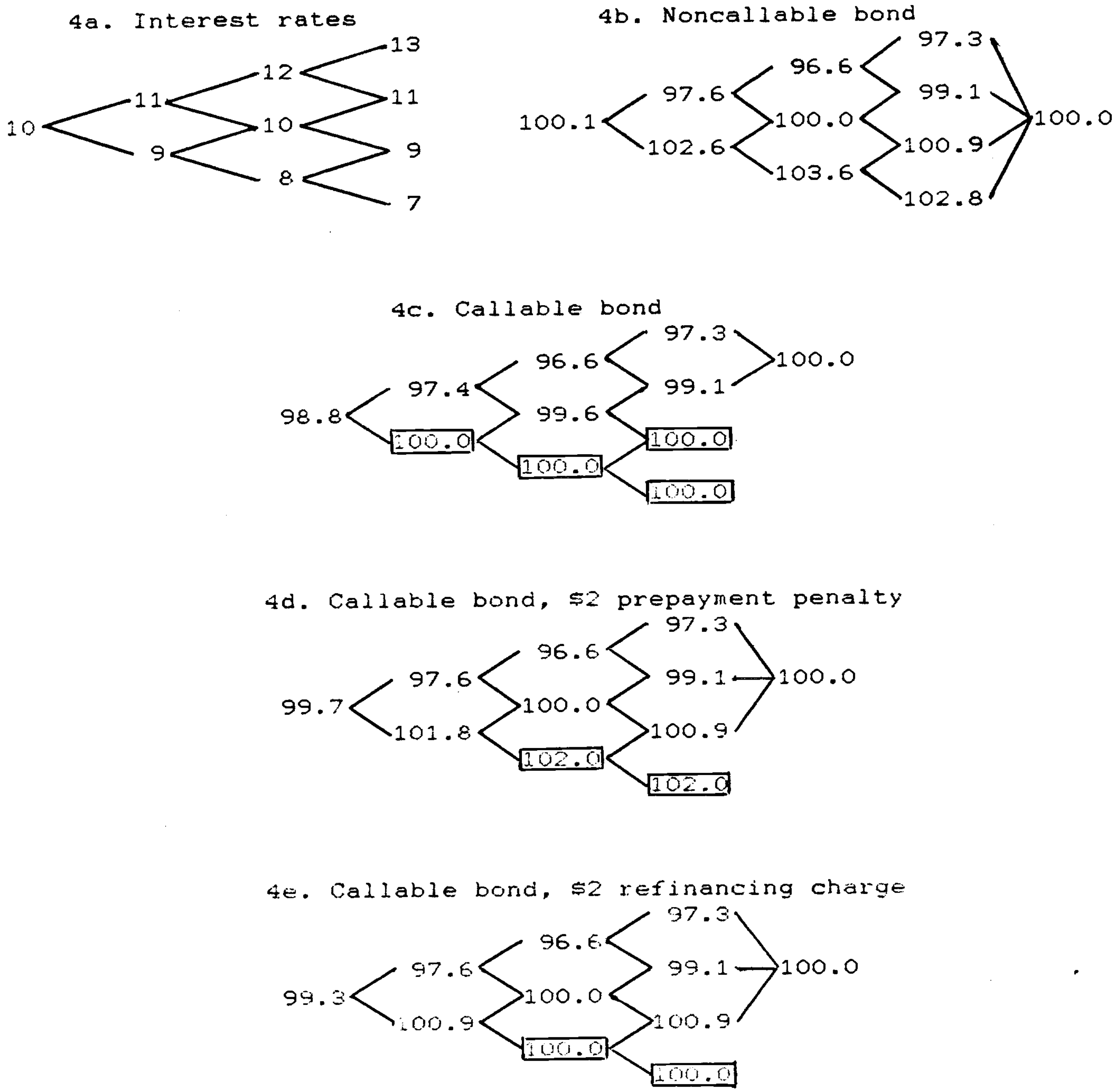


Figure 3: Mortgage Pricing with Transaction Costs.

Figure 4: Simple Binomial Model



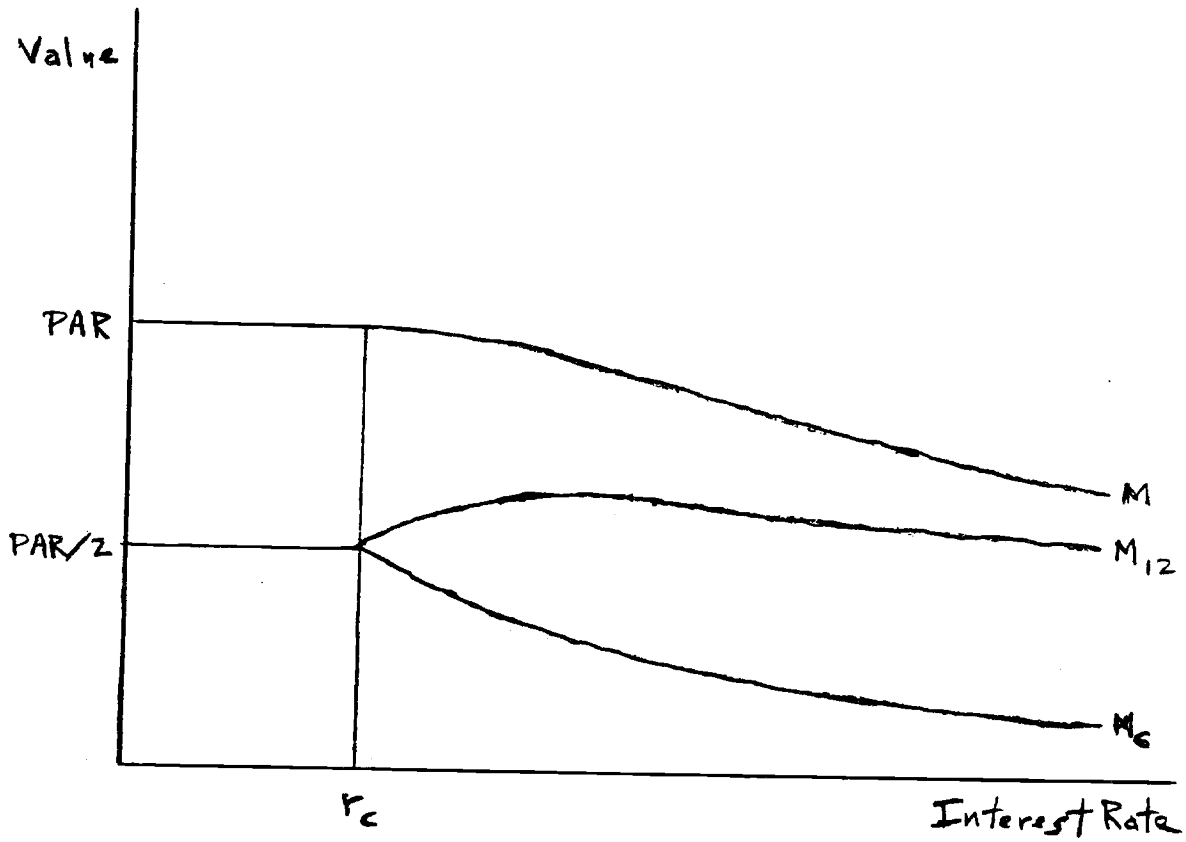


Figure 5: Values of 6 Percent and 12 Percent Strips and the Underlying MBS

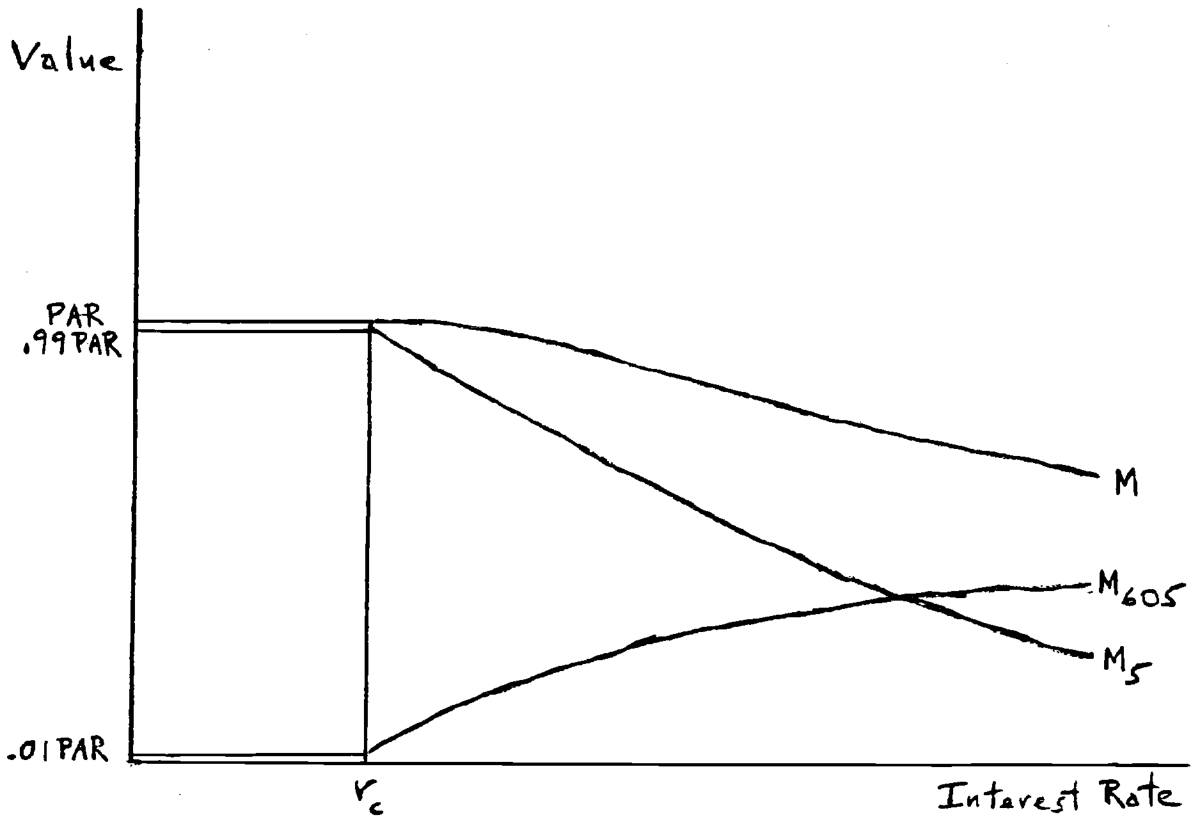


Figure 6: Values of 5 Percent and 605 Percent Strips and the Underlying MBS

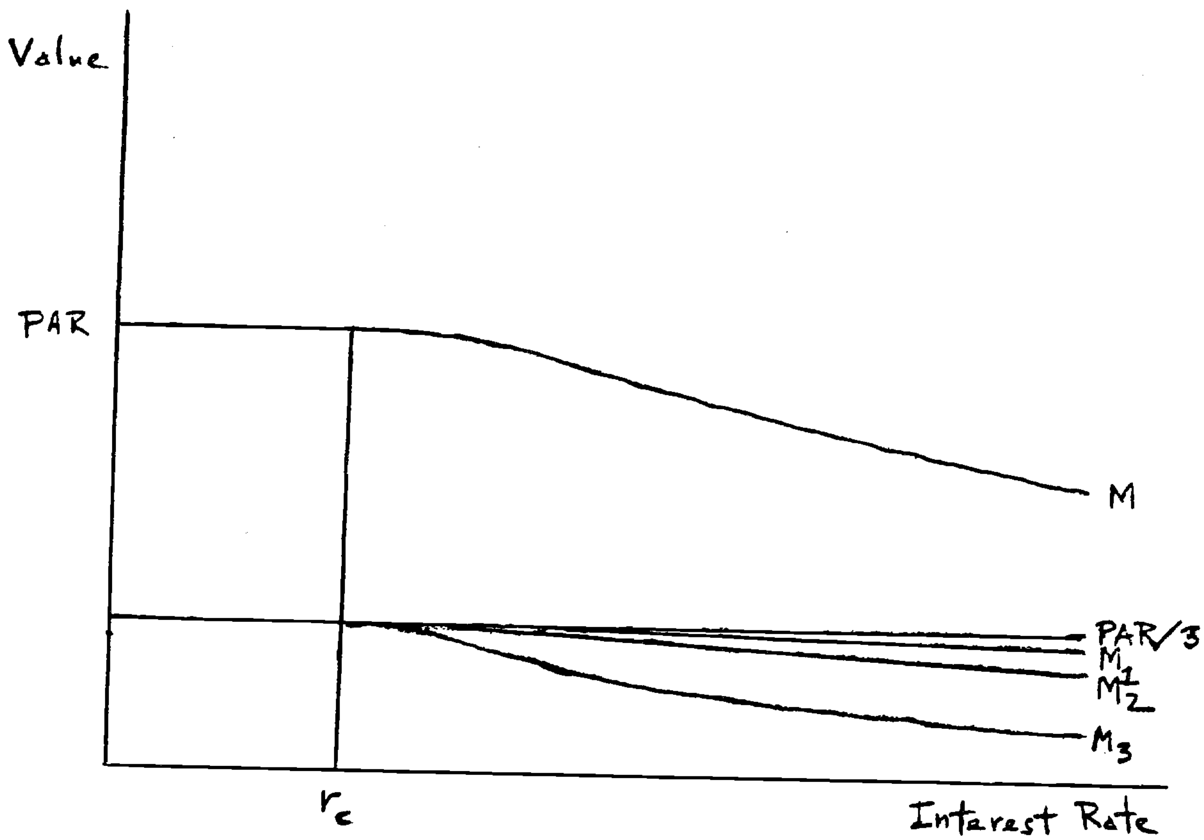


Figure 7: Values of CMO Tranches and the Underlying MBS

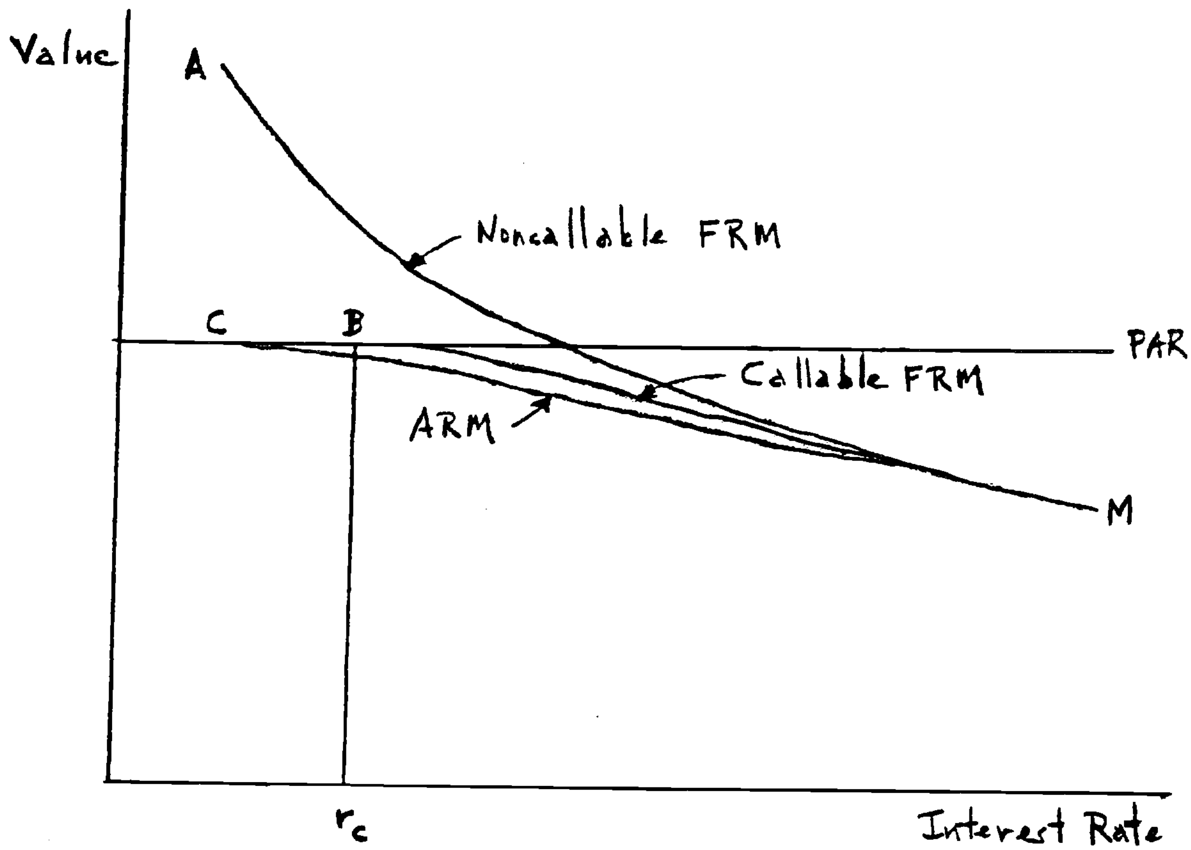


Figure 8: Value Functions for Callable and Noncallable FRMs and an ARM Whose Coupon Plus Life of Loan Cap Equals the Coupon on the FRMs

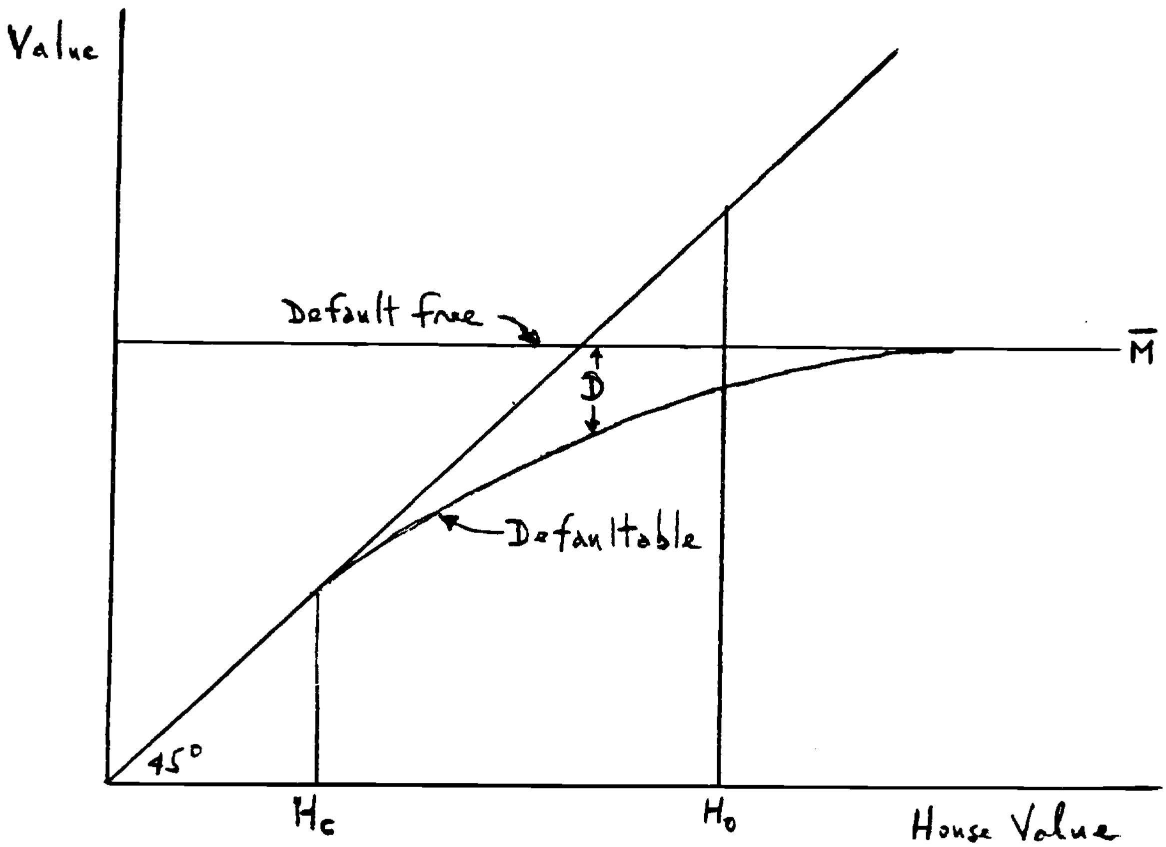


Figure 9: Price of a Defaultable FRM as a Function of House Value

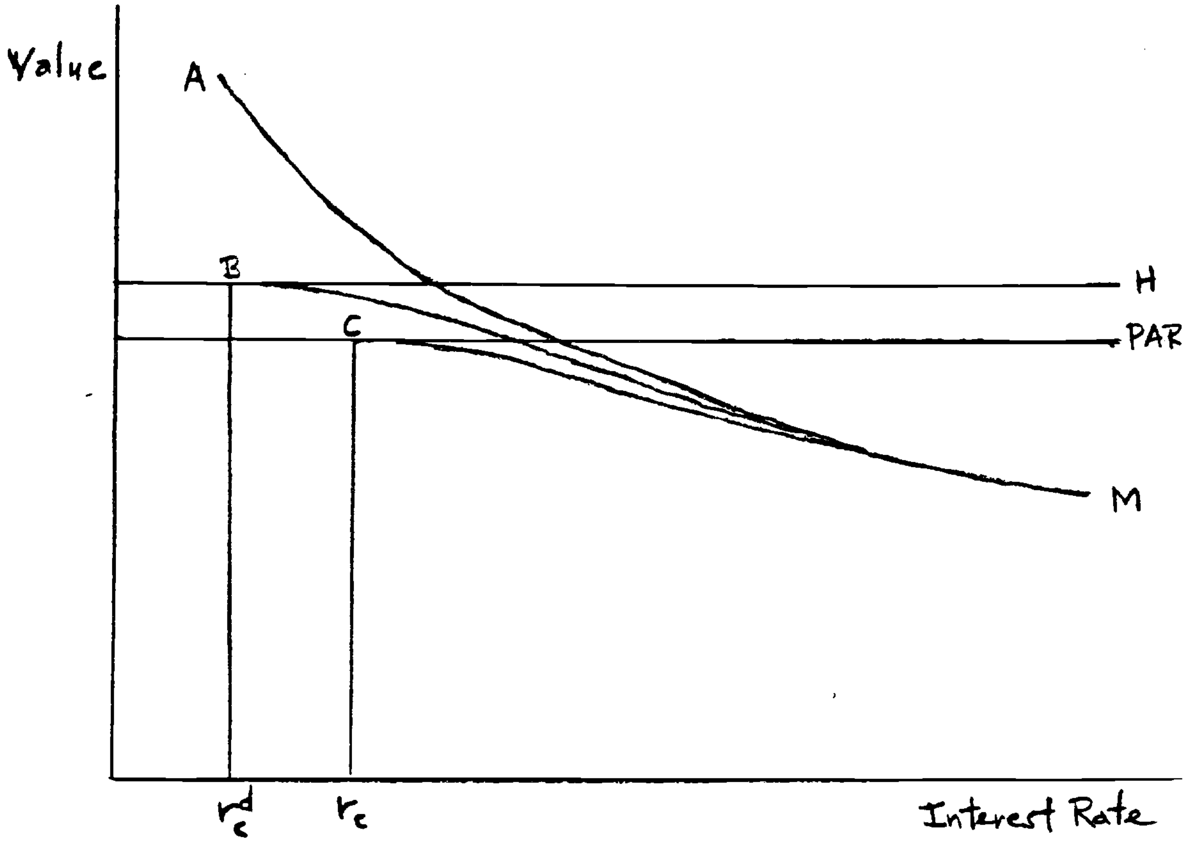


Figure 10: Price of Defaultable FRM as a Function of the Interest Rate

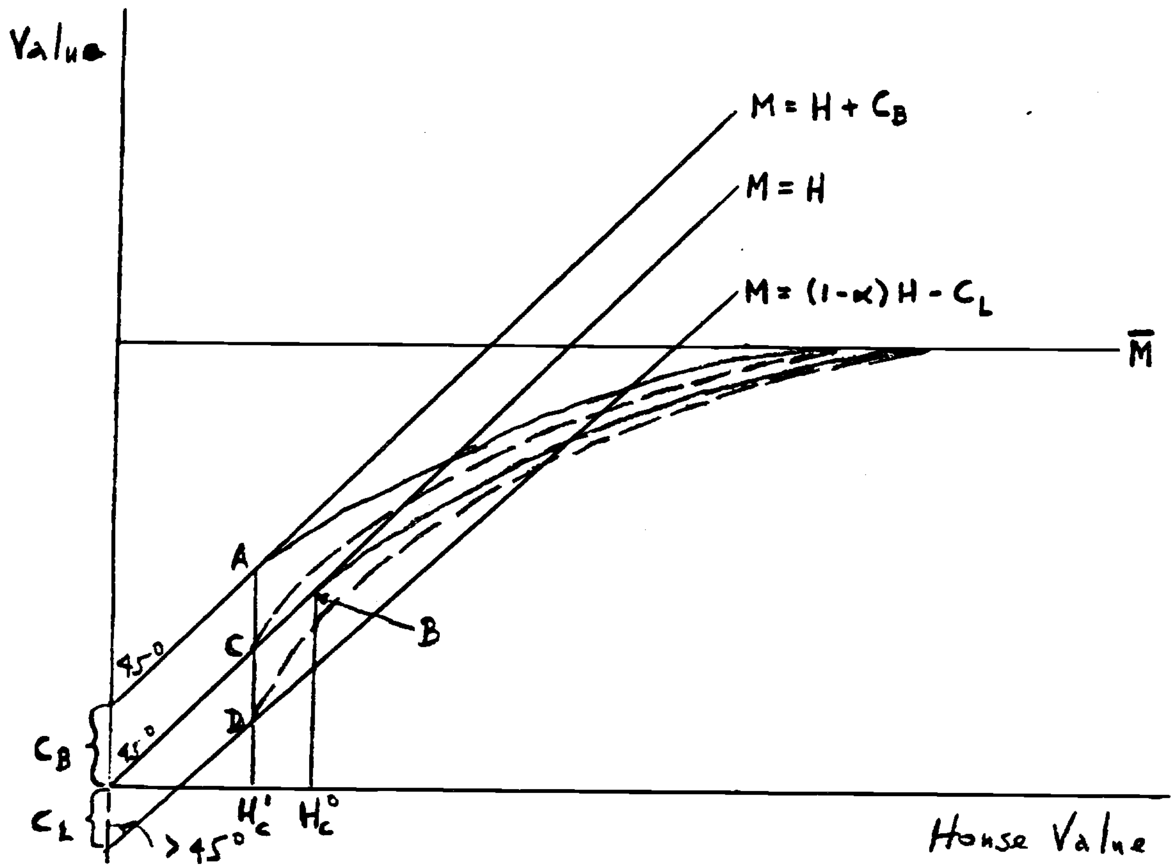


Figure 11: Pricing Default with Transaction Costs