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## PRICING OF EXPORTS AND EXCHANGE RATE UNCERTAINTY

by

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## I. Introduction

International transactions in commodities involve currency translation. These transactions spread over longer periods of time than domestic transactions. In a regime of fixed exchange rates or flexible exchange rates but with fully anticipated changes, the mere fact that international transactions spread over long periods of time, has no real consequences for the behavior of exporters and importers of commodities, and therefore on the volume of trade. However, for more than a decade there has been a significant degree of exchange rate uncertainty. Typically, exchange rate uncertainty inhibits international trade. Consequently various types of institutions and payment mechanisms have emerged to reduce the adverse effects of exchange rate uncertainty on international transactions. Ethier (1972), Clark (1973), Baron (1976b), have examined the impact of forward markets on the behavior of exporting and importing firms, in an environment of exchange rate uncertainty. Exchange rate uncertainty resulted in commodity price uncertainty because prices were quoted in foreign currency. In practice, however exporting (importing) firms set prices either in their own currency or in the buyer's (seller's) currency. This is well documented in the empirical investigation conducted by Grassman (1973), (1976), Magee (1974), Page (1977), Van Nieuwkerk (1979) and Carse, Williamson and Wood (1980). Grassman (1973) found that in $1968,66 \%$ of the Swedish exports contracts to the U.S. and Canada were denominated in Kroners and $25 \%$ in dollars; $95 \%$ of the Swedish imports from the U.S. had contracts denominated in dollars. For the period 1971-1973, Magee (1974) examined the currency of denomination of the contracts of Japanese and German exporting firms to the U.S. He found that $79 \%$ of the contracts for U.S. imports from Germany were denominated in the currency of the German ex-
porters and $17 \%$ in dollars. However, for Japanese exports, $63 \%$ of the contracts were denominated in the currency of the U.S. importers and $37 \%$ in the currency of the exporters. Page (1977) and Carse et.al. (1980) investigated the invoicing practices of U.K. exporters in the 1970's. They found that 76\% of U.K. exports were invoiced in British pounds and $17 \%$ in the importer's currency. For the U.K. imports they found that $51 \%$ were invoiced in the seller's currency and $30 \%$ in the British/importer currency. Van Nieuwkerk (1979) found that during 1973 and 1976, about $50 \%$ of Netherlands exports were invoiced in their own currency while $30 \%$ of its imports were invoiced in the exporter's currencies.

The empirical evidence thus suggests that both the exporter's and importer's currencies are used for invoicing. Furthermore, Carse et.al. conducted interviews with British exporting firms during 1970 and found that the consensus was that the currency of invoice is a matter of choice and that the exporters play the dominant role in determining the currency of invoice.

Theoretical investigations on the choice of currency of invoice are almost inexistent; the exceptions are Baron (1976a) and Giovannini (1988). Baron examined how the invoicing of an exporting firm which sells solely in the foreign market affects its export pricing behavior and its profitability. Baron assumed that the exporters output and thus sales decisions are made ex-ante when the relevant spot exchange rate is unknown. He focused in particular on the impact of risk aversion on the exporters pricing decisions, and on the volume of trade. Giovannini examined the choice of currency denomination of export prices and showed that deviations from the law of one price could be attributed to ex-ante price discrimination between domestic and foreign markets. ${ }^{1}$

The assumption which is common to Baron's study and the theoretical studies mentioned earlier is that production, pricing and export decisions are made at the same point in time. The empirical evidence gathered by Magee (1974) suggests that because of production, transportation and entry lags the currency contract period spreads over many months. The period of contract from the date of the exporter's acceptance of the order till the date of receiving delivery by the importer averages 96 days for U.S. imports from Japan and 76 days for imports from Germany. ${ }^{2,3}$ Therefore, we believe that the assumption that the exporter makes all the ex-ante decisions at the same time, is not realistic. A more accurate description of the exporter's decision making process, allows for a sequential decision process. That is production, pricing and sales allocation decisions are made at different points in time. By doing so one is able to capture and thus examine how the arrival of new information affects the exporter's expectations about the spot exchange rate that will prevail at the time of payment. This in turn affects the exporter's decision on output, pricing and distribution of sales. Furthermore, these decisions will differ depending on the strategy of pricing-cum-invoicing that is chosen by the firm: in the seller's/exporter's currency or in the buyer's/importer's currency.

In this paper we provide a framework which focuses on the dynamic aspects of the exporters decisions and highlight the importance of the invoicing strategies in the context of a sequential decision process regarding the firm's choices of output and allocation of sales across markets. We pay special attention to differences in the level of production, prices, expected quantity of domestic and foreign sales that arise across the two invoicing strategies. The analysis also highlights what sort of conditions result in the superiority of one invoicing strategy over the other.

Some of the main findings are:
(a) The exporting firm's expected profits are higher when exports are invoiced in the importer's currency if some elasticity condition about the demand holds.
(b) The total output produced by an exporting firm is larger when exports are invoiced in the buyer's (importer's) currency rather than in the seller's currency if the marginal revenue functions are concave (which includes the linear case).
(c) The expected level of exports is larger, expected domestic sales are smaller, when exports are invoiced in the buyer's/importer's rather than in the seller's/exporter's currency, if the domestic marginal revenue function is convex and the foreign marginal revenue function is concave.
(d) Finally, when pricing decisions are made ex-post, i.e., after the spot exchange rate is known, (production decisions are still made ex-ante) the exporting firm produces more and its (expected) exports are larger than under either one of the above pricing-cum-invoicing strategies.

## II. The Model

Consider an exporting firm that produces a good which is sold in the domestic and a foreign market. The firm has monopoly power in both markets. Because of impediments to trade the two markets are segregated and thus prices can differ across markets. Lags are inherent in international transactions; they include production lag, transportation lag and the entry lag. ${ }^{4}$ To capture the various lags we distinguish between three decision points in time: Date $t_{0}$ : the time when output decisions are made

Date $t_{1}$ : pricing decisions are made
Date $t_{2}$ : allocation of sales decisions are made.

At each stage the decision is based upon the updated information currently available about the (random) exchange rate that will prevail at the time of payment by the foreign buyer to the exporting firm. Specifically, at time $t_{0}$ the information available to the firm is reflected by the prior-distribution over the random exchange rate $\tilde{e}, F(\tilde{e})$, where $e$ is defined in units of domestic currency per unit of foreign currency. Based on this information the firm selects the optimal level of output. At time $t_{1}$, new information about the exchange rate becomes available; for example, new projections about the trade balance etc. are released. The new information is represented by the parameter $\alpha$, which may take positive or negative values. For instance if the trade deficit (surplus) is larger (smaller) than previously thought the parameter $\alpha$ takes a positive value, and conversely otherwise. Once the value of $\alpha$ is known, expectations about the exchange rate are updated. The remaining uncertainty about the exchange rate $\tilde{e}$, is depicted by the random variable $\bar{\theta}$ with a known distribution. We denote its expected value $\mathrm{E}_{\theta}(\bar{\theta})=\bar{\theta}$. We shall assume that the exchange rate based on the information available at time $t_{1}$, i.e., when $\alpha$ is given, by

$$
\tilde{\mathrm{e}}(\alpha)=\alpha+\tilde{\theta}
$$

At time $t_{1}$ the exporting firm chooses prices to be set in each market. We now introduce some further notation and assumptions. Let $Q$ denote total output, $q$ denotes the quantity of exports, hence $Q-q$ is the quantity sold in the domestic market. The total revenue from domestic sales is $R(Q-q)$ and $R^{*}(q)$ is the total revenue from exports denominated in foreign currency. We assume that $R(\cdot)$ and $R^{*}(\cdot)$ are concave functions. The total cost function in domestic currency is $C(Q)$ and we assume that $C^{\prime}(Q)>0, C^{\prime \prime}(Q)>0$.

Now we examine the behavior of this firm under two pricing-cum-invoicing strategies; first when export prices are set in the importer's currency and second when the prices are set in the exporter's currency. In either case the firm allocates its output selected at $t_{0}$ in a way that satisfies the foreign demand according to the price that was set. Thus the amount sold in the domestic market is the residual quantity (since no apriori commitment is made in this market).

When the firm sets its price for exports in the buyer's currency, the quantity to be exported is known (at date $t_{1}$ ) but the revenue $\tilde{e r}^{*}(q)$ is random since it depends upon the realization of $\bar{\theta}$. However, when the price for the exported good $\hat{p}$ is denominated in the exporter's currency $R^{*}(q)$ is still random since the quantity to be sold in the foreign market at $t_{2}$ depends upon the realization of $\bar{\theta}$, i.e., $q=h\left(\frac{\hat{p}}{\alpha+\theta}\right)$, where $h(\cdot)$ is the foreign demand function. The two cases, obviously, yield different results unless $\bar{\theta}$ is a constant. ${ }^{5}$

In the sequel we assume that the firm sells positive quantities for all possible realizations of the exchange rate $\tilde{e}$; the modification to the case where we allow for zero sale in some market in some states of nature, is basically straightforward.

## A. Invoicing in the Importer's Currency

When making its decision about the output $Q$ (at date $t_{0}$ ) the firm takes into account that the price $p$ it is going to set for exports at date $t_{1}$ will depend upon the signal $\alpha$ (to be observed at $t_{1}$ ), i.e., $p=p(\alpha)$. Since the price is quoted in the currency of the importer and demand has to be satisfied, setting price is equivalent to a commitment of exporting the quantity $\mathrm{q}(\alpha)=\mathrm{h}(\mathrm{p}(\alpha))$ for any $\alpha$. Assuming that the monopolist is an ex-
pected profits maximizer, i.e., the firm is risk neutral, we can write the optimization problem that the firm solves as follows

$$
\begin{equation*}
\max _{Q, p(\alpha)} E_{e}\left[R(Q-h(p(\alpha)))+\overline{\mathrm{e}} \mathrm{R}^{*}(\mathrm{~h}(\mathrm{p}(\alpha)))-\mathrm{C}(\mathrm{Q})\right] \tag{1}
\end{equation*}
$$

In the above maximization problem the expectation operator is over the prior distribution of $\overline{\mathrm{e}}$. Since the firm is risk neutral the way in which the random variable $\bar{\theta}$ affects output and pricing decisions is through its mean $\bar{\theta}$ only; hence integrating over $\theta$ we obtain that (1) can be rewritten as

$$
\begin{equation*}
\max _{\mathrm{Q}, \mathrm{p}(\alpha)} \mathrm{E}_{\alpha}\left[\mathrm{R}(\mathrm{Q}-\mathrm{h}(\mathrm{p}(\alpha)))+(\alpha+\bar{\theta}) \mathrm{R}^{*}(\mathrm{~h}(\mathrm{p}(\alpha)))-\mathrm{C}(\mathrm{Q})\right] \tag{1'}
\end{equation*}
$$

Given the assumption that $R(\cdot), R^{*}(\cdot)$ and $-C(\cdot)$ are concave in their respective arguments, the necessary and sufficient conditions of the above optimization problem are,

$$
\begin{align*}
& E_{\alpha}\left[R^{\prime}\left(Q^{*}-h\left(p^{*}(\alpha)\right)\right)-C^{\prime}\left(Q^{*}\right)\right]=0  \tag{2}\\
& R^{\prime}\left(Q^{*}-h\left(p^{*}(\alpha)\right)\right)-(\alpha+\bar{\theta}) R^{* \prime}\left(h\left(p^{*}(\alpha)\right)\right)=0, \text { for all } \alpha \tag{3}
\end{align*}
$$

where the optimal solution to (2)-(3) is denoted by $Q^{*}$ and $p^{*}(\alpha)$.
The economic interpretation of conditions (2) and (3) are straightforward. By (2) the firm chooses the level of output $Q^{*}$ in a way that equates marginal cost of production with the expected marginal revenue from domestic sales, (which is dependent upon the realization of $\alpha$ through the quantity to be exported. This results from our assumption that the firm sells in both markets in probability 1). From (3) we see that for each realization of $\alpha$, the firm selects the price of export, $\mathrm{p}^{\star}(\alpha)$, in a manner that equates marginal revenue from domestic sales with expected marginal revenue from exports.

## B. Invoicing in the Seller's Currency

As in the previous section the firm chooses its total output $Q$ at date $t_{0}$. In the subsequent stage, once the realization of $\alpha$ is known, the firm sets its export price $\hat{p}(\alpha)$, where now the price is quoted in the exporter's currency. In contrast to the previous case, the exporter does not know at time $t_{1}$ the actual quantity that will be demanded by importers at time $t_{2}$. This quantity is contingent upon the realization of $\tilde{\theta}$ which will determine the actual price in the importers/buyer currency, i.e., $\frac{\hat{p}(\alpha)}{\alpha+\theta}$. Thus, after the realization of $\theta$, the monopolist allocates sales across markets according to $\hat{q}(\theta)=h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)$ and $(Q-\hat{q}(\theta))$ for all $\alpha$. The monopolist is assumed to honour the precommitment to satisfy foreign demand at the prevailing price, i.e., to provide the foreign buyers with the quantity $\hat{q}(\theta)=h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)$ for all $\theta$. Such a precommitment is sensible only if breaching the contract is not profitable, i.e.,

$$
R^{\prime}(\hat{Q}-\hat{q}(\theta)) \leq \hat{p}(\alpha) \quad \text { for all } \theta
$$

Namely for all realizations of $\theta$ the domestic marginal revenue does not exceed the price of exports. Such a condition is satisfied, for example, if the spread of $\theta$ is small.

In view of the above discussion, the firm's optimization problem is,

$$
\begin{equation*}
\max _{Q, p(\alpha)} E_{e}\left[R\left(Q-h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)\right)+\hat{p}(\alpha) h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)-C(Q)\right] \tag{4}
\end{equation*}
$$

In this maximization problem the forward looking monopolist selects at date $t_{0}$ output and at date $t_{1}$ the optimal pricing rule $\hat{p}(\alpha)$, for each $\alpha$, taking into account the impact of the price on the quantity of exports to be demanded in the last stage at $t_{2}$ once $\bar{\theta}$ has realized.

Necessary conditions for optimality of problem (4) are,

$$
\begin{align*}
& E_{e}\left[R^{\prime}\left(Q-h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)\right)-C^{\prime}(Q)\right]=0  \tag{5}\\
& E_{\theta}\left[-\frac{\hat{p}(\alpha)}{\alpha+\theta}\right) \\
& \left.\quad+\frac{\hat{p}(\alpha)}{\alpha+\theta} h^{\prime}\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)\right]=0 \quad \text { for all } \alpha . \tag{6}
\end{align*}
$$

We denote by $\hat{Q}$ and $\hat{p}(\alpha)$ the solution to (5) and (6). Clearly the solution of (5) and (6) differs from the solution of (2) and (3) since in the former the distribution of $\bar{\theta}$ matters while in the latter case only $\bar{\theta}$ matters.

For later use, let us note here that condition (6) can be replaced, using the notation $\hat{q}(\theta)=h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)$, by the equation

$$
\mathrm{E}_{\theta}\left[\mathrm{R}^{\prime}(\hat{\mathrm{Q}}-\hat{\mathrm{q}}(\theta))-(\alpha+\theta) \mathrm{R}^{\prime \prime}(\hat{\mathrm{q}}(\theta))\right]=0 \quad \text { for all } \alpha
$$

This can be verified by either directly rearrangement of (6) or by maximizing (4) with respect to $Q$ and $\hat{q}(\theta)$ rather than $\hat{p}(\alpha)$.

We first examine the profitability of the two invoicing strategies. Specifically we highlight the conditions which lead to the dominance of one invoice strategy over the other. For later use we define the elasticity of the slope of the demand for exports, $\eta f(e)=-\frac{h^{\prime \prime}(\cdot)}{h^{\prime}(\cdot)} \frac{\hat{p}(\alpha)}{\alpha+\theta}$.

Proposition 1. The exporting monopolist will prefer to invoice exports in the importer's currency rather than in its own currency if the following two conditions hold:
(a) the elasticity of the slope of the demand for exports $\eta^{f}(e) \leq 2$ for all e
(b) the marginal revenue from domestic sales does not exceed the price of exports in all states of nature, i.e., $R^{\prime}\left(\hat{Q}-h\left(\frac{\hat{p}(\alpha)}{\alpha+\theta}\right)\right) \leq \hat{p}(\alpha)$ for all e.

Remark: We note from the optimality condition (6') and the fact that $\mathrm{R}^{* \prime}(\cdot) \leq \frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}$ for all $\theta$, that we obtain,

$$
\begin{equation*}
\mathrm{E}_{\theta} \mathrm{R}^{\prime}(\hat{\mathrm{Q}}-\hat{\mathrm{q}}(\theta)) \leq \mathrm{E}_{\theta}(\alpha+\tilde{\theta}) \frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}=\hat{\mathrm{p}}(\alpha) \tag{6"}
\end{equation*}
$$

Hence assumption (b) in Proposition 1 strengthens the condition stated in (6") since the assumption requires that marginal revenue from domestic sales should be less than the price of exports for each realization of $\theta$ rather than for the expected value. Moreover, for the situations where $\alpha$ and $\theta$ are revealed simultaneously (i.e., $t_{0}$ and $t_{1}$ coincide) the condition ( $6^{\prime \prime}$ ) is satisfied automatically.

Proof: First we need to show that when conditions (a) and (b) are satisfied, the total revenue obtained when the price of exports is set in the seller's currency is a concave function. Specifically, for any $\alpha$, we define $\Delta(\theta)$ as

$$
\begin{equation*}
\Delta(\theta)=R\left(\left(\hat{\mathrm{Q}}-\mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}\right)\right)+\hat{\mathrm{p}}(\alpha) \mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}\right) .\right. \tag{7}
\end{equation*}
$$

Differentiating $\Delta(\theta)$ with respect to $\theta$ yields:
$\Delta^{\prime}(\theta)=-R^{\prime}(\cdot) h^{\prime}(\cdot)\left(-\frac{\hat{p}(\alpha)}{(\alpha+\theta)^{2}}\right)+\hat{p}(\alpha) h^{\prime}(\cdot)\left(-\frac{\hat{p}(\alpha)}{(\alpha+\theta)^{2}}\right)$
$\Delta^{\prime \prime}(\theta)=R^{\prime \prime}(\cdot)\left[h^{\prime}(\cdot)\right]^{2}\left[\frac{-\hat{p}(\alpha)}{(\alpha+\theta)^{2}}\right]^{2}-R^{\prime}(\cdot) h^{\prime \prime}(\cdot)\left[\frac{-\hat{p}(\alpha)}{(\alpha+\theta)^{2}}\right]^{2}-$
$-\mathrm{R}^{\prime}(\cdot) \mathrm{h}^{\prime}(\cdot)\left[\frac{2 \hat{\mathrm{p}}(\alpha)}{(\alpha+\theta)^{3}}\right]+\hat{\mathrm{p}}(\alpha) \mathrm{h}^{\prime \prime}(\cdot)\left[\frac{\hat{\mathrm{p}}(\alpha)}{(\alpha+\theta)^{2}}\right]^{2}+\hat{\mathrm{p}}(\alpha) \mathrm{h}^{\prime}(\cdot) \frac{2 \hat{\mathrm{p}}(\alpha)}{(\alpha+\theta)^{3}}=$
$=h^{\prime}(\cdot)\left[\hat{\mathrm{p}}(\alpha)-\mathrm{R}^{\prime}(\cdot)\right] \frac{\hat{\mathrm{p}}(\alpha)}{(\alpha+\theta)^{3}}\left[\frac{\mathrm{~h}^{\prime \prime}(\cdot)}{\mathrm{h}^{\prime}(\cdot) \frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}}+2\right]+\mathrm{R}^{\prime \prime}(\cdot)\left[\mathrm{h}^{\prime}(\cdot)\right]^{2} \frac{\hat{\mathrm{p}}(\alpha)^{2}}{(\alpha+\theta)^{4}}$

Inspection of (8) reveals that $\Delta^{\prime \prime}(\theta)<0$ since by assumption $R^{\prime \prime}(\cdot)<0$, and by conditions (a) and (b), $\eta^{f}(e)=-\frac{h^{\prime \prime}(\cdot)}{h^{\prime}(\cdot)} \frac{\hat{p}(\alpha)}{\alpha+\theta} \leq 2$ and $\hat{p}(\alpha) \geq R^{\prime}(\cdot)$. The concavity of $\Delta(\theta)$ enables us to write the following chain of inequalities:

$$
\begin{align*}
& \mathrm{E}\left[\mathrm{R}\left(\hat{\mathrm{Q}}-\mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}\right)\right)+\hat{\mathrm{p}}(\alpha) \mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\theta}\right)-\mathrm{C}(\hat{\mathrm{Q}})\right] \leq \\
& \mathrm{E}_{\alpha}\left[\mathrm{R}\left(\hat{\mathrm{Q}}-\mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\bar{\theta}}\right)+\hat{\mathrm{p}}(\alpha) \mathrm{h}\left(\frac{\hat{\mathrm{p}}(\alpha)}{\alpha+\bar{\theta}}\right)-\mathrm{C}(\hat{\mathrm{Q}})\right]=\right. \\
& \mathrm{E}_{\alpha}[\mathrm{R}(\hat{\mathrm{Q}}-\mathrm{h}(\overline{\mathrm{p}}(\alpha))+(\alpha+\bar{\theta}) \overline{\mathrm{p}}(\alpha) \mathrm{h}(\overline{\mathrm{p}}(\alpha))-\mathrm{C}(\hat{\mathrm{Q}})] \leq \\
& \max _{\mathrm{p}(\alpha)} \mathrm{E}_{\alpha}[\mathrm{R}(\hat{\mathrm{Q}}-\mathrm{h}(\mathrm{p}(\alpha))+(\alpha+\bar{\theta}) \mathrm{p}(\alpha) \mathrm{h}(\mathrm{p}(\alpha))-\mathrm{C}(\hat{\mathrm{Q}})] \leq \\
& \max _{\mathrm{Q}, \mathrm{p}(\alpha)} \mathrm{E}_{\alpha}[\mathrm{R}(\mathrm{Q}-\mathrm{h}(\mathrm{p}(\alpha)))+(\alpha+\bar{\theta}) \mathrm{p}(\alpha) \mathrm{h}(\mathrm{p}(\alpha))-\mathrm{C}(\mathrm{Q})] . \tag{9}
\end{align*}
$$

The first expression in (9) is the maximum expected profits when the firm invoices exports in its own currency. The first inequality is due to the concavity of $\Delta(\theta)$ in $\theta$. The following equality holds since $\hat{p}(\alpha)=(\alpha+\bar{\theta}) \bar{p}(\alpha)$. However, $\overline{\mathrm{p}}(\alpha)$ is not the optimal price when $\theta=\bar{\theta}$ and thus the next inequality follows. The last inequality is self-explanatory. The last expression in (9) depicts the maximum profits attained when the price of exports is set in the importer's currency. ${ }^{6}$
Q.E.D.

We now proceed to compare the levels of output, exports and domestic sales under these two invoicing strategies. For the remainder of this section we assume that the firm selects its output after it knows the realization of $\alpha$; that is we combine the dates $t_{0}$ and $t_{1}$ into one date. Let us emphasize here that linear functions are concave and convex functions as well.

Proposition 2. Invoicing exports in the importer's currency entails a larger level of production than when the exports are invoiced in the exporter's currency, i.e., $Q^{*}>\hat{Q}$, if the domestic and foreign marginal revenue functions are both concave.

Proof: Suppose to the contrary that $Q^{*}<\hat{Q}$. Using equations (2)-(6) and the concavity of $R^{\prime}(\cdot)$ yields:

$$
\begin{align*}
0 & <C^{\prime}(\hat{Q})-C^{\prime}\left(Q^{*}\right)=E_{\theta}\left[R^{\prime}(\hat{Q}-\hat{q}(\theta))-R^{\prime}\left(Q^{*}-q^{*}\right)\right] \leq \\
& \leq R^{\prime \prime}\left(Q^{*}-q^{*}\right)\left[\left(\hat{Q}-E_{\theta} \hat{q}(\theta)\right)-\left(Q^{*}-q^{*}\right)\right] \tag{10}
\end{align*}
$$

Since $R^{\prime \prime}\left(Q^{*}-q^{*}\right)<0$, it follows from (10) that

$$
\begin{equation*}
\hat{\mathrm{Q}}-\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)<\mathrm{Q}^{*}-\mathrm{q}^{*} \tag{11}
\end{equation*}
$$

From the first order conditions (2), (3) and (5), ( $6^{\prime}$ ) in conjunction with the assumption that $\hat{Q}>Q^{*}$ we obtain that

$$
\begin{align*}
& (\alpha+\bar{\theta}) \mathrm{R}^{*} \cdot\left(\mathrm{q}^{*}\right)<\mathrm{E}_{\theta}(\alpha+\tilde{\theta}) \mathrm{R}^{*} \cdot(\hat{\mathrm{q}}(\theta)) \leq \\
& \leq(\alpha+\bar{\theta}) \mathrm{E}_{\theta} \mathrm{R}^{*} \cdot(\hat{\mathrm{q}}(\theta)) \leq(\alpha+\bar{\theta}) \mathrm{R}^{*} \cdot\left(\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)\right) \tag{12}
\end{align*}
$$

The first inequality in (12) is due to the fact that $\hat{q}(\theta)$ is increasing in $\theta$ and $\operatorname{cov}\left[\tilde{\theta}, \mathrm{R}^{*}(\hat{\mathrm{q}}(\theta))\right]<0$, and the second inequality follows from the concavity of $R^{* \prime}(\cdot)$. The above chain of inequalities in (12) imply that

$$
\begin{equation*}
\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)<\mathrm{q}^{*} \tag{13}
\end{equation*}
$$

Now (11), (13) and the assumption that $Q^{*}<\hat{Q}$ imply

$$
\hat{\mathrm{Q}}=\left[\hat{\mathrm{Q}}-\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)\right]+\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)<\left(\mathrm{Q}^{\star}-\mathrm{q}^{\star}\right]+\mathrm{q}^{\star}=\mathrm{Q}^{\star}
$$

Since this is a contradiction it follows that $Q^{*}>\hat{Q}$.
Q.E.D.

Proposition 3. If the domestic marginal revenue is convex and the foreign marginal revenue is concave, then invoicing foreign sales in the importer's currency rather than in the exporter's currency results in (a) more exports, i.e., $q^{*}>E_{\theta} \hat{q}(\theta)$ and (b) lower domestic sales, i.e., $Q^{*}-q^{*}<\hat{Q}-E_{\theta} \hat{q}(\theta)$.

Proof: Like in the proof of Proposition 2 we use the first order conditions (2) $-\left(6^{\prime}\right)$. Consider first the case where $Q^{*}>\hat{Q}$. Then due to the convexity of $R^{\prime}(\cdot)$,
$0>C^{\prime}(\hat{Q})-C^{\prime}\left(Q^{*}\right)=E_{\theta}\left[R^{\prime}(\hat{Q}-\hat{q}(\theta))-R^{\prime}\left(Q^{*}-q^{*}\right)\right] \geq$

$$
\begin{equation*}
\geq E_{\theta}\left[R^{\prime \prime}\left(Q^{*}-q^{*}\right)\left((\hat{Q}-\hat{q}(\theta))-\left(Q^{*}-q^{*}\right)\right)=R^{\prime \prime}\left(Q^{*}-q^{*}\right)\left[\hat{Q}-E_{\theta} \hat{q}(\theta)-\left(Q^{*}-q^{*}\right)\right]\right. \tag{14}
\end{equation*}
$$

Since $\hat{Q}<Q^{*}$ it follows from (14) that $Q^{*}-\mathrm{q}^{*}<\hat{\mathrm{Q}}-\mathrm{E}_{\theta} \hat{\mathrm{Q}}(\theta)$ which in turn also implies that $\mathrm{q}^{*}>\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)$.

Consider now the case where $Q^{*}<\hat{Q}$. Then using equations (2)-(7) and the concavity of $R^{* \prime}(\cdot)$ we obtain that,

$$
\begin{aligned}
& 0<C^{\prime}(\hat{\mathrm{Q}})-C^{\prime}\left(\mathrm{Q}^{*}\right)=\mathrm{E}_{\theta}\left[\mathrm{R}^{\prime}(\hat{\mathrm{Q}}-\hat{\mathrm{q}}(\theta))-\mathrm{R}^{\prime}\left(\mathrm{Q}^{*}-\mathrm{q}^{*}\right)\right]= \\
& =\mathrm{E}_{\theta}(\alpha+\bar{\theta})\left[\mathrm{R}^{* \prime}(\hat{\mathrm{q}}(\theta))-\mathrm{R}^{{ }^{\prime}}\left(\mathrm{q}^{*}\right)\right]= \\
& =(\alpha+\bar{\theta})\left[\mathrm{E}_{\theta} \mathrm{R}^{* \prime}(\hat{\mathrm{q}}(\theta))-\mathrm{R}^{{ }^{\prime}}\left(\mathrm{q}^{*}\right)\right]+\operatorname{cov}\left(\theta, \mathrm{R}^{* \prime}(\hat{\mathrm{q}}(\theta))\right)< \\
& <(\alpha+\bar{\theta})\left[\mathrm{R}^{* \prime}\left(\mathrm{E}_{\theta} \hat{\mathrm{q}}(\theta)\right)-\mathrm{R}^{{ }^{\prime}}\left(\mathrm{q}^{*}\right)\right]
\end{aligned}
$$

The last inequality is due to the fact that the covariance is negative. But this implies that $\mathrm{E}_{\theta} \hat{\mathrm{q}}(\cdot)<\mathrm{q}^{*}$; thus, for the case considered now, this also implies that $\hat{Q}-E_{\theta} \hat{q}>Q^{*}-q^{*}$.
Q.E.D.

## C. Spot Transactions

The investigation conducted in the preceding section presumed that the time lag between the pricing and sales allocation decision is significant. Consequently, the firm had to precommit first to a pricing strategy, and second to provide buyers the quantity demanded once the exchange rate is fully known.

There are however situations like international transactions within the EEC where the lag involved between shipping and the arrival of the product is
relatively short; this implies that the pricing and the sales allocation are made when the actual exchange rate is known. In this environment the firm does not need to precommit to price. After the exchange rate $e$ is observed, there is no uncertainty and the firm distributes output across markets to maximize total revenue. However, the firm's output decision is made ex-ante as in the previously examined cases.

It is interesting to compare the firm's behavior, namely its output, prices and quantity of exports when the transportation and distribution lags are significant and therefore it has to precommit to price relative to the situation of no precommitment.

To this end we first derive the optimal behavior under no priceprecommitment. To simplify our analysis we shall assume as before that for all realizations of $\bar{e}$, the firm finds it profitable to serve both markets. At time $t_{1}$ after the exchange rate $e$ is observed, the firm allocates total output $Q$ (chosen at time $t_{0}$ ) to equate marginal revenues in both markets. ${ }^{7}$ The firm's maximization problem at time $t_{0}$ is,

$$
\max _{Q, p(e)} E\left[R(Q-h(p(\tilde{e})))+\tilde{e} R^{*}(h(p(\tilde{e})))-C(Q)\right]
$$

where $0 \leq q(e)=h(p(e)) \leq Q$ for all $e$ and $\bar{e}^{*}{ }^{*}(h(p(\bar{e}))=\tilde{e} p(\bar{e}) h(p(\bar{e}))$.
The necessary and sufficient conditions for optimum with positive sales in both markets, are

$$
\begin{align*}
& E_{e}\left[R^{\prime}(\bar{Q}-h(\bar{p}(e)))-C^{\prime}(\bar{Q}]=0\right.  \tag{15}\\
& R^{\prime}(\bar{Q}-h(\bar{p}(e)))-e R^{\prime \prime}(h(\bar{p}(e)))=0 \quad \text { for all } e . \tag{16}
\end{align*}
$$

Denote the solution to (15) and (16), by $\bar{Q}$ and $\overline{\mathrm{p}}(\mathrm{e})$, and note that it is the optimum to the above maximization.

We now examine the differences between the price-precommitment strategy and the case of no price precommitment. Specifically we focus on the firms decisions about output, quantity of exports and profitability when the precommited price is invoiced in the currency of the importer and when it does not precommit to price.

Proposition 4. If the marginal revenue function in the domestic market is convex and the marginal revenue function in the foreign market is concave, then in the absence of price-precommitment the firm produces more, and exports more than in the case with price-precommitment, i.e., $\overline{\mathrm{Q}}>\mathrm{Q}^{*}$ and $E_{\mathrm{e}} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e}))>\mathrm{h}\left(\mathrm{p}^{*}(\alpha)\right)$ for all $\alpha$.

Proof: The assumption that domestic marginal revenue is convex and foreign marginal revenue is concave imply that for each $\alpha$
(i) $\quad E_{\theta} R^{\prime}(\bar{Q}-h(\bar{p}(\bar{e}))) \geq R^{\prime}\left(\bar{Q}-E_{\theta} h(\overline{\mathrm{p}}(\tilde{\mathrm{e}}))\right)$
(ii) $\mathrm{E}_{\theta}(\alpha+\bar{\theta}) \mathrm{R}^{{ }^{\prime \prime}}(\mathrm{h}(\overline{\mathrm{p}}(\mathrm{e})))<(\alpha+\bar{\theta}) \mathrm{E}_{\theta} \mathrm{R}^{\mathrm{A}^{\prime}}(\mathrm{h}(\overline{\mathrm{p}}(\mathrm{e})))<$

$$
\begin{equation*}
<(\alpha+\bar{\theta}) \mathrm{R}^{{ }^{\prime}}\left(\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\tilde{\mathrm{e}}))\right) \tag{17}
\end{equation*}
$$

The inequality in (i) is due to convexity of $R^{\prime}(\cdot)$, and the inequalities in (ii) are due to concavity of $\mathrm{R}^{* \prime}(\cdot)$ which implies that the $\operatorname{cov}\left(e, \mathrm{R}^{* \prime}(\mathrm{~h}(\overline{\mathrm{p}}(\mathrm{e}))\right.$ ) $<0$. Since $e=\alpha+\theta$, (15) and (17) imply,

$$
\begin{align*}
& \mathrm{E}_{\alpha}\left[\mathrm{R}^{\prime}\left(\overline{\mathrm{Q}}-\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\tilde{\mathrm{e}}))\right)-\mathrm{C}^{\prime}(\overline{\mathrm{Q}})\right] \leq 0  \tag{18}\\
& \mathrm{R}^{\prime}\left(\overline{\mathrm{Q}}-\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e}))\right)-(\alpha+\bar{\theta}) \mathrm{R}^{* \prime}\left(\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e}))\right)<0 \quad \text { for all } \alpha . \tag{19}
\end{align*}
$$

Now comparing (2) and (3) with (18) and (19) leads to $\bar{Q}>Q^{*}$. To show that assume to the contrary $\overline{\mathrm{Q}}<\mathrm{Q}^{*}$. This assumption and (19) imply that $\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e})) \leq \mathrm{q}^{\star}(\alpha)$ for all $\alpha$. Since $\mathrm{R}^{\prime}(\cdot)$ is decreasing, (3) and (18) imply that $\mathrm{E}_{\alpha} \mathrm{R}^{\prime}\left(\overline{\mathrm{Q}}-\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e})) \geq \mathrm{E}_{\alpha} \mathrm{R}^{\prime}\left(\mathrm{Q}^{*}-\mathrm{q}^{*}\right)\right.$. Hence by (2) and (18) we obtain $C^{\prime}(\bar{Q}) \geq C^{\prime}\left(Q^{*}\right)$ and thus $\bar{Q} \geq Q^{*}$ which is a contradiction. Thus $\bar{Q}>Q^{*}$ and $\mathrm{E}_{\theta} \mathrm{h}(\overline{\mathrm{p}}(\mathrm{e}))>\mathrm{h}\left(\mathrm{p}^{*}(\alpha)\right)$ for all $\alpha$.
Q.E.D.

To establish that expected profits are higher when the firm does not precommit to price, let us suppose that the total output $\bar{Q}$ that has been chosen is equal to $Q^{*}$. In the absence of price precommitment the firm maximizes profits by choosing the export price function $\overline{\mathrm{p}}(\mathrm{e})$ which depends on e . Suppose that in the no precommitment case the firm sets a price equal to $\mathrm{p}^{*}(\alpha)$, i.e., the optimal price when exports are invoiced in the buyer's currency, regardless of the realization of $\theta$. Since the feasible set, i.e. the pairs $(\overline{\mathrm{Q}}, \overline{\mathrm{p}}(\mathrm{e}))$, in the no-precommitment case includes the set of all ( $\mathrm{Q}, \mathrm{p}(\alpha)$ ) where the firm precommits to price, the optimum in the former case is higher. It can also be shown that no precommitment dominates the strategy to precommit to a price which is quoted in the exporter's currency. In particular, under the conditions of Proposition 1 the later strategy was shown to be dominated by precommitment to a price set in the importer's currency.

## III. Concluding Remarks

The purpose of this paper was to examine the dynamic aspects of the exporters decisions regarding production, pricing and sales distribution when faced with fluctuating exchange rates. Because of the lags inherent in international transactions exporting firms need to precommit to export prices be-
fore the exchange rate is known. We examined two types of priceprecommitment: the price is quoted in the importer's currency versus the case where the price is quoted in the exporter's currency. Our analysis showed that the currency in which the export prices are invoiced has a significant impact on the exporting firm's levels of output, expected exports and domestic sales. Specifically, we found that if certain reasonable conditions hold, the profitability of the exporting firm is enhanced and total production and expected exports are larger when exports are invoiced in the importer's currency rather than in its own currency. Furthermore the need to precommit to a price, regardless of the currency of invoice, reduces the profitability of the exporting firm and lowers its total production and exports relative to the case of no price precommitment.

## Footnotes

1. Magee and Rao (1980) and Bilson (1983) also examined the issue of currency invoicing between exporters and importers. They assume that the quantity of the traded good is exogenously given and focus on the effects of risk aversion on the currency composition of the contract.
2. Magee (1974) investigated the length of the period contract for exports from Japan and Germany to the U.S. He found that on the average it takes 96 days from the time of the exporter acceptance of the order to delivery for U.S. imports from Japan and 76 days for U.S. imports from Germany. The distribution of contracts were skewed to the right with a maximum length of 22 months from imports from Japan and 19 months from Germany. Carse, et.al. (1980) looked at the overall average length of the period of contract of exports and imports of U.K. and found it to be six months for exports and four months for imports. In the same study these authors looked also at the length of the credit period, that is the time between registration of goods at customs and payment being received by exporters. They found it to be 2 months for U.K. exports and $1 / 2$ months for U.K. imports.
3. Magee also found that the distribution of contract lengths were skewed to the right with a maximum length of 22 months for imports from Japan and 19 months for import from Germany.
4. The sequence of decision making that we examine differs from that suggested by Giovannini (1988) who assumes that prices are set before the realization of the exchange rate whereas production is adjusted ex-post to fulfill realized demands.
5. The conclusion that the exporting firm prefers to set export prices in the importer's currency appears to be quite robust. Baron (1976) and Giovannini (1988) have arrived at a similar result even though their assumption on the sequence of decision making differs from ours.
6. Here invoicing exports in the importer's or seller's currency is inconsequential since the actual exchange rate is known.

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