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Iftekhar Hasan – Cristiano Zazzara

# Pricing risky bank loans in the new Basel II environment




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## **Pricing risky bank loans in the new Basel II environment**

The views expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland.

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# Pricing risky bank loans in the new Basel II environment

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## Abstract

Recently, banking literature has had a quest for appropriate pricing of bank loans under the new Basel II rules and has been in pursuit of possible outcomes for undertaking such credit risk. In this paper, we propose a simplified formula to price bank's corporate loans, aiming at making bank managers aware of the creation/destruction of shareholder value. We show that the mathematical treatability of the proposed formula and its easy feeding with internal and market inputs allow simple implementation by the final user.

Key words: Basel II, rating, pricing, exposure at default, EVA

JEL classification numbers: C63, G12, G21, G28

# Riskillisten pankkilainojen hinnoittelu uudessa Basel II -järjestelmässä

Suomen Pankin tutkimus  
Keskustelualoitteita 3/2006

Iftekhar Hasan – Cristiano Zazzara  
Rahapolitiikka- ja tutkimusosasto

## Tiivistelmä

Viimeaikaisessa pankkitutkimuksessa on intensiivisesti etsitty Basel II -sääntöjen alaisille pankkiluotoille sopivaa hinnoittelua sekä tarkasteltu näihin lainoihin liittyvän luottoriskin mahdollisia seurauksia. Tässä työssä esitetään yksinkertaistettu malli pankkien myöntämien yrityslainojen hinnoitteluksi. Mallin perimmäinen tarkoitus on tehdä pankinjohtajat tietoisiksi osakkeenomistajan varallisuuden luomiseen ja tuhoamiseen vaikuttavista tekijöistä. Työssä osoitetaan, että esitetyn mallin tekninen helppokäyttöisyys ja vaivaton yhdistäminen markkinoilta ja pankista saatavaan tietoon mahdollistavat sen, että loppukäyttäjä voi mutkattomasti toteuttaa mallin mukaista hinnoittelua käytännössä.

Avainsanat: Basel II, reittaus (riskiluokitus), hinnoittelu, TAL (taloudellinen arvonlisä)

JEL-luokittelu: C63, G12, G21, G28



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# 1 Introduction

The measurement of a bank counterparty risk is a widely discussed topic both in practice and in literature. The new Basel II rules<sup>1</sup>, based on the recognition of individual credit risk through internal rating systems and portfolio risk through a simplified mathematical formula<sup>2</sup>, make the estimation and pricing of credit risk official in the Banking environment. Notwithstanding the many critiques of the Basel II rules by the academic and financial community, and – *last but not least* – by leading Institutions, it cannot be denied that the Basel Committee has contributed to increase risk awareness in the financial world.

In order to create value for their shareholders and subordinated note-holders, banks' managers must correctly measure risk and price it accordingly. Defining and measuring the link between risk and pricing is a successful key for banking business, especially in the activity of customer loans, where clients represent the main asset of a commercial bank. Today, the need of risk adjusted pricing and, consequently, of risk adjusted performance measures (RAPM)<sup>3</sup>, is mainly due to two reasons: 1) The new Basel II capital requirements are directly associated with risk and therefore it is necessary to correctly estimate the riskiness of credit exposures to avoid unjustified increase in capital, which will become an even scarcer resource; 2) Shareholders always expect high returns thus demanding a specific and focused business policy aimed at value creation.

In this research, we propose a methodology to estimate risk adjusted rates and spreads for banks' corporate loans, making use of the same inputs needed to calculate the new Basel II capital requirements. Furthermore, we will set out the relation between our risk adjusted spread and the EVA<sup>TM</sup> and RAROC indicators, showing why our internal measure can be considered as a real benchmark of economic value creation for a bank.

The structure of the research is as follows. In section 2 we define the main factors of a risk-adjusted pricing formula. Furthermore, we present the mathematical formulas to derive interest rates and spreads for each of the three credit risk components (expected loss, unexpected loss, and liquidity cost). In section 3 we set out the relation between our internal risk-adjusted spread and two very well known and used performance indicators: EVA<sup>TM</sup> and RAROC. In section 4, we provide an application of the proposed methodology on corporate loans, arriving at determining the internal risk-adjusted spreads for rating classes and maturities, with reference to the Corporate segments proposed by Basel II

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<sup>1</sup> See Basel Committee on Banking Supervision (1999, 2001, 2003, 2005).

<sup>2</sup> The formula for calculating the Basel II capital requirement has been developed by Gordy (2003).

<sup>3</sup> See Saunders, Allen (2002) and Saita (2003) for a clear description of the issues related to the measurement of banks' performances based on credit risk.

(*Corporate, SME-Corporate and SME-Retail*). In section 5, we compare our internal risk-adjusted spreads with those of the European Corporate Bond Market, highlighting and commenting on the possible differences. In section 6, we conclude with some remarks on the potential for further developments of the proposed methodology. Finally, in Appendices 1 and 2 we provide further clarifications on the methodology we have used while in Appendix 3 we derive the risk-adjusted interest rates for corporate loans (with fixed exposures) characterized by specific repayment plans.

## 2 The loan pricing methodology: a distinction between ‘technical’ and ‘commercial’ pricing

An appropriate pricing formula for a credit exposure has to consider various risk factors, such as the counterparty’s Probability of Default (PD), the Recovery Rate (RR) related to the granted facility (mortgage, loan and loan commitment, etc...), the maturity (M), the Exposure in the event of Default (EAD) and the amount of Regulatory Capital requested by the supervisory authorities (RC).

The credit risk components to be analyzed are detailed below:

- **Expected Loss:** such losses are related to the counterparty’s probability of default and the recovery rate on the specific credit facility, whose estimates stem from the Bank’s internal rating system. Since these losses are expected, they must be covered by appropriate accounting provisions.
- **Unexpected Loss:** such losses are a function of the PD volatility and consequently of the bank’s portfolio correlation. The bank has to use economic capital to hedge these losses. An estimate of these losses can be obtained through a portfolio model – internal, external or regulatory – based on a methodology *à-la-VaR*.

In the next years, when Basel II will become effective, banks should remunerate the the capital absorbed by loans according to a new methodology, which is derived from the modern literature on portfolio risk measurement. The regulatory capital will therefore be more aligned to the economic capital estimated by internal models and will constitute a risk sensitive component of cost. As reported in Exhibit 1, the new regulatory capital proposed by the Basel Committee will be calibrated only to the unexpected losses of the loan portfolio<sup>4</sup>.

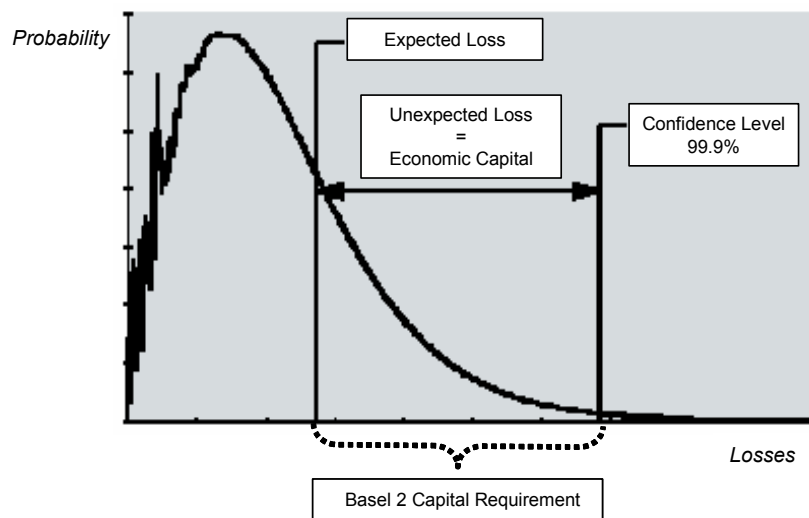
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<sup>4</sup> This proposal was set out by the Committee in a consultative document released on January 30, 2004 (see Basel Committee on Banking Supervision, (2004)), and later confirmed on November 2005 to be part of the final version of the text of the new Basel II framework. In the original proposal, the Basel II regulatory capital was meant to cover both expected and unexpected losses.

- **Operational Cost:** this component refers to the disbursement and monitoring of loans, and depends on a variety of factors. Among these latter, the most important is surely the average size of the borrower, which renders less onerous the granting of a loan to a large corporate than the granting of a limited loan to a small business. The allocation of such costs to individual loans, far from being effortless, may be carried out through a detailed analysis of the data coming from the two-dimension borrower/facility accounting.
- **Liquidity Cost:** this component refers exclusively to commitment loans where borrowers at their request may call the committed amount, thus generating liquidity costs for the bank<sup>5</sup>. In fact, the bank will have to maintain financial resources to meet with the possible usage by borrowers, therefore incurring in an opportunity cost (ie, the cost of liquidity) due to the sacrifice of a return surplus offered by alternative and more profitable investments. An estimate of this liquidity component is related to the analysis of the ratio between drawn and undrawn amount of the commitments, which leads to the estimation of the *Exposure At Default*.

Exhibit 1.

**The Loss Distribution of the loan portfolio and the meaning of the Basel II capital requirement**



Source: Our elaborations on Basel Committee on Banking Supervision (2004).

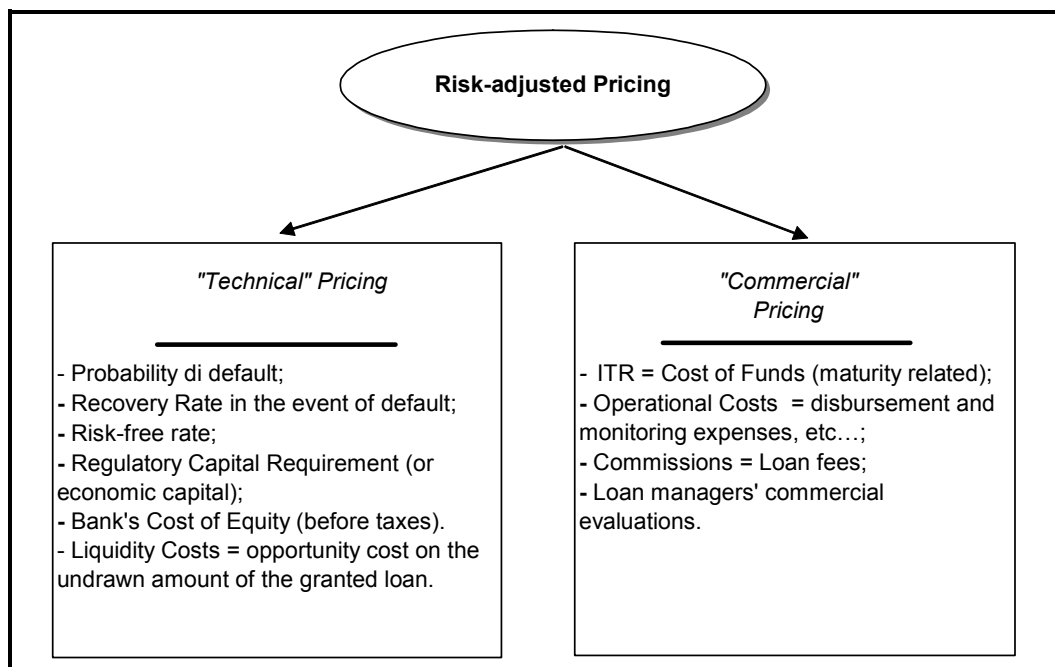
Although the above-mentioned components are all equally important to adequately price credit risk, the formula we propose here only refers to the Expected and Unexpected Loss components for loans with fixed exposures, with

<sup>5</sup> On the contrary, on fixed credit exposures – such as mortgages and personal loans – the liquidity cost is zero, being known with certainty the amount of the exposure at default.

the addition of the Liquidity Cost component for loans with variable exposures. These components represent the ‘technical’ portion of the total *pricing*, directly and fully derivable from an internal rating model. The other portion, which includes operational costs, as well as commissions and other subjectively allocated costs (based on considerations that lie outside objective information on the loan), may be defined as ‘commercial’ and need not an explicit mathematical modeling. From our perspective, the *pricing* may thus be ideally split in two portions: one ‘technical’, of which we will provide the calculation details, and one ‘commercial’ which we refer to further analyses. In Exhibit 2 below we schematize this pricing risk-adjusted framework.

Exhibit 2.

**The pricing risk-adjusted framework:  
the ‘technical’ and ‘commercial’ portions**



Our risk-adjusted pricing formula considers therefore three components (see Exhibit 3), and for each we propose a specific methodological approach. For the first component, related to the Expected Loss, we employ a risk-neutral approach, where a without risk investment, remunerated a default risk-adjusted rate, is equivalent to a risky one remunerated at a default risk-adjusted rate. For the second component, concerning the Unexpected Loss, we use the new formulas on Basel II capital requirements. As previously reported, such formulas represent the capital requirement of a credit exposure as a function of risk – expressed by four main factors: *Probability of Default* (PD), *Loss Given Default* (LGD), *Exposure at*

*Default* (EAD), *Maturity* (M) – in the form of the unexpected loss<sup>6</sup>. Such component should be properly remunerated beginning from the New Basel Accord's coming into force, expected by the end of 2007<sup>7</sup>. For the third component, regarding the Liquidity Cost, we estimate the Usage Given Default (UGD) on the undrawn amount of the loan and the respective *Exposure At Default* (EAD), so as to quantify the opportunity cost on the variable exposure of the loan.

Exhibit 3.

### The 'technical' remuneration of credit losses

	Components to remunerate for credit risk		
<b>Loan Typology</b> <i>Fixed Exposures</i> <i>Variable Exposures</i>	Expected Loss	Unexpected Loss	...
	Expected Loss	Unexpected Loss	Liquidity Cost
<b>Methodological Approach</b>	Risk-Neutrality	Basel 2 Capital Requirement	EAD estimate

In the next sections we will describe the methodology to estimate the risk-adjusted rates and spreads with reference to loans with repayment on maturity (so-called, *zero-coupon* plan). In Appendix 3 we will extend our analysis considering two typical amortization plans: 1) periodical repayment of interest and repayment on maturity of principal (so-called, *bullet* plan); 2) repayment in equal instalments at end-period, inclusive of both the interest and the principal share (so-called, *straight-line amortization*).

## 2.1 The cost of the expected loss: the risk-neutral approach

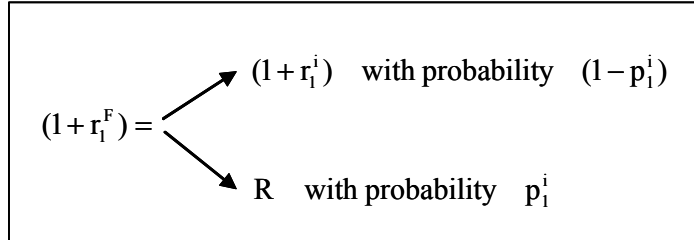
In a risk-neutrality framework, with reference to an *i*-th loan (*i*) and to a 1 year horizon, the present value – at the risk-adjusted interest rate ( $r_1^i$ ) – of a risk-free cash flow of 1 € should be equal to the present value – at the risk-free rate ( $r_1^F$ ) – of a risky cash flow of 1 €. As shown in the following Exhibit, the risk at issue is

<sup>6</sup> In the Basel II formulas, it is assumed zero correlation between the probability of default and the recovery rate (which is equal to 1-LGD). However, this hypothesis has been questioned by recent empirical evidence (see Altman, Brady, Resti, Sironi (2004)).

<sup>7</sup> The Basel Committee considers this implementation date for the most advanced approaches, while year-end 2006 will be the starting date of Basel II for the basic approaches.

a function of both the probability of default ( $p_1^i$ ) within the considered time horizon, and the recovery rate in the event of default ( $R$ )<sup>8</sup>.

Exhibit 4. **The risk-neutrality approach to gauge a loan's yield (1-year time horizon)**



Therefore, the risky loan will return the same future value of a risk-free loan, on the basis of the following equation

$$(1 + r_1^F) = \underbrace{(1 + r_1^i)(1 - p_1^i)}_{a)} + \underbrace{p_1^i R}_{b)} \quad (2.1)$$

where parts a) and b) represent the future value of the loan in case of survival and the recovered amount of the loan in case of default respectively.

With some algebraic manipulations we get the formula for the 1-year risk-adjusted interest rate, which expresses the remuneration for the loan's expected loss granted to the i-th borrower

$$r_1^i = \frac{r_1^F + p_1^i(1 - R)}{1 - p_1^i} \quad (2.2)$$

Using the Basel II's notation, formula (2.2) may also be rewritten as follows

$$r_1^i = \frac{r_1^F + p_1^i(1 - R)}{1 - p_1^i} = \frac{r_1^F + PD \cdot LGD}{1 - PD} \quad (2.3)$$

To get the term structure of the risk-adjusted interest rates (at two, three, ..., n years), at first we extend to the second year the above described 1-year framework, successively generalizing the same logic to the n-year perspective.

---

<sup>8</sup> In this paper, the recovery rate is meant as the present value of the future cash flows of the loan during the work-out period. Furthermore, for the sake of simplicity, we consider the recovery rate variable as deterministic, disregarding its volatility for the meantime. However, this assumption does not invalidate the underlying logic of our methodology.



Thus, the risk-neutrality equation of the 2-nd year will be equal to

$$(1 + r_1^F)(1 + f_1) = \underbrace{(1 + r_2^i)^2(1 - p_2^i)}_{a)} + \underbrace{Rp_2^i}_{b)} \quad (2.4)$$

where

$(1 + f_1)$  = compounding factor at the 1-year *forward* rate. This rate represents the 1-year *spot* rate prevailing in the period between the 1-st and the 2-nd year.

According to the expectation theory of interest rates,<sup>9</sup>  $(1 + f_1) = \frac{(1 + r_2^F)^2}{(1 + r_1^F)}$ .

$(1 + r_1^F)(1 + f_1)$  = future value of a 2-year risk-free loan.

$p_2^i$  = 2-year cumulative probability of default.

$(1 + r_2^i)^2(1 - p_2^i)$  = future value of a risky loan in case of survival at the end of year 2.

$Rp_2^i$  = recovered amount in the event of default at the end of year 2.

Making explicit the forward rate expression, formula (2.4) may be rewritten in the following way

$$(1 + r_2^F)^2 = (1 + r_2^i)^2(1 - p_2^i) + Rp_2^i \quad (2.5)$$

The above equation may be generalized to n periods, obtaining

$$(1 + r_n^F)^n = (1 + r_n^i)^n(1 - p_n^i) + Rp_n^i \quad (2.6)$$

from which we get the risk-adjusted interest rate at n-year (annualized<sup>10</sup>), expressing the remuneration for the expected loss component (REL)

$$REL = r_n^i = \left[ \frac{(1 + r_n^F)^n - Rp_n^i}{1 - p_n^i} \right]^{\frac{1}{n}} - 1 \quad (2.7)$$

Finally, the *risk-adjusted* spread that remunerates for such loss component (SEL) is easily derived as the difference between the risk-adjusted interest rate (the above REL) and the relative risk-free rate

<sup>9</sup> This theory has been originally developed by Irving Fisher (1930).

<sup>10</sup> The interest rates in question, both those *risk-free* and *risk-adjusted*, are in fact calculated on an annual basis.

$$SEL = (r_n^i - r_n^F) = \left[ \frac{(1 + r_n^F)^n - R p_n^i}{1 - p_n^i} \right]^{\frac{1}{n}} - 1 - r_n^F \quad (2.8)$$

As mentioned before, an adequate *pricing* has to remunerate not only the expected loss, but also the unexpected loss so as to properly reward the bank's stakeholders (share and subordinated debt holders). This issue is covered in the next section.

## 2.2 The cost of the unexpected loss: a regulatory approach based on the Basel II capital requirement

As pointed out above, the unexpected loss of a credit exposure should be covered by the bank's economic capital. From a risk measurement perspective, the unexpected loss is a systematic risk factor, which is a function of the default correlation between the portfolio's exposures and may be estimated through an internal portfolio model. The expected loss is instead an idiosyncratic risk factor, a direct function of the borrower's risk, and may be estimated through an internal rating model (which exactly measures the specific borrower's default risk).

The advent of sophisticated internal portfolio models can be traced back to 1997, when some of the major international investment banks and consulting companies released technical documents of their methodological approaches: *Creditmetrics* (JP Morgan)<sup>11</sup>, *CreditRisk+* (Credit Suisse First Boston)<sup>12</sup>, *Credit Portfolio View* (McKinsey's)<sup>13</sup>, and *Portfolio Manager* (KMV)<sup>14</sup>.

Although theoretically elegant, these models soon showed their drawbacks in the risk measurement of illiquid assets (such as, for example, banking loans), since they rely on market inputs (mostly stock prices) for the estimate of correlations and on very elaborate calculation engine based on simulation algorithm to estimate the loan loss portfolio distribution function<sup>15</sup>. For these reasons, there have been many proposals to adapt such models in order to take

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<sup>11</sup> Gupton, Finger, Bhatia (1997).

<sup>12</sup> Credit Suisse Financial Products (1997).

<sup>13</sup> Wilson (1997).

<sup>14</sup> KMV (1997).

<sup>15</sup> The only exception is the *CreditRisk+* model which – on the basis of specific assumption on the functional form of the statistical distribution of the number of defaults and of the portfolio losses – proposes a 'closed formula' analytic approach without resorting to Monte Carlo simulations.

into account the structural characteristics of the European market, which are very peculiar compared to those of the US market<sup>16</sup>.

To consider these evolutions in the risk measurement field, in 2001 the Basel Committee on Banking Supervision proposed a new measure of regulatory capital based on ratings (*Internal Ratings-Based*) which replicates the result of the above cited models on the basis of specific assumptions. Particularly, the Basel Committee proposed a simple ‘closed’ formula<sup>17</sup> to assess the regulatory capital. Such measure represents a bank’s economic capital under the assumption of an average asset return correlation between portfolio exposures, which is in turn a function of the segment, the probability of default and the size of the borrower<sup>18</sup>.

Such economic capital estimate will be our measure of unexpected loss to be included in the remuneration for credit risk of an exposure, in addition to the remuneration for the expected loss already described in the previous section.

In line with Grippa and Viviani (2001), the remuneration for the unexpected loss is equal to the product of the economic capital and its cost. In formula

$$RUL = CEC_n \cdot B2_n^i \quad (2.9)$$

where, with reference to an i-th borrower and an n-th period:

RUL = Remuneration for the Unexpected Loss;

CEC = Cost of the Economic Capital;

B2 = Basel II Regulatory Capital. To determine the regulatory capital beyond the 1-year horizon, we use the relative annualized PD as an input;<sup>19</sup>

Furthermore, assuming that:

- a bank’s economic capital is composed of 2/3 by core capital (Tier 1) and 1/3 by subordinated debt (Tier 2),<sup>20</sup>
- these components are remunerated at the bank’s expected ROE<sup>21</sup> (the return requested by shareholders) and at the bank’s price of the subordinated debt issues,

---

<sup>16</sup> For example, Resti (2000) proposed a simplified version of the *Creditmetrics* model adapted to the European environment, while Zazzara (2002) proposed a portfolio model with specific reference to the Italian credit market.

<sup>17</sup> Easily calculable with Excel™.

<sup>18</sup> In the Appendix 2 we report the Basel II capital requirement formulas related to the *Corporate*, *SME-Corporate*, and *SME-Retail* segments.

<sup>19</sup> On the relation between cumulative and marginal default rates see Altman, Caouette, Narayan (1998). However, this assumption needs closer examination, since the Basel II formula in the IRB-Foundation Approach is calibrated to a time horizon of 2.5 years.

<sup>20</sup> These figures represent a hypothetical average of the distribution of a bank’s economic capital between Tier 1 and Tier 2. In real applications they will be different and specific for each bank.

<sup>21</sup> The expected ROE should incorporate the bank’s growth prospects and expected risk, thus representing a ‘market’ indicator and not a book value. For a review of the methods employed to estimate a bank’s cost of equity, see Maccario, Sironi, Zazzara (2002).

formula (2.9) takes the following form

$$RUL = \frac{2}{3} ROE_{\text{expected}} \cdot B2_n^i + \frac{1}{3} (\text{Spread}_{\text{sub}} + r_n^F) B2_n^i \quad (2.10)$$

where:

$r_n^F$  = Risk Free Rate;

$\text{Spread}_{\text{sub}}$  = Average spread of the subordinated debt issues.

After some algebraic manipulations, the previous (2.10) formula becomes

$$RUL = B2_n^i \left[ \frac{2}{3} ROE_{\text{expected}} + \frac{1}{3} (\text{Spread}_{\text{sub}} + r_n^F) \right] \quad (2.11)$$

Finally, from (2.11) we derive the formula for the *spread* on the unexpected loss

$$SUL = B2_n^i \left[ \frac{2}{3} (ROE_{\text{expected}} - r_n^F) + \frac{1}{3} \text{Spread}_{\text{sub}} \right] \quad (2.12)$$

It is now possible to assess the term structure of the risk-adjusted interest rates and relative spreads for loans with fixed exposures, adding up the remunerations for the expected and unexpected loss respectively

'Technical' Risk-adjusted Interest Rate ( $r_{n,\text{adj}}^i$ ) = REL + RUL (2.13)

$$\left[ \frac{(1 + r_n^F)^n - Rp_n^i}{1 - p_n^i} \right]^{\frac{1}{n}} - 1$$

REL  
Remuneration for the  
Expected Loss

$$+ B2_n^i \left[ \frac{2}{3} ROE_{\text{expected}} + \frac{1}{3} (\text{Spread}_{\text{sub}} + r_n^F) \right]$$

RUL  
Remuneration for the Unexpected Loss

$$\text{'Technical' Risk-adjusted Spread} = \text{SEL} + \text{SUL} \quad (2.14)$$

$$\left[ \frac{(1 + r_n^F)^n - Rp_n^i}{1 - p_n^i} \right]^{\frac{1}{n}} - 1 - r_n^F + B2_n^i \left[ \frac{2}{3} (\text{ROE}_{\text{expected}} - r_n^F) + \frac{1}{3} \text{Spread}_{\text{sub}} \right]$$

SEL
RUL  
Spread on the Expected Loss
Spread on the Unexpected Loss

In the next section we will make use of the above described term structure of the risk-adjusted interest rates to estimate the Liquidity Cost component for loans with variable exposures.

### 2.3 The estimate of the liquidity cost for loans with variable exposures

For loans with variable exposures there is a further component to be remunerated, the Liquidity Cost (LQC), which is a function of the actual value of the exposure in the event of default (EAD).

This category of loans<sup>22</sup> gives in fact the counterparty the option to borrow the granted amount on the basis of her needs. In practice, a loan with variable exposure is formed by a fixed exposure (the drawn amount) and by a call option on the remaining portion of the loan (the undrawn amount). In formula

$$\text{EAD} = D + \text{UGD} \cdot \text{UA}$$

where:

$D$  = *Drawn Amount*, which represents the portion of the total granted loan actually used by the borrower;

$\text{UGD}$  = *Usage Given Default*, which is a function of the borrower riskiness;

$\text{UA}$  = *Undrawn Amount*, equal to the difference between the Granted ( $G$ ) and the Drawn amount ( $D$ ).

<sup>22</sup> These typical facilities take the form of loan commitments, overdrafts and credits subject to collection (discounts, deposit loans, etc...).

According to Basel II, the estimate of the EAD is different in the two IRB-*Foundation* and IRB-*Advanced* approaches. In the base approach, the EAD is equal to the drawn amount plus a 75% rate of usage in the event of default (UGD) of the undrawn amount, whereas in the *Advanced* approach banks may estimate internally the UGD rate. Particularly, in this latter case banks estimate for each rating class of their internal risk classification system the average drawn amount and percentage of ‘usage’ of the undrawn amount, thus arriving at estimating the EAD (*Exposure at Default*) on the basis of the counterparty risk.

By way of an example, we report Citibank’s historical evidence on EAD for loans with variable exposures (precisely, loan commitments), under the two IRB approaches proposed by Basel II.

Table 1. **Estimates of the EAD per rating class on Citibank’s Loan commitments**

Rating Class	Drawn Amount (a)	UGD (b)	EAD	
			IRB- <i>Foundation</i> (c) = (a) + 75%[1-(a)]	IRB- <i>Advanced</i> (c) = (a) + (b)[1-(a)]
AAA	0.1%	69%	75.03%	69.03%
AA	1.6%	73%	75.40%	73.43%
A	4.6%	71%	76.15%	72.33%
BBB	20%	65%	80.00%	72%
BB	46.8%	52%	86.70%	74.46%
B	63.7%	48%	90.93%	81.12%
CCC	75%	44%	93.75%	86%

Source: our elaborations on Asarnow, Marker (1995)’s data.

Under this framework, the *pricing* is then subdivided into two parts: one with respect to the actual exposure (EAD), which is remunerated at the risk-adjusted rate, and the other against the unused granted loan (1-EAD), which is invested at the risk-free rate to face prospective borrower’s usage. On this latter portion, the bank incurs an opportunity-cost equal to the surplus return to which it gives up not investing such funds at the risk-adjusted rate. This component represents the Liquidity Cost (LQC) of the granted loan and is known as ‘fee on the undrawn amount’.

In our approach, we consider the exposure at risk of default (EAD) – estimated per each class of rating – as a loan with fixed exposure, that has accordingly to be priced at the risk-adjusted rate  $r_{n,adj}^i$  (according to formula (2.13)). On the residual portion of the granted loan (1-EAD), the bank applies a surcharge for the liquidity cost of the unused line of credit. This surplus is equal to the *spread risk-adjusted* and represents a margin to which the bank gives up in order to maintain at the borrower’s disposal such amount of money.

Summarizing:

- on the EAD, the bank applies the equivalent of the *risk-adjusted rate*  $r_{n,adj}^i$  of a fixed exposure loan;
- on the portion  $(1-EAD)$ , the bank applies a surcharge resulting from the difference between the future value the bank would earn if invested such amount at the *risk-adjusted rate* – equal to  $(1-EAD) \times (1+r_{n,adj}^i)^n$  – and the future value the bank would earn investing such amount at the *risk-free rate*, this latter being equal to  $(1-EAD) \times (1+r_n^F)^n$ .

The difference between these two future values represents the Liquidity Cost (LQC) on the loan with variable exposure

$$LQC = (1 - EAD) \times [(1 + r_{n,adj}^i)^n - (1 + r_n^F)^n] \quad (2.15)$$

that, when  $n = 1$ , reduces to the following formula

$$LQC = (1 - EAD) \times (r_{1,adj}^i - r_1^F) \quad (2.16)$$

It is easy to verify that when  $EAD = 1$  (therefore, when the granted loan is fully drawn) we fall back in the case of a loan with fixed exposure, where the Liquidity Cost is obviously equal to zero.<sup>23</sup>

After having analyzed and derived the loan losses' risk factors, in the next section we will describe the relationships between the 'technical' risk-adjusted spread and two of the major financial indicators of performance, the EVA<sup>TM</sup> and the RAROC.

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<sup>23</sup> We highlight this is a static model, since the EAD is estimated a priori, before the granting of the loan. If the EAD changed during the loan's time horizon, the price should obviously take it into account, even if at the regulatory level the effect on the loan's *pricing* would be exactly equal to that described in this section.

### 3 The relationship between the ‘technical’ risk-adjusted spread and the EVA<sup>TM</sup> and RAROC indicators

In formula (2.14) we derived the ‘technical’ risk-adjusted spread on a loan as the difference between the risk-adjusted interest rate (which remunerates for the expected and the unexpected loss) and the risk-free rate. If such spread weren’t fully ‘charged’ on the loan price, the bank would be destroying shareholders’ value, net of possible gains on the side of fees and commissions, of operational costs and, for loans with variable exposures, of liquidity costs. It is therefore clear the link between the ‘technical’ risk-adjusted spread and two well known performance indicators, such as the EVA<sup>TM</sup> and the RAROC.

We start by analyzing the relation between the EVA<sup>TM</sup> (*Economic Value Added*),<sup>24</sup> a classic indicator measuring shareholder value, and the RAROC (*Risk Adjusted Return on Capital*). This latter represents the return on the economic capital absorbed by a financial asset (in this case, a bank loan) and is comparable to the Sharpe Index<sup>25</sup> on securities

$$\text{RAROC} = \frac{S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} - \text{SEL}}{\text{EC}} = \frac{S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} - \text{SEL}}{\text{B2}} \quad (3.1)$$

where:

$S_{\text{appl}}$  = Spread charged on the loan;

COM = Commissions and Fees;

OC = Operational Costs;

LQC = Liquidity Costs;

SEL = ‘Technical’ Spread on the Expected Loss;

EC = Economic Capital, in regulatory terms equal to the Basel II capital requirement.

As it is easy to guess, the relationship between the RAROC and the EVA<sup>TM</sup> is the following

$\text{If RAROC} > (\text{ROE}_{\text{expected}} - r_n^F) \quad \text{Then EVA} > 0 \quad (3.2)$
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<sup>24</sup> For further details on the EVA indicator, see Stewart (1990) and his internet site <http://www.sternstewart.com>.

<sup>25</sup> The Sharpe Index measures the relation ‘risk-return’ of a security portfolio and is equal to the ratio between the additional return on the risk-free rate and its volatility. For further details, see Prof. Sharpe’s Home page: <http://www.stanford.edu/~wfsarpe/art/sr/sr.htm>.



Therefore,  $EVA > 0$ . If

$$\frac{S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} - \text{SEL}}{B2} > (\text{ROE}_{\text{expected}} - r_n^F) \quad (3.3)$$

which is equivalent to the following expression

$$S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} - \text{SEL} - (\text{ROE}_{\text{expected}} - r_n^F)B2 > 0 \quad (3.4)$$

Moving to the right the last two terms of the inequality (3.4), we get

$$S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} > \text{SEL} + (\text{ROE}_{\text{expected}} - r_n^F)B2 \quad (3.5)$$

If we assume – for the sake of simplicity – all the bank’s economic capital is held by shareholders,<sup>26</sup> the (positive) EVA may be expressed as a function of the ‘technical’ risk-adjusted spread

$$S_{\text{appl}} + \text{COM} - \text{OC} - \text{LQC} > \text{'Technical' Risk-Adjusted Spread} \quad (3.6)$$

Of course, for loans with fixed exposures, the above expression reduces to

$$S_{\text{appl}} + \text{COM} - \text{OC} > \text{'Technical' Risk-Adjusted Spread} \quad (3.7)$$

We have thus demonstrated the ‘technical’ risk-adjusted spread represents a real *benchmark* of economic value creation/destruction for a bank.

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<sup>26</sup> Therefore,  $SUL = (\text{ROE}_{\text{expected}} - r_n^F)B2$ .

## 4 An ‘internal ratings-based’ application to estimate the risk-adjusted spreads

Our methodology will be now employed to estimate the term structure of the ‘technical’ risk-adjusted spreads for loans with fixed exposures and repayment on maturity (*zero-coupon*), with regard to the corporate segments proposed by Basel II.

Resorting to formula (2.14), we calculate the spreads for each rating class, and at different time horizons, on the basis of:

- A multi-period rating Master Scale which gives the probabilities of default for each rating class ( $p_1, \dots, p_n$ ). In our case, we adopt a rating Master Scale which is the result of statistical elaborations on the empirical evidence released by Moody’s (2003), in order to obtain estimates in line with our business and economic intuitions;<sup>27</sup>
- A term structure of the ‘zero-coupon’ risk-free rates ( $r_1^F, \dots, r_n^F$ ), which we derive from the corresponding structure of the marked-to-market *swap* rates.<sup>28</sup> In actual fact, the appropriate risk-free rates would be those internal provided by the bank’s Treasury Department (ITR, Internal Transfer Rates);<sup>29</sup>
- A measure of the Recovery Rate in the event of default ( $R$ ), which we assume constant and equal to 55%<sup>30</sup> for all rating classes and each maturity. On the basis of evidence on the actual recovery carried out on specific loan work-out processes, each bank will be able to define an appropriate term structure (per rating class) of the recovery rates;
- An estimate of the expected ROE which we set to a 20% gross rate, approximately equivalent to a 14% rate after tax under the assumption of an average European tax rate of 30%. An actual ROE measure is easily taken from a bank’s business plan;
- A value of the spread on the subordinated debt issues, which we set to 0.75% on the basis of the empirical evidence based on European banks provided by Sironi (2001). Actually, each bank will apply values consistent with its own subordinated debt issues;

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<sup>27</sup> We refer to Appendix 2 for details on these elaborations.

<sup>28</sup> We qualify in this case the *risk-free* concept has a rather broad meaning. In fact, the *swap* rate we employ is the interest rate prevailing on the interbank deposit market, therefore expressing approximately the risk of a AA rating counterparty.

<sup>29</sup> The ITR considers a component related to the cost of funds, usually expressed as a spread on the risk-free rate, which also includes the credit risk of the bank raising funds. Such component changes from bank to bank and represents a *quid* to add to the ‘technical’ rate in order to arrive at determining the ‘final’ interest rate of the loan.

<sup>30</sup> Since  $LGD = (1-R)$ , a recovery rate of 55% coincides with an LGD of 45%. This measure is equivalent to that proposed by Basel II in the *IRB-Foundation* approach.

- A measure of the economic capital absorbed by loan exposures, which we assume equal to the Basel II regulatory capital requirement.<sup>31</sup> In this case, we will refer to the risk weight functions proposed for corporates, which apply to the following segments: *Corporate* (corporates with turnover greater than € 50 million), *Small Business-SME Corporate* (corporates with turnover less than € 50 million and managed as *Corporate*) and *Small Business-SME Retail* (corporates managed as *Retail* and with exposure less than € 1 million).<sup>32</sup>

In the following table we report the multi-period rating Master Scale and the term structure of ‘zero-coupon’ risk-free rates, which will form the basis of our case study.

Table 2. **The multi-period rating Master Scale and the term structure of ‘zero-coupon’ risk-free interest rates (cumulative probability of default; ‘zero-coupon’ risk-free rates derived from swap rates as of December 22, 2003)**

	1	2	3	4	5	6	7	8	9	10
<i>AAA</i>	0.01%	0.03%	0.06%	0.10%	0.13%	0.17%	0.21%	0.25%	0.29%	0.33%
<i>AA+</i>	0.02%	0.07%	0.13%	0.19%	0.26%	0.33%	0.41%	0.49%	0.57%	0.66%
<i>AA</i>	0.03%	0.09%	0.16%	0.23%	0.31%	0.39%	0.48%	0.57%	0.66%	0.76%
<i>AA-</i>	0.04%	0.11%	0.19%	0.27%	0.36%	0.45%	0.55%	0.65%	0.75%	0.86%
<i>A+</i>	0.05%	0.16%	0.28%	0.42%	0.57%	0.73%	0.89%	1.06%	1.23%	1.41%
<i>A</i>	0.06%	0.18%	0.31%	0.46%	0.62%	0.79%	0.96%	1.14%	1.32%	1.51%
<i>A-</i>	0.09%	0.24%	0.40%	0.58%	0.77%	0.97%	1.17%	1.38%	1.59%	1.81%
<i>BBB+</i>	0.13%	0.48%	0.91%	1.40%	1.92%	2.47%	3.05%	3.64%	4.25%	4.87%
<i>BBB</i>	0.16%	0.54%	1.00%	1.52%	2.07%	2.65%	3.25%	3.87%	4.51%	5.16%
<i>BBB-</i>	0.39%	1.00%	1.69%	2.42%	3.20%	3.99%	4.81%	5.64%	6.48%	7.33%
<i>BB+</i>	0.67%	1.91%	2.97%	4.71%	6.15%	8.08%	8.49%	9.60%	10.71%	11.05%
<i>BB</i>	1.17%	2.34%	4.67%	7.27%	9.37%	11.01%	12.83%	14.44%	15.61%	15.92%
<i>BB-</i>	2.03%	4.97%	7.36%	11.22%	14.27%	18.32%	19.39%	22.00%	24.61%	24.79%
<i>B+</i>	3.51%	8.01%	11.60%	17.31%	21.74%	27.57%	29.32%	33.32%	37.32%	37.13%
<i>B</i>	6.08%	12.92%	18.27%	26.71%	33.12%	37.94%	40.40%	42.57%	44.96%	47.37%
<i>B-</i>	10.54%	20.83%	28.79%	37.92%	44.40%	49.26%	53.64%	58.21%	61.39%	62.60%
<i>CCC</i>	18.27%	33.58%	45.35%	55.61%	60.99%	66.16%	69.72%	74.94%	78.07%	81.73%
<i>Risk-Free</i>	2.37%	2.78%	3.16%	3.47%	3.71%	3.92%	4.10%	4.26%	4.39%	4.49%

Source: Our elaborations on data from Moody’s (2003) and Bloomberg™.

<sup>31</sup> On this point, we refer to the remarks reported in footnote 5.

<sup>32</sup> See Appendix 2 for mathematical details of these risk weight functions.

Feeding formula (2.14) with the above mentioned inputs, we get the following term structures of the risk-adjusted spreads for the corporate segments proposed by Basel II

Table 3. **The term structure of the risk-adjusted spreads for the ‘Corporate’ segment**  
(gross expected ROE equal to 20%; spread on the subordinated debt issues equal to 0.75%)

	1	2	3	4	5	6	7	8	9	10
<i>AAA</i>	0.08%	0.11%	0.12%	0.14%	0.14%	0.15%	0.16%	0.16%	0.17%	0.17%
<i>AA+</i>	0.12%	0.17%	0.19%	0.21%	0.22%	0.24%	0.24%	0.25%	0.26%	0.27%
<i>AA</i>	0.15%	0.20%	0.22%	0.24%	0.25%	0.26%	0.27%	0.28%	0.29%	0.29%
<i>AA-</i>	0.19%	0.22%	0.25%	0.26%	0.28%	0.29%	0.30%	0.30%	0.31%	0.32%
<i>A+</i>	0.21%	0.28%	0.32%	0.35%	0.37%	0.39%	0.41%	0.42%	0.43%	0.44%
<i>A</i>	0.24%	0.31%	0.34%	0.37%	0.39%	0.41%	0.43%	0.44%	0.45%	0.46%
<i>A-</i>	0.31%	0.37%	0.41%	0.43%	0.45%	0.47%	0.48%	0.50%	0.51%	0.52%
<i>BBB+</i>	0.39%	0.58%	0.68%	0.75%	0.81%	0.86%	0.90%	0.94%	0.97%	1.00%
<i>BBB</i>	0.45%	0.63%	0.72%	0.79%	0.85%	0.89%	0.94%	0.98%	1.01%	1.04%
<i>BBB-</i>	0.78%	0.91%	1.00%	1.06%	1.11%	1.16%	1.20%	1.24%	1.28%	1.32%
<i>BB+</i>	1.07%	1.35%	1.40%	1.60%	1.68%	1.83%	1.74%	1.77%	1.80%	1.74%
<i>BB</i>	1.49%	1.52%	1.86%	2.12%	2.23%	2.26%	2.33%	2.37%	2.36%	2.26%
<i>BB-</i>	2.09%	2.45%	2.52%	2.89%	3.04%	3.35%	3.21%	3.32%	3.42%	3.25%
<i>B+</i>	3.01%	3.48%	3.55%	4.12%	4.37%	4.88%	4.71%	4.94%	5.19%	4.83%
<i>B</i>	4.61%	5.19%	5.29%	6.26%	6.74%	6.94%	6.72%	6.55%	6.48%	6.46%
<i>B-</i>	7.48%	8.22%	8.43%	9.33%	9.66%	9.75%	9.87%	10.17%	10.20%	9.76%
<i>CCC</i>	12.85%	14.05%	14.89%	16.09%	15.76%	15.89%	15.66%	16.56%	16.68%	17.36%

Source: Our elaborations on the basis of formula (2.14), with a recovery rate of 55%.

Table 4.

**The term structure of the risk-adjusted spreads for the ‘SME-Corporate’ segment**  
(gross expected ROE equal to 20%; spread on the subordinated debt issues equal to 0.75%; average turnover equal to € 25 millions)

	1	2	3	4	5	6	7	8	9	10
<i>AAA</i>	0.07%	0.10%	0.11%	0.12%	0.13%	0.13%	0.14%	0.14%	0.15%	0.15%
<i>AA+</i>	0.11%	0.15%	0.17%	0.19%	0.20%	0.21%	0.22%	0.23%	0.23%	0.24%
<i>AA</i>	0.14%	0.18%	0.20%	0.21%	0.22%	0.23%	0.24%	0.25%	0.26%	0.26%
<i>AA-</i>	0.16%	0.20%	0.22%	0.24%	0.25%	0.26%	0.27%	0.27%	0.28%	0.29%
<i>A+</i>	0.19%	0.25%	0.29%	0.31%	0.33%	0.35%	0.37%	0.38%	0.39%	0.40%
<i>A</i>	0.21%	0.28%	0.31%	0.33%	0.35%	0.37%	0.39%	0.40%	0.41%	0.42%
<i>A-</i>	0.28%	0.33%	0.37%	0.39%	0.41%	0.42%	0.44%	0.45%	0.46%	0.47%
<i>BBB+</i>	0.35%	0.53%	0.62%	0.69%	0.74%	0.78%	0.82%	0.86%	0.89%	0.93%
<i>BBB</i>	0.40%	0.57%	0.66%	0.72%	0.78%	0.82%	0.86%	0.90%	0.93%	0.96%
<i>BBB-</i>	0.71%	0.84%	0.91%	0.97%	1.02%	1.07%	1.11%	1.15%	1.19%	1.22%
<i>BB+</i>	0.98%	1.24%	1.30%	1.49%	1.57%	1.71%	1.63%	1.65%	1.68%	1.63%
<i>BB</i>	1.38%	1.41%	1.73%	1.98%	2.09%	2.12%	2.19%	2.23%	2.22%	2.13%
<i>BB-</i>	1.95%	2.30%	2.37%	2.72%	2.88%	3.18%	3.05%	3.15%	3.26%	3.09%
<i>B+</i>	2.84%	3.29%	3.37%	3.93%	4.18%	4.69%	4.51%	4.75%	4.99%	4.64%
<i>B</i>	4.40%	4.98%	5.07%	6.04%	6.51%	6.71%	6.51%	6.33%	6.27%	6.25%
<i>B-</i>	7.24%	7.98%	8.19%	9.09%	9.42%	9.51%	9.63%	9.93%	9.96%	9.53%
<i>CCC</i>	12.59%	13.80%	14.64%	15.84%	15.51%	15.64%	15.41%	16.31%	16.43%	17.11%

Source: Our elaborations on the basis of formula (2.14), with a recovery rate of 55%.

Table 5.

**The term structure of the risk-adjusted spreads for the ‘SME-Retail’ segment**  
(gross expected ROE equal to 20%; spread on the subordinated debt issues equal to 0.75%).

	1	2	3	4	5	6	7	8	9	10
AAA	0.02%	0.04%	0.05%	0.05%	0.06%	0.06%	0.06%	0.07%	0.07%	0.07%
AA+	0.04%	0.07%	0.08%	0.09%	0.10%	0.11%	0.11%	0.12%	0.12%	0.13%
AA	0.06%	0.08%	0.10%	0.11%	0.12%	0.12%	0.13%	0.14%	0.14%	0.15%
AA-	0.08%	0.10%	0.11%	0.12%	0.13%	0.14%	0.14%	0.15%	0.16%	0.16%
A+	0.09%	0.13%	0.16%	0.18%	0.19%	0.20%	0.22%	0.23%	0.24%	0.24%
A	0.11%	0.15%	0.17%	0.19%	0.20%	0.22%	0.23%	0.24%	0.25%	0.26%
A-	0.15%	0.19%	0.21%	0.23%	0.24%	0.26%	0.27%	0.28%	0.29%	0.30%
BBB+	0.20%	0.33%	0.40%	0.46%	0.50%	0.54%	0.58%	0.61%	0.64%	0.67%
BBB	0.24%	0.36%	0.43%	0.49%	0.53%	0.57%	0.61%	0.64%	0.67%	0.70%
BBB-	0.47%	0.58%	0.64%	0.70%	0.74%	0.78%	0.82%	0.86%	0.89%	0.93%
BB+	0.70%	0.93%	0.98%	1.15%	1.22%	1.35%	1.28%	1.31%	1.34%	1.29%
BB	1.04%	1.07%	1.36%	1.59%	1.69%	1.73%	1.79%	1.83%	1.83%	1.75%
BB-	1.54%	1.86%	1.92%	2.24%	2.39%	2.67%	2.56%	2.66%	2.77%	2.62%
B+	2.31%	2.71%	2.79%	3.29%	3.53%	4.00%	3.86%	4.09%	4.32%	4.01%
B	3.64%	4.17%	4.28%	5.16%	5.62%	5.83%	5.66%	5.52%	5.48%	5.47%
B-	6.18%	6.90%	7.12%	8.00%	8.34%	8.44%	8.57%	8.89%	8.92%	8.53%
CCC	11.33%	12.52%	13.37%	14.57%	14.26%	14.40%	14.18%	15.08%	15.21%	15.89%

Source: Our elaborations on the basis of formula (2.14), with a recovery rate of 55%.

From the above tables we see a slight difference between the *spreads* of the *Corporate* and *SME-Corporate* segments, due probably to the fact for the latter we consider an average turnover of € 25 millions.<sup>33</sup>

On the contrary, we see lower spreads within the *SME-Retail* segment, particularly pronounced for the *Investment Grade* rating classes and slightly lower, but however significant, for the *Speculative Grade* classes.<sup>34</sup> To make clear the differences among the spreads of these three corporate segments it is important noting that:

- On average, the corporate size is negatively correlated with risk. Therefore, larger companies have a lower probability of default than that of smaller companies and, as a result, a lower risk-adjusted spread as well;
- The assumption of a constant and undifferentiated recovery rate for the various corporate segments is unrealistic. In fact, larger companies are able to offer more qualified guarantees and collaterals compared to smaller

<sup>33</sup> Actually, even using the minimum turnover threshold proposed by Basel II for the *SME-Corporate* segment (€ 5 millions), we didn't find notable differences in the risk-adjusted spreads.

<sup>34</sup> In the rating jargon, the rating classes from AAA to BBB are considered *Investment Grades*, whereas the ratings from BB to CCC are considered *Speculative Grades*.

companies. This is also due to the intragroup relationships that characterize larger companies and which are in practice non-existent among the smaller domestic companies;

- Finally, if we move from the ‘technical’ risk-adjusted spreads on the ‘commercial’ spreads, we should consider the operational cost component and the commissions/fees applied to large borrowers are decidedly lower than those applied to smaller corporates (for reasons related to both economies of scale, and to the bargaining power of the counterparties).

In the end, we give an example of the EVA calculation on a loan with fixed exposure granted to a company with a turnover of € 60 millions and with the following features

<b>Loan amount</b>	<b>2 mln. Euro</b>
Maturity	1 year
Rating class	BBB+
Probability of default	0.13%
Spread ( $S_{\text{appl}}$ )	0.65%
Commissions/Fees (COM)	0.15%
Operational cost (OC)	0.20%

With the risk-free rate reported in the rating Master Scale (see Table 2) as an input and using formula (2.14) with reference to the *Corporate* segment, we get a ‘technical’ risk-adjusted spread equal to 0.39% (this result is directly inferable from Table 3).

Applying formula (3.7)

$$S_{\text{appl}} + \text{COM} - \text{OC} > \text{'Technical' Risk-Adjusted Spread}$$

we can assess the creation/destruction of shareholder value through the EVA™ indicator, obtaining the following result

$$0.65\% + 0.15\% - 0.20\% > 0.39\%$$

$$\Rightarrow 0.60\% > 0.39\%$$

$$\Rightarrow \text{EVA} = +0.21\% \Rightarrow \text{Value Creation}$$

## 5 A comparison between the ‘technical’ risk-adjusted and bond market spreads

To test the goodness of our ‘technical’ risk-adjusted spreads, we compared them with the prevailing spreads on the corporate bond market, with specific reference to the Industrial sector. To that end, we used the data released by Bloomberg<sup>TM</sup>, a well know financial information provider.

### **Overview on the credit spreads provided by Bloomberg<sup>TM</sup>**

With reference to recent issues of corporate bonds, Bloomberg<sup>TM</sup> provides on a daily basis yield term structures for these securities, differentiated by currency, industry sector, and rating class (so-called *Fair Market Curves*, FMC<sup>35</sup>). *Credit spreads* for each ‘credit class’ (currency-sector-rating) are derived by difference between the FMCs and the reference term structure of risk-free rates.

The FMCs are estimated from yields of a security basket – daily updated – on the basis of new issues of each specific ‘credit class’.<sup>36</sup> The reference securities are mostly in the form of *bullet* bonds, while the remaining are *callable* and bonds.

The marked-to-market interest rate for each maturity is a swap rate (different from the *zero coupon* rate), which refers to a security with periodical repayment of interest and repayment on maturity of principal (*bullet* plan). Therefore, the reference term structure of the *risk free* rates for the estimation of credit spreads is that of the market *swap* rates.

### **The comparison between spreads<sup>37</sup>**

Since Bloomberg’s credit spreads refer to *Bullet* bonds, also ‘technical’ *risk-adjusted spreads* should refer to loans with the same repayment plan. Thus, here we will not use the spreads estimated in the previous section (referred to *zero-coupon* loans), but those estimated in Appendix 3 (to which we refer to for further details).

In Table 6 below we report – with reference to four rating classes – the comparison between the risk-adjusted spreads on *Bullet* loans (for the Basel II

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<sup>35</sup> These yield curves can be found at the Bloomberg’s page ‘FMC’ (*Fair Market Curves*).

<sup>36</sup> Technically, the Fair Market Curves are estimated through a best fitting procedure.

<sup>37</sup> We note the analysis based on market credit spreads may suffer from various drawbacks: a) bond prices are not always available or significant for each credit class and maturity; b) bond prices are strongly affected by liquidity factors, in terms of trading of the underlying securities.



Corporate segment) and the market *credit spreads* derived from the FMCs as of December 22, 2003 (Bloomberg™’s Industrial sector).<sup>38</sup>

Table 6. **The comparison of spreads on Bullet securities for major rating classes**

	AA		A		BBB		BB	
	“Technical” spread	Bloomb. spread	“Technical” spread	Bloomb. spread	“Technical” spread	Bloomb. spread	“Technical” spread	Bloomb. spread
<b>1</b>	0.18%	0.01%	0.28%	0.13%	0.52%	0.32%	1.68%	1.46%
<b>2</b>	0.23%	0.09%	0.36%	0.30%	0.72%	0.52%	1.70%	1.85%
<b>3</b>	0.26%	0.18%	0.40%	0.36%	0.82%	0.52%	2.03%	2.02%
<b>4</b>	0.27%	0.11%	0.42%	0.29%	0.89%	0.62%	2.27%	2.38%
<b>5</b>	0.29%	0.19%	0.45%	0.38%	0.94%	0.64%	2.37%	2.39%
<b>6</b>	0.30%	0.20%	0.46%	0.39%	0.99%	0.68%	2.39%	2.45%
<b>7</b>	0.31%	0.21%	0.48%	0.40%	1.02%	0.72%	2.44%	2.51%
<b>8</b>	0.31%	0.22%	0.49%	0.45%	1.05%	0.77%	2.47%	2.67%
<b>9</b>	0.32%	0.19%	0.50%	0.45%	1.08%	0.72%	2.46%	2.59%
<b>10</b>	0.32%	0.22%	0.51%	0.48%	1.11%	0.80%	2.39%	2.56%

Source: Our elaborations on the basis of formula (A3.1) and on Bloomberg™ data.

From the above Table we see a widespread concordance between the two categories of spreads, with reference to both rating classes and maturities. In fact, the correlation between the term structures of the ‘technical’ spreads and the ‘Bloomberg’ ones ranges from a minimum of 92.51% to a maximum of 98.26%. However, having a look at the above data we point out that:

- For the AA and BBB rating classes, we find an average difference between the values of the two term structures equal to 0.12% for the AA rating and to 0.28% for the BBB rating. For the AA class such difference is due to our choice of assigning a minimum probability of default of 0.03% in the rating Master Scale (see Table 2, section 4). Probably, the market spreads include the empirical probabilities of default that – as inferred by the Matrix of cumulative probabilities released by Moody’s in 2003 (see Table 7, Appendix 1) – are lower than those employed in our internal estimates. On the contrary, for the BBB rating class the difference does not seem to stem from misalignments between the term structures of the ‘internal’ and ‘empirical’ probabilities of default. A plausible explanation could be related to the role

<sup>38</sup> As concerns the Industrial sector, Bloomberg proposes yield term structures for corporate bonds relatively to various rating classes, in addition to those considered in our analysis. Strictly speaking, Bloomberg provides FMCs for bonds rated AA–, BBB+ and BBB– as well. However, leaving these latter out does not invalidate our results.

played by liquidity factors and the reference economic environment on the market prices of issues with similar creditworthiness.

- For the A and BB rating classes, we find slight differences between the two spread term structures. For the A rating, we note a difference in the spreads only at the 4th year, attributable to a rather irregular ‘leap’ of the market spread curve. For the BB rating, finally, the spread term structures are in step with each other across rating classes and maturities, both showing a downward trend after a certain maturity. This phenomenon is due to the fact that, for a loan or bond with low creditworthiness, the longer the residual maturity, the higher its probability to migrate towards better rating classes. On the contrary, for a loan or bond with high creditworthiness is more likely that with the passing of time its rating class may worsen.<sup>39</sup>

## 6 Conclusions

The risk remuneration of the banks’ lending activity is playing an increasingly significant role, especially as a consequence of an explicit policy of value requested by shareholders and stakeholders in general. To take into account this phenomenon, we proposed a simple formula to calculate a loan’s ‘technical’ risk-adjusted interest rate and the corresponding spread, moreover making clear the existing relationship between the risk-adjusted spread and the EVA<sup>TM</sup> and RAROC indicators. From this perspective, such ‘technical’ *spread* plays the role of a real benchmark of economic value creation for a bank. The results obtained with our methodology seem rather reassuring, also in the light of the comparison carried out between the ‘internal’ spreads on loans and the ‘market’ spreads on corporate *bonds* of the Industrial sectors.

However, some caveats on the proposed methodology and the consequent results are necessary. A first signal is that at the same rating (and, therefore, at the same level of default probability) and recovery rate, ‘technical’ spreads are lower for smaller companies than those for larger companies; this is due to the impact of the new Basel II’ IRB-Foundation capital requirements on the proposed pricing formula, which are more favourable for smaller companies. However, with regard to this result we point out that:

- all financial, behavioural and qualitative information being equal, a larger company’s rating will usually be better than that of a smaller company, since banks develop different internal rating systems for borrower segments,

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<sup>39</sup> The Basel Committee on Banking Supervision reported similar remarks with regard to the maturity adjustment of the new capital requirement. See Basel Committee on Banking Supervision (2001), paragraphs 183–184.

‘calibrated’ at different average levels of probability of default (so-called ‘anchor points’);<sup>40</sup>

- the assumption of a ‘flat’ recovery rate for all corporate segments, as implied by the *IRB-Foundation* approach, seems rather unrealistic, since in practice larger companies are able to offer more qualified guarantees and collaterals compared to smaller companies, benefiting from lower expected and unexpected losses. Such differences will be instead incorporated into the *IRB-Advanced* approach, which will be adopted by sophisticated banks in the new Basel II environment;
- the assumption of a non-stochastic recovery rate (ie absence of volatilità) is simplistic as well, especially in the light of the empirical international evidence.<sup>41</sup> Further research is needed in order to assess the impact of the recovery rate volatility on the risk-adjusted spreads of the various corporate segments, as concerns the rating and the maturity dimensions.

To summarize, our results confirm the existence of a significant relationship between risk and spreads of loans, spurring further studies in this field. Particularly, more sophisticated banks will have to adequately value the guarantees and collaterals offered by their counterparties with respect to prospective loans, as well as the impact of the historical recovery rate estimates deriving from their complete loan work-out processes.

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<sup>40</sup> An internal rating ‘calibration’ procedure consists in defining the average default level of the bank’s credit portfolio. During an economic cycle, smaller companies record – on average – a higher default probability than that of larger companies.

<sup>41</sup> See Gupton, Gates, Carty (2000).

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## Appendix 1

### The estimation of the multi-period rating master scale from the rating agencies' empirical evidence

The major international rating agencies (*Moody's Investors Service, Standard & Poor's, etc...*) regularly release statistics on corporate bond issuers' defaults and migrations (*upgrades/downgrades*) per rating classes at different time horizons. Evidence is usually reported in tabular form and refers to both numbers of issuers and amounts of relative issues. We report below the matrix of the cumulative average default rates (by numbers), for years 1 to 10, published in a recent annual report by Moody's (2003)

Table A1.1 **Cumulative Average Default Rates in the years 1983–2002 (by numbers)**

	1	2	3	4	5	6	7	8	9	10
AAA	0.00%	0.00%	0.00%	0.05%	0.17%	0.24%	0.31%	0.40%	0.40%	0.40%
AA+	0.00%	0.00%	0.00%	0.17%	0.17%	0.28%	0.28%	0.28%	0.28%	0.28%
AA	0.00%	0.00%	0.05%	0.15%	0.33%	0.40%	0.48%	0.57%	0.68%	0.81%
AA-	0.05%	0.07%	0.13%	0.21%	0.29%	0.38%	0.38%	0.38%	0.38%	0.48%
A+	0.00%	0.02%	0.24%	0.37%	0.47%	0.57%	0.62%	0.72%	0.78%	0.93%
A	0.03%	0.09%	0.24%	0.48%	0.68%	0.89%	1.04%	1.41%	1.73%	1.86%
A-	0.04%	0.21%	0.34%	0.47%	0.62%	0.84%	1.15%	1.34%	1.57%	1.75%
BBB+	0.21%	0.60%	1.02%	1.40%	1.80%	2.10%	2.39%	2.56%	2.77%	2.90%
BBB	0.15%	0.46%	0.84%	1.56%	2.24%	2.89%	3.47%	3.99%	4.61%	5.50%
BBB-	0.50%	1.27%	2.05%	3.15%	4.23%	5.40%	6.52%	7.55%	8.25%	8.97%
BB+	0.70%	2.11%	3.76%	5.82%	7.61%	9.64%	10.93%	12.23%	13.01%	13.96%
BB	0.65%	2.34%	4.72%	7.30%	9.42%	11.01%	13.00%	14.44%	15.61%	15.92%
BB-	2.38%	6.60%	11.49%	16.22%	20.70%	24.98%	28.59%	32.32%	36.05%	39.29%
B+	3.33%	9.73%	16.14%	22.05%	27.56%	32.77%	38.42%	42.50%	46.26%	49.97%
B	7.14%	15.99%	23.43%	29.57%	34.49%	37.94%	40.40%	42.57%	44.96%	47.37%
B-	11.97%	21.97%	30.41%	37.92%	44.40%	49.26%	53.64%	58.21%	61.39%	62.60%
CCC	23.65%	36.95%	47.47%	55.61%	60.99%	66.16%	69.72%	74.94%	78.07%	81.73%

Source: Moody's (2003), Exhibit 46, page 39.

The above Moody's empirical data, referred to a fairly long period (twenty years), reveal some counterintuitive results, which could also reflect in the loan pricing:

- For the first three years, the empirical default rates of the better rating classes (AAA and AA) are equal to zero – that is, absence of credit risk –, implying a *technical spread* of zero as well on granted loans;

- For a few years, some companies have better ratings than others but their risk is higher. For example, for the first year, companies with a rating of AA– are riskier than companies with ratings A+, A and A–. Such an empirical anomaly may be classified as absence of \*vertical\* monotony (namely, between default rates of the various rating classes for a specific year) within the matrix of cumulative default rates.<sup>42</sup>

Therefore, some adjustments to the above Table are needed in order to make it more consistent with our economic intuition.<sup>43</sup> Particularly, to resolve the first issue we assumed a minimum PD for the AAA and AA rating classes, with a floor of 1 basis point (0.01%) for the former class. Whereas, to tackle the second issue we interpolated the default rate curves for each year through exponential functions, imposing a maximum threshold for worse rating classes and, on an ad hoc basis, a minimum threshold for better rating classes. This method leads to the results reported in Table 2 (section 4).

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<sup>42</sup> Obviously, since the matrix in question refers to cumulative default rates, the *\*horizontal\** monotony (namely, between default rates at various years for a specific rating class) is always satisfied.

<sup>43</sup> However, we don't want here to negate the facts. After all, these are the actual default rates of companies rated by Moody's in the last twenty years.



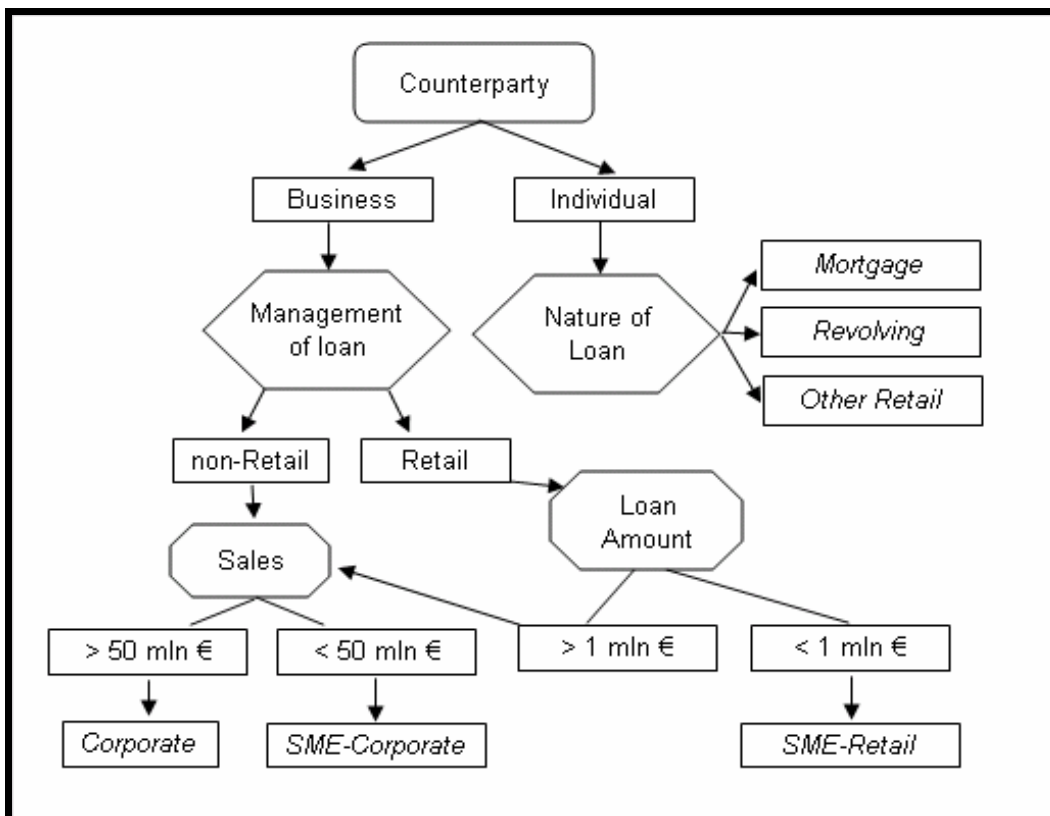
## Appendix 2

### The formulas to calculate the Basel II capital requirements for corporates<sup>44</sup>

As concerns the IRB approaches to capital requirements, the Basel Committee on Banking Supervision subdivides corporates in three segments: *Corporate* (corporates with turnover greater than € 50 million), *Small Business-SME Corporate* (corporates with turnover less than € 50 million and managed as *Corporate*) and *Small Business-SME Retail* (corporates managed as *Retail* and with exposure less than € 1 million). In the flow chart below, we show the criteria applied to segment banking counterparties under the Basel II proposal.

Exhibit A2.1

#### The segmentation criteria under Basel II



Source: Basel Committee on Banking Supervision (2002), *QIS 3 Frequently Asked Questions*, December ([http://www.bis.org/bcbs/qis/qis3qa\\_1.htm](http://www.bis.org/bcbs/qis/qis3qa_1.htm)).

<sup>44</sup> As recalled in the previous footnote 4, the Basel Committee recently decided to move to an Unexpected Loss approach to calculate regulatory capital, removing the Expected Loss portion from the risk weight functions. For further details, see Basel Committee on Banking Supervision (2004).

For each counterparty segment, different mathematical formulas are proposed, these latter being function of the Probability of Default (PD), the Loss Given Default (LGD), the Maturity (M), the Turnover or Sales (S) and the Average Correlation (R) between loans according to a given state of the global economy. Below we give the details of these formulas – which we made use of to estimate the loan risk-adjusted pricing –, clarifying that:

- K denotes the capital requirement;
- R denotes the correlation;
- b denotes the maturity adjustment factor;
- N denotes the cumulative distribution function for a standard normal random variable; G denotes the inverse cumulative distribution function for a standard normal random variable.

### Corporate

$$K = \text{LGD} \left\{ N \left\{ \frac{1}{(1-R)^{0.5}} G(\text{PD}) + \left[ \frac{R}{(1-R)} \right]^{0.5} G(0.999) \right\} - \text{PD} \right\} \frac{1}{[1-1.5 \cdot b(\text{PD})]} [1 + (M-2.5)b(\text{PD})]$$

$$R = 0.12 \left[ \frac{1 - \text{EXP}(-50\text{PD})}{1 - \text{EXP}(-50)} \right] + 0.24 \left[ \frac{1 - [1 - \text{EXP}(-50\text{PD})]}{1 - \text{EXP}(-50)} \right]$$

$$b = [0.11852 - 0.05478 \text{Log}(\text{PD})]^2$$

### SME-Corporate

$$K = \text{LGD} \left\{ N \left\{ \frac{1}{(1-R)^{0.5}} G(\text{PD}) + \left[ \frac{R}{(1-R)} \right]^{0.5} G(0.999) \right\} - \text{PD} \right\} \frac{1}{[1-1.5b(\text{PD})]} [1 + (M-2.5)b(\text{PD})]$$

$$R = 0.12 \left[ \frac{1 - \text{EXP}(-50\text{PD})}{1 - \text{EXP}(-50)} \right] + 0.24 \left[ \frac{1 - [1 - \text{EXP}(-50\text{PD})]}{1 - \text{EXP}(-50)} \right] - 0.04 \left[ 1 - \frac{S-5}{45} \right]$$

$$b = [0.11852 - 0.05478 \text{Log}(\text{PD})]^2$$

### **SME-Retail**

$$K = \text{LGD} \left\{ N \left\{ \frac{1}{(1-R)^{0.5}} G(\text{PD}) + \left[ \frac{R}{(1-R)} \right]^{0.5} G(0.999) \right\} - \text{PD} \right\}$$

$$R = 0.03 * \left[ \frac{1 - \text{EXP}(-35\text{PD})}{1 - \text{EXP}(-35)} \right] + 0.16 \left[ \frac{1 - [1 - \text{EXP}(-35\text{PD})]}{1 - \text{EXP}(-35)} \right]$$

## Appendix 3

### The pricing of loans with fixed exposures<sup>45</sup> under specific repayment plans

The pricing model presented in the main text refers to loans with repayment plan on maturity, where interest and principal are repaid in a single sum on a set maturity.<sup>46</sup> In practice, however, banking loans are granted under different amortization plans, with the repayment of interest and capital being distributed during the loan's time horizon.

Therefore, firstly, we extend our methodological approach in order to price loans with fixed exposures under two typical repayment plans for banking loans:

- *Bullet Loan*, which schedules the repayment of interest at regular intervals (for example, semi-annually, annually, etc...) and of principal on maturity;
- *Equal-Instalment Loan* ('Straight-Line Amortization'), which schedules the loan's repayment in equal instalments, inclusive of both the interest and the principal share, at end-period regular intervals up to maturity.

Secondly, with reference to the Basel II *Corporate* segment, we compare the *risk-adjusted* rates of loans under the two above amortization plans with those of *Zero-Coupon* loans (full repayment on maturity).

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<sup>45</sup> In this Appendix we will not derive the rates of loans with variable exposures, since this is a straightforward extension of this case.

<sup>46</sup> In technical jargon, securities or loans without coupons are called as *Zero-Coupon*.

*A Note.* From now on, we will refer exclusively to risk-adjusted interest rates and not to spreads, since the presence of specific amortization plans changes the

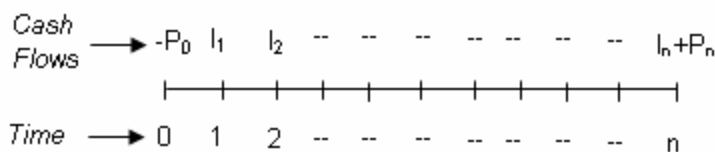
actual average maturity of the loan, which is equal to  $\frac{\sum_{i=1}^n \text{Cash Flow}_i * i}{\sum_{i=1}^n \text{Cash Flow}_i}$  (where  $i = \text{maturity}$ ).

In this case, in fact, getting the proper spreads would require a definition of the reference risk-free rates. By way of an example, for an Equal-Instalment Loan with a maturity of 10 years, the risk-adjusted spread shouldn't be referred to a 10-year risk-free rate, but to a rate reflecting the loan's actual average maturity, that in this case would be equal to 5.5 years. As concerns the term structure of risk-free rates shown in the main text, we see the difference between these risk-free rates is rather considerable, being equal to about 69 basis points.<sup>47</sup>

We recall that under the category of loans with fixed exposures falls not only the typical credit forms such as mortgages and personal loans, but also guaranty loans. In fact, when the guaranty is called, the exposure is known with certainty, being equal to the agreed amount at the guaranty loan origination.

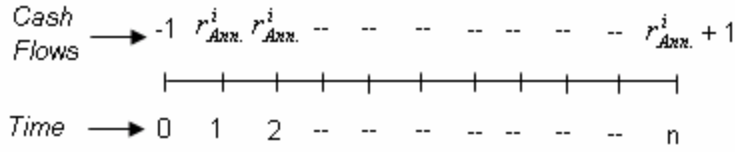
### A. Bullet Loan

An amortization plan with periodical repayment of interest and full repayment of principal on maturity may be depicted as follows



where  $I = \text{Interest}$  and  $P = \text{Principal}$ . When  $P = 1$ , the above diagram turns into

<sup>47</sup> As shown in Table 2 of the main text, the 10-year risk-free rate is in fact equal to 4.49%, while the 5.5-year is approximately equal to 3.80% (this latter calculated as the average of the risk-free rates at 5-year, 3.71%, and at 6-year, 3.92%).



where  $r_{Ann.}^i$  = annualized *risk-adjusted* interest rate, which is our unknown with regard to an *i*-th borrower.

The Loan value at time 0, denoted as  $L_0$ , is therefore equal to the sum of the Loan cash flows' present values, discounted at the *risk-adjusted* interest rates related to different periods ( $r_{j,adj}^i$ ). Each cash flow may be in fact considered as a 'stand-alone' maturity loan, discountable at an appropriate risk-adjusted interest rate. The term structure of such rates is that derived in section 4 of the main text.

In formula

$$L_0 = -1 + \frac{r_{Ann.}^i}{(1 + r_{1,adj}^i)} + \frac{r_{Ann.}^i}{(1 + r_{2,adj}^i)^2} + \dots + \frac{r_{Ann.}^i + 1}{(1 + r_{n,adj}^i)^n}$$

The above equation can also be rewritten as

$$1 = r_{Ann.}^i \sum_{j=1}^n \frac{1}{(1 + r_{j,adj}^i)^j} + \frac{1}{(1 + r_{n,adj}^i)^n}$$

from which we derive the annualized risk-adjusted interest rate

$$r_{Ann.}^i = \frac{1 - \frac{1}{(1 + r_{n,adj}^i)^n}}{\sum_{j=1}^n \frac{1}{(1 + r_{j,adj}^i)^j}} \tag{A3.1}$$

In the following Table we report the risk-adjusted interest rates of *Bullet* and *Zero-Coupon Loans* for 5 time intervals (1, 3, 5, 7 and 10 years), as concerns the *Corporate* Basel II segment and under the same assumptions formulated in the main text (section 4). As expected, for horizons beyond 1 year, the *risk-adjusted* interest rates of *Bullet Loans* are slightly lower than those of *Zero-Coupon Loans*, because of the progressive repayment of interest during the loan term in the former amortization plan. Furthermore, the differences between these interest rates increase as a function of both riskiness and maturity.

Table A3.1

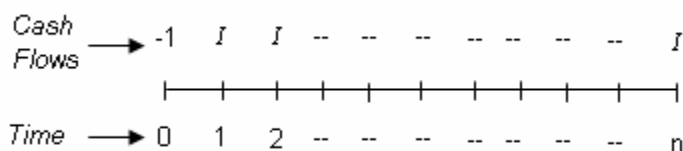
**Comparison between the risk-adjusted interest rate term structures of Bullet and Zero-coupon Loans** ('Corporate' Basel II segment; gross expected ROE equal to 20%; spread on the subordinated debt issues equal to 0.75%)

	Bullet	Zero Coupon	Bullet	Zero Coupon	Bullet	Zero Coupon	Bullet	Zero Coupon	Bullet	Zero Coupon
	1	1	3	3	5	5	7	7	10	10
AAA	2.46%	2.46%	3.29%	3.31%	3.84%	3.88%	4.21%	4.29%	4.56%	4.69%
AA+	2.51%	2.51%	3.37%	3.39%	3.93%	3.97%	4.30%	4.39%	4.67%	4.81%
AA	2.55%	2.55%	3.40%	3.42%	3.96%	4.01%	4.33%	4.42%	4.70%	4.84%
AA-	2.59%	2.59%	3.43%	3.45%	3.99%	4.04%	4.36%	4.45%	4.73%	4.86%
A+	2.62%	2.62%	3.52%	3.54%	4.09%	4.15%	4.48%	4.57%	4.86%	5.01%
A	2.65%	2.65%	3.54%	3.56%	4.12%	4.17%	4.50%	4.59%	4.88%	5.03%
A-	2.73%	2.73%	3.61%	3.63%	4.18%	4.24%	4.57%	4.66%	4.95%	5.09%
BBB+	2.82%	2.82%	3.92%	3.95%	4.57%	4.64%	5.01%	5.12%	5.44%	5.63%
BBB	2.89%	2.89%	3.97%	3.99%	4.61%	4.68%	5.05%	5.17%	5.48%	5.67%
BBB-	3.26%	3.26%	4.27%	4.30%	4.90%	4.97%	5.33%	5.46%	5.77%	5.97%
BB+	3.59%	3.59%	4.71%	4.74%	5.49%	5.58%	5.90%	6.03%	6.23%	6.42%
BB	4.04%	4.04%	5.18%	5.22%	6.04%	6.16%	6.47%	6.64%	6.76%	6.97%
BB-	4.67%	4.67%	5.87%	5.91%	6.86%	7.00%	7.36%	7.56%	7.72%	7.98%
B+	5.63%	5.63%	6.92%	6.97%	8.16%	8.36%	8.79%	9.09%	9.23%	9.59%
B	7.28%	7.28%	8.69%	8.76%	10.43%	10.78%	10.78%	11.15%	10.91%	11.26%
B-	10.22%	10.22%	11.85%	11.96%	13.35%	13.76%	13.81%	14.34%	14.10%	14.61%
CCC	15.66%	15.66%	18.20%	18.50%	19.38%	19.91%	19.65%	20.19%	20.59%	22.28%

Source: Our elaborations on the basis of formulas (A3.1) and (2.13), with a recovery rate of 55%.

### B. Equal-Instalment Loan ('Straight-Line Amortization')

A 'Straight-Line Amortization' plan, scheduling the loans's repayment in equal instalments (inclusive of both the interest and the principal share), may be represented as follows



where  $I$  = Loan's Equal Instalment.

Following the usual reasoning, the Loan value at time 0 ( $L_0$ ) is equal to the sum of the Loan cash flows' present values, where in this case each cash flow coincides with the equal instalment  $I$ .

In formula

$$L_0 = -1 + \frac{I}{(1+r_{1,adj}^i)} + \frac{I}{(1+r_{2,adj}^i)^2} + \dots + \frac{I}{(1+r_{n,adj}^i)^n}$$

also equal to

$$1 = I * \sum_{j=1}^n \frac{1}{(1+r_{j,adj}^i)^j}$$

from which

$$I = \frac{1}{\sum_{j=1}^n \frac{1}{(1+r_{j,adj}^i)^j}} \quad (A3.2)$$

However, to get the interest rate  $r_{Ann.}^i$  we have to further derive another expression for I as a function of the unknown rate  $r_{Ann.}^i$ . To that end, it is known I may also be expressed as follows

$$1 = I * d_{n|r_{Ann.}^i}, \text{ where } d_{n|r_{Ann.}^i} = \frac{1 - \frac{1}{(1+r_{Ann.}^i)^n}}{r_{Ann.}^i}$$

from which

$$I = \frac{1}{d_{n|r_{Ann.}^i}} = \frac{1}{\frac{1 - \frac{1}{(1+r_{Ann.}^i)^n}}{r_{Ann.}^i}} \quad (A3.3)$$

Now, combining formulas (A3.2) and (A3.3) we obtain

$$I \frac{1 - \frac{1}{(1+r_{Ann.}^i)^n}}{r_{Ann.}^i} = I \sum_{j=1}^n \frac{1}{(1+r_{j,adj}^i)^j} = 1$$

Finally, dividing by I both members of the previous equation, we get



$$\frac{1 - \frac{1}{(1 + r_{Ann.}^i)^n}}{r_{Ann.}^i} = \sum_{j=1}^n \frac{1}{(1 + r_{j,adj}^i)^j} \quad (A3.4)$$

Formula (A3.4) allows us to derive the rate  $r_{Ann.}^i$ , resorting to numerical methods<sup>48</sup> for loans with a maturity beyond 1 year.

In the following Table we report the risk-adjusted interest rates of *Equal-Instalment* and *Zero-Coupon Loans* for the usual 5 time intervals. In this case, the differences between these rates are greater than those previously shown for *Bullet* and *Zero-Coupon Loans*, because of the larger cash flows implied by the ‘Straight-Line Amortization’ plan.

Table A3.2 **Comparison between the risk-adjusted interest rate term structures of Equal-Instalment and Zero-coupon Loans** (‘Corporate’ Basel II segment; gross expected ROE equal to 20%; spread on the subordinated debt issues equal to 0.75%)

	Equal Instal.	Zero Coupon	Equal Instal.	Zero Coupon	Equal Instal.	Zero Coupon	Equal Instal.	Zero Coupon	Equal Instal.	Zero Coupon
	1	1	3	3	5	5	7	7	10	10
AAA	2.46%	2.46%	3.03%	3.31%	3.45%	3.88%	3.77%	4.29%	4.13%	4.69%
AA+	2.51%	2.51%	3.10%	3.39%	3.54%	3.97%	3.86%	4.39%	4.23%	4.81%
AA	2.55%	2.55%	3.13%	3.42%	3.57%	4.01%	3.90%	4.42%	4.26%	4.84%
AA-	2.59%	2.59%	3.16%	3.45%	3.60%	4.04%	3.92%	4.45%	4.29%	4.86%
A+	2.62%	2.62%	3.23%	3.54%	3.69%	4.15%	4.03%	4.57%	4.40%	5.01%
A	2.65%	2.65%	3.26%	3.56%	3.71%	4.17%	4.05%	4.59%	4.43%	5.03%
A-	2.73%	2.73%	3.33%	3.63%	3.78%	4.24%	4.12%	4.66%	4.49%	5.09%
BBB+	2.82%	2.82%	3.58%	3.95%	4.11%	4.64%	4.50%	5.12%	4.92%	5.63%
BBB	2.89%	2.89%	3.63%	3.99%	4.15%	4.68%	4.54%	5.17%	4.97%	5.67%
BBB-	3.26%	3.26%	3.95%	4.30%	4.46%	4.97%	4.83%	5.46%	5.26%	5.97%
BB+	3.59%	3.59%	4.39%	4.74%	4.98%	5.58%	5.40%	6.03%	5.80%	6.42%
BB	4.04%	4.04%	4.76%	5.22%	5.46%	6.16%	5.90%	6.64%	6.33%	6.97%
BB-	4.67%	4.67%	5.54%	5.91%	6.25%	7.00%	6.78%	7.56%	7.26%	7.98%
B+	5.63%	5.63%	6.58%	6.97%	7.44%	8.36%	8.09%	9.09%	8.70%	9.59%
B	7.28%	7.28%	8.32%	8.76%	9.47%	10.78%	10.12%	11.15%	10.52%	11.26%
B-	10.22%	10.22%	11.43%	11.96%	12.51%	13.76%	13.12%	14.34%	13.70%	14.61%
CCC	15.66%	15.66%	17.47%	18.50%	18.70%	19.91%	19.20%	20.19%	19.90%	22.28%

Source: Our elaborations on the basis of formulas (A3.4) and (2.13), with a recovery rate of 55%.

<sup>48</sup> In Excel, for example, this equation may be solved by the ‘Solver’ add-in.

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