



# Primal/Dual Linear Programming and Statistical Atlases for Cartilage Segmentation



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## Idea: Automatic Cartilage Segmentation by Constructing a Probabilistic Atlas

### Probabilistic Atlas Construction

- $\mathcal{V}_{\mathcal{M}}$  : optimal representative of training set
- $\sigma_{\mathcal{M}}$  : variance map / agreement between atlas and training set
- $p_{\mathbf{x}}(i)$  : pdf defined at each voxel, e.g. Gaussian density

**Minimization problem:**

$$E(\mathcal{V}_{\mathcal{M}}, \sigma_{\mathcal{M}}) = \int_{\Omega} \sum_{i=1}^n \left[ \log(\sigma_{\mathcal{M}}^2(\mathbf{x})) + \frac{(\mathcal{V}_i(\mathbf{x}) - \mathcal{V}_{\mathcal{M}}(\mathbf{x}))^2}{2\sigma_{\mathcal{M}}^2(\mathbf{x})} \right] d\mathbf{x}$$

### Automatic Cartilage Segmentation

- Novel framework for deformable registration based on discrete optimization [1] is used for atlas matching
- Primal/Dual linear programming [2] is used for efficient optimization of the atlas matching problem

**Matching criteria:**

$$\rho_{\mathcal{M}}(\mathcal{V}(\mathcal{T}(\mathbf{x}))) = \left[ \log(\sigma_{\mathcal{M}}^2(\mathbf{x})) + \frac{(\mathcal{V}(\mathcal{T}(\mathbf{x})) - \mathcal{V}_{\mathcal{M}}(\mathbf{x}))^2}{2\sigma_{\mathcal{M}}^2(\mathbf{x})} \right]$$

### Atlas Matching Framework

- Grid-based deformation model (e.g. Free Form Deformation)

$$\mathcal{T}(\mathbf{x}) = \mathbf{x} + \mathcal{D}(\mathbf{x}), \quad \mathcal{D}(\mathbf{x}) = \sum_{p \in \mathcal{G}} \eta(|\mathbf{x} - \mathbf{p}|) \mathbf{d}_p$$

- Deformable registration is formulated as a discrete labeling problem

$$\mathcal{L} = \{u^1, \dots, u^i\} \quad \Theta = \{d^1, \dots, d^i\} \quad \mathcal{D}(\mathbf{x}) = \sum_{p \in \mathcal{G}} \eta(|\mathbf{x} - \mathbf{p}|) d^{u_p}$$

- Markov Random Field energy formulation of the discrete labeling

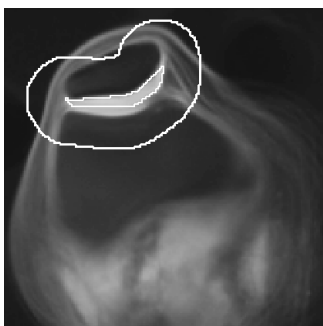
$$E_{\text{total}}(u) = \sum_{p \in \mathcal{G}} V_p(u_p) + \sum_{p, q \in \mathcal{E}(p)} V_{pq}(u_p, u_q).$$

$$E_{\text{data}}(u) = \sum_{p \in \mathcal{G}} \int_{\Omega} \eta^{-1}(|\mathbf{x} - \mathbf{p}|) \rho_{\mathcal{M}}(\mathcal{V}(\mathcal{T}(\mathbf{x}))) d\mathbf{x} \approx \sum_{p \in \mathcal{G}} V_p(u_p)$$

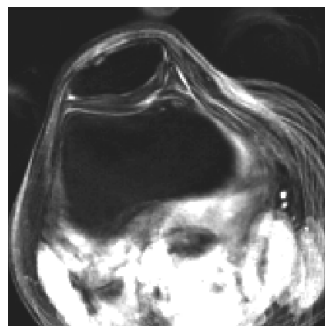
$$E_{\text{smooth}}(u) = \sum_{p, q \in \mathcal{E}(p)} V_{pq}(u_p, u_q), \quad V_{pq}(u_p, u_q) = \min(|d^{u_p} - d^{u_q}|, T)$$

### Conclusion

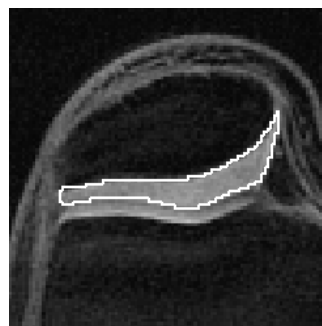
- Automatic segmentation in less than 20 seconds
- Average surface distance of 0.49 ( $\pm 0.23$ ) millimeters
- Significant improvement of automatic segmentation initialization for applications with high accuracy constraints
- Flexible and general framework can be used for other anatomy
- Powerful linear programming algorithm allows to incorporate complex priors in future work



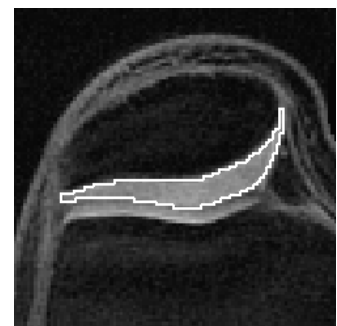
Mean Intensity Image



Variance Map



Automatic Segmentation



Expert Segmentation

## Experiments / Results

Method	DSC	Sensitivity	Specificity	Interaction	Cartilage
Grau <i>et al.</i>	0.90 (0.01)	90.03 %	99.87 %	5-10 min	Tibia, Femur, Patella
Dam <i>et al.</i>	0.92 (n/a)	93.00 %	99.99 %	Max 10 min	Tibia, Femur
Cheong <i>et al.</i>	0.64 (0.15)	74.00 %	n/a	0	Medial Tibia
Cheong <i>et al.</i>	0.72 (0.09)	79.00 %	n/a	0	Lateral Tibia
Folkesson <i>et al.</i>	0.80 (0.03)	90.01 %	99.80 %	0	Tibia, Femur
<b>Our Approach</b>	<b>0.83 (0.06)</b>	<b>93.77 %</b>	<b>99.94 %</b>	<b>0</b>	<b>Patella</b>

### Image Data

- 56 data sets of patella cartilage
- 28 data sets used for atlas generation / 28 for evaluation
- Protocol: T1-w 3D FLASH water excitation
- Scanner: Siemens MAGNETOM Symphony 1.5T
- Resolution: 256 x 256 x 20 (.625 x .625 x 3mm)

### References

- [1] Glocker, B. et al.: Inter and Intra-modal deformable registration: Continuous Deformations meet efficient linear programming. IPMI 2007, Kerkrade, Netherlands.  
 [2] Komodakis, N. et al.: Fast, approximately optimal solutions for single and dynamic MRFs. CVPR 2007, Minneapolis, Minnesota, USA.

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