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PRIME LABELING FOR SOME HELM RELATED GRAPHS

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Abstract: A graph with vertex set V is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,3 ... $|V|$ such that for edge xy the labels assigned to x and y are relatively prime. A graph which admits prime labeling is called a prime graph. In this paper we investigate prime labeling for some helm related graphs. We also discuss prime labeling in the context of some graph operations namely fusion and duplication in Helm H_n

Keywords: Prime Labeling, Fusion, Duplication.

I. INTRODUCTION

In this paper, we consider only finite simple undirected graph. The graph G has vertex set $V = V(G)$ and edge set $E = E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to Bondy and Murthy [1].

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A (1982 P 365-368). [2] Many researchers have studied prime graph for example Fu.H. (1994 P 181-186) [5] Have proved that path P_n on n vertices is a Prime graph.

Deretsky.T (1991 P359 – 369) [4] have proved that the C_n on n vertices is a prime graph. Lee.S (1998 P.59-67) [2] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all tress have prime labeling which is not settled till today. The prime labeling for planner grid is investigated by Sundaram.M (2006 P205-209) [6]

In [8] S.K.Vaidhya and K.K.Kanmani have proved the prime labeling for some cycle related graphs.

II. DEFINITION

Definition 1.1

Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f: V(G) \rightarrow \{1,2,\ldots p\}$ is called a prime labeling if for each edge $e = uv, gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2

Fusion: Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by identifying (fusing) two vertices u and v by a single vertex x in such that every edge which was incident with either u or v in G now incident with x in G.

Definition: 1.3

Duplication: Duplication of a vertex v_k of a graph G produces a new graph G_k by adding a vertex $v_{k'}$ with $(v_{k'}) = N(v_k)$.

In other words a vertex $v_{k'}$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_{k'}$.

Definition: 1.4

Switching: A vertex switching G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Definition: 1.5 (Path Union)

Let $G_1, G_2, \ldots. G_n, n \geq 2$ be n copies of a fixed graph G. The graph obtained by adding an edge between G_i *and* G_{i+1} for $i = 1,2, \ldots.. n-1$ is called the path union of G.

Definition: 1.6

The helm H_n is a graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

In this paper we have proved that the helm H_n, the graph obtained by fusing the vertices v_1 and v_k on the rim, the graph obtained by duplication of any vertex of H_n, the graph obtained by switching of any vertex of H_n and the graph obtained by the path union of two copies of H_n by a path of length k are all prime graphs.

III. THEOREM

Theorem: 1

The helm H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(H_n) = \{\, cv_i \,/\, 1 \le i \le n \,\} U \,\{v_i v_i^{'} / \, 1 \le i \le n \,\} U \,\{v_i v_{i+1} / \, 1 \le i \le n-1 \,\} U \; v_1 v_n$

Where c is the centre vertex

Here $|V(H_n)| = 2n + 1$

We consider two cases,

Case (i):

When $n \ne 3k + 1$where k is any integer,

Define a labeling $f: V(H_n) \to \{1,2,3 \dots 2n + 1\}$ as follows

$f(c) = 1$

$f(v_i) = 2i + 1 \qquad for\ 1 \le i \le n$

$f(v_i^{'}) = 2i \qquad for\ 1 \le i \le n$

Then f admits prime labeling.

Case (ii):

When $n = 3k + 1$ where k is any integer,

Define a labeling $f: V(H_n) \to \{1,2,3 \dots 2n + 1\}$ as follows

$f(c) = 1\,; \qquad f(v_1) = 2$

$f(v_1^{'}) = 3$

$f(v_i) = 2i + 1 \qquad for\ 2 \le i \le n$

$f(v_i^{'}) = 2i \qquad for\ 2 \le i \le n$

Then f admits prime labeling.

Thus H_n is a prime graph.

Theorem 2:

The graph obtained by fusing the vertex v_2 with v_1 (or any two consecutive vertices) in a helm graph H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(H_n) = \{\, cv_i \,/\, 1 \le i \le n \,\} U \,\{v_i v_i^{'} / \, 1 \le i \le n \,\} U \,\{v_i v_{i+1} / \, 1 \le i \le n-1 \,\} U \; v_1 v_n$

Let G_2be the graph obtained by fusing v_1 and v_2.

Here $|V(G_2)| = 2n$

Case (i):

When $n \ne 3k - 1$where k is any integer,

Define a labeling $f: V(G_2) \to \{1,2,3 \dots 2n\}$ as follows

Let $f(c) = 1$

$f(v_1) = 2n - 1\,; f(v_1^{'}) = 2n - 2$

$f(v_2^{'}) = 2n\,; \qquad f(v_i) = 2i - 3 \qquad for\ 3 \le i \le n$

$f(v_i^{'}) = 2i - 4 \qquad for\ 3 \le i \le n$

Then f admits prime labeling.

Case (ii):

When $n = 3k - 1$ where k is any integer,

Define a labeling $f: V(G_2) \to \{1,2,3 \dots 2n\}$ as follows

Let $f(c) = 1\,; f(v_3) = 2$

$f(v_3^{'}) = 3\,;$

$f(v_i) = 2i - 3 \qquad for\ 4 \le i \le n$

$f(v_i^{'}) = 2i - 4 \qquad for\ 4 \le i \le n$

$f(v_1) = 2n - 1\,; f(v_1^{'}) = 2n - 2$

$f(v_2^{'}) = 2n$

Then f admits prime labeling.

Thus G_2 is a prime graph.

Theorem 3:

The graph obtained by fusing the vertex v_1 with v_3 in a helm graph H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1', v_2' \dots v_n'\}$
$E(H_n) = \{ cv_i \,/\, 1 \le i \le n \} U \{v_i v_i' / 1 \le i \le n\} U \{v_i v_{i+1} / 1 \le i \le n-1\} U\, v_1 v_n$
Let G_3 be the graph obtained by fusing v_1 and v_3 in H_n.
Here $|V(G_3)| = 2n$

Case (i):
When $n \ne 3k-1$ and $2n-1$ is not a multiple of 5,
Define a labeling $f: V(G_3) \to \{1,2,3 \dots 2n\}$ as follows
Let $f(c) = 1;$
$f(v_1) = 2n-1;$ $f(v_1') = 2n-2$
$f(v_3') = 2n;$ $f(v_2) = 3$
$f(v_2') = 2;$
$f(v_i) = 2i-3$ $for\ 4 \le i \le n$
$f(v_i') = 2i-4$ $for\ 4 \le i \le n$
Then f admits prime labeling.

Case (ii):
When $n = 3k-1$ and $2n-1$ is not a multiple of 5,
Define a labeling $f: V(G_3) \to \{1,2,3 \dots 2n\}$ as follows
Let $f(c) = 1;$ $f(v_1) = 2n-1$
$f(v_1') = 2n-2;$ $f(v_3') = 2n$
$f(v_2) = 2;$ $f(v_2') = 3$
$f(v_i) = 2i-3$ $for\ 4 \le i \le n$
$f(v_i') = 2i-4$ $for\ 4 \le i \le n$
Then f admits prime labeling.

Case (iii):
When $n \ne 3k-1$ and $2n-1$ is a multiple of 5,
Define a labeling $f: V(G_3) \to \{1,2,3 \dots 2n\}$ as follows
$f(c) = 1;$ $f(v_1) = 2n-1$
$f(v_1') = 2n-2;$ $f(v_3') = 2n$
$f(v_2) = 3;$ $f(v_2') = 2$
$f(v_4) = 4;$ $f(v_4') = 5$
$f(v_i) = 2i-3$ $for\ 5 \le i \le n$
$f(v_i') = 2i-4$ $for\ 5 \le i \le n$
Then f admits prime labeling.

Case (iv):
When $n = 3k-1$ and $2n-1$ is a multiple of 5,
Define a labeling $f: V(G_3) \to \{1,2,3 \dots 2n\}$ as follows
$f(c) = 1;$ $f(v_1) = 2n-1$
$f(v_1') = 2n-2;$ $f(v_3') = 2n$
$f(v_2) = 3;$ $f(v_2') = 2$
$f(v_4) = 4;$ $f(v_4') = 5$
$f(v_i) = 2i-3$ $for\ 5 \le i \le n$
$f(v_i') = 2i-4$ $for\ 5 \le i \le n$
Then f admits prime labeling.

Theorem 4:
The graph obtained by fusing the vertex v_1 with v_4 in a helm graph H_n is a prime graph.

Proof:
Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1', v_2' \dots v_n'\}$
$E(H_n) = \{ cv_i \,/\, 1 \le i \le n \} U \{v_i v_i' / 1 \le i \le n\} U \{v_i v_{i+1} / 1 \le i \le n-1\} U\, v_1 v_n$
Let G_4 be the graph obtained by fusing v_1 and v_4 in H_n.
Then $|V(G_4)| = 2n$

Case (i):
When $n \ne 3k-1$ and $2n-1$ is not a multiple of 5 and $2n-1$ is not a multiple of 7,
Define a labeling $f: V(G_4) \to \{1,2,3 \dots 2n\}$ as follows
Let $f(c) = 1;$ $f(v_2) = 3$
$f(v_2') = 2;$ $f(v_3) = 5$
$f(v_3') = 4$
$f(v_i) = 2i-3$ $for\ 5 \le i \le n$
$f(v_i') = 2i-4$ $for\ 5 \le i \le n$

$f(v_1) = 2n - 1$; $f(v_1^{'}) = 2n - 2$
$f(v_4^{'}) = 2n$

Then f admits prime labeling.

Case (ii)

If $n = 3k - 1$ but $2n - 1$ is not a multiple of 5 and not a multiple of 7, then in the above labeling f defined in case (i) interchange the labels of v_2 and $v_2^{'}$. The resulting labeling f^{I} is a prime labeling.

Case (iii)

If $n = 3k - 1$ but $2n - 1$ is a multiple of 5 but not a multiple of 7, then in the above labeling f defined in case (i) interchange the labels of v_2 and $v_2^{'}$ and also interchange the labels of v_3 and $v_3^{'}$. The resulting labeling f^{II} is a prime labeling.

Case (iv):

When $n = 3k - 1$ and $2n - 1$ is a multiple of 5 and $2n - 1$ is a multiple of 7,
Define a labeling $f: V(G_4) \to \{1,2,3 \dots 2n\}$ as follows

Let $f(c) = 1$; $f(v_2) = 2n - 1$
$f(v_2^{'}) = 2n$; $f(v_3) = 2$
$f(v_3^{'}) = 3$; $f(v_5) = 4$
$f(v_5^{'}) = 5$
$f(v_i) = 2i - 5$ for $6 \le i \le n$
$f(v_i^{'}) = 2i - 6$ for $6 \le i \le n$
$f(v_1) = 2n - 3$; $f(v_1^{'}) = 2n - 4$
$f(v_4^{'}) = 2n - 2$

Then f admits prime labeling.

Case (v):

If $n = 3k - 1$ and $2n - 1$ is not a multiple of 5 and also a multiple of 7, then in the above labeling f defined in case (iv) interchange the labels of v_5 and $v_5^{'}$. The resulting labeling f^{III} is a prime labeling.

Case (vi)

If $n \ne 3k - 1$ but $2n - 1$ is a multiple of 5 and also a multiple of 7, then in the above labeling f defined in case (iv) interchange the labels of v_3 and $v_3^{'}$. The resulting labeling $f^{(iv)}$ is a prime labeling.

Case (vii):

If $n \ne 3k - 1$ and $2n - 1$ is not a multiple of 5 but $2n - 1$ is a multiple of 7, then in the above labeling f defined in interchange the labels of v_3 and $v_3^{'}$, v_5 and $v_5^{'}$. The resulting labeling $f^{(v)}$ is a prime labeling.

Case (viii)

If $n \ne 3k - 1$ and $2n - 1$ is a multiple of 5 but not a multiple of 7, then in the above labeling f defined in case (i) interchange the labels of v_5 and $v_5^{'}$. The resulting labeling $f^{(vi)}$ is a prime labeling.

Thus in all the cases G_4 admits prime labeling, hence G_4 is a prime graph.

Theorem 5:

The graph obtained by fusing the vertex v_1 with v_5 in a helm graph H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$
$E(H_n) = \{ cv_i \,/\, 1 \le i \le n \} U \{v_i v_i^{'} / 1 \le i \le n \} U \{v_i v_{i+1} / 1 \le i \le n - 1 \} U\, v_1 v_n$

Let G_5 be the graph obtained by fusing v_1 and v_5 in H_n.
Then $|V(G_5)| = 2n$

Case (i):

When $n \ne 3k - 1$ and $2n - 1$ is not a multiple of 7,
Define a labeling $f: V(G_5) \to \{1,2,3 \dots 2n\}$ as follows

Let $f(c) = 1$, $f(v_2) = 3$
$f(v_2^{'}) = 2$; $f(v_3) = 5$; $f(v_3^{'}) = 4$
$f(v_4) = 7$; $f(v_4^{'}) = 6$;
$f(v_i) = 2i - 3$ for $6 \le i \le n$
$f(v_i^{'}) = 2i - 4$ for $6 \le i \le n$
$f(v_1) = 2n - 1$; $f(v_1^{'}) = 2n - 2$
$f(v_5^{'}) = 2n$

Then f admits prime labeling.

Case (ii)

If $n = 3k - 1$ but $2n - 1$ is not a multiple of 7, then in the above labeling f defined in case (i) interchange the labels of v_2 and $v_2^{'}$ and the label v_6 and $v_6^{'}$. The resulting labeling f^{I} is a prime labeling.

Case (iii)

If either $n = 3k - 1$or $n \ne 3k - 1$ but $2n - 1$ is a multiple of 7.

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Define a labeling $f: V(G_5) \rightarrow \{1,2,3 \ldots 2n\}$ as follows

Let $f(c) = 1$; $f(v_1) = 2n - 5$
$f(v_1') = 2n - 4$; $f(v_2) = 2n - 3$
$f(v_2') = 2n - 2$; $f(v_3) = 2n - 1$; $f(v_3') = 2n$; $f(v_4) = 2$; $f(v_4') = 3$;
$f(v_i) = 2i - 7$ $for\ 6 \leq i \leq n$
$f(v_i') = 2i - 8$ $for\ 6 \leq i \leq n$

Then f admits prime labeling.

Thus G_5 is a prime graph.

Remark In a similar way we can prove that the graph obtained by fusing the vertices v_1 and v_k in a helm graph H_n is a prime graph.

III. EXAMPLES

Example for theorem 1:

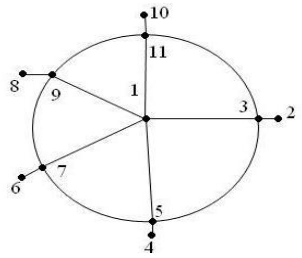

Fig.1 prime labeling for $H_5 (n \neq 3k - 1)$

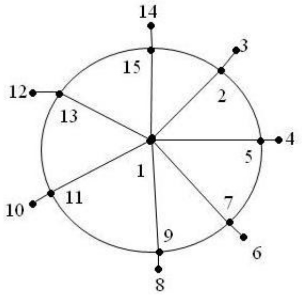

Fig.2 prime labeling for $H_7 (n = 3k - 1)$

Example for theorem 2:

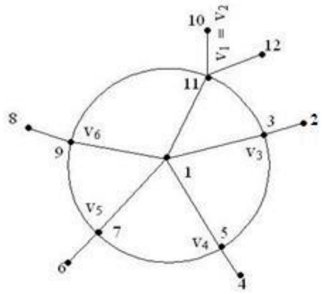

Fig.3 prime labeling for fusion of v_1 and v_2 in H_6

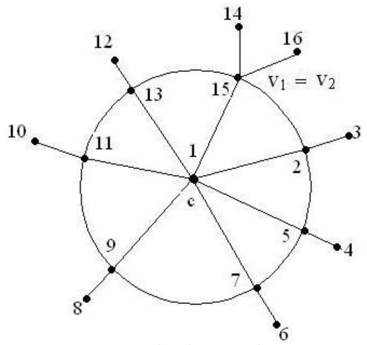

Fig.4 prime labeling for fusion of v_1 and v_2 in H_8

Example for theorem 3:

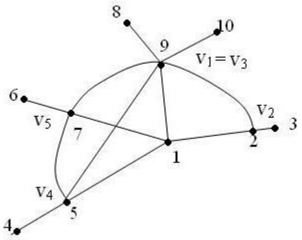

Fig.5 prime labeling for fusion of $v_1$and v_3 in $H_5(n = 3k - 1)$

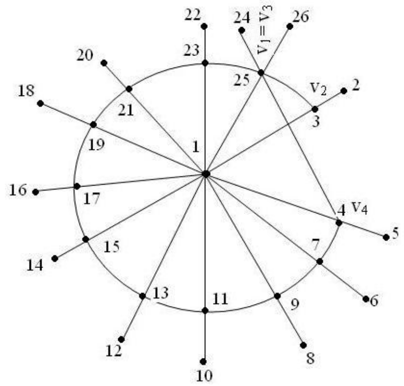

Fig.6 prime labeling for fusion of $v_1$and v_3 in $H_{13}((n \neq 3k - 1)$, 2n-1 is a multiple of 5)

Example for theorem 4:

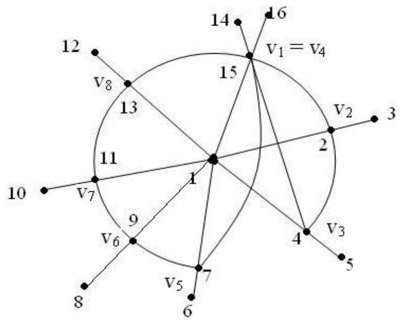

Fig.7 prime labeling for fusion of $v_1$and v_4 in $H_8((n = 3k - 1)$, 2n-1 is a multiple of 5)

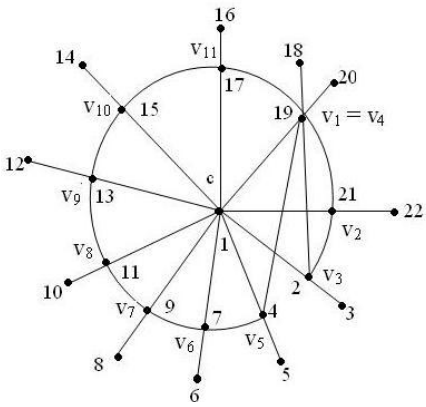

Fig.8 prime labeling for fusion of $v_1$and v_4 in $H_{11}((n = 3k - 1)$, 2n-1 is a multiple of 7)

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Example for theorem 5:

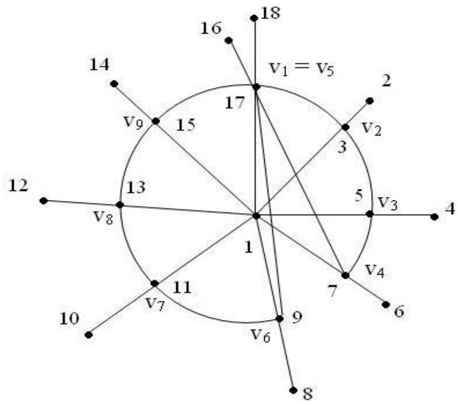

Fig.9 prime labeling for fusion of $v_1$and v_5 in $H_9((n \neq 3k-1)$, 2n-1 is not a multiple of 7)

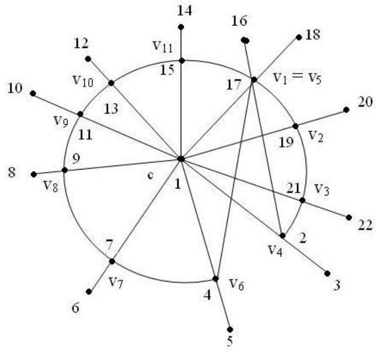

Fig.10 prime labeling for fusion of $v_1$and v_5 in $H_{11}((n = 3k-1)$, 2n-1 is a multiple of 7)

Theorem 6:

The graph obtained by duplicating a vertex v_k in the rim of the helm H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \ldots v_n, v_1', v_2' \ldots v_n'\}$

$E(H_n) = \{ cv_i \,/\, 1 \leq i \leq n \} U \{v_i v_i' / \, 1 \leq i \leq n \} U \{v_i v_{i+1} / \, 1 \leq i \leq n-1 \} U \, v_1 v_n$

Let G_k be the graph obtained by duplicating the vertex v_k in H_n.

Then$|V(G_k)| = 2n+2$, Let the new vertex be v_k^*

Define a labeling $f: V(G_k) \rightarrow \{1,2,3 \ldots 2n+2\}$ as follows

Let $f(c) = 1$, $f(v_k) = 2$

$f(v_k^*) = 4$

$f(v_k') = 3$

$f(v_{k+1}) = 5$

$f(v_{k+1}) = 5 + (2i-2)$ $\quad for\ 2 \leq i \leq n-k$

$f(v_i) = f(v_n) + 2i$ $\quad for\ 1 \leq i \leq k-1$

$f(v_i') = f(v_i) + 1$ $\quad for\ 1 \leq i \leq n, i \neq k$

Then f admits prime labeling.

Thus G_k is a prime graph.

Example:

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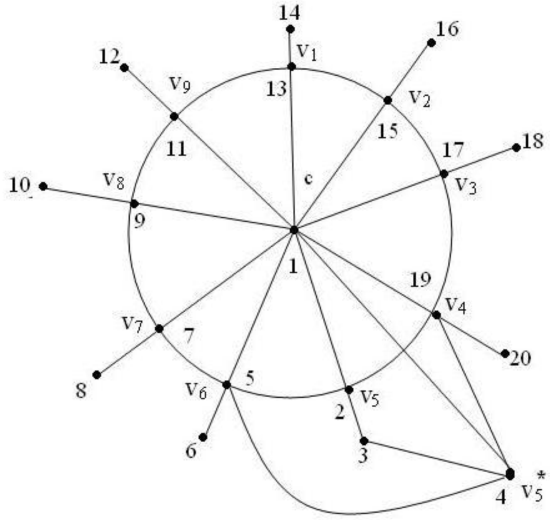

Fig.11 prime labeling for duplication of v_5 in H_9

Theorem 7:

The graph G_k obtained by switching of any vertex v_k in the rim of the helm H_n is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(H_n) = \{ cv_i \ / \ 1 \le i \le n \} U \left\{v_i v_i^{'} / \ 1 \le i \le n \right\} U \{v_i v_{i+1} / \ 1 \le i \le n-1 \} U \ v_1 v_n$

Let G_k be the graph obtained by switching the vertex v_k in H_n.

Then$|V(G_k)| = 2n + 1$

Define a labeling $f: V(G) \to \{1,2,3 \dots 2n+1\}$ as follows

Let $f(c) = 2$

$f(v_k) = 1$

$f\left(v_k^{'}\right) = 2n + 1$

$f(v_{k+1}) = 3$

$f(v_{k+i}) = 3 + (2i - 2) \qquad for\ 2 \le i \le n - k$

$f(v_i) = f(v_n) + 2i \qquad for\ 1 \le i \le k - 1$

$f\left(v_i^{'}\right) = f(v_i) + 1 \qquad for\ 1 \le i \le n, i \ne k$

Then f admits prime labeling.

Thus G_k is a prime graph.

Example for theorem 7:

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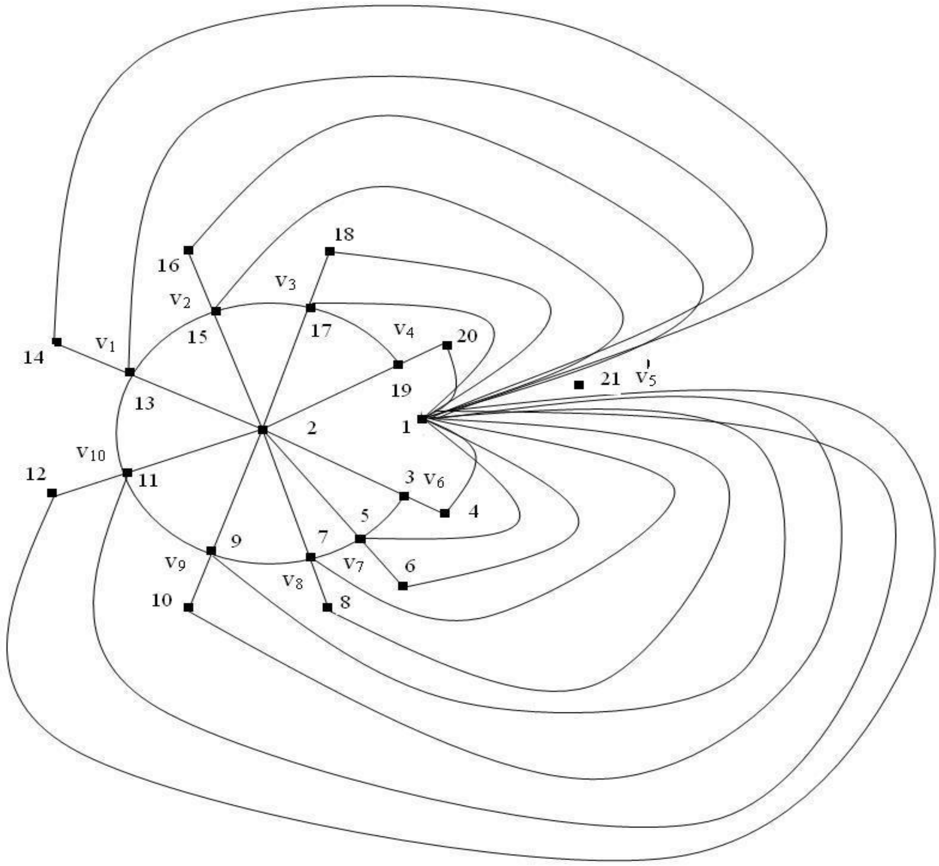

Fig.12 Prime Labeling for switching of v_5 in H_{10}.

Theorem 8:

Let G_c be the graph obtained by switching the centre vertex c in the helm H_n then G_c is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1', v_2' \dots v_n'\}$

$E(H_n) = \{ cv_i \,/\, 1 \le i \le n \} U \{v_i v_i' / 1 \le i \le n\} U \{v_i v_{i+1} / 1 \le i \le n-1\} U \, v_1 v_n$

Let G_c be the graph obtained by switching the centre vertex c in H_n.

Then$|V(G_c)| = 2n + 1$

Case (i):

When $n \neq 3k + 1$ where k is any integer,

Define a labeling $f: V(G) \to \{1,2,3 \dots 2n + 1\}$ as follows

Let $f(c) = 1$

$f(v_i) = 2i + 1 \qquad for\ 1 \le i \le n$

$f(v_i') = 2i \qquad for\ 1 \le i \le n$

Then f admits prime labeling.

Case (ii)

If $n = 3k + 1$ where k is any integer, then in the above labeling *f* defined in case (i) interchange the labels of v_1 and v_1'. The resulting labeling is a prime labeling.

Thus G_c is a prime graph.

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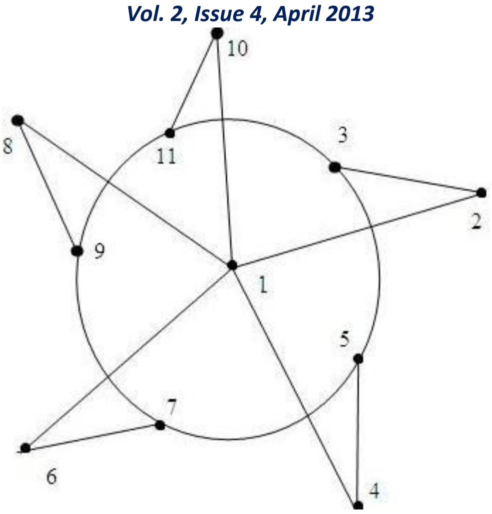

Fig.13 prime labeling for switching c in H_5

Theorem 9:
Let G be the graph obtained by the path union of two pieces of helm graph H_n. Then G is a prime graph if $n \neq 5k + 1$.
Proof:
Consider two copies H_n and H_n^* of helm graph
Let $V(H_n) = \{c, v_1, v_2, v_3 \ldots v_n, v_1^{'}, v_2^{'} \ldots v_n^{'}\}$
$E(H_n) = \{\, cv_i \,/\, 1 \leq i \leq n \,\} U \,\{v_i v_i^{'} / \,1 \leq i \leq n\,\} U\, \{v_i v_{i+1} / \,1 \leq i \leq n-1\,\} U\, v_1 v_n$
$V(H_n^*) = \{c^{'}, w_1, w_2, w_3 \ldots w_n, w_1^{'}, w_2^{'} \ldots w_n^{'}\}$
$E(H_n^*) = \{\, c^{'} w_i \,, 1 \leq i \leq n \,\}\, U\, \{w_i w_i^{'}, 1 \leq i \leq n\,\}\, U\, \{w_i w_{i+1}, 1 \leq i \leq n-1\,\} U w_1 w_n$
Let $V(G) = V(H_n) U\, V(H_n^*)$
$E(G) = E(H_n) U\, E(H_n^*) U\, \{v_1 w_1\}$
Define a labeling $f: V(G) \rightarrow \{1,2,3 \ldots 4n + 2\}$ as follows

Let $f(c) = 1$
$f(c^{'}) = 2$
$f(v_1) = 4$
$f(v_1^{'}) = 3$
$f(v_i) = 2i + 1$ $\quad$ $for\ 2 \leq i \leq n$
$f(v_i^{'}) = 2i + 2$ $\quad$ $for\ 2 \leq i \leq n$
$f(w_i) = (2n + 1) + 2i$ $\quad$ $for\ 1 \leq i \leq n$
$f(w_i^{'}) = 2(n + 1 + i)$ $\quad$ $for\ 1 \leq i \leq n$

Then f admits prime labeling.
Thus G is a prime graph.
Remark:
1. If $n = 5k + 1$ then G is not a prime graph.
2. The path union of more than two copies H_n of is also not prime labeling.

Example for theorem 9:

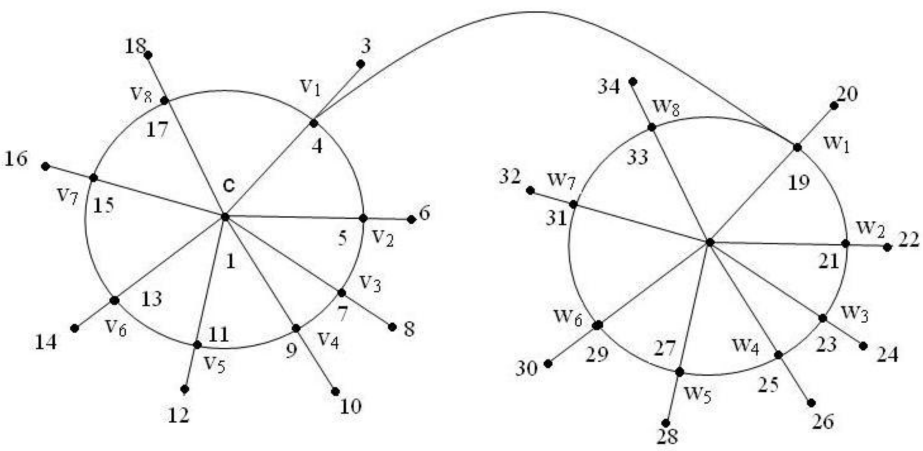

Fig.14 prime labeling for path union of two copies of H_8

Theorem 10:

The union of Helm graph and star graph $H_n U K_1, n$ is a prime graph.

Proof:

Let $V(H_n) = \{c, v_1, v_2, v_3 \dots v_n, v_1^{'}, v_2^{'} \dots v_n^{'}\}$

$E(H_n) = \{ cv_i \,/\, 1 \le i \le n \} U \{v_i v_i^{'} / \, 1 \le i \le n \} U \{v_i v_{i+1} / \, 1 \le i \le n-1 \} U\, v_1 v_n$

$V(K_1, n) = \{c^{'}, w_1, w_2, w_3 \dots w_n\}$

$E(K_1, n) = \{ c^{'} w_i \,, 1 \le i \le n \}$

Clearly, $V(H_n U K_1, n) = V(H_n) U\, V(K_1, \text{n})$

$E(H_n U K_1, n) = E(H_n) U\, E(K_1, \text{n})$

$|V(H_n U K_1, n)| = 3n + 2$

Define a labeling $f: V(G) \to \{1,2,3 \dots 3n+2\}$ as follows

Let $f(c) = 1$

$f(v_1) = 2$

$f(v_1^{'}) = 3$

$f(v_i) = 2i + 1 \qquad for\ 2 \le i \le n$

$f(v_i^{'}) = 2i \qquad for\ 2 \le i \le n$

And let k be the smallest prime number greater than $2n + 1$

$f(c^{'}) = k$

$f(w_i) = (2n+1) + i \qquad for\ 1 \le i \le k - (2n+1) - 1$

$f(w_i) = (2n+1) + i + 1 \qquad for\ k - (2n+1) \le i \le n$

Then f admits prime labeling.

Thus $H_n U K_1, n$ is a prime graph.

Example for Theorem 10:

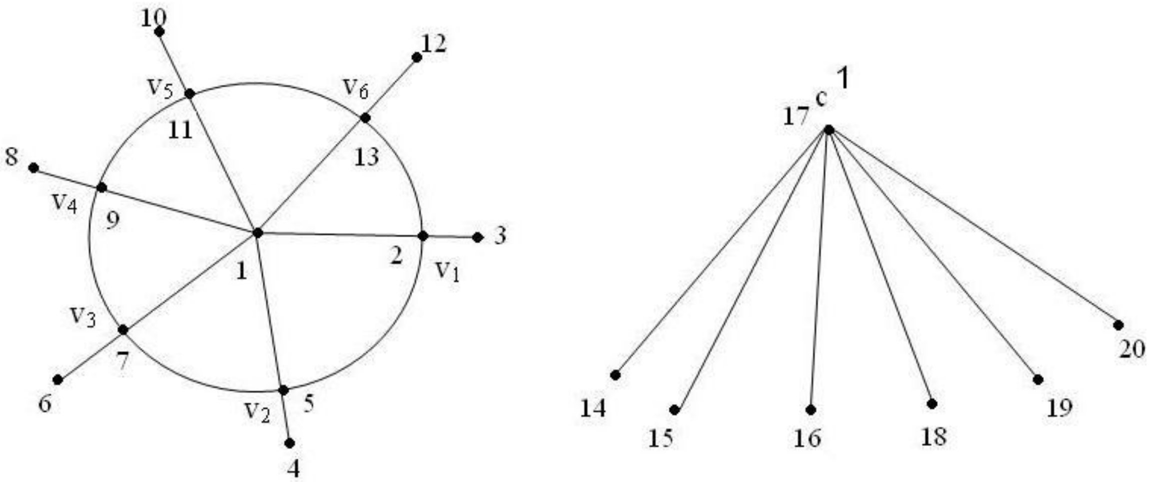

Fig.15 prime labeling for $H_n U K_1, 6$ (Here $k = 17$)

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