PRIMO: Probability Interpretation of Moments for Delay Calculation†

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Abstract

Moments of the impulse response are widely used for interconnect delay analysis, from the explicit Elmore delay (first moment of the impulse response) expression, to moment matching methods which create reduced order transimpedance and transfer function approximations. However, the Elmore delay is fast becoming ineffective for deep submicron technologies, and reduced order transfer function delays are impractical for use as early-phase design metrics or as design optimization cost functions. This paper describes an approach for fitting moments of the impulse response to probability density functions so that delays can be estimated from probability tables. For RC trees it is demonstrated that the incomplete gamma function provides a provably stable approximation. The step response delay is obtained from a one-dimensional table lookup.

1: Introduction

Due to CMOS technology trends, it is no longer possible to model all of the delays in terms of gates driving capacitive loads, since the RC charging delays for the interconnect can be the dominant delay component. Model order reduction methods using moments [10][15] or Krylov subspace methods [4][14][19] have effectively addressed the interconnect modeling problem for back-end design verification, however, all of these methods produce transfer function models for which a transcendental equation must be solved to obtain the delay. The implicit nature of these models makes them impractical for most front-end design applications or for use in the inner-loop during design optimization.

The Elmore delay [3], or first moment of the impulse response, provides a simple, explicit, delay approximation that is the *defacto*-standard metric for performance driven design applications. Elmore proposed in 1948 to approximate the median of the impulse response (50% delay of the step response) by the mean of the impulse response by noting the similarity between non-negative impulse responses and probability density functions. But the accuracy of this metric is sometimes unacceptable for RC interconnect problems encountered with today's CMOS technologies. The Elmore delay was shown to be an upper bound on the 50% step and ramp response delays for RC trees[6], but with relative accuracy that can be quite poor at times. Moreover, it can be shown that the first moment of the impulse response is sometimes incapable of even approximating delay sensitivities with respect to changing path resistances, which is becoming increasingly important due to resistive shielding effects. This lack of correlation between front-end delay metrics and back-end model order reduction methods can produce convergence problems for top-down design methodologies.

Some attempts have been made to formulate explicit solutions of 2nd order (two time constant) models, which would seem to be the obvious compromise between an Elmore approximation and a complete reduced-order model via moment matching. To apply such a two pole model, however, requires a moment matching formulation that characterizes the poles in a provably stable manner. Horowitz proposed the first stable two pole model in [1][7], but while it was stable, it can produce complex pole pairs for RC circuits. In addition, this two pole model requires nonlinear iterations to solve for the delay. Some two pole models were proposed in [8] and [21] which can be evaluated by closed-form approximations, however, these methods are lacking in terms of accuracy and generality.

This paper proposes an extension of Elmore's approximation to include matching of higher order moments of the probability density function. Specifically, using a time-shifted incomplete Gamma function approximation for the impulse responses of RC trees, the three parameters of this model are fitted by matching the first three central moments (mean, variance, skewness), which is equivalent to matching the first three moments of the circuit response (m1, m2, m3). Importantly, it is proven that such a gamma fit is guaranteed to be realizable and stable for the moments of an RC tree. Once the moments are fitted to characterize the Gamma function, the step response delay is obtained via a one-dimensional table lookup, thereby providing the same explicitness as the Elmore approximation.

2: Background

We begin with a review of the relationship between moments of a linear circuit response and moments of a random variable.

2.1: Moments of a Linear Circuit Response

Let $h(t)$ be a circuit impulse response in the time domain and let $H(s)$ be the corresponding transfer function. By definition, $H(s)$ is

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the Laplace transform of $h(t)$:

$$H(s) = \int_0^{\infty} h(t) e^{-st} dt \quad (1)$$

Applying a Taylor series expansion of e^{-st} about $s = 0$ yields:

$$H(s) = \int_0^{\infty} h(t) \left\{ 1 - st + \frac{1}{2!}s^2t^2 - \frac{1}{3!}s^3t^3 + \dots \right\} dt =$$

$$\sum_{i=0}^{\infty} \frac{(-1)^i}{(i)!} s^i \int_0^{\infty} t^i h(t) dt \quad (2)$$

Using the terminology in [15], the *i*-th circuit-response moment, $\tilde{m}_i$ is defined as:

$$\tilde{m}_i = \frac{(-1)^i}{(i)!} \int_0^{\infty} t^i h(t) dt \quad (3)$$

From (2) and (3), the transfer function $H(s)$ can be expressed as:

$$H(s) = \tilde{m}_0 + \tilde{m}_1 s + \tilde{m}_2 s^2 + \tilde{m}_3 s^3 + \tilde{m}_4 s^4 + \dots \quad (4)$$

Asymptotic Waveform Evaluation (AWE)[15] demonstrated how impulse response moments could be calculated recursively for a lumped, linear RLC circuit. Following the first moment calculation, all subsequent moments are calculated recursively from the same dc equivalent circuit. Path tracing algorithms, such as those used in RICE[17], can be used to calculate moments with extreme efficiency for interconnect circuits.

For RC trees, Penfield and Rubenstein proposed in [18] a path tracing algorithm to calculate the first moment, or Elmore delay. This was done by traversing the tree topology and summing up contributions of branches along a path P_i from the root to the node of interest. The contribution of branch j is the product of the branch resistance R_j and the total down stream capacitance Cds_j. Denoting $ds(j)$ the set of capacitances downstream of j, the Elmore delay ED_i of path i is

$$ED_i = \sum_{j \in P_i} R_j \cdot Cds_j = \sum_{j \in P_i} \left[R_j \cdot \sum_{k \in ds(j)} C_k \right] \quad (5)$$

In a similar manner, the n^{th} moment of the impulse response along path *i*, m_i^n, is obtained by scaling the downstream capacitors at all nodes, *k*, by their *n-1*th moments, $C_k m_k^{n-1}$, yielding the following recursive expression:

$$\tilde{m}_i^n = - \sum_{j \in P_i} \left[R_j \cdot \sum_{k \in ds(j)} C_k \tilde{m}_k^{n-1} \right] \quad (6)$$

Any set of moments can be computed by two traversals of the RC tree in linear time.

It should be noted that the impulse response moments (series coefficients) in (3) are related to the probability theory moments by a $\frac{(-1)^i}{(i)!}$ term. It is this relation which forms the basis of PRIMO.

2.2: Moments of Probability Density Functions

A *probability function* is a real valued set function where the domain is a subset of the sample space, S, and the range is a real number in the interval $[0, 1]$. Generally, a function $Pr\{\circ\}$ should satisfy the three Kolmogorov axioms [11], or equivalent conditions, in order to be considered a probability function: (i) $Pr\{S\} = 1$; (ii) $Pr\{A\} \geq 0$ for all $A \subset S$; (iii) $Pr\{A \cup B\} = Pr\{A\} + Pr\{B\}$ if $A \cap B = \varnothing, A \subset S, B \subset S$.

The *distribution function* of a continuous random variable T - denoted $F_T(t)$ - provides the value of $Pr\{T \leq t\}$ for any real number $-\infty \leq t \leq \infty$. The associated *probability-density-function* (pdf) - denoted $f_T(t)$ - is the derivative of the distribution function with respect to t, thus

$$f_T(t) = \frac{dF_T(t)}{dt} \quad (7)$$

and

$$F_T(t) = \int_{-\infty}^{t} f_T(\tau) d\tau \quad (8)$$

The median, $t_{(0.5)}$, is defined by:

$$Pr\{T \leq t_{(0.5)}\} = F_T(t_{(0.5)}) = \int_{-\infty}^{t_{(0.5)}} f_T(t)\ dt = 0.5 \quad (9)$$

Whereas, the expected value or mean, $E[T]$, of a continuous random variable T with distribution $f_T(t)$ is

$$E[T] = \int_{-\infty}^{\infty} t f_T(t)\ dt \quad (10)$$

The mean is also the first moment of the distribution (or pdf). In general, the *i*-th moment m_i of the distribution is

$$m_i = E[T^i] = \int_{-\infty}^{\infty} t^i f_T(t)\ dt \quad (11)$$

Fig. 1 summarizes these distribution definitions. Note that we use '~' (tilda) to distinguish between the probability moments m_1, m_2, m_3 and circuit moments $\tilde{m}_1, \tilde{m}_2, \tilde{m}_3$.

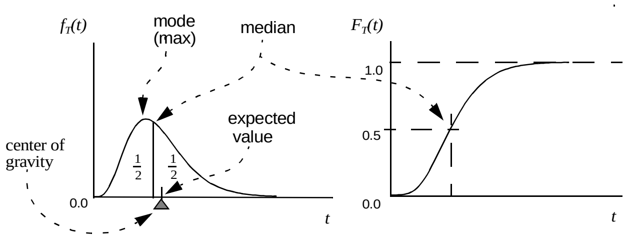

FIGURE 1. A probability density function (left) and corresponding distribution function (right).

2.3: The Central Moments

The moments of a pdf in (11) are more frequently viewed in terms of moments about the origin. The central moments of a distribution - denoted μ_i - are the moments around the expected value. More formally, central moments of a random variable T with probability density function $f_T(t)$ and expected value $E[T]$ are

$$\mu_i = \int_{-\infty}^{\infty} (t - E[T])^i f_T(t) \; dt = E[(T - E[T])^i] \qquad (12)$$

It is straight forward to show from (12) that the central moments as a function of moments about the origin are[6]:

$$\begin{aligned} \mu_0 &= 1 \\ \mu_1 &= 0 \\ \mu_2 &= m_2 - (m_1)^2 \\ \mu_3 &= m_3 - 3m_1 m_2 + 2(m_1)^3 \\ \mu_4 &= m_4 - 4m_3 m_1 + 6m_2(m_1)^2 - 3(m_1)^4 \end{aligned} \qquad (13)$$

From (3) and (11),

$$m_1 = -\tilde{m}_1 \,;\; m_2 = 2! \cdot \tilde{m}_2 \,;\; m_3 = -3! \cdot \tilde{m}_3 \qquad (14)$$

From which we can express the central moments - variance μ_2 and skewness μ_3 - in terms of the circuit response moments:

$$\begin{aligned} \mu_2 &= 2\tilde{m}_2 - (\tilde{m}_1)^2 \\ \mu_3 &= -6\tilde{m}_3 + 6\tilde{m}_1\tilde{m}_2 - 2(\tilde{m}_1)^3 \end{aligned} \qquad (15)$$

Unlike the moments of the impulse response, the central moments have geometrical interpretations:

- μ_0 is the area under the curve. It is generally unity, or else a simple scaling factor is applied.
- μ_2 is the *variance* of the distribution which measures the spread of the curve from the center. A larger variance reflects a larger spread of the curve.
- μ_3 is a measure of the *skewness* of the distribution; for a unimodal function its sign determines if the mode (global maximum) is to the left or to the right of the expected value (mean). Its magnitude is a measure of the distance between the mode and the mean.
- μ_4 is a measure of the *kurtosis* or *peakedness* of the distribution; for a unimodal function its relation to μ_2 reflects the portion of the total area under the curve that is attributable to the tails of the curve.

2.4: Connection Between Probability Density Functions and Circuit Responses

Any function $f(t)$ can be treated as a probability density function if it is defined in the range $[a, b]$ and satisfies

$$\begin{cases} f(t) \geq 0 \qquad \forall t \\ \int_a^b f(t) \; dt = 1 \end{cases} \qquad (16)$$

If $f(t)$ is equal to zero outside of the range $[a,b]$, we can replace the integration limits in (16) with $-\infty$ and ∞. Elmore[3] was the first to apply moments for delay approximation of a limited class of circuit responses by observing that the impulse response of a circuit can be treated as a probability density function. He used this observation to justify the approximation of the 50% point of a monotonic step response (the median point of the impulse response) by the first moment (mean of the impulse response).

In [6] it was shown that the impulse response corresponding to an RC tree is unimodal with positive skew. From this it follows that the mode is less then the median which is less then the mean and vice versa [9] [13]:

$$(skew > 0) \quad \text{if and only if} \quad (mode < median < mean)$$

This proved that the Elmore delay is an upper bound for the 50% step response delay[6], and was shown to hold for finite input signal rise time.

An important observation from [6] is that because of the variation in impulse response shapes along an interconnect path (*e.g.* from driver to load), the relative accuracy of the Elmore delay bound can be quite poor. Especially for the interconnects associated with deep submicron technologies, more than one moment is needed to capture the waveform shape-characteristics.

3: PRIMO

The approach proposed here is related to the work of Pearson [2] for approximating a histogram of discrete experimental data with a continuous function. The most important step in this process is finding the representative distribution family. The Gamma distribution, depicted in Fig. 2, is reasonably representative of the impulse responses in RC trees since it provides good “coverage” of bell shaped curves which are bounded on the left and exponentially decaying to the right.

The probability density function, $g_{\lambda, n}(t)$, of Gamma distribution is a function of one variable t and two parameters λ and n (real numbers):

$$g_{\lambda, n}(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{\Gamma(n)} \quad , \quad \Gamma(x) = \int_0^{\infty} y^{x-1} e^{-y} \, dy \qquad (17)$$

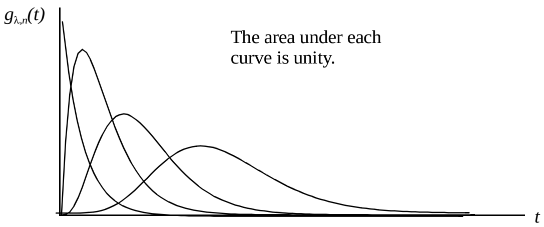

FIGURE 2. Examples of several members from the Gamma distribution family. Each family member corresponds to specific values of the distribution parameters λ and n.

From the moment generating function [11][20] of the Gamma distribution,

$$m_G(s) = \left(\frac{\lambda}{\lambda - s}\right)^n \qquad (18)$$

we can express the moments in terms of the parameters n and λ:

$$\begin{aligned} m_1 &= \frac{d}{ds} m_G(s) \bigg|_{s=0} = \frac{n}{\lambda} \\ m_2 &= \frac{d^2}{ds^2} m_G(s) \bigg|_{s=0} = \frac{n(n+1)}{\lambda^2} \\ m_3 &= \frac{d^3}{ds^3} m_G(s) \bigg|_{s=0} = \frac{n(n+1)(n+2)}{\lambda^3} \end{aligned} \qquad (19)$$

Since Gamma has only two parameters, λ and n, matching two moments would completely characterize this model. Therefore, to match the third moment and capture the skewness of the distribution, we add a third variable, Δ, to include a degree of freedom in terms of time-shifting the function.

To begin, the shape information that is encapsulated in μ_2 and μ_3 can be used to fit the Gamma-pdf parameters λ and n. Moment fitting λ and n in this way corresponds to a system of two equations and two unknowns:

$$\begin{aligned} \mu_2 &= \frac{n(n+1)}{\lambda^2} - \left(\frac{n}{\lambda}\right)^2 = \frac{n}{\lambda^2} \\ \mu_3 &= \frac{n(n+1)(n+2)}{\lambda^3} - 3\left(\frac{n}{\lambda}\right)\frac{n(n+1)}{\lambda^2} + 2\left(\frac{n}{\lambda}\right)^3 = \frac{2n}{\lambda^3} \end{aligned} \qquad (20)$$

Rearranging (20) results in:

$$\lambda = \frac{2\mu_2}{\mu_3}; \qquad n = \frac{4(\mu_2)^3}{(\mu_3)^2} \qquad (21)$$

The first moment is matched by translating the Gamma distribution by a time-shift Δ that is equal to $m_1 - \frac{n}{\lambda}$, as shown in Fig. 3. From (13) it is recognized that fitting μ_2, μ_3 and m_1 is equivalent to fitting the first three moments, m_1, m_2, m_3. This is equivalent to moment matching, in the AWE sense[15], to the moments of a time-shifted gamma function:

$$e^{-\Delta s}\left[1 - \frac{n}{\lambda}s + \frac{n(n+1)}{2! \cdot \lambda^2}s^2 - \frac{n(n+1)(n+2)}{3! \cdot \lambda^3}s^3 + \ldots\right] \qquad (22)$$

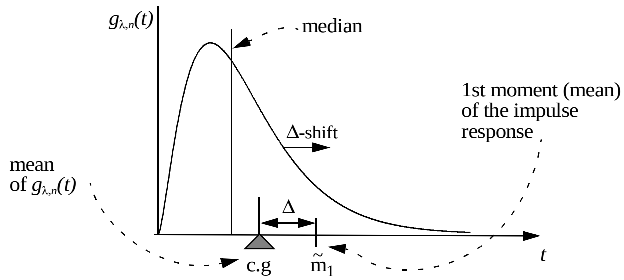

FIGURE 3. The variance and skewness, which are invariants of the origin location, are used to fit the shape of the curve. Then the curve is shifted such that its mean is aligned with the first moment of the impulse response.

Once the values of Δ, n and λ are obtained by matching the circuit response moments, the step response delay can be estimated from the probability density function $g_{\lambda, t}(t)$. For example, the approximate step response $\hat{y}(t)$ is given by

$$\hat{y}(t - \Delta) = \int_0^t g_{\lambda, n}(\tau) \, d\tau \qquad (23)$$

Finding the delay point $t_{(\alpha)}$ for a particular waveform threshold voltage $\alpha \cdot V_{max}$ (*e.g.* if $\alpha = 0.5$ it is the 50% delay), is equivalent to finding a percentile α of a distribution,

$$\hat{y}(t_{(\alpha)}) = \alpha \qquad (24)$$

Therefore, it is straightforward to use pre-compiled tables of Gamma distribution percentiles (with respect to the parameters n and λ) to solve (24) at the runtime cost of a *single* table-access. Moreover, we can normalize time by λ so that the percentile value can be obtained from a one-dimensional table in terms of n. An efficient computation scheme for the Gamma distribution function can be found in [16].

4: Properties and Implementation Issues

4.1: Stability

The Gamma probability density function in (17) is stable if $\lambda > 0$. From (21), $sgn(\lambda) = sgn(\mu_2) \cdot sgn(\mu_3)$. It was proven in [6] that both μ_2 and μ_3 are both positive for any RC tree. It was further shown in [5] that the point at which the skew changes from positive to negative for an RLC tree can be used as a measure of the response damping. We use these results to construct the following theorem:

Theorem of Stability: *The Gamma approximation is valid and stable for any RLC topology which has a response for which its derivative is a positively skewed bell shape distribution.*

4.2: Accuracy

Although it is provably stable, the shifted Gamma function may display significant error at times, particularly for the fitting of the early portion of the impulse response. As expected, fitting moments will tend to be most accurate for the tail portion of the response, since moments represent the series expansion about s=0. Fortunately, the response near t=0 is the least important portion of the response for delay calculation purposes. In addition, a large time-shift value, Δ, provides some indication of when the Gamma function is not fitting the impulse response in the expected manner.

4.3: Finite Rise Time Effects

Finite rise time effects can be captured by using two time-shifted gamma functions, separated by the risetime, to approximate the pulse response. The pre-compiled 2-D table is now a function of n and the risetime[12].

5: Experimental Results

As a first example, consider the RC tree in Fig. 4. We compare the

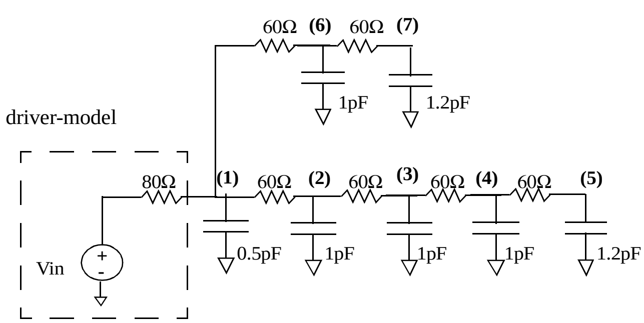

FIGURE 4. An RC tree example.

delays obtained from SPICE with those found using the Gamma approximation. The results for the 50% delay and 90% delay are summarized in Table 1.

Note that the differences between the PRIMO delays and the SPICE delays at the leaf nodes is about 1% or less. Excluding the driving point node, the largest error at any downstream node is about 4%. This worst case occurs for nodes that are closest to the root (node 2). These nodes correspond to the responses with the highest frequency content, hence one would expect the largest moment matching error there.

TABLE 1. Comparison of the 50% and 90% step response delays from SPICE and from PRIMO-Gamma (time in ns).

Node	SPICE (50%)	PRIMO (50%)	SPICE (90%)	PRIMO (90%)
2	0.477	0.497	2.02	2.01
3	0.700	0.699	2.27	2.27
4	0.845	0.836	2.42	2.43
5	0.919	0.909	2.50	2.50
6	0.375	0.376	1.75	1.74
7	0.452	0.450	1.82	1.82

The impulse response at the driving point node does not follow the bell shape, but rather starts out with a non-zero value and asymptotically approach zero in a multi-exponential decay form. Other distribution families may permit matching higher order moments, or more naturally capture these driving point response shapes. But as with all moment matching problems, the greatest challenge is to find a model that is provably stable and realizable.

The Beta function, for example, can be characterized in terms of its moments just like Gamma. But even with more shape fitting parameters, the Beta function is not always superior to the Gamma function for fitting downstream nodes. Some examples for driving point and near-end loads are shown in Table 2. Importantly, the utility of Beta is questionable, since the moment matching is sometimes non-realizable.

TABLE 2. The 50% step response delays obtained from Gamma and Beta approximations at the highest frequency nodes.

Node	SPICE	Gamma	Beta
1	0.196	0.243	0.193
2	0.477	0.498	0.489
6	0.375	0.378	0.355

An RLC Line

Since PRIMO requires a bell-shaped distribution for the impulse response, overdamped and critically damped RLC circuit responses can be approximated with this method. As a simple example, consider the 2000 micron distributed RLC line in Fig. 5, which was modeled with 20 lumped RLC sections. The results are compared with SPICE in Table 3.

TABLE 3. Comparison of SPICE and PRIMO for RLC line.

R_{driver}	C_{load}	Slope	SPICE		Gamma	
			50%	90%	50%	90%
25Ω	1pF	step	under-damped		under-damped	
35Ω	1pF	step	40.1ps	99.3ps	39.6ps	100ps

PRIMO matched the SPICE simulation very well for the over-damped responses. The underdamped cases can be recognized prior to analysis by a negative value for the third central moment[5]. Since delay is difficult to define for an underdamped response (since it may cross a threshold point multiple times), and because underdamped responses generally require design/circuit changes, simply detecting this condition is sufficient for many applications.

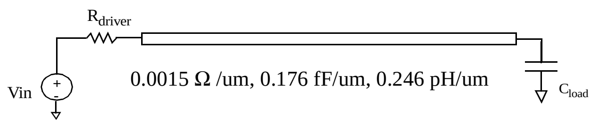

FIGURE 5. A distributed RLC line with a linear driver and capacitance load.

0.25 micron RC Tree Example

As a final example we considered a large, 10-fanout RC tree taken from a 0.25 micron commercial design. PRIMO is compared with SPICE and a 1-pole approximation in Table 4. Even for a step approximation the PRIMO results match SPICE in most cases, and is significantly better than a 1-pole approximation in almost all cases. The largest errors are observed at the near-end fanouts for both the PRIMO and the 1-pole models.

TABLE 4. Comparison of the 50% step response delays from SPICE, PRIMO (Gamma) and a 1-pole approximation (in ps).

Fan-out	SPICE	PRIMO	1-Pole	PRIMO % error	1-Pole % error
1	137.19	137.85	137.25	0.48	0.04
2	141.76	142.02	140.02	0.19	1.22
3	112.90	116.85	123.82	3.50	9.67
4	112.89	116.84	123.81	3.50	9.67
5	112.77	116.71	123.72	3.50	9.72
6	48.46	59.05	84.68	21.9	74.74
7	19.08	29.09	63.28	52.5	232.0
8	59.50	68.79	91.31	15.6	53.45
9	53.22	63.22	87.53	18.8	64.5
10	191.93	189.98	172.43	1.02	10.2

6: Conclusion

This paper describes a new delay metric based on a probability interpretation of moments of the circuit response. A time-shifted Gamma function is shown to provide good approximations of RC tree impulse responses so that one-dimensional probability tables can be used to evaluate the delays. Therefore, this new metric is able to capture higher order moment effects but with runtime efficiency that is comparable to that for the Elmore delay.

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