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SOLITOGENESIS: PRIMORDIAL ORIGIN OF NON–TOPOLOGICAL SOLITONS

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ABSTRACT

We discuss the formation of nontopological solitons in a second-order phase transition in the early Universe. Ratios of dimensionless coupling constants in the Lagrangian determine their abundance and mass. For a large range of parameters, non-topological solitons can be cosmologically significant, contributing a significant fraction of the present mass density of the Universe.

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Non-topological soliton solutions of classical field theories were introduced a number of years ago by Rosen,[1] and by Friedberg, Lee, and Sirlin.[2] In the recent literature, variations on this theme include Q-balls,[3] cosmic neutrino balls,[4] quark nuggets,[5] and soliton stars.[6] Unlike magnetic monopoles and cosmic strings, which arise in theories with non-trivial vacuum topology, non-topological solitons (hereafter, NTSs) are rendered stable by the existence of a conserved Noether charge carried by fields confined to a finite region of space. The minimum charge of the stable soliton depends upon ratios of coupling constants,[2] and in principle can be very small (of order one). Although the properties of non-topological solitons have been studied by a number of authors,[1–6] scenarios for actually producing such objects in the Universe have not been discussed. In this letter, we consider the possibility of forming NTSs during a phase transition in the early Universe.

In the context of renormalizable theories,[7] the simplest NTS solution arises from the interaction between a real scalar field σ and a complex scalar ϕ with Lagrangian $\mathcal{L} = |\partial_\mu \phi|^2 + (1/2)(\partial_\mu \sigma)^2 - U$, where

$$U(|\phi|,\sigma) = \frac{\lambda_1}{8}(\sigma^2 - \sigma_0^2)^2 + h|\phi|^2(\sigma - \sigma_0)^2 + \frac{\lambda_2}{3}(\sigma - \sigma_0)^3\sigma_0 + g|\phi|^4 + \Lambda \; ; \quad (1)$$

the constant Λ is adjusted to give $U = 0$ at the absolute minimum of the potential.

An important feature of this potential is the explicit breaking of the discrete symmetry $\sigma \leftrightarrow -\sigma$ driven by the ϕ–σ coupling term. It is this term that requires us to include the cubic term for the real scalar field and the $|\phi|^4$ term in the Lagrangian (even if they are absent at tree level). Although these terms are traditionally neglected in analyses of NTSs,[2,6] as we discuss below, inclusion of the cubic term is crucial for solitogenesis. To understand the structure of the NTS it is sufficient to consider the limit $g = 0$. Notice that while ϕ is massless at the local minimum of the potential ($\sigma = \sigma_0$), at the global minimum ($\sigma = \sigma_-$) the field ϕ has a mass $m_\phi^2 = h(\sigma_- - \sigma_0)^2$. Thus, a configuration of massless ϕ particles trapped inside a region with $\sigma = \sigma_0$ (the local minimum), separated from the true vacuum $\sigma = \sigma_-$ by a wall of thickness $\sim \sigma_0^{-1}$ will be stable. The larger mass in the $\sigma = \sigma_-$ vacuum prevents the ϕ particles from escaping the $\sigma = \sigma_0$ bag. In order to make this statement more quantitative, we have to compare the energy of the soliton of charge Q with the energy of Q free ϕ particles propagating in the $\sigma = \sigma_-$ vacuum. This can be done by introducing the dimensionless variables[2] A and B such that, $\sigma(t, \mathbf{r}) = \sigma_0 A(\mathbf{r})$, $\phi(t, \mathbf{r}) = 2^{-1/2}\sigma_0 B(\mathbf{r})e^{-i\omega t}$. In terms of A and B the energy of the NTS configuration is

$$E = \frac{\omega Q}{2} + 4\pi\lambda_1\sigma_0^4 \int [\frac{1}{2\lambda_1\sigma_0^2}(\frac{dA}{dr})^2 + \frac{1}{2\lambda_1\sigma_0^2}(\frac{dB}{dr})^2 + \frac{1}{8}(A^2-1)^2 + \frac{h}{\lambda_1}B^2(A-1)^2 + \frac{a}{3}(A-1)^3 + C]r^2 dr, \quad (2)$$

(assuming spherical symmetry) where $a \equiv \lambda_2/\lambda_1$, and we have used the definition of the conserved charge $Q = 4\pi\omega\sigma_0^2 \int B^2 r^2 dr$. The constant $C \equiv \Lambda/\lambda_1\sigma_0^4 = -(1/8)(A_-^2 - 1)^2 - (a/3)(A_- - 1)^3$, where $A_- \equiv \sigma_-/\sigma_0 = -(1+2a)/2 - [(1+2a)^2 + 8a]^{1/2}/2$ is the scaled σ field in the true vacuum. For an estimate of the energy we introduce the trial functions $B = (B_0/r)\sin\omega r\ \theta(R-r)$; $A = 1$ (for $r < R$), $A = (1 - A_-)e^{-(r-R)/l} + A_-$ (for $r \geq R$). R is the "radius" of the NTS, given in terms of ω by $\omega R = \pi$. The above trial functions satisfy exactly the equations of motion inside the "bag". Using these trial functions we find that the energy must satisfy the inequality

$$E \leq \frac{\pi Q}{R} + \frac{4\pi}{3}\Lambda R^3 + O(R^2 \lambda_1^{1/2} \sigma_0^3). \tag{3}$$

The three terms in Eq. (3) represent the kinetic energy of the confined complex field, the false vacuum energy of the NTS interior, and the surface energy of the wall separating the interior from the true vacuum. For large Q, unless $\lambda_2/\lambda_1 \ll 1$, the volume energy dominates the surface energy. In this limit (which we will assume henceforth) the NTS has radius and mass found by minimizing the energy: $R = (Q/4\Lambda)^{1/4}$, $M = (4\pi/3)\sqrt{2}Q^{3/4}\Lambda^{1/4}$.

The next step is to compare the energy in Eq. (3) with the energy of Q free ϕ's in the true vacuum, $E_{\text{free}} = Qh^{1/2}\sigma_0|A_- - 1|$. The NTS will be stable so long as its energy is less than E_{free}. This occurs whenever

$$Q \geq Q_{min} = 1231 \frac{C}{(A_- - 1)^4} \frac{\lambda_1}{h^2}. \tag{4}$$

If $a = 0.15$, then $\Lambda = 0.6\lambda_1\sigma_0^4$, and we find $Q_{min} = 18\lambda_1/h^2$, $M_{min} = 46(\lambda_1/h^{3/2})\sigma_0$, $R_{min} = 1.7h^{-1/2}\sigma_0^{-1}$. The potential for $a = 0.15$ is shown in Fig. 1.

A few comments are in order. It is possible that quantum corrections will give a small mass for the ϕ particles inside the bag (i.e., $g \neq 0$). The above calculation may be easily extended to cover this case,[8] but here we will only consider the case with massless particles inside the bag. The condition that the soliton interior is in the false vacuum limits the range of allowed coupling constants, although the range is not very restrictive.[8]

Now consider the evolution of the vacuum through the cosmological phase transition. In the limit $h = \lambda_2 = 0$, the reflection symmetry is exact, and it is well known that once the temperature of the Universe drops below the critical value ($T_c = 2\sigma_0$), infinite (and finite) regions of minima $\sigma = \pm\sigma_0$ form with stable domain walls separating domains of degenerate vacua. These domain walls soon dominate the energy density of the Universe, leading to contradictions with the observed isotropy of the cosmic radiation background (unless $\sigma_0 \lesssim 10$ MeV).[9] However, a small energy density difference between the two vacua ($\lambda_2 \neq 0$) causes regions of false vacuum to shrink, leading to the eventual disappearance of the wall system, with the Universe everywhere in the true vacuum.[9,10] If the shrinking walls survive to sufficiently low temperatures, they become impermeable to the passage of ϕ particles from the false to true vacuum. As a result, if the number of ϕ particles inside a contracting bag is larger than Q_{min}, the outward kinetic pressure exerted on the walls will halt the collapse, rendering the false vacuum bags (NTSs) stable.

As the temperature drops below $T \sim T_c$, thermal fluctuations of the σ field become large, with regions rapidly (compared to the expansion timescale) interconverting between (+) (false vacuum, $\sigma = +\sigma_0$) and (−) (true vacuum, $\sigma = \sigma_-$). These fluctuating regions typically have volume $V_\xi = (2\xi)^3$, where ξ is the correlation length. Below T_c the transition rate between the two vacua is proportional to $\exp(-F_M/T)$, where F_M is the free energy of the fluctuation, $F_M = U_M \times V_\xi$, and U_M is the energy barrier separating the two vacuum states (see Fig. 1). When the temperature drops below the Ginzburg temperature, T_G, the transition rate becomes less than the expansion rate: the fluctuations "freeze out" because there is insufficient thermal energy available to drive the transition.[11] At this temperature, the correlation length is[10] $\xi(T_G) \simeq \lambda_1^{-1} T_G^{-1}$. If, for simplicity, we approximate the potential below $T = T_c$ by its zero-temperature form, we find the Ginzburg temperature is $T_G \simeq V_\xi U_M$; below this temperature the thermal transition rate is exponentially suppressed.[12]

The relative probability of a fluctuation of the σ field ending up in a $(+)$ domain (denoted as p_+) or in a $(-)$ domain (denoted as p_-) is very sensitive to the energy difference between the two vacua ($\equiv \Lambda$, see Fig. 1); the false vacuum, with larger free energy, becomes progressively more improbable as Λ grows. So long as the system is in equilibrium, the relative population is given by the Boltzmann formula, $p_+/p_- = \exp(-\Delta F/T)$ where $\Delta F = \Lambda \times V_\xi$ is the difference in free energies of the two minima. This relation holds so long as the system is in equilibrium, which requires that the transition rate between vacua is greater than the expansion rate, i.e., $T > T_G$. Below T_G the relative probability is frozen at its value at T_G, $p_+/p_- = \exp(-\Delta F(T_G)/T_G)$. Since $\Delta F = \Lambda \times V_\xi$, and $T_G = V_\xi \times U_M$, below T_G, $p_+/p_- = \exp(-\Lambda/U_M)$. The argument of the exponent will depend upon ratios of dimensionless coupling constants in the Lagrangian.

The structure of vacuum domains below T_G is well-known from percolation studies. If the probability to be in a given vacuum state is greater than a critical probability, p_c ($p_c = 0.31$ for a simple cubic lattice), an infinite cluster of that vacuum will appear.[13] It is clear that the structure formed in the phase transition changes entirely if both vacuum domains or only one is above percolation threshold. For example, in the case of exact degeneracy ($h = \lambda_2 = 0$) both vacua are equally probable, $p_+ = p_- = 0.5$, and percolate, creating the familiar infinite domain wall problem.[9] On the other hand, if only the true vacuum percolates, the Universe would be filled with only finite clusters of false vacuum. Given the above value of p_c, the false vacuum percolates whenever $\Lambda/U_M \leq 0.8$; for the potential of Eq. 1, this occurs for $a \lesssim 0.13$.

First consider the case where $p_+ < 0.31$. Since this is below percolation threshold, isolated bags of false vacuum are formed in the true vacuum "sea". If r is the number of false vacuum cells in a cluster ($r \simeq (L/2\xi)^3$, where L is the cluster "diameter"), the density of r-clusters per lattice site is known[13] from Monte Carlo simulations to be $f(r) = br^{-1.5}\exp(-cr)$. The constants b and c are not known, but for $p_+ \to p_c$, $c \to 0$, and for $p_+ \to 0$, $b \to 0$. For intermediate values of p_+ we expect both b, $c \sim 1$. The number density of r-clusters produced at $T = T_G$ is then simply $n(r) = f(r)/V_\xi$. Thus, the typical size of a false vacuum bubble is $L \simeq 2\xi$, with larger bubbles exponentially suppressed.

Once formed, the bubbles will be acted upon by several forces: 1.) a surface tension due to the domain walls proportional to $\lambda_1^{1/2}$, which tends to straighten out curved walls; 2.) a vacuum pressure $p_{vac} = \Lambda$, which acts to collapse regions of (+) (false) vacuum; 3.) a thermal pressure p_ϕ due to the massless ϕ's in the (+) vacuum (assuming $T \leq m_\phi$), which expands regions of (+) vacuum. The evolution of the system of domain walls and vacuum bubbles is quite complicated, and the dynamics depends upon ratios of coupling constants λ_1, λ_2, and h. To elucidate a scenario for production of NTSs, we will assume two conditions are satisfied: *(i)* $T_G \lesssim m_\phi$, i.e., h is not too small, so that ϕ particles are trapped inside the (+) domains at the Ginzburg temperature; *(ii)* $p_\phi = (\pi^2/45)T_G^4 \lesssim p_{\rm vac} = \Lambda$, i.e., λ_2 is not too small, so that the thermal pressure due to the massless ϕ's in the (+) domain is always smaller than the vacuum pressure. (If condition(*ii*) is not satisfied, the (+) domains will grow and possibly percolate; for this case, see the discussion below.) Given (*i*) and (*ii*), (+) domains formed in the transition with $Q \geq Q_{min}$ will survive to form stable NTSs, while those with $Q \leq Q_{min}$ will evaporate and disappear. Since large domains of (+) vacuum are exponentially rare for $p_+ < p_c$, to first approximation the only surviving domains have $Q = Q_{min}$.

The typical number of relativistic ϕ particles inside an r-cluster is $N(r) = r\eta_{\rm eff} n_\phi V_\xi$, where $\eta_{\rm eff}$ is the effective excess ratio of particles over anti-particles,[14] and at T_G, $n_\phi \simeq \varsigma(3)T_G^3/\pi^2$. Setting $N(r_{min}) = Q_{min}$ gives $r_{min} = Q_{min}\lambda_1^3/\eta_{\rm eff}$. We find that the ratio between the number density of NTSs with $N(r_{min})$ produced at the Ginzburg temperature to the entropy density, $s = 2\pi^2 g_* T_G^3/45$, is (with $g_* = 100$)

$$\frac{n(Q_{min})}{s} = \frac{n(r_{min})}{s} = 3\times 10^{-3} b\left(\frac{\eta_{\rm eff}}{\lambda_1 Q_{min}}\right)^{3/2} \exp[-cQ_{min}\lambda_1^3/\eta_{\rm eff}]. \qquad (5)$$

The present energy density of NTSs (with $Q = Q_{min}$), $\rho_{\rm NTS} = M_{min} n(Q_{min})$, contributes a fraction of closure density

$$\Omega_{\rm NTS} {h_0}^2 = 5\times 10^9 b\left(\frac{\eta_{\rm eff}}{\lambda_1}\right)^{3/2} Q_{min}^{-3/4}\left(\frac{\Lambda}{\rm TeV^4}\right)^{1/4} \exp[-cQ_{min}\lambda_1^3/\eta_{\rm eff}], \qquad (6)$$

where h_0 reflects the uncertainty in the Hubble constant ($1 \geq h_0 \geq 1/2$). For example, if we set $b = c = 1$ and, as before, take $a = 0.15$, then $\Omega_{\rm NTS} {h_0}^2 \simeq 6 \times 10^8 (h\eta_{\rm eff}/\lambda_1^2)^{3/2} (\sigma_0/{\rm TeV}) \exp[-18\lambda_1^4/h^2\eta_{\rm eff}]$. We consider two possibilities: a) If the effective asymmetry[14] is comparable to the baryon asymmetry ($\eta_{\rm eff} = 10^{-9}$)

for the NTS density in the range $2 \times 10^{-3} \lesssim \Omega h_0^2 \lesssim 2$, we find the constraint $1.7 \times 10^{-5} \lesssim (\lambda_1^2/h) \lesssim 2.5 \times 10^{-5}$ for $\sigma_0 = 1$ TeV, and $4.2 \times 10^{-5} \lesssim (\lambda_1^2/h) \lesssim 4.6 \times 10^{-5}$ for $\sigma_0 = 10^{12}$ TeV. The corresponding masses are $M_{\rm NTS} = 0.2h^{-1}$ TeV and $M_{\rm NTS} = 3 \times 10^{11} h^{-1}$ TeV. b) If $\eta_{\rm eff} = 1$, for the NTS density to lie in the above range requires $1.0 \lesssim (\lambda_1^2/h) \lesssim 1.2$ for $\sigma_0 = 1$ TeV, and $1.6 \lesssim (\lambda_1^2/h) \lesssim 1.7$ for $\sigma_0 = 10^{12}$ TeV. We note that since the density is exponentially sensitive to the ratio λ_1^2/h, for fixed $\eta_{\rm eff}$ the range of parameter space for cosmologically significant NTSs is rather narrow. By the same token, the mass scale σ_0 required to produce abundant NTSs is essentially unconstrained.

Next we consider the case where both the (+) and (−) domains percolate, $0.31 < p_+ < 0.69$. In a given region, the Universe will be composed primarily of two interlocking infinite (+) and (−) domains of complicated topology. The typical distance between the walls, as well as the typical curvature radius, is initially $L(t) \simeq 2\xi(T_G)$.

In the evolution of the wall system, the early motion of the walls is dominated by the surface tension, which rapidly (compared to the expansion time) acts to increase the wall separation $L(t)$. This continues until the vacuum pressure, $p_{vac} = \Lambda \simeq (8/3)\lambda_2\sigma_0^4$, becomes comparable to the surface pressure, $p_s \simeq (2/3)\lambda_1^{1/2}\sigma_0^3/L$, i.e., when $L(t) = L_0 \simeq \lambda_1^{1/2}/4\lambda_2\sigma_0$. At this point, the vacuum pressure begins to accelerate the walls into the false vacuum regions and the infinite (+) domain will initially be pinched off into a series of finite (+) bubbles of typical size L_0.[10] The trapped charge in a typical bubble is thus $N(L_0) \simeq (4\pi/3)L_0^3 n_\phi(T_G) \simeq 8 \times 10^{-3}\eta_{\rm eff}\lambda_1^{3/2}/\lambda_2^3$, while, from Eq. (4), the minimum charge for a stable NTS is $Q_{min} = 205\lambda_2/h^2$. $N(L_0)$ will be larger than

Q_{min} if $h\lambda_1^{3/4}\eta_{\rm eff}^{1/2}/\lambda_2^2 \gtrsim 160$. If condition (*i*) is imposed, h cannot be too small. We also note that if h is much larger than λ_1 and λ_2, quantum corrections driven by the $h|\phi|^2(\sigma-\sigma_0)^2$ term will dominate the potential at finite temperature, so we assume that neither λ_2 nor λ_1 is much smaller than h. With these assumptions, it is unlikely that the charge inside a bubble of radius L_0 will exceed Q_{min}. If this is so, then as in the 'below percolation case' stable NTS will be formed from rare large clusters consisting of many cells, and the only qualitative difference in above and below percolation is that the effective cell size above percolation is L_0 rather than ξ. In the limit $\lambda_2 \ll \lambda_1$, $U_M = (\lambda_1/8)\sigma_0^4$, and $\xi = \sigma_0^{-1}\lambda_1^{-1/2}$. The fundamental cell volume above percolation is a factor of $(L_0/\xi)^3 \simeq \lambda_1^3/64\lambda_2^3$ larger (above percolation $\lambda_2/\lambda_1 \ll 1$), and r_{min}, the minimum number of cells necessary for a cluster to have $N \geq Q_{min}$, is correspondingly smaller.

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Figure Caption

Fig. 1: The potential of Eq. (1) with $a = \lambda_2/\lambda_1 = 0.15$ and $\langle|\phi|^2\rangle = 0$. $A_- \equiv \sigma_-/\sigma_0$ where σ_- is the global minimum of the potential.

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[7] NTSs also arise in the theory of a single complex scalar with a non-renormalizable effective Lagrangian (Refs. 3,6), and in theories with scalar-fermion interactions (Refs. 2, 4, 6). Our results below can be extended to cover these cases as well (see Ref. 8).

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[12] In a more exact treatment, one would follow the evolution of the scalar field in the finite temperature effective potential; e.g., in the limit $\lambda_2 = h = 0$, $\langle\sigma(T)\rangle = \sigma_0[1-(T/T_c)^2]^{1/2}$. In general, one then finds $T_G \sim T_c \simeq \sigma_0$. These modifications do not qualitatively change our order of magnitude results.

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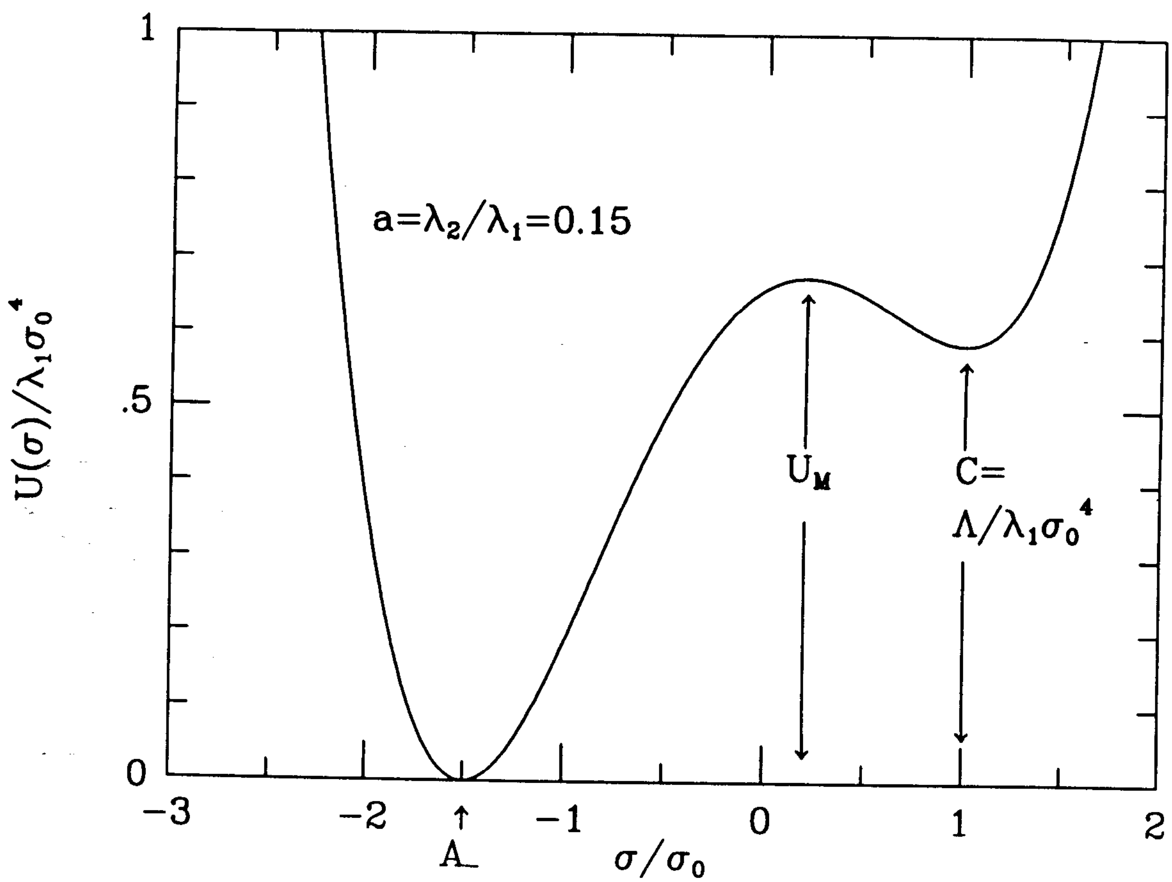
[14] If the ϕ's are able to annihilate inside the NTSs, $\eta_{\rm eff}$ depends upon a cosmic asymmetry between ϕ and $\bar{\phi}$, $\eta_{\rm eff} = |n_\phi - n_{\bar{\phi}}|/|n_\phi + n_{\bar{\phi}}|$. If there are no open annihilation channels for the massless ϕ's, $\eta_{\rm eff}$ would be unity. If ϕ particles are stable in the true vacuum, their cosmic asymmetry is constrained by the requirement that the Universe not be overclosed by the density of ϕ's.


1
.5
0
−3
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1
2
$U(\sigma)/\lambda_1\sigma_0^4$
$a=\lambda_2/\lambda_1=0.15$
U_M
$C=$
$\Lambda/\lambda_1\sigma_0^4$
A_-
σ/σ_0

Fig. 1