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Principal and Expert Agent

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Abstract

This paper analyses principal-agent contracts when the agent's action generates information not directly verifiable but used by the agent to make a risky decision. It considers a more general formulation than those studied previously, focusing on the impact on the decision made and the contract between principal and agent. It establishes a precise sense in which distorting decisions reduces the risk borne by a risk-averse agent and conditions under which implementing an optimal decision rule imposes no substantive restrictions on the contract. The paper also uses an application to bidding to supply a good or service to illustrate those results and derive additional ones. A risk-neutral agent with limited liability may optimally choose lower, less risky bids or higher, more risky bids, according to which relaxes the limited liability constraint. There are also natural conditions under which optimal contracts are monotone, possibly with flat sections, like stock option rewards.

Keywords: Principal-agent contracts, project selection, optimal bidding, portfolio selection, limited liability, risk aversion, asymmetric information

JEL classification: D82

1 Introduction

In the classic principal-agent problem studied by Mirrlees (1999), Holmström (1979) and Grossman and Hart (1983), the agent takes unverifiable action that directly affects the return to the principal. But in many practical applications, the agent's action generates information that is used to make a decision influencing that return. Managers are expected to investigate profitability before deciding how much to invest or which project to undertake. Those bidding to supply goods and services are expected to find out about the probability that any given bid will be accepted before deciding what to bid. Portfolio managers are expected to find out about potential investments before choosing portfolios for clients. General medical practitioners are expected to assess what specialist services a patient requires before arranging for them. This paper is concerned with two questions about such settings:

1. What effect does the agency relationship have on the decision made?
2. When does the addition of a decision affect the optimal contract between principal and agent?

If the information generated by the agent's action is verifiable, information generation can be treated as a standard one-dimensional principal-agent problem with the agent's "output" the information itself, and the decision treated separately. But in many of the applications, the information itself is not directly verifiable. What can be verified is the return to the principal resulting from the agent's decision and in some applications the agent's decision itself, but not the information on which the decision was based. Demski and Sappington (1987) refer to this as *delegated expertise*. In such situations, the contract between principal and agent must provide incentives for the agent not only to acquire information but also to make an appropriate decision on the basis of that information. Typically that affects the optimal contract between principal and agent but there are interesting cases in which it does not.

There are a number of papers in the literature concerned with such settings but they typically consider very special cases. Demski and Sappington (1987) has the most general formulation but does not consider the first of the questions of concern here. Other papers consider formulations that are restricted in a number of respects. Lambert (1986), Biais and Casamatta (1999) and Feess and Walzl (2004) consider no more than three levels of return to the principal on which payment to the agent can be based, so the same incentives can be achieved by different forms of contract. Moreover, in all those papers, as in Core and Qian (2002), Barron and Waddell (2003), and Gromb and Martimort (2003), the agent can choose between only two decisions, which limits the scope in answering question 1 above. Some papers limit the distribution of returns for each decision to those characterised by two parameters, the mean and riskiness of a portfolio (for example, Palomino and Prat (2003), in which it is also assumed that agent action gives access to different possible decisions, not information about the distribution of returns to those decisions always available) or to normal distributions for which those two parameters characterise the whole distribution (Stoughton (1993), Demski and Dye (1999), Barron and Waddell (2003), and Core and Qian (2002)). That limits the ways in which the agent's decision can affect the return to the principal. Demski and Dye (1999), Stoughton (1993), and Feltham and Wu (2001) limit contracts to certain predetermined forms rather than derive results on optimal forms. In the literature on managerial compensation, a common restriction is to debt, equity and stock options. Diamond (1998), Biais and Casamatta (1999), Palomino and Prat (2003), Feess and Walzl (2004),and Gromb and Martimort (2003) are concerned only with a risk-neutral agent who has limited liability. Innes (1990) analyses the classic principal-agent problem with the output of the agent's action verifiable and no additional decision but with the restrictions that the rewards to both agent and principal are monotone in the output. Such monotonicity restrictions arise naturally in some cases in which the agent's action generates unverifiable information that is used subsequently in making a decision, though Innes (1990) does not derive them in that way. But again the analysis is restricted to a risk-neutral agent with limited liability. Moreover, in all these papers, the only verifiable information is the return to the principal arising from the decision, not the decision itself, despite it being natural in many contexts for the decision (for example, the bid made in a takeover battle or the stocks chosen for a portfolio) to be known to the principal.[1]

The present paper adopts a more general formulation. The agent can be risk averse or risk neutral with limited liability, with no restriction on either the number of possible levels of return to the principal or the number of possible decisions by the agent, and with decisions influencing the distribution of returns in quite general ways. The contract between principal and agent is restricted only by the verifiability of information, allowing for the agent's decision itself to be verifiable, which is natural in some applications. The paper illustrates its results with an important economic application, a principal who wishes an agent to tender a bid to supply a good or service to a buyer after first taking an action to acquire information about the probability of different bids being accepted. That application has a continuum of possible decisions because the agent can choose any bid. Moreover, there is positive probability that any bid

[1]Melumad et al. (1995) and Aghion and Tirole (1997) also consider an agent who makes decisions in addition to taking action. There, however, the principal can monitor (and possibly over-rule) the agent directly rather than having to rely on inducing the agent to reveal information. In Osband (1989), the agent is a forecaster whom the principal wishes to induce to incur costs to refine the forecast and also to truthfully report the forecast estimate but the forecaster makes no further decisions affecting the principal's payoff. Povel and Raith (2001) also analyse a model in which an agent observes information that is not verifiable and then makes a choice that affects the principal's payoff. But the choice in that case concerns only a transfer, how much debt to repay. Crémer and Khalil (1992), Lewis and Sappington (1997), Crémer et al. (1998*a*), Crémer et al. (1998*b*) and Szalay (2004) are concerned with incentives for a supplier to acquire information in an otherwise standard procurement context *before* accepting a contract with the principal, not *after* as in the other papers cited here.

within some interval will be accepted, so the possible returns correspond to that interval plus zero (because bids may also be rejected). Thus returns are not restricted to isolated points for which different contract forms may have the same incentive effects.

On question 1 above, the paper shows that it is not, in general, optimal for the principal to induce the agent to adopt an *efficient* decision rule that corresponds to what the principal would have decided if the information were verifiable. The reason is that deviations from such a rule have only second-order effects on the principal's expected return but can have a first-order effect on the expected payment to the agent. With a risk-averse agent, as one would expect, profitable deviations reduce the risk the agent bears but this is in a sense the paper makes precise that depends not only on the risk characteristics of the decisions themselves but also on the characteristics of the contract with the principal. Thus, even where decisions can be ordered by second-order stochastic dominance, it is not necessarily the case that the principal gains by deviating to a dominating decision. Deviating to a dominating decision does, however, increase the principal's payoff when optimal contracts for implementing an efficient decision rule have certain characteristics. Those characteristics apply to the bidding application with a risk-averse agent and unlimited liability. In that case, a risk-averse agent is unambiguously induced to bid less aggressively (that is set a lower price for supply with a correspondingly higher probability the bid is accepted) than if the principal were able to verify the agent's information and so make the decision directly.

For a risk-neutral agent with binding limited liability, of course, there is no gain to deviating from an efficient decision rule in order to change the risk borne by the agent. In that case, deviations are worthwhile only if they relax the limited liability constraint. Which deviations have this effect is far from obvious in the general case. Even for the more restricted bidding application, optimal deviations can induce either higher, more risky bids or lower, less risky bids. In their application to delegated portfolio choice, Palomino and Prat (2003) reach a similar conclusion that optimal deviations may go in the direction of either more, or less, risky decisions, but their result relies on the "first-order approach" being valid, which they cannot guarantee. The result here does not suffer from this limitation. In the case that limited liability results in *less* risky decisions, managers are induced to make decisions in a way that makes it look as if they are risk-averse — limited liability not only reduces the profits of firms but also biases their decisions in a risk-averse direction.

As Grossman and Hart (1983) showed, there is little that can be said in general about the characteristics of optimal principal-agent contracts. For that reason, the approach to question 2 taken here is not to solve directly for an optimal contract but to ask when a decision rule will be chosen by the agent if not imposed as a constraint on the optimization. For a contract conditioned only on the return to the principal, it will be if the rule is first-order stochastically dominant and an appropriate monotone likelihood ratio property (MLRP) holds. Essentially, the MLRP ensures that the contract has the reward to the agent increasing in the return to the principal and, with the reward increasing in the return, the agent always prefers a decision rule that is first-order stochastically dominant. Under certain conditions, a decision will also be chosen if it is second-order stochastically dominant. Those conditions provide an interesting link to the literature on the "first-order approach" to solving the classic principal-agent problem. The first-order approach is valid if the optimal contract makes the agent's utility a concave function of the return to the principal, see Jewitt (1988). But, with a utility function concave in the return, the agent always prefers a decision rule that is second-order stochastically dominant. The case of first-order stochastic dominance applies quite naturally to the bidding application when limited liability is not binding and the principal wishes the agent always to choose the highest bid that will certainly be successful given the information received. Even where the MLRP does not hold, payment to the agent in the bidding application is still monotone non-decreasing in the return to the principal. Thus, in contrast to the classic principal-agent problem for which monotonicity holds only when the MLRP holds, see Hart and Holmström (1987), the application gives reasons for contracts to be monotone even when that property does not hold. Moreover, the monotone contracts may have flat segments and there are plausible circumstances in which a flat segment occurs for the lowest returns, so the agent is protected from downside risks beyond a certain level, as with compensation in the form of stock options that a manager does not have to exercise.

Binding limited liability limits the applicability of these stochastic dominance results. This is illustrated for a risk-neutral agent in the bidding application, a case sufficiently straightforward to solve explicitly for the optimal contract when the MLRP holds and the principal wants the agent to make the highest bid that will certainly be successful. In the absence of the additional decision, it is optimal for the principal to attach all payment to the return that has highest likelihood given the optimal action choice. With the additional decision, payments are distributed over all returns. But the optimal contract is not a debt contract, as derived by Innes (1990) when monotonicity of rewards for both principal and agent is simply assumed. Nor is it a combination of debt, equity and share options that Biais and Casamatta (1999) found optimal with just two possible decisions and three possible levels of return to the principal. It is not even a linear contract which, as in Diamond (1998), also induces a risk-neutral agent to make efficient decisions. Diamond (1998) shows that, when a linear contract is the only contract that ensures efficient decisions and the cost of inducing action is small relative to the principal's concern with making efficient decisions, a fully linear contract becomes near-optimal. In this version of the bidding application, however, there are other contracts that ensure efficient decisions at lower cost to the principal, so Diamond's result does not apply.

The remainder of this paper is organised as follows. Section 2 sets out the model and the application to bidding. Section 3 analyses optimal deviations from efficient decision rules. Section 4 considers contracts to induce the agent to adopt decision rules. Section 5 contains concluding remarks.

2 The model

2.1 General framework

In the framework used in this paper, a risk-neutral principal employs an agent to both choose an action $a \in A$ and make a decision $b \in B$ with uncertain net monetary return $y \in [\underline{y}, \overline{y}]$. The decision may, for example, concern which of a set of projects to undertake. The agent's action may affect the monetary return to the principal directly, as in the classic principal-agent model, but also yields the agent a signal $s \in [\underline{s}, \overline{s}]$ providing information about the distributions of monetary returns to the possible decisions and, hence, about which decision it is optimal to make. The probability density function for s given a is denoted $f(s; a)$ which, in order that action choice is potentially valuable, is assumed to have $f(s; a') \neq f(s; a'')$ for some $s \in [\underline{s}, \overline{s}]$ and $a', a'' \in A$. The probability distribution function of the monetary return y given decision b and signal s is denoted $G(y; b, s)$. A useful benchmark is an *efficient decision rule* $b^*(.)$ that the principal would use if receiving the signal s directly. Such a rule satisfies

$$b^*(s) \in \arg\max_{b \in B} \int_{\underline{y}}^{\overline{y}} y dG(y; b, s), \text{ for all } s \in [\underline{s}, \overline{s}]. \tag{1}$$

As conventional in principal-agent models, the agent's utility is additively separable in income and action. The utility from being paid P and taking action a is denoted $u(P) - v(a)$ with the standard properties

$$u'(P) > 0; u''(P) \leq 0; v(a) \text{ strictly increasing and strictly convex.} \tag{2}$$

The agent's reservation utility for accepting a contract with the principal is denoted $\underline{U}$. For simplicity, the analysis is restricted to cases in which the set of possible actions is either an interval, so $A = [\underline{a}, \overline{a}]$, or binary, so $A = \{\underline{a}, \overline{a}\}$.

The sequence of events is as follows. As in the classic principal-agent model, the principal makes a "take it or leave it" offer of a contract to the agent. If accepted, the agent chooses an action a, receives a signal s, and makes a decision b, in that order. The return from the decision is then realised and the agent is paid according to the contract. The essential difference from the classic principal-agent model is that the signal received by the agent is not itself verifiable. If it were, the incentive issues here would reduce to those of the classic model.

2.2 An application to bidding

An important economic application that illustrates the results is that of a principal who wishes the agent to tender a bid to supply a good or service to a purchaser whose reservation value is unknown. (It is straightforward to reverse the analysis for a principal who is a purchaser, as in a takeover bid or in the standard procurement models in Laffont and Tirole (1993), though in the latter case with the cost of supply not verifiable after the purchase has been completed.) By taking an action, the agent acquires information about the purchaser's reservation value and thus about the optimal price to bid. That information corresponds to the signal s. The price to bid corresponds to the decision b measured, for notational simplicity, net of the cost of supply so that b is the net return to the principal from a successful bid. The net return from an unsuccessful bid is zero. Because the focus here is less on the choice of action than on the additional decision, the number of actions is restricted to two, $\underline{a} < \overline{a}$, so $A = \{\underline{a}, \overline{a}\}$. For there to be a role for incentives for action, the principal is assumed to wish to induce the agent to choose $\overline{a}$. Without loss of generality, signals can be ordered so that the likelihood ratio $LR(s) \equiv f(s; \underline{a}) / f(s; \overline{a})$ is non-increasing in s.

Denote by $\pi(b; s)$ the probability that a bid of $b \in [0, \overline{b}]$ is successful given signal $s \in [\underline{s}, \overline{s}]$. In terms of the more general set-up, $\pi(b; s) = 1 - G(0; b, s)$ for $b > 0$. The economic context requires $\pi(b; s)$ non-increasing in b for given s — a higher b corresponds to setting a higher price for supply, so the probability the bid is successful cannot increase — so it is natural to define $\pi(0; s) = 1$. It is convenient to assume that $\pi(b; s)$ is strictly decreasing in b where feasible (that is, $\pi_b(b; s) \equiv \partial\pi(b; s) / \partial b < 0$ for all (b, s) such that $\pi(b; s) > 0$), twice differentiable with respect to b and s except possibly for $\pi(b; s) = 0$ and $\pi(b; s) = 1$ and, to avoid trivial bidding, has the property that for each $s \in [\underline{s}, \overline{s}]$ there exists some $b \in (0, \overline{b}]$ for which $\pi(b; s) > 0$.

In this application, an efficient decision rule $b^*(.)$ satisfies

$$b^*(s) \in \arg\max_{b \in [0, \overline{b}]} b\pi(b; s), \text{ for all } s \in [\underline{s}, \overline{s}]. \tag{3}$$

It is immediate that $b^*(s) > 0$ and $\pi(b^*(s), s) > 0$ for all $s \in [\underline{s}, \overline{s}]$ because the principal's payoff $b\pi(b; s)$ from bidding $b = 0$ is zero and, by assumption, for each s there exists some $b > 0$ for which $\pi(b; s) > 0$.

2.3 Contracts between principal and agent

A contract between principal and agent specifies the payment to the agent as a function of verifiable outcomes. The outcomes here are a, s, y and b. In keeping with the classical principal-agent literature, a is taken to be information private to the agent — without that, the contractual issues of interest here disappear. In keeping with the underlying motivation of the paper that the outcome of the agent's action is unverifiable information, s is also taken to be information private to the agent. In all the literature discussed in the Introduction, it is assumed that the only verifiable outcome is y. But that is not essential to the underlying motivation. There are obvious economic contexts in which it is natural that the agent's decision is public information—for example, bids in a takeover battle. Thus the potentially verifiable outcomes on which the principal may wish to contract in the present context are y and b.[2] For the notation to cover all cases, let x denote the verifiable subset of these, $x \subseteq \{y, b\}$, and X the set of all possible verifiable outcomes. A contract $P(.)$ specifies the payment $P(x)$ from the principal to the agent in the event of verifiable outcome $x \in X$. To cover the possibility that the agent has limited liability, the contract is required to have $P(x) \geq \underline{P}$ for all x, though it may be that $\underline{P}$ is so low that this constraint never binds.

An agent facing contract $P(.)$ who receives signal s and makes decision b has expected utility from the monetary compensation given by

$$\hat{u}(b, s, P(.)) \equiv \int_{\underline{y}}^{\overline{y}} u(P(x))\, dG(y; b, s). \tag{4}$$

The agent's expected utility before the signal s is known from adopting the decision rule $b(.)$ for given action a is then

$$U(a, b(.), P(.)) \equiv \int_{\underline{s}}^{\overline{s}} \hat{u}(b(s), s, P(.))\, f(s; a)\, ds - v(a). \tag{5}$$

The principal's expected payoff from decision b given signal s and contract $P(.)$ is

$$r(b, s, P(.)) \equiv \int_{\underline{y}}^{\overline{y}} [y - P(x)]\, dG(y; b, s), \tag{6}$$

that from the agent choosing action a and decision rule $b(.)$

$$R(a, b(.), P(.)) = \int_{\underline{s}}^{\overline{s}} r(b(s), s, P(.))\, f(s; a)\, ds. \tag{7}$$

An optimal contract maximises the principal's payoff subject to feasibility, individual rationality and incentive compatibility for the agent. The optimal contract problem can be written in the way standard with principal-agent problems as

[2] The contract might also be conditioned on a message the agent sends to the principal after observing the signal. That possibility is not considered in the literature referred to in the Introduction and so is not included in the main analysis here but some implications are mentioned in footnotes. The possibility of messages also raises the broader question of whether the principal would do better by, where possible, making the decision personally on the basis of the agent's message. (It may not always be possible, for example, in a medical emergency there may not be time to consult the principal.) That issue is discussed in the separate literature on delegation, for example Aghion and Tirole (1997).

$$\max_{a,b(.),P(.)} R\left(a,b\left(.\right),P\left(.\right)\right) \qquad \text{subject to} \tag{8}$$

$$U\left(a,b\left(.\right),P\left(.\right)\right) \geq \underline{U}; \tag{9}$$

$$a \in \arg\max_{a' \in A} U\left(a',b\left(.\right),P\left(.\right)\right); \tag{10}$$

$$b\left(s\right) \in \arg\max_{b \in B} \hat{u}\left(b,s,P\left(.\right)\right) \text{ for all } s \in \left[\underline{s},\bar{s}\right]; \tag{11}$$

$$P\left(x\right) \geq \underline{P}, \text{ for all } x \in X. \tag{12}$$

Constraint (9) is the individual rationality condition that the agent has expected utility from the decision rule $b\left(.\right)$ when choosing action a under contract $P\left(.\right)$ no lower than from not agreeing to the contract in the first place. Constraint (10) ensures that the agent receives at least as high expected utility from choosing action a as from choosing any other action $a' \in A$. Constraint (11) ensures that the decision rule $b\left(.\right)$ maximises the agent's payoff for each s. (Writing the constraints this way implicitly assumes that an agent who is indifferent between two values of a or b chooses that preferred by the principal.) Finally, constraint (12) ensures that any limited liability requirement for the agent is satisfied. In what follows, it is assumed that an optimal contract exists.

3 Optimal departures from efficient decision rules

3.1 Optimal decision rules in the general framework

Any efficient decision rule $b^*\left(.\right)$ can be implemented by a contract $P\left(.\right)$ that satisfies

$$u\left[P\left(y\right)\right] = \underline{u} + ky, \qquad \text{for any constants } \underline{u}, k \text{ with } k > 0, \tag{13}$$

because replacing y in (1) by $u\left[P\left(y\right)\right]$ from (13) does not change the values of b that achieve a maximum. (The same is true for $k = 0$ if an agent indifferent between two decisions makes that preferred by the principal but $k = 0$ gives no incentive for the agent to take any action $a > \underline{a}$.) But, given sufficient continuity, it is, in general, worthwhile for the principal to have the agent deviate from an efficient decision rule because a marginal deviation has only a second-order effect on the expected return to the principal whereas any reduction in expected payment to the agent has a first-order effect. It is natural to expect that, with a risk-averse agent, the deviation will be in the direction of reducing the risk borne by the agent. But that risk depends on the contract with the principal, not just on the inherent risk characteristics of the decisions, so changes in risk based on such measures as stochastic dominance of the underlying decisions do not capture the full impact. The first result makes precise the sense in which a deviation from an efficient decision rule affects the risk borne by the agent.

For this result, consider the optimal way for the principal to induce the agent to adopt an implementable decision rule $b(.)$, as given by the problem in (8)-(12) but with the decision rule fixed at the specified $b(.)$. Call an optimal solution, contract and payoff for that problem *constrained optimal* for that $b(.)$. As already noted, an efficient decision rule $b^*(.)$ can always be implemented by a contract that satisfies (13).

Theorem 1 *Suppose $B = \left[\underline{b}, \overline{b}\right]$ and $dG(y; b, s)$ is differentiable with respect to b for all (y, s). Let $P(.)$ be a constrained optimal contract that implements action a and efficient decision rule $b^*(.)$ and is differentiable with respect to $b(s)$ in the neighbourhood of $b^*(s)$ for all $s \in [\underline{s}, \overline{s}]$.*[3] *Then, if limited liability is not binding, a necessary and sufficient condition for a marginal implementable change in the decision rule from $b^*(.)$ to increase the principal's payoff above the constrained optimal level is that the change satisfies*

$$\int_{\underline{s}}^{\overline{s}} \left[\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} P(x)\, dG(y; b, s)\, db\right]_{b=b^*(s)} f(s; a)\, ds < 0. \tag{14}$$

Proof. Let $V(b(.))$ denote the maximum value function for a constrained optimal solution for implementable decision rule $b(.)$. Note that, in view of the definition in (5), the constraints (9) and (10) can be expressed entirely in terms of $\hat{u}(b(s), s, P(.))$ and $v(a)$. Note also that, from (11), $b(s)$ maximises $\hat{u}(b, s, P(.))$ over all b. It follows from differentiability of $dG(y; b, s)$ and $P(x)$ with respect to b that, if limited liability is not binding for a constrained optimal contract, a marginal change in $b(s)$ in the neighbourhood of $b^*(s)$ has no first-order effect on any of the constraints. Thus, by the envelope theorem and the definitions in (6) and (7),

$$\begin{aligned} dV(b(.)) &= \int_{\underline{s}}^{\overline{s}} \left[\frac{\partial}{\partial b} r(b, s, P(.))\, db\right]_{b=b(s)} f(s; a)\, ds \\ &= \int_{\underline{s}}^{\overline{s}} \left[\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} [y - P(x)]\, dG(y; b, s)\, db\right]_{b=b(s)} f(s; a)\, ds. \end{aligned}$$

Moreover, from (1), $b^*(s)$ maximises $\int_{\underline{y}}^{\overline{y}} y dG(y; b, s)$ so, evaluated at $b^*(.)$,

$$dV(b^*(.)) = -\int_{\underline{s}}^{\overline{s}} \left[\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} P(x)\, dG(y; b, s)\, db\right]_{b=b^*(s)} f(s; a)\, ds, \tag{15}$$

from which the result follows. ■

The result in Theorem 1 is a direct consequence of the envelope theorem applied in two ways: (1) in a solution that implements $b(.)$, the contract $P(.)$ must be such

[3] Conditioning the contract on a message $m(s)$ the agent sends to the principal revealing the signal s does not affect the result in Theorem 1 because the constraint $m(s) = s$ is unaffected by a change in b, so the envelope theorem result used in the proof continues to apply.

that $b(s)$ maximises the agent's expected utility given s so, by the envelope theorem, a marginal change in $b(s)$ by itself leaves the agent's expected utility unchanged; and (2) because $b^*(s)$ is, by definition, a decision that maximises the principal's expected return gross of payments to the agent, a marginal deviation from $b^*(s)$ leaves that expected return unchanged, again as a result of the envelope theorem. Together these imply that, in the absence of limited liability, distorting the decision rule marginally away from $b^*(.)$ affects the principal's payoff only by the direct effect on the principal's expected payment to the agent. The expression on the left-hand side of (14) corresponds to that direct effect. It is clear that its sign is driven by risk aversion — for a risk-neutral agent, it is necessarily zero because the term in square brackets is just $\hat{u}_b(b^*(s), s, P(.))$ and, by (11), $b^*(s)$ maximises $\hat{u}(b, s, P(.))$ over b. That is to be expected: for a risk-neutral agent without binding limited liability, it is straightforward to achieve the first-best outcome, so distorting the decision rule away from efficiency cannot increase the principal's payoff. In particular, any reduction in the principal's expected payment to a risk-neutral agent must reduce the expected payoff of the agent and thus, in the absence of binding limited liability, violate the agent's individual rationality constraint (9). For a risk-averse agent, the effect of any particular distortion depends not only on $dG(y; b, s)$, which is determined by the underlying risk characteristics of the decisions, but also on the contract with the principal. Theorem 1 makes clear that the contract for a constrained optimal solution is the appropriate contract for assessing the effect of a change in the decision rule on the risk borne by the agent.

When limited liability is binding, distorting the decision rule may also relax the limited liability constraint (12), thus providing an additional way the principal can benefit. With a risk-neutral agent for whom the effect in (14) is zero, relaxing the limited liability constraint is the only way the principal can benefit. For a risk-neutral agent, limited liability can be binding only when the individual rationality constraint (9) is satisfied with strict inequality. Then relaxing a binding limited liability constraint by a distortion in decision rule can reduce the principal's expected payment to the agent because, although it also reduces the agent's payoff, doing that is feasible when the individual rationality constraint is not binding. But, in general, it is not obvious how even to characterise the direction of the distortion. The direction is explored below in the context of the bidding application. For a strictly risk-averse agent with binding limited liability, both this effect and that captured in (14) are at work. In that case, the combined effects must increase the principal's payoff for the principal to gain from distorting the decision rule away from $b^*(.)$.

Standard measures of riskiness of decisions are concerned with a decision maker whose payoff depends only on the return to the decision y. The following result relates the result in Theorem 1 to one such measure, second-order stochastic dominance.

Theorem 2 *Suppose the agent is strictly risk averse and the contract is conditioned only on*

y, so $x = y$. Then for any $s \in [\underline{s}, \overline{s}]$ for which $b^*(s) \in \left(\underline{b}, \overline{b}\right)$ and $G(y; b, s)$ satisfies either

$$\int_{\underline{y}}^{y} G_b\left(\theta; b^*(s), s\right) d\theta \geq 0, \ \forall y \in \left[\underline{y}, \overline{y}\right], \text{ with strict inequality for some } y, \tag{16}$$

or

$$\int_{\underline{y}}^{y} G_b\left(\theta; b^*(s), s\right) d\theta \leq 0, \ \forall y \in \left[\underline{y}, \overline{y}\right], \text{ with strict inequality for some } y, \tag{17}$$

the following properties hold.

1. *For any contract $P(.)$ twice-differentiable with respect to y that implements $b^*(s)$, $P''(y) > 0$ for some y for which strict inequality holds in (16) or (17).*
2. *If $P''(y) > 0$ for all $y \in \left[\underline{y}, \overline{y}\right]$,*

$$\begin{aligned} \frac{\partial}{\partial b} \int_{\underline{y}}^{\overline{y}} P(y)\, dG\left(y; b^*(s), s\right) &> 0, \text{ if (16) holds;} \\ &< 0, \text{ if (17) holds.} \end{aligned} \tag{18}$$

Proof. Consider any twice-differentiable function $\xi\left[P(x)\right]$. When $x = y$,

$$\frac{\partial}{\partial b} \int_{\underline{y}}^{\overline{y}} \xi\left[P(x)\right] dG(y; b, s) = \int_{\underline{y}}^{\overline{y}} \xi\left[P(x)\right] dG_b(y; b, s).$$

Integration of the right-hand side by parts twice gives

$$\begin{aligned} & \int_{\underline{y}}^{\overline{y}} \xi\left[P(x)\right] dG_b(y; b, s) \\ = & \left[\xi\left[P(x)\right] G_b(y; b, s)\right]_{y=\underline{y}}^{y=\overline{y}} - \int_{\underline{y}}^{\overline{y}} \xi'\left[P(x)\right] P'(x)\, G_b(y; b, s)\, dy \\ = & -\int_{\underline{y}}^{\overline{y}} \xi'\left[P(x)\right] P'(x)\, G_b(y; b, s)\, dy \\ = & -\left[\xi'\left[P(x)\right] P'(x) \int_{\underline{y}}^{y} G_b(\theta; b, s)\, d\theta\right]_{y=\underline{y}}^{y=\overline{y}} \\ & + \int_{\underline{y}}^{\overline{y}} \left[\xi'\left[P(x)\right] P''(x) + \xi''\left[P(x)\right] P'(x)^2\right] \left[\int_{\underline{y}}^{y} G_b(\theta; b, s)\, d\theta\right] dy \\ = & -\xi'\left[P(\overline{y})\right] P'(\overline{y}) \int_{\underline{y}}^{\overline{y}} G_b(y; b, s)\, dy \\ & + \int_{\underline{y}}^{\overline{y}} \left[\xi'\left[P(x)\right] P''(x) + \xi''\left[P(x)\right] P'(x)^2\right] \left[\int_{\underline{y}}^{y} G_b(\theta; b, s)\, d\theta\right] dy, \end{aligned} \tag{19}$$

the second line following because $G\left(\underline{y};b,s\right)=0$ and $G\left(\overline{y};b,s\right)=1$ for all (b,s). The first-order necessary conditions derived from (1) and (11) for $b^*(s)\in\left(\underline{b},\overline{b}\right)$ are

$$\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} y dG\left(y;b^*(s),s\right) = 0,\ \forall s, \tag{20}$$

$$\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} u\left[P(x)\right] dG\left(y;b^*(s),s\right) = 0,\ \forall s. \tag{21}$$

For $\xi\left[P(x)\right]=P(x)=y$, so $\xi'\left[P(x)\right]=P'(x)=1$ and $\xi''\left[P(x)\right]=P''(x)=0$, (19) implies

$$\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} y dG\left(y;b,s\right) = -\int_{\underline{y}}^{\overline{y}} G_b\left(y;b,s\right)dy,\ \forall s,$$

which, together with (20), implies

$$\int_{\underline{y}}^{\overline{y}} G_b\left(y;b^*(s),s\right)dy=0,\ \forall s. \tag{22}$$

For $\xi(.)=u(.)$, (19) and (22) imply

$$\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} u\left[P(x)\right] dG\left(y;b^*(s),s\right)$$
$$=\int_{\underline{y}}^{\overline{y}}\left[u'\left[P(x)\right]P''(x)+u''\left[P(x)\right]P'(x)^2\right]\left[\int_{\underline{y}}^{y} G_b\left(\theta;b^*(s),s\right)d\theta\right]dy.$$

By (21), this must equal zero. Given (16) or (17), that can hold only if the term in the first square bracket under the integral on the right-hand side is neither strictly negative nor strictly positive for all y for which strict inequality holds in (16) or (17). But with $u'(.)>0$ and $u''(.)<0$, this can be the case only if $P''(x)>0$ for some such x. That establishes Part 1.

To establish Part 2, note that the expression in (18) whose sign is to be evaluated corresponds to that in (19) when $\xi\left[P(x)\right]=P(x)$, so $\xi'\left[P(x)\right]=1$ and $\xi''\left[P(x)\right]=0$. With the use of (22), that gives, when evaluated at $b=b^*(s)$,

$$\frac{\partial}{\partial b}\int_{\underline{y}}^{\overline{y}} P(y) dG\left(y;b^*(s),s\right)=\int_{\underline{y}}^{\overline{y}} P''(y)\left[\int_{\underline{y}}^{y} G_b\left(\theta;b^*(s),s\right)d\theta\right]dy.$$

Part 2 follows directly. ■

Conditions (16) and (17) correspond to $b(s)$ marginally higher than $b^*(s)$ respectively being stochastically dominated by $b^*(s)$, and stochastically dominating $b^*(s)$, in the second-order sense. Since $P(.)$ is a payment from principal to agent, the principal's payoff is strictly concave when the contract has $P''(.)>0$. Thus increasing $b(s)$

marginally from $b^*(s)$ reduces the principal's payoff in the first case and increases it in the second. Combined with Theorem 1, therefore, Theorem 2 implies the following.

Corollary 1 *Suppose the agent is strictly risk averse, the contract is conditioned only on y, and limited liability is not binding for a constrained optimal contract $P(.)$ that implements $b^*(.)$. Then, for any s for which $P''(y) > 0$ for all $y \in \left[\underline{y}, \overline{y}\right]$, the principal gains by distorting the decision $b(s)$ in a direction that second-order stochastically dominates $b^*(s)$.*[4]

This conclusion is intuitive. Distorting the decision to be implemented for any s in a direction that second-order stochastically dominates $b^*(s)$ reduces the inherent riskiness of the decision but it may not itself benefit the principal for some contracts. If, however, the contract makes payment to the agent strictly convex in the return y, the principal's payoff is strictly concave in that return so the principal gains from the reduction in risk. Note that Part 1 of Theorem 2 guarantees that the contract is strictly convex for *some* y. For the linear utility form in (13) that always induces the agent to choose $b^*(s)$ for all s, the contract is actually strictly convex for *all* y. To see this note that, with $P(.)$ of the form in (13),

$$u'[P(y)]\,P'(y) = k \text{ or } P'(y) = \frac{k}{u'[P(y)]} > 0, \text{ for } k > 0,$$

and

$$u''[P(y)]\,P'(y)^2 + u'[P(y)]\,P''(y) = 0 \text{ or } P''(y) = -\frac{u''[P(y)]\,P'(y)^2}{u'[P(y)]}.$$

With $P'(y) \neq 0$ and the agent strictly risk-averse (so $u''[P(y)] < 0$), it follows directly that $P''(.) > 0$. As shown in Section 4.2, the linear utility form in (13) is not necessarily constrained optimal for $b^*(.)$. But in some cases, as shown below for a version of the bidding application, it is the *only* form of contract that implements $b^*(.)$, so it is necessarily constrained optimal for $b^*(.)$ and Theorem 2 can then be applied directly.

3.2 Optimal bidding rules in the bidding application

The results of Theorem 2 and Corollary 1 can be illustrated by the bidding application. The assumption used for that is the following.

Assumption 1 *For the bidding application of Section 2.2, $\pi(b;s)$ satisfies $\pi(b;s) \in (0,1)$ for all $b \in \left(0, \overline{b}\right)$, $\pi\left(\overline{b};s\right) = 0$, and $\pi_b(b;s)/\pi(b;s)$ is continuous non-increasing in b for all $b \in [0, \overline{b})$, for all $s \in [\underline{s}, \overline{s}]$. The principal chooses a contract with the agent conditioned only on y for which $a = \overline{a}$ is optimal.*

[4]Conditioning the contract on a message $m(s)$ the agent sends revealing the signal s does not affect the results in Theorem 2 and Corollary 1 because specifying $x = (y, m)$ does not affect the proofs.

Note that, given the other assumptions of the model, this assumption guarantees $\pi_b(b;s) < 0$ for all s and $b < \overline{b}$. The assumption that a bid of $\overline{b}$ has zero probability of success is not seriously restrictive — it will be the case for a sufficiently high bid if the buyer places a finite value on supply. More restrictive is the implication that the probability of success drops to zero at the same bid level for all s but this avoids the complication of needing to be concerned with boundary solutions. The following lemma gives some useful properties of an efficient decision rule under Assumption 1.

Lemma 1 *For the bidding application of Section 2.2, suppose Assumption 1 holds. Then*

1. $b^*(s) \in \left(0, \overline{b}\right)$ *for all* $s \in [\underline{s}, \overline{s}]$;
2. $b^*(s)$ *is the unique solution to the first-order condition*

$$b^*(s)\,\pi_b(b^*(s);s) + \pi(b^*(s);s) = 0, \text{ for all } s \in [\underline{s}, \overline{s}]; \tag{23}$$

3. $b^*(s)$ *is strictly increasing (decreasing) if* $\pi_b(b;s)/\pi(b;s)$ *is strictly increasing (decreasing) in* s *for all* $s \in [\underline{s}, \overline{s}]$ *and all* $b \in \left[0, \overline{b}\right]$.

Proof. From (3), $b^*(s)$ satisfies

$$b^*(s) \in \arg\max_{b \in [0,\overline{b}]} b\pi(b;s), \quad \forall s \in [\underline{s}, \overline{s}]. \tag{24}$$

The maximand in this is zero for $b = 0$ and, because by Assumption 2 $\pi\left(\overline{b};s\right) = 0$, also for $b = \overline{b}$. By assumption, for each s there exists some $b > 0$ for which $\pi(b;s) > 0$, so the payoff from selecting some $b \in \left(0, \overline{b}\right)$ is strictly positive, establishing Part 1.

In view of Part 1, the first-order condition for the problem in (24) holds with equality and hence takes the form in (23). Since $\pi_b(b;s) < 0$ for $\pi(b;s) \in (0,1)$ and hence for $b \in \left(0, \overline{b}\right)$, that condition can be written

$$b^*(s) = -\frac{\pi(b^*(s);s)}{\pi_b(b^*(s);s)}, \quad \forall s. \tag{25}$$

By the assumptions, the right-hand side is strictly positive and strictly less than $\overline{b}$ as $b^*(s) \to \overline{b}$ for all s. Moreover, $\pi_b(b;s)/\pi(b;s)$ is non-increasing in b and hence so is $-\pi(b;s)/\pi_b(b;s)$. Thus (25) has a unique solution. To see that this unique solution is a maximum, note first that $\pi_b(b;s)/\pi(b;s)$ non-increasing in b implies

$$\frac{\partial}{\partial b}\left(\frac{\pi_b(b;s)}{\pi(b;s)}\right) = \frac{\pi(b;s)\,\pi_{bb}(b;s) - \pi_b(b;s)^2}{\pi(b;s)^2} \le 0,$$

or

$$\pi_{bb}(b;s) \le \frac{\pi_b(b;s)^2}{\pi(b;s)}. \tag{26}$$

The second derivative with respect to b of the maximand in (24) evaluated at $b^*(s)$ is

$$\begin{aligned}
& b^*(s)\,\pi_{bb}(b^*(s);s) + 2\pi_b(b^*(s);s) \\
&= -\frac{\pi(b^*(s);s)}{\pi_b(b^*(s);s)}\pi_{bb}(b^*(s);s) + 2\pi_b(b^*(s);s) \\
&\leq \pi_b(b^*(s);s) < 0,
\end{aligned}$$

the equality following from use of the first-order condition (25) and the weak inequality from (26) given $\pi_b(b^*(s);s) < 0$. Thus the second-order sufficient condition for a maximum is satisfied at the unique solution to (23), establishing Part 2.

Part 3 follows directly from (25) and Assumption 1. ■

This result establishes that there is a unique efficient bidding rule under Assumption 1 and specifies some of its characteristics. The next lemma gives results on implementing bidding rules that are helpful for characterising an optimal contract.

Lemma 2 *For the bidding application of Section 2.2, suppose Assumption 1 holds.*

1. *A bidding rule $b(.)$ is implementable by the contract $P(.)$ only if, for all $s \in [\underline{s}, \overline{s}]$, $b(s) \in \left(0, \overline{b}\right)$, $P(b(s)) > P(0)$, and $P(b)$ is non-decreasing for $b = b(s)$.*
2. *Suppose $\pi_b(b;s)/\pi(b;s)$ is either strictly increasing or strictly decreasing in s for all $s \in [\underline{s}, \overline{s}]$ and $b \in \left(0, \overline{b}\right)$. Then necessary and sufficient conditions for a differentiable contract $P(.)$ accepted by the agent to implement $b(.)$ are that $b(s)$: (a) satisfies the first-order condition*

$$u'[P(b(s))]\,P'(b(s))\,\pi(b(s);s) + \{u[P(b(s))] - u[P(0)]\}\,\pi_b(b(s);s) = 0, \tag{27}$$

for all $s \in [\underline{s}, \overline{s}]$, and (b) is non-decreasing (non-increasing) if $\pi_b(b;s)/\pi(b;s)$ is strictly increasing (decreasing) in s.[5]

Proof. Part 1. Under the assumptions of the lemma, the condition corresponding to (11) for the agent to choose $b(s)$ given signal s is

$$b(s) \in \arg\max_{b \in [0, \overline{b}]} u(P(b))\,\pi(b;s) + u(P(0))\,[1 - \pi(b;s)]\,, \text{ for all } s \in [\underline{s}, \overline{s}]\,.$$

To induce the agent to take action $a = \overline{a}$, it must be that $P\left(\hat{b}\right) > P(0)$ for some $\hat{b}$ for which $\pi\left(\hat{b};s\right) > 0$. Consider $b(s)$ that, for some s, has $P(b(s)) = P(0)$ or $b(s) = \overline{b}$. Then the agent's expected utility from choosing $b(s)$ given s is $u[P(b(s))] = u(P(0))$,

[5]This result also holds if "non-decreasing" and "non-increasing" are replaced by "strictly increasing" and "strictly decreasing" respectively but $b'(s) = 0$ is in fact inconsistent with the first-order condition (27) holding for all s. The result is stated as in the text because it is more convenient later to work with a weak, rather than a strict, inequality.

in the former case because $b(s) = 0$, in the latter because $\pi\left(\bar{b};s\right) = 0$. But the agent could have chosen $\hat{b}$ when observing s and this would have yielded expected utility

$$u(P(0)) + \left[u\left(P\left(\hat{b}\right)\right) - u(P(0))\right]\pi\left(\hat{b};s\right),$$

which is greater than $u(P(0))$ given $\pi\left(\hat{b};s\right) > 0$. Thus $b(s)$ is not implementable. Finally, if $P(b(s))$ is strictly decreasing in b, the agent's payoff is strictly increased by choosing some $b < b(s)$ because $\pi(b;s)$ is non-increasing in b for each s.

Part 2. Recall that $\pi(b(s);s)$ is differentiable for $\pi(b(s);s) \in (0,1)$. Necessity of the first-order condition (27) then follows from $b(s)$ interior to $\left[0,\bar{b}\right]$ and differentiability of $P(b)$. Let $z(b) = u(P(b)) - u(P(0))$ and

$$W(b(s),s) = z'(b(s)) + z(b(s))\frac{\pi_b(b(s);s)}{\pi(b(s);s)}.$$

The first-order condition (27) can then be written $\pi(b(s);s)W(b(s),s) = 0$ for all s. By Part 1, $b(.)$ is implementable only if $b(s) \in \left(0,\bar{b}\right)$ for all s and, by Assumption 1, $\pi(b(s);s) \in (0,1)$ for all such $b(s)$. So to implement $b(.)$ requires $W(b(s),s) = 0$ for all s. It is sufficient for (27) to have at most one solution for each s, and for this solution to be a maximum, that

$$\pi_b(b(s);s)W(b(s),s) + \pi(b(s);s)W_b(b(s),s) < 0 \text{ for all } s.$$

With $W(b(s),s) = 0$ and $\pi(b(s);s) > 0$, that will certainly hold if $W_b(b(s),s) < 0$ for each s. Because (27) must hold for all s, its total derivative with respect to s must equal zero. With $W(b(s),s) = 0$ and $\pi(b(s);s) > 0$ for all s, that implies

$$\frac{dW(b(s),s)}{ds} = W_b(b(s),s)\,b'(s) + W_s(b(s),s) = 0. \tag{28}$$

From the definition of $W(b(s),s)$,

$$W_s(b(s),s) = z(b(s))\frac{\partial}{\partial s}\left(\frac{\pi_b(b(s);s)}{\pi(b(s);s)}\right),$$

which, given $P(b(s)) > P(0)$ and hence $z(b(s)) > 0$, is either strictly positive or strictly negative under the conditions in Part 2(b). Thus (28) cannot be satisfied if $b'(s) = 0$. It can therefore be written

$$\begin{aligned} W_b(b(s),s) &= -W_s(b(s),s)/b'(s) \\ &= -\frac{z(b(s))}{b'(s)}\frac{\partial}{\partial s}\left(\frac{\pi_b(b(s);s)}{\pi(b(s);s)}\right). \end{aligned}$$

Thus $W_b(b(s), s) > 0$ if $b'(s)$ and $\frac{\partial}{\partial s}(\pi_b(b(s); s) / \pi(b(s); s))$ have opposite signs, so any solution to the first-order condition cannot be a maximum, establishing necessity of Part 2(b). Moreover, $W_b(b(s), s) < 0$ under the conditions in Part 2(b), which establishes sufficiency of those and the first-order conditions. ■

This lemma establishes conditions under which the first-order condition for the agent's bid decision is sufficient, as well as necessary, for an optimum so that one can adopt a "first-order approach" to the principal-agent problem studied here — the requirement that the agent's bid must be optimal can be characterised by the first-order condition for optimality. The underlying idea is the same as in the application of the "first-order approach" in Rogerson (1985) and Jewitt (1988) but there the application is to the agent's choice of action, not the agent's decision based on a signal that results from that action. Lemma 2 makes use of the property that the agent's first-order condition must hold for *all* s. This imposes restrictions on how the agent's expected utility varies with s, which in turn imposes restrictions on how that expected utility can vary with b given the function $b(s)$. The conditions on $\pi_b(b; s) / \pi(b; s)$ are properties of the exogenously given probability distribution. Thus in the principal's optimization problem the only constraint that needs to be applied to ensure incentive compatibility in addition to the first-order condition is that on the appropriate sign of $b'(s)$. This is a standard, and straightforward, constraint to impose. It is also one that can be expected to bind only in perverse cases because, as shown in Part 3 of Lemma 1, it corresponds to the bid $b(s)$ changing with s in the same direction as $b^*(s)$, the bid the principal would make given the same signal s as the agent.

To apply the results of the previous section to the bidding application under Assumption 1, consider what contracts implement the efficient bidding rule $b^*(.)$. From Lemma 1, $b^*(s) \in \left(0, \bar{b}\right)$ and satisfies (25). Moreover, the first-order condition for the agent (27) must hold for all s for which the agent is to use the rule $b^*(.)$. Suppose that is on the interval $[s', s'']$. It is then clear from comparison with (25) that the agent chooses bid $b^*(s)$ for all $s \in [s', s'']$ only if

$$\frac{u'(P(b))\, P'(b)}{u(P(b)) - u(P(0))} = \frac{1}{b}, \text{ for all } b \in \left[b^*(s'), b^*(s'')\right]. \tag{29}$$

Define $z(b) = u(P(b)) - u(P(0))$ for given $P(0)$. Then (29) can be written

$$\frac{z'(b)}{z(b)} = \frac{1}{b}, \text{ for all } b \in \left[b^*(s'), b^*(s'')\right],$$

which, by integration, has solution

$$z(b) = kb, \text{ for all } b \in \left[b^*(s'), b^*(s'')\right], \tag{30}$$

for some constant of integration k. That leads to the following result.

Proposition 1 *Suppose, in the bidding application of Section 2.2, Assumption 1 holds and $P(.)$ is differentiable.*

1. *A necessary condition for the agent to make an efficient bid $b^*(s)$ for all $s \in [\underline{s}, \overline{s}]$ is that $P(.)$ satisfies*

$$u(P(b)) = u(P(0)) + kb, \text{ for all } b \in [b^*(\underline{s}), b^*(\overline{s})], \text{ with } k > 0. \tag{31}$$

2. *If the agent is strictly risk-averse, marginally reducing $b(s)$ below $b^*(s)$ for any $s \in [\underline{s}, \overline{s}]$ increases the principal's payoff above the constrained optimal level.*

Proof. Part 1 follows because the solution for $u(P(b))$ implied by (30) for $s' = \underline{s}$ and $s'' = \overline{s}$ is that given by (31) and $k > 0$ from Part 1 of Lemma 2.

To establish Part 2, recall that the form of contract in (31) implies $P''(b) > 0$ for all b. When Assumption 1 holds

$$dG(y; b, s) = \begin{cases} 1 - \pi(b; s), & \text{for } y = 0 \ (\equiv \underline{y}); \\ \pi(b; s), & \text{for } y = b; \\ 0, & \text{otherwise}; \end{cases}$$

so

$$G(y; b, s) = \begin{cases} 1 - \pi(b; s), & \text{for } y \in [0, b); \\ 1, & \text{for } y \in \left[b, \overline{b}\right]; \end{cases}$$

and

$$G_b(y; b, s) = \begin{cases} -\pi_b(b; s), & \text{for } y \in [0, b); \\ 0, & \text{for } y \in \left[b, \overline{b}\right]. \end{cases}$$

It follows that $\int_{\underline{y}}^{y} G_b(\theta; b^*(s), s)\, d\theta > 0$ for all $y \in \left[\underline{y}, \overline{y}\right]$ because $\pi_b(b; s) < 0$ for all $b \in \left(0, \overline{b}\right)$. It then follows from Theorem 2 that the principal's payoff is increased by marginally reducing $b(s)$ below $b^*(s)$ for any s. ■

Part 1 of Proposition 1 establishes that the form of contract that makes the agent's utility linear in y is not only, as shown at the start of Section 3.1, sufficient to induce the agent to adopt $b^*(.)$ but also necessary when Assumption 1 holds. That allows Theorem 2 to be applied directly to the bidding application because the form of contract in (31) has $P''(b) > 0$ for all b. That the principal's payoff is increased by reducing $b(s)$ below $b^*(s)$ is to be expected. Reducing $b(s)$ reduces the spread between the payments for a bid's success and failure. Moreover, the probability of success is decreasing in b, so implementing $b(s) < b^*(s)$ increases the probability of success. This result contrasts with results in Lambert (1986) in which a risk-averse agent always makes an efficient decision for extreme values of the information signal. In that paper, the agent has only two possible decisions to choose from, a risky project and a safe project. It is then not surprising that it is optimal for the principal to induce even a risk-averse agent to select the risky project if the expected return is sufficiently high and to select the safe project if that expected return is sufficiently low. With a continuum of possible decisions, as in the bidding application, the choice is not as stark. The principal always has the possibility of inducing the agent to take a decision marginally less risky than is efficient for which the cost in terms of efficiency is only second-order.

With a risk-neutral agent, the motivation for distorting the agent's decision is not to reduce the risk the agent bears but, as discussed in Section 3.1, to reduce the effect of limited liability. In general, it is not obvious what direction the distortion takes. The bidding application can, however, be used to show that the distortion may be towards either more risky decisions or less risky decisions. To show this, it is convenient to make use of the first-order approach. Part 2 of Lemma 2 established that, when Assumption 1 holds, the first-order condition (27), together with the condition that $b(s)$ is non-decreasing if $\pi_b(b;s)/\pi(b;s)$ is strictly increasing in s or the condition that $b(s)$ is non-increasing if $\pi_b(b;s)/\pi(b;s)$ is strictly decreasing in s, is necessary and sufficient to induce the agent to select $b(s)$ for all s. Imposing the conditions (27) and either $b(s)$ non-decreasing or $b(s)$ non-increasing as appropriate, ensures, therefore, that the first-order approach is valid. That approach is used in an appendix to demonstrate the following result about how the bid optimally implemented by the principal is related to the efficient bid.

Proposition 2 *For the bidding application of Section 2.2, suppose Assumption 1 holds and there is a binding lower bound on payments to the agent.*[6] *Then, an optimal contract for a risk-neutral agent implements:*

1. $b(s) > b^*(s)$ *for all* $s \in (\underline{s}, \overline{s})$, $b(\underline{s}) = b^*(\underline{s})$ *and* $b(\overline{s}) \geq b^*(\overline{s})$ *if* $\pi_b(b;s)/\pi(b;s)$ *is strictly increasing in* s *for all* $s \in [\underline{s}, \overline{s}]$;
2. $b(s) < b^*(s)$ *for all* $s \in (\underline{s}, \overline{s})$, $b(\underline{s}) = b^*(\underline{s})$ *and* $b(\overline{s}) \leq b^*(\overline{s})$ *if* $\pi_b(b;s)/\pi(b;s)$ *is strictly decreasing in* s *for all* $s \in [\underline{s}, \overline{s}]$.

The essence of the result in Part 1 of Proposition 2 is illustrated in Figure 1. (That for Part 2 is similar.) For a risk-neutral agent, it follows from Proposition 1 that only a contract linear in b with positive slope induces the agent both to take action $\overline{a}$ and to select bid $b^*(s)$ for all s. Let $\hat{P}(b)$ in the figure illustrate the linear contract with the *least steep* slope that satisfies both the incentive compatibility constraint for the agent to choose action $\overline{a}$ and the individual rationality constraint with equality. To have a binding lower bound on payments implies $\underline{P} > \hat{P}(0)$. One possible response of the principal to such a bound would be to continue to implement $b^*(s)$ for all s by using a linear contract with the same slope but with $P(0)$ increased to $\underline{P}$. That corresponds

[6]This proposition is also conditional on an appropriate constraint qualification being satisfied.

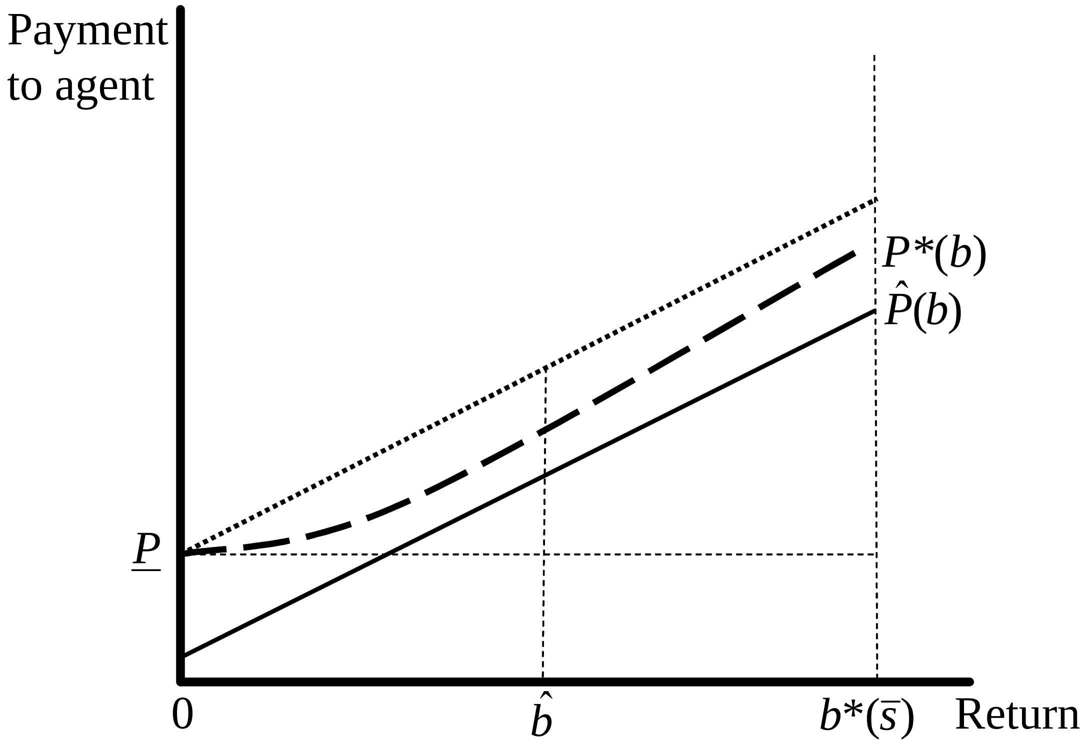

Figure 1: Illustration of Proposition 2

to the dotted line in Figure 1. The incentive compatibility condition (corresponding to (10) in the general framework) for a risk-neutral agent to choose action $\bar{a}$ is

$$P(0) + \int_{\underline{s}}^{\bar{s}} [P(b(s)) - P(0)]\, \pi(b(s);s)\, f(s;\bar{a})\, ds - v(\bar{a})$$
$$\geq P(0) + \int_{\underline{s}}^{\bar{s}} [P(b(s)) - P(0)]\, \pi(b(s);s)\, f(s;\underline{a})\, ds - v(\underline{a}), \quad (32)$$

so adding the same constant to $P(b)$ for all b leaves it still satisfied. But the expected payment to the agent is given by the left-hand side of (32), so adding a positive constant involves paying the agent more than required to satisfy the individual rationality constraint. However, in view of the definition $LR(s) \equiv f(s;\underline{a})/f(s;\bar{a})$, (32) can, when limited liability binds so that $P(0) = \underline{P}$, be written

$$\int_{\underline{s}}^{\bar{s}} [P(b(s)) - \underline{P}]\, \pi(b(s);s)\, f(s;\bar{a})\, [1 - LR(s)]\, ds \geq v(\bar{a}) - v(\underline{a}). \quad (33)$$

Thus the expected payment to the agent can be reduced while still satisfying (33) by reducing payments for positive returns for which the likelihood ratio $LR(s)$ is greater than 1. Since s is ordered so that $LR(s)$ is non-increasing and, by Lemma 1, $b^*(s)$ is strictly increasing when $\pi_b(b;s)/\pi(b;s)$ is strictly increasing in s, this corresponds to reducing payments for low, positive b. Because that increases the left-hand side of (33), it enables payments to be reduced for $LR(s) < 1$ too but never such as to make the contract concave because then a less steep linear contract would also have satisfied the constraint. Suppose the optimal contract corresponds to $P^*(b)$ in Figure 1. The effect on the bid for given s can be seen from the agent's first-order condition (27) which, for a risk-neutral agent with binding limited liability ($P(0) = \underline{P}$), can be written

$$-\frac{\pi_b(b(s);s)}{\pi(b(s);s)} = \frac{P'(b(s))}{P(b(s)) - \underline{P}}. \tag{34}$$

Consider the point $\hat{b}$ in Figure 1 at which the dashed line has a steeper slope than the dotted line and let s' and s'' denote the values of s for which $\hat{b}$ is bid under the contracts represented by the dashed and the dotted lines respectively. At $\hat{b}$, the denominator on the right hand side of (34) is smaller for the contract represented by the dashed line and the numerator is larger. Thus, from (34), s' and s'' must satisfy

$$-\frac{\pi_b\left(\hat{b};s'\right)}{\pi\left(\hat{b};s'\right)} > -\frac{\pi_b\left(\hat{b};s''\right)}{\pi\left(\hat{b};s''\right)}. \tag{35}$$

When $\pi_b(b;s)/\pi(b;s)$ is strictly increasing in s, that implies $s' < s''$. Moreover, with $b(s)$ and $b^*(s)$ both increasing and $\hat{b} = b(s') = b^*(s'')$, it follows that $b(s'') > b^*(s'')$. The proof of Proposition 2 consists of showing that the same applies for all $s \in (\underline{s}, \bar{s})$.

Proposition 2 shows the direction of the distortion of the optimal bid $b(s)$ from the efficient bid $b^*(s)$. Specifically, $b(s)$ is distorted above $b^*(s)$ when $\pi_b(b;s)/\pi(b;s)$ is strictly increasing in s for all $s \in [\underline{s}, \bar{s}]$ and below $b^*(s)$ when $\pi_b(b;s)/\pi(b;s)$ is strictly decreasing in s for all $s \in [\underline{s}, \bar{s}]$. In view of Part 3 of Lemma 1, that corresponds to bids being increased above $b^*(s)$ when $b^*(s)$ is increasing and below it when $b^*(s)$ is decreasing. The former corresponds to a higher, more risky bid, the latter to a lower, less risky bid. The conclusion that the direction of the distortion from efficiency can go either way mirrors that in Palomino and Prat (2003) in their application to delegated portfolio choice. Their result, however, relies on the "first-order approach" being valid, which they cannot guarantee. For the bidding application used here, it has been established that the first-order approach used is indeed valid.

Part 2 of Proposition 2 has an interesting implication. When limited liability binds, even a risk-neutral agent is induced, for all values of the information signal apart from those at the two extremes of its support, to make a lower, less risky bid, with a lower return and a higher probability of success, than the principal would choose given the same information. Thus the principal chooses a contract that more than compensates for the incentive an agent otherwise has to make a more risky decision if limited liability is imposed on a contract that would be optimal in the absence of limited liability. In that case, managers with limited liability are induced to take decisions in a way that makes it look as if they are risk-averse — limited liability not only reduces the profits of firms but also biases their decisions in a less risky direction.

Proposition 1 implies that, when Assumption 1 holds, only a contract with payment linear in y induces a risk-neutral agent to select $b^*(s)$ for all s, a condition under which Diamond (1998) establishes the following "near-linearity" result. For a risk-neutral agent with binding limited liability, an optimal contract to induce $a > \underline{a}$ converges to linearity as the ratio of the return to the principal (gross of payment to the agent) to the cost to the agent of taking actions $a > \underline{a}$ increases. The essence of the result is that, as this ratio increases, the cost of inducing an action becomes smaller relative to the gains from implementing efficient decisions and, if an efficient decision rule is implemented only by a linear contract, an optimal contract approaches linearity. But, as Diamond (1998) recognises and will be shown in the next section, a linear contract is not necessarily the only contract that implements an efficient rule, in which case his result may not apply.

4 Contracts to implement decision rules

The previous section was concerned with the decision rules it is optimal for the principal to implement. This section is concerned with contracts to implement decision rules. Some papers in the literature, for example, Demski and Sappington (1987), Diamond (1998) and Biais and Casamatta (1999), have been concerned with contracts to implement efficient decision rules. But, as Theorem 1 showed, efficient decision rules may not be optimal, so the analysis here is not restricted to efficient rules.

4.1 Results for the general framework

In general, adding a decision made by the agent to the classic principal-agent problem affects the contract between principal and agent. A striking example is that with a continuum of possible decisions, so $B = [\underline{b}, \overline{b}]$, and the probability distribution $G(y; b, s)$ for the return y given decision b and signal s taking the form $\hat{G}(y; s - b)$. In the absence of the additional decision, b is a fixed parameter. This gives rise to a standard principal-agent problem in which the agent's action a affects s which, in turn, affects the distribution of returns y. Inducing the agent to choose $a > \underline{a}$ involves, in the usual way, a trade-off between providing the agent with incentives to take the action and insuring the agent's income. With the additional decision, however, an efficient decision rule satisfies, from (1),

$$b^*(s) \in \arg\max_{b \in [\underline{b}, \overline{b}]} \int_{\underline{y}}^{\overline{y}} y d\hat{G}(y; s - b), \text{ for all } s \in [\underline{s}, \overline{s}],$$

which is satisfied for all s by $b^*(s) = s - \beta^*$ for constant β^* that is a solution to

$$\beta^* \in \arg \max_{\beta \in [\underline{s} - \overline{b}, \overline{s} - \underline{b}]} \int_{\underline{y}}^{\overline{y}} y d\hat{G}(y; \beta).$$

The principal's expected return is then $\int_{\underline{y}}^{\overline{y}} y d\hat{G}(y; \beta^*)$, which is independent of s. Thus the principal no longer has reason to be concerned with the value of s and so has no reason to provide the agent with incentives for action to influence its distribution. If, moreover, the payment to the agent is made independent of the outcome, the agent is indifferent as to which decision is made and hence will use an efficient decision rule $b^*(.)$. Thus, in this case, the possibility of a decision in addition to an action removes any need for the principal to use an incentive contract at all. Moreover, it does not require the agent's decision to be verifiable. This example is clearly special in that the possibility of the decision makes any one signal as good as any other. But it serves to illustrate the dramatic impact on incentive contracts that adding a decision may have.

Inducing the agent to implement a decision rule does not, however, necessarily affect the contract between principal and agent. Suppose that, for each s, the decision $b(s)$ the principal wishes the agent to take stochastically dominates any other decision in either the first-order or the second-order sense. If the stochastic dominance is first-order then, from a standard result, the agent prefers $b(s)$ to any other decision for each s provided only that the contract ensures the agent's utility is increasing in y. If the stochastic dominance is second-order then, again from a standard result, the agent prefers $b(s)$ to any other decision for each s provided only that the contract ensures the agent's utility is increasing and concave in y. With these forms of stochastic dominance, the principal thus has substantial freedom in choosing an incentive contract to implement $b(.)$. To explore this formally, let

$$Y(a, b(.)) = \left\{ y \in \left[\underline{y}, \overline{y}\right] \mid \int_{\underline{s}}^{\overline{s}} dG(y; b(s), s) f(s; a) \, ds > 0 \right\}, \text{ for all } a, b(.). \tag{36}$$

$Y(a, b(.))$ is the set of returns y that have strictly positive probability density given action a and decision rule $b(.)$. Next, let

$$L(y; a, b(.)) = \begin{cases} \frac{\int_{\underline{s}}^{\overline{s}} dG(y; b(s), s) f_a(s; a) ds}{\int_{\underline{s}}^{\overline{s}} dG(y; b(s), s) f(s; a) ds}, & \text{for } y \in Y(a, b(.)), \text{ if } A = [\underline{a}, \overline{a}]; \\ \frac{\int_{\underline{s}}^{\overline{s}} dG(y; b(s), s)[f(s; a) - f(s; \underline{a})] ds}{\int_{\underline{s}}^{\overline{s}} dG(y; b(s), s) f(s; a) ds}, & \text{for } y \in Y(a, b(.)), \text{ if } A = \{\underline{a}, \overline{a}\}. \end{cases} \tag{37}$$

$L(y; a, b(.))$ is the relative rate of change with respect to a of the likelihood of y occurring, conditional on using the decision rule $b(.)$. Finally, let

$$\hat{B}(s;a,b(.)) = \begin{cases} B, \text{ if limited liability binds;} \\ \\ \Big\{ b \in B \mid \int_{y'}^{y''} dG(y;b,s) = 0 \text{ for any } y', y'' \in \left[\underline{y}, \overline{y}\right] \text{ with} \\ \quad y'' > y' \text{ such that, for all } y \in [y', y''], y \notin Y(a,b(.)) \Big\}, \\ \qquad \text{if limited liability does not bind.} \end{cases} \tag{38}$$

$\hat{B}(s;a,b(.))$ excludes, when limited liability is not binding, decisions with positive probability mass of returns for which there is zero probability mass under $b(.)$.

Theorem 3 *Suppose the agent is strictly risk-averse and the principal implements action $a > \underline{a}$ and decision rule $b(.)$ with a contract conditioned only on y. Then, the conditions (11) for the agent to adopt $b(.)$ do not constrain optimal contract payments for returns $y \in Y(a,b(.))$ if either of the following two sets of conditions hold:*

1. *(a) for each $s \in [\underline{s}, \overline{s}]$, $G(y;b(s),s)$ first-order stochastically dominates $G(y;b,s)$ for all decisions $b \in \hat{B}(s;a,b(.))$; and (b) $L(y;a,b(.))$ is non-decreasing in y for $y \in Y(a,b(.))$ for every $a \in A$; in this case, $P(y)$ is non-decreasing for all $y \in Y(a,b(.))$;*

2. *(a) for each $s \in [\underline{s}, \overline{s}]$, $G(y;b(s),s)$ second-order stochastically dominates $G(y;b,s)$ for all decisions $b \in \hat{B}(s;a,b(.))$; (b) $L(y;a,b(.))$ is non-decreasing and concave in y for $y \in Y(a,b(.))$ for every $a \in A$; (c) limited liability is not binding; and (d) the agent's utility function is such that*

$$u\left[u'^{-1}\left(\frac{1}{z}\right)\right] \textit{ is concave as a function of } z \textit{ for } z > 0. \tag{39}$$

Proof. A contract conditioned only on y takes the form $P(y)$ independent of b. The principal cannot do better than set $P(y) = \underline{P}$ for $y \notin Y(a,b(.))$. For limited liability not binding, that rules out decisions $b \notin \hat{B}(s;a,b(.))$ without restricting the contract for $y \in Y(a,b(.))$. For limited liability binding, there are no decisions $b \notin \hat{B}(s;a,b(.))$.

Given those payments for $y \notin Y(a,b(.))$, suppose for $y \in Y(a,b(.))$ the principal were to ignore the constraints (11). Then, for $A = [\underline{a}, \overline{a}]$, the principal's first-order necessary condition with respect to $P(y)$ for an optimal contract is, for all $y \in Y(a,b(.))$ for which limited liability is not binding,

$$-\int_{\underline{s}}^{\overline{s}} dG(y;b(s),s) f(s;a)\, ds + \lambda u'(P(y)) \int_{\underline{s}}^{\overline{s}} dG(y;b(s),s) f(s;a)\, ds$$
$$+ \mu u'(P(y)) \int_{\underline{s}}^{\overline{s}} dG(y;b(s),s) f_a(s;a)\, ds = 0,$$

where $\lambda > 0$ and μ are Lagrange multipliers corresponding to the constraints (9) and (10) respectively. For $A = \{\underline{a}, \overline{a}\}$, the corresponding first-order condition is

$$-\int_{\underline{s}}^{\overline{s}} dG\left(y; b\left(s\right), s\right) f\left(s; a\right) ds + \lambda u'\left(P\left(y\right)\right) \int_{\underline{s}}^{\overline{s}} dG\left(y; b\left(s\right), s\right) f\left(s; a\right) ds$$
$$+ \mu u'\left(P\left(y\right)\right) \int_{\underline{s}}^{\overline{s}} dG\left(y; b\left(s\right), s\right) \left[f\left(s; a\right) - f\left(s; \underline{a}\right)\right] ds = 0.$$

With the definition of $L\left(y; a, b\left(.\right)\right)$ in (37), in both cases the relevant necessary condition can be written in the standard optimal risk-sharing form

$$\frac{1}{u'\left(P\left(y\right)\right)} = \lambda + \mu L\left(y; a, b\left(.\right)\right). \tag{40}$$

With $u'\left(.\right)$ strictly decreasing and $L\left(y; a, b\left(.\right)\right)$ non-decreasing in y, it follows from (40) that $P\left(y\right)$ is non-decreasing for all $y \in Y\left(a, b\left(.\right)\right)$ for which limited liability does not bind if $\mu > 0$ or non-increasing for all $y \in Y\left(a, b\left(.\right)\right)$ for which limited liability does not bind if $\mu < 0$. For y such that limited liability binds, the left-hand side of (40) must be no greater than the right-hand side given $P\left(y\right) = \underline{P}$. For $L\left(y; a, b\left(.\right)\right)$ non-decreasing in y, this can apply only to an interval $[\underline{y}, \hat{y}]$ for some $\hat{y} \geq \underline{y}$ when $\mu > 0$ and an interval $[\hat{y}, \overline{y}]$ for some $\hat{y} \leq \overline{y}$ when $\mu < 0$. That implies $P\left(y\right)$ non-decreasing for all $y \in Y\left(a, b\left(.\right)\right)$ or non-increasing for all $y \in Y\left(a, b\left(.\right)\right)$ according as $\mu > 0$ or $\mu < 0$. Then, by an argument attributed to Lambert (see Rogerson (1985, footnote 8)), $\mu > 0$ since otherwise $P\left(y\right)$ is non-increasing for all $y \in Y\left(a, b\left(.\right)\right)$ and the agent would have no incentive to choose $a > \underline{a}$ given decision rule $b\left(.\right)$. Thus $P\left(y\right)$ is non-decreasing for all $y \in Y\left(a, b\left(.\right)\right)$. It follows that $u\left(P\left(y\right)\right)$ is non-decreasing as a function of y for $y \in Y\left(a, b\left(.\right)\right)$. Thus, by the standard result in Laffont (1989, p. 32), an agent receiving signal s always prefers $b\left(s\right)$ if it is stochastically dominant in the first-order sense to any other decision for which the return is always some $y \in Y\left(a, b\left(.\right)\right)$. With the payments specified for $y \notin Y\left(a, b\left(.\right)\right)$, decisions $b \in \hat{B}\left(s; a, b\left(.\right)\right)$ for which the return is not always some $y \in Y\left(a, b\left(.\right)\right)$ are certainly no more attractive than if $P\left(y\right)$ for $y \notin Y\left(a, b\left(.\right)\right)$ were increased to ensure $P\left(y\right)$ non-decreasing for all y. Thus, given first-order stochastic dominance, $b\left(s\right)$ is also preferable to all such decisions. This establishes that $b\left(s\right)$ satisfies (11) for all s, as claimed in Part 1.

The proof of Part 2 follows that of Theorem 1 in Jewitt (1988). Given $\mu > 0$, it follows from (40) and $L\left(y; a, b\left(.\right)\right)$ non-decreasing concave in y that $1/u'\left(P\left(y\right)\right)$ is non-decreasing concave in y for all $y \in Y\left(a, b\left(.\right)\right)$. The condition in (39) ensures that $u\left(P\right)$ is a concave transformation of $1/u'\left(P\right)$ and hence $u\left(P\left(y\right)\right)$ is non-decreasing concave in y for all $y \in Y\left(a, b\left(.\right)\right)$. With the payments specified for $y \notin Y\left(a, b\left(.\right)\right)$, decisions $b \in \hat{B}\left(s; a, b\left(.\right)\right)$ for which the return is not always some $y \in Y\left(a, b\left(.\right)\right)$ are certainly no more attractive than if $P\left(y\right)$ for $y \notin Y\left(a, b\left(.\right)\right)$ were increased to ensure $P\left(y\right)$ non-decreasing concave for all y. It then follows from the standard result in Laffont (1989,

p. 32-33) that an agent receiving signal s always prefers $b(s)$ if it stochastically dominates other $b \in \hat{B}(s; a, b(.))$ in the second-order sense. ■

The cases covered by Theorem 3 are ones in which there is no conflict between implementing the decision rule $b(.)$ and providing the agent with efficient incentives for action. In those cases, the requirement that the agent use the decision rule $b(.)$ does not constrain the optimal contract for returns $y \in Y(a, b(.))$ and, since returns $y \notin Y(a, b(.))$ are never realised, any constraints on the contract for these returns do not affect the principal's payoff. The essential reason is the following. A standard result from principal-agent theory without the additional decision is that the shape of the optimal contract is determined by the likelihood ratio of returns as a function of the action — when that likelihood ratio is monotone, payment is non-decreasing in the return to the principal. The corresponding likelihood term with the additional decision included here is $L(y; a, b(.))$ defined in (37). When this is monotone, the payment to the agent is non-decreasing in the return to the principal, so the agent will make the desired decision if it is stochastically dominant in the first-order sense. When the payment to the agent also results in expected utility that is concave, the agent will make the desired decision if it is stochastically dominant in the second-order sense. The restriction to contracts conditioned only on y does not necessarily rule out application to cases in which the agent's decision is itself verifiable because, as discussed below in connection with the bidding application, the decision may convey no useful additional information. But that restriction in any case applies to all the related papers cited in the Introduction.

Proposition 3 in Demski and Sappington (1987) proves a related result on first-order stochastic dominance. Part 1 of Theorem 3 extends that result in three significant respects. The first is that it does not use an assumption on convexity of the underlying distribution function. Such an assumption has been used in the literature on the validity of the first-order approach to principal-agent problems to ensure that the agent's first-order condition for choice of action is sufficient for a maximum, as well as necessary. But that is not required here because the proof relies only on conditions that are necessary for any $a > \underline{a}$. Since it is widely recognised that such an assumption is unappealing, see Jewitt (1988), removing the need for it is a worthwhile gain. The second is that it extends the result to cases with binding limited liability. The third is the extension to stochastic dominance that applies after excluding those decisions for which there is positive probability mass of returns for which there is zero probability mass under $b(.)$. From a pure theory perspective, that is straightforward. But it is significant for applications. For example, as shown below, it makes the theorem applicable to the bidding application of Section 2.2 when it would not otherwise be. There is no counterpart in Demski and Sappington (1987) to Part 2 of Theorem 3.

4.2 Implementing bidding rules in the bidding application

The bidding application from Section 2.2 provides an illustration of the results in Theorem 3. Suppose the principal wants to be sure that the bid is successful at a price no less than is necessary to achieve this, as specified in the following assumption.

Assumption 2 *For the bidding application of Section 2.2, the principal chooses a contract to ensure that the agent chooses* $a = \overline{a}$ *and uses a bidding rule* $b(.)$ *that, for each* $s \in [\underline{s}, \overline{s}]$, *selects the highest bid* $b(s)$ *that will certainly be successful.*

Such a bidding rule is of interest only when there is some positive bid that will certainly be successful. For an example, let b' denote the highest bid that will actually be successful and suppose $f(s; a)$ has support $[ab', b']$ for $a \in \{\underline{a}, \overline{a}\}$ with $0 < \underline{a} < \overline{a} < 1$. Then the signal s is a random draw from a distribution with both lower and upper supports no greater than b' and a bid $b = s$ will thus certainly be successful. Moreover, higher a results in a signal s drawn from a distribution concentrated closer to b' and, as a approaches one, s corresponds to b'. This is illustrated for the case of $f(s; a)$ uniform in Figure 2. Under straightforward conditions, $b = s$ is also an efficient decision. Since $\pi(b; s)$ has not been assumed differentiable with respect to b for (b, s) such that $\pi(b; s) = 1$, let $\pi_{b_+}(b; s)$ denote its right-hand derivative. Then if

$$s\pi_{b_+}(s; s) + 1 < 0,$$

or equivalently $\pi_{b_+}(b; s) < -1/s$, a bid $b = s$ is a local optimum for the maximand in (3). Under straightforward conditions, it is also a global optimum. Then $b^*(s) = s$, so an efficient bidding rule ensures that the bid always succeeds. Provided the cost of inducing the agent to implement this rule is not too high, it will also be the optimal bidding rule for the principal to implement. (Theorem 1 does not apply when $\pi(b; s)$ is not differentiable at $b = b(s)$.)

From the set of bids that will be successful for sure, the principal's payoff is obviously increased by having the agent choose the highest. In this case, the only useful information in a signal is the highest bid b that will surely be successful. Thus, without loss of generality, attention can be restricted to the case in which there is a one-to-one correspondence between b and s and the inverse function $b^{-1}(b)$ exists. Several things are worth noting about this case. First, there is no loss to the principal from restricting the contract to one in which payment does not depend explicitly on the bid made — the return from a successful bid necessarily reveals the bid and, since the bid the agent is induced to make never actually fails, there is no loss from imposing the same penalty for failure whatever bid is made. Second, there is no loss to the principal in restricting the contract to one in which payment is a non-decreasing function of that return. Recall that, for the bidding application, b is the return to the principal if the bid is successful so, if the bidding rule to be implemented ensures that bids are always

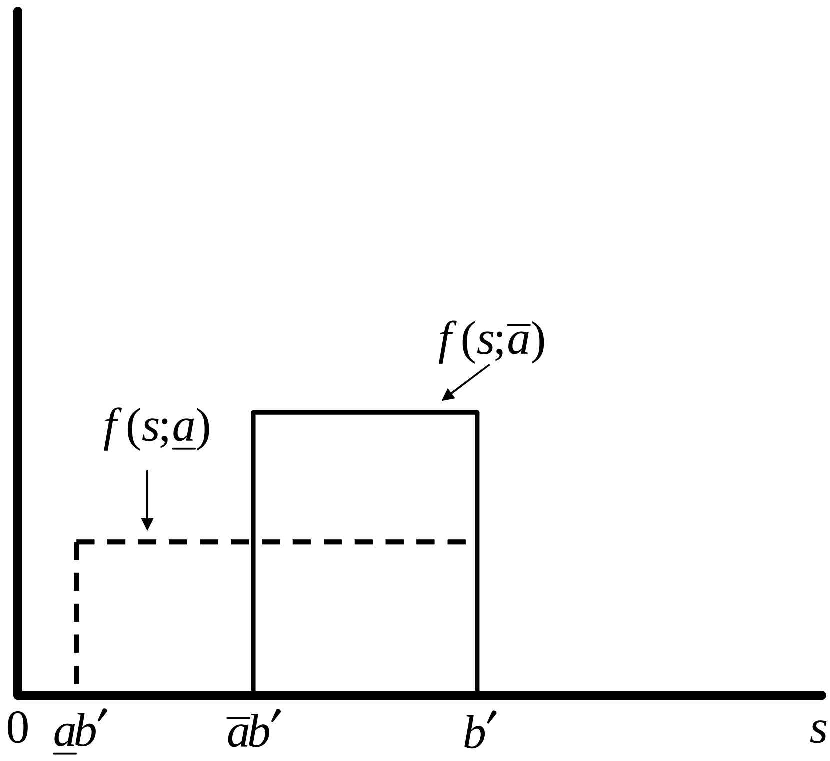

Figure 2: Example of $f(s;a)$ for case of certain success

successful, a contract can be written $P(b)$. Suppose $P(b)$ were decreasing at some point b'. Then the agent would always choose some bid $b < b'$ in preference to b' because, if b' would be successful for sure, so would b and b would also result in a higher payment to the agent. But b would result in a lower return to the principal, so the principal would do better to raise $P(b')$ to the same level as $P(b)$ if the agent is to choose b' for any s. Even if the agent is not to choose b' for any s, the principal does not lose by doing this. Such a contract may have a flat section, as illustrated in Figure 3, but is non-decreasing everywhere.[7]

[7]There is also nothing to be gained from making payment depend on a message sent by the agent after observing s. Formally, suppose the agent is required to send a message $m \in M$ and, if bid b is made and message m sent, receives payment $\hat{P}(b,m)$ if the bid is successful, $\hat{P}_0(b,m)$ if not. Then, given signal s, the agent's bid $b(s)$ and message $m(s)$ must satisfy

$$\{b(s), m(s)\} \in \arg\max_{b\in[0,\bar{b}], m\in M} u\left(\hat{P}(b,m)\right)\pi(b;s) + u\left(\hat{P}_0(b,m)\right)[1-\pi(b;s)], \quad s \in [\underline{s},\bar{s}].$$

But this would result in exactly the same bid and payoffs as a contract independent of the message that, for $b^{-1}(b)$ the signal (if any) for which bid b is chosen, satisfies

$$P(b) = \begin{cases} \hat{P}\left(b, m\left(b^{-1}(b)\right)\right) & \text{if } \ b = b(s) \ \text{ for some } s \in [\underline{s},\bar{s}]; \\ \min_{m\in M}\hat{P}(b,m) & \text{if } \ b \neq b(s) \ \text{ for any } s \in [\underline{s},\bar{s}]; \end{cases}$$

$$P_0(b) = \begin{cases} \hat{P}_0\left(b, m\left(b^{-1}(b)\right)\right) & \text{if } \ b = b(s) \ \text{ for some } s \in [\underline{s},\bar{s}]; \\ \min_{m\in M}\hat{P}_0(b,m) & \text{if } \ b \neq b(s) \ \text{ for any } s \in [\underline{s},\bar{s}]. \end{cases}$$

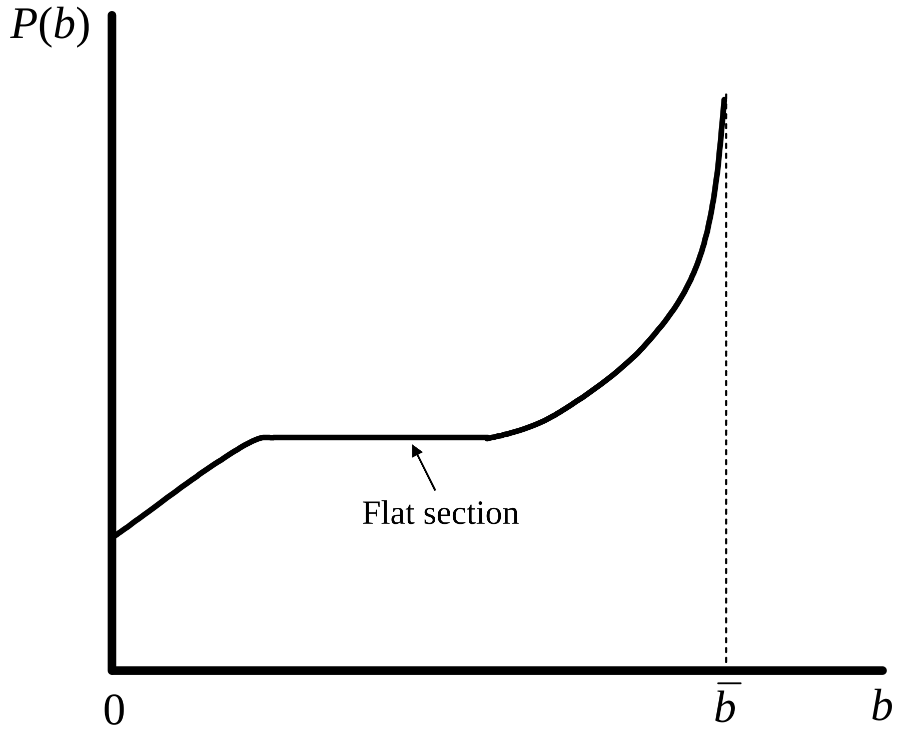

Figure 3: Optimal contract in the bidding application

The first of these properties allows Theorem 3 to be applied. When the bidding rule $b(.)$ to be implemented requires bids that are always successful, a return of zero has probability zero if the agent adopts that rule and hence is not in the set $Y(\bar{a}, b(.))$. Thus, provided limited liability is not binding, any bid that may be unsuccessful given s is not in the set $\hat{B}(s; \bar{a}, b(.))$ and can be ruled out by a sufficiently large penalty for an unsuccessful outcome. Moreover, the highest bid that guarantees success stochastically dominates all lower bids. Thus the stochastic dominance properties of Part 1(a) of Theorem 3 are satisfied. Moreover, with $y = b$ for a successful bid b,

$$dG(y; b(s), s) = \begin{cases} 1, & \text{if } y = b(s); \\ 0, & \text{otherwise.} \end{cases}$$

Application of this to (37) gives

$$L(b; \bar{a}, b(.)) = \frac{f\left(b^{-1}(b); \bar{a}\right) - f\left(b^{-1}(b); \underline{a}\right)}{f\left(b^{-1}(b); \bar{a}\right)} = 1 - \frac{f\left(b^{-1}(b); \underline{a}\right)}{f\left(b^{-1}(b); \bar{a}\right)}. \tag{41}$$

If this expression is non-decreasing in b, the remaining condition of Part 1 of Theorem 3, Part 1(b), is also satisfied and Theorem 3 applies. Given that s is ordered such that $LR(s) \equiv f(s; \underline{a}) / f(s; \bar{a})$ is non-increasing, $L(b; \bar{a}, b(.))$ is non-decreasing in b as long as $b(s)$ is non-decreasing because then $b^{-1}(b)$ is also non-decreasing. These results are collected here for convenience.

Proposition 3 *For the bidding application of Section 2.2, suppose Assumption 2 holds.*

1. *There is no loss to the principal in restricting the contract to a form in which payment is a non-decreasing function of the return to the principal and does not depend directly on the bid made.*

2. *The conditions (11) for the agent to adopt $b(.)$ do not constrain an optimal contract for positive returns if (a) the agent is strictly risk-averse; (b) limited liability is not binding; and (c) $b(.)$ is non-decreasing.*

That $b(.)$ is non-decreasing is a natural characteristic under Assumption 2. The only useful information in the signal s is about the highest bid that will certainly be successful. Suppose s' reveals that bid b will certainly be successful. Then one would not expect a signal $s > s'$ with a higher relative likelihood of arising from greater action by the agent to result in uncertainty about whether b will be successful. And provided it does not, $b(.)$ will be non-decreasing. But it is worth emphasising that, even if $b(.)$ is decreasing for some s, Part 1 of Proposition 3 still ensures that the payment schedule is non-decreasing, though with a flat segment as in Figure 3. When that segment corresponds to an interval at the lower end of values of b, the payment schedule looks very much like a salary plus a performance-related bonus for performance above some specified level. Stock options too are a reward with a flat section for an interval of low returns.

With binding limited liability, the result in Part 2 of Proposition 3 may not hold because the worst penalty the principal can impose may not be sufficient to deter the agent from making a bid that may be unsuccessful, and such bids do not satisfy the first-order stochastic dominance property in Part 1 of Theorem 3. To explore the implications further, note that, in view of Part 1 of Proposition 3, if limited liability binds it does so for $P(0)$. Thus, with binding limited liability the condition corresponding to (11) for the agent to choose $b(s)$ given signal s is

$$b(s) \in \arg\max_{b \in [0,\bar{b}]} u(P(b))\,\pi(b;s) + u(\underline{P})\left[1-\pi(b;s)\right], \text{ for all } s \in [\underline{s},\bar{s}]. \tag{42}$$

Under Assumption 2, with $b(.)$ such that $\pi(b(s);s) = 1$ for all s, and hence $\pi(b;s) = 1$ for all $b < b(s)$, this condition is satisfied for all $b \le b(s)$ if $P(b)$ is non-decreasing. If the signal s reveals the purchaser's reservation value precisely (that is, it reveals not only that bid $b(s)$ will certainly be successful but also that any higher bid will certainly be unsuccessful), then $\pi(b;s) = 0$ for $b > b(s)$. In that case, (42) is satisfied for $b > b(s)$ as long as $P(b(s)) \ge \underline{P}$. Then, as with non-binding limited liability, all that is required to implement $b(.)$ is that $P(b)$ is non-decreasing and, as in Proposition 3, implementing $b(.)$ does not constrain optimal contract payments for positive returns if $b(s)$ is non-decreasing.

But a signal that reveals the purchaser's reservation value precisely is clearly a rather special case. More generally, even if the signal reveals a bid level that will certainly be successful, it will not rule out that a higher bid may also be successful. Formally, that corresponds to $\pi(b(s)+\varepsilon; s) \in (0,1)$ for sufficiently small $\varepsilon > 0$ for all $s \in [\underline{s}, \bar{s})$. Under Assumption 2, the bidding rule $b(.)$ to be implemented always satisfies $\pi(b(s); s) = 1$. Thus a necessary condition for $b(s)$ to be preferred to any $b > b(s)$ for signal s is that

$$u'[P(b(s))] P'(b(s)) + \{u[P(b(s))] - u(\underline{P})\} \pi_{b_+}(b(s); s) \leq 0, \quad \text{for all } s \in [\underline{s}, \bar{s}]. \tag{43}$$

This necessary condition can be re-written

$$P'(b(s)) \leq -\pi_{b_+}(b(s); s) \frac{u[P(b(s))] - u(\underline{P})}{u'[P(b(s))]}, \text{ for all } s \in [\underline{s}, \bar{s}]. \tag{44}$$

It is shown in an appendix that this condition is also sufficient provided $b(s)$ and $\pi_b(b; s) / \pi(b; s)$ are either both non-decreasing in s or both non-increasing in s for all $b \in \left[0, \bar{b}\right]$ and $s \in [\underline{s}, \bar{s}]$. The implication is that, to ensure the agent makes a bid that is always successful, $P(b)$ must not only be non-decreasing but also not increase too fast. If it does, with $\pi(b(s)+\varepsilon; s) > 0$, the agent's payoff is increased by making a slightly higher bid, with a higher reward if successful but with a probability of success strictly less than one.

The impact this has on the optimal contract can be seen most clearly with a risk-neutral agent. Suppose the constraint (42) is not binding. Then limited liability reduces the principal's payoff only when it is not possible to satisfy the incentive compatibility condition for the agent to choose action $\bar{a}$ while satisfying the agent's individual rationality constraint with equality. The incentive compatibility condition in this case (corresponding to (33) with $\pi(b(s); s) = 1$) when binding can be written

$$\int_{\underline{s}}^{\bar{s}} P(b(s)) f(s; \bar{a}) [1 - LR(s)] \, ds = v(\bar{a}) - v(\underline{a}). \tag{45}$$

As discussed above in connection with Proposition 2, the expected payment to the agent can be reduced while keeping the incentive compatibility condition (45) satisfied by re-allocating rewards from $b(s)$ for which $LR(s)$ is high to $b(s)$ for which $LR(s)$ is low. Since s is ordered so that $LR(s)$ is non-increasing, that corresponds to increasing the payments for low b if $b(s)$ is decreasing and for high b if $b(s)$ is increasing. In the former case, the principal would wish to focus all payments above $\underline{P}$ on the lowest b, but that would clearly violate the constraint that $P(b)$ be non-decreasing. In the latter case, the principal would wish to focus all payments above $\underline{P}$ on the $b(s)$ corresponding to the highest s, but that would necessarily violate the constraint (44) that $P(b)$ not increase too fast.[8] Thus in both cases implementing $b(.)$ necessarily imposes a restriction on the optimal contract.

If $b(s)$ is increasing (which corresponds to the monotone likelihood ratio property (MLRP) that the expression in (41) is non-increasing), the constraint (44) necessarily binds for all b and thus so does (43). That is sufficient to determine the form of the optimal contract. Define $z(b) = u(P(b)) - u(\underline{P})$. Then (43) can be written

$$z'(b) + z(b)\,\pi_{b_+}\left(b; b^{-1}(b)\right) \leq 0, \text{ for all } b \in [b(\underline{s}), b(\bar{s})],$$

which, when it is binding for all b, can be re-written

$$\frac{z'(b)}{z(b)} = -\pi_{b_+}\left(b; b^{-1}(b)\right), \text{ for all } b \in [b(\underline{s}), b(\bar{s})].$$

This has solution

$$z(b) = K \exp\left[-\int_{b(\underline{s})}^{b} \pi_{b_+}\left(b; b^{-1}(b)\right) db\right], \text{ for all } b \in [b(\underline{s}), b(\bar{s})],$$

with $K = z(b(\underline{s}))$. Translating that solution back into the original notation for a risk-neutral agent gives the following result.

Proposition 4 *For the bidding application of Section 2.2, suppose Assumption 2 holds, the agent is risk-neutral with limited liability binding, and $\pi(b(s)+\varepsilon; s) \in (0,1)$ for sufficiently small $\varepsilon > 0$ for all $s \in [\underline{s}, \bar{s})$. Then the following form of contract is optimal if $b(s)$ is increasing and $\pi_b(b;s)/\pi(b;s)$ is non-decreasing in s for all $s \in [\underline{s}, \bar{s}]$:*

$$P(b) = \begin{cases} \underline{P} + [P(b(\bar{s})) - \underline{P}] \exp\left[-\int_{b(\underline{s})}^{b} \pi_{b_+}\left(b; b^{-1}(b)\right) db\right], & \textit{for} \quad b \in [b(\underline{s}), b(\bar{s})]; \\ \underline{P}, & \textit{for} \quad b \notin [b(\underline{s}), b(\bar{s})]; \end{cases} \tag{46}$$

with $P(b(\bar{s}))$ ensuring that the incentive compatibility condition (45) is satisfied.

This result gives the formula for an optimal contract under the conditions specified. Everything in that formula is data for the model apart from $P(b(\bar{s}))$, which is set so that the incentive compatibility condition (45) is satisfied when the formula is substituted for $P(b)$. The formula is linear in the exponential term and so will not, in general, result in a contract linear in b. If, for example, $\pi_{b_+}\left(b; b^{-1}(b)\right)$ were constant,

[8] Formally, there is a standard open set issue in choosing an optimal contract when s is a continuous variable. The principal's payoff is increased by increasing $P(b(\bar{s}))$ and reducing $P(b(s))$ for $s \neq \bar{s}$ in such a way that incentive compatibility is maintained but, if $P(b(s)) = \underline{P}$ for all $s \neq \bar{s}$, incentive compatibility ceases to hold because $P(b(\bar{s}))$ is received with probability only of order ds. The contract described in the text is approximately optimal.

$P(b)$ would be exponentially increasing in b. That makes the contract very different from the one with all payments above $\underline{P}$ concentrated on the $b(s)$ for which $LR(s)$ is lowest that the principal would use if incentive compatibility of decisions were not an issue. Note that the top line in (46) approaches a constant as $\pi_{b_+}\left(b; b^{-1}(b)\right)$ approaches zero (because then the exponential term approaches one), in which case the contract approaches one with provision for just two levels of payment, $\underline{P}$ and $P(b(\bar{s}))$, the former of which is never actually paid because it applies only to returns that do not occur. That is like a fixed salary $P(b(\bar{s}))$, with $\underline{P}$ corresponding to the threat of being fired for a return that cannot occur if the agent uses the decision rule $b(.)$. But that limit can never actually be reached; for $\pi_{b_+}\left(b; b^{-1}(b)\right) = 0$, it would not be optimal for the principal to implement $b(.)$ because it would always be profitable to accept a small risk of an unsuccessful outcome in order to relax the limited liability constraint.

It is instructive to compare the result in Proposition 4 with results in three papers in which optimal contract forms are derived explicitly for a risk-neutral agent with limited liability, Diamond (1998), Innes (1990), and Biais and Casamatta (1999). Diamond (1998) proves the near-linearity result described at the end of Section 3.2: if the only contracts that induce the agent to select an efficient decision rule are linear, an optimal contract converges to linearity as the ratio of the return to the principal (gross of the payment to the agent) to the cost of actions $a > \underline{a}$ increases. Proposition 4 illustrates that this near-linearity result does not apply more generally. Although a linear contract would implement an efficient decision rule $b^*(.)$, the non-linear contract in (46) does so at lower cost to the principal, no matter what the magnitude of the agent's disutility of action $\bar{a}$ (conditional, of course, on it being worthwhile to employ the agent to take that action).

In his Proposition 1, Innes (1990) shows that a debt contract is optimal when the monotone likelihood ratio property holds and it is required that the principal's (as well as the agent's) reward is monotonic in the return. In the notation used here, a debt contract has $P(b) = \underline{P}$ (and hence $P'(b) = 0$) for $b < \hat{b}$ and $P'(b) = 1$ for $b > \hat{b}$ for some $\hat{b}$. The contract in (46) clearly does not correspond to that. The reason for the difference lies in the nature of the constraint on how fast $P(b)$ can increase. A contract monotonic for the principal implies that the reward to the agent cannot increase faster than the realised return, that is, $P'(b) \leq 1$. Thus the constraint $P'(b) \leq 1$ replaces the constraint (44). As discussed in connection with Proposition 4, under the monotone likelihood ratio property, payment to the agent is reduced while maintaining incentives for action by transferring rewards from lower values of b to higher values, which results in the constraint $P'(b) \leq 1$ becoming binding. As long as limited liability is not so tight as to make that constraint bind for all b, a debt contract, with a flat section for low b, is optimal. Thus, although the contracts differ, the underlying rationale for a debt contract in Innes (1990) is similar to that for the contract in Proposition 4. The difference is that the constraint (44) is not simply imposed but is derived from the need to make the bidding rule incentive compatible for the agent.

In Biais and Casamatta (1999), there is only a single decision that is ever worthwhile to the principal, no matter what signal the agent receives. Thus there is no issue of trading off a different decision for a lower expected payment to the agent. They show that an optimal contract can be implemented in terms of three instruments, debt, equity and share options. Thus their result, like that in Proposition 4, serves to emphasize that a debt contract is not in general optimal when a risk-neutral agent makes a decision as well as choosing how hard to work. But with only three possible outcomes in their model, it is not surprising that just three instruments can implement an optimal contract. It remains to be seen what standard instruments (if any) can do so when, as in the model studied here, the optimal decision for the principal depends on the signal privately received and there is a continuum of possible outcomes.

5 Concluding remarks

This paper has been concerned with a principal-agent problem in which the agent's action reveals information that is not itself verifiable but is used by the agent to make a decision on which the return is verifiable. It has used a formulation more general than those in the literature. In such settings, the agent has to be induced not only to take action to acquire information but also to make an appropriate decision given that information. The analysis has focused on two issues: (1) the effect on the decision made and (2) the effect on the contract between principal and agent.

On the second issue, the paper gives conditions for which inducing the agent to adopt a decision rule that is first-order or second-order stochastically dominant imposes no substantive restriction on a contract that is optimal for inducing the agent to take the optimal action to acquire the information. On the first, it is shown that the decision is distorted in a direction that reduces the risk borne by a risk-averse agent in a sense that is made precise. It differs from standard definitions of reducing risk because it depends not only on the risk characteristics of the decisions themselves but also on the contract with the principal but, for certain contract forms, is satisfied by second-order stochastic dominance. The paper uses an application to bidding to supply a good or service to show how those results can be applied directly. That application has also been used to derive additional results. If the principal wishes the agent to choose the highest bid that will always be successful or if payment to the agent can depend only on the return to the principal, then an optimal contract is monotone — payment to the agent is a non-decreasing function of the return to the principal from the decision — but it may have flat sections like rewards to managers that take the form of stock options. For a risk-neutral agent with binding limited liability, the precise form of the optimal contract is derived under certain conditions. It is not in general the debt contract that Innes (1990) found to be an optimal monotone contract. Nor is it the combination of debt, equity and share options that Biais and Casamatta (1999) found to be optimal in a model in which the agent makes decisions based on information acquired by action but with a simpler decision framework and fewer outcomes than the model used here. It does not even have the property discussed by Diamond (1998) of approaching a linear contract as the ratio of the principal's payoff to the cost of action increases. Moreover, when the probability of a bid's success lies strictly between zero and one, the optimal bid by a risk-neutral manager under limited liability may be distorted in the direction of either lower, less risky bids or higher, more risky bids, depending on which relaxes the limited liability constraint. Thus limited liability for managers may induce firms to make either more risky or less risky decisions.

There are numerous examples of agency in which the agent takes action to acquire information that is subsequently used to make decisions. Much managerial activity takes this form: investigating profitability before deciding how much to invest or which project to undertake, making bids for the supply or purchase of goods and services after investigating the probability that a given bid will be successful, making portfolio decisions after acquiring information about stocks, and so on. Agency relationships of this type are a fundamental part of economic life. The results derived here throw light on the implications of these types of relationships.

Appendix A Bidding application with Assumption 1

A.1 Agent and principal payoffs

The agent's expected utility from bidding rule $b(s)$ *given* signal s under contract $P(.)$ is

$$\hat{u}(b(s), s, P(.)) = u(P(b(s)))\,\pi(b(s); s) + u(P(0))\,[1 - \pi(b(s); s)]. \tag{47}$$

For notational convenience, let $h(b, s; a) = \pi(b; s)\, f(s; a)$ for all (b, s, a). Note, for future reference that, since $\pi(0; s) = 1$ for all s, $h(0, s; a) = f(s; a)$ and

$$\frac{h_b(b, s; a)}{h(b, s; a)} = \frac{\pi_b(b; s)}{\pi(b; s)}, \text{ for all } (b, s, a). \tag{48}$$

The expected utility *before* s is revealed from bidding rule $b(.)$ for given a is

$$\begin{aligned} U(a, b(.), P(.)) &= \int_{\underline{s}}^{\bar{s}} \hat{u}(s, b(s), P(.)) f(s; a)\, ds - v(a) \qquad (49) \\ &= \int_{\underline{s}}^{\bar{s}} \Big\{ u(P(b(s)))\, \pi(b(s); s) \\ &\quad + u(P(0)) [1 - \pi(b(s); s)] \Big\} f(s; a)\, ds - v(a) \\ &= \int_{\underline{s}}^{\bar{s}} \Big\{ u(P(b(s)))\, h(b(s), s; a) + u(P(0)) [h(0, s; a) \\ &\quad - h(b(s), s; a)] \Big\} ds - v(a), \qquad (50) \end{aligned}$$

the final equality following from the definition of $h(.)$.

The principal's payoff conditional on a, $b(.)$ and the contract $P(.)$ is

$$\begin{aligned} R(a, b(.), P(.)) &= \int_{\underline{s}}^{\bar{s}} \Big\{ [b(s) - P(b(s))]\, \pi(b(s); s) - P(0) [1 - \pi(b(s); s)] \Big\} f(s; a)\, ds \\ &= \int_{\underline{s}}^{\bar{s}} \Big\{ [b(s) - P(b(s))]\, h(b(s), s; a) \\ &\quad - P(0) [h(0, s; a) - h(b(s), s; a)] \Big\} ds, \end{aligned}$$

with again the final equality following from the definition of $h(.)$.

A.2 The principal's problem of contract choice

Under the Assumption 1, the optimal contract problem of Section 2.3 can be written

$$\begin{aligned} \max_{b(.), P(.)} R(\bar{a}, b(.), P(.)) & \qquad \text{subject to} \\ U(\bar{a}, b(.), P(.)) &\geq \underline{U}; \\ U(\bar{a}, b(.), P(.)) &\geq U(\underline{a}, b(.), P(.)); \qquad (51) \\ b(s) = \arg\max_{b} u(P(b))\, \pi(b; s) & \quad + u(P(0)) [1 - \pi(b; s)] \text{ for all } s \in [\underline{s}, \bar{s}]; \\ P(b) &\geq \underline{P}, \text{ for all } b \in \left[0, \bar{b}\right]. \end{aligned}$$

By Part 2 of Lemma 2, the first-order condition (27) and a condition on $b(s)$ (that $b(s)$ is non-decreasing if $\pi_b(b; s) / \pi(b; s)$ is strictly increasing in s or non-increasing if $\pi_b(b; s) / \pi(b; s)$ is strictly decreasing in s) are necessary and sufficient to induce the agent to select $b(s)$ for all s. Imposing the conditions (27) and $b(s)$ non-decreasing or non-increasing (as appropriate) on the principal's optimization problem ensures, therefore, that the first-order approach is valid. To implement the first-order approach, the constraints (27) and $b'(s) \geq 0$ or $b'(s) \leq 0$ for all $s \in [\underline{s}, \bar{s}]$ replace the arg max constraint in (51).

One complication in the principal's problem (51) is that $P(.)$ is a function of $b(.)$ which is in turn a function of the variable of integration s. One strategy for handling that is the following. Define the variables $\zeta(s)$ and $\chi(s)$ by

$$\zeta(s) = P(b(s)), \quad s \in [\underline{s}, \bar{s}], \tag{52}$$

$$\chi(s) = b'(s), \quad s \in [\underline{s}, \bar{s}], \tag{53}$$

so that

$$\zeta'(s) = P'(b(s))\, b'(s). \tag{54}$$

Choosing $\zeta(s)$ and $b(s)$ optimally will determine $P(b)$ for those values of b chosen for some s but not for those values of b not chosen for any s. Part 1 of Lemma 2 showed that $b = 0$ can never be implemented. $P(0)$ plays a central role in the first-order condition (27), so an optimal contract must specify a value for it. For other values of b not to be implemented for any s, it is sufficient to set $P(b) = P(0)$. For the optimization, this issue can be handled by introducing a parameter P_0 specified in the contract and setting $P(b) = P_0$ for any b (including $b = 0$) that is not to be implemented for any s. With this notation, the first-order condition (27) can be written

$$\begin{aligned} \zeta'(s) &= -\chi(s) \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))} \frac{\pi_b(b(s); s)}{\pi(b(s); s)} \\ &= -\chi(s) \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))} \frac{h_b(b(s), s; \bar{a})}{h(b(s), s; \bar{a})}, \end{aligned} \tag{55}$$

the second equality following from (48). The condition $b'(s) \geq (\leq)\, 0$ can, in this notation, be written

$$\chi(s) \geq (\leq)\, 0, \text{ for all } s \in [\underline{s}, \bar{s}]. \tag{56}$$

Imposing the condition $P_0 \geq \underline{P}$ is sufficient to ensure $\zeta(s) \geq \underline{P}$ for all s because, by Part 1 of Lemma 2, the first-order condition ensures b will be selected for s only if $P(b) > P_0$.

In writing the remaining constraints in terms of $\zeta(s)$, rather than $P(b)$, it is convenient to define the following function for the agent's expected utility:

$$\begin{aligned} Z(a, b(.), \zeta(.), P_0) \equiv \int_{\underline{s}}^{\bar{s}} \Big\{ & u(\zeta(s))\, h(b(s), s; a) + u(P_0) [h(0, s; a) \\ & - h(b(s), s; a)] \Big\} ds - v(a). \end{aligned} \tag{57}$$

The principal's problem (51) can then be written

$$\max_{b(.),\zeta(.),\chi(.),P_0} \int_{\underline{s}}^{\overline{s}} \left\{ \left[b\left(s\right) - \zeta\left(s\right) \right] h\left(b\left(s\right), s; \overline{a}\right) \; - P_0 \left[h\left(0, s; \overline{a}\right) - h\left(b\left(s\right), s; \overline{a}\right) \right] \right\} ds \tag{58}$$

subject to the dynamic constraints

$$\zeta'\left(s\right) = -\chi\left(s\right) \frac{u\left(\zeta\left(s\right)\right) - u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)} \frac{h_b\left(b\left(s\right), s; \overline{a}\right)}{h\left(b\left(s\right), s; \overline{a}\right)}, \text{ for all } s \in \left[\underline{s}, \overline{s}\right]; \tag{59}$$

$$b'\left(s\right) = \chi\left(s\right), \text{ for all } s \in \left[\underline{s}, \overline{s}\right]; \tag{60}$$

and the inequality constraints

$$Z\left(\overline{a}, b\left(.\right), \zeta\left(.\right), P_0\right) - \underline{U} \geq 0; \tag{61}$$

$$Z\left(\overline{a}, b\left(.\right), \zeta\left(.\right), P_0\right) - Z\left(\underline{a}, b\left(.\right), \zeta\left(.\right), P_0\right) \geq 0; \tag{62}$$

$$P_0 - \underline{P} \geq 0; \tag{63}$$

$$\text{for } \frac{h_b\left(b, s; \overline{a}\right)}{h\left(b, s; \overline{a}\right)} \text{ strictly increasing in } s: \quad \chi\left(s\right) \geq 0, \text{ for all } s \in \left[\underline{s}, \overline{s}\right]; \tag{64}$$

$$\text{for } \frac{h_b\left(b, s; \overline{a}\right)}{h\left(b, s; \overline{a}\right)} \text{ strictly decreasing in } s: \quad \chi\left(s\right) \leq 0, \text{ for all } s \in \left[\underline{s}, \overline{s}\right]; \tag{65}$$

with free boundaries at $s = \underline{s}$ and $s = \overline{s}$.

A.3 Hamiltonian and first-order conditions

To solve this problem, define the Hamiltonian in the standard way:

$$\begin{aligned} &H\left(\overline{a}, b\left(s\right), \zeta\left(s\right), P_0, s\right) \\ &= \left[b\left(s\right) - \zeta\left(s\right) + P_0\right] h\left(b\left(s\right), s; \overline{a}\right) - P_0 h\left(0, s; \overline{a}\right) \\ &- \psi_1\left(s\right) \chi\left(s\right) \frac{u\left(\zeta\left(s\right)\right) - u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)} \frac{h_b\left(b\left(s\right), s; \overline{a}\right)}{h\left(b\left(s\right), s; \overline{a}\right)} + \psi_2\left(s\right) \chi\left(s\right), \end{aligned} \tag{66}$$

where $\psi_1\left(s\right)$ and $\psi_2\left(s\right)$ are multipliers attached to the constraints (59) and (60) respectively. Then, provided the appropriate constraint qualification is satisfied, the Maximum Principle conditions, in which λ, μ, ϕ and $\nu\left(s\right)$ are multipliers attached in this order to the inequality constraints (61)–(64), are:

$$
\begin{aligned}
\psi_1'(s) &= -\frac{\partial H(\overline{a},.)}{\partial \zeta(s)} - \lambda \frac{\partial Z(\overline{a},.)}{\partial \zeta(s)} - \mu \left[\frac{\partial Z(\overline{a},.)}{\partial \zeta(s)} - \frac{\partial Z(\underline{a},.)}{\partial \zeta(s)} \right] \\
&= h(b(s),s;\overline{a}) - \lambda u'(\zeta(s)) h(b(s),s;\overline{a}) \\
&\quad -\mu u'(\zeta(s)) [h(b(s),s;\overline{a}) - h(b(s),s;\underline{a})] \\
&\quad +\psi_1(s)\chi(s) \frac{u'(\zeta(s))^2 - [u(\zeta(s)) - u(P_0)] u''(\zeta(s))}{u'(\zeta(s))^2} \frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}; \qquad (67) \\
\psi_2'(s) &= -\frac{\partial H(\overline{a}.)}{\partial b(s)} - \lambda \frac{\partial Z(\overline{a},.)}{\partial b(s)} - \mu \left[\frac{\partial Z(\overline{a},.)}{\partial b(s)} - \frac{\partial Z(\underline{a},.)}{\partial b(s)} \right] \\
&= -h(b(s),s;\overline{a}) - [b(s) - \zeta(s) + P_0] h_b(b(s),s;\overline{a}) \\
&\quad -\lambda [u(\zeta(s)) - u(P_0)] h_b(b(s),s;\overline{a}) \qquad (68) \\
&\quad -\mu [u(\zeta(s)) - u(P_0)] [h_b(b(s),s;\overline{a}) - h_b(b(s),s;\underline{a})] \\
&\quad +\psi_1(s)\chi(s) \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))} \frac{h(b(s),s;\overline{a}) h_{bb}(b(s),s;\overline{a}) - h_b(b(s),s;\overline{a})^2}{h(b(s),s;\overline{a})^2}; \\
0 &= \frac{\partial H(\overline{a},.)}{\partial \chi(s)} + \lambda \frac{\partial Z(\overline{a},.)}{\partial \chi(s)} + \mu \left[\frac{\partial Z(\overline{a},.)}{\partial \chi(s)} - \frac{\partial Z(\underline{a},.)}{\partial \chi(s)} \right] \\
&= -\psi_1(s) \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))} \frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} + \psi_2(s) + \nu(s); \qquad (69) \\
0 &= \frac{\partial}{\partial P_0} \int_{\underline{s}}^{\overline{s}} H(\overline{a},.)\, ds + \lambda \frac{\partial Z(\overline{a},.)}{\partial P_0} + \mu \left[\frac{\partial Z(\overline{a},.)}{\partial P_0} - \frac{\partial Z(\underline{a},.)}{\partial P_0} \right] + \phi \\
&= \int_{\underline{s}}^{\overline{s}} \Bigg\{ -[h(0,s;\overline{e}) - h(b(s),s;\overline{e})] + \lambda u'(P_0)[h(0,s;\overline{a}) - h(b(s),s;\overline{a})] \\
&\quad +\mu u'(P_0) \{[h(0,s;\overline{a}) - h(b(s),s;\overline{a})] - [h(0,s;\underline{a}) - h(b(s),s;\underline{a})]\} \\
&\quad +\psi_1(s)\chi(s) \frac{u'(P_0)}{u'(\zeta(s))} \frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} \Bigg\} ds + \phi; \qquad (70)
\end{aligned}
$$

plus the original equality constraints

$$
\begin{aligned}
\zeta'(s) &= -\chi(s) \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))} \frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}; \qquad (71) \\
b'(s) &= \chi(s); \qquad (72)
\end{aligned}
$$

the complementary slackness conditions

$$
\begin{aligned}
Z(\overline{a},.)-\underline{U}\geq 0; &\qquad \lambda\geq 0; \qquad \lambda\left[Z(\overline{a},.)-\underline{U}\right]=0; &(73)\\
Z(\overline{a},.)-Z(\underline{a},.)\geq 0; &\qquad \mu\geq 0; \qquad \mu\left[Z(\overline{a},.)-Z(\underline{a},.)\right]=0; &(74)\\
P_0-\underline{P}\geq 0; &\qquad \phi\geq 0; \qquad \phi\left[P_0-\underline{P}\right]=0; &(75)
\end{aligned}
$$

$$
\text{for } \frac{h_b(b,s;\overline{a})}{h(b,s;\overline{a})} \text{ strictly increasing in } s \ :
$$

$$
\chi(s)\geq 0; \qquad \nu(s)\geq 0; \qquad \nu(s)\chi(s)=0,\ \text{for all } s\in[\underline{s},\overline{s}]; \tag{76}
$$

$$
\text{for } \frac{h_b(b,s;\overline{a})}{h(b,s;\overline{a})} \text{ strictly decreasing in } s \ :
$$

$$
\chi(s)\leq 0; \qquad \nu(s)\leq 0; \qquad \nu(s)\chi(s)=0,\ \text{for all } s\in[\underline{s},\overline{s}]; \tag{77}
$$

and the boundary conditions

$$
\psi_1(\underline{s}) \;=\; \psi_2(\underline{s})=0; \tag{78}
$$

$$
\psi_1(\overline{s}) \;=\; \psi_2(\overline{s})=0. \tag{79}
$$

It is convenient to re-arrange some of these conditions in order to derive some preliminary results before proving the propositions in the main text. First, (67) and (68) can be re-arranged respectively as

$$
\begin{aligned}
&\frac{1}{u'(\zeta(s))}\left[1-\frac{\psi_1'(s)}{h(b(s),s;\overline{a})}\right]=\lambda+\mu\left(1-\frac{f(s;\underline{a})}{f(s;\overline{a})}\right) &(80)\\
&\qquad -\frac{\psi_1(s)\chi(s)}{h(b(s),s;\overline{a})}\frac{1}{u'(\zeta(s))}\left[1-\frac{\left[u(\zeta(s))-u(P_0)\right]u''(\zeta(s))}{u'(\zeta(s))^2}\right]\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})};\\
&\frac{h(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}+\frac{\psi_2'(s)}{h_b(b(s),s;\overline{a})} &(81)\\
&\qquad =-\left[b(s)-\zeta(s)+P_0\right]-\left[\lambda+\mu\left(1-\frac{f(s;\underline{a})}{f(s;\overline{a})}\right)\right]\left[u(\zeta(s))-u(P_0)\right]\\
&\qquad\qquad +\frac{\psi_1(s)\chi(s)}{h(b(s),s;\overline{a})}\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\left[\frac{h_{bb}(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}-\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right].
\end{aligned}
$$

Second, note that $\int_{\underline{s}}^{\overline{s}} h(0,s;a)\,ds=1$ for all a because $h(0,s;a)=f(s;a)$ by definition. Use of this in (70) allows that condition to be re-arranged as

$$
\begin{aligned}
\left[1-\int_{\underline{s}}^{\overline{s}} h(b(s),s;\overline{a})\,ds\right] \;=\;& \lambda u'(P_0)\left[1-\int_{\underline{s}}^{\overline{s}} h(b(s),s;\overline{a})\,ds\right]\\
&-\mu u'(P_0)\left[\int_{\underline{s}}^{\overline{s}} h(b(s),s;\overline{a})\,ds-\int_{\underline{s}}^{\overline{s}} h(b(s),s;\underline{a})\,ds\right]\\
&+\int_{\underline{s}}^{\overline{s}} \psi_1(s)\chi(s)\frac{u'(P_0)}{u'(\zeta(s))}\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}ds+\phi. \qquad (82)
\end{aligned}
$$

Lemma 3 *(69), (71), (72), (80) and (81) imply*

$$\pi_b\left(b\left(s\right);s\right)f\left(s;\overline{a}\right)\left\{-\left[\frac{\pi\left(b\left(s\right);s\right)}{\pi_b\left(b\left(s\right);s\right)}+b\left(s\right)\right]+\left[\zeta\left(s\right)-P_0-\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\right]\right\}$$
$$+\nu'\left(s\right)=\psi_1\left(s\right)\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\frac{\partial}{\partial s}\left(\frac{\pi_b\left(b\left(s\right);s\right)}{\pi\left(b\left(s\right);s\right)}\right),\ \textit{for all } s\in\left[\underline{s},\overline{s}\right]. \tag{83}$$

Proof. Add $\left[u\left(\zeta\left(s\right)\right)-u\left(P_0\right)\right]$ times (80) to (81) noting that, from Lemma 2, $\zeta\left(s\right)>P_0$ for all s and from (72) that $\chi\left(s\right)=b'\left(s\right)$, to get

$$\begin{aligned}
&\frac{\left[u\left(\zeta\left(s\right)\right)-u\left(P_0\right)\right]}{u'\left(\zeta\left(s\right)\right)}\left[1-\frac{\psi_1'\left(s\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\right]+\frac{h\left(b\left(s\right),s;\overline{a}\right)}{h_b\left(b\left(s\right),s;\overline{a}\right)}+\frac{\psi_2'\left(s\right)}{h_b\left(b\left(s\right),s;\overline{a}\right)}\\
&=-\left[b\left(s\right)-\zeta\left(s\right)+P_0\right.\\
&+\frac{\psi_1\left(s\right)b'\left(s\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\left\{\frac{h_{bb}\left(b\left(s\right),s;\overline{a}\right)}{h_b\left(b\left(s\right),s;\overline{a}\right)}-\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\right.\\
&\left.\left.-\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\left[1-\frac{\left[u\left(\zeta\left(s\right)\right)-u\left(P_0\right)\right]u''\left(\zeta\left(s\right)\right)}{u'\left(\zeta\left(s\right)\right)^2}\right]\right\}\right]\\
&=-\left[b\left(s\right)-\zeta\left(s\right)+P_0\right.\\
&+\psi_1\left(s\right)\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\frac{b'\left(s\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\left\{\frac{h_{bb}\left(b\left(s\right),s;\overline{a}\right)}{h_b\left(b\left(s\right),s;\overline{a}\right)}\right.\\
&\left.\left.+\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\left[\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\frac{u''\left(\zeta\left(s\right)\right)}{u'\left(\zeta\left(s\right)\right)}-2\right]\right\}\right].
\end{aligned} \tag{84}$$

From (69), it follows that

$$\psi_2\left(s\right)=\psi_1\left(s\right)\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}-\nu\left(s\right) \tag{85}$$

so, differentiating with respect to s,

$$\begin{aligned}
\psi_2'\left(s\right)&=\psi_1'\left(s\right)\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}-\nu'\left(s\right)\\
&+\psi_1\left(s\right)\left\{\frac{u'\left(\zeta\left(s\right)\right)^2-\left[u\left(\zeta\left(s\right)\right)-u\left(P_0\right)\right]u''\left(\zeta\left(s\right)\right)}{u'\left(\zeta\left(s\right)\right)^2}\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\zeta'\left(s\right)\right.\\
&+\frac{u\left(\zeta\left(s\right)\right)-u\left(P_0\right)}{u'\left(\zeta\left(s\right)\right)}\left[\frac{h\left(b\left(s\right),s;\overline{a}\right)h_{bb}\left(b\left(s\right),s;\overline{a}\right)-h_b\left(b\left(s\right),s;\overline{a}\right)^2}{h\left(b\left(s\right),s;\overline{a}\right)^2}b'\left(s\right)\right.\\
&\left.\left.+\frac{\partial}{\partial s}\left(\frac{h_b\left(b\left(s\right),s;\overline{a}\right)}{h\left(b\left(s\right),s;\overline{a}\right)}\right)\right]\right\}.
\end{aligned} \tag{86}$$

The expression for $\psi_2'(s)$ in (86) can be re-arranged to give

$$\begin{aligned}
\psi_2'(s) &= \psi_1'(s)\,\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} - \nu'(s) \\
&\quad + \psi_1(s)\left\{\left[1 - \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))}\right]\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\zeta'(s)\right. \\
&\quad + \frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\left[\left(\frac{h_{bb}(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} - \frac{h_b(b(s),s;\overline{a})^2}{h(b(s),s;\overline{a})^2}\right)b'(s)\right. \\
&\quad \left.\left. + \frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)\right]\right\}. \qquad (87)
\end{aligned}$$

Substitution for $\zeta'(s)$ from (71) and use of (72) gives

$$\begin{aligned}
\psi_2'(s) &= \psi_1'(s)\,\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} - \nu'(s) \\
&\quad + \psi_1(s)\,\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\left\{\left[\left(\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))} - 1\right)\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)^2\right.\right. \\
&\quad \left.\left. + \left(\frac{h_{bb}(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} - \frac{h_b(b(s),s;\overline{a})^2}{h(b(s),s;\overline{a})^2}\right)\right]b'(s) + \frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)\right\} \\
&= \psi_1'(s)\,\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})} - \nu'(s) \\
&\quad + \psi_1(s)\,\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\left\{\left[\left(\frac{u(\zeta(s)) - u(P_0)}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))} - 2\right)\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)^2\right.\right. \\
&\quad \left.\left. + \frac{h_{bb}(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right]b'(s) + \frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)\right\}.
\end{aligned}$$

Use of this in (84) gives

$$
\begin{aligned}
&\frac{[u(\zeta(s))-u(P_0)]}{u'(\zeta(s))}\left[1-\frac{\psi_1'(s)}{h(b(s),s;\overline{a})}\right]+\frac{h(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}+\frac{\psi_1'(s)}{h(b(s),s;\overline{a})}\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\\
&\quad-\frac{v'(s)}{h_b(b(s),s;\overline{a})}=-\left[b(s)-\zeta(s)+P_0\right]\\
&\qquad+\psi_1(s)\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\left\{\frac{b'(s)}{h(b(s),s;\overline{a})}\left[\frac{h_{bb}(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}\right.\right.\\
&\qquad\left.+\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\left(\frac{[u(\zeta(s))-u(P_0)]}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))}-2\right)\right]\\
&\qquad-\frac{1}{h_b(b(s),s;\overline{a})}\left[\left[\left(\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))}-2\right)\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)^2\right.\right.\\
&\qquad\left.\left.\left.+\frac{h_{bb}(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right]b'(s)+\frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)\right]\right\}.
\end{aligned}
$$

or, re-arranging and cancelling terms,

$$
\begin{aligned}
&\frac{[u(\zeta(s))-u(P_0)]}{u'(\zeta(s))}+\frac{h(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}-\frac{v'(s)}{h_b(b(s),s;\overline{a})}\\
&\quad=-\left[b(s)-\zeta(s)+P_0\right]\\
&\quad+\frac{\psi_1(s)b'(s)}{h(b(s),s;\overline{a})}\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\left\{\frac{h_{bb}(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}\right.\\
&\quad+\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\left[\frac{[u(\zeta(s))-u(P_0)]}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))}-2\right]\\
&\quad-\left[\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\frac{u''(\zeta(s))}{u'(\zeta(s))}-2\right]\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\\
&\quad\left.-\frac{h_{bb}(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}\right\}-\frac{\psi_1(s)}{h_b(b(s),s;\overline{a})}\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right)
\end{aligned}
$$

from which all the terms in $\psi_1(s)\,b'(s)$ cancel to give

$$
\begin{aligned}
&\frac{[u(\zeta(s))-u(P_0)]}{u'(\zeta(s))}+\frac{h(b(s),s;\overline{a})}{h_b(b(s),s;\overline{a})}-\frac{v'(s)}{h_b(b(s),s;\overline{a})}\\
&=-\left[b(s)-\zeta(s)+P_0\right]-\frac{\psi_1(s)}{h_b(b(s),s;\overline{a})}\frac{u(\zeta(s))-u(P_0)}{u'(\zeta(s))}\frac{\partial}{\partial s}\left(\frac{h_b(b(s),s;\overline{a})}{h(b(s),s;\overline{a})}\right).
\end{aligned}
$$

In view of (48), this can be re-arranged to give (83). ■

Lemma 4 *For the bidding application of Section 2.2, suppose Assumption 1 holds. Then, provided the constraint $b'(s)\geq 0$ ($b'(s)\leq 0$) is not binding in the neighbourhoods of $\underline{s},\overline{s}$, an* *optimal contract implements $b(s)$ with the property $b(s) < b^*(s)$ for $s = \underline{s}, \overline{s}$ for a risk-averse agent and $b(s) = b^*(s)$ for $s = \underline{s}, \overline{s}$ for a risk-neutral agent.*

Proof. Any solution for $b(s)$ must satisfy the the Maximum Principle conditions (67)-(79), so Lemma 3 applies. By hypothesis, $\frac{\partial}{\partial s}(\pi_b(b(s);s)/\pi(b(s);s))$ is either strictly positive for all s or strictly negative for all s and, by Lemma 2, $\zeta(s)\,[\equiv P(b(s))] > P_0$ for all s. Thus, the right-hand side of (83) in Lemma 3 equals zero if and only if $\psi_1(s) = 0$. It follows from (78) that $\psi_1(\underline{s}) = 0$, so the right-hand side of (83) equals zero for $s = \underline{s}$. It follows from (76) that, provided the constraint on $b'(\underline{s})\,[\equiv \chi(\underline{s})]$ is not binding in the neighbourhood of $\underline{s}$, $\nu(s) = 0$ in that neighbourhood and thus $\nu'(\underline{s}) = 0$ also. A corresponding argument applies for $s = \overline{s}$. Since $\pi_b(b(s);s) = 0$ is ruled out by Assumption 1 and Lemma 2, it must be that, for both $\underline{s}$ and $\overline{s}$, the term in braces on the left-hand side of (83) equals zero.

For a risk-averse agent with $\zeta(s) > P_0$ for all s,

$$\zeta(s) - P_0 < [u(\zeta(s)) - u(P_0)]\,/u'(\zeta(s))$$

by strict concavity of $u(.)$, so the term in the second square bracket on the left-hand side of (83) is negative. Thus it must be the case that an optimal $b(s)$ satisfies

$$\frac{\pi(b(s);s)}{\pi_b(b(s);s)} + b(s) < 0 \text{ for } s = \underline{s}, \overline{s}. \tag{88}$$

For $b(s) = b^*(s)$ and $0 < \pi(b^*(s);s) < 1$, it follows from (25) that the left-hand side of (88) would have to be zero. But $\pi_b(b;s)/\pi(b;s)$ is non-increasing in b, so the left-hand side of (88) is increasing in $b(s)$. It follows from (25) that $b(s) < b^*(s)$ for $s = \underline{s}, \overline{s}$.

For a risk-neutral agent

$$\zeta(s) - P_0 = [u(\zeta(s)) - u(P_0)]\,/u'(\zeta(s)),$$

so both terms in square brackets on the left-hand side of (83) are zero for $s = \underline{s}, \overline{s}$. It then follows from (25) that $b(s) = b^*(s)$ for $s = \underline{s}, \overline{s}$. ∎

A.4 Proofs

Proof of Proposition 2. Any solution for $b(s)$ must satisfy the the Maximum Principle conditions (67)-(79), so Lemma 3 applies. Integration of (67) over $s \in [\underline{s}, \overline{s}]$ gives

$$\begin{aligned}
\int_{\underline{s}}^{\overline{s}} \psi_1'(s)\, ds &= \int_{\underline{s}}^{\overline{s}} \Bigg\{ h(b(s), s; \overline{a}) - \lambda u'(\zeta(s))\, h(b(s), s; \overline{a}) \\
&\quad - \mu u'(\zeta(s)) \left[h(b(s), s; \overline{a}) - h(b(s), s; \underline{a}) \right] \\
&\quad + \psi_1(s)\, \chi(s) \frac{u'(\zeta(s))^2 - [u(\zeta(s)) - u(P_0)]\, u''(\zeta(s))}{u'(\zeta(s))^2} \frac{h_b(b(s), s; \overline{a})}{h(b(s), s; \overline{a})} \Bigg\} ds \\
&= \int_{\underline{s}}^{\overline{s}} h(b(s), s; \overline{a})\, ds - \lambda \int_{\underline{s}}^{\overline{s}} u'(\zeta(s))\, h(b(s), s; \overline{a})\, ds \\
&\quad - \mu \int_{\underline{s}}^{\overline{s}} u'(\zeta(s)) \left[h(b(s), s; \overline{a}) - h(b(s), s; \underline{a}) \right] ds \qquad (89) \\
&\quad + \int_{\underline{s}}^{\overline{s}} \left[\psi_1(s)\, \chi(s) \frac{u'(\zeta(s))^2 - [u(\zeta(s)) - u(P_0)]\, u''(\zeta(s))}{u'(\zeta(s))^2} \frac{h_b(b(s), s; \overline{a})}{h(b(s), s; \overline{a})} \right] ds.
\end{aligned}$$

Now $\int_{\underline{s}}^{\overline{s}} \psi_1'(s)\, ds = \psi_1(\overline{s}) - \psi_1(\underline{s})$ and, from (78) and (79), $\psi_1(\overline{s}) = \psi_1(\underline{s}) = 0$. Thus, (89) can be used to substitute for $\int_{\underline{s}}^{\overline{s}} h(b(s), s; \overline{a})\, ds$ in (82) to get

$$\begin{aligned}
&-1 + \lambda \int_{\underline{s}}^{\overline{s}} u'(\zeta(s))\, h(b(s), s; \overline{a})\, ds + \mu \int_{\underline{s}}^{\overline{s}} u'(\zeta(s)) \left[h(b(s), s; \overline{a}) - h(b(s), s; \underline{a}) \right] ds \\
&\quad - \int_{\underline{s}}^{\overline{s}} \left\{ \psi_1(s)\, \chi(s) \frac{u'(\zeta(s))^2 - [u(\zeta(s)) - u(P_0)]\, u''(\zeta(s))}{u'(\zeta(s))^2} \frac{h_b(b(s), s; \overline{a})}{h(b(s), s; \overline{a})} \right\} ds \\
&\qquad + \lambda u'(P_0) \left[1 - \int_{\underline{s}}^{\overline{s}} h(b(s), s; \overline{a})\, ds \right] - \mu u'(P_0) \left[\int_{\underline{s}}^{\overline{s}} h(b(s), s; \overline{a})\, ds \right. \\
&\qquad \left. - \int_{\underline{s}}^{\overline{s}} h(b(s), s; \underline{a})\, ds \right] + \int_{\underline{s}}^{\overline{s}} \psi_1(s)\, \chi(s) \frac{u'(P_0)}{u'(\zeta(s))} \frac{h_b(b(s), s; \overline{a})}{h(b(s), s; \overline{a})} ds + \phi = 0,
\end{aligned}$$

or

$$\begin{aligned}
&-1 + \lambda u'(P_0) \left\{ 1 + \int_{\underline{s}}^{\overline{s}} \left[\frac{u'(\zeta(s))}{u'(P_0)} - 1 \right] h(b(s), s; \overline{a})\, ds \right\} \\
&\qquad + \mu u'(P_0) \int_{\underline{s}}^{\overline{s}} \left[\frac{u'(\zeta(s))}{u'(P_0)} - 1 \right] \left[h(b(s), s; \overline{a}) - h(b(s), s; \underline{a}) \right] ds \\
&+ \int_{\underline{s}}^{\overline{s}} \psi_1(s)\, \chi(s) \frac{h_b(b(s), s; \overline{a})}{h(b(s), s; \overline{a})} \left[\frac{u'(P_0)}{u'(\zeta(s))} - 1 + \frac{[u(\zeta(s)) - u(P_0)]\, u''(\zeta(s))}{u'(\zeta(s))^2} \right] ds + \phi = 0.
\end{aligned}$$

For a risk-neutral agent with $u'(.)$ constant and $u''(.) = 0$, this reduces to

$$\lambda u'(P_0) = 1 - \phi. \qquad (90)$$

Consider first values of s for which $\psi_1(s) = 0$. Use of (90) in (80) for a risk-neutral agent (with the normalization $u'(.) = 1$) for any such s allows (80) to be written

$$-\frac{\psi_1'(s)}{h(b(s), s; \bar{a})} = -\phi + \mu \left[1 - \frac{f(s; \underline{a})}{f(s; \bar{a})}\right]. \tag{91}$$

Since s is ordered such that $f(s; \underline{a}) / f(s; \bar{a})$ is non-increasing in s, there exists $\hat{s}$ such that

$$\frac{f(s; \underline{a})}{f(s; \bar{a})} \geq 1 \text{ for } s \leq \hat{s}; \qquad \frac{f(s; \underline{a})}{f(s; \bar{a})} < 1 \text{ for } s > \hat{s}.$$

Note that $\phi, \mu > 0$ if the lower bound on P_0 is binding. Thus the right-hand side of (91) is negative for $s = \underline{s}$ and increasing in s. So, either the right-hand side of (91) is negative for all s or there exists an $\tilde{s} > \hat{s}$ such that it is negative for $s \leq \tilde{s}$ and positive for $s > \tilde{s}$. It follows either that, for any $s \leq \tilde{s}$ for which $\psi_1(s) = 0$, $\psi_1'(s) > 0$ and, for any $s > \tilde{s}$ for which $\psi_1(s) = 0$, $\psi_1'(s) < 0$, or that $\psi_1'(s) > 0$ for all s for which $\psi_1(s) = 0$. The second of these cannot, however, be the case because we know from (78) and (79) that $\psi_1(\underline{s}) = \psi_1(\bar{s}) = 0$. Thus, as s increases from $\underline{s}$, $\psi_1(s)$ becomes positive and cannot change sign because, to do so, $\psi_1'(s)$ would have to become negative and, since $\psi_1'(s)$ would remain negative for all higher s, $\psi_1(s)$ would not be able to satisfy $\psi_1(\bar{s}) = 0$. Thus $\psi_1(s) > 0$ for all $s \in (\underline{s}, \bar{s})$.

With a risk-neutral agent, the second square bracket on the left-hand side of (83) in Lemma 3 is always zero. When $\psi_1(s) > 0$ and $\zeta(s) > P_0$ for all $s \in (\underline{s}, \bar{s})$, the right-hand side of (83) is positive if $\pi_b(b; s) / \pi(b; s)$ is everywhere strictly increasing in s and negative if $\pi_b(b; s) / \pi(b; s)$ is everywhere strictly decreasing in s. The combined term multiplying the brace on the left-hand side is negative under Assumption 1. Thus, provided the constraint on $b'(s)$ $[\equiv \chi(s)]$ is not binding for any s, so that $\nu(s) = 0$ for all s from (76) and thus also $\nu'(s) = 0$ for all s, it must be that

$$\begin{aligned}
\frac{\pi(b(s); s)}{\pi_b(b(s); s)} + b(s) &> 0 \text{ for } s \in (\underline{s}, \bar{s}) \text{ if } \frac{\partial}{\partial s}\left(\frac{\pi_b(b; s)}{\pi(b; s)}\right) > 0 \text{ for all } s; \\
\frac{\pi(b(s); s)}{\pi_b(b(s); s)} + b(s) &< 0 \text{ for } s \in (\underline{s}, \bar{s}) \text{ if } \frac{\partial}{\partial s}\left(\frac{\pi_b(b; s)}{\pi(b; s)}\right) < 0 \text{ for all } s.
\end{aligned}$$

But $\pi_b(b; s) / \pi(b; s)$ is non-increasing in b, so the left-hand sides of these conditions are increasing in $b(s)$. By the argument in the proof of Lemma 4, It follows from (25) that $b(s) > b^*(s)$ for $s \in (\underline{s}, \bar{s})$ if $\pi_b(b; s) / \pi(b; s)$ is everywhere strictly increasing in s and $b(s) < b^*(s)$ for $s \in (\underline{s}, \bar{s})$ if $\pi_b(b; s) / \pi(b; s)$ is everywhere strictly decreasing in s.

That establishes the result for all $s \in (\underline{s}, \bar{s})$ provided the constraint on $b'(s)$ does not bind for any s (and hence $\nu(s) = \nu'(s) = 0$ for all s). To show that the result is unaffected by having that constraint bind for some s, start with the case $\pi_b(b; s) / \pi(b; s)$ everywhere strictly increasing in s in which that constraint takes the form $b'(s) \geq 0$.

Consider first $s = \bar{s}$. It follows from Lemma 4 that, if the constraint did not bind, $b(\bar{s}) = b^*(\bar{s})$. Thus the constraint can bind only if $b(\bar{s}-\varepsilon) > b^*(\bar{s})$ as $\varepsilon \to 0$ from above and the effect of the constraint cannot be to reduce $b(\bar{s})$. Thus certainly $b(\bar{s}) \geq b^*(\bar{s})$. Moreover, by Part 3 of Lemma 1, $b^*(s)$ is strictly increasing so, if $b(\bar{s}-\varepsilon) = b(\bar{s}) \geq b^*(\bar{s})$, it is certainly the case that $b(\bar{s}-\varepsilon) > b^*(\bar{s}-\varepsilon)$. In addition, for any s such that $b(s) > b^*(s)$, a similar argument implies $b(s-\varepsilon) > b^*(s-\varepsilon)$ as $\varepsilon \to 0$ from above even when the constraint binds. Thus $b(s) > b^*(s)$ for all $s \in (\underline{s}, \bar{s})$ whether or not the constraint binds. Finally, $b(\underline{s}+\varepsilon) > b^*(\underline{s}+\varepsilon)$ as $\varepsilon \to 0$ from above implies $b(\underline{s}+\varepsilon) > b^*(\underline{s})$, so having $b(\underline{s}) = b^*(\underline{s})$, as implied by Lemma 4 if the constraint does not bind, cannot result in the constraint binding. Thus $b(\underline{s}) = b^*(\underline{s})$. That completes the proof of Part 1 of the proposition.

Now consider the case $\pi_b(b;s)/\pi(b;s)$ everywhere strictly decreasing in s in which the constraint on $b'(s)$ takes the form $b'(s) \leq 0$. It follows from Lemma 4 that, if the constraint does not bind at $s = \bar{s}$, then $b(\bar{s}) = b^*(\bar{s})$. Thus the constraint can bind at $s = \bar{s}$ only if $b(\bar{s}-\varepsilon) < b^*(\bar{s})$ as $\varepsilon \to 0$ from above and the effect of the constraint cannot be to increase $b(\bar{s})$. Thus certainly $b(\bar{s}) \leq b^*(\bar{s})$. Moreover, by Part 3 of Lemma 1, $b^*(s)$ is strictly decreasing so, if $b(\bar{s}-\varepsilon) = b(\bar{s}) \leq b^*(\bar{s})$, it is certainly the case that $b(\bar{s}-\varepsilon) < b^*(\bar{s}-\varepsilon)$. In addition, for any s such that $b(s) < b^*(s)$, a similar argument implies $b(s-\varepsilon) < b^*(s-\varepsilon)$ as $\varepsilon \to 0$ from above even when the constraint binds. Thus $b(s) < b^*(s)$ for all $s \in (\underline{s}, \bar{s})$ whether or not the constraint binds. Finally, $b(\underline{s}+\varepsilon) < b^*(\underline{s}+\varepsilon)$ as $\varepsilon \to 0$ from above implies $b(\underline{s}+\varepsilon) < b^*(\underline{s})$, so having $b(\underline{s}) = b^*(\underline{s})$, as implied by Lemma 4 if the constraint does not bind, cannot result in the constraint binding. Thus $b(\underline{s}) = b^*(\underline{s})$. That completes the proof of Part 2. ■

Appendix B Bidding application with Assumption 2

This appendix shows that condition (44) is sufficient, as well as necessary, for the agent to bid $b(s)$ in the bidding application under Assumption 2 of Section 4.2.

Lemma 5 *Suppose $b(s)$ satisfies, for $s, s', s'' \in [\underline{s}, \bar{s}]$,*

$$u(\underline{P}) + \left[u\left(P\left(b\left(s'\right)\right)\right) - u(\underline{P})\right]\pi\left(b\left(s'\right);s'\right) \\ \geq u(\underline{P}) + \left[u\left(P\left(b\left(s''\right)\right)\right) - u(\underline{P})\right]\pi\left(b\left(s''\right);s'\right); \tag{92}$$

$$u(\underline{P}) + \left[u\left(P\left(b\left(s''\right)\right)\right) - u(\underline{P})\right]\pi\left(b\left(s''\right);s''\right) \\ \geq u(\underline{P}) + \left[u\left(P\left(b\left(s\right)\right)\right) - u(\underline{P})\right]\pi\left(b\left(s\right);s''\right). \tag{93}$$

Then, for $s \leq s'' \leq s'$ or $s' \leq s'' \leq s$,

$$u\left(\underline{P}\right) + \left[u\left(P\left(b\left(s'\right)\right)\right) - u\left(\underline{P}\right)\right]\pi\left(b\left(s'\right);s'\right) \geq u\left(\underline{P}\right) + \left[u\left(P\left(b\left(s\right)\right)\right) - u\left(\underline{P}\right)\right]\pi\left(b\left(s\right);s'\right) \tag{94}$$

if $b(s)$ and $\pi_b(b;s)/\pi(b;s)$ are either both non-decreasing in s or both non-increasing in s for all $s \in [\underline{s}, \bar{s}]$ and all $b \in \left[0, \bar{b}\right]$.

Proof. The two inequalities hypothesised in the lemma respectively imply

$$\begin{aligned}
\left[u\left(P\left(b\left(s'\right)\right)\right) - u\left(\underline{P}\right)\right]\pi\left(b\left(s'\right);s'\right) &\geq \left[u\left(P\left(b\left(s''\right)\right)\right) - u\left(\underline{P}\right)\right]\pi\left(b\left(s''\right);s'\right), \\
\left[u\left(P\left(b\left(s''\right)\right)\right) - u\left(\underline{P}\right)\right] &\geq \left[u\left(P\left(b\left(s\right)\right)\right) - u\left(\underline{P}\right)\right]\frac{\pi\left(b\left(s\right);s''\right)}{\pi\left(b\left(s''\right);s''\right)}.
\end{aligned}$$

Together these imply

$$\left[u\left(P\left(b\left(s'\right)\right)\right) - u\left(\underline{P}\right)\right]\pi\left(b\left(s'\right);s'\right) \geq \left[u\left(P\left(b\left(s\right)\right)\right) - u\left(\underline{P}\right)\right]\frac{\pi\left(b\left(s''\right);s'\right)}{\pi\left(b\left(s''\right);s''\right)}\pi\left(b\left(s\right);s''\right).$$

Thus the lemma certainly holds if

$$\frac{\pi\left(b\left(s''\right);s'\right)}{\pi\left(b\left(s''\right);s''\right)}\pi\left(b\left(s\right);s''\right) \geq \pi\left(b\left(s\right);s'\right), \text{ for } s \leq s'' \leq s' \text{ and } s' \leq s'' \leq s,$$

or, equivalently,

$$\frac{\pi\left(b\left(s''\right);s'\right)}{\pi\left(b\left(s''\right);s''\right)} \geq \frac{\pi\left(b\left(s\right);s'\right)}{\pi\left(b\left(s\right);s''\right)}, \text{ for } s \leq s'' \leq s' \text{ and } s' \leq s'' \leq s. \tag{95}$$

This clearly holds with equality for $s = s''$. But

$$\begin{aligned}
\frac{d}{ds}\left(\frac{\pi\left(b\left(s\right);s'\right)}{\pi\left(b\left(s\right);s''\right)}\right) &= \frac{b'\left(s\right)}{\pi\left(b\left(s\right);s''\right)^2}\left[\pi\left(b\left(s\right);s''\right)\pi_b\left(b\left(s\right);s'\right) - \pi\left(b\left(s\right);s'\right)\pi_b\left(b\left(s\right);s''\right)\right] \\
&= b'\left(s\right)\frac{\pi\left(b\left(s\right);s'\right)}{\pi\left(b\left(s\right);s''\right)}\left[\frac{\pi_b\left(b\left(s\right);s'\right)}{\pi\left(b\left(s\right);s'\right)} - \frac{\pi_b\left(b\left(s\right);s''\right)}{\pi\left(b\left(s\right);s''\right)}\right],
\end{aligned}$$

which is non-positive for $s' \leq s''$ and non-negative for $s'' \leq s'$ if $b(s)$ and $\pi_b(b;s)/\pi(b;s)$ are either both non-decreasing in s or both non-increasing in s for all $s \in [\underline{s}, \bar{s}]$ and all $b \in \left[0, \bar{b}\right]$. This implies (95). ■

An implication of this result is the following. Suppose $P(b)$ is set such that $P(b) = \underline{P}$ for any b that is not to be selected for any s, as a result of which no such b is chosen for any s. Then if, under the conditions given, $P(b)$ is such that the agent prefers $b(s')$ to $b(s'')$ given signal s' and $b(s'')$ to $b(s)$ given signal s'' for any signal s the opposite side of s'' from s', the agent also prefers $b(s')$ to $b(s)$ given signal s'. Letting s'' approach s' establishes that, under the conditions of Lemma 5, a local optimum is also a global optimum. As a result, (43) (or alternatively, (44)) is sufficient to ensure that no $b > b(s)$ is preferred to $b(s)$ when $b(s)$ and $\pi_b(b;s)/\pi(b;s)$ are either both non-decreasing in s or both non-increasing in s for all $s \in [\underline{s}, \bar{s}]$ and all $b \in \left[0, \bar{b}\right]$.

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