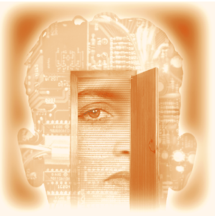
Principles and Applications of Chaotic Systems

William Ditto and Toshinori Munakata

This overview of the behavioral elements of chaos introduces a wide array of commercial applications to enhance, manipulate, or better control many current technical functions.

There lies a behavior between rigid regularity and randomness based on pure chance. It's called a *chaotic system,* or *chaos* for short [5]. Chaos is all around us. Our notions of physical motion or dynamic systems have encompassed the precise clock-like ticking of periodic systems and the vagaries of dice-throwing chance, but have often been overlooked as a way to account for the more commonly observed chaotic behavior between these two extremes. When we see irregularity we cling to randomness and disorder for explanations. Why should this be so? Why is it that when the ubiquitous irregularity of engineering, physical, biological, and other systems are studied, it is assumed to be random and the whole vast machinery of probability and statistics is applied? Rather recently, however, we have begun to realize that the tools of chaos theory can be applied toward the understanding, manipulation, and control of a variety of systems, with many of the practical applications coming after 1990. To understand why this is true, one must start with a working knowledge of how chaotic systems behave—profoundly, but sometimes subtly different, from the behavior of random systems.



As with many terms in science, there is no standard definition of chaos. The typical features of chaos include:

- *Nonlinearity.* If it is linear, it cannot be chaotic.
- *Determinism.* It has *deterministic* (rather than probabilistic) underlying rules every future state of the system must follow.
- *Sensitivity to initial conditions.* Small changes in its initial state can lead to radically different behavior in its final state. This "butterfly effect" allows the possibility that even the slight perturbation of a butterfly flapping its wings can

dramatically affect whether sunny or cloudy skies will predominate days later.

- Sustained *irregularity* in the behavior of the system. Hidden order including a large or infinite number of unstable periodic patterns (or motions). This hidden order forms the infrastruture of irregular chaotic systems—*order in disorder* for short.
- Long-term prediction (but not control!) is mostly impossible due to sensitivity to initial conditions, which can be known only to a finite degree of precision.

A simple example of a chaotic system in computer science is a pseudo-random number generator. The underlying rule in this case is a simple deterministic formula (e.g., $x_{n+1} = cx_n \bmod m$). However, the resulting solutions, such as the pseudo-random numbers are very irregular and unpredictable (the more unpredictable, the closer to random). We also note that a small change in the initial condition (seed) can yield a significantly different sequence of random numbers. These pseudo-random number generators are chaotic but also periodic with certain periods. Such generators viewed carefully yield the hidden order characteristic of chaos. (For more on fundamentals, see [1, 2, 14]).

While this random number generator is an artificial chaotic system, there are numerous chaotic systems in nature. While engineers typically shun chaos and irregularity, nature may indeed treasure and exploit it. For example, normal brain activity may be chaotic, and pathological *order* may indeed be the cause of such diseases as epilepsy. It has been speculated that too much periodicity in heart rates might indicate disease. Perhaps the chaotic characteristics of the human body are better adapted to its chaotic environment. It is even suspected that biological systems exploit chaos to store, encode, and decode information.

Table 1. Major Historical Developments in the Study of Chaos

1890	King Oscar II of Sweden. Announced a prize for the first person who could solve the n-body problem to determine the orbits of n-celestial bodies and thus prove the stability of the solar system. As of 1995, no one has solved the problem.
1890	Henri Poincaré. Won first prize in King Oscar's contest by being closest to solve the n-body problem. Discovered that the orbit of three or more interacting celestial bodies can exhibit unstable and unpredictable behavior. Thus is chaos born (but not yet named!).
1963	Edward Lorenz. Irregularity in a toy model of the weather displays first chaotic or strange attractor [9].
1975	Tien-Yien Li and James A. Yorke. In paper "Period Three Implies Chaos," introduced the term "Chaos Theory."
1976	Robert M. May. Application of the logistic equation to ecology, showing chaotic population behavior.
1978	Mitchell Feigenbaum. Universal number associated with the way systems approach chaos.
1980	Benoit Mandelbrot. Fractal geometry with application to computer graphics and image compression.
1990	Ed Ott, Celso Grebogi and James Yorke. Beginning of chaos control theory.
1990	Lou Pecora. Synchronization of chaotic systems.

Generally, the boundary between deterministic chaos and probabilistic random systems may not always be clear since seemingly random systems could involve deterministic underlying rules yet to be found. Nonlinear problems are very difficult to solve; various approximation techniques have been employed. These techniques include linearization, which is a simplification of the problem, and fuzzy theory, which can be used as an approximation method.

Historically, the study of chaos started in mathematics and physics. It then expanded into engineering, and more recently into information and social sciences (see Table 1). For the past five years or so in particular, there has been growing interest in commercial and industrial applications of chaotic systems. One key element of this recent interest is the power of easily accessible computer hardware for its speed and memory capacity. Although the history of chaotic systems research is not new, the computer revolution gave life to their practical applications. There are various types of potential commercial and industrial applications, based on different aspects of chaotic systems (see Tables 2 and 3). For simplicity, we classify the application types into the categories stabilization, synthesis, and analysis [11].

Stabilization and control. The extreme sensitivity of chaotic systems to tiny perturbations can be manipulated to stabilize or control the systems. The fundamental idea is that tiny perturbations can be artificially incorporated either to keep a large system stable (stabilization) or to direct a large chaotic system into a desired state (control). For example, although it is still problematic, small, carefully chosen chaos control interventions might lead to more efficient airplane wings, power delivery systems, turbines, chemical reactions in industrial plants, implantable defibrillators, brain pacemakers, conveyer belts, eco-

Table 2. Potential Application Types of Chaos

Category	Applications
Control	First application of chaos is control of irregular behavior in devices and systems. List of applications is included in Table 3.
Synthesis	Potential control of epilepsy, improved dithering of systems, such as ring laser gyroscopes. Switching of packets in computer networks.
Synchronization	Secure communications, chaotic broad band radio, encryption.
Information Processing	Encoding, decoding, and storage of information in chaotic systems, such as memory elements and circuits. Better performance of neural networks. Pattern recognition.
Short Term Prediction	Contagious diseases, weather, economy.

Table 3. Sample Potential Application Areas of Chaos

Category	Applications
Engineering	Vibration control, stabilization of circuits, chemical reactions, turbines, power grids, lasers, fluidized beds, combustion, and many more.
Computers	Switching of packets in computer networks. Encryption. Control of chaos in robotic systems.
Communications	Information compression and storage. Computer network design and management.
Medicine and Biology	Cardiology, heart rhythm (EEG) analysis, prediction and control of irregular heart activity (chaos-aware defibrillator).
Management and Finance	Economic forecasting, restructuring, financial analysis, and market prediction and intervention.
Consumer Electronics	Washing machines, dishwashers, air conditioners, heaters, mixers.

nomic planning, and even computer networks.

Synthesis of chaotic systems. Artificially generated chaotic systems may be applied to certain types of problems to make the systems, whether chaotic or nonchaotic, work better. The fundamental idea is that regularity is not always the best, depending on the type of problem. Artificially stimulated chaotic brain waves may someday help inhibit epileptic seizures [16]. We can synthetically generate chaotic output for consumer products, such as air conditioners and fan heaters, to make temperature changes feel more natural for human comfort. (See Aihara and Katayama in this issue.) Whether the chaotic output outperforms random output in these consumer products has yet to be studied, but it appears to be better than homeostasis in certain applications.

Two identical sequences of chaotic signals (called "synchronized") can be used for encryption by superposing a message on one sequence. Only a person with the other sequence can decode the message by subtracting the chaotic masking component. In communications, artificially generated chaotic signals can follow a prescribed sequence, thus enabling them to transmit information. Artificially generated chaotic fluctuations can be used to stimulate trapped solutions so they escape from local minima for optimization problems or for learning, as in neural networks.

Analysis and prediction of chaotic systems. How do you tell if a system is random or chaotic? The study of chaos may lead to better detection and prediction algorithms for chaotic systems. Amazingly, the lack of long-term predictability in chaotic systems does not imply that short-term prediction is impossible. In counterpoint to purely random systems, chaotic systems can be predicted for a short interval into the future. For example, many people believe that financial markets may exhibit chaotic behavior. Obviously, if the market were chaotic rather than random, there would exist the possibility of reliably predicting market behavior in the short term. Therefore, the identification of chaos in systems can potentially, at least in the market, lead to huge rewards.

Stabilization and Control of Chaotic Systems

As described earlier, the underlying principle discussed here is to control or stabilize chaotic systems by using their extreme sensitivities.

NASA satellite control. In 1978, a spacecraft called the International Sun-Earth Explorer 3 (ISEE-3) was launched toward a halo orbit around a Sun-Earth libration point. In 1983, ISEE-3 was retargeted for an interception with a comet a million miles away across the solar system. NASA engineers performed this orbital acrobatic feat by a combination of propulsive maneuvers, lunar swingbys, and the effective use of solar perturbations. Although the term "chaos control" did not exist at that time, this event demonstrated the idea. Through clever burns of fuel, the engineers exploited the chaotic sensitivity to perturbations exhibited by the three-body problem (shown by Poincaré, see Table 1) to use small amounts of fuel (all that was available) to nudge the spacecraft near the comet they wanted to investigate. If the system had been nonchaotic, large perturbations, and consequently large expenditures of fuel would have made such a mission impossible [3, 17].

Multimode laser. The *green problem* consists of the desire to have a cheap, stable laser capable of generating stable green laser light. The problem is that stable green lasers, unlike stable infrared or red lasers, are expensive to manufacture. One solution is to use frequency-doubling crystals to convert cheap 1024- nm laser light into cheap 512-nm laser light. Unfortunately, such crystals can induce chaotic fluctuations in the intensity of the doubled light and thus limit its usefulness. Raj Roy, at Georgia Tech, applied a chaotic controller developed by Earl Hunt at Ohio University to this problem. Hunt devised an Analog Controller to implement a chaos control technique that automatically selects out and controls unstable periodic behaviors in chaotic diode resonator circuits through small feedback signals by exploiting the extreme sensitivity of the system. The chaos controller worked beautifully in controlling the chaotic intensity fluctuations in Roy's solid-state laser [15]. Control-of-chaos techniques have since been successfully applied to nuclear magnetic resonance laser systems.

Belousov-Zhabotinsky chemical reaction. Chaos control was accepted as a viable technique for lasers, circuits, and chemical systems, but the consensus in 1992 was that chemical reactions were too difficult to apply chaos control. While the timid were doubting, Kenneth Showalter and associates promptly exhibited beautiful control in a chaotic chemical reaction: the Belousov-Zhabotinsky chemical reaction [13]. Under continuous flow of chemicals into the tank, the concentrations of the chemicals exhibit steady-state, periodic, and chaotic behavior. Small perturbations (applied according to chaos control theory) in the rate at which the reactant chemicals were fed into the tank converted chaotic oscillations in the ion concentrations into periodic behavior. Such chaos control of chemical reactions can lead to improved efficiency in industrial plants, with potentially huge benefits.

Heart arrhythmias. When one thinks of possible candidates for chaos in biological systems, the irregular beating of hearts in the form of arrhythmias and fibrillations comes to mind. A simple system that exhibits an interesting irregular arrhythmia similar to that seen in humans consists of a rabbit heart septum preparation that is induced to beat chaotically through perfusion with drugs (such as a fast-acting version of digitalis, an overdose of which can cause arrhythmias). The chaotic beating of the rabbit heart tissue was converted to periodic beating through electrical stimuli applied at irregular timing dictated by chaos control [4]. This application of chaos control has been patented by the inventors and licensed by a major medical company. Human trials are currently being performed.

Many efforts to control spatio-temporal complexity are being expended. The new Holy Grail of the chaos community is to be able to understand, characterize, and control such complexity in systems, including chemical mixing, combustion, fibrillation in hearts, and seizures in brain tissue (Figure 1).

Synthesized Chaos: Control and Adaptation

The ability to control chaos provides at least one way to exploit chaos for applications. It may now be advantageous to violate one of the classic dogmas of the applied scientist—linearity,—and design devices using, rather than avoiding, nonlinearity and chaos. Through the exploitation of chaos we may be able to replace many linear systems that do one thing well with fewer and more flexible nonlinear systems that exploit chaos. Thus, someday we may be able to make physical devices that mimic the flexibility of biological systems and make ailing biological systems mimic the regularity of physical devices. However, the advantages of these nonlinear systems will have to outweigh their increased complexity.

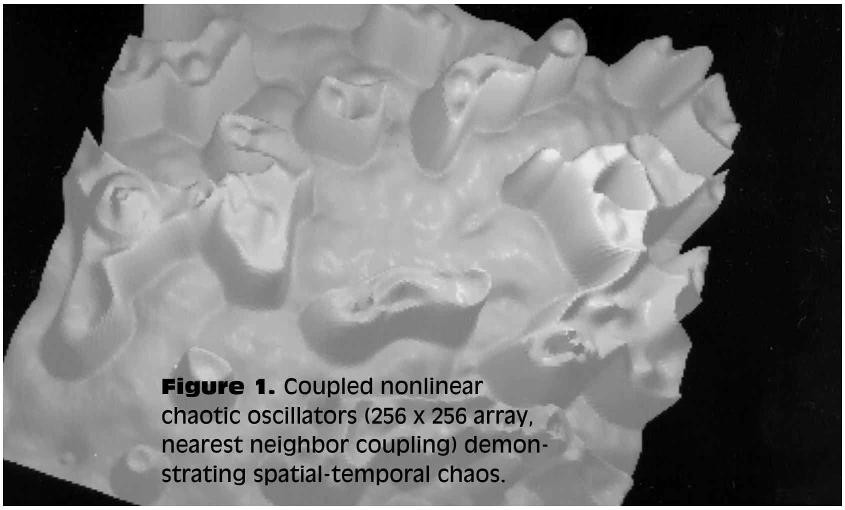


Figure 1. Coupled nonlinear chaotic oscillators (256 x 256 array, nearest neighbor coupling) demonstrating spatial-temporal chaos.

Biological neural networks. Brain slices taken from the hippocampal regions of rats, be kept alive in a dish, exhibit synchronous bursting neuronal behavior that has been linked to epileptic seizures of a type seen in human brains. Neurosurgeons and physicists have managed to both control the chaotic irregularity of such bursting behavior and, in a potentially more useful treatment, make the irregularity more chaotic through a technique termed *antichaos control,* which essentially keeps the systems away from an undesired periodicity [16]. While the efficacy of this method for preventing or quenching seizures in humans has yet to be tested, it shows much promise as a novel therapeutic technique for intervention in seizures.

Synchronized Chaos

Two identical chaotic systems started at nearly the same initial conditions quickly become uncorrelated. It is thus difficult to construct synchronized chaotic systems.

Lou Pecora and Tom Carroll, at the Naval Research Lab, considered how two identical or almost identical chaotic systems can be synchronized [12]. They take a chaotic system and produce a subsystem of the original by duplicating a part of the original system. They then supply a signal from the original to the subsystem. The original is unaware of the duplicate subsystem, but the duplicate is being driven by a signal from the original. Under certain conditions this subsystem behaves chaotically but is in complete synchronization with its counterparts in the original system. What we have built is a new system with more dynamical variables than the original, which has a subsystem that always follows the original. The overall behavior is chaotic, but the subsystem is stable. Its stability guarantees that any noise or perturbations will be damped out and it will continue to track the input signal, despite its complexity.

We have begun to realize that **the tools of chaos theory can be applied toward the understanding, manipulation, and control** *of a variety of systems, with many of the practical applications coming after 1990.*

Until this discovery, no one considered subsystems of any system, chaotic or not, as potentially separate systems with their own stability (or instability). Nor had anyone thought that a nonlinear system would be stable when driven with a chaotic signal. The trick is to find the subsystems stable to the input chaotic signal. Note that systems that are stable when driven by one type of signal may not be stable under the influence of another type. The stability depends on the driving signals as well as on the subsystem. Today, the stability can be estimated if a good mathematical model is available, but finding the stable subsystems of a general chaotic system remains a difficult problem. Much work remains to be done on this.

On a practical level, the engineering aspects of designing optimal synchronized chaotic systems are in their infancy, but several electronic circuits have been realized. The first synchronized chaos circuit was built by Carroll, a collaborator of Pecora, in 1989. He took a circuit that already showed chaotic behavior and built a subcircuit that was stable to a chaotic signal coming in from the original circuit. His final circuit had two parts, in which voltages and currents fluctuated in a chaotic fashion but whose voltages and currents were always in step with each other. Carroll and Pecora were able to build simple chaotic circuits that clearly demonstrated this synchronization of chaos.

Encryption. Since the subsystem to be chaotically synchronized could be located anywhere (in another country, for example), it is possible to use the chaos to communicate hidden messages. Simply take one of the chaotic voltages from the original system, add your information signal to it, and send it to a receiver synchronized with your circuit. The receiver can strip away the chaos and recover the information. Hopefully, anyone intercepting the signals will see only a noisy, chaotic signal. This type of communication setup was built by Neff and Carroll [10]. Although secure communications using synchronized chaos would probably have to be more sophisticated than simply adding your signal to chaos to hide it, these accomplishments represent the first steps in using chaos in encryption.

Leon Chua and collaborators at the University of California at Berkeley have built chaotic analog phase-locked-loop synchronizing circuits. Lichtenberg and Leiberman and their team at the same institute are using chaotic systems made from digital phase-locked loops to test new secure communications. They have built the circuits and begun some preliminary testing. Workers at Al Oppenheim's Digital Signal Processing Group at the Massachusetts Institute of Technology have constructed synchronized chaotic systems with feedback mechanisms to mix the chaos and information and extract it on the receiving end. Their knowledge of signal processing married with nonlinear/chaotic dynamics is yielding new approaches to issues in communications. Others are now duplicating these results and moving on to more sophisticated approaches.

Chaos radio and locked loop. Pecora has continued to exploit synchronized chaotic systems by generalizing a well-known electronic idea. A phase-locked-loop component in a circuit allows a receiver to follow the change in frequency of an incoming signal. This is the component that makes FM radio work. A frequency is just a parameter in a dynamic system (in this case a cyclic system). Pecora is exploiting cascades of synchronized subsystems to build chaotic systems that follow the change in a parameter of a transmitter analogous to the phase-locked loop we could call a parameter-locked loop.

He starts with a chaotic system withat least two stable subsystems. The two subsystems are cascaded in such a way that the first, which is missing the part of the original system where the drive signal is produced, feeds the second, which has a duplicate of the part of the original system where the synchronizing drive signal is produced. Both subsystems will synchronize so the second subsystem will duplicate the incoming drive signal. This assumes the parameters in all systems are the same. Remember that parameters are dynamic variables that usually don't change in time. However, they "turn a knob" and change a parameter in the original system, causing both subsystems to go slightly out of synchronization with the incoming signal. The detection of this effect allows them to tune the parameters in the receiver corresponding to the original system's parameters so they again get the same output from the second subsystem as the input driving system.

They were the first to build an actual operating circuit based on these ideas. It automatically followed the variation of the transmitter's parameters, proving the idea is robust and practical. This continual tuning of the receiver's parameters to keep things in synchronization and follow the original system's parameter variations is the chaotic analog of the phase-locked loop. It may also afford a way to send encrypted signals and allow the transmission of information in terms of parameter variations over broad band signals, which are more resistant to noise and interference. Since it allows a receiver to identify a hidden parameter in a remote sending system, it may be useful in sending identification signals of the friend-or-foe type.

Chaos in Communications

Researchers at the Army research labs have exploited the control of chaos by using small perturbations to make chaotic circuits follow prescribed symbol sequences [6]. Hayes and collaborators state, "The natural complexity thus provides a vehicle for information transmission in the usual sense." They exploit a nonlinear electrical oscillator producing large-amplitude chaotic signals. Their system produces a seemingly random sequence of positive and negative peaks; by assigning a positive peak a value of 1 and a negative peak a value of 0, the signal yields a binary sequence. Hayes' group uses a chaos control method to cause the signal to follow an orbit whose binary sequence represents the information they wish to encode. (For example, the letter "c," whose alphabetical position, 3, may be encoded as the binary 00011.) The transmitted signal is then detected and decoded. They even speculate that such control and sequences may be a possible mechanism by which biological systems transmit information.

Analysis and Prediction of Chaotic Systems

The basic idea is that a successful analysis of the time series of a chaotic system allows prediction or forecasting of the system's behavior in the near future. Such analysis is generally still very difficult, however, and much more work may be needed for massive applications.

Chaos prediction contest. A recent contest was held in which a portion of a chaotic laser time series of 1,000 data points was made public and a prize presented to the person who could most accurately predict the next 100 data points. Various methods, including neural networks, were used, and the very best did a nice job of prediction. Obviously, the ramifications of such prediction gives the identification of systems as either random (unpredictable) or chaotic (short term predictable) enormous importance.

Prediction and chaotic diseases. Prediction algorithms have also been used to determine whether the diseases have random (unpredictable) or chaotic (short-term predictable) incidence. Although the claims based on these algorithms are sometimes controversial, due to a susceptibility to measurement error, researchers using such techniques claim to have identified measles as a chaotic disease but chicken pox as nonchaotic [18].

Other prediction problems. Other application domains, many of which are speculative, include cardiology, ecology, financial markets, economy, fluid flow, weather, and climate (e.g., the El Nino oscillation).

Hybrid Systems

Other areas of AI, such as (artificial) neural networks, fuzzy logic, and genetic algorithms can be employed together with chaotic systems (e.g., neural networks + chaos, or neural networks + fuzzy + chaos.) Recently, there have been extensive research works on these topics whose introductions are beyond the scope of this article. The fundamental concept of such hybrid systems is that the components complement each other's strengths, creating new approaches to solve problems.

Neural networks + chaos. Neural networks are modeled on biological neural networks, or the human brain, and learn by themselves from patterns. This learning can then be applied to classification, prediction, or control.

There has been a surge of activity in work on neural network models that can behave as adaptively as biological systems. It is quite apparent that as we seek to make computer systems and neural networks behave with the flexibility of biological systems (such as speech production, speech and handwriting recognition, motor control), we must deliver neural networks that are adaptive, and chaotic behavior provides a rich library of behaviors to aid such adaptation. Computer scientists have started using neural network architectures and learning algorithms using chaos and chaotic circuits for associative memory storage of analog patterns and improvement of handwriting recognition systems using chaotic neural networks. Novel computing architectures have been constructed of recurrently interconnected associative memory modules using chaotic circuits.

Architectural variations employ selective synchronization of modules with chaotic behaviors that communicate through broad spectrum chaotic signals. We can construct or train a neural network so that its output exhibits a dynamic behavior of chaos [19].

From an engineering point of view, such a chaotic neural network may be applied for prediction and control. From a scientific point of view, such a network may lead to a better understanding of a biological neural network where the normal brain exhibits chaos. As discussed earlier, artificially generated chaotic fluctuations can be used to escape from local minima for certain types of neural networks. (See Aihara and Katayama in this issue.)

Fuzzy logic + chaos. Fuzzy systems are suitable for uncertain or approximate reasoning, especially for the system with a rigorous mathematical model that is difficult to derive.

They also allow us to represent descriptive or qualitative expressions.

Fuzzy logic may be employed to describe a chaotic dynamical system. Fuzzy logic can be useful for complex dynamic system for which a common mathematic modeling does not work well. From the application point of view, control is probably the most promising domain of chaos-fuzzy hybrid systems since it has been the most successful in both fuzzy and chaotic systems.

Genetic algorithms + chaos. Genetic algorithms are computer models based on genetics and evolution. Their basic idea is that the program works toward finding better solutions to problems, just as species evolve to better adapt to their environments. Genetic algorithms may be particularly useful for hard problems of optimization and machine learning. As is the case for the use of fuzzy logic, genetic algorithms can be a useful tool to describe a complex chaotic system where common mathematical modeling is difficult.

Other potential applications include the use of chaos as a tool to enhance genetic algorithms. For example, certain chaotic functions, rather than random numbers, might be used in the processes of crossover. This might alter the characteristics of genetic algorithm solutions---hopefully toward more desirable situations, such as avoiding premature convergence. This may be interpreted as the use of artificially generated chaotic functions to escape from local minima.

Chaos modeling of genetic algorithms can be another example of the potential use of chaos as a tool to analyze genetic algorithms. Genetic algorithms, especially those that generate chaotic solutions, may be analyzed by a chaos model. For example, the changing behavior of a population over generations can be visualized through computer graphics similar to the Mandelbrot set [8].

Acknowledgments

The authors are indebted to Kazuyuki Aihara and Ryu Katayama for their valuable information.

References

1. Cambel, A.B. *Applied Chaos Theory: A Paradigm for Complexity.* Academic Press, San Diego, Calif., 1993.
2. Devaney, R.L. *A First Course in Chaotic Dynamical Systems Theory and Experiment.* Addison-Wesley, Reading, Mass., 1992.
3. Farquhar, R., Muhonen, D., and Church, L.C. Trajectories and orbital maneuvers for the ISEE-3/ICE comet mission. *J. Astronaut. Sci. 33*, 3 (July–Sept., 1985), 235–254.
4. Garfinkel, A., Spano, M. L., Ditto, W. L., and Weiss, J. A. Controlling cardiac chaos. *Science 257,* (1992), 1230–1235.
5. Gleick, J. *Chaos: Making a New Science.* Viking Penguin, New York, 1987.
6. Hayes, S., Grebogi, C., and Ott, E. Communicating with chaos, *Phys. Rev. Lett. 70* (1993), 3031–3034.
7. Hunt, E., and Johnson, G. Keeping chaos at bay. Chaotic dynamics and control in electronic circuits (Advanced Technology/Tutorial). *IEEE Spectrum 30* (Nov. 1993), 32–36.
8. Juliany, J., and Vose, M.D. The genetic algorithm fractal. In *The Fifth International Conference on Genetic Algorithms* (Urbana-Champaign, 1993), p. 639.
9. Lorenz, E.N. Deterministic non-periodic flow. *J. Atmos. Sci. 20,* (1963), 130–141.
10. Neff, J., and Carroll, T. Circuits that get chaos in sync. *Sci. Am. 269,* (Aug., 1993), 120–122.
11. Ott, E., Sauer, T., and Yorke, J.A. Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems. Wiley, New York 1994.
12. Pecora, L.M., and Carroll, T.L. Synchronization in chaotic systems. *Phys. Rev. Lett. 64,* (1990), 821–824.
13. Petrov, V., Gaspar, V,. Masere, J., and Showalter, K. Controlling chaos in the Belousov-Zhabotinsky reaction. *Nature 361,* (1993), 240–243.
14. *Proceedings of the IEEE,* Special issue on chaotic systems *75,* 8 (Aug. 1987).
15. Roy, R., Murphy, T. W., Maier, T. D., Gills, Z., and Hunt, E. R. Dynamical control of a chaotic laser: Experimental stabilization of a globally coupled system. *Phys. Rev. Lett. 68* (1992), 1259–1262.
16. Schiff, S. J., Jerger, K., Duong, D. H., Chang, T., Spano, M. L., and Ditto, W.L. Controlling chaos in the brain. *Nature, 370* (Aug. 25, 1994), 615–620.
17. Shinbrot, T., Grebogi, C., Ott, E., and Yorke, J. A. Using small perturbations to control chaos. *Nature 363* (June 3, 1993), 411–417.
18. Sugihara, G., and May, R. M. Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series. *Nature 344* (Apr. 1990), 734–740.
19. Wasserman, P.D. *Advanced Methods in Neural Computing.* Van Nostrand Reinhold, New York, 1993.

About the Authors:

WILLIAM DITTO is an assistant professor of physics at Georgia Tech where he and his colleagues have pioneered methods for controlling chaotic behavior. Current research interests include exploitation in control of chaos in physical and biological systems. **Author's Present Address:** Applied Chaos Lab, School of Physics, Georgia Institute of Technology, Atlanta, GA 30332; email: wditto@acl1.physics.gatech.edu; www: http://acl1.physics.gatech.edu/aclhome.html.

TOSHINORI MUNAKATA is a professor in the Computer and Information Science Department at Cleveland State University. Current research interests include applied artificial intelligence in the areas of fuzzy systems, rough set control, neural networks, genetic algorithms, and logic programming; and analysis of algorithms. **Author's Present Address:** Computer and Information Science Department, Cleveland State University, Cleveland, OH 44115; email: munakata@cis.csuohio.edu; www: httop://cis.csuhio.edu/~munakata/