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ABSTRACT

Multiple matrix sampling is a psychometric procedure in which a set of test items is subdivided randomly into subtests of items with each subtest administered to different subgroups of examinees selected at random from the examinee population. Although each examinee receives only a proportion of the complete set of items, the statistical model employed permits the researcher to estimate the mean, variance and frequency distribution of test scores which would have been obtained by testing all examinees on all items. Contained herein is a detailed description of multiple matrix sampling. The topics covered range from an introductory discussion to the listing with expanded writeup of the computer program used to analyze the data. Throughout this Report an attempt has been made to keep the practitioner clearly in mind. (Author)

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PRINCIPLES AND PROCEDURES OF MULTIPLE MATRIX SAMPLING

David M. Shoemaker

I

Introduction

Multiple matrix sampling or, more popularly, item-examinee sampling, is a psychometric procedure whose time has come. It is the Zeitgeist. Descriptions of multiple matrix sampling procedures and explorations into areas of application are scattered over a multitude of technical journals. There is no single book or article which describes, studies, and unifies all of this material. Yet there is a need for such a document both as a reference source and as a textbook.

Although statisticians have dealt for several decades with incomplete data problems in the design of experiments and data analysis, the psychometrician responsible primarily for the derivation of statistical procedures in multiple matrix sampling and the application of such procedures to problems in psychology and education is Frederic M. Lord. Lord and Novick discuss multiple matrix sampling in Chapter 11 in _Statistical theories of mental test scores_ but the chapter does not encompass the degree of detail and depth of explanation required by the majority of educational research practitioners who desire to implement this research procedure in a particular investigation. This Technical Report has been designed to remedy this situation.

Throughout this Report an attempt has been made to keep the practitioner clearly in mind. The emphasis is on the _why_, _when_, and _how_ to use multiple matrix sampling. The topics covered range from an introduction to multiple matrix sampling to the listing with expanded writeup of the computer program used to analyze the data. All discussions and guidelines contained in the monograph reflect theoretical and empirical results reported in the literature as well as personal experiences of the author in implementing multiple matrix sampling in a variety of applied situations.

II

Characteristics, Advantages, And Applications Of Multiple Matrix Sampling

The majority of contemporary psychometric procedures reflect strongly the original impetus of the psychometric movement, that is, the measurement of individual differences. Historically, individual differences have been investigated, and appropriately so, using the matched-items model in which a single set of test items is administered in a standardized procedure to all, or a sample, of the examinee population under consideration. One exemplar of such methodology is the anthropometric laboratory of Sir Francis Galton established at the International Health Exhibition in England in 1884. Galton measured individuals ranging in age from five to eighty on such dimensions as standing height, sitting height, arm span, weight, breathing capacity and strength of pull "to supply information on the methods, practice, and uses of human measurement." Understandably so and undoubtedly for lack of a reasonable alternative, procedures appropriate for the assessment of _individual_ differences have been transferred completely to investigations concerned primarily with the measurement of _group_ differences. An example of a research design emphasizing the assessment of group differences is found in an investigation which contrasts treatment effects through administering each treatment to a group of examinees selected randomly from the examinee population. Given treatments A, B, and C, for example, the researcher is interested primarily in the behavior of _group_ A as contrasted with _group_ B as contrasted with _group_ C. Differences among individual examinees are of little concern. The point to be made is simply this: the methodology employed successfully in the assessment of individual differences is neither the appropriate nor the most efficient methodology for group assessment. Multiple matrix sampling or, more popularly, item-examinee sampling, has been demonstrated theoretically and empirically to be the appropriate procedure for group assessment and a procedure superior to the matched-items model.

The matched-items model and the multiple matrix sampling model are contrasted readily by considering the data base which would be generated if the entire testable population of N examinees were administered the complete set of K test items. Such a data base is illustrated in Figure 2.1 and the arrangement is referred to commonly as an item-examinee matrix. Test items are scored dichotomously frequently and such is the case in Figure 2.1. For example, examinee 1 passed item 1, failed item 2, passed items 3 and 4, and failed item 5. Within the

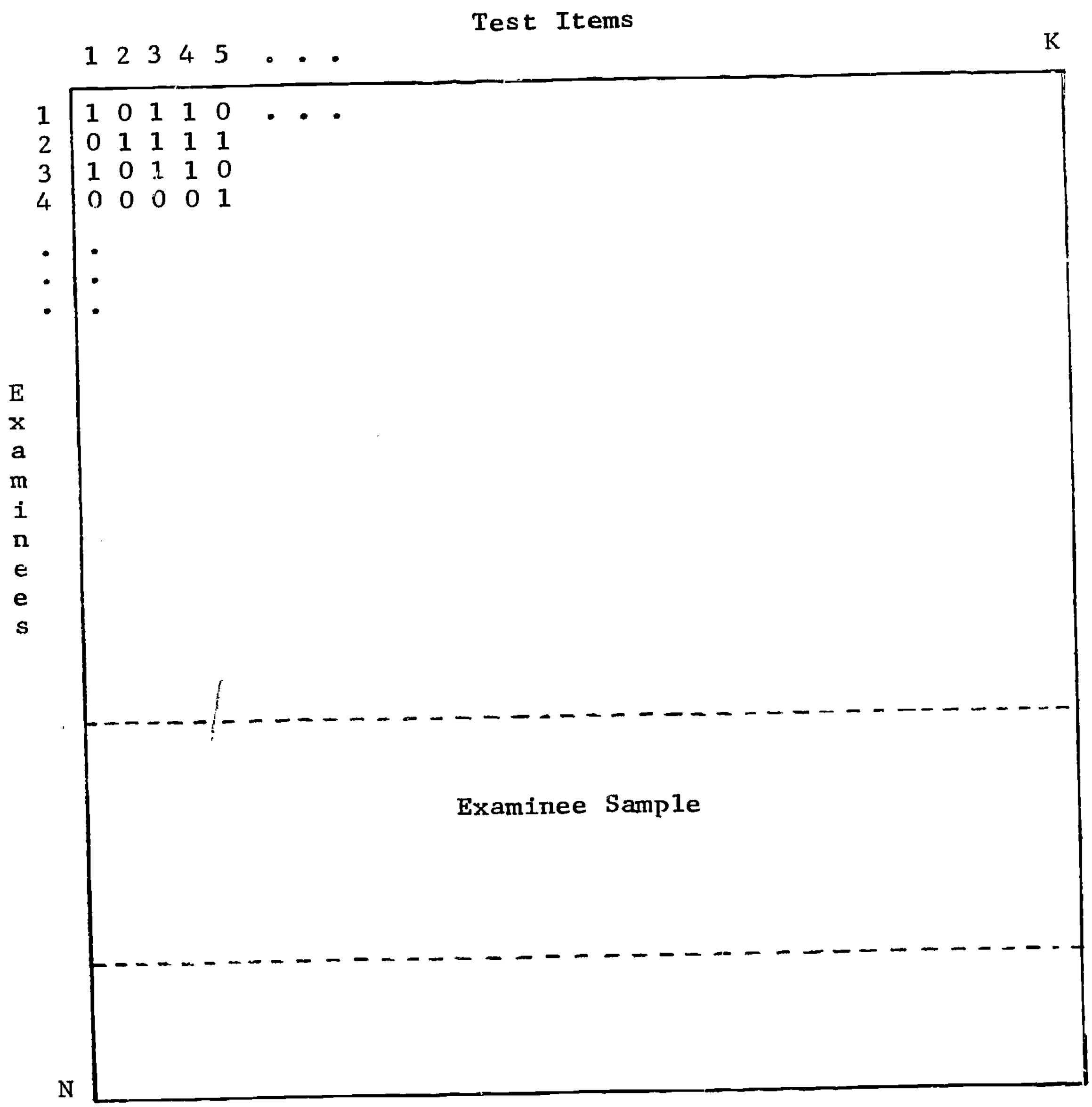

Figure 2.1: Item-examinee matrix illustrating examinee-sampling.

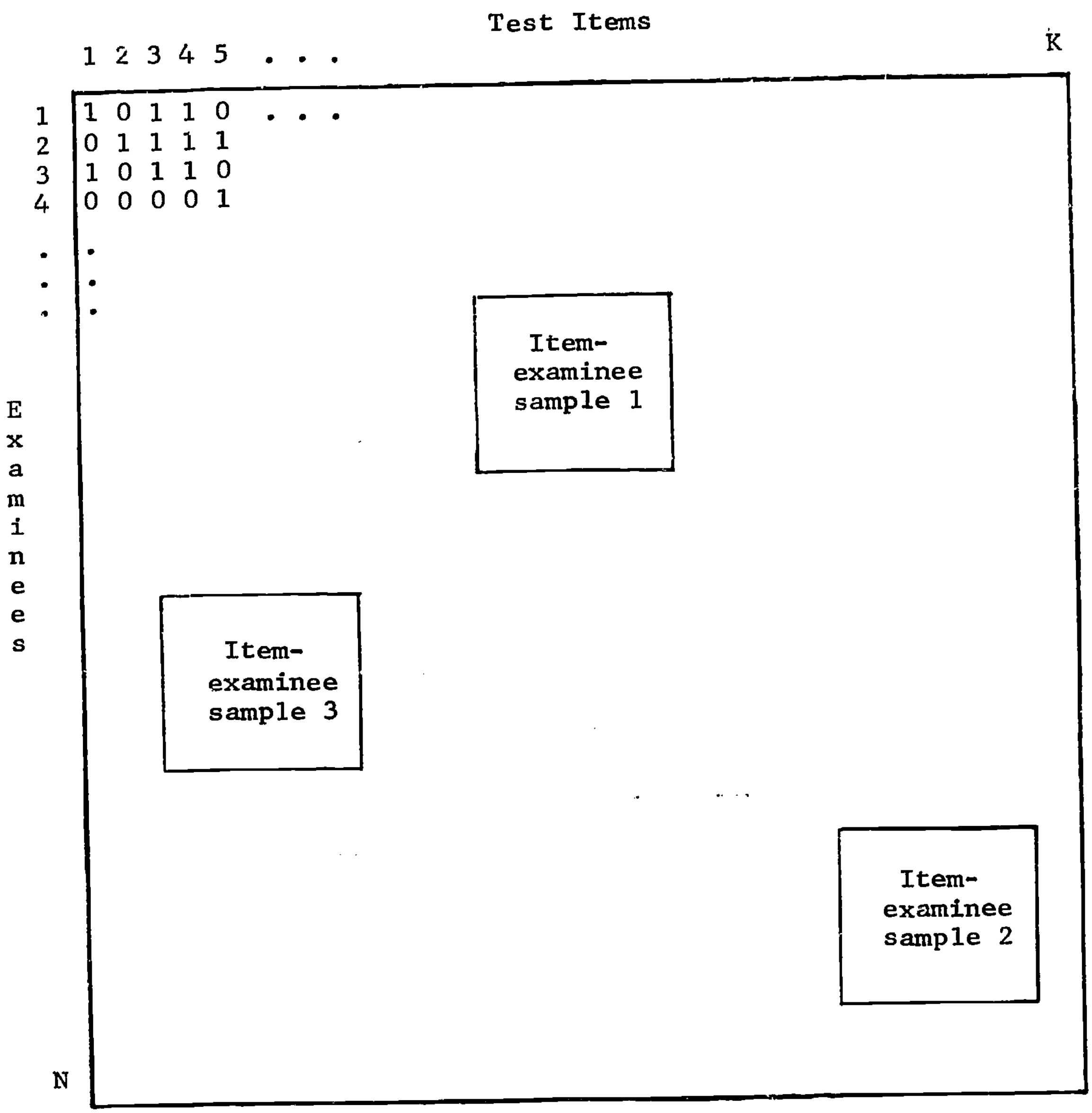

Figure 2.2: Item-examinee matrix illustrating multiple matrix sampling.

framework of the item-examinee matrix, the matched-items model used in the assessment of individual differences is referred to as the examinee-sampling model because all test items are administered to a subgroup of examinees selected at random from the population of N examinees. By contrast, multiple matrix sampling involves the joint sampling of examinee subgroups and item subtests as illustrated in Figure 2.2. Data from item-examinee sample 1 were obtained by administering a set of items selected at random from the population of K test items and administering these and only these items to a subgroup of examinees selected randomly from the population of N examinees. Replicating this procedure produces item-examinee samples 2 and 3 and suggests, concomitantly, the derivation of the expression "multiple matrix sampling." Statistics obtained from examinee-sampling and from multiple matrix sampling are used to estimate parameters of the N by K item-examinee matrix. It must be remembered, however, that the N by K item-examinee matrix illustrated in Figures 2.1 and 2.2 is a hypothetical matrix the parameters of which are estimated from the subset of data gathered in practice through examinee-sampling or multiple matrix sampling.

Advantages of Multiple Matrix Sampling

A concept important in discussing the advantages of multiple matrix sampling and one mentioned frequently herein is the standard error of estimate. Assume that two experimental procedures have been developed for measuring weight and each procedure is used to obtain in a standardized manner 1000 independent measurements of the weight of a given object. Hypothetical measurements so acquired have been assembled into frequency distributions and are given in Figure 2.3. The standard error of estimate associated with procedure M is the standard deviation of the 1000 values for the weight obtained using procedure M; the standard error of estimating the weight for procedure E is determined identically. The difference in standard errors of estimate depicted in Figure 2.3 illustrates an important advantage of multiple matrix sampling over examinee-sampling in group assessment. Lord and Novick (1968) have demonstrated algebraically that, when subtests are constructed by sampling items without replacement from the K-item population, the standard error in estimating the group mean test score using multiple matrix sampling is less than the standard error obtained with examinee-sampling. Furthermore, the minimum standard error of estimate under multiple matrix sampling is found by administering one item to each of K random samples of examinees. A conclusion such as this is of major significance because the parameter of primary importance in many investigations is the group mean test score.

To clarify this point, consider how such a result could have been determined empirically through post mortem item-examinee sampling. In post mortem item-examinee sampling, an existing N by K item-examinee data base is taken to be the population of scores and item scores from item-examinee samples selected randomly from this base are used to

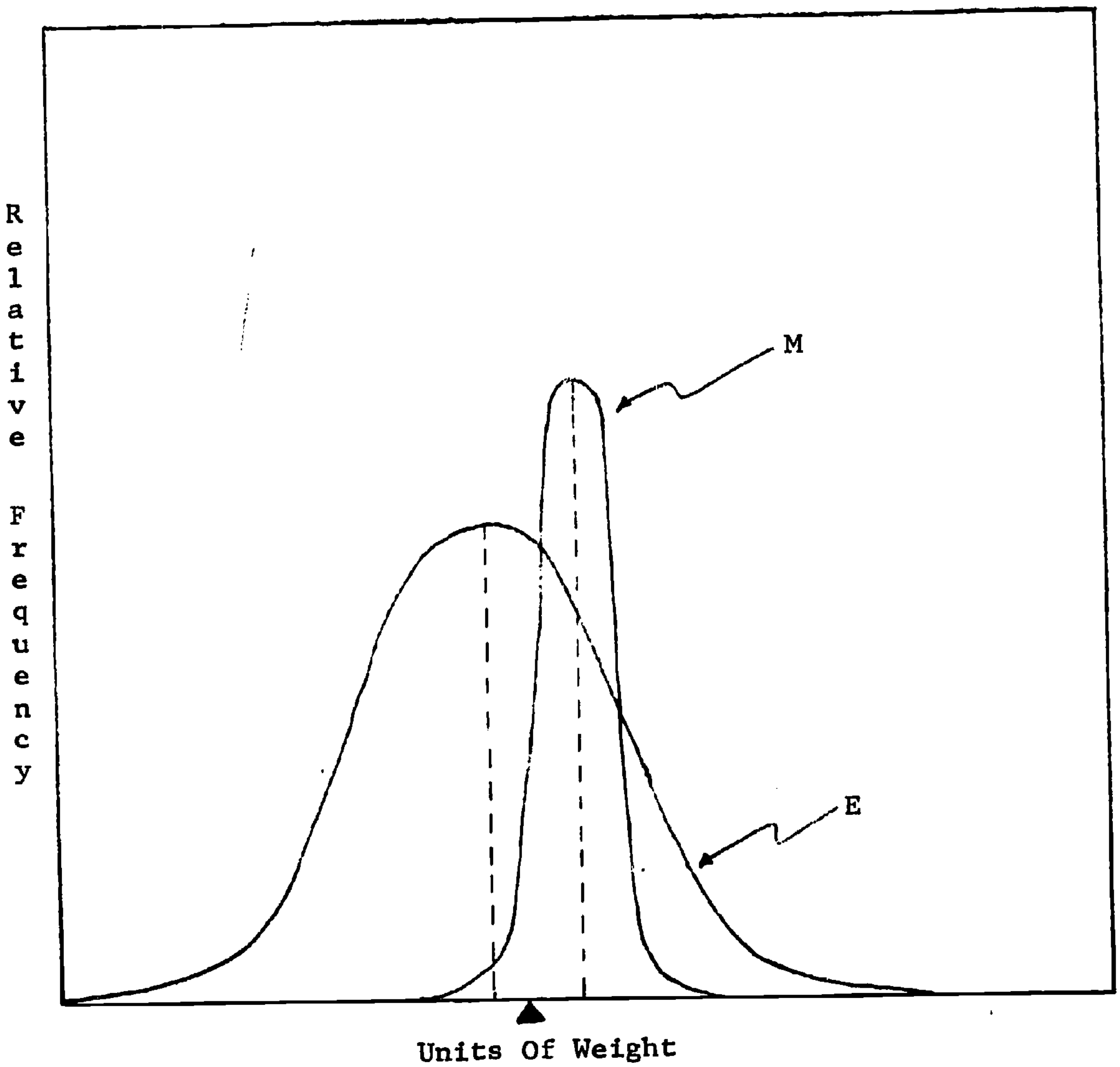

Figure 2.3: Hypothetical distributions of weight measurements resulting from 1000 replications of procedure M and 1000 replications of procedure E.

estimate parameters of interest. Although all examinees have responded to all items, in post mortem item-examinee sampling the investigator acts as if individual examinees had responded only to specific items. The standard error of estimate under examinee-sampling could be approximated, for example, by selecting at random 1000 examinee subgroups and testing each subgroup over K items. Data from each examinee subgroup provide an estimate of the mean score over N examinees and the standard deviation calculated over these 1000 estimates is the standard error of estimating the population mean under examinee-sampling. A single estimate of the mean test score under multiple matrix sampling is obtained, for example, by dividing randomly the set of K test items into t non-overlapping subtests containing K/t items each and administering each subtest to a subgroup of examinees selected at random from the population of N examinees. A single estimate of the population mean is obtained by pooling the t estimates obtained from each item-examinee sample. Replicating this procedure 1000 times provides 1000 pooled estimates of the population mean test score and hence the standard error of estimate associated with the particular item-examinee sampling plan used. (All computational formulas used in multiple matrix sampling are explained in detail in Chapter IV.)

The advantages of multiple matrix sampling have thus far been focused on the standard error of estimating the mean test score. Important also is the expected value or mean of the estimates of the population mean test score over replications. In Figure 2.3, the standard error of procedure M is less than the standard error of procedure E; however, on the average, the values obtained using procedure E are more accurate than those obtained using procedure M (assuming that the true weight is the value on the abscissa indicated by the pointer). A consideration such as this prompts an examination of the mean estimate of the population mean test score obtained under multiple matrix sampling. The results of several empirical investigations (Johnson & Lord, 1958; Lord, 1962; Plumlee, 1964; Stufflebeam & Cook, 1967; Shoemaker, 1970a, 1970b) using post mortem item-examinee sampling support the conclusion that, on the average, estimates of the mean test score are extremely accurate. (Results such as these are to be expected since the mean of a random sample is always an unbiased estimator of the population mean and estimates of the mean test score obtained through multiple matrix sampling are no exception.) Shoemaker (1970b) has demonstrated that this conclusion is appropriate, additionally, for estimates of the population standard deviation.

In addition to the statistical advantages of multiple matrix sampling in estimating group achievement, there are other advantages of practical import: (a) The testing time per examinee is reduced under multiple matrix sampling. This is, indeed, an important consideration as the time necessary for testing K items per examinee is frequently difficult or impossible to obtain. (b) Under multiple matrix sampling, the costs of scoring each test are reduced. (c) Multiple matrix sampling as a procedure may be accepted more readily in certain situations than the matched-items design. In a company, for example, supervisors fearing that test results may be used against their employees may be assured more convincingly if each employee takes only a part of a test and different employees take different parts. (d) Given a limited amount of available testing time per examinee, performance on a larger number of test items can be approximated through multiple matrix sampling than through a matched-items design. (e) With multiple matrix sampling it is possible to estimate simultaneously parameters of several tests. To the examinee, the test so constructed is merely another test; however, to the test constructor, the composite is a collection of several tests each having parameters estimated through multiple matrix sampling.

Limitations of Multiple Matrix Sampling

Although advantages of multiple matrix sampling are more numerous than limitations, the latter do exist. Estimating parameters through multiple matrix sampling assumes that the responses of an examinee to an item sample are exactly those which would have been obtained had the examinee responded to those items embedded in the K-item test. Although the data available (Sirotnik, 1970; Shoemaker, 1970c) suggest that multiple matrix sampling is relatively immune to a context effect, there is one important exception: using multiple matrix sampling to estimate parameters of speeded tests. In this case, an examinee's response is not independent of the context of the test and multiple matrix sampling should not be used.

An insidious variation of the context effect occurs when multiple matrix sampling is used to estimate parameters for a test which is impossible to administer in practice. For example, parameters of a 500-item vocabulary test designed for grade one students could be estimated readily through multiple matrix sampling by forming 25 subtests having 20 items each with each subtest administered to one class of grade one students. Although all students could respond appropriately to the 20-item test, data from each subtest would be used to estimate the results which would have been obtained had all grade one children taken the 500-item test. The problem is that no individual grade one student could have tolerated the 500-item test.

A potentially serious limitation of multiple matrix sampling is found in the logistics involved in giving different tests to different subgroups of examinees. If test items are administered individually, problems are minimal. If, however, each item requires oral instructions by the test administrator and different tests are to be distributed among the examinees in the testing room, serious problems occur. In this situation, the examinees must be segregated and isolated according to subtest before administering each test. If the instructions to each item are written on the test booklet, administering different tests to different examinees within the testing room is accomplished with relative ease.

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III

Procedural Guidelines in Multiple Matrix Sampling

Multiple matrix sampling as a procedure involves basically three steps: (a) a K-item test is subdivided through random or stratified-random sampling into subtests each having typically the same number of items, (b) each subtest is administered to a group of examinees selected randomly from the examinee population, and (c) test parameters are estimated from subtest results. Although the procedure is described easily, implementing it produces many interesting questions. For example: How many subtests should be formed? To how many examinees should each subtest be administered? Is it more appropriate to administer a few subtests containing a large number of items or a large number of subtests containing few items? These are only a few of the questions encountered frequently when using multiple matrix sampling. Described herein are general guidelines for answering these and other related questions.

Let t denote the number of subtests, k the number of items per subtest and n the number of examinees to which each subtest is administered. A specific sampling plan is denoted by (t/k/n). For example, (2/25/60), (10/5/60) and 10/20/30) are three sampling plans which could be used to estimate the parameters of a 50-item test. With the first plan, 2 subtests are formed containing 25 items each with each subtest administered to 60 examinees; with the second plan, 10 tests with 5 items each with each subtest administered to 60 examinees; and with the third, 10 tests with 20 items each with each subtest administered to 30 examinees. The third plan introduces an important variable in multiple matrix sampling, namely, the procedure used to sample items in constructing subtests. With (2/25/60) and (10/5/60) subtests are formed by sampling test items <u>without replacement</u> from the pool of 50 items. With (10/20/30), items are sampled <u>without replacement</u> for a given subtest but <u>with replacement</u> among subtests; consequently, an individual item will often be included in more than one subtest, but no item will be included twice in the same subtest. The rule is this: if the product tk is less than or equal to K, the sampling of items for subtests is always <u>without replacement</u>; when tk is greater than K, the sampling of items is <u>without replacement</u> for each subtest and <u>with replacement</u> between subtests. Selecting items for two subtests using the latter sampling procedure is demonstrated easily with a deck of cards numbered consecutively from 1 to K: (a) the deck of K cards is shuffled thoroughly, (b) k cards are selected at random from the deck with the numbers on the cards indicating those items to be included in subtest i, (c) the k cards are returned to the deck, (d) the card deck is reshuffled, and

(e) k cards are selected at random for subtest j. Although a multitude of sampling plans are possible, it is generally the case that tk is equal to or greater than K.

Although constructing subtests having overlapping item subsets is desirable in that it increases the number of observations acquired by the sampling plan (and, hence, decreases generally the standard error of estimate associated with that sampling plan), it is of critical importance that, when tk is greater than K, tk be an integer multiple of K, and items are sampled randomly but subject to the restriction that each item appear with equal frequency among subtests. With (10/20/30), for example, the multiple is 4 and each of the 50 items should appear in exactly 4 subtests. Any deviation from this procedure results in a marked increase in the standard error of estimate.

An important characteristic of any sampling plan used in multiple matrix sampling is the number of observations acquired by that plan. Defining one observation as the score received by one examinee on one item, the number of observations acquired by a sampling plan is equal to the product tkn. For example, 3000 observations are acquired by (2/25/60) and by (10/5/60) while 6000 observations are acquired by (10/20/30). The number of observations per sampling plan is an important concept in multiple matrix sampling and one mentioned frequently herein.

In multiple matrix sampling, a variety of sampling plans are possible with the selection of a particular sampling plan being typically the result of both practical and statistical considerations. Determining the relative merits of individual sampling plans is accomplished readily through a consideration of the standard error of estimate for each parameter for each sampling plan. Shoemaker (1970a, 1970b, 1971a, 1971b) has determined empirically, through post mortem item-examinee sampling, standard errors of estimate for selected parameters as a function of variations in (a) the number of observations acquired by the sampling plan, (b) t, k, and n, (c) test reliability of the normative distribution of test scores, (d) the variance of item difficulty indices, and (e) degree of skewness in the normative test score distribution. The following are general guidelines in multiple matrix sampling resulting from these and other investigations (Shoemaker & Osburn, 1968; Osburn, 1969):

1. The number of observations acquired by the sampling plan is an important variable. In general, as the number of observations increases, the standard error of estimating parameters decreases. (The major exception to this guideline occurs when guideline 4 is not followed.)

2. Increasing the number of examinees per subgroup is least effective in reducing the standard error of estimate.

3. For normal normative distributions, increases in the number of items per subtest are most effective in reducing standard errors of estimate; for negatively-skewed distributions, increases in the number of subtests are most effective.

4. When tk is greater than K, tk should be an integer multiple of K and items should be selected randomly but subject to the restriction that among subtests each item appears with equal frequency.

5. In general, fewer observations are required to estimate parameters of a skewed normative distribution than of a normal normative distribution.

6. If subtest items are being selected according to a stratified-random sampling plan instead of a random sampling plan, items should be stratified according to difficulty level and not according to content.

7. As the reliability of the normative distribution of test scores increases, it becomes increasingly difficult to estimate parameters. For this reason, it is true generally that a relatively large number of observations is required by the sampling plan when estimating parameters of a distribution having high reliability. This is true also when the variance of item difficulty indices is large.

8. If no information concerning the normative distribution of test scores is available, select a sampling plan having the number of subtests equal to the square root of the total number of test items (rounded to the nearest integer) with each subtest having approximately the same number of test items.

Guidelines such as these are concerned primarily with relative standard errors of estimate in multiple matrix sampling. Although Lord and Novick (1968, equation 11.12.3) have determined algebraically the standard error of estimating the mean proportion correct score in multiple matrix sampling given nonoverlapping random samples of dichotomously-scored items drawn without replacement from the item population, the standard error of estimate for any parameter using any and all sampling plans may be determined easily and effectively through use of the simulation model for multiple matrix sampling described in detail in Chapter V.

Multiple Matrix Sampling Step by Step

Step 1: Construct or select the K-item test. If possible, assemble the items into strata according to difficulty level.

Step 2: Determine the limitations and restrictions which must be imposed upon the test administration procedure.

Step 3: Select a sampling plan which is appropriate in view of the known characteristics of the normative distribution, the restrictions and limitations inherent in the test administration procedure, and guidelines 1 through 8.

Step 4: Administer subtests to examinees in a standardized procedure. Avoid confounding subtests with examinee subgroups, i.e., make every attempt to have examinee subgroups homogeneous.

Step 5: Compute estimates of parameters using equations 4.1, 4.2, 4.4, 4.5, 4.7 and 4.9 with the computer program given in Appendix A.

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IV

Computational Formulas in Multiple Matrix Sampling

Computational formulas used in multiple matrix sampling are applied easily in practice and are detailed and sequenced appropriately in the following application of the procedure. It should be noted initially that all formulas assume uniform item scoring procedures; for example, some items cannot be scored dichotomously and other trichotomously.

An Application of Multiple Matrix Sampling

A spelling program is being designed for kindergarten students and the word and rule content of this program is to be related closely to the reading program used by these students. Before constructing such a program it is necessary to determine the spelling proficiency of those students who have used the reading program but have not had formal spelling instruction on the related words. Although there were 78 unique words introduced in the particular reading program under consideration, technical considerations dictated that only words having regular spellings be included in the word population. As a result, the original word population was reduced from 78 words to 50 words. The modified word population was then subdivided through random sampling without replacement into 5 subtests containing 10 words each. (This is one of many procedures which could have been used. Alternative procedures are discussed in detail in Chapter III.) Three kindergarten classes were selected randomly from the pool of 9 classes. Students within each class were divided at random into 5 groups and each group was assigned at random to one of the 5 subtests. Each test was administered individually. All items were scored dichotomously (1 = pass, 0 = fail) with the results of each subtest given in Tables 4.1 through 4.5.

Estimating Parameters From Subtest Results

In multiple matrix sampling, subtest results are of secondary interest. Of chief concern is the estimation of parameters, that is, the results which would have been obtained had all students been tested over the entire set of 50 items comprising the word population. The results of each subtest, however, can be used to provide estimates of parameters of interest. For example, from subtest 1 it is possible to obtain an estimate of several parameters, i.e., μ (the population mean test score), σ (the standard deviation of test scores), σ^2 (the variance of test scores), μ_3 (the third moment about the arithmetic mean), μ_4 (the fourth moment about the arithmetic mean), α_{21} (the coefficient of reliability), g_1 (the index of skewness), and g_2 (the degree of kurtosis). All of these parameters are not independent, but each can be estimated from the results of one subtest. In multiple matrix sampling, multiple subtests are used and, hence, multiple estimates of each parameter are obtained. A more accurate estimate of each parameter is obtained by combining or pooling the estimates obtained from each subtest.

Although it is possible to estimate several parameters, the majority of investigations are interested primarily in estimating μ, σ^2, and α_{21}. The appropriate formulas for estimating these parameters from subtest i are

$$\hat{\mu}_i = \frac{K\bar{T}_i}{k_i}, \qquad (4.1)$$

$$\hat{\sigma}_i^2 = \frac{n_i K\{(K-1)s_i^2 - (K-k_i)\sum^{k_i} v_i\}}{k_i(k_i-1)(n_i-1)}, \qquad (4.2)$$

and,

$$\hat{\alpha}_{21_i} = \frac{K}{K-1}\left[1 - \frac{\hat{\mu}_i - \frac{\hat{\mu}_i^2}{K}}{\hat{\sigma}_i^2}\right], \qquad (4.3)$$

where,

K = the total number of items in the population,

k_i = the number of items in subtest i,

n_i = the number of examinees receiving subtest i,

Table 4.1

Results And Computations For Subtest 1

		Item										
Examinee	Class	03	09	11	16	21	27	37	42	44	50	T
01	1	1	0	1	1	1	1	1	1	1	0	8
02	1	1	1	1	1	1	0	1	0	1	1	8
03	1	1	1	1	0	1	0	0	0	0	0	4
04	1	1	1	1	0	0	0	0	0	0	0	3
05	1	0	0	0	0	0	0	0	0	0	0	0
06	1	1	1	0	1	0	1	0	0	0	1	5
07	2	1	1	1	0	1	0	1	0	1	0	6
08	2	1	1	0	0	1	0	0	0	0	0	3
09	2	0	0	0	0	0	1	1	0	1	0	3
10	2	0	1	0	0	0	0	0	0	0	0	1
11	2	1	1	1	1	1	0	1	0	1	0	7
12	2	1	0	0	1	0	0	1	0	0	0	3
13	3	0	1	0	0	1	0	0	0	0	0	2
14	3	0	1	1	1	1	0	0	0	1	0	5
15	3	0	0	0	0	0	0	0	0	0	0	0
16	3	0	0	0	0	0	0	0	0	0	0	0
17	3	1	1	0	1	0	0	1	1	1	0	6
18	2	0	0	0	0	0	0	0	0	0	0	0
	p	.5556	.6111	.3889	.3889	.4444	.1667	.3889	.1111	.3889	.1111	
	p(1 - p)	.2469	.2377	.2377	.2377	.2469	.1389	.2377	.0988	.2377	.0988	

$K = 50$ $\quad \overline{T}_1 = 3.5556$

$k_1 = 10$

$n_1 = 18$ $\quad s_1^2 = 7.1358$

$\sum T = 64$ $\quad \sum p(1-p) = 2.0188$

$\sum T^2 = 356$

$$\hat{\mu}_1 = \frac{(50)(3.5556)}{10} = 17.7780$$

$$\hat{\sigma}_1^2 = \frac{(18)(50)((50 - 1)(7.1358) - (50 - 10)(2.0188))}{10(10 - 1)(18 - 1)} = 158.1778$$

Table 4.2

Results And Computations For Subtest 2

Examinee	Class	08	12	18	23	30	33	38	40	43	46	T
		Item										
01	1	1	1	1	1	1	1	0	0	1	1	8
02	1	1	0	1	1	1	0	0	0	0	0	4
03	1	1	1	0	0	1	0	1	0	0	0	4
04	1	1	1	1	1	1	1	0	0	0	0	6
05	1	1	0	1	1	1	1	0	0	1	0	6
06	1	1	1	1	0	1	1	0	0	0	0	5
07	2	1	0	1	0	1	0	0	0	0	0	3
08	2	1	1	1	1	1	1	0	0	1	0	7
09	2	1	1	0	1	1	1	1	0	1	0	7
10	2	1	1	1	1	1	1	0	0	0	0	6
11	3	0	0	0	1	1	1	0	0	0	0	3
12	3	1	1	1	1	1	1	1	1	0	0	8
13	3	1	1	0	1	0	0	0	0	0	0	3
14	3	0	0	1	1	1	1	0	0	0	0	4
	p	.8571	.6429	.7143	.7857	.9286	.7143	.2143	.0714	.2857	.0714	
	p(1 - p)	.1225	.2296	.2041	.1684	.0663	.2041	.1684	.0663	.2041	.0663	

$K = 50$

$k_2 = 10$

$n_2 = 14$

$\sum T = 74$

$\sum T^2 = 434$

$\overline{T}_2 = 5.2857$

$s_2^2 = 3.0612$

$\sum p(1-p) = 1.5001$

$$\hat{\mu}_2 = \frac{(50)(5.2857)}{10} = 26.4285$$

$$\hat{\sigma}_2^2 = \frac{(14)(50)((50 - 1)(3.0612) - (50 - 10)(1.5001))}{10(10 - 1)(14 - 1)} = 53.8430$$

Table 4.3

Results And Computations For Subtest 3

Examinee	Class	Item 01	05	06	14	15	20	25	26	29	32	T
01	1	1	1	0	1	1	1	1	1	0	1	8
02	1	0	1	0	1	0	0	1	0	0	0	3
03	1	1	0	0	1	0	1	1	1	0	1	6
04	1	0	0	1	0	0	1	0	0	0	0	2
05	1	1	1	1	1	1	1	1	1	1	1	10
06	1	1	1	0	1	1	1	0	1	0	0	6
07	2	0	1	0	0	0	0	0	1	0	0	2
08	2	0	1	1	1	1	1	1	1	1	0	8
09	2	1	0	0	0	0	0	0	0	0	0	1
10	2	1	0	0	1	0	0	0	0	0	0	2
11	3	1	1	1	1	1	1	1	1	0	0	8
12	3	1	1	1	1	0	0	0	1	0	0	5
13	3	0	0	0	0	0	0	0	0	0	0	0
	p	.6154	.6154	.3846	.6923	.3846	.5385	.4615	.6154	.1538	.2308	
	p(1 - p)	.2367	.2367	.2367	.2130	.2367	.2485	.2485	.2367	.1301	.1775	

$K = 50$

$k_3 = 10$

$n_3 = 13$

$\sum T = 61$

$\sum T^2 = 411$

$\overline{T}_3 = 4.6923$

$s_3^2 = 9.5976$

$\sum p(1-p) = 2.2011$

$$\hat{\mu}_3 = \frac{(50)(4.6923)}{10} = 23.4615$$

$$\hat{\sigma}_3^2 = \frac{(13)(50)((50 - 1)(9.5976) - (50 - 10)(2.2011))}{10(10 - 1)(13 - 1)} = 230.0509$$

Table 4.4

Results And Computations For Subtest 4

Examinee	Class	Item 02	04	07	10	19	24	31	39	41	47	T
01	1	1	1	0	1	0	0	1	1	0	0	5
02	1	0	0	1	0	0	0	1	0	0	0	2
03	1	1	1	1	1	0	1	1	0	1	1	8
04	1	1	1	1	1	1	1	1	1	1	1	10
05	1	1	1	1	1	0	1	1	1	1	1	9
06	2	0	1	1	1	0	1	0	0	0	0	4
07	2	1	1	1	1	0	1	1	1	1	1	9
08	2	0	0	0	0	0	0	0	0	0	0	0
09	2	0	1	1	0	0	0	1	1	0	0	4
10	3	0	0	1	1	0	0	1	0	0	0	3
11	3	1	1	1	1	0	0	1	1	1	0	7
12	3	0	0	1	1	0	0	1	0	0	0	3
13	3	0	0	0	0	0	0	0	0	0	0	0
	p	.4615	.6154	.7692	.6923	.0769	.3846	.7692	.4615	.3846	.3077	
	p(1 - p)	.2485	.2367	.1775	.2130	.0710	.2367	.1775	.2485	.2367	.2130	

$K = 50$

$k_4 = 10$

$n_4 = 13$

$\sum T = 64$

$\sum T^2 = 454$

$\overline{T}_4 = 4.9231$

$s_4^2 = 10.6864$

$\sum p(1-p) = 2.0591$

$$\hat{\mu}_4 = \frac{(50)(4.9231)}{10} = 24.6155$$

$$\hat{\sigma}_4^2 = \frac{(13)(50)((50 - 1)(10.6864) - (50 - 10)(2.0591))}{10(10 - 1)(13 - 1)} = 265.5789$$

Table 4.5

Results And Computations For Subtest 5

Examinee	Class	Item 13	17	22	28	34	35	36	45	48	49	T
01	1	1	0	0	1	0	0	1	0	1	0	4
02	1	0	0	0	0	0	0	0	0	0	0	0
03	1	1	0	0	0	0	0	0	0	0	0	1
04	1	0	0	0	0	0	0	0	0	0	0	0
05	2	0	1	0	1	1	1	1	0	1	0	6
06	2	1	1	1	0	0	1	0	1	0	0	5
07	2	0	0	0	0	0	0	0	0	0	0	0
08	2	0	0	0	0	0	0	0	0	0	0	0
09	3	1	0	1	1	1	1	1	1	0	0	7
10	3	0	0	0	0	0	0	0	0	0	0	0
11	3	0	0	0	0	0	0	0	0	0	0	0
12	3	0	0	0	0	0	0	0	0	0	0	0
	p	.3333	.1667	.1667	.2500	.1667	.2500	.2500	.1667	.1667	.0000	
	p(1 - p)	.2222	.1389	.1389	.1875	.1389	.1875	.1875	.1389	.1389	.0000	

$K = 50$

$k_5 = 10$

$n_5 = 12$

$\sum T = 23$

$\sum T^2 = 127$

$\overline{T}_5 = 1.9167$

$s_5^2 = 6.9097$

$\sum p(1-p) = 1.4792$

$$\hat{\mu}_5 = \frac{(50)(1.9167)}{10} = 9.5835$$

$$\hat{\sigma}_5^2 = \frac{(12)(50)((50 - 1)(6.9097) - (50 - 10)(1.4792))}{10(10 - 1)(12 - 1)} = 169.3378$$

$\bar{T}_i$ = the mean test score on subtest i,

$s_i^2 = \sum^{n_i} (T - T_i)^2/n_i$, the variance of test scores on subtest i,

and

$\sum^{k_i} v_i$ = the sum of the k_i item variances in subtest i. If items are scored dichotomously, the variance of item j is equal to $p_j(1 - p_j)$ where p_j is the proportion of examinees answering item j correctly.

The computational formula for $\hat{\sigma}^2$ was derived from an associated formula given by Sirotnik (1970) in which it was assumed that the number of examinees and number of items in the population were both finite. Formula 4.2 is based on the assumption that the number of examinees in the population is infinite and that the number of items in the population is finite.

The results of each subtest provide an estimate of μ and σ^2 and a pooled estimate of μ and σ^2 is obtained by combining the t subtest estimates using

$$\hat{\mu}_{pooled} = \frac{\sum^{t} o_i \hat{\mu}_i}{\sum o_i} \quad (4.4)$$

and

$$\hat{\sigma}^2_{pooled} = \frac{\sum^{t} o_i \hat{\sigma}^2_i}{\sum o_i}, \quad (4.5)$$

where,

$$o_i = n_i k_i, \quad (4.6)$$

the number of observations obtained from subtest i. If the total number of examinees $\sum^{t} n_i = N$ is less than 500, $\hat{\sigma}^2_{pooled}$ should be multiplied by $(N - 1)/N$. Pooled estimates of μ and σ^2 for the word spelling project are given in Table 4.6. The pooled estimate of the mean test score on the 50-item test is 20.4287. On the basis of this result, the conclusion was made that kindergarten students can spell correctly approximately 40 per cent of words having regular spelling in the reading program without having had any formal spelling instruction.

Although individual estimates of the reliability of the 50-item test could have been obtained from each subtest and then combined into a single estimate, a simpler procedure for estimating α_{21} is one using the pooled estimates of μ and σ^2. Specifically,

$$\hat{\alpha}_{21} = \frac{K}{K - 1}\left[1 - \frac{\hat{\mu}_{pooled} - \frac{\hat{\mu}^2_{pooled}}{K}}{\hat{\sigma}^2_{pooled}}\right]. \qquad (4.7)$$

For the word spelling test α_{21} for the 50-item test was estimated from 4.7 to be .9479. The exact computations are given in Table 4.7 where $\hat{\alpha}_{21}$ is computed as an intermediate step in approximating the normative test score distribution with a probability distribution.

<u>Direct Calculation of SE($\hat{\mu}_{pooled}$)</u>

A more meaningful interpretation of $\hat{\mu}_{pooled}$ is possible if SE($\hat{\mu}_{pooled}$) is known. Although SE($\hat{\mu}_{pooled}$) and SE($\hat{\sigma}_{pooled}$) may be determined for all sampling plans through use of the simulation model described in Chapter V, Lord and Novick (1968, equation 11.12.3) have derived an equation for determining the standard error of the mean proportion correct score given (a) items are scored dichotomously, (b) items are sampled randomly and without replacement from the item population, and (c) examinees are sampled randomly and without replacement from the examinee population. Restrictions (b) and (c) produce item subsets and examinee subgroups which are nonoverlapping, i.e., no item is found in more than one subtest and no examinee

Table 4.6

Pooled Estimates Of Parameters From Subtest Results

Subtest	Number of Observations	n	k	$\hat{\mu}$	$\hat{\sigma}^2$
1	180	18	10	17.7780	158.1778
2	140	14	10	26.4285	53.8430
3	130	13	10	23.4615	230.0509
4	130	13	10	24.6155	265.5789
5	120	12	10	9.5835	169.3378
	700	70			

$$\hat{\mu}_{pooled} = \frac{(180)(17.7780) + (140)(26.4285) + \ldots + (120)(9.5835)}{180 + 140 + \ldots + 120}$$

$$= 20.4287$$

$$\hat{\sigma}^2_{pooled} = \frac{(180)(158.1778) + (140)(53.8430) + \ldots + (120)(169.3378)}{180 + 140 + \ldots + 120}$$

$$= 172.5178$$

$N \angle 500$

$$= 172.5178\ [(70 - 1)/70]$$

$$= 170.0533$$

is found in more than one subgroup. Equation 11.12.3 when modified to give the standard error of the mean test score is

$$\mathrm{VAR}(\hat{\mu}_{pooled}) = [\frac{1}{tkn}] [\frac{1}{(K-1)(N-1)}] \cdot$$

$$[K^2 N \sigma_p^2 \{(K-k)(n-1) - kn(t-1)\} + \tag{4.8}$$

$$K \sigma^2 \{(N-n)(k-1) - kn(t-1)\} +$$

$$\hat{\mu}(K-\hat{\mu}) \{(K-k)(N-n) + kn(t-1)\}] ,$$

where K refers to the total number of test items,

N to the total number of examinees,

σ^2 to the population variance,

σ_p^2 to the variance of item difficulty indices, and

$\hat{\mu}$ to the estimate of the population mean obtained from multiple matrix sampling.

In practice, σ^2 and σ_p^2 are estimated; t, k, and n are parameters defining the sampling plan. Of course, $\mathrm{SE}(\hat{\mu}_{pooled}) = \mathrm{VAR}(\hat{\mu}_{pooled})^{\frac{1}{2}}$. No equation is given by Lord and Novick for $\mathrm{SE}(\hat{\sigma}_{pooled})$ under multiple matrix sampling.

Approximating the Normative Distribution

In addition to estimating individual parameters through multiple matrix sampling, it is possible to estimate the entire normative frequency distribution of test scores which would have been obtained by testing all students on all 50 items. The negative hypergeometric distribution has been shown by Keats and Lord (1962) to provide a reasonably good fit for a wide variety of test score distributions when the test score is the number of correct responses. The negative hypergeometric distribution is a function of the mean test score μ, the variance of the test scores σ^2 and the total number of items in the test K. Lord (1962) and Shoemaker (1970) have demonstrated that the negative hypergeometric distribution with parameters estimated by multiple matrix sampling can be used satisfactorily to approximate normative distributions of number correct test scores. The formula for the negative hypergeometric distribution is

$$h(T) = \frac{c(-K)_T(a)_T}{(-b)_T T!} \quad \text{for } T = 0, 1, 2, \ldots, K \qquad (4.9)$$

where,

$$a = (-1 + 1/\hat{\alpha}_{21})\hat{\mu}_{pooled}$$

$$b = -a - 1 + K/\hat{\alpha}_{21}$$

$$c = \frac{b^{[K]}}{(a + b)^{[K]}}$$

noting that,

$$b^{[K]} = b(b - 1)(b - 2) \ldots (b - K + 1)$$

$$(a)_T = a(a + 1)(a + 2) \ldots (a + T - 1)$$

$$(a)_0 = b^{[0]} = 1$$

$$T! = T(T - 1)(T - 2) \ldots (2)(1).$$

Using estimates of μ and σ^2 obtained from the word spelling project, the calculations necessary for approximating the normative distribution on the 50-item test with the negative hypergeometric distribution are illustrated in Table 4.7 with complete results given in Table 4.9. The computations involved in estimating μ and σ^2 and approximating the normative distribution by the negative hypergeometric distribution are more laborious than difficult. A computer program has been developed which performs all the necessary computations and output for the word spelling project is given in Tables 4.8 and 4.9. A detailed writeup and listing of the computer program is given in Appendix A.

An examination of the estimates of parameters given in Table 4.8 suggests that individual subtests were not equally difficult, particularly subtest 5. Although the words included in subtest 5 were selected randomly from the 50-word population and administered to subgroups of examinees selected at random from each class, the results merely confirm the well-known fact that extreme cases do occur through random sampling. An obvious advantage, then, of multiple matrix sampling over any individual item-examinee sample is that the estimates obtained in the former case are based on a composite and hence less subject to sampling extremities. Stated more precisely, the standard error associated with the pooled estimate of the mean test score is less than the standard error associated with any of the estimates of the mean obtained from subtests. The results for μ and σ given in Chapter V illustrate adequately the difference in standard errors of estimate described here.

The relative frequencies given in Table 4.9 are actually the individual probabilities associated with all possible test scores. For example, the probability of an examinee spelling correctly 20 words out of 50 is .023. An equally appropriate interpretation is that 2.3 per cent of the examinees in the population would spell correctly 20 words. As should be the case, the relative frequencies in Table 4.9 sum to unity. An estimate of the number of examinees receiving each test score is obtained by multiplying the total number of examinees in the population by the probability associated with each test score. For example, if there were 1000 students in the population of kindergarteners, 23 students would be expected to spell correctly 20 of the 50 words on the test.

Although equations 4.1 and 4.2 are appropriate for all item scoring procedures, the negative hypergeometric distribution is used only when the test score is the number of correct answers. This is, of course, the case when items are scored 1 = pass and 0 = fail. When items are <u>not</u> scored dichotomously, the normative frequency distribution may be approximated by a Pearson curve using the first moment about the origin and the second, third and fourth moments about the mean. There are 12 curves in the family of Pearson curves and the procedure for selecting the appropriate curve and making the necessary calculations to approximate the normative distribution are given by Elderton (1938, pp. 38-127) and by Kendall (1952, pp. 137-145). Lord (1960) has suggested that a Pearson Type I curve may be an appropriate selection. It should be mentioned, however, that such a procedure is not a casual undertaking. Before such procedures can be used, computational formulas for estimating μ_3 and μ_4 must be derived. Guidelines for estimating these moments are given by Hooke (1956).

Table 4.7

Computations For Negative Hypergeometric Distribution

$$\hat{\alpha}_{21} = (50/(50 - 1))(1 - (20.4286 - 20.4286^2/50)/170.0552) = .9479$$

$$a = (-1 + 1/.9479)(20.4287) = 1.1226$$

$$b = -1.1226 - 1 + 50/.9479 = 50.6250$$

$$c = \frac{50.6250^{[50]}}{51.7476^{[50]}} = \frac{50.6250(50.6250-1)(50.6250-2)\ \ldots\ (50.6250-49)}{51.7476(51.7476-1)(51.7476-2)\ \ldots\ (51.7476-49)} = .0214$$

$$h(0) = (.0214)\frac{(-50)_0(1.1226)_0}{(-50.6250)_0\ 0!} = (.0214)\frac{(1)(1)}{(1)(1)} = .0214$$

$$h(1) = (.0214)\frac{(-50)_1(1.1226)_1}{(-50.6250)_1\ 1!} = (.0214)\frac{(-50)(1.1226)}{(-50.6250)(1)} = .0237$$

$$h(2) = (.0214)\frac{(-50)_2(1.1226)_2}{(-50.6250)_2\ 2!} = (.0214)\frac{(-50)(-49)(1.1226)(2.1226)}{(-50.6250)(-49.6250)(2)} = .0248$$

Table 4.8

Estimates Of Parameters For Word Spelling Project

Sample	Estimate Of Parameter	
	Mean	Variance
1	17.7777770	158.1844600
2	26.4285710	53.8461540
3	23.4615380	230.0498600
4	24.6153840	265.5769300
5	9.5833331	169.3392200

POOLED MEAN = 20.4285710

POOLED VARIANCE = 170.0552200

Table 4.9

Estimated Relative Frequency Per Test Score On The 50-Item Test Using The Negative Hypergeometric Distribution

Score	Relative Frequency	Score	Relative Frequency
0	.0213564		
1	.0236785	26	.0216387
2	.0248133	27	.0211852
3	.0254953	28	.0207151
4	.0259319	29	.0202280
5	.0262115	30	.0197236
6	.0263806	31	.0192016
7	.0264667	32	.0186613
8	.0264872	33	.0181020
9	.0264544	34	.0175230
10	.0263766	35	.0169233
11	.0262602	36	.0163017
12	.0261101	37	.0156567
13	.0259298	38	.0149867
14	.0257223	39	.0142896
15	.0254900	40	.0135627
16	.0252347	41	.0128031
17	.0249578	42	.0120066
18	.0246608	43	.0111683
19	.0243444	44	.0102815
20	.0240095	45	.0093369
21	.0236568	46	.0083215
22	.0232868	47	.0072158
23	.0228997	48	.0059869
24	.0224960	49	.0045728
25	.0220756	50	.0028209

Computational Irregularities

In estimating parameters from subtests having a small number of items and examinees, it happens frequently that $\hat{\sigma}^2$ is equal to zero or is less than zero for one or more subtests. Although uninterpretable, estimates such as these should not be discarded or set equal to zero in computing $\hat{\sigma}^2_{pooled}$. It must be remembered that results of any subtest are relatively unimportant; what is important is the accuracy of the pooled estimate of σ^2. Any procedure which ignores part of the data produces an estimate of σ^2 which is biased, i.e., it would not approach the true value even if the number of subtests was increased indefinitely. Sirotnik (1970) has verified empirically this conclusion.

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V

Computer Simulation of Multiple Matrix Sampling

In evaluating a particular sampling plan or contrasting the relative merits of several plans used in multiple matrix sampling, statistics of primary importance are the standard error of estimate and the mean estimate for each parameter given that sampling plan. For example, if an investigator were estimating parameters of a 50-item test using multiple matrix sampling, one sampling plan might be (5/20/30); another, (10/10/60). In the first sampling plan, the 50-item test is subdivided through random sampling without replacement within subtests and with replacement between subtests into 5 subtests containing 20 items each with each subtest administered to 30 examinees; in the second plan, similarly, 10 subtests containing 10 items each with each subtest administered to 60 examinees. The sampling plan selected will be used only once in an investigation; yet, in selecting the particular plan to be used, the investigator must be aware of the standard error of estimate associated with each sampling plan under consideration. Lord and Novick (1968, equation 11.12.3) have derived algebraically the standard error of estimating the mean proportion correct score given nonoverlapping random samples of dichotomously-scored items drawn without replacement from the item population. No comparable equation is given by them for computing the standard error of estimating the population standard deviation under multiple matrix sampling. What is required, however, are equations for estimating standard errors of estimate per parameter for all potentially useful sampling plans, not just those plans involving nonoverlapping random samples of items from the item population. The computational difficulties in such a task are not minor; however, the results of such equations are approximated readily and to any desired degree of accuracy through the computer simulation model described herein. The remaining sections of this chapter are devoted to a detailed discussion of a simulation model for multiple matrix sampling. The reader uninterested in such matters can bypass safely this chapter without a loss of continuity. However, several of the guidelines for multiple matrix sampling given in Chapter III are based on results obtained through use of this model and, it must be stressed, that the results obtained are only as good as the simulation model used.

Simulated Post Mortem Sampling

The algorithm used within the model is described most appropriately as simulated post mortem multiple matrix sampling. In post mortem sampling, item-examinee samples are taken from an N by K data base obtained by testing N examinees over K items; in simulated post mortem sampling, the N by K data base is computer-generated by a simulation model. Generating data bases with prescribed parameters is essential in investigating hypotheses in multiple matrix sampling because it is difficult, if not impossible, to locate existing data bases having the necessary variation in test parameters. For example, if the standard error of estimate were being investigated as a function of variation in item difficulty indices for a given test reliability and test length, it would be difficult locating data bases with $\sigma_p^2 = .00$, .05, and .08 all having $\alpha_{20} = .80$ and the same test length. Such a problem is, however, handled easily with a simulation model. As an overview, the computer program generates a data base, selects multiple item-examinee samples from this data base, performs all calculations necessary for estimating parameters, and replicates this procedure as many times as specified before computing the standard error of estimate and mean estimate per parameter over replications. The computer program is restricted to data bases having dichotomously-scored items and, in multiple matrix sampling, to subtests having an equal number of items and examinees.

Generation of Data Bases

In simulating multiple matrix sampling, generation of the data base is of primary importance. Although one procedure might be that of generating an N by K matrix and storing it in memory, a more appropriate procedure is one in which the item scores on the K-item test are generated for one and only one individual at a time. All that is stored in memory are the K item scores for <u>one</u> individual. The procedure, however, for generating item scores must be one such that, <u>over any number of hypothetical examinees generated</u>, the items and test scores have prescribed characteristics. In this procedure, the population of examinees N is countably infinite. The test parameters subject to manipulation within the program are: (a) K, the number of items in the item population, (b) μ, the mean test score over examinees, (c) σ^2, the variance of test scores over examinees, (d) α_{20}, the coefficient of reliability for the K-item test, (e) σ_p^2, the variance of the item difficulty indices, where, the difficulty index p_i for item i is the proportion of examinees answering correctly item i, and (f) the degree of skewness in the distribution of test scores for examinees on the K-item test. In the computer program, values for K, μ and σ_p^2 must be specified by the user. The maximum value for K is 150; μ is, therefore, restricted to values $0 < \mu < K$. If α_{20} is specified, σ^2 is determined by the well-known relationship

$$\alpha_{20} = \frac{K}{K - 1}\left[\frac{\sigma^2 - \mu + \frac{\mu^2}{K} + K\sigma_p^2}{\sigma^2}\right] \tag{5.1}$$

derived originally by Tucker (1949). If σ^2 is specified by the user, α_{20} is determined consequently. Such an arrangement has been incorporated within the program to facilitate hypothesis testing where either σ^2 or α_{20} is to be controlled across levels of K. Of course, $\bar{p} = \mu/K$ is determined once μ has been specified. The degree of skewness in the normative distribution is simulated by using the lognormal or normal probability distribution functions to generate test score distributions. The lognormal distribution with two parameters is used to generate positively-skewed test score distributions while the three parameter lognormal distribution is used for negatively-skewed distributions. The lognormal distribution is described in detail by Aitchison and Brown (1957) and a detailed explanation of simulating stochastic variates with the lognormal distribution is given by Naylor, Balintfy, Burdick and Chu (1966). The normal density function is, of course, used to simulate normal test score distributions. Density functions for the two and three parameter lognormal probability distributions are, respectively,

$$\Lambda\ (T \mid \mu, \sigma^2) = \frac{1}{T\sigma\sqrt{2\Pi}} \exp\left[- \frac{\{\ln(T) - \mu\}^2}{2\sigma^2} \right] \tag{5.2}$$

$$\Lambda\ (T' \mid T' = K - T, \mu, \sigma^2) = \frac{1}{T\sigma\sqrt{2\Pi}} \exp\left[- \frac{\{\ln(T) - \mu\}^2}{2\sigma^2} \right] \tag{5.3}$$

for T = 0, 1, 2, ... , K.

For the normal distribution, the density function is

$$N(T \mid \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\Pi}} \exp\left[- \frac{(T - \mu)^2}{2\sigma^2} \right]. \tag{5.4}$$

The constants μ and σ^2 in 5.4 are equal, respectively, to the desired mean and variance in the normative distribution; however, in 5.2 and 5.3, μ and σ^2 are a function of the desired mean and variance in the normative distribution. If the desired mean and variance of the normative distribution are denoted, respectively, by α and β, μ and σ^2 in 5.2 and 5.3 are computed by

$$\mu = \ln(\alpha) - \frac{\ln(\beta^2/\alpha^2 + 1)}{2} \qquad (5.5)$$

and

$$\sigma^2 = \ln(\beta^2/\alpha^2 + 1). \qquad (5.6)$$

The appropriate derivations for 5.5 and 5.6 are given by Naylor, Balintfy, Burdick and Chu (1966). If z is a random normal deviate $N(0,1)$, test scores T having lognormal distributions are generated by

$$T_i = \exp(\mu + \sigma z_i) \quad i=1,2, \ldots , N \qquad (5.7)$$

for positively-skewed distributions, and

$$T_i = K - \exp(\mu + \sigma z_i) \quad i=1,2, \ldots , N \qquad (5.8)$$

for negatively-skewed distributions. For normal distributions,

$$T_i = \mu + \sigma z_i \quad i=1,2, \ldots , N \qquad (5.9)$$

The T scores computed in 5.7, 5.8 and 5.9 will be continuous variables. Because items are scored dichotomously, the T score must be rounded to the nearest integer value. The midpoint of each score interval is taken to be that point above which one-half of the area in that score interval is found. This point is found by integrating via trapezoid rule the area under the appropriate normal or lognormal curve. If the T score is equal to or greater than the midpoint, the score is rounded up; if not, the score is rounded down.

Item scores are related to test scores. Specifically, if X_{ij} is the item score for examinee i on item j, $\sum_{j}^{K} X_{ij} = T_j$. Also, $\bar{p} = \bar{T}/K$. If σ_p^2 is greater than zero, individual item difficulty indices are generated by

$$p_i = \bar{p} + \sigma_p z_i \quad i=1,2, \ldots , K \tag{5.10}$$

where z_i is a random normal deviate. When σ_p^2 is not equal to zero, the distribution of p_i values will be approximately normal. If σ_p^2 is equal to zero, $p_i = \bar{p}$ for all values of i. With skewed distributions, σ_p^2 is typically $0 < \sigma_p^2 < .001$ and, because of this, σ_p^2 is set to zero for all skewed distributions generated by the simulation model. After the item difficulty indices have been generated within the program, deciding if an examinee passes or fails each item is relatively simple. Item difficulty indices are computed for all items generated. An examinee "passes" those items which will bring the computed item difficulty indices most closely to the desired item difficulty indices. For example, if the computed item difficulty for item i were less than the desired item difficulty for item i, examinee j would pass item i; if the computed difficulty were equal to or greater than the desired item difficulty, he "fails" item i. In the program, the desired item difficulty indices are sorted in descending order. If, in following the algorithm from the first through the K<u>th</u> item, $\sum^{K} X_{ij} \neq T_j$, the first $T_j - \sum^{K} X_{ij} = d$ items not already passed by examinee j are scored by the program as items answered correctly by him.

The validity of the simulation model is found in its ability to generate the desired data base. Two examples of data bases generated by the model are given in Tables 5.1 and 5.2. Although the discrepancies in Table 5.2 are minor, it should be noted that the magnitude of the discrepancies decreases with increases in K.

Simulation of Multiple Matrix Sampling

Subtests are constructed within the program by sampling at random items from the K-item population. For example, if K equals 50 and a (5/10/30)

Table 5.1

Results Obtained From Simulation Model For 3000 Examinees When K = 20 With The Normative Distribution Distributed Normally

Parameter	Input	Output
K	20	20
μ	10.0000	10.0150
σ^2		computed
α_{20}	.8000	.7999
$\bar{p}$	.5000	.5010
σ^2_p	.0800	.0799
N	3000	3000

Item Difficulty Indices

Item	Input	Output
1	.987	.987
2	.901	.901
3	.857	.857
4	.782	.787
5	.750	.750
6	.737	.738
7	.702	.703
8	.634	.634
9	.565	.566
10	.518	.519
11	.458	.458
12	.442	.442
13	.430	.430
14	.374	.374
15	.275	.275
16	.208	.209
17	.139	.140
18	.113	.114
19	.112	.113
20	.026	.026

Obtained Frequency Dist.

Score	Frequency
0	16
1	17
2	36
3	47
4	96
5	136
6	182
7	225
8	307
9	266
10	328
11	327
12	256
13	210
14	191
15	122
16	104
17	67
18	38
19	16
20	13

Table 5.2

Results Obtained From Simulation Model For 3000 Examinees When K = 20 With The Normative Distribution Negatively-Skewed (Three Parameter Lognormal Distribution)

Parameter	Input	Output
K	20	20
μ	17.5000	17.6150
σ^2		computed
α_{20}	.8000	.7570
$\bar{p}$	.8750	.8810
σ_p^2	.0000	.0002
N	3000	3000

Item Difficulty Indices			Obtained Frequency Dist.	
Item	Input	Output	Score	Frequency
			0	16
1	.875	.930	1	3
2	.875	.908	2	2
3	.875	.888	3	5
4	.875	.876	4	4
5	.875	.876	5	3
6	.875	.876	6	3
7	.875	.876	7	6
8	.875	.876	8	7
9	.875	.876	9	8
10	.875	.876	10	15
11	.875	.876	11	32
12	.875	.876	12	32
13	.875	.876	13	43
14	.875	.876	14	84
15	.875	.876	15	135
16	.875	.876	16	200
17	.875	.876	17	352
18	.875	.876	18	670
19	.875	.875	19	885
20	.875	.875	20	495

sampling plan were used, 5 subtests would be formed by sampling without replacement from the 50-item pool 10 items for each subtest. If (10/10/30) were used, 10 subtests would be formed containing 10 items each; however, the sampling plan for items requires sampling without replacement for each subtest but with replacement between subtests. In (10/10/30), several items will be common to more than one subtest. Taking (10/10/30) as an example, item scores on the K-item test would be generated by the program for 300 examinees. For subtest 1, the data from the first 30 examinees would be processed for only those items included in subtest 1. An identical procedure is followed for subtest 2 through subtest 10. The computations performed on each item-examinee sample are identical to those outlined in Chapter IV. If the user opts r replications of a particular sampling plan, r pooled estimates of each parameter will be produced and the standard error of estimate per parameter with that sampling plan is the standard deviation of the r pooled estimates for each parameter. Sample output for the (10/15/30) plan with 5 replications is given on page 42 through 49 for the normal normative distribution case.

Uses for the Simulation Model

It is anticipated that the computer program for simulating multiple matrix sampling described herein, and listed with expanded writeup in Appendix B, will facilitate readily a detailed examination of the relative merits of one or more sampling plans in multiple matrix sampling. In multiple matrix sampling questions asked frequently are "How do I do it?" and "If I sample this way, how accurate will the estimates be?" Questions such as these are answered easily through use of the simulation model. The results obtained from the program are reasonable to the degree that the normative distributions can be described adequately by the normal and lognormal probability distributions. It is commonly known that achievement test scores are frequently normally distributed. However, the scores on criterion-referenced tests, i.e., end-of-program tests, are frequently markedly negatively-skewed and resemble closely a three parameter lognormal distribution. It is anticipated that the simulation model will prove to be an asset in test theory and test construction courses permitting the student to have a working familiarity with sampling procedures used in multiple matrix sampling.

REFERENCES

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Naylor, T. H., Balintfy, J. L., Burdick, D. S. & Chu, K. Computer simulation techniques. New York: Wiley, 1966.

Tucker, L. R. A note on the estimation of test reliability by the Kuder-Richardson Formula 20. Psychometrika, 1949, 14, 117-119.

PROBLEM NO. 2265.06

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PARAMETERS INPUTTED

MEAN	10.000
VARIANCE	0.0
K	20
ALPHA 20	0.700
VAL (P)	0.0

ITEM-EXAMINEE SAMPLING PLAN

NE	[illegible]
IPT	[illegible]
NSPT	[illegible]
NREPS	5

SWITCHES

ITEM-SAMPLING PLAN	0
NEGATIVE HYPERGEOMETRIC	0
NORMATIVE DISTRIBUTION	1
INTERMEDIATE PRINTOUT	1

COMPUTED SIGMA = 3.86334

ITEM DIFFICULTY INDICES

[illegible] [illegible] [illegible] 0.500 0.500 0.500 0.500 0.500 0.500 [illegible]
[illegible] [illegible] [illegible] 0.500 0.500 0.500 0.500 0.500 0.500 [illegible]

DISTRIBUTION ORDINATES

N	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
0	0.0036	0.0039	0.0041	0.0044	0.0047	0.0050	0.0054	0.0057	0.0061	0.0064	0.0068
1	0.0068	0.0073	0.0077	0.0082	0.0087	0.0092	0.0097	0.0103	0.0109	0.0115	0.0121
2	0.0121	0.0128	0.0135	0.0142	0.0149	0.0157	0.0165	0.0173	0.0182	0.0191	0.0200
3	0.0200	0.0210	0.0219	0.0230	0.0240	0.0251	0.0262	0.0273	0.0285	0.0297	0.0309
4	0.0309	0.0322	0.0335	0.0348	0.0361	0.0375	0.0389	0.0403	0.0417	0.0432	0.0447
5	0.0447	0.0462	0.0477	0.0493	0.0508	0.0524	0.0540	0.0556	0.0572	0.0588	0.0604
6	0.0604	0.0620	0.0637	0.0653	0.0659	0.0685	0.0701	0.0717	0.0733	0.0748	0.0764
7	0.0764	0.0779	0.0794	0.0809	0.0823	0.0838	0.0851	0.0865	0.0878	0.0891	0.0903
8	0.0903	0.0915	0.0926	0.0937	0.0948	0.0958	0.0967	0.0976	0.0984	0.0992	0.0999
9	0.0999	0.1005	0.1011	0.1016	0.1020	0.1024	0.1027	0.1030	0.1031	0.1032	0.1033
10	0.1033	0.1032	0.1031	0.1030	0.1027	0.1024	0.1020	0.1016	0.1011	0.1005	0.0999
11	0.0999	0.0992	0.0984	0.0976	0.0967	0.0958	0.0948	0.0937	0.0926	0.0915	0.0903
12	0.0903	0.0891	0.0878	0.0865	0.0851	0.0838	0.0823	0.0809	0.0794	0.0779	0.0764
13	0.0764	0.0748	0.0733	0.0717	0.0701	0.0685	0.0669	0.0653	0.0637	0.0620	0.0604
14	0.0604	0.0588	0.0572	0.0556	0.0540	0.0524	0.0508	0.0493	0.0477	0.0462	0.0447
15	0.0447	0.0432	0.0417	0.0403	0.0389	0.0375	0.0361	0.0348	0.0335	0.0322	0.0309
16	0.0309	0.0297	0.0285	0.0273	0.0262	0.0251	0.0240	0.0230	0.0219	0.0210	0.0200
17	0.0200	0.0191	0.0182	0.0173	0.0165	0.0157	0.0149	0.0142	0.0135	0.0128	0.0121
18	0.0121	0.0115	0.0109	0.0103	0.0097	0.0092	0.0087	0.0082	0.0077	0.0073	0.0068
19	0.0068	0.0064	0.0061	0.0057	0.0054	0.0050	0.0047	0.0044	0.0041	0.0039	0.0036

AREA = 0.9903524

ROUNDING RULE	SCORE	ROUND	SCORE
	0	0.58	1
	1	1.57	2
	2	2.57	3
	3	3.56	4
	4	4.55	5
	5	5.54	6
	6	6.53	7
	7	7.53	8
	8	8.52	9
	9	9.51	10
	10	10.50	11
	11	11.49	12
	12	12.48	13
	13	13.48	14
	14	14.47	15
	15	15.46	16
	16	16.45	17
	17	17.44	18
	18	18.44	19
	19	19.43	20

SUBTEST	ITEMS														
1	2	8	15	20	3	19	13	9	14	7	4	6	1	12	16
2	10	12	1	2	5	11	7	16	17	20	3	15	9	19	13
3	7	9	13	19	2	3	18	10	20	12	11	4	14	6	17
4	8	2	15	20	16	5	3	13	11	18	14	17	4	7	1
5	10	11	17	3	9	14	19	16	1	12	6	13	8	7	18
6	18	14	20	17	15	7	13	16	3	6	9	2	4	12	1
7	16	9	13	17	4	2	19	20	10	6	7	8	18	15	1
8	20	11	7	4	2	1	12	3	15	5	17	13	3	18	9
9	20	2	12	14	15	16	1	6	8	9	7	5	4	13	19
10	9	11	5	3	17	18	14	7	13	19	8	2	1	16	10

ESTIMATE OF PARAMETER

SAMPLE	MEAN	VARIANCE	STANDARD DEV
1	10.6666660	16.0153198	4.0019140
2	9.7333317	10.1582947	3.1872072
3	9.5555534	14.6546278	3.8281355
4	9.5999985	14.3192110	3.7840729
5	10.1777763	17.9327850	4.2347116
6	9.3333330	19.4001007	4.4045544
7	11.1111097	16.0897675	4.0112047
8	9.4666653	13.4410610	3.6662045
9	10.0000000	9.3607025	3.0595264
10	10.7999983	12.0836411	3.4761524

REPLICATION NO.	1	EST. OF MEAN	10.044
		EST. OF STANDARD DEVIATION	3.791

SUBTEST	ITEMS														
1	17	18	1	2	10	4	12	5	8	15	20	9	14	11	7
2	14	4	19	1	20	10	2	5	11	13	6	17	8	7	15
3	6	4	12	14	18	7	3	19	11	20	8	2	5	1	16
4	8	10	7	20	17	2	3	16	19	5	14	13	4	12	15
5	19	6	7	18	4	11	10	13	1	15	14	17	20	5	2
6	14	8	6	1	17	11	18	12	19	7	2	16	4	15	3
7	5	7	14	8	18	[illegible]	20	19	1	17	12	2	15	16	4
8	18	12	14	17	2	5	10	20	8	9	13	3	4	16	7
9	9	14	3	12	8	11	13	15	17	10	4	2	7	19	6
10	8	19	6	5	20	1	7	15	9	4	11	12	10	13	17

ESTIMATE OF PARAMETER

SAMPLE	MEAN	VARIANCE	STANDARD DEV
1	10.6666660	16.0547180	4.0068340
2	9.5555534	16.5243530	4.0650158
3	11.0222216	16.0278015	4.00347[illegible]
4	8.9333315	13.3120995	3.6485748
5	10.2222204	11.9441700	3.4560337
6	10.8444433	9.0143509	3.002390[illegible]
7	10.5333319	9.9500961	3.1543770
8	9.9111090	16.5915680	4.0732746
9	9.4666653	17.8132477	4.2205734
10	10.5333319	18.1821594	4.2640543

REPLICATION NO. 2 EST. OF MEAN 10.169
EST. OF STANDARD DEVIATION 3.807

SUBTEST	ITEMS														
1	4	19	7	6	18	17	3	10	9	15	14	2	1	11	13
2	11	7	10	13	1	4	16	2	6	3	8	15	17	5	19
3	9	20	4	2	16	15	6	1	11	13	7	3	12	17	14
4	5	8	6	3	9	4	14	15	17	11	18	19	1	20	2
5	18	9	14	11	19	17	16	10	7	13	1	15	2	5	3
6	1	16	10	19	6	7	8	15	5	18	12	13	4	14	9
7	9	13	17	3	1	16	10	2	14	19	5	15	20	12	7
8	5	9	7	6	18	11	19	10	13	20	3	1	15	12	4
9	11	12	5	8	16	2	15	14	19	9	7	18	13	17	20
10	15	7	13	14	11	1	8	5	20	12	17	16	19	9	6

ESTIMATE OF PARAMETER

SAMPLE	MEAN	VARIANCE	STANDARD DEV
1	10.7999983	7.2483969	2.6922846
2	9.2888870	21.0778351	4.5910597
3	10.6222200	9.0833139	3.0138531
4	9.8666649	14.9670553	3.8687277
5	9.1999989	21.3140564	4.6167145
6	10.5777760	14.8761902	3.8569660
7	9.4666653	15.3042278	3.9120617
8	9.4666653	16.3124237	4.0388641
9	10.0000000	16.4783630	4.0593548
10	10.2666655	10.9070606	3.3025837

REPLICATION NO. 3 — EST. OF MEAN 9.956
EST. OF STANDARD DEVIATION 3.835

SUBTEST	ITEMS														
1	4	2	1	5	7	16	15	13	18	10	8	14	6	9	3
2	17	18	1	3	12	11	13	4	8	15	20	2	6	5	10
3	3	13	16	7	20	17	2	5	9	12	8	1	19	14	4
4	4	9	15	13	6	18	19	11	16	3	5	20	12	10	2
5	4	10	5	6	12	18	8	13	17	14	15	7	16	19	11
6	3	17	18	14	5	8	4	10	7	13	11	9	16	1	6
7	11	15	9	7	18	12	6	10	14	13	4	19	17	8	3
8	6	9	19	1	15	2	14	11	20	5	7	12	16	10	3
9	16	13	20	5	7	2	10	8	12	17	18	3	15	19	11
10	3	17	16	9	12	8	5	11	4	15	18	1	6	2	19

ESTIMATE OF PARAMETER

SAMPLE	MEAN	VARIANCE	STANDARD DEV
1	10.2222204	22.0361176	4.6942635
2	10.4888868	18.7351990	4.3284163
3	11.5555534	14.8910809	3.8588963
4	9.2444420	15.0130234	3.8745634
5	10.1333323	28.0562439	5.2968140
6	10.6222200	12.2294531	3.4970627
7	10.8888874	14.8166456	3.8492393
8	9.5555534	10.1718702	3.1893368
9	9.9555540	15.4686365	3.9330187
10	9.5111103	15.8550692	3.9818470

REPLICATION NO.	4	EST. OF MEAN	10.218
		EST. OF STANDARD DEVIATION	4.083

SUBTEST	ITEMS														
1	18	9	15	14	8	4	17	11	20	12	13	5	19	16	1
2	12	11	16	19	13	8	20	1	6	14	7	17	18	10	5
3	3	8	18	17	15	5	12	13	11	6	2	9	16	4	1
4	15	19	9	3	18	16	10	14	2	12	5	7	11	4	8
5	8	3	7	17	2	19	20	13	4	12	14	10	16	18	1
6	8	5	20	13	19	3	7	14	12	9	18	10	15	6	17
7	8	15	17	12	3	5	19	6	20	7	19	1	16	4	9
8	8	9	6	16	5	11	1	20	2	10	15	4	7	13	3
9	9	4	8	14	16	10	20	13	12	3	7	18	19	1	11
10	1	9	7	4	12	2	10	11	6	20	5	16	3	15	18

ESTIMATE OF PARAMETER

SAMPLE	MEAN	VARIANCE	STANDARD DEV
1	10.7555552	11.9008207	3.4497557
2	10.3111086	21.6545105	4.6534405
3	10.0888892	17.2166290	4.1492920
4	10.5777760	14.3824806	3.7924232
5	10.0444431	21.9667206	4.6868668
6	10.9777756	13.8876839	3.7266178
7	11.0222216	17.3239136	4.1622000
8	9.2888870	12.2294531	3.4970627
9	10.1333323	15.0863810	3.8841190
10	11.1555538	14.1539202	3.7621689

REPLICATION NO. 5 EST. OF MEAN 10.436

EST. OF STANDARD DEVIATION 3.991

AVERAGE POOLED MEAN OVER REPS 10.1644287 SE OF POOLED MEAN OVER REPS 0.1643227

AVERAGE POOLED SD OVER REPS 3.8394379 SE OF POOLED SD OVER REPS 0.1175046

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VI

Hypothesis Testing and Multiple Matrix Sampling

Parameters estimated through multiple matrix sampling are integrated easily into a variety of hypothesis testing procedures. For example, one-sample and two-sample t-tests can be performed readily with estimates of μ and σ^2 obtained by multiple matrix sampling. Specifically,

$$t_{df=N-1} = \frac{\hat{\mu}_{pooled} - \mu_{standard}}{(\hat{\sigma}^2_{pooled}/N)^{\frac{1}{2}}} \qquad (6.1)$$

and

$$t_{df=N_1+N_2-2} = \frac{\hat{\mu}_{1_{pooled}} - \hat{\mu}_{2_{pooled}} - (\mu_1 - \mu_2)}{\left[\frac{(N_1-1)\hat{\sigma}^2_{1_{pooled}} + (N_2-1)\hat{\sigma}^2_{2_{pooled}}}{N_1+N_2-2}\right]^{\frac{1}{2}}\left[\frac{1}{N_1}+\frac{1}{N_2}\right]} . \qquad (6.2)$$

The t-test for the difference between two independent means given in 6.2 can be extended to completely randomized and factorial analysis of variance designs where the dependent variable is a mean test score. Although analysis of variance designs with mean scores as the dependent variable are found infrequently in the literature, the frequently occuring circumstances in which mean scores are preferable to raw scores in such analyses are detailed most succinctly by Peckham, Glass and Hopkins (1969).

Consider the design in which the relative merits of four experimental training programs are being contrasted through end-of-program test scores obtained from students participating in each procedure. Through an analysis of pretest scores given to all students, ten classes have been selected for each training program such that, across training programs, the four groups of 10 classes are approximately homogeneous at the start of instruction. The mean achievement test score for each class is estimated easily through multiple matrix sampling. The statistical layout and sources of variation are given in Table 6.1. If an additional variable, such as school district, were added to the design, the statistical layout and sources of variation are modified slightly as seen in Table 6.2. After the measurement on the dependent variable is accomplished, computations in the analysis of variance proceed in the usual manner. The novelty herein is in estimating the class mean test score through multiple matrix sampling.

Testing homogeneity of variance hypotheses of the form $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma^2$ is accomplished for two variances by

$$F_{(N_1-1,N_2-1)} = \frac{\hat{\sigma}^2_{1_{pooled}}}{\hat{\sigma}^2_{2_{pooled}}}, \qquad (6.3)$$

and for more than two variances by, for example,

$$F_{max} = \frac{\hat{\sigma}^2_{largest}}{\hat{\sigma}^2_{smallest}}. \qquad (6.4)$$

Tables for the F_{max} statistic have been constructed by Hartley and are given in Winer (1962, p. 653). Another simple test for homogeneity of variance developed by Cochran which lends itself to multiple matrix sampling is

$$C = \frac{\hat{\sigma}^2_{\text{largest}}}{\sum \hat{\sigma}^2} \qquad (6.5)$$

and the necessary tables for the C statistic are given in Winer (1962, p. 654). The procedures in 6.3, 6.4, and 6.5 are not the only tests possible, but they are used frequently and illustrate the concept.

The normative distribution approximated by the negative hypergeometric distribution with parameters estimated through multiple matrix sampling provides the basic data for several goodness-of-fit tests. For example, the Kolmogorov-Smirnov one-sample test (Siegel, 1956, pp. 47-52) provides a test of the hypothesis that the approximated distribution of scores came from a population of scores having a specified theoretical distribution. The test involves specifying the cumulative frequency distribution which would occur under the theoretical distribution and comparing that with the approximated cumulative frequency distribution. The cumulative frequency distribution is, of course, obtained readily after the individual frequencies have been determined by multiplying the number of examinees in the population by the relative frequency per test score approximated by the negative hypergeometric distribution. A simple extension of the Kolmogorov-Smirnov one-sample test is the Kolmogorov-Smirnov two-sample test (Siegel, 1956, pp. 127-136) which is concerned with the agreement between two approximated frequency distributions.

The tests of hypotheses mentioned herein do not constitute an exhaustive listing of statistical tests to which estimates of parameters obtained through multiple matrix sampling are applicable. The intent is merely that of suggesting the applicability of a novel technique to traditional hypothesis testing procedures. It should be noted that the t-tests given in 6.1 and 6.2 are to be considered conservative tests of the hypotheses under consideration. The standard errors of estimate given in the denominators are those for the matched-items design and there is evidence (Osburn, 1967) suggesting that the corresponding standard errors under multiple matrix sampling will be less. In the algebraic derivation supporting this conclusion, Osburn was considering a form of multiple matrix sampling in which k items were selected at random from the population of items for <u>each</u> examinee.

Table 6.1

Statistical Layout For One-Way Analysis Of Variance Problem With The Dependent Variable Being A Mean Achievement Test Score Estimated Through Multiple Matrix Sampling

Program			
1	2	3	4
$\hat{\mu}_{1_{pooled}}$	$\hat{\mu}_{11_{pooled}}$	$\hat{\mu}_{21_{pooled}}$	$\hat{\mu}_{31_{pooled}}$
$\hat{\mu}_{2_{pooled}}$	$\hat{\mu}_{12_{pooled}}$	$\hat{\mu}_{22_{pooled}}$	$\hat{\mu}_{32_{pooled}}$
...	...	...	...
$\hat{\mu}_{10_{pooled}}$	$\hat{\mu}_{20_{pooled}}$	$\hat{\mu}_{30_{pooled}}$	$\hat{\mu}_{40_{pooled}}$

Source Of Variation	Degrees Of Freedom
Programs	3
Classes Within Programs	36
Total	39

Table 6.2

Statistical Layout For Factorial (Two-Way) Analysis Of Variance Problem With The Dependent Variable Being A Mean Achievement Test Score Estimated Through Multiple Matrix Sampling

	Program			
District A	$\hat{\mu}_{1_{pooled}}$	$\hat{\mu}_{11_{pooled}}$	$\hat{\mu}_{21_{pooled}}$	$\hat{\mu}_{31_{pooled}}$
	...	...	...	...
	$\hat{\mu}_{5_{pooled}}$	$\hat{\mu}_{15_{pooled}}$	$\hat{\mu}_{25_{pooled}}$	$\hat{\mu}_{35_{pooled}}$
District B	$\hat{\mu}_{6_{pooled}}$	$\hat{\mu}_{16_{pooled}}$	$\hat{\mu}_{26_{pooled}}$	$\hat{\mu}_{36_{pooled}}$
	...	...	...	...
	$\hat{\mu}_{10_{pooled}}$	$\hat{\mu}_{20_{pooled}}$	$\hat{\mu}_{30_{pooled}}$	$\hat{\mu}_{40_{pooled}}$

Source Of Variation	Degrees Of Freedom
Programs	3
Districts	1
Programs x Districts	3
Classes Within Programs x Districts	32
Total	39

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VII

Unique Applications of Multiple Matrix Sampling

Multiple matrix sampling has been used traditionally to estimate parameters of standardized tests where the total test score is equal to the sum of the item scores. For investigations focused primarily on group assessment, multiple matrix sampling has been demonstrated empirically to be an important and valuable procedure. Multiple matrix sampling, however, is applicable to a broader range of research problems than that suggested by the current literature. Four unique and important applications of multiple matrix sampling are described in this chapter. As is the case with most psychometric procedures and is certainly the case with multiple matrix sampling, the range of applications is determined solely by the degree of inventiveness in the individual researcher.

Design of Experiments

In the evaluation of instructional programs, the pre-post paradigm is used frequently and, as is traditionally the case, an individual test is administered to all examinees at both the start and end c instruction. Given an item population related to the instructional progr under evaluation, a research design such as this is improved easily w the addition of multiple matrix sampling. In place of using the sa test pre and post, random or stratified-random parallel tests are us d with parameters for both tests estimated through multiple matrix sam ing. A procedure such as this could be expanded further to include intermediate testing using additional parallel tests. An example of a design such as this and one demonstrating the concomitant benefits is given by Osburn and Shoemaker (1968). In the evaluation of instructional programs it should be noted that a researcher is seldom interested in individual test items, individual tests, or individual examinees but is interested primarily in group behavior over time with regard to some specified item population. As such, multiple matrix sampling in conjunction with random or stratified-random parallel tests is an ideal measurement procedure.

Estimation of Covariance and Correlation Matrices

Item and test covariance matrices (and, hence, correlation matrices) are estimated readily through multiple matrix sampling. A modified sampling plan is required such that all possible pairs of items or tests are included in one or more subtests or subbatteries. For example, consider estimating the elements in a covariance matrix for a 5-item test. To compute the covariance of Item 1 with Item 4, there must be a subgroup of examinees responding to both Item 1 and Item 4. If the examinee subgroup is sampled randomly from the population of examinees, COV(1, 4) computed over those examinees is an estimate of COV(1, 4) which would have been obtained by testing all examinees over both items. All remaining entries in the covariance matrix are estimated identically. A test covariance matrix is determined similarly with items being replaced by tests. A procedure such as this sets the stage for multiple matrix sampling playing an important role in a variety of multivariate procedures as, for example, factor analysis. Although little has been done in this area, some important preliminary research and a few of the relevant equations for estimating parameters have been reported by Lord (1960), Ray, Hundleby and Goldstein (1962), Knapp (1968) and Timm (1970).

Questionnaires and Surveys

A perennial problem with questionnnaires and surveys is the disappointingly low rate of completions or returns. Return rates of 20 to 30 per cent are not uncommon. Although examinees fail to return questionnaires for a multitude of reasons, one factor is undoubtedly the length of the questionnaire and the time required to complete all questions. If the measurement required is the proportion of examinees in each category, results can be approximated through multiple matrix sampling by administering questions selected randomly to a random sample of examinees. For example, if an 8-page questionnaire were to be administered to all elementary school teachers within a particular city, the questions contained therein could be divided into 8 subquestionnaires (each of which would require no more than the front of one piece of paper) with each subquestionnaire administered to a random sample of teachers. The time for completing each subquestionnaire is minimal and, as such, may increase the rate of returns. The point to be made is simply this: a little data from a large number of teachers is better than a lot of data from few teachers. It must be remembered, however, that questions within questionnaires are interrelated frequently (If "No" on Question 13, go to Question 20.) and complications such as these must be incorporated in constructing subquestionnaires.

Measurement in the Affective Domain

It is frequently the case that an investigator is interested in scaling the preferences or affect of a group of individuals for a particular set of objects. Although there are several procedures which could be used, the method of paired-comparisons is one encountered frequently in the literature (e.g., Snider (1960) and Holliman (1970). In the

Table 7.1

Example F-matrix And P-matrix Obtained By Method Of Paired-Comparisons For 6 Stimuli

	F-matrix						P-matrix					
	1	2	3	4	5	6	1	2	3	4	5	6
1		6	5	7	10	7		.353	.294	.412	.588	.412
2	11		9	10	12	10	.647		.529	.588	.706	.588
3	12	8		12	12	9	.706	.471		.706	.706	.529
4	10	7	5		14	6	.588	.412	.294		.824	.353
5	7	5	5	3		5	.412	.294	.294	.176		.294
6	10	7	8	11	12		.588	.412	.471	.647	.706	

method of paired-comparisons, all possible combinations of the objects taking two at a time are presented individually and for each pair the examinee is asked to indicate his preference. For example, if 6 stimuli were being scaled by the method of paired-comparisons, the test so constructed would contain 6(6-1)/2 = 15 items, for 12 stimuli, 66 items. After all pairs have been administered to all examinees, the preliminary analysis of the data involves the computation of the F-matrix and subsequent P-matrix. The P-matrix is the base from which the scale values per stimulus are computed and it is in estimating the values in the P-matrix that an application of multiple matrix sampling is found. Relevant preliminary research in this area has been reported by McCormick and Roberts (1952), McCormick and Bachus (1952) and Bursack and Cook (1970). If the s stimuli are numbered consecutively from 1 to s, the F-matrix is an s by s matrix with entries denoting the frequency with which the column stimulus was judged more favorable than the row stimulus. An example of an F-matrix and associated P-matrix are given in Table 7.1. Dividing each entry in the F-matrix by the total number of examinees, which is in this case equal to 17, produces the corresponding entry in the P-matrix labeled appropriately as the proportion of examinees selecting the column stimulus over the row stimulus. In estimating the entries in the P-matrix through multiple matrix sampling, paired-comparisons are selected at random from the pool of all possible pairs and administered to samples of examinees selected randomly from the testable population.

Shoemaker (1971) using a post mortem item-examinee sampling design has explored systematically the feasibility of using multiple matrix sampling to estimate scale values obtained by the method of paired-comparisons. The major conclusions reached in this investigation were that (a) scale values can be approximated satisfactorily through multiple matrix sampling, and (b) the similarity between the estimated scale values and the normative scale values increases with increases in the number of observations acquired by the sampling plan, with the converse true. The specific procedure used to estimate the P-matrix from subtest results is detailed in the following 5 steps. Each step is illustrated with results from one replication of a (3/10/15) sampling plan. (In the Shoemaker investigation, the data base consisted of responses made by 407 primary grade students to a 15-item test designed to scale degree of affect to 6 stimuli.)

Step 1: Three subtests containing 10 items each are formed by sampling items randomly and without replacement within subtests but with replacement between subtests.

Subtest	Items									
1	8	14	11	9	6	2	3	12	15	10
2	14	6	3	12	4	1	2	11	13	10
3	14	13	2	9	6	11	4	15	3	12

Step 2: Three subgroups of examinees containing 15 examinees each are formed by sampling randomly and without replacement from the 407-examinee population.

Subgroup	Examinees														
1	359	22	280	272	139	206	169	321	323	23	271	66	221	109	100
2	345	367	281	390	366	70	361	250	154	168	8	138	279	335	399
3	342	220	276	125	382	219	217	327	401	385	113	62	77	192	156

Step 3: Pairing subtest _i_ with subgroup _i_, an f-matrix is formed for each subtest using only the responses made by the corresponding examinee subgroup on the items contained in that subtest. Each f-matrix is constructed in conjunction with a link-matrix containing the code numbers of stimuli paired within each test item. For the data base considered herein, the link-matrix was

Test Item	Stimulus Pair	
01	1	2
02	4	3
03	5	6
04	2	6
05	1	3
06	4	5
07	2	3
08	1	5
09	4	6
10	1	4
11	2	5
12	3	6
13	3	5
14	1	6
15	2	4

The f-matrices for the 3 subtests used in (3/10/15) are

$$\text{f-matrix 1} = \begin{bmatrix} & 0 & 0 & 9 & 14 & 8 \\ 0 & & 0 & 8 & 15 & 0 \\ 0 & 0 & & 5 & 0 & 4 \\ 6 & 7 & 10 & & 12 & 9 \\ 1 & 0 & 0 & 3 & & 2 \\ 7 & 0 & 11 & 6 & 13 & \end{bmatrix}$$

$$\text{f-matrix 2} = \begin{bmatrix} & 13 & 0 & 5 & 0 & 10 \\ 2 & & 0 & 0 & 11 & 4 \\ 0 & 0 & & 4 & 6 & 2 \\ 10 & 0 & 11 & & 11 & 0 \\ 0 & 4 & 9 & 4 & & 1 \\ 5 & 11 & 13 & 0 & 14 & \end{bmatrix}$$

$$\text{f-matrix 3} = \begin{bmatrix} & 0 & 0 & 0 & 0 & 7 \\ 0 & & 0 & 9 & 14 & 8 \\ 0 & 0 & & 6 & 6 & 4 \\ 0 & 6 & 9 & & 13 & 11 \\ 0 & 1 & 9 & 2 & & 3 \\ 8 & 7 & 11 & 4 & 12 & \end{bmatrix}.$$

Step 4: In pooling the f-matrices to obtain the P-matrix, an accounting-matrix is required to distinguish between items omitted in the construction of subtests and items to which all examinees in a particular subgroup responded identically. For the f-matrices given in step 3, the accounting-matrix is

$$\text{accounting-matrix} = \begin{bmatrix} 0 & 1 & 0 & 2 & 1 & 3 \\ 1 & 0 & 0 & 2 & 3 & 2 \\ 0 & 0 & 0 & 3 & 2 & 3 \\ 2 & 2 & 3 & 0 & 3 & 2 \\ 1 & 3 & 2 & 3 & 0 & 3 \\ 3 & 2 & 3 & 2 & 3 & 0 \end{bmatrix}.$$

Off-diagonal zeros are of critical importance in pooling subtest results. In each f-matrix, $f(i,j) + f(j,i) = n$ for those stimulus pairs contained within the subtest and $f(j,i)$, for example, could be zero for two reasons: (a) the item containing stimulus pair (i,j) was not included in the subtest, or (b) all examinees in that particular subgroup selected stimulus i over stimulus j. This distinction must be maintained in pooling the f-matrices to produce the P-matrix.

Step 5: The P-matrix is formed by pooling across corresponding entries in the f-matrices after each entry in the f-matrix has been divided by the number of examinees in the corresponding subgroup. The sum of proportions is then divided by the corresponding number in the accounting-matrix. As an example, consider computing the (1,6) and (5,1) entries in the P-matrix:

$$P(1,6) = \frac{8/15 + 10/15 + 7/15}{3} = .556$$

$$P(5,1) = \frac{1/15 + \text{no data} + \text{no data}}{1} = .067$$

If the number of examinees per subgroup is unequal, the proportions are combined by a weighted arithmetic mean and the corresponding entry in the accounting-matrix is equal to the number of examinees for which data existed. Elements in the P-matrix are set equal to .5 if the corresponding entry in the accounting-matrix is equal to zero. In this example, the P-matrix is

$$\text{P-matrix} = \begin{bmatrix} .500 & .867 & .500 & .467 & .933 & .556 \\ .133 & .500 & .500 & .567 & .889 & .400 \\ .500 & .500 & .500 & .333 & .400 & .222 \\ .533 & .433 & .667 & .500 & .800 & .667 \\ .067 & .111 & .600 & .200 & .500 & .133 \\ .444 & .600 & .778 & .333 & .867 & .500 \end{bmatrix} .$$

After the P-matrix has been formed, scale values per stimulus are computed as if all examinees had responded to all items using computational procedures detailed in Edwards (1957). Using Thurstone's Model V scaling procedure, the resultant scale values from the P-matrix given in step 5 and those obtained from using all 407 examinees over all items are

	S_1	S_2	S_3	S_4	S_5	S_6
(3/10/15)	.000	.442	.688	.172	1.184	.184
Norm(5)	.000	.075	.638	.215	1.023	.193

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APPENDIX A

Listing And Expanded Writeup Of Computer Program For Estimating Test Parameters Through Multiple Matrix Sampling And For Approximating Normative Distributions With The Negative Hypergeometric Distribution

A Fortran IV Program For Estimating Test Parameters Through Multiple Matrix Sampling And For Approximating A Normative Distribution Of Test Scores With The Negative Hypergeometric Distribution

The negative hypergeometric distribution provides a reasonably good fit for a variety of test score distributions when the test score is the number of correct answers. The negative hypergeometric distribution is a function of the mean test score, the variance of the test scores and the total number of items in the test. The first two parameters may be approximated efficiently by multiple matrix sampling. Furthermore, the negative hypergeometric distribution with parameters estimated by multiple matrix sampling can be used satisfactorily to approximate a normative distribution of number correct test scores.

In multiple matrix sampling, a set of K test items is randomly divided into subsets of items. Each subset of items is then randomly assigned to a group of examinees. Although each examinee receives only a proportion of the complete set of test items, the statistical model permits one to estimate the mean and variance of the total test score distribution for all examinees over the complete set of test items. Multiple matrix sampling is an efficient procedure for approximating a normative distribution when it is not possible or is economically unfeasible to administer the complete set of K items to all examinees in the testable population.

The Fortran IV program which approximates the normative distribution with the negative hypergeometric distribution is relatively machine-independent and has been implemented easily on an IBM 7040, IBM S360/50, IBM S360/91 and a UNIVAC 1108. The program has been designed to approximate test score distributions involving at maximum 500 items. However, this restriction may be easily modified. The number of subtests and number of examinees per subtest are limited only by the amount of computer time available.

Organization Of Control Cards And Data Cards

	columns	(all integers right-justified)
Card Set 1 (1 card)	1-72	Alphanumeric title of project
Card Set 2 (1 card)	1-5	Integer number of examinee groups
	6-10	Integer number of items in each subtest
	11-15	Integer number of examinees per subgroup

	columns	(all integers right-justified)
	51-55	Punch 00000 if there is only one format card by which all item scores are to be inputted Punch 00001 if there is to be a different format card for each item-examinee sample within a data set
Format Card Set (k cards, optional)		Standard Fortran IV format punched in columns 1-72 on each card and enclosed in parentheses for inputting item scores for each examinee in each item-examinee sample. The number of format cards may not exceed 9 for each item-examinee data set. The first card after the format cards must contain END OF FORMAT in columns 1-13. Example: (5X,25F1.0) END OF FORMAT
Data Card Set (k cards, optional)		The responses of each examinee per item-examinee sample must be sequenced by examinee group and within each group by examinee.

Acceptable Input Data Structures

Plan 1	Plan 2	Plan 3
Fortran Source Deck Card Set 1 Card Set 2	Fortran Source Deck Card Set 1 Card Set 2 Format Card Set Data Cards	Fortran Source Deck Card Set 1 Card Set 2 Format Card Set 1 Data From Sample 1 Format Card Set 2 Data From Sample 2 ... Format Card Set t Data From Sample t

Plan 4	Plan 5
Fortran Source Deck Card Set 1 Card Set 2 Format Card Set Card with no. of examinees and items for subtest 1 Data from subtest 1 Card with no. of examinees and items for subtest 2 Data from subtest 2 ... Card with no. of examinees and items for subtest t Data from subtest t	Fortran Source Deck Card Set 1 Card Set 2 Format Card Set 1 Card with no. of examinees and items for subtest 1 Data from subtest 1 Format Card Set 2 Card with no. of examinees and items for subtest 2 Data from subtest 2 ... Format Card Set t Card with no. of examinees and items for subtest t Data from subtest t

Plan 1: Mean and variance of test scores are inputted on card set 2. No item scores are required.

Plan 2: Mean and variance of test scores are to be estimated from item-examinee samples. All item scores in each item-examinee sample are organized in the same manner on the data card and are to be inputted with one format card.

Plan 3: Same as Plan 2 with exception that item scores for each item-examinee sample are not organized on data cards in same manner. Each sample requires an individual set of format cards describing how item scores are organized for that particular sample.

Plan 4: Same as Plan 2 with exception that number of examinees and number of items per subtest are not constant across subtests. Same format card is used for each data set.

Plan 5: Same as Plan 3 with exception that number of examinees and number of items per subtest are not constant across subtests. In addition, different format cards are used for each data set.

```
C*****************************************************************************
C
C   APPROXIMATION OF FREQUENCY DISTRIBUTION OF TEST SCORES
C   BY NEGATIVE HYPERGEOMETRIC DISTRIBUTION
C
C   REFERENCE
C
C   LORD, F.M. AND NOVICK, M.R.  STATISTICAL THEORIES OF MENTAL TEST
C        SCORES.  READING, MASS.  ADDISON-WESLEY,1968,CHAPTER 23.
C
C   DAVID M. SHOEMAKER
C
C        NTS  = NUMBER OF ITEMS PER SUBTEST
C        NTP  = NUMBER OF ITEMS IN TEST ITEM POPULATION
C        NSM  = NUMBER OF SUBTESTS
C        NSS  = NUMBER OF EXAMINEES PER SUBGROUP
C        NSP  = NUMBER OF EXAMINEES IN EXAMINEE POPULATION
C       XBAR  = ESTIMATE OF MEAN TEST SCORE
C        VAR  = ESTIMATE OF TEST SCORE VARIANCE
C
C*****************************************************************************
C
      COMMON DUMMY(500),P(500)
      DIMENSION TITLE(18)
C
C   INPUT PROBLEM PARAMETERS
C
 1000 READ (5,1,END=5000) (TITLE(I),I=1,18),
     1NSM,NTS,NSS,NTP,NSP,XBAR,VAR,NGPH,NFMT
      WRITE (6,5) (TITLE(I),I=1,18)
      IF ( IFIX(VAR*1000.) .NE. 0 ) WRITE (6,9) XBAR,VAR
C
C   ESTIMATE MEAN AND VARIANCE FROM SUBTESTS
C
      IF (IFIX(VAR*1000.).EQ.0) CALL POOL(NSS,NTS,NSM,NTP,XBAR,VAR,NF
      IF ( NSP .EQ. 0 ) NSP=1000
      WRITE (6,10) NSP
C
C   COMPUTE PARAMETERS FOR NEGATIVE HYPERGEOMETRIC DISTRIBUTION
C
      S=NTP
      A21= (S/(S-1.))*(1.-XBAR*(S-XBAR)/(S*VAR))
      IF ( A21 .GT. 0. ) GO TO 40
      WRITE (6,7) A21
      GO TO 1000
   40 CONTINUE
      A=(-1.+1./A21)*XBAR
      B=-A-1.+S/A21
```

```
      SLOG1=0.
      SLOG2=0.
      C=A+B
      DO 50 I=1,NTP
      SLOG1=SLOG1+ALOG10(B-I+1.)
 50   SLOG2=SLOG2+ALOG10(C-I+1.)
      C=10.**(SLOG1-SLOG2)
      WRITE (6,3) A21,A,B,C
C
C  COMPUTE NEGATIVE HYPERGEOMETRIC DISTRIBUTION
C
      N3=NTP+1
      WRITE (6,4)
      CK=0.
      DO 100 I=1,N3
      K=I-1
      CALL NEGHGR (K,A,B,C,S,NSP,HX,HFX)
      CK=CK+HX
      P(I)=HX
      WRITE (6,2) K,HX,HFX,CK
 100  CONTINUE
C
C  PLOT NEGATIVE HYPERGEOMETRIC DISTRIBUTION
C
      IF ( NGPH .EQ. 0 ) CALL PLOT (NTP)
      GO TO 1000
 5000 WRITE (6,8)
      CALL EXIT
 1    FORMAT (18A4/5I5,2F10.0,2I5)
 2    FORMAT (I10,3F30.7)
 3    FORMAT (/7H KR21 =F12.3,6X,3HA =F15.7/25X,3HB =F15.7/25X,3HC =E18.
     17////)
 4    FORMAT (////5X,5HSCORE,22X,4HH(X),26X,6HN*H(X),24X,6HCUM HX//)
 5    FORMAT (1H1,18A4//)
 7    FORMAT (43H KR21 NEGATIVE OR ZERO ... DATA SET ABORTED,5X,6HKR21 =
     1F10.4)
 8    FORMAT (1H1,20X,19HALL INPUT PROCESSED)
 9    FORMAT (//7H XBAR =F12.3//7H  VAR =F12.3)
 10   FORMAT (/4X,3HN =I8)
      END
```

```
      SUBROUTINE NEGHGR (K,A,B,C,S,NSUB,HX,HFX)
C
C  NEGATIVE HYPERGEOMETRIC FUNCTION
C
      IF ( K .EQ. 0 ) GO TO 150
      SLOG1=0.
      SLOG2=0.
      SLOG3=0.
      SLOG4=0.
      DO 100 I=1,K
      SLOG1=SLOG1+ALOG10(S-I+1.)
      SLOG2=SLOG2+ALOG10(A+I-1.)
      SLOG3=SLOG3+ALOG10(B-I+1.)
  100 SLOG4=SLOG4+ALOG10(FLOAT(I))
      HX=C*10.**(SLOG1+SLOG2-SLOG3-SLOG4)
  125 HFX=HX*NSUB
      RETURN
  150 HX=C
      GO TO 125
      END


      SUBROUTINE RDFMT(FMT)
C
C  SUBROUTINE FOR INPUTTING VARIABLE FORMAT
C
C  INPUT STRUCTURE
C     FORMAT (ENCLOSED IN PARENTHESES) COL 1-72
C          CONTINUE ON CARD 2 IF NECESSARY
C          CONTINUE ON CARD 3 IF NECESSARY
C          ETC.
C          MAXIMUM NUMBER OF FORMAT CARDS IS 9
C       'END OF FORMAT'   NECESSARY ...  PUNCH IN COLUMNS 1 - 1
C
      DIMENSION FMT(200)
      DATA END/3HEND/
      N=1
      DO 100 I=1,10
      M=N+17
      READ (5,1) (FMT(J),J=N,M)
      IF ( FMT(N) .EQ. END ) RETURN
  100 N=N+18
      WRITE (6,2)
      STOP
1     FORMAT (18A4)
2     FORMAT (37H *** EXCESSIVE NUMBER OF FORMAT CARDS)
      END
```

```
 SUBROUTINE POOL (NSUB,NITEMS,NSAM,NTP,XBAR,VAR,NFMT)
ETERMINATION OF POOLED ESTIMATE OF POPULATION MEAN TEST SCORE AND
ARIANCE
  COMMON P(500),X(500)
  DIMENSION FMT(200)
  IF ( NFMT .EQ. 0 ) CALL RDFMT(FMT)
  WRITE (6,1)
  NTEST=NSUB*NITEMS
  SESTM=0.
  SESTV=0.
  NSM=0
  SWGHT=0.
  DO 1000 I=1,NSAM
  IF ( NFMT .NE. 0 ) CALL RDFMT(FMT)
  IF ( NTEST .EQ. 0 ) READ (5,6) NSUB,NITEMS
  SY=0.
  SYY=0.
  DO 50 J=1,NITEMS
  P(J)=0.
  DO 500 J=1,NSUB
  READ (5,FMT) (X(K),K=1,NITEMS)
  Y=0.
  DO 510 K=1,NITEMS
  P(K)=P(K)+X(K)
  Y=Y+X(K)
  SY=SY+Y
  SYY=SYY+Y*Y
  XBR=SY/NSUB
  VR=(SYY-SY*SY/NSUB)/NSUB
  SPQ=0.
  DO 520 J=1,NITEMS
  PP=P(J)/NSUB
0 SPQ=SPQ+PP*(1.-PP)
  NSM=NSM+NSUB
  WGHT=NSUB*NITEMS
  ESTM=NTP*XBR/NITEMS
  ESTV=(NSUB*NTP*((NTP-1.)*VR-(NTP-NITEMS)*SPQ))/
 1(NITEMS*(NITEMS-1.)*(NSUB-1.))
  SESTM=SESTM+ESTM*WGHT
  SESTV=SESTV+ESTV*WGHT
  SWGHT=SWGHT+WGHT
  WRITE (6,2) I,ESTM,ESTV
00 CONTINUE
  XBAR=SESTM/SWGHT
  VAR=SESTV/SWGHT
  IF ( NSM .LT. 500 ) VAR=VAR*(NSM-1.)/NSM
  WRITE (6,3) XBAR,VAR
  RETURN
  FORMAT (////24X,21HESTIMATE OF PARAMETER///5X,6HSAMPLE,10X,4HMEAN
 1,16X,8HVARIANCE///)
  FORMAT (I10,2F20.7)
  FORMAT (//14H POOLED XBAR =F20.7//18H POOLED VARIANCE =F16.7////)
  FORMAT (2I5)
  END
```

```
      SUBROUTINE PLOT (NITEMS)
      REAL N
      COMMON N(500),P(500)
      DIMENSION BCD(10)
      DATA BCD/1H0,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/
      DATA BLK,DOT,XX/1H ,1H.,1HX/
C
C  LOCATE MAXIMUM VALUE FOR H(X)
C
      NNN=NITEMS+1
      T=0.
      DO 50 I=1,NNN
      IF ( P(I) .GT. T ) T=P(I)
 50   CONTINUE
C
C  DETERMINATION OF APPROPRIATE SCALE FACTOR FOR H(X) PLOT
C
      J=0
      DO 60 I=1,6
      K=I-1
      J=T*10.**K
      IF ( J .EQ. 0 ) GO TO 60
      J=K-1
      WRITE (6,3) J
      GO TO 70
 60   CONTINUE
C
C  SCALE H(X) BEFORE PLOTTING
C
 70   DO 75 I=1,NNN
 75   P(I)=P(I)*10.**J
C
C  LABEL ORDINATE
C
      WRITE (6,1)
      DO 500 I=1,100
 500  N(I)=BLK
      N(101)=BCD(2)
      WRITE (6,2) (N(J),J=1,101)
      NN=0
      DO 550 I=1,10
      DO 575 J=1,10
 575  N(NN+J)=BCD(I)
      NN=NN+10
 550  CONTINUE
      N(101)=BCD(1)
      WRITE (6,2) (N(J),J=1,101)
      DO 580 I=1,101,10
      DO 590 J=1,10
      K=J-1
 590  N(I+K)=BCD(J)
 580  CONTINUE
```

```
      N(101)=BCD(1)
      WRITE (6,2) (N(J),J=1,101)
      DO 595 I=1,101
 595  N(I)=DOT
      WRITE (6,2) (N(J),J=1,101)
C
C  PLOT VALUES OF SCALED H(X)
C
      DO 100 I=1,NNN
      K=I-1
      L=P(I)*100.+1.5
      DO 105 J=1,101
 105  N(J)=BLK
      DO 110 J=1,L
 110  N(J)=XX
      DO 120 J=11,101,10
      IF ( N(J) .EQ. BLK ) N(J)=DOT
 120  CONTINUE
      WRITE (6,4) K,(N(J),J=1,101)
 100  CONTINUE
C
C  END OF GRAPH
C
      DO 900 I=1,101
 900  N(I)=DOT
      WRITE (6,2) (N(J),J=1,101)
      RETURN
 1    FORMAT (1H1,50X,40HPROPORTION OF POPULATION RECEIVING SCORE//)
 2    FORMAT (14X,101A1)
 3    FORMAT (////5X,19HH(X) SCALED BY 10 E13,9H IN GRAPH//)
 4    FORMAT (I10,4X,101A1)
      END
```

Sample Output

```
Card     00000000011111111112222222222333333333344444444445555555555556
column   12345678901234567890123456789012345678901234567890123456789 0

         FIRST YEAR WORD SPELLING PROJECT       SHOEMAKER/OKADA
         0000500000000000050                                    00000
         (10X,10F1.0)
         END OF FORMAT
         0001800010
         01 1 1     1011111110
         02 1 1     1111101011
         03 1 1     1110100000
         04 1 1     1110000000
         05 1 1     0000000000
         06 1 1     1101010001
         07 2 1     1110101010
         08 2 1     1100100000
         09 2 1     0000011010
         10 2 1     0100000000
         11 2 1     1111101010
         12 2 1     1001001000
         13 3 1     0100100000
         14 3 1     0111100010
         15 3 1     0000000000
         16 3 1     0000000000
         17 3 1     1101001110
         18 2 1     0000000000
         0001400010
         01 1 2     1111110011
         02 1 2     1011100000
         03 1 2     1100101000
         04 1 2     1111110000
         05 1 2     1011110010
         06 1 2     1110110000
         07 2 2     1010100000
         08 2 2     1111110010
         09 2 2     1101111010
         10 2 2     1111110000
         11 3 2     0001110000
         12 3 2     1111111100
         13 3 2     1101000000
         14 3 2     0011110000
```

```
Card      000000000111111111122222222223333333333444444444455555555556
column    123456789012345678901234567890123456789012345678901234567890
          0001300010
          01 1 3    1101111101
          02 1 3    0101001000
          03 1 3    1001011101
          04 1 3    0010010000
          05 1 3    1111111111
          06 1 3    1101110100
          07 2 3    0100000100
          08 2 3    0111111110
          09 2 3    1000000000
          10 2 3    1001000000
          11 3 3    1111111100
          12 3 3    1111000100
          13 3 3    0000000000
          0001300010
          01 1 4    1101001100
          02 1 4    0010001000
          03 1 4    1111011011
          04 1 4    1111111111
          05 1 4    1111011111
          06 2 4    0111010000
          07 2 4    1111011111
          08 2 4    0000000000
          09 2 4    0110001100
          10 3 4    0011001000
          11 3 4    1111001110
          12 3 4    0011001000
          13 3 4    0000000000
          0001200010
          01 1 5    1001001010
          02 1 5    0000000000
          03 1 5    1000000000
          04 1 5    0000000000
          05 2 5    0101111010
          06 2 5    1110010100
          07 2 5    0000000000
          08 2 5    0000000000
          09 3 5    1011111100
          10 3 5    0000000000
          11 3 5    0000000000
          12 3 5    0000000000
```

APPENDIX B

Computer Program For Simulating Multiple Matrix Sampling

Computer Program For Simulating Multiple Matrix Sampling

The computer program for simulating multiple matrix sampling is described in detail in Chapter V. A listing of the Fortran IV program is given in this appendix for those readers who may want to implement the model on the computer configuration available to them. The program given herein was written originally for a UNIVAC 1108 and a modified version has been implemented on an IBM S360/91. In modifying the program for the S360/91, the only changes made were those involving the uniform (.00 to .99) random number generator RUNIF. On the 1108, RUNIF is initialized by RINITL. Calling RINITL with BASE as the argument causes BASE to be used as the starting value or seed in the algorithm used by RUNIF in generating uniform random numbers. Because RINITL is specific to UNIVAC 1108, readers should consult the local computing center staff to determine the subprogram and calling procedures at that installation comparable to the RINITL/RUNIF system. The conversion process was relatively simple for the S360/91 and it is anticipated that such will be the case with other hardware and software systems. Input values to the program are made on one parameter card. The organization of the card is described at the beginning of the program listing. Examples of parameter cards are found on page 97 of this appendix. Sample output from the program is given in Chapter V.

```
C
C*********************************************************************************
C
C
C   COMPUTER SIMULATION OF ITEM-EXAMINEE SAMPLING
C   DAVID M. SHOEMAKER
C
C
C   PARAMETER CARD (THERE IS JUST ONE)
C
C       COLUMNS        (ALL INTEGERS RIGHT-JUSTIFIED)
C
C        01-03         INTEGER NUMBER OF ITEMS IN TOTAL TEST
C
C        04-09         DESIRED MEAN TEST SCORE IN POPULATION
C                      (MUST BE SPECIFIED, WITH DECIMAL POINT PUNCHED ON CARD)
C
C        10-15         DESIRED VARIANCE OF TEST SCORES IN POPULATION
C                      (WITH DECIMAL POINT PUNCHED ON CARD)
C
C                      NOTE ... IF VARIANCE IS OMITTED, RELIABILITY MUST
C                      BE SPECIFIED.
C
C        16-21         DESIRED VARIANCE OF ITEM DIFFICULTY INDICES OVER
C                      POPULATION OF EXAMINEES.  THE ITEM DIFFICULTY
C                      INDEX FOR ITEM I IS THE PROPORTION OF EXAMINEES
C                      ANSWERING ITEM I CORRECTLY.
C                      (MUST BE SPECIFIED WITH DECIMAL POINT PUNCHED ON CARD)
C
C                      WITH SKEWED DISTRIBUTIONS, VARIANCE OF ITEM
C                      DIFFICULTY INDICES IS ASSUMED TO BE EQUAL TO ZERO.
C
C
C        22-27         DESIRED RELIABILITY OF TEST SCORES IN POPULATION
C                      (WITH DECIMAL POINT PUNCHED ON CARD)
C
C                      NOTE ... IF RELIABILITY IS OMITTED, VARIANCE MUST
C                      BE SPECIFIED.
C
C        28-31         INTEGER NUMBER OF SUBTESTS IN ITEM-EXAMINEE SAMPLING
C
C        32-35         INTEGER NUMBER OF ITEMS PER SUBTEST
C                      (CONSTANT ACROSS SUBTESTS)
C
C        36-39         INTEGER NUMBER OF EXAMINEES PER SUBTEST
C                      (CONSTANT ACROSS SUBTESTS)
C
C        40-43         INTEGER NUMBER OF INDEPENDENT REPLICATIONS OF ITEM-
C                      EXAMINEE SAMPLING PLAN
C
C          44          SAMPLING PLAN FOR ITEMS
C
C                            0 = SAMPLING WITH REPLACEMENT
C                                (USED WHEN TK IS GREATER THAN K)
C
C                            1 = SAMPLING WITHOUT REPLACEMENT
C                                (USED WHEN TK IS LESS THAN OR EQUAL TO K)
```

2 = SAMPLING WITHOUT REPLACEMENT BUT SUBJECT TO RESTRICTION THAT ITEMS OCCUR WITH EQUAL FREQUENCY AMONG SUBTESTS (USED WHEN TK IS GREATER THAN K)

45 INTERMEDIATE PRINTOUT OPTION

0 = NO INTERMEDIATE PRINTOUT
1 = INTERMEDIATE PRINTOUT WANTED

46 NEGATIVE HYPERGEOMETRIC DISTRIBUTION OPTION

0 = NO NEG. HYPER. DIST. WANTED
1 = COMPUTE NEG. HYPER. DIST.

47 DEGREE OF SKEWNESS IN NORMATIVE DISTRIBUTION

1 = NORMALLY DISTRIBUTED
2 = POSITIVELY SKEWED
3 = NEGATIVELY SKEWED

48-53 SEED FOR UNIFORM RANDOM NUMBER GENERATOR (ODD NUMBER)

55 GENERATE ITEM DIFFICULTY INDICES

0 = GENERATE NEW ITEM DIFFICULTY INDICES
1 = USE ITEM DIFFICULTY INDICES GENERATED BY PREVIOUS DATA CARD

RESTRICTIONS

MAXIMUM NUMBER OF ITEMS IS 150 (EASILY MODIFIED, HOWEVER)
ITEMS SCORED DICHOTOMOUSLY

PROGRAM WILL PROCESS REPEATED PARAMETER CARDS (NUMBER LIMITED ONLY BY AMOUNT OF COMPUTER TIME ALLOCATED)

```
*****************************************************************************

      REAL N,M,MPOP
      COMMON N(150),M(150),LT(3000)
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,INTPRT
      COMMON /BLOCK2/ RND(150)
      COMMON /BLOCK3/ P(150),Q(150),NSUB

 2000 READ (5,1,END=5000) KPOP,MPOP,VPOP,PVAR,A20,NT,IPT,NSPT,NREPS,
     1                    ISAMP,INTPRT,NHPER,NDIST,BASE,ISAVE
      IF ( NDIST .GT. 1 ) PVAR=0.
      WRITE (6,2) BASE,MPOP,VPOP,KPOP,A20,PVAR,NT,IPT,NSPT,NREPS,
     1ISAMP,NHPER,NDIST,INTPRT

C     INITIALIZE RANDOM NUMBER GENERATOR (UNIQUE TO UCC)
      CALL RINITL(BASE)
```

```
C
C
      NSUB=0
C
C  CHECK ON PARAMETERS
C
      IF ( ISAMP .NE. 1 ) GO TO 30
      IF ( NT*IPT .GT. KPOP ) GO TO 55
 30   IF ( A20 .LT. 0. .OR. A20 .GT. 1. ) GO TO 55
      IF ( NDIST .GT. 3 .OR. NDIST .LT. 1 ) GO TO 55
      IF (IFIX(A20*1000.).EQ.0.AND.IFIX(VPOP*1000.).EQ.0) GO TO 55
      IF (IFIX(MPOP*1000.).GE.KPOP*1000) GO TO 55
      IF ( PVAR .LT. .2 ) GO TO 70
 55   WRITE (6,3)
      GO TO 2000
C
C  COMPUTE NECESSARY PARAMETERS
C
 70   TEMP=MPOP*(KPOP-MPOP)-KPOP*KPOP*PVAR
      IF ( IFIX(VPOP) .EQ. 0 ) VPOP=TEMP/(KPOP-(KPOP-1.)*A20)
      SPOP=SQRT(VPOP)
      WRITE (6,13) SPOP
C
C
      IF ( ISAVE .EQ. 1 ) GO TO 102
C
C
C  GENERATE ITEM DIFFICULTY INDICES (PROPORTION OF EXAMINEES ANSWERING
C  ITEM CORRECTLY)
C
      IF (IFIX((KPOP-MPOP)*1000.).EQ.0) GO TO 55
      D1=1000.
      PBAR=MPOP/KPOP
      DO 173 I=1,KPOP
 173  Q(I)=0.
      IF ( IFIX(PVAR*1000.) .GT. 0 ) GO TO 66
      DO 65 I=1,KPOP
 65   P(I)=PBAR
      GO TO 102
 66   PSD=SQRT(PVAR)
C
C
      DO 100 IJ=1,100
      DO 74 I=1,KPOP
      CALL RANDND (Z)
      Q(I)=Z*PSD +PBAR
      IF ( Q(I) .LT. 0. ) Q(I)=0.
 74   IF ( Q(I) .GT. 1. ) Q(I)=1.
C
C  DETERMINE INITIAL MEAN AND VARIANCE OF GENERATED ITEM DIFFICULTY
C  INDICES
C
      SP=0.
      SPP=0.
      DO 81 I=1,KPOP
      PP=Q(I)
      SP=SP+PP
```

```
 81    SPP=SPP+PP*PP
       PVR=(SPP-SP*SP/KPOP)/KPOP
       CVR=SQRT(PVAR/PVR)
C
C  SCALE VARIANCE OF ITEM DIFFICULTY INDICES TO STANDARD
C
       SP=0.
       SPP=0.
       DO 82 I=1,KPOP
       Q(I)=Q(I)*CVR
       IF ( Q(I) .GT. 1. ) Q(I)=1.
       PP=Q(I)
       SP=SP+PP
 82    SPP=SPP+PP*PP
       PVR=(SPP-SP*SP/KPOP)/KPOP
C
C  SCALE MEAN OF ITEM DIFFICULTY INDICES
C
       D4=SP/KPOP-PBAR
       SP=0.
       SPP=0.
       DO 84 I=1,KPOP
       Q(I)=Q(I)-D4
       IF ( Q(I) .LT. 0. ) Q(I)=0.
       IF ( Q(I) .GT. 1. ) Q(I)=1.
       PP=Q(I)
       SP=SP+PP
 84    SPP=SPP+PP*PP
       PVR=(SPP-SP*SP/KPOP)/KPOP
       PBR=SP/KPOP
       D2=ABS(PVAR-PVR)
       D3=ABS(PBAR-PBR)
       I1=(D1+.0005)*1000.
       I2=(D2+.0005)*1000.
       I3=(D3+.0005)*1000.
       IF ( I2 .LE. 5 .AND. I3 .LE. 5 ) GO TO 103
       IF ( I2 .GE. I1 ) GO TO 100
       D1=D2
       DO 90 I=1,KPOP
 90    P(I)=Q(I)
 100   CONTINUE
 103   DO 104 I=1,KPOP
 104   P(I)=Q(I)
       NSTOP=KPOP-1
       DO 110 I=1,NSTOP
       JJ=I+1
       DO 110 J=JJ,KPOP
       IF ( IFIX(P(I)*1000.).GE. IFIX(P(J)*1000.) ) GO TO 110
       TEMP=P(J)
       P(J)=P(I)
       P(I)=TEMP
 110   CONTINUE
 102   IF ( INTPRT .EQ. 1 ) WRITE (6,4) (P(I),I=1,KPOP)
C
C  COMPUTATION OF CONSTANTS FOR GENERATION OF LOGNORMAL DISTRIBUTION
C
       YSD=0.
```

```
      IF ( NDIST .EQ. 1 ) GO TO 111
      IF ( NDIST .EQ. 3 ) MPOP=KPOP-MPOP
      YVAR=ALOG(VPOP/(MPOP*MPOP)+1.)
      YBAR=ALOG(MPOP)-YVAR/2.
      YSD=SQRT(YVAR)
C
C  COMPUTE ROUNDING VALUES FOR EACH TEST SCORE INTERVAL
C
 111  IF ( ISAVE .EQ. 0 ) CALL ROUND
C
C  REPLICATION OF ITEM-EXAMINEE SAMPLING PARADIGM
C
      SXM=0.
      SXS=0.
      SXXM=0.
      SXXS=0.

      DO 7000 IJK=1,NREPS
      CALL ALLOC (NT,IPT,ISAMP)
      IF ( INTPRT .EQ. 0 ) GO TO 113
      WRITE (6,5)
      J=0
      N1=NT*IPT
      DO 112 I=1,N1,IPT
      KK=I+IPT-1
      J=J+1
 112  WRITE (6,6) J,(LT(K),K=I,KK)
 113  CALL POOL (NSPI,IPT,NT,XBAR,XVAR)

      XSD=0.
      IF ( XVAR .GT. 0. ) XSD=SQRT(XVAR)
      IF ( INTPRT .EQ. 1 ) WRITE (6,7) IJK,XBAR,XSD
      SXM=SXM+XBAR
      SXS=SXS+XSD
      SXXM=SXXM+XBAR*XBAR
      SXXS=SXXS+XSD*XSD
C
C  COMPUTATION OF CONSTANTS FOR NEGATIVE HYPERGEOMETRIC DISTRIBUTION OPTIO
C
      IF ( NHPER .EQ. 0 ) GO TO 7000
      A21=(KPOP/(KPOP-1.))*(1.-XBAR*(KPOP-XBAR)/(KPOP-XVAR))
      IF ( A21 .GT. 0. ) GO TO 120
      WRITE (6,8) A21
      GO TO 2000
 120  A=(-1.+1./A21)*XBAR
      B=-A-1.+KPOP/A21
      SLOG1=0.
      SLOG2=0.
      C=A+B
      DO 140 I=1,KPOP
      SLOG1=SLOG1+ALOG10(B-I+1.)
 140  SLOG2=SLOG2+ALOG10(C-I+1.)
      C=10.**(SLOG1-SLOG2)
C
C  GENERATION OF NEGATIVE HYPERGEOMETRIC DISTRIBUTION
C
      WRITE (6,9)
```

```
      N3=KPOP+1
      CK=0.
      DO 160 I=1,N3
      K=I-1
      CALL NEGHGR(K,A,B,C,HX)
      CK=CK+HX
60    WRITE(6,10) K,HX,CK


000   CONTINUE
  COMPUTE STANDARD ERRORS OF ESTIMATE OVER REPLICATIONS

      BARS=SXS/NREPS
      BARM=SXM/NREPS
      SES=SQRT((SXXS-SXS*SXS/NREPS)/NREPS)
      SEM=SQRT((SXXM-SXM*SXM/NREPS)/NREPS)
      WRITE (6,11) BARM,SEM,BARS,SES
      GO TO 2000

  EXIT GRACEFULLY

000   WRITE (6,12)
      CALL EXIT

  FORMAT STATEMENTS

1      FORMAT (I3,4F6.0,4I4,4I1,F6.0,I2)
2      FORMAT (12H1PROBLEM NO.F9.2///20H PARAMETERS INPUTTED///4X,4HMEAN,
     17X,F8.3//4X,8HVARIANCE,3X,F8.3//4X,1HK,10X,I4//4X,8HALPHA 20,3X,
     2F8.3//4X,6HVAR(P),5X,F8.3////1X,27HITEM-EXAMINEE SAMPLING PLAN///
     34X,2HNT,5X,I4 //4X,3HIPT,4X,I4//4X,4HNSPT,3X,I4//4X,5HNREPS,2X,
     4I4////9H SWITCHES///4X,18HITEM-SAMPLING PLAN,8X,I2//4X,23HNEGATIVE
     5 HYPERGEOMETRIC,3X,I2//4X,22HNORMATIVE DISTRIBUTION,I6//4X,21HINTE
     6RMEDIATE PRINTOUT,I7//)
3      FORMAT (28H *** ERROR ON PARAMETER CARD///12X,2HOR//29H NEED MORE
     1INFO ON PARAMETERS)
4      FORMAT (////24H ITEM DIFFICULTY INDICES//(10F7.3))
5      FORMAT (////8H SUBTEST,5X,5HITEMS///)
6      FORMAT (1X,I5,5X,20I4/(11X,20I4))
7      FORMAT (//16H REPLICATION NO.,I5,5X,12HEST. OF MEAN,F24.3/ 26X,26H
     1EST. OF STANDARD DEVIATION,F10.3//)
8      FORMAT (43H KR21 NEGATIVE OR ZERO ... DATA SET ABORTED,5X,6HKR21 =
     1F10.4)
9      FORMAT (////5X,5HSCORE,22X,4HH(X),26X,8HCUM H(X)///)
10     FORMAT (I10,2F30.7)
11     FORMAT (30H AVERAGE POOLED MEAN OVER REPSF15.7,3X,27HSE OF POOLED
     1MEAN OVER REPSF15.7//30H AVERAGE POOLED SD   OVER REPSF15.7,3X,
     227HSE OF POOLED SD   OVER REPSF15.7)
1      FORMAT (20H ALL INPUT PROCESSED)
1      FORMAT (//17H COMPUTED SIGMA =F10.5//)
      END
```

```
      SUBROUTINE POOL (NSPT,IPT,NT,XBAR,XVAR)
C
C  DETERMINATION OF POOLED ESTIMATE  OF POPULATION MEAN TEST SCORE AND
C  VARIANCE
C
      REAL MPOP
      COMMON P(150),X(150),LT(3000)
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,IPRT
      DIMENSION TEST(150)
      IF ( IPRT .EQ. 1 ) WRITE (6,1)
      SESTM=0.
      SESTV=0.
      NSM=0
      DO 1000 I=1,NT
      SY=0.
      SYY=0.
      DO 508 K=1,IPT
  508 P(K)=0.
      ISTART=IPT*(I-1)+1
      ISTOP=IPT*I
      DO 500 J=1,NSPT
      CALL DATA (TEST)
      LL=0
      DO 505 K=ISTART,ISTOP
      KK=LT(K)
      LL=LL+1
  505 X(LL)=TEST(KK)
      Y=0.
      DO 510 K=1,IPT
      T=X(K)
      P(K)=P(K)+T
  510 Y=Y+T
      SY=SY+Y
  500 SYY=SYY+Y*Y
      XBR=SY/NSPT
      VR=(SYY-SY*SY/NSPT)/NSPT
      SPQ=0.
      DO 520 J=1,IPT
      PP=P(J)/NSPT
  520 SPQ=SPQ+PP*(1.-PP)
      NSM=NSM+1
      ESTM=KPOP*XBR/IPT
      ESTV=(NSPT*KPOP*((KPOP-1.)*VR-(KPOP-IPT)*SPQ))/(IPT*(IPT-1.)*
     1(NSPT-1.))
      SESTM=SESTM+ESTM
      SESTV=SESTV+ESTV
      ESTS=0.
      IF ( ESTV .GT. 0. ) ESTS=SQRT(ESTV)
      IF ( ESTV .LT. 0. ) ESTS=-1.*SQRT(ABS(ESTV))
      IF ( IPRT .EQ. 1 ) WRITE (6,2) I,ESTM,ESTV,ESTS
 1000 CONTINUE
```

```
      XMAR=SESTM/NSM
      XVAR=SESTV/NSM
      M=NSPT*NSM
      IF ( M .LT. 500 ) XVAR=XVAR*(M-1.)/M
      RETURN
      FORMAT (///38X,22H ESTIMATE OF PARAMETER///5X,6HSAMPLE,10X,
     14HMEAN,16X,8HVARIANCE,12X,12HSTANDARD DEV//)
      FORMAT (I10,3F20.7)
      END
```

```
      SUBROUTINE DATA (X)
C
C  GENERATION OF ITEM SCORES AND TEST SCORE FOR HYPOTHETICAL EXAMINEE
C
      REAL MPOP
      INTEGER TSCORE
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,INTPRT
      COMMON /BLOCK2/ RND(150)
      COMMON /BLOCK3/ P(150),Q(150),NSUB
      DIMENSION X(150)
C
C  GENERATE TOTAL TEST SCORE
C
      NSUB=NSUB+1
      CALL RANDND (Z)
      GO TO (215,220,220),NDIST
215   TEMP=Z*SPOP+MPOP
      GO TO 230
220   TEMP=EXP(Z*YSD+YBAR)
230   IF ( TEMP .LT. 0. ) TEMP=0.
      IF ( TEMP .GT. FLOAT(KPOP) ) TEMP=KPOP
      KK=TEMP+1.
      IF ( KK .GT. KPOP ) KK=KPOP
      TSCORE=TEMP+(KK-RND(KK))
      IF ( TSCORE .LT. 0 ) TSCORE=0
      IF ( TSCORE .GT. KPOP ) TSCORE=KPOP
      IF ( NDIST .EQ. 3 ) TSCORE=KPOP-TSCORE
C
C  GENERATE ITEM SCORES FOR EXAMINEE
C
      DO 240 J=1,KPOP
240   X(J)=0.
      IF ( TSCORE .EQ. 0 ) GO TO 300
      IF ( TSCORE .LT. KPOP ) GO TO 248
      DO 242 J=1,KPOP
242   X(J)=1.
      GO TO 300
248   KOUNT=0
      DO 250 J=1,KPOP
      IF ( IFIX(Q(J)*1000.) .GT. IFIX(P(J)*1000.) ) GO TO 250
      KOUNT=KOUNT+1
      IF ( KOUNT .GT. TSCORE ) GO TO 300
      X(J)=1.
250   CONTINUE
      DO 260 J=1,KPOP
      IF ( IFIX(X(J)) .EQ. 1 ) GO TO 260
      KOUNT=KOUNT+1
      IF ( KOUNT .GT. TSCORE ) GO TO 300
      X(J)=1.
260   CONTINUE
300   DO 320 J=1,KPOP
320   Q(J)=(Q(J)*(NSUB-1.)+X(J))/NSUB
      RETURN
      END
```

```
      SUBROUTINE NEGHGR (K,A,B,C,HX)
C
C   NEGATIVE HYPERGEOMETRIC FUNCTION
C
      REAL N,M,MPOP
      COMMON N(150),M(150),LT(3000)
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,INTPRT
      IF ( K .EQ. 0 ) GO TO 150
      S=KPOP
      SLOG1=0.
      SLOG2=0.
      SLOG3=0.
      SLOG4=0.
      DO 100 I=1,K
      SLOG1=SLOG1+ALOG10(S-I+1.)
      SLOG2=SLOG2+ALOG10(A+I-1.)
      SLOG3=SLOG3+ALOG10(B-I+1.)
  100 SLOG4=SLOG4+ALOG10(FLOAT(I))
      HX=C*10.**(SLOG1+SLOG2-SLOG3-SLOG4)
      RETURN
  150 HX=C
      RETURN
      END
```

```
      SUBROUTINE ALLOC (NT,IPT,ISAMP)
C
C  RANDOM ASSIGNMENT OF ITEMS TO SUBTESTS
C
      REAL MPOP
      COMMON X(300),LT(3000)
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,INTPRT
      DIMENSION L(150),KNTR(150)
      DO 100 I=1,KPOP
      KNTR(I)=0
 100  L(I)=I
      NN=NT*IPT
      IF ( ISAMP .EQ. 2 ) GO TO 200
 130  K=0
      DO 150 I=1,NN
 165  R=RUNIF(BASE)
      JJ=R*KPOP+1.
      IF ( JJ .LT. 1 ) JJ=1
      IF ( JJ .GT. KPOP ) JJ=KPOP
      IF ( L(JJ) .GT. 0 ) GO TO 170
      GO TO 165
 170  LT(I)=L(JJ)
      K=K+1
      IF ( ISAMP .NE. 1 ) GO TO 180
      L(JJ)=-L(JJ)
      GO TO 150
 180  IF ( K .LT. IPT ) GO TO 185
      K=0
      DO 183 II=1,KPOP
 183  L(II)=IABS(L(II))
      GO TO 150
 185  L(JJ)=-L(JJ)
 150  CONTINUE
      RETURN
 200  NMULT=NN/KPOP
      IF (IFIX((FLOAT(NN)/KPOP)*10.).NE.IFIX(FLOAT(NMULT)*10.)) GOTO400
      K=0
      NSTOP=NN-IPT
      DO 300 I=1,NSTOP
 210  R=RUNIF(BASE)
      JJ=R*KPOP+1.
      IF ( JJ .LT. 1 ) JJ=1
      IF ( JJ .GT. KPOP ) JJ=KPOP
      IF ( L(JJ) .GT. 0 .AND. KNTR(JJ) .LT. NMULT ) GO TO 220
      GO TO 210
 220  LT(I)=L(JJ)
      K=K+1
      KNTR(JJ)=KNTR(JJ)+1
      IF ( K .LT. IPT ) GO TO 250
      K=0
```

```
DO 230 J=1,KPOP
L(J)=IABS(L(J))
GO TO 300
L(JJ)=-L(JJ)
CONTINUE
DO 350 I=1,KPOP
IF ( KNTR(I) .EQ. NMULT ) GO TO 350
NSTOP=NSTOP+1
LT(NSTOP)=I
CONTINUE
RETURN
IF ( NN .GT. KPOP ) ISAMP=0
IF ( NN .EQ. KPOP ) ISAMP=1
WRITE (6,1) ISAMP
GO TO 130
FORMAT (29H TK NOT INTEGER MULTIPLE OF K//30H ITEM-SAMPLING SWITCH
 RESET TO15//)
END
```

```
      SUBROUTINE RANDND (X)
      REAL MPOP
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,INTPRT
      DIMENSION C(290),C1(90),C2(85),C3(45),C4(60),C5(10)
      EQUIVALENCE (VTAIL,C(200)),(C1(1),C(1)),(C2(1),C(91)),
     1 (C3(1),C(176)),(C4(1),C(221)),(C5(1),C(281))
      DATA C1/
     1 .2,.2,.3,.3,.3,.3,.3,.5,.6,.6,
     2 .6,.6,.6,.8,.8,.8,1.,1.,1.5,0.,
     3 0.,0.,0.,0.,0.,0.,.1,.1,.1,.1,
     4 .1,.1,.1,.2,.2,.2,.2,.2,.3,.3,
     5 .4,.4,.4,.4,.4,.4,.5,.5,.5,.5,
     6 .5,.6,.7,.7,.7,.7,.7,.8,.8,.9,
     7 .9,.9,.9,1.,1.,1.1,1.1,1.1,1.2,1.2,
     8 1.2,1.3,1.3,1.4,1.4,1.5,1.6,1.7,1.8,.0,
     9 .4,.4,.7,.9,.9,.9,1.1,1.1,1.1,1.1 /
      DATA C2/
     1 1.3,1.3,1.3,1.3,1.3,1.3,1.4,1.4,1.6,1.6,
     2 1.6,1.6,1.6,1.6,1.7,1.7,1.7,1.8,1.9,1.9,
     3 1.9,1.9,1.9,1.9,1.9,1.9,2.,2.,2.,2.,
     4 2.,2.,2.,2.1,2.1,2.1,2.1,2.1,2.1,2.2,
     5 2.2,2.2,2.2,2.3,2.3,2.3,2.4,2.4,2.5,2.6,
     6 .7,1.1,1.3,.4,1.,1.9,1.4,.9,.8,.6,
     7 .5,1,2,1.6,1.7,.3,1.5,2.,1.8,2.2,.2,
     8 2.5,2.3,2.4,2.1,.1,2.7,0.,2.6,2.8,2.9,
     9 .943216507, .946409288, .949496939, .952578378, .955556764 /
      DATA C3/
     1 .958489620, .961388536, .964198279, .966788825, .969367756,
     2 .971936598, .974474970, .976942627, .979212915, .981233554,
     3 .983249373, .985020795, .986448314, .987806989, .989110415,
     4 .990207369, .991260517, .992236259, .993158205, .994021949,
     5 .994845636, .995501310, .995889739, .996268373, .997300203,
     6 .942278196, .945572077, .948551446, .951165313, .954986329,
     7 .956691427, .960485017, .963804134, .966571775, .968916970,
     8 .971291678, .974201251, .976132812, .978422883, .980579525,
     9 .983065206, .984224076, .986325151, .987141582, .988832851  /
      DATA C4/
     1 .989490775, .990781611, .991730598, .993063286, .993813410,
     2 .994262546, .995110801, .995805552, .996077866, .996413834,
     3 .973,.996,.992,.920,.998,.982,.990,.996,.985,.959,
     4 .942,.994,.986,.985,.890,.988,.980,.983,.977,.843,
     5 .973,.975,.974,.978,.755,.970,.501,.971,.968,.967,
     6 12.5,8.2052333, 6.9186539, 20.,9.0325579,
     7 4.6444448,6.4086308,10.,11.111111,14.735714,
     8 16.666666,7.5104139,5.5743498,5.2288616,25.,
     9 5.9645244,4.3951201,4.9208132,3.9631786,33.333333 /
      DATA C5/
     1 3.4427955,3.7748844,3.6020289,4.1690656,50.,
     2 3.1592514,100.,3.2956424,3.0324898,2.9143782 /
      SGN=1.
      U=RUNIF(BASE)
      IF ( U .LT. .5 ) SGN=-1.
      U=RUNIF(BASE)
      IV2=1000.*U
```

```
      IV1=IV2/10
      V=100.*U-.1*FLOAT(IV2)
      IF ( U .GE. .79 ) GO TO 10
      X=(C(IV1+1)+V)*SGN
      RETURN
10    IF ( U .GE. .94 ) GO TO 20
      X=(C(IV2-70)+V)*SGN
      RETURN
20    IF ( U .GE. VTAIL ) GO TO 30
      J=170
21    J=J+1
      IF ( U .GE. C(J) ) GO TO 21
      IF ( U .LT. C(J+30) ) GO TO 23
      U=RUNIF(BASE)
      X=(C(J-30)+.1*U)*SGN
      RETURN
23    U=RUNIF(BASE)
      V=RUNIF(BASE)
      U1=AMIN1(U,V)
      U2=AMAX1(U,V)
      IF ( U2 .GE. C(J+60) ) GO TO 25
24    X=(C(J-30)+.1*U1)*SGN
      RETURN
25    W=-.5*(.1*U1-.1)*(2.*C(J-30)+.1*U1+.1)
      IF ( (EXP(W)-1.)*C(J+90)-U2+U1 ) 23,23,24
30    U1=RUNIF(BASE)
      U2=RUNIF(BASE)
      S=U1*U1+U2*U2
      IF ( S .GE. 1. ) GO TO 30
      T=SQRT((9.-2.*ALOG(S))/S)
      IF ( U1*T .LE. 3. ) GO TO 32
      X=U1*T*SGN
      RETURN
32    IF ( U2*T .LT. 3. ) GO TO 30
      X=U2*T*SGN
      RETURN
      END
```

```
      SUBROUTINE ROUND
C
C     ROUNDING SUBROUTINE FOR NORMAL AND LOGNORMAL FUNCTIONS.  AREA
C     BETWEEN ADJACENT INTEGER SCORES IS COMPUTED BY MEANS OF THE
C     TRAPEZOID FORMULA.  THE ROUNDING VALUE IS THAT CONTINUOUS TEST
C     SCORE SUCH THAT ONE-HALF OF THE AREA WITHIN THE SCORE INTERVAL I
C     ABOVE THAT POINT.
C
      REAL MPOP
      COMMON /BLOCK1/ YBAR,YSD,MPOP,SPOP,KPOP,NDIST,BASE,IPRT
      COMMON /BLOCK2/ RND(150)
      DIMENSION Y(101)
C
C
      DELTA=.01
      YVAR=YSD*YSD
      VPOP=SPOP*SPOP
      PI=3.1415927
      GO TO (40,50,50), NDIST
 40   CND1=1./SQRT(2.*PI*VPOP)
      CND2=2.*VPOP
      GO TO 60
 50   CLN1=1./SQRT(2.*PI*YVAR)
      CLN2=2.*YVAR
 60   CONTINUE
C
C
      SAREA=0.
      IF ( IPRT .EQ. 1 ) WRITE (6,7)
      DO 100 I=1,KPOP
      N=I-1
      DO 150 J=1,101
      K=J-1
      X=N+K*DELTA
      GO TO (130,131,131), NDIST
 130  Y(J)=CND1*EXP(-((X-MPOP)**2)/CND2)
      GO TO 150
 131  IF ( X .GT. 0. ) GO TO 135
      Y(J)=0.
      GO TO 150
 135  Y(J)=(CLN1/X)*EXP(-((ALOG(X)-YBAR)**2)/CLN2)
 150  CONTINUE
      IF (IPRT .EQ. 1) WRITE (6,5) N,(Y(J),J=1,101,10)
      N1=Y(1)*10000.
      N2=Y(101)*10000.
      IF ( N1 .GT. 0 .OR. N2 .GT. 0 ) GO TO 153
      RND(I)=N+.5
      GO TO 100
 153  AREA=0.
      DO 155 J=2,100
 155  AREA=AREA+Y(J)
```

```
      AREA=DELTA*((Y(1)+Y(101))/2.+AREA)
      SAREA=SAREA+AREA
      P=0.
      DO 160 J=1,100
      K=J+1
      P=P+((Y(J)+Y(K))*DELTA/2.)/AREA
      IF ( P .LT. .5 ) GO TO 160
      RND(I)=N+J*DELTA
      GO TO 100
160   CONTINUE
      WRITE (6,3)
      CALL EXIT
100   CONTINUE
      IF ( IPRT .EQ. 0 ) RETURN
      WRITE (6,6) SAREA
      WRITE (6,1)
      DO 200 I=1,KPOP
      J=I-1
200   WRITE (6,2) J,RND(I),I
      RETURN
1     FORMAT (///22H ROUNDING RULE    SCORE,5X,5HROUND,5X,5HSCORE//)
2     FORMAT (17X,I5,F10.2,5X,I5)
3     FORMAT (34H PROBLEM IN ROUND SUBR EXIT CALLED)
5     FORMAT (1X,I3,11F8.4)
6     FORMAT (7H AREA =F15.7//)
7     FORMAT (//23H DISTRIBUTION ORDINATES//
     1         3X,1HN,3X,2H.0,6X,2H.1,6X,2H.2,6X,2H.3,6X,2H.4,6X,2H.5,
     26X,2H.6,6X,2H.7,6X,2H.8,6X,2H.9,5X,3H1.0//)
      END
```

Examples Of Parameter Cards

```
Card      00000000011111111112222222222333333333344444444445555555555666666666677777777778
column    12345678901234567890123456789012345678901234567890123456789012345678901234567890

           80 76.            .00    .85   02  10 030   51103  7777 0
           80 40.            .05    .85   02  10 090   51101  1234 0
           80 76.            .00    .70   02  10 030   51103  1111 0
           60 40.            .05    .70   05  10 060   51101  7120 0
           80 40.            .00    .70   02  10 090   51101  8054 0
           80 40.            .00    .70   10  05 090   51101  1004 0
           40 20.            .05    .70   02  15 120   51101  9276 0
           40 20.            .00    .70   02  15 120   51101  2988 0
```