

**Principles of  
Digital Transmission  
With Wireless Applications**

# **Information Technology: Transmission, Processing, and Storage**

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Sergio Benedetto and Ezio Biglieri

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**Sergio Benedetto and  
Ezio Biglieri**

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# Preface

*Quelli che s'innamoran di pratica senza scienza,  
son come 'I nocchieri ch'entra in navilio senza timone o bussola,  
che mai ha la certezza dove si vada.*  
*Leonardo da Vinci, Codex G, Bibliothèque de l'Institut de France, Paris.*

This book stems from its ancestor *Digital Transmission Theory*, published by Prentice-Hall in 1987 and now out of print. Following the suggestion of several colleagues who complained about the unavailability of a textbook they liked and adopted in their courses, two out of its three former authors have deeply revised and updated the old book, laying a strong emphasis on wireless communications. We hope that those who liked the previous book will find again its flavor here, while new readers, untouched by nostalgia, will judge it favorably.

In keeping with the cliché “every edition is an addition,” we started planning what new topics were needed in a textbook trying to provide a substantial covering of the discipline. However, we immediately became aware that an in-depth discussion of the many things we deemed appropriate for inclusion would quickly make this book twice the size of the previous one. It would certainly be nice to write, as in the **Mahābhārata**, “what is in this book, you can find somewhere else; but what is not in it, you cannot find anywhere.” Yet such a book, like Borges’ map drawn to 1:1 scale, would not hit the mark. For this reason we aimed at writing an entirely new book, whose focus was on (although not totally restricted to) wireless digital transmission, an area whose increasing relevance in these days need not be stressed. Even with this shift in focus, we are aware that many things were left out, so that the reader should not expect an encyclopedic coverage of the discipline, but rather a relatively thorough coverage of some important parts of it.

Some readers may note with dismay that in a book devoted, at least partially, to wireless communications, there is no description of wireless *systems*. If we were to choose an icon for this book, we would choose Carroll’s Cheshire Cat of Wonderland. As Martin Gardner notes in his “Annotated Alice,” the phrase “grin without a cat” is not a bad description of pure mathematics. Similarly, we think

of this phrase as a good description of “communication theory” as contrasted to “communication systems.” A book devoted to communication systems alone would be a cat without a grin: thus, due to the practical impossibility of delivering both, we opted for the grin. Another justification is that, as the Cheshire Cat is identified only by its smile, so we have characterized communications by its theoretical foundations.

Our goal is primarily to provide a textbook for senior or beginning-graduate students, although practicing engineers will probably find it useful. We agree with Plato, who in his *Seventh Letter* contrasts the dialectic method of teaching, exemplified by Socrates’ personal, interactive mode of instruction, with that afforded by the written word. Words can only offer a shallow form of teaching: when questioned, they always provide the same answer, and cannot convey ultimate truths. Instruction can only take place within a dialogue, which a book can never offer. Yet, we hope that our treatment is reflective enough of our teaching experience so as to provide a useful tool for self-study.

We assume that the reader has a basic understanding of Fourier transform techniques, probability theory, random variables, random processes, signal transmission through linear systems, the sampling theorem, linear modulation methods, matrix algebra, vector spaces, and linear transformations. However, advanced knowledge of these topics is not required.

This book can serve as a text in either one-semester or two-semester courses in digital communications. We outline below some possible, although not exhaustive, roadmaps.

1. *A one-term basic course in digital communications:*  
Select review sections in Chapters 2, 3, 4, and 5, parts of Chapters 7 and 9.
2. *A one-term course in advanced digital communications:*  
Select review sections in Chapters 4 and 5, then Chapters 6, 7, 8, 9, and 13.
3. *A one-term course in information theory and coding:*  
Chapters 3, 9, 10, 11, 12, and parts of 13.
4. *A two-term course sequence in digital communications and coding:*  
(A) Select review sections in Chapters 2, 3, 4, 5, 6, and 7.  
(B) Chapters 9, 10, 11, 12, 13, and 14.

History tells us that Tolstoy’s wife, Sonya, copied out “War and Peace” seven times. Since in these days wives are considerably less pliable than in 19th-century Russia, we produced the whole book by ourselves using  $\text{\LaTeX}$ : this implies that we are solely responsible not only for technical inaccuracies, but

also for typos. We would appreciate it if the readers who spot any of them would write to us at <benedetto, biglieri>@polito.it. An errata file will be kept and sent to anyone interested.

As this endeavor is partly the outcome of our teaching activity, it owes a great deal to our colleagues and students who volunteered to read parts of the book, correct mistakes, and provide criticism and suggestions for its improvement. We take this opportunity to acknowledge Giuseppe Caire, Andrea Carena, Vittorio Curri, G. David Forney, Jr., Roberto Garello, Roberto Gaudino, Jørn Justesen, Guido Montorsi, Giorgio Picchi, Pierluigi Poggiolini, S. Pas Pasupathy, Fabrizio Pollara, Bixio Rimoldi, Giorgio Taricco, Monica Visintin, Emanuele Viterbo, and Peter Willett. Participation of E.B. in symposia with Tony Ephremides, Ken Vastola, and Sergio Verdú, even when not strictly related to digital communications, was always conducive to scholarly productivity. Luciano Brino drew most of the figures with patience and skill.

namo *Gaṇeśāya vighnēśvarāya*

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