

Principles of Plasma Physics for Engineers and Scientists

This unified introduction provides the tools and techniques needed to analyze plasmas, and connects plasma phenomena to other fields of study. Combining mathematical rigor with qualitative explanations, and linking theory to practice with example problems, this is a perfect textbook for senior undergraduate and graduate students taking a one-semester introductory course in plasma physics.

For the first time, material is presented in the context of unifying principles, illustrated using organizational charts, and structured in a successive progression from single-particle motion to kinetic theory and average values, through to the collective phenomena of waves in plasma. This provides students with a stronger understanding of the topics covered, their interconnections, and when different types of plasma models are applicable. Furthermore, mathematical derivations are rigorous yet concise, so physical understanding is not lost in lengthy mathematical treatments. Worked examples illustrate practical applications of theory, and students can test their new knowledge with 90 end-of-chapter problems.

Umran Inan is a Professor of Electrical Engineering at Stanford University, where he has led pioneering research on very low frequency studies of the ionosphere and radiation belts, space plasma physics, and electromagnetics for over 30 years. He also currently serves as President of Koç University in Istanbul, Turkey. As a committed teacher, he has supervised the Ph.D. dissertations of 42 students and has authored two previous books that have become standard textbooks for electromagnetics courses, as well as receiving numerous awards including the Tau Beta Pi Excellence in Undergraduate Teaching Award and the Outstanding Service Award from the Electrical Engineering Department for excellence in teaching. He is a Fellow of the Institute for Electrical and Electronics Engineers (IEEE), the American Geophysical Union (AGU), and the American Physical Society (APS), and is the recipient of the 2008 Appleton Prize from the International Union of Radio Science and the Royal Society, the 2007 Allan Cox Medal of Stanford for Faculty Excellence in fostering undergraduate research, and the 2010 Special Science Award given by the Scientific and Technological Research Council of Turkey.

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Principles of Plasma Physics for Engineers and Scientists

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**To my parents, my beautiful wife Elif, my dear children
Ayşe and Ali, and my very special granddaughter Ayla.
USI**

**To my father, who taught me to appreciate physics,
my mother, who taught me to appreciate writing, and
my wife, who gave me the support and motivation to
finish this project.
MG**

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I have been teaching Introductory Plasma Physics to senior undergraduates and beginning graduate students for many years, and I find the level of the presentation of material, the order that the topics are presented, and the overall length of the book to be an excellent match for my needs in a textbook.

David Hammer, Cornell University

The authors have done an excellent job in introducing the vast scope of plasma physics for basic plasma physics courses. The schematic illustrations and flow charts used are especially helpful in understanding the complexities involved in the hierarchical nature of plasmas. Mathematics is kept at just the right level for the intended readers and the descriptions of the physical processes are clear. Although this book is targeted to advanced undergraduate or beginning graduate students, it will be a good addition to the personal library of every plasma physicist.

Gurudas Ganguli, Naval Research Laboratory

This new book provides an excellent summary of the basic processes occurring in plasmas together with a comprehensive introduction to the mathematical formulation of fluid (MHD) and kinetic theory. It provides an excellent introduction to the subject suitable for senior undergraduate students or entry-level graduate students.

Richard M. Thorne, University of California at Los Angeles

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Preface

This book is intended to provide a general introduction to plasma phenomena at a level appropriate for advanced undergraduate students or beginning graduate students. The reader is expected to have had exposure to basic electromagnetic principles including Maxwell's equations and the propagation of plane waves in free space. Despite its importance in both science and engineering the body of literature on plasma physics is often not easily accessible to the non-specialist, let alone the beginner. The diversity of topics and applications in plasma physics has created a field that is fragmented by topic-specific assumptions and rarely presented in a unified manner with clarity. In this book we strive to provide a foundation for understanding a wide range of plasma phenomena and applications. The text organization is a successive progression through interconnected physical models, allowing diverse topics to be presented in the context of unifying principles. The presentation of material is intended to be compact yet thorough, giving the reader the necessary tools for further specialized study. We have sought a balance between mathematical rigor championed by theorists and practical considerations important to experimenters and engineers. Considerable effort has been made to provide explanations that yield physical insight and illustrations of concepts through relevant examples from science and technology.

The material presented in this book was initially put together as class notes for the EE356 Elementary Plasma Physics course, newly introduced and taught by one of us (USI) at Stanford University in the spring quarter of 1998. The course was then taught regularly every other year, for graduate students from the departments of Electrical Engineering, Materials Science, Mechanical Engineering, Applied Physics, and Physics. Over the years, several

PhD students, including Nikolai Lehtinen, Georgios Veronis, Jacob Bortnik, Michael Chevalier, Timothy Chevalier, and Prajwal Kulkarni, contributed to the course in their work as teaching assistants. The course was co-taught by Prajwal Kulkarni and one of us (MG) in the Spring of 2008, and by Brant Carlson in the Spring of 2010. We offer our thanks to each of these colleagues for their enthusiastic help and contributions, as well as to the many students enrolled in the course who helped improve its content with their contributions.

More generally, we owe considerable gratitude to all the other researchers and students of the Very Low Frequency Group at Stanford University who have been a source of valuable feedback and expertise, and to our administrative assistants, Shaolan Min and Helen Wentong Niu, for their contributions. We would like specifically to acknowledge Dr. Prajwal Kulkarni, for his pedagogical insights that have helped shape this text, and Dr. Brant Carlson, for valuable help in editing the manuscript.