Priority-based Merging Operator without Distance Measures ⋆

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Abstract. This paper proposes a refinement of the PS-Merge merging operator, which is an alternative merging approach that employs the notion of partial satisfiability rather than the usual distance measures. Our approach will add to PS-Merge a mechanism to deal with a kind of priority based on the quantity of information of the agents. We will refer to the new operator as Pr-Merge. We will also analyze its logical properties as well its complexity by conceiving an algorithm with a distinct strategy from that presented for PS-Merge.

1 Introduction

Information fusion or *merging* consists in techniques of how to merge or combine information provided by multiple sources, taking into account possible inconsistencies and letting the result as reliable as possible. Different kinds of information may be merged: knowledge, belief, preference, rule, etc; each one with its own specificity and intuition [8].

Most of the works introduced in the literature focus especially on belief and preference merging [1,3,8,13]. Belief (preference) merging is concerned with the process of combining the information contained in a set of belief (preference) bases obtained from different sources to produce a single consistent belief (preference) base. It is an important issue in Artificial Intelligence and Databases, and its applications are many and diverse [2].

There is a slight difference between the approaches of belief and preference merging. Beliefs are information held by human or artificial agents about the world. Preferences represent human or artificial agents' goals, desires and plans about the world. They both can be false, uncertain, exhibit an elementary nature, susceptible to changes or involve a complex logical structure. Syntactically, they can be represented in the same way, but semantically, it is needed to consider their own characteristics, inherited by the nature of its information.

Under this assumption, several merging operators have been defined and characterized in a logical way. Among them, model-based merging operators [13] obtain a belief/preference base from a set of interpretations with the help of a

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distance measure on interpretations and an aggregation function. Other merging operators, syntax-based (or formula-based) ones [13], are based on the selection of some consistent subsets of the set-theoretic union of the belief/preference bases.

The major problem with distance-based merging operators is that evaluating the closeness between interpretations as a number may lead to lose too much information [7]. For example, the widely used Hamming distance [4] (also known as Dalal distance) assumes not only that propositional symbols are equally relevant to determining a distance between interpretations, but also that they are independent from each other and that nothing else is relevant to the determination of the distance between interpretations. These assumptions are restrictive and give the Hamming distance very little flexibility [14].

To overcome this issue, some characterizations of model-based merging operators were achieved by modifying the distance measure [6,7,10,14]. In addition, merging operators without distance measures were also conceived. An alternative method of merging was proposed in [16,17,18], which uses the notion of Partial Satisfiability instead of a distance measure, to define PS-Merge, a model-based merging operator which depends on the syntax of the belief bases [15].

In this paper, we will consider mainly the problem of preference merging without distance measures, by refining the definition of PS-Merge (which is characterized originally considering belief merging) through the weighting of the information in the preference bases. We will name our approach of Pr-Merge. Intuitively, we are concerned in representing priority information among the agents, that will be provided according of how the preference bases are organized.

The paper is organized as follows: in Section 2, we will introduce the Pr-Merge. In Section 3, we will discuss about its logical properties. In Section 4, we will exhibit its computational complexity results. Finally, in Section 5 we will conclude the paper.

2 Priority-based Merging Operator

In this section we introduce the priority-based merging operator Pr-Merge. Basically, the idea of priority consists in ranking the importance of each outcome, based on the preferences of each agent. In our work, we will measure the importance of an outcome by considering the number of propositions' appearance in the agents' goal bases.

Example 1. The application of this merging is relevant in the following scenario: suppose that three friends are going to share a meal in a restaurant, which is constituted of a main dish and a drink. One person is very restrictive with relation to his/her preferences, e.g., he/she prefers vegetarian food, while the others two have more choices to make than the first one, since they are non-vegetarian and there is a greater diversity of choices to make for both, and these possible options are considered equally satisfactory for them. Since the choices are more restricted and objective for the first person, it is natural that we need to give more priority to his/her desires, but without forgetting completely the desires of the other two people.

The merging operator introduced in this section will consider this aspect: it will give more importance and priority to the agents which express their preferences in a simplified, objective or restricted way. On the other hand, it is extremely plausible to think in a context where we should give more priority to the agents that express more preferences (this kind of view can be achieved later by changing a definition in the merging operator). The details about this approach will be explained during this section.

In the following lines, we will present some preliminary notions and the definition of the Pr-Merge. As said previously, we considered the definitions and intuitions of PS-Merge to define our approach. More details about PS-Merge can be found in [16,17,18].

First, we will consider a propositional language $\mathcal{L}$ defined from a finite set of propositional variables $\mathcal{P}$ and the usual connectives $\neg, \wedge$ and $\vee$. A literal is a propositional variable from $\mathcal{P}$ or its negation.

Definition 1. *A profile $E = \{K_1, \ldots, K_n\}$ represents sets of goal bases K_i, for $1 \leq i \leq n$. For a goal base $K_i = \{c_1, \ldots, c_m\}$, each c_j, where $1 \leq j \leq m$, denotes the set of preferences of the agent i.*

A goal base K_i is a finite and consistent set of propositional formulas. In this work, we restrict each goal base K_i to a DNF (Disjunctive Normal Form) formula, i.e., it can be viewed as $K_i = (c_1 \vee \cdots \vee c_m)$ and $c_l = (x_1 \wedge \cdots \wedge x_k)$, where $x_1, \ldots, x_k$ are literals. We chose the DNF format in order to represent the agents' preferences/choices of a simplified way.

Example 2 (Borrowed from [19]). Let us consider the academic example of a teacher who asks his three students which among the following languages *SQL* (denoted by s), O_2 (denoted by o) and *Datalog* (denoted by d) they would like to learn. The first student wants to only learn *SQL* or O_2, that is, $K_1 = (s \vee o) \wedge \neg d$. The second wants to learn either *Datalog* or O_2 but not both, i.e., $K_2 = (\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)$. For the last, the third one wants to learn the three languages: $K_3 = (s \wedge d \wedge o)$.

First of all, we need to convert these preferences to the DNF format. We shall have $K_1 = (s \wedge \neg d) \vee (o \wedge \neg d)$, and consequently, $K_1 = \{c_1, c_2\}$, where $c_1 = (s \wedge \neg d)$ and $c_2 = (o \wedge \neg d)$. For the goal bases K_2 and K_3, we shall have $K_2 = \{c_3, c_4\}$ and $K_3 = \{c_5\}$, where $c_3 = (\neg s \wedge d \wedge \neg o)$, $c_4 = (\neg s \wedge \neg d \wedge o)$ and $c_5 = (s \wedge d \wedge o)$. We can view in this example that the third agent has only one preferable choice $(s \wedge d \wedge o)$, while the first and second ones have both two preferable choices (for K_1, it is $(s \wedge \neg d)$ or $(o \wedge \neg d)$, and for K_2, it is $(\neg s \wedge d \wedge \neg o)$ or $(\neg s \wedge \neg d \wedge o)$). We can say that K_3 is more certain/restricted about his/her choices.

Definition 2. *An outcome or interpretation is a function $\omega : \mathcal{P} \rightarrow \{0, 1\}$. The values 0 and 1 are identified with the classical truth values false and true, respectively.*

For instance, when $\omega(s) = 1$, we say that the interpretation of the propositional variable s is true, whereas when $\omega(s) = 0$, we say that its interpretation is false. We have that $\omega(s) = 1 \Leftrightarrow \omega(\neg s) = 0$.

Example 3. With respect to the previous example, we have three propositional variables: s, d and o. The set of all possible outcomes/interpretations is $\Omega = \{\omega_1, \ldots, \omega_8\}$, where: $\omega_1 = \neg s \neg d \neg o$, $\omega_2 = \neg s \neg do$, $\omega_3 = \neg sd \neg o$, $\omega_4 = \neg sdo$, $\omega_5 = s \neg d \neg o$, $\omega_6 = s \neg do$, $\omega_7 = sd \neg o$ and $\omega_8 = sdo$.

Slightly abusing the notation, the interpretation $\omega_1 = \neg s \neg d \neg o$ may be viewed as $\omega_1(\neg s) = 1, \omega_1(\neg d) = 1$ and $\omega_1(\neg o) = 1$.

Before proceeding with the rest of the definitions, let us make a little detour in the subject. As said previously, several merging operators have been defined and characterized in a logical way. Among them, model-based merging operators [13] obtain a belief/preference base from a set of interpretations with the help of a distance measure on interpretations and an aggregation function. Formally, a distance measure between an interpretation and a goal base is defined as $d(\omega, K) = \min_{\omega' \models K} d(\omega, \omega')$, where $d(\omega, \omega')$ is the distance between interpretations. In the first works on model-based merging, the distance used was the Hamming distance between interpretations [4], but any other distance may be used as well.

To be considered a distance measure, a function needs to satisfy the following conditions:

Definition 3 (Distance). *A distance measure between interpretations is a total function d from $\Omega \times \Omega$ to $\mathbb{N}$ such that for every $\omega_1, \omega_2 \in \Omega$,*

- $d(\omega_1, \omega_2) = d(\omega_2, \omega_1)$*, and*
- $d(\omega_1, \omega_2) = 0$ *if and only if* $\omega_1 = \omega_2$.

The Hamming distance between interpretations characterizes the number of propositional variables that they differ. For example, the Hamming distance (denoted d_H) between $\omega_1 = \neg s \neg d \neg o$ and $\omega_6 = s \neg do$ is $d_H(\omega_1, \omega_6) = 2$ (i.e., they differ in two propositional variables).

Basically, the distance gives the closeness between an interpretation and each formula of a goal base. However, this measure between interpretations may lead to lose information and not to discriminate them [6,7]. In order to try to avoid this problem, merging operators without distance measures were conceived. An alternative method of merging was proposed in [16,17,18], which uses the notion of Partial Satisfiability instead of a distance measure. In this work, we will exploit the notion of Partial Satisfiability for the purpose of describing the priority preferences.

We can now begin with the notion of preference priority. In order to do this, we will work in two levels: the partial satisfiability of a specific agent (to each $K_i \in E$) and the preference priorities of a group of agents E (based on the partial satisfiability of each agent). These definitions are inspired in the work of the PS-Merge operator [16,17,18].

Definition 4 (Partial Satisfiability). *Let $K = \{c_1, \ldots, c_m\}$ be a goal base. The partial satisfiability of the interpretation ω w.r.t. K is given by:*

$$\omega(K) = max\{\omega(c_1), \ldots, \omega(c_m)\},$$

where for each $c_i = (x_1 \wedge \cdots \wedge x_k)$, $1 \leq i \leq k$:

$$\omega(c_i) = \sum_{l=1}^{k} \left\{ \frac{\omega(x_l)}{k} \right\}.$$

The partial satisfiability of an interpretation in a clause indicates the rate of the occurrences of its literals in the DNF formula. The higher an interpretation appears in a clause the higher will be its partial satisfiability. We assume that each literal in a clause must have the same weight in the evaluation, i.e., no propositional variable has priority over another one. For example, in the clause $(s \wedge d \wedge o)$ of K_3, the propositions s, d and o have the same weight of $\frac{1}{3}$, since the sum of the weights of propositional variables needs to be equal to 1; and in the clause $(s \wedge \neg d)$ of K_1, the propositions s and $\neg d$ have the same weight of $\frac{1}{2}$.

Example 4. From the *Example 2*, we have $K_1 = \{(s \wedge \neg d), (o \wedge \neg d)\}$, $K_2 = \{(\neg s \wedge d \wedge \neg o), (\neg s \wedge \neg d \wedge o)\}$ and $K_3 = \{(s \wedge d \wedge o)\}$. The partial satisfiability of each interpretation w.r.t. K_1, K_2 and K_3 is computed as:

Ω	$\omega(K_1)$	$\omega(K_2)$	$\omega(K_3)$
$\omega_1 = \neg s \neg d \neg o$	1/2	2/3	0
$\omega_2 = \neg s \neg d o$	1	1	1/3
$\omega_3 = \neg s d \neg o$	0	1	1/3
$\omega_4 = \neg s d o$	1/2	2/3	2/3
$\omega_5 = s \neg d \neg o$	1	1/3	1/3
$\omega_6 = s \neg d o$	1	2/3	2/3
$\omega_7 = s d \neg o$	1/2	2/3	2/3
$\omega_8 = s d o$	1/2	1/3	1

To define the preference priority in our framework, we will assume that each clause of a goal base shares the same weight in the preference evaluation. For example, the formula $(s \wedge d \wedge o)$ of the goal base K_3 will have a priority weight 1 (because there is only one clause in the goal base), while the clauses $(s \wedge \neg d)$ and $(o \wedge \neg d)$ of the goal base K_1 will have both the priority weight $\frac{1}{2}$ (the sum of weights needs to be equal to 1). Formally, we will define this idea in two different ways.

Definition 5 (Preference Priority (sum)). *Let $E = \{K_1, \ldots, K_n\}$ be a profile and ω an interpretation. The priority of ω w.r.t. E is given by:*

$$\omega_+(E) = \sum_{i=1}^{n} \frac{1}{a_i} \times \omega(K_i),$$

where a_i is the number of clauses in the goal base K_i.

This step reflects the preference priority of the group of agents, which will be a prioritized sum of the partial satisfiability of each individual goal base of the group. Intuitively, The higher is the number of choices made by an agent, the lower will be his/her preference priority among the group of agents. Another characterization of the preference priority can be defined as:

Definition 6 (Preference Priority (product)). *Let $E = \{K_1, \ldots, K_n\}$ be a profile and ω an interpretation. The priority of ω w.r.t. E is given by:*

$$\omega_\times(E) = \prod_{i=1}^{n} (\omega(K_i))^{\frac{1}{a_i}},$$

where a_i is the number of clauses in the goal base K_i.

Example 5. Finally, considering the sum operation, the preference priority of the profile $E = \{K_1, K_2, K_3\}$ is:

Ω	$\omega_+(E)$
$\omega_1 = \neg s \neg d \neg o$	$1/4 + 1/3 + 0 = 7/12 \simeq \mathbf{0.583}$
$\omega_2 = \neg s \neg d o$	$1/2 + 1/2 + 1/3 = 4/3 \simeq \mathbf{1.333}$
$\omega_3 = \neg s d \neg o$	$0 + 1/2 + 1/3 = 5/6 \simeq \mathbf{0.833}$
$\omega_4 = \neg s d o$	$1/4 + 1/3 + 2/3 = 5/4 = \mathbf{1.25}$
$\omega_5 = s \neg d \neg o$	$1/2 + 1/6 + 1/3 = 6/6 = \mathbf{1}$
$\omega_6 = s \neg d o$	$1/2 + 1/3 + 2/3 = 3/2 = \mathbf{1.5}$
$\omega_7 = s d \neg o$	$1/4 + 1/3 + 2/3 = 5/4 = \mathbf{1.25}$
$\omega_8 = s d o$	$1/4 + 1/6 + 1 = 17/12 \simeq \mathbf{1.416}$

By considering the product, the preference priority of the profile E is:

Ω	$\omega_\times(E)$
$\omega_1 = \neg s \neg d \neg o$	$0.707 \times 0.816 \times 0 = \mathbf{0}$
$\omega_2 = \neg s \neg d o$	$1 \times 1 \times 0.333 \simeq \mathbf{0.333}$
$\omega_3 = \neg s d \neg o$	$0 \times 1 \times 0.333 = \mathbf{0}$
$\omega_4 = \neg s d o$	$0,707 \times 0.816 \times 0.666 \simeq \mathbf{0.384}$
$\omega_5 = s \neg d \neg o$	$1 \times 0.577 \times 0.333 \simeq \mathbf{0.192}$
$\omega_6 = s \neg d o$	$1 \times 0.816 \times 0.666 \simeq \mathbf{0.544}$
$\omega_7 = s d \neg o$	$0,707 \times 0.816 \times 0.666 \simeq \mathbf{0.384}$
$\omega_8 = s d o$	$0,707 \times 0.577 \times 1 \simeq \mathbf{0.407}$

For the sake of information, if we consider in giving more priority to the agents that are expressing more choices, we must make a little change in the definitions above. In this case, we shall have $\omega_+(E) = \sum_{i=1}^{n} a_i \times \omega(K_i)$ and $\omega_\times(E) = \prod_{i=1}^{n} (\omega(K_i))^{a_i}$, We will follow the examples using the former definitions, but we want to highlight that, although these two approaches express different ideas, they share similar properties (the logical properties of the merging operator will be explored in the next section).

After compute the preference priorities, we can rank the interpretations and decide which one is the best option for the group.

Definition 7. *The binary relations* $\leq_E^{pr,+}$ *and* $\leq_E^{pr,\times}$ *are defined as*

$$\omega \leq_E^{pr,+} \omega' \text{ if and only if } \omega_+(E) \leq \omega'_+(E) \text{ and}$$
$$\omega \leq_E^{pr,\times} \omega' \text{ if and only if } \omega_\times(E) \leq \omega'_\times(E)$$

Here, an outcome ω' is preferred to ω if the preference priority of ω' is greater or equal to the priority of ω.

Example 6. After computing the preference priority of the group of agents we can rank the interpretations as:

$$\omega_1 \leq_E^{pr,+} \omega_3 \leq_E^{pr,+} \omega_5 \leq_E^{pr,+} \{\omega_4, \omega_7\} \leq_E^{pr,+} \omega_2 \leq_E^{pr,+} \omega_8 \leq_E^{pr,+} \omega_6 \text{ and}$$
$$\{\omega_1, \omega_3\} \leq_E^{pr,\times} \omega_5 \leq_E^{pr,\times} \omega_2 \leq_E^{pr,\times} \{\omega_4, \omega_7\} \leq_E^{pr,\times} \omega_8 \leq_E^{pr,\times} \omega_6.$$

The best outcome in this example is the interpretation ω_6. Comparing our approach (with the sum operation) to the one presented by the PS-Merge (which is defined with the help of the sum), we will have:

	Pr-Merge	PS-Merge
Ω	$\omega_+(E)$	$\omega(E)$
$\omega_1 = \neg s \neg d \neg o$	0.583	1.16
$\omega_2 = \neg s \neg d o$	1.333	**2.33**
$\omega_3 = \neg s d \neg o$	0.833	1.5
$\omega_4 = \neg s d o$	1.25	1.83
$\omega_5 = s \neg d \neg o$	1	1.67
$\omega_6 = s \neg d o$	**1.5**	**2.33**
$\omega_7 = s d \neg o$	1.25	1.83
$\omega_8 = s d o$	1.416	1.83

Note that, in general, the preferences between the outcomes are very similar. The difference appears in the results of the outcomes ω_2 and ω_8. The goal base $K_3 = (s \wedge d \wedge o)$ have a preference priority greater than the other bases, which will influence in the result of ω_8 (an interpretation that satisfies K_3), increasing its final result, whereas it will decrease the result of the outcome ω_2, because it is not a good outcome to K_3 (ω_2 satisfies only one propositional variable of K_3). We can define this process as a merging operator in the following model-theoretical way:

Definition 8 (Pr-Merge). *Let* $E = \{K_1, \ldots, K_n\}$ *be a profile and* μ *an integrity constraint, the merging operator* $\Delta_\mu^{pr,op}(E)$ *is defined as:*

$$Mod(\Delta_\mu^{pr,op}(E)) = max(Mod(\mu), \leq_E^{pr,op}),$$

where $op \in \{+, \times\}$ *and* $max(Mod(\mu), \leq_E^{pr,op})$ *is the set of interpretations that satisfy* μ *and are the maximal with respect to the relation* $\leq_E^{pr,op}$.

An integrity constraint μ is a formula that the result of the merging process has to obey, i.e., they cannot be inconsistent. When we do not consider an integrity constraint in the process, we assume that $\mu = \top$.

Example 7. The merging operator $\Delta_{\mu}^{pr,op}(E)$ for the previous example, when $\mu = \top$ and $op \in \{+, \times\}$, shall result in:

$$Mod(\Delta_{\mu}^{pr,op}(E)) = \omega_6 = (s \wedge \neg d \wedge o).$$

If we restrict the result of merging, considering that only one programming language will be taught, i.e., $\mu_1 = (s \wedge \neg d \wedge \neg o) \vee (\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)$, the result is:

$$Mod(\Delta_{\mu_1}^{pr,op}(E)) = \omega_2 = (\neg s \wedge \neg d \wedge o).$$

To conclude this section, we want to emphasize our choice with respect to the partial satisfiability approach. The approach introduced in this paper is not restricted only to PS-Merge, i.e., it can be used with distance-based merging operators too. Indeed, the distance-based merging with priorities may be viewed as a particular case of the weighted sum aggregation function [9].

Formally, it can be defined in the following way: as said previously, the distance measure between an interpretation and a goal base is defined as $d(\omega, K) = \min_{\omega' \models K} d(\omega, \omega')$, where $d(\omega, \omega')$ is the distance between interpretations. Using the *sum* as an aggregation function we define the distance measure between an interpretation and a profile $E = \{K_1, \ldots, K_n\}$ as $d(\omega, E) = \sum_{i=1}^{n} \{d(\omega, K_i)\}$. When the weighted *sum* is considered as the aggregation function we have $d(\omega, E) = \sum_{i=1}^{n} a_i \times d(\omega, K_i)$, where a_i is the number of clauses in the goal base K_i in our work. Consequently, the merging operator $\Delta_{\mu}^{d,op}(E)$, where $op \in \{sum, wsum\}$, is defined as $Mod(\Delta_{\mu}^{d,op}(E)) = min(Mod(\mu), \leq_E^{d,op})$. The comparison between distance-based and partial satisfiability merging is showed below (when $d = d_H$):

Ω	$\Delta_{\mu}^{d_H,sum}$	$\Delta_{\mu}^{ps,+}$	$\Delta_{\mu}^{d_H,wsum}$	$\Delta_{\mu}^{pr,+}$
$\omega_1 = \neg s \neg d \neg o$	5	1.16	4	0.583
$\omega_2 = \neg s \neg d o$	**2**	**2.33**	2	1.333
$\omega_3 = \neg s d \neg o$	4	1.33	3	0.833
$\omega_4 = \neg s d o$	3	1.83	2	1.25
$\omega_5 = s \neg d \neg o$	4	1.66	3	1
$\omega_6 = s \neg d o$	**2**	**2.33**	**1.5**	**1.5**
$\omega_7 = s d \neg o$	3	1.83	2	1.25
$\omega_8 = s d o$	3	1.83	**1.5**	1.416

In short, we can see that a partial satisfiability-based merging is richer than a distance-based merging, since it gives us a more detailed evaluation of the interpretations. Another important point that we want to highlight is that the partial satisfiability allows us to employ the product as an aggregation function, which is not possible when a distance is considered.

3 Logical Properties

A main requirement for adhering to a merging operator is that it offers the expected properties of what intuitively merging means. This calls for sets of rationality postulates and this has been addressed in several papers [5,6,7,10,11]. The more postulates satisfied the more rational the operator. We will look in the sequence the characterization of Integrity Constraints (IC) merging operators.

Definition 9 (IC merging operators [11]). *Let E, E_1, E_2 be profiles, K_1, K_2 be consistent goal bases, and μ, μ_1, μ_2 be propositional formulas. Δ is an IC merging operator if and only if it satisfies the following postulates:*

- **(IC0)** $\Delta_\mu(E) \models \mu$.
- **(IC1)** If μ is consistent, then $\Delta_\mu(E)$ is consistent.
- **(IC2)** If $\bigwedge E$ is consistent with μ, then $\Delta_\mu(E) \equiv \bigwedge E \wedge \mu$.
- **(IC3)** If $E_1 \equiv E_2$ and $\mu_1 \equiv \mu_2$, then $\Delta_{\mu_1}(E_1) \equiv \Delta_{\mu_2}(E_2)$.
- **(IC4)** If $K_1 \models \mu$ and $K_2 \models \mu$, then $\Delta_\mu(\{K_1, K_2\}) \wedge K_1$ is consistent if and only if $\Delta_\mu(\{K_1, K_2\}) \wedge K_2$ is consistent.
- **(IC5)** $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2) \models \Delta_\mu(E_1 \sqcup E_2)$.
- **(IC6)** If $\Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$ is consistent, then $\Delta_\mu(E_1 \sqcup E_2) \models \Delta_\mu(E_1) \wedge \Delta_\mu(E_2)$.
- **(IC7)** $\Delta_{\mu_1}(E) \wedge \mu_2 \models \Delta_{\mu_1 \wedge \mu_2}(E)$.
- **(IC8)** If $\Delta_{\mu_1}(E) \wedge \mu_2$ is consistent, then $\Delta_{\mu_1 \wedge \mu_2}(E) \models \Delta_{\mu_1}(E)$.

The meaning of the properties is the following: **(IC0)** ensures that the result of merging satisfies the integrity constraint. **(IC1)** states that, if the integrity constraint is consistent, then the result of merging will be consistent. **(IC2)** states that if there is no inconsistencies among the goal bases, the result of merging is simply the conjunction of the goal bases with the integrity constraint. **(IC3)** is the principle of irrelevance of syntax: the result of merging has to depend only on the expressed opinions and not on their syntactical presentation. **(IC4)** is a fairness postulate meaning that the result of merging of two goal bases should not give preference to one of them. It is a condition that aims at ruling out operators that can give priority to one of the bases. **(IC5)** expresses the following idea: if profiles are viewed as expressing the beliefs/preferences of the members of a group, then if E_1 (corresponding to a first group) compromises on a set of alternatives which A belongs to, and E_2 (corresponding to a second group) compromises on another set of alternatives which contains A too, then A has to be in the chosen alternatives if we join the two groups. **(IC5)** and **(IC6)** together state that if one could find two subgroups which agree on at least one alternative, then the result of the global merging will be exactly those alternatives the two groups agree on. **(IC7)** and **(IC8)** state that the notion of closeness is well-behaved, i.e., that an alternative that is preferred among the possible alternatives will remain preferred if one restricts the possible choices.

Proposition 1. $\Delta_{\mu}^{pr,op}$ *satisfies* ***(IC0)-(IC3)*** *and* ***(IC5)-(IC8)****.*

Proof. **(IC0)** By definition, $Mod(\Delta_{\mu}^{pr,op}(E)) \subseteq Mod(\mu)$.

(IC1) The functions $\omega_+(E)$ and $\omega_\times(E)$ map to values in $\mathbb{R}$, so if $Mod(\mu) \neq \emptyset$, there is a model ω of μ such that for every model ω' of μ, $\omega_+(E) \geq \omega'_+(E)$ (or $\omega_\times(E) \geq \omega'_\times(E)$). So $\omega \models \Delta_{\mu}^{pr,op}(E)$ and $\Delta_{\mu}^{pr,op}(E) \not\models \perp$.

(IC2) By assumption, $\bigwedge E$ is consistent and without loss of generality let $E = \{K_1, \ldots, K_n\}$. There exists ω such that $\omega \models (c_{11} \vee \cdots \vee c_{1k}) \wedge \cdots \wedge (c_{n1} \vee \cdots \vee c_{nm})$, where $K_1 = \{c_{11}, \ldots, c_{1k}\}, \ldots, K_n = \{c_{n1}, \ldots, c_{nm}\}$. By definition, $\omega(K_1) = max\{\omega(c_{11}), \ldots, \omega(c_{1n})\}$ and as $\omega \models (c_{11} \vee \cdots \vee c_{1n})$, there is a clause c_{1j} such that $\omega \models c_{1j}$. It is easy to see that this clause has the maximum value, i.e. $\omega(c_{ij}) = 1$ (see the Definition 4). Thus, $\omega(K_1)$ will also receive the maximum possible value. The same idea holds for every K_i, $1 \leq i \leq n$. Hence, as $\omega_+(E) = \sum_{i=1}^{n} \frac{1}{a_i} \times \omega(K_i)$, for every ω', $\omega_+(E) \geq \omega'_+(E)$ (the same holds for $\omega_\times(E)$). So $\omega \models \Delta_{\mu}^{pr,op}(E)$ if and only if $\omega \models \bigwedge E \wedge \mu$.

(IC3) Assume that $E_1 \equiv E_2$ and $\mu_1 \equiv \mu_2$, where $E_1 = \{K_1, \ldots, K_n\}$ and $E_2 = \{K'_1, \ldots, K'_n\}$. We want to prove that $\Delta_{\mu_1}^{pr,op}(E_1) \equiv \Delta_{\mu_2}^{pr,op}(E_2)$. For this, it is sufficient to guarantee that $\omega(K_i) \leq \omega'(K_i) \Rightarrow \omega(K'_i) \leq \omega'(K'_i)$, for any ω, ω'. It is possible to show this using the notion of Hamming distance [10]. The Hamming distance between interpretations, denoted as $d_H(\omega, \omega')$, characterizes the number of propositional variables that they differ. The distance between an interpretation and a goal base is defined as: $d(\omega, K_i) = \min_{\omega' \models K_i} d(\omega, \omega')$.

We have that if $\omega(K_i) \leq \omega'(K_i)$ then $d(\omega', K_i) \leq d(\omega, K_i)$ (it is easy to show this by contradiction). By hypothesis, $K_i \equiv K'_i$, and therefore we have $\omega(K_i) \leq \omega'(K_i)$ then $d(\omega', K'_i) \leq d(\omega, K'_i)$. We need to show now that $d(\omega', K'_i) \leq d(\omega, K'_i) \Rightarrow \omega(K'_i) \leq \omega'(K'_i)$. By contradiction, suppose that $d(\omega', K'_i) \leq d(\omega, K'_i)$ and $\omega(K'_i) > \omega'(K'_i)$. In this case we would have $d(\omega', K'_i) > d(\omega, K'_i)$ (by the consequence of $\omega(K'_i) > \omega'(K'_i)$), which is a contradiction.

To end this proof, note that definition of $\omega_+(E) = \sum_{i=1}^{n} \frac{1}{a_i} \times \omega(K_i)$ (and $\omega_\times(E) = \prod_{i=1}^{n} (\omega(K_i))^{\frac{1}{a_i}}$)does not alter the results showed above, i.e., $\omega_+(E_1) \leq \omega'_+(E_1) \Rightarrow \omega_+(E_2) \leq \omega'_+(E_2)$ (resp. $\omega_\times(E_1) \leq \omega'_\times(E_1) \Rightarrow \omega_\times(E_2) \leq \omega'_\times(E_2)$), due the properties of the sum (resp. product). As $\mu_1 \equiv \mu_2$, finally we have that $\Delta_{\mu_1}^{pr,op}(E_1) \equiv \Delta_{\mu_2}^{pr,op}(E_2)$.

(IC5) In order to show that the operator satisfy **(IC5)**, it is enough to guarantee that the following property holds: if $\omega_{op}(E_1) \geq \omega'_{op}(E_1)$ and $\omega_{op}(E_2) \geq \omega'_{op}(E_2)$, then $\omega_{op}(E_1 \sqcup E_2) \geq \omega'_{op}(E_1 \sqcup E_2)$, for $op \in \{+, \times\}$. We can see clearly that this is satisfied.

(IC6) In order to show that the operator satisfy **(IC6)**, it is enough to guarantee that the following property holds: if $\omega_{op}(E_1) > \omega'_{op}(E_1)$ and $\omega_{op}(E_2) \geq$

$\omega'_{op}(E_2)$, then $\omega_{op}(E_1 \sqcup E_2) > \omega'_{op}(E_1 \sqcup E_2)$, for $op \in \{+, \times\}$. We can see clearly that this is satisfied.

(IC7) Suppose that $\omega \models \Delta^{pr,op}_{\mu_1}(E) \wedge \mu_2$. For any $\omega' \models \mu_1$, we have $\omega_{op}(E) \geq \omega'_{op}(E)$. Hence, for any $\omega' \models \mu_1 \wedge \mu_2$, we have $\omega_{op}(E) \geq \omega'_{op}(E)$. Subsequently $\omega \models \Delta^{pr,op}_{\mu_1 \wedge \mu_2}(E)$.

(IC8) Suppose that $\Delta^{pr,op}_{\mu_1}(E) \wedge \mu_2$ is consistent. Then there exists a model ω' of $\Delta^{pr,op}_{\mu_1}(E) \wedge \mu_2$. Consider a model ω of $\Delta^{pr,op}_{\mu_1 \wedge \mu_2}(E)$ and suppose that $\omega \not\models \Delta^{pr,op}_{\mu_1}(E)$. In this case $\omega'_{op}(E) > \omega_{op}(E)$, and since $\omega' \models \mu_1 \wedge \mu_2$, we have $\omega \notin Mod(\Delta^{pr,op}_{\mu_1 \wedge \mu_2}(E)) = max(Mod(\mu_1 \wedge \mu_2), \leq^{pr,op}_E)$, hence $\omega \not\models \Delta^{pr,op}_{\mu_1 \wedge \mu_2}(E)$. Contradiction. □

Proposition 2. *$\Delta^{pr,op}_{\mu}$ does not satisfy **(IC4)**.*

Proof. In general, $\Delta^{pr,op}_{\mu}$ does not satisfy **(IC4)**. Let us give a counter-example: suppose that $\mu = \top$, $K_1 = \{(a \wedge \neg b) \vee (\neg a \wedge b)\}$ and $K_2 = \{(a \wedge b)\}$. The result of the merging is $\Delta^{pr,op}_{\mu}(\{K_1, K_2\}) = (a \wedge b)$, when $op \in \{+, \times\}$. $\Delta^{pr,op}_{\mu}(\{K_1, K_2\}) \wedge K_2$ is consistent, but $\Delta^{pr,op}_{\mu}(\{K_1, K_2\}) \wedge K_1$ is not. □

Since **(IC4)** is not satisfied, it means that this merging operator tends to give preference to some specific goal bases. This is not a bad result, since we intended from the beginning to give more priority to some agents.

The merging operators $\Delta^{pr,+}_{\mu}$ and $\Delta^{pr,\times}_{\mu}$ share the same logical properties so far, but intuitively, they express different ideas. Two main subclasses of merging operators are described by analyzing others characteristics: majority operators which are related to utilitarianism, and arbitration operators which are related to egalitarianism. In other words, majority operators solve conflicts using majority wishes, i.e., they try to satisfy the group as a whole. Whereas arbitration operators have a more consensual behavior, trying to satisfy each agent as far as possible.

Besides these nine postulates presented above, we will also consider these two important sub-classes of merging operators: IC majority operator and IC arbitration operator. We will show in the sequel that $\Delta^{pr,+}_{\mu}$ and $\Delta^{pr,\times}_{\mu}$ do not agree with both postulates.

Definition 10 (IC majority operator). *A merging operator is a majority operator if it satisfies*

– **(Maj)** $\exists n \Delta_\mu(E_1 \sqcup \underbrace{E_2 \sqcup \cdots \sqcup E_2}_{n}) \models \Delta_\mu(E_2)$.

This postulate states that if an information has a majority audience, then it will be the choice of the group.

Proposition 3. *$\Delta^{pr,+}_{\mu}$ satisfies **(Maj)**.*

Proof. Showing that the operator satisfies **(Maj)** is easy from the properties of sum. Since $\omega_+(E) = \sum_{i=1}^{n} \frac{1}{a_i} \times \omega(K_i)$, without loss of generality we can assume two cases: (i) let ω be a model for $\Delta_{\mu}^{pr,+}(E_1 \sqcup E_2)$ and for all ω', $\omega_+(E_2) \geq \omega'_+(E_2)$. In this case, we also have that ω is a model for $\Delta_{\mu}^{pr,+}(E_2)$, and for every n, $\Delta_{\mu}^{pr,+}(E_1 \sqcup E_2^n) \models \Delta_{\mu}^{pr,+}(E_2)$; (ii) let ω be a model for $\Delta_{\mu}^{pr,+}(E_1 \sqcup E_2)$ and there is a ω' such that $\omega_+(E_2) < \omega'_+(E_2)$. In this case we can always find a number n of repetitions to E_2 such that ω' will be a model for $\Delta_{\mu}^{pr,+}(E_1 \sqcup E_2^n)$, i.e., $\omega'_+(E_2) \times n + \omega'_+(E_1) > \omega_+(E_2) \times n + \omega_+(E_1)$. Consequently, $\Delta_{\mu}^{pr,+}(E_1 \sqcup E_2^n) \models \Delta_{\mu}^{pr,+}(E_2)$. □

As a consequence of this postulate, we can state that although it is given more priority to some goal bases in the merging process of $\Delta_{\mu}^{pr,+}$, it will not be always the case that these goal bases will be satisfied by the results of the merging operator.

Proposition 4. *$\Delta_{\mu}^{pr,\times}$ does not satisfy **(Maj)**.*

Proof. We can find a counter-example where the repetition of one base does not change the result. Consider the following counter-example: Let $\mu = \top$, $E_1 = \{K_1\} = \{\{a \wedge b\}\}$ and $E_2 = \{K_2\} = \{\{\neg a \wedge \neg b\}\}$. Clearly, we have $\Delta_{\mu}^{pr,\times}(E_1 \sqcup \underbrace{E_2 \sqcup \cdots \sqcup E_2}_{n}) \not\equiv \Delta_{\mu}^{pr,\times}(E_2)$ for any $n \in \mathbb{N}$. □

Definition 11 (IC arbitration operator). *A merging operator is an arbitration operator if it satisfies*

- **(Arb)**

$$\left.\begin{array}{r} \Delta_{\mu_1}(\{K_1\}) \equiv \Delta_{\mu_2}(\{K_2\}) \\ \Delta_{\mu_1 \leftrightarrow \neg\mu_2}(\{K_1, K_2\}) \equiv (\mu_1 \leftrightarrow \neg\mu_2) \\ \mu_1 \not\models \mu_2 \\ \mu_2 \not\models \mu_1 \end{array}\right\} \Rightarrow \Delta_{\mu_1 \vee \mu_2}(\{K_1, K_2\}) \equiv \Delta_{\mu_1}(\{K_1\}).$$

Unlike the majority operator, an arbitration operator tries to satisfy each agent as possible. According to [12] this postulates ensures that this is the median of possible choices that are preferred.

Proposition 5. *$\Delta_{\mu}^{pr,+}$ does not satisfy **(Arb)**.*

Proof. To show that $\Delta_{\mu}^{pr,+}$ does not satisfy **(Arb)**, consider the following counter-example: $K_1 = \{\{a \wedge b\}\}, K_2 = \{\{\neg a \wedge \neg b\}\}, \mu_1 = \neg(a \wedge b)$ and $\mu_2 = a \vee b$. We have that $\Delta_{\mu_1 \vee \mu_2}^{pr,+}(\{K_1, K_2\}) \not\equiv \Delta_{\mu_1}^{pr,+}(\{K_1\})$. □

We can note that, it may be the case where a goal base has more priority than the other ones, and the result of the merging will only favor it rather than the others.

Proposition 6. *$\Delta_{\mu}^{pr,\times}$ satisfies **(Arb)**.*

Proof. We can see that **(Arb)** holds since the stronger following property is true: if $\Delta_{\mu_1}^{pr,\times}(K_1) \equiv \Delta_{\mu_2}^{pr,\times}(K_2)$, then $\Delta_{\mu_1 \vee \mu_2}^{pr,\times}(\{K_1, K_2\}) \equiv \Delta_{\mu_1}^{pr,\times}(K_1)$. □

The weighted product considers relevant the partial satisfiability of each agent to compute the preference priority of the group. It is different from the weighted sum in the sense that every agent is relevant to the final result and this result tries to satisfy the whole group as much as possible. In other terms, we can say that, although the merging gives priority to some specific agents, the product operator tries to consider important the opinion of each agent to the result of the merging.

To finish this section, we remind that regardless the strategy used in the priority merging, the logical properties remain the same, i.e., we can use the same proofs of this section to the case where we give more priority to the agents with more clauses in the goal bases.

4 Computational Complexity

Let us now consider the complexity issue of the merging operator $\Delta_{\mu}^{pr,op}$. Formally, the decision problem MERGE($\Delta_{\mu}^{pr,op}$) is defined as:

- **Input:** A triple $\langle E, \mu, \alpha \rangle$ where $E = \{K_1, \dots, K_n\}$ is a profile and μ and α are propositional formulas.
- **Question:** Does $\Delta_{\mu}^{pr,op}(E) \models \alpha$ hold?

In this section, we will give an alternative algorithm to Pr-Merge, instead of using the one presented for PS-Merge in [16].

Proposition 7. *MERGE($\Delta_{\mu}^{pr,op}$) is PTIME.*

This result is consequence of the following two lemmas:

Lemma 1. *For any $\omega \in \Omega$ the number of possible values of $\omega_{op}(E)$ is bounded by the value $h(|E|)$ (where h is a function with values in $\mathbb{N}$), which is polynomial.*

Proof. Let $E = \{K_1, \dots, K_n\}$ be a profile and $|V| = m$ be the number of propositional variables of E. For each $K_i \in E$, the number of possible values that $\omega(K_i)$ may receive is bounded by $m + (m-1) + \cdots + 1 = m.(m+1)/2 = O(m^2)$, i.e., the scenario where K_i has clauses of size $m, m-1, \dots, 2$ and 1 (if a clause has size m, then the quantity of values that it can obtain is m). Thus, for the profile E, the number of possible values is $O(n.m^2)$. □

Lemma 2. *Given a profile E and an integrity constraint μ, the problem of determining the $\max\limits_{\omega \models \mu} \omega_{op}(E)$ is PTIME.*

Proof. $\max\limits_{\omega \models \mu} \omega_{op}(E)$ can be computed using binary search on $L = \{0, \dots, h(|E|)\}$ (the list of possible values for $\omega_{op}(E)$), but first we shall change slightly the representation of L. Assuming that $E = \{K_1, \dots, K_n\}$, each $l_i \in L$ is represented as $l_i = [l_{i1}, \dots, l_{in}]$, where l_{ij} denotes a possible value of the base K_j and $l_i = l_{i1} + \cdots + l_{in}$ (when $op = +$) or $l_i = l_{i1} \times \cdots \times l_{in}$ (when $op = \times$). For instance, considering $op = +$, we have that the first element of the list is $0 = [0, 0, \ldots, 0]$, and according to *Example 2*, the last element of the list would be $2 = [\frac{2}{4}, \frac{2}{4}, \frac{3}{3}]$ (the maximum value of ω for $K_1 = \{(s \wedge \neg d), (o \wedge \neg d)\}$ is $\frac{2}{4}$, $K_2 = \{(\neg s \wedge d \wedge \neg o), (\neg s \wedge \neg d \wedge o)\}$ is $\frac{2}{4}$ and $K_3 = \{(s \wedge d \wedge o)\}$ is $\frac{3}{3}$).

Generating the list L can be made in the following way: Consider $E = \{K_1, \ldots, K_n\}$, and $(K_i) = [m, [m_1, \ldots, m_m]]$, where m is the number of clauses of K_i and for $1 \leq j \leq m$, m_j is the number of literals in the j-th clause. With respect to the weighted sum operator, the set of possible values of K_i is $\{0, \frac{1}{m.m_1}, \frac{2}{m.m_1}, \ldots, \frac{m_1}{m.m_1}, \ldots, \frac{1}{m.m_m}, \ldots, \frac{m_m}{m.m_m}\}$. In consideration with the weighted product, the set of possible values of K_i is $\{0, (\frac{1}{m_1})^{\frac{1}{m}}, (\frac{2}{m_1})^{\frac{1}{m}}, \ldots, (\frac{m_1}{m_1})^{\frac{1}{m}}, \ldots, (\frac{1}{m_m})^{\frac{1}{m}}, \ldots, (\frac{m_m}{m_m})^{\frac{1}{m}}\}$. For instance, in the *Example 2*, for $op = +$, the set of possible values of $K_1 = \{(s \wedge \neg d), (o \wedge \neg d)\}$, where $(K_1) = [2, [2, 2]]$ is $\{0, \frac{1}{4}, \frac{2}{4}, \frac{1}{4}, \frac{2}{4}\} = \{0, \frac{1}{4}, \frac{2}{4}\}$.

Let us assume now that L is ordered by the value of the l_i, where $l_i = l_{i1} + \cdots + l_{in}$ or $l_i = l_{i1} \times \cdots \times l_{in}$ (this sorting can be done in polynomial time) and that $E = \{K_1, \ldots, K_n\}$ is also ordered by the number of clauses in the bases (i.e., K_1 is the base with the least number of clauses), in order to simplify the execution of the algorithm.

It is sufficient to consider the following algorithms:

1. The first step is ask whether $\max_{\omega \models \mu} \omega_{op}(E) \geq l$, for a given $l \in L$.
2. For a given $l = [l_1, \ldots, l_n]$, pick K_1 and find the interpretations ω in which $\omega(K_1) = l_1$ and $\omega \models \mu$. As each l_i is a number of the form $(p/q.m)$, an interpretation ω is given by the outcome that satisfies p elements in the clause with q literals. These interpretations can be found in polynomial time, since K_1 is in DNF.
3. For every $K_j \in E$, check if $\omega(K_j) = l_j$, for any ω found in the previous step. If it is true, then $\max_{\omega \models \mu} \omega_{op}(E) \geq l$. This step can be done in polynomial time.
4. To compute $\max_{\omega \models \mu} \omega_{op}(E)$, we can make a binary search on $L = \{0 = [0, \ldots, 0], \ldots, l_k = [l_{k1}, \ldots, l_{kn}]\}$. We start with l_k and ask if $\max_{\omega \models \mu} \omega_{op}(E) \geq l_k$. The $\max_{\omega \models \mu} \omega_{op}(E)$ will be the highest l_i which $\max_{\omega \models \mu} \omega_{op}(E) \geq l_i$ holds. Consequently all ω that satisfies this statement are results from merging. Clearly, we can see that this step is polynomial, since the binary search needs at most $log_2 h(|E|)$ steps and the procedure of $\max_{\omega \models \mu} \omega_{op}(E) \geq l$ is polynomial.
5. Lastly, we only have to check if $\omega \models \alpha$, for any ω found in the previous step. This can be done in linear time. $\Box$

This result shows that Pr-Merge is computationally easier (as well as the PS-Merge) than usual merging operators, which are usually at the first level of the polynomial hierarchy [10]. This is given mainly because the goal bases are represented in DNF formulas and the computation of the preference priority ω can be done in polynomial time.

5 Conclusion

In this work, we described a refined version of the merging operator PS-Merge by introducing the notion of priority information between goal bases. This new operator was named Pr-Merge, which was defined in two versions: one with a weighted sum and another one with a weighted product. The weighted sum has a characteristic of majority priority, whereas the weighted product shows the characteristic of priority combined with some aspects of egalitarianism. We analyzed their logical properties and computational complexity. With respect to the complexity, we exhibited an alternative algorithm from that presented to PS-Merge, which has a polynomial time complexity.

Regarding the logical properties, Pr-Merge satisfies all postulates in general, except **(IC4)**. The loss that we have in using Pr-Merge is that our approach does not satisfy the fairness condition, i.e., our merging approach can give priority to some goal bases, which is an expected result to us. When the weighted sum is considered as the aggregation function, Pr-Merge satisfies **(Maj)**. In other terms, we can say that, even the priority given to some agents, a group of agents can influence the result of the merging. When the weighted product is considered, Pr-Merge satisfies **(Arb)**, i.e., the priority merging tries to satisfy each agent as far as possible.

Following the proposal presented by PS-Merge, this paper focus in researching a merging operator without using distance measures. There is still too much to be done in this area. A possible line of research is to characterize a family of merging operators using the notion of partial satisfiability employed by PS-Merge, through different aggregation functions, and their relationships. Another open question is to discover the relationship between Partial Satisfiability-based and distance-based merging. Lastly, another interesting subject is to find out other alternative ways of doing information merging without using distance measures.

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