

Probabilistic and Randomized Methods for Design under Uncertainty

Giuseppe Calafiore and Fabrizio Dabbene (Eds.)

Probabilistic and Randomized Methods for Design under Uncertainty

With 21 Figures

Giuseppe Calafiore, PhD
Dipartimento di Automatica e Informatica
Politecnico di Torino
Corso Duca degli Abruzzi, 24
10129 Torino
Italy

Fabrizio Dabbene, PhD
IEIIT-CNR
Politecnico di Torino
Corso Duca degli Abruzzi, 24
10129 Torino
Italy

British Library Cataloguing in Publication Data
Probabilistic and randomized methods for design under
uncertainty
1. Engineering design - Mathematical models 2. Mathematical
optimization 3. Stochastic systems 4. Sampling (Statistics)
5. Uncertainty (Information theory)
I. Calafiore, Giuseppe, 1969- II. Dabbene, Fabrizio, 1968-
620'.0042'015196
ISBN-10: 184628094X

Library of Congress Control Number: 2005935660

ISBN-10: 1-84628-094-X e-ISBN 1-84628-095-8 Printed on acid-free paper
ISBN-13: 978-1-84628-094-8

© Springer-Verlag London Limited 2006

Apart from any fair dealing for the purposes of research or private study, or criticism or review, as permitted under the Copyright, Designs and Patents Act 1988, this publication may only be reproduced, stored or transmitted, in any form or by any means, with the prior permission in writing of the publishers, or in the case of reprographic reproduction in accordance with the terms of licences issued by the Copyright Licensing Agency. Enquiries concerning reproduction outside those terms should be sent to the publishers.

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant laws and regulations and therefore free for general use.

The publisher makes no representation, express or implied, with regard to the accuracy of the information contained in this book and cannot accept any legal responsibility or liability for any errors or omissions that may be made.

Printed in Germany

9 8 7 6 5 4 3 2 1

Springer Science+Business Media
springer.com

To our old and new families

Preface

Solum certum nihil esse certi
Plinius, *Naturalis Historia*

A central issue in many engineering design endeavors is the presence of *uncertainty* in the problem description. Different application fields employ different characterizations of the uncertainty (for instance, deterministic unknown-but-bounded description *vs* stochastic description) and correspondingly adopt different techniques to devise ‘designs’ that are in some way insensitive, or *robust*, with respect to uncertainty.

The classes of problems considered in this book refer to uncertain control systems, as well as to generic decision or optimization problems in which the data are not exactly known.

In the area of robust control, the approach relying on a purely deterministic unknown-but-bounded description of the uncertainty spawned researches that yielded significant results in the last thirty years. However, this deterministic, or worst-case, approach also showed some inherent limitations that can be resumed in the fundamental tradeoff between computational complexity and ‘conservatism’.

From a more philosophical perspective, a worst-case design, even if it could come at a cheap computational cost, may not be a desirable design in practice, since it contemplates *all* possible uncertainty scenarios, including those that are extremely unlikely to happen. On the other hand, one might think that a worst-case approach is necessary in situations where even rare events may lead to disastrous consequences. It should nevertheless be noticed that a design based on an unknown-but-bounded description of the uncertainty may lead to a ‘false’ belief of safety, since it provides no guarantees for uncertainty outcomes that, for some unforeseen reason, happen to fall outside the *a-priori* assumed uncertainty set. In this respect, an alternative approach based on a stochastic description of the uncertainty makes it clear right from the outset that, in reality, no statement can be given with absolute certainty.

In the field of optimization, the stochastic approach is indeed a classical one, dating back to the late fifties with the work of Dantzig [96] on linear programming under uncertainty and Charnes and Cooper [80] on chance-constrained optimization. In the robust control area, instead, the stochas-

tic approach still enjoys limited attention, since this area has been dominated lately by the worst-case deterministic viewpoint; early exceptions include [192, 195, 342] and later [207, 290, 347, 402]. From an historical point of view, we notice that the two areas of optimization and control followed two different routes: the optimization area has always been dominated by the stochastic paradigm, and only recently the works of El Ghaoui and Lebret [121] and Ben-Tal and Nemirovski [35] brought the worst-case approach into this area. Conversely, since the early eighties robust control has been mainly based on the worst-case paradigm; see the seminal works of Zames [407] on \mathcal{H}_∞ control, of Kharitonov [183] on parametric uncertainty, and the structured singular value theory of Doyle [114] and Safonov [316]. Lately, however, the probabilistic paradigm gained new interest among control researchers, see, *e.g.*, [24, 74, 252, 359, 381] and the many references therein.

Book Scope and Structure

This book brings together leading researchers from both the optimization and control areas, with the intent of highlighting the interactions between the two fields, and with a focus on randomized and probabilistic techniques for solving design problems in the presence of stochastic uncertainty.

The book is divided into three parts. The first part presents three contributions dealing with the general theory and solution methodologies for probability-constrained and stochastic optimization problems and a contribution on the theory of risk measures. The second part of the book contains five chapters devoted to explicit robust design methods based on uncertainty randomization and sampling. The first chapter of the third part of the book presents a novel statistical learning theory framework for system identification, whereas the other six chapters in this part focus on applications of randomized methods for analysis and design of robust control systems.

Acknowledgments

We sincerely thank all the outstanding researchers and friends that made this project possible with their contributions. The financial support of the Italian Ministry of University and Research through an FIRB grant is gratefully acknowledged.

This book goal will be attained if it will stimulate further exchange and dialogue among scientists from the control and the optimization areas, and if it will foster further interest and attract new researchers to these fields.

G. Calafiore
F. Dabbene

Torino, November 2005

Contents

List of Contributors	xi
----------------------------	----

Part I Chance-Constrained and Stochastic Optimization

1 Scenario Approximations of Chance Constraints	
<i>Arkadi Nemirovski, Alexander Shapiro</i>	3
2 Optimization Models with Probabilistic Constraints	
<i>Darinka Dentcheva</i>	49
3 Theoretical Framework for Comparing Several Stochastic Optimization Approaches	
<i>James C. Spall, Stacy D. Hill, David R. Stark</i>	99
4 Optimization of Risk Measures	
<i>Andrzej Ruszczyński, Alexander Shapiro</i>	119

Part II Robust Optimization and Random Sampling

5 Sampled Convex Programs and Probabilistically Robust Design	
<i>Giuseppe Calafiore, Marco C. Campi</i>	161
6 Tetris: A Study of Randomized Constraint Sampling	
<i>Vivek F. Farias, Benjamin Van Roy</i>	189
7 Near Optimal Solutions to Least-Squares Problems with Stochastic Uncertainty	
<i>Giuseppe Calafiore, Fabrizio Dabbene</i>	203

8 The Randomized Ellipsoid Algorithm for Constrained Robust Least Squares Problems	
<i>Stoyan Kanev, Michel Verhaegen</i>	223
9 Randomized Algorithms for Semi-Infinite Programming Problems	
<i>Vladislav B. Tadić, Sean P. Meyn, Roberto Tempo</i>	243
<hr/>	
Part III Probabilistic Methods in Identification and Control	
<hr/>	
10 A Learning Theory Approach to System Identification and Stochastic Adaptive Control	
<i>Mathukumalli Vidyasagar, Rajeeva L. Karandikar</i>	265
11 Probabilistic Design of a Robust Controller Using a Parameter-Dependent Lyapunov Function	
<i>Yasuaki Oishi</i>	303
12 Probabilistic Robust Controller Design: Probable Near Minimax Value and Randomized Algorithms	
<i>Yasumasa Fujisaki, Yasuaki Kozawa</i>	317
13 Sampling Random Transfer Functions	
<i>Constantino M. Lagoa, Xiang Li, Maria Cecilia Mazzaro, Mario Sznaiar</i>	331
14 Nonlinear Systems Stability via Random and Quasi-Random Methods	
<i>Peter F. Hokayem, Silvia Mastellone, Chaouki T. Abdallah</i>	365
15 Probabilistic Control of Nonlinear Uncertain Systems	
<i>Qian Wang, Robert F. Stengel</i>	381
16 Fast Randomized Algorithms for Probabilistic Robustness Analysis	
<i>Xinjia Chen, Kemin Zhou, and Jorge Aravena</i>	415
References	433
Index	455

List of Contributors

Chaouki T. Abdallah

Dept. Electrical & Computer Eng.
University of New Mexico
Albuquerque - New Mexico,
87131-0001
chaouki@ece.unm.edu

Jorge Aravena

Dept. Electrical & Computer Eng.
Louisiana State University
Baton Rouge - Louisiana, 70803
aravena@ece.lsu.edu

Giuseppe Calafiore

Dip. Automatica e Informatica
Politecnico di Torino
Torino - Italy, 10129
giuseppe.calafiore@polito.it

Marco C. Campi

Dip. Elettronica per l'Automazione
Università di Brescia
Brescia - Italy, 25123
campi@ing.unibs.it

Xinjia Chen

Dept. Electrical & Computer Eng.
Louisiana State University
Baton Rouge - Louisiana, 70803
chan@ece.lsu.edu

Fabrizio Dabbene

IEEIT-CNR
Politecnico di Torino
Torino - Italy, 10129
fabrizio.dabbene@polito.it

Darinka Dentcheva

Dept. Mathematical Sciences
Stevens Institute of Technology
Castle Point on Hudson
Hoboken - New Jersey, 07030
ddentche@stevens-tech.edu

Vivek F. Farias

Dept. Electrical Engineering
Stanford University
vff@stanford.edu

Yasumasa Fujisaki

Dept. Computer & Systems
Engineering
Kobe University
Nada, Kobe - Japan, 657-8501
fujisaki@cs.kobe-u.ac.jp

Stacy D. Hill

Applied Physics Laboratory
The Johns Hopkins University
11100 Johns Hopkins Road
Laurel - Maryland, 20723-6099
stacy.hill@jhuapl.edu

Peter F. Hokayem

Coordinated Science Laboratory
University of Illinois
1308 W. Main Street
Urbana - Illinois, 61801
hal@uiuc.edu

Stoyan Kanev

DCSC, TU-Delft
Mekelweg 2
CD Delft - the Netherlands, 2628
s.kanev@dcsc.tudelft.nl

Rajeeva L. Karandikar

Indian Statistical Institute
S.J.S. Sansawal Marg
New Delhi - India, 110 016
rlk@isid.ac.in

Yasuaki Kozawa

Grad. School Science & Technology
Kobe University
Nada, Kobe - Japan, 657-8501

Constantino M. Lagoa

Dept. Electrical Engineering
The Pennsylvania State University
University Park - Penn., 16802
lagoa@engr.psu.edu

Xiang Li

Dept. Electrical Engineering
The Pennsylvania State University
University Park - Penn., 16802
xiangli@psu.edu

Silvia Mastellone

Coordinated Science Laboratory
University of Illinois
1308 W. Main Street
Urbana - Illinois 61801
smastel2@uiuc.edu

Maria Cecilia Mazzaro

Dept. Electrical Engineering
The Pennsylvania State University
University Park - Penn., 16802
cmazzaro@gandalf.ee.psu.edu

Sean P. Meyn

Dept. Electrical & Computer Eng.
Coordinated Science Laboratory
University of Illinois
Urbana-Champaign - Illinois, 61801
meyn@uiuc.edu

Arkadi Nemirovski

Technion
Israel Institute of Technology
Haifa - Israel, 32000
nemirovs@ie.technion.ac.il

Yasuaki Oishi

Dept. Mathematical Informatics
Grad. School Inf. Science & Tech.
The University of Tokyo
Tokyo - Japan, 113-8656
oishi@mist.i.u-tokyo.ac.jp

Andrzej Ruszczyński

Rutgers University
Piscataway - New Jersey, 08854
rusz@rutcor.rutgers.edu

Alexander Shapiro

Georgia Institute of Technology
Atlanta - Georgia, 30332-0205
ashapiro@isye.gatech.edu

James C. Spall

Applied Physics Laboratory
The Johns Hopkins University
11100 Johns Hopkins Road
Laurel - Maryland, 20723-6099
james.spall@jhuapl.edu

David R. Stark

Applied Physics Laboratory
The Johns Hopkins University
11100 Johns Hopkins Road
Laurel - Maryland, 20723-6099
david.stark@jhuapl.edu

Robert F. Stengel

Mechanical & Aerospace Engineering
Princeton University
Princeton - New Jersey, 08544
stengel@princeton.edu

Mario Sznaier

Dept. Electrical Engineering
The Pennsylvania State University
University Park - Penn., 16802
msznaier@frodo.ee.psu.edu

Vladislav B. Tadić

Dept. Aut. Control & Sys. Eng.
University of Sheffield
Sheffield - United Kingdom, S1 3JD
v.tadic@sheffield.ac.uk

Roberto Tempo

IEEIT-CNR
Politecnico di Torino
Torino - Italy, 10129
tempo@polito.it

Benjamin Van Roy

Dept. Electrical Engineering
Stanford University
bvr@stanford.edu

Michel Verhaegen

DCSC, TU-Delft
Mekelweg 2,
CD Delft - the Netherlands, 2628
m.verhaegen@dcsc.tudelft.nl

Mathukumalli Vidyasagar

Tata Consultancy Services
No. 1, Software Units Layout
Hyderabad - India, 500 081
sagar@atc.tcs.co.in

Qian Wang

Mechanical Engineering
The Pennsylvania State University
University Park - Penn., 16802
quw6@psu.edu

Kemin Zhou

Dept. Electrical & Computer Eng.
Louisiana State University
Baton Rouge - Louisiana, 70803
kemin@ece.lsu.edu