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Probabilistic assessment of stability of a cut slope in residual soil

H. EL-RAMLY,* N. R. MORGENSTERN† and D. M. CRUDEN†

A probabilistic slope analysis methodology based on Monte Carlo simulation using Microsoft Excel and @Risk software is applied to investigate the failure of the Shek Kip Mei cut in Hong Kong. The study demonstrates the techniques used in quantifying uncertainties in shear strength of granitic soils based on a large database of triaxial tests. Probabilistic back-analyses of the failure are applied to estimate the probability distribution of the pore water pressure. Using the back-calculated pore pressure, the inclination of the Shek Kip Mei slope is redesigned to a flatter inclination, and the probability of unsatisfactory performance and reliability index are estimated.

KEYWORDS: laboratory tests; landslides; residual soils; slopes; statistical analysis

Nous appliquons une méthodologie d'analyses de pente probabiliste basée sur la simulation de Monte Carlo et utilisant des logiciels de simulation Microsoft Excel et @Risk afin d'enquêter sur la rupture de la faille de Shek Kip Mei à Hong Kong. L'étude montre les techniques utilisées dans la quantification des incertitudes sur la résistance au cisaillement des sols granitiques, techniques basées sur une grande base de données d'essais triaxiaux. Nous appliquons des rétro-analyses probabilistes de la rupture pour évaluer la distribution probabiliste de la pression d'eau de pores. En utilisant la pression de pore rétro-calculée, nous reformulons l'inclinaison de la pente de Shek Kip Mei pour la rendre plus plate et nous estimons la probabilité d'une performance et d'un indice de fiabilité non satisfaisants.

INTRODUCTION

The reluctance of practising engineers to apply probabilistic methods of slope stability in practice is attributed, among other factors, to the lack of published studies illustrating the implementation and benefits of probabilistic analyses and the absence of a consistent probabilistic design criterion.

Starting with basic field and laboratory data, this study demonstrates the application of probabilistic techniques to the assessment of a cut slope failure in Hong Kong. The study is one of a series of case histories (El-Ramly *et al.*, 2002a, 2002b, 2003a) illustrating the value of probabilistic techniques, and providing guidelines for acceptable probabilities of unsatisfactory performance (or failure probabilities). The latter objective is achieved through comparison of computed failure probabilities with observed field performances of slopes.

Hong Kong is characterised by intense urbanisation around steep slopes. Heavy seasonal rainfall there triggers landslides, which result in economic losses, fatalities and injuries. Hong Kong soils are mainly residual soils formed by decomposition of granite and volcanic rocks. Variations in the mineralogy and grain size of the parent rock and in processes of chemical weathering, physical disintegration, hydrothermal alteration, and leaching result in heterogeneous soils. This heterogeneity is a major source of uncertainty in estimating operational shear strengths, and in identifying potential slip surfaces and failure mechanisms in slopes.

Pore water pressure is controlled by rainfall intensity, duration and frequency, infiltration rate, soil macro-permeability and joint structure and infilling. So predictions of pore water pressures at failure are educated guesses at best. For example, Brand (1985) reported that the piezometric head in a slope increased by 5 m in only 18 h during a rainstorm in June 1982. The head dropped quickly when the rain stopped. Sweeney & Robertson (1979) reported a 12 m increase in water level in one piezometer during an intense rainstorm, whereas another piezometer nearby showed only a 3 m increase. Such an environment of extreme uncertainty led Kay (1993) to question the applicability of deterministic factors of safety to slopes in Hong Kong.

PROBABILISTIC SLOPE ANALYSIS METHODOLOGY

El-Ramly *et al.* (2002a) developed a probabilistic slope analysis methodology based on Monte Carlo simulation using Microsoft Excel (Microsoft, 1997) and @Risk (Palisade, 1996) software. The slope geometry, stratigraphy, soil properties, critical slip surfaces and selected method of slope analysis are modelled in an Excel spreadsheet. Uncertain input parameters are identified and treated as random variables. At any location, i, within a statistically homogeneous domain, an input variable x_i is divided into a trend component t_i, a residual component ε, and a bias correction factor, B:

$$x_i = B(t_i + \varepsilon) \tag{1}$$

The mean and variance of the bias correction factor are evaluated by experience and comparison with field performance or other more accurate tests. The mean of the trend component, a function of location, is estimated from available data using regression techniques. The uncertainty in the mean trend due to limited data is evaluated from statistical theory (Ang & Tang, 1975; Neter *et al.*, 1990). The residual component, ε, characterises the random spatial variation of the input variable. It has a mean of zero and a constant variance, $\sigma^2[\varepsilon]$, independent of location. The variance, $\sigma^2[\varepsilon]$, is estimated from the scatter of observations around the mean trend. The residual component at a given location is spatially correlated with the residual components at surrounding locations.

El-Ramly *et al.* (2002a) modelled the spatial variability of the residual component along the slip surface using a one-dimensional, stationary random field (Vanmarcke, 1977). They divided the portion of the slip surface within each soil layer into segments of lengths l_i and considered the spatial

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variability of the residual component averaged over slip surface segments. By taking segment lengths l_i equal to or less than the scale of fluctuation, δ, the variance of the local average approaches the point variance, and the correlation coefficients between most of the local averages approach zero (Vanmarcke, 1977, 1983).

Each uncertain input parameter is represented in the spreadsheet by variables corresponding to bias factor, trend component and local averages of the residual component. Using @Risk built-in functions, each variable is assigned a probability distribution function, and the correlations among variables are defined. In performing Monte Carlo simulation, @Risk draws at random a value for each input variable from within its defined probability distribution, maintaining the correlations between variables. Each set of sampled input values is used to solve the spreadsheet and calculate the corresponding factor of safety. The process is repeated sufficient times to estimate the statistical distribution of the factor of safety. The mean and variance of the factor of safety, the probability of unsatisfactory performance, and the reliability index can then be estimated.

UNCERTAINTY IN SHEAR STRENGTH OF HONG KONG GRANITIC SOILS

The slope investigated is cut in residual soils formed by decomposition of granite. As it weathers, Hong Kong granite disintegrates into soils ranging from coarse sand to silty clayey sand. Boulders and corestones may also be present. Brand (1985) noted that the majority of cut slope failures in Hong Kong are in soils classified as highly decomposed granite (HDG) or completely decomposed granite (CDG), in accordance with the weathering grade system used by the Geotechnical Engineering Office (GEO, 1988). The assessment presented here is limited to these two weathering classes. Uncertainty about shear strengths of decomposed granites, which often significantly affect slope assessments, is attributed to spatial variability in material properties.

The petrography of Hong Kong granitic soils indicated that shear strength depends on the soil microfabric derived from weathering and hydrothermal alteration (Massey *et al.*, 1989), and is poorly correlated with gradation and void ratio (Lumb, 1962; Pun & Ho, 1996). Weathering and alteration effects are controlled by grain size and mineralogy of the parent rock, microcrack regimes and joint spacing, rainfall and water infiltration, leaching, and history and duration of weathering. These complex and random factors cause weathering effects to vary significantly over short distances, even within uniform geologic units.

The random processes that form Hong Kong granitic soils suggest that the shear strength of decomposed granite can be statistically homogeneous over large areas. So probability distributions of shear strength parameters can be estimated by pooling data from different sites. The uncertainty in shear strength of HDG and CDG can be evaluated from a large database of triaxial tests collected from different sources and localities (Lumb, 1965; Hencher *et al.*, 1983; Siu & Premchitt, 1988; Shelton & Cooper, 1984; Pun & Ho, 1996). All tests were conducted on saturated samples using standard triaxial testing procedures. Available data are first screened to meet the criteria below.

Slope failures in Hong Kong are generally shallow, with low effective stresses acting on the slip surface; typically 30–200 kPa (Brand, 1985). For convenience, triaxial tests are usually conducted at higher confining stresses. Brand (1985) and Massey *et al.* (1989) noted that the strength envelopes of Hong Kong residual soils are curvilinear, and that inferences based on linear projections of tests at high stress levels underestimate available shear strengths for slope assessments. Hence a mean effective stress threshold of $p' = (\sigma'_1 + \sigma'_3)/2 = 400$ kPa is set; all tests conducted under higher stresses are discarded.

Some results were obtained through multi-stage tests, where the same specimen is consolidated and sheared several times under increasing confining stresses. Pun & Ho (1996) analysed a large database of multi-stage and single-stage triaxial tests on completely decomposed granite from the Kowloon Pluton. For comparison, they calculated the shear strength at a normal stress of 200 kPa for each test using the effective cohesion and friction angle obtained from that test. The shear strengths from the single-stage tests were higher than those of the multi-stage tests. Pun & Ho (1996) noted that multi-stage tests are sometimes performed to overcome the problem of sample variability, but that single-stage tests are carried out more often because of concerns of progressive loss of inherent structure and residual bonding in multi-stage shearing. Hence only the results of single-stage tests and the first stages of multi-stage tests are considered here.

In recent years, researchers and practitioners have made significant advances in quantifying and addressing uncertainties in input parameters used in geotechnical analyses. Progress in evaluating and accounting for the limitations of theories, models and hypotheses used in performance predictions (model uncertainty) has, however, been much less notable. Morgenstern (1995) commented that no statistical analyses can overcome the limitations of fundamentally flawed models. One common example of model uncertainty in Hong Kong is shown by structurally controlled slope failures, where the orientations of joint systems, joint strengths, types of infill materials, and disruptions of drainage patterns by impervious joint infills have significant impacts on performance projections. The assessment presented here focuses on slope problems dominated by uncertainties in input parameters, rather than analytical models. The case study we present was selected because failure was mostly through the soil matrix of decomposed granite. To obtain representative probability distributions of shear strength parameters, the results of triaxial tests in which specimens failed along relict joint planes are discarded.

Following the above screening, 62 consolidated undrained tests and 39 consolidated drained tests remain; each test comprised two to four specimens. Comparative studies on granitic soils (Lumb, 1965; Massey, 1983; Shen, 1985) showed no significant difference between effective shear strength parameters obtained from consolidated drained tests and consolidated undrained tests. Hence, in assessing the shear strength parameters, the results of the two tests are combined.

The principal stresses at failure, σ'_1 and σ'_3, were available for 59 specimens from three localities, 37 of which are HDG and 22 are CDG. Figure 1 shows the $p'-q$ plots of the specimens grouped by location and weathering class. The plot indicates that strength does not vary much from one location to another (Fig. 1(a)), consistent with the assumed statistically homogeneous random field. The strength of the HDG is only slightly higher than that of the CDG (Fig. 1(b)). Furthermore, each site has samples classified as HDG and as CDG. This is attributed to the continuous spectrum of weathering process, the fact that a distinct boundary between the two classes is only hypothetical, and the dependence of the classification into highly or completely decomposed on subjective individual judgement. Although rock decomposition generally decreases with depth, the triaxial data do not indicate higher strengths with depth. Based on these observations, regional probability distributions of the effective friction angle and cohesion are developed using triaxial test results of both the HDG and the CDG from all

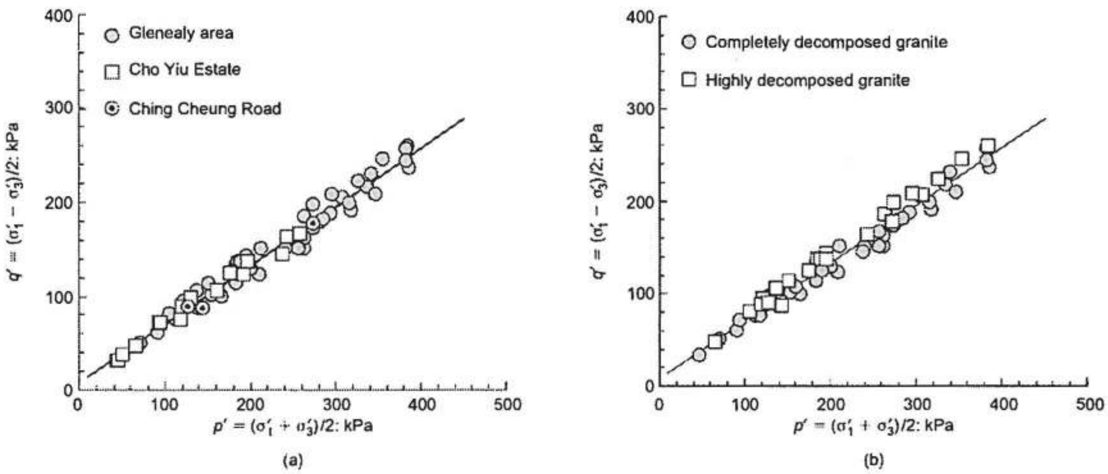

Fig. 1. p'–q plots of triaxial test results of granitic soils: (a) based on location; (b) based on weathering class

locations. The statistical populations used in estimating probability distributions, means and standard deviations comprise the cohesions and friction angles from individual tests.

The data show that the effective friction angles of granitic soils vary between 28° and 49°. The mean and standard deviation are 37·8° and 4·5°. Figure 2 shows the probability histogram and the cumulative distribution function of the friction angle. A log-normal parametric distribution with the experimental mean and standard deviation plotted on the same graph matches the experimental distribution function very closely (Fig. 2).

The effective cohesion ranges between zero and 25 kPa. The mean and standard deviation are both equal to 5·6 kPa, giving a coefficient of variation of 1·0 (Fig. 3). The correlation between the tangent of the friction angle and the cohesion intercept is possibly negative, but definitely very weak.

The estimated probability distributions of the friction angle and cohesion, based on this large database from different localities, seem a reasonable representation of the variability of shear strength parameters of typical HDG and CDG in Hong Kong. However, the mean values and variances of shear strength parameters at a specific site may differ from regional distributions. The 'within-site variability' is usually less than the variability of regional data (Zhang *et al.*, 2004). Where no site-specific information is available, the uncertainty in shear strength can be represented by the regional distributions (Figs 2 and 3). Where site-specific observations are available, the regional distributions can be updated using Bayesian approach (Zhang *et al.*, 2004).

SHEK KIP MEI SLIDE

To illustrate the implementation of probabilistic techniques, the methodology by El-Ramly *et al.* (2002a) is applied to investigate the failure of the Shek Kip Mei cut in Hong Kong. First the failed slope is back-analysed to estimate pore water pressure at failure. Then the slope is redesigned, hypothetically, to a lower inclination, and the probability of unsatisfactory slope performance is estimated.

Background

On 25 August 1999, the cut slope located 5 m behind housing Block No. 36 of the Shek Kip Mei Estate in Hong Kong failed. The height of the slope was about 21 m. The slope comprised five batters, each dipping at 55° (0·7h : 1v)

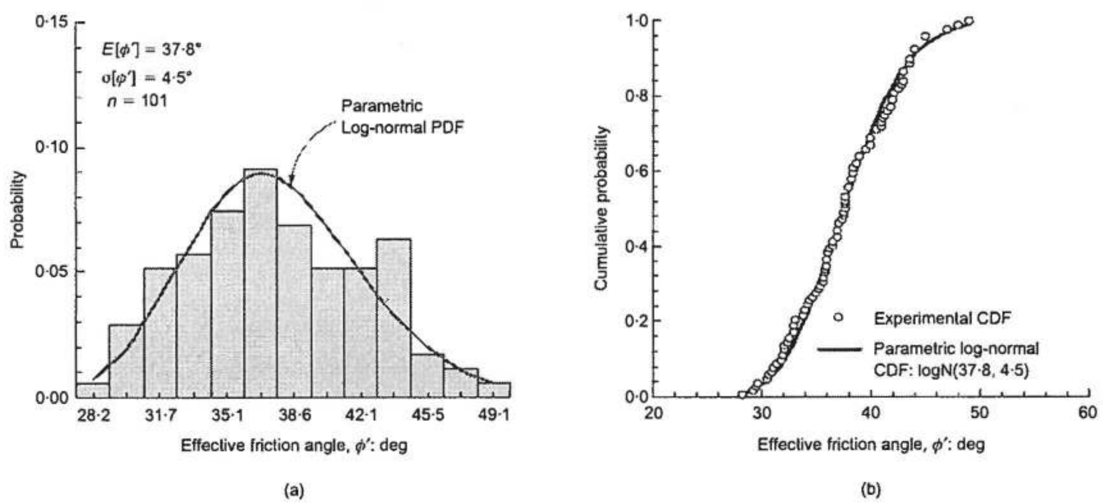

Fig. 2. (a) Probability histogram and (b) probability distribution function (CDF) of effective friction angle of HDG and CDG

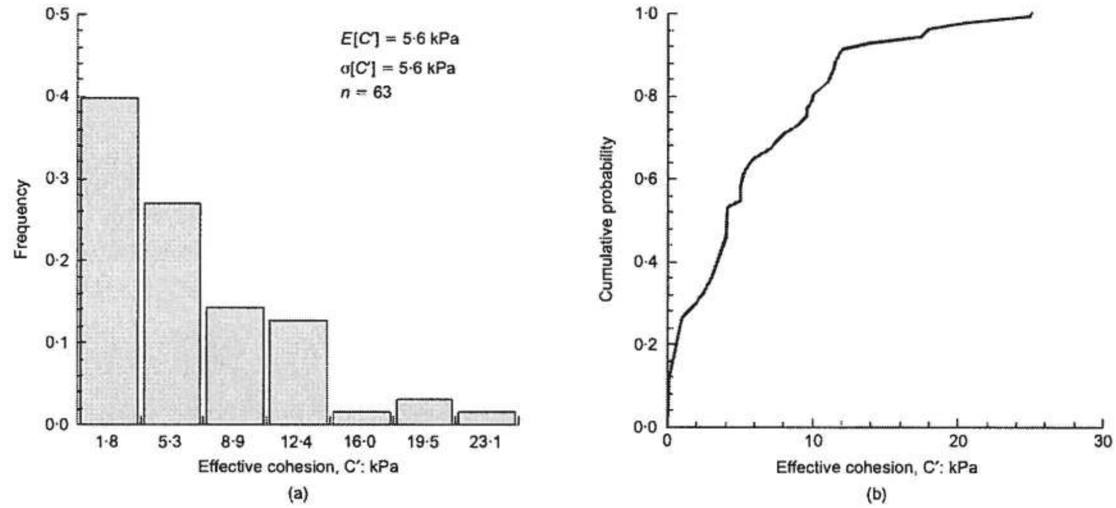

Fig. 3. (a) Histogram and (b) probability distribution function of effective cohesion of HDG and CDG

with berms 1–2 m wide (Fig. 4). The displaced mass, 2500 m^3 in volume and 37 m wide, remained largely intact. The failure, largely translational, left a well-developed scarp and most of the material on the slope. A comprehensive investigation of the failure (FMSW, 2000) forms the basis of our review.

Local geology

The site is underlain by completely decomposed granite with corestones of moderately to slightly decomposed granite. Several sets of closely spaced discontinuities at different dips were mapped; some were infilled with kaolinite and manganese deposits. The depth of the interface between weathered and fresh granites ranged from 15 m below the crest of the slope to 5 m at the toe (Fig. 4).

Rainfall and groundwater conditions

Failure occurred on the last day of a four-day intense rainstorm. An automatic rain gauge 1 km from the slide indicated 115·5 mm and 133·5 mm of rain over the 12 h and 24 h before failure. The total rainfall during the storm was 690 mm. For two months after failure seepage was observed along the toe of the slope. Field observations and assessment of slope hydrology indicated a permanent groundwater table within or close to the surface of the bedrock. An upper preferential groundwater regime within the decomposed granite was also possible. Tensiometers installed in the decomposed granite indicated suctions ranging between 25 kPa and 80 kPa within the upper 5·5 m. FMSW (2000) suggested that failure occurred because of a combination of suction reduction by infiltration of rainwater and a rise in the base groundwater table. Notwithstanding the detailed investigation, groundwater conditions and suctions at failure remain uncertain.

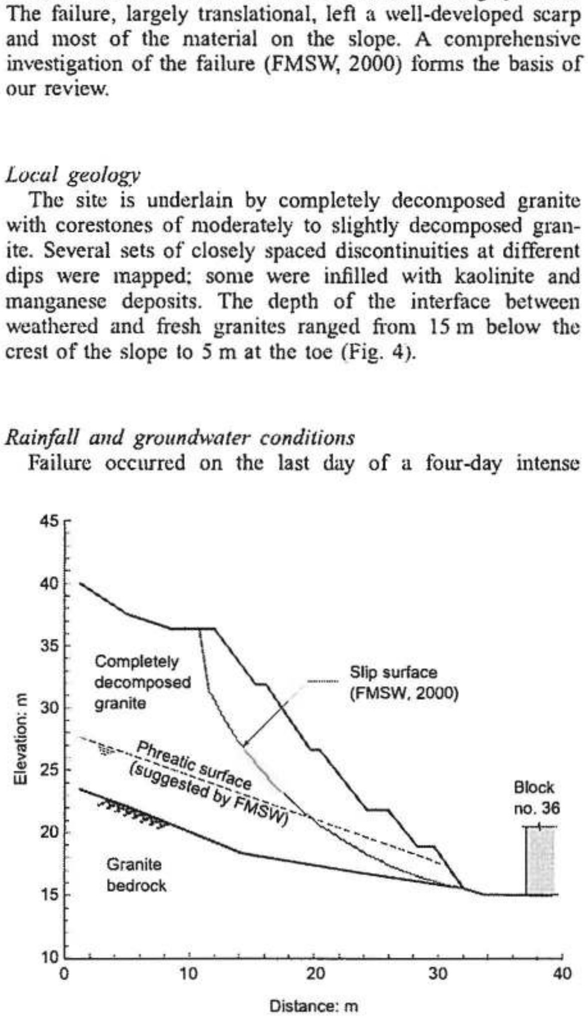

Fig. 4. Geometry and stratigraphy of the Shek Kip Mei Estate landslide

Rupture surface

The slide had a fully developed rupture surface with a main scarp and a distinct toe. Core samples retrieved from boreholes through the slide, and test pits and trial trenches provided important information about the geometry and depth of the rupture surface. Along the toe, the slip surface was planar within a soft clay layer dipping at 6–20°. At the slope crest, the rupture surface was partially along a shallow, soil-infilled tension crack. The majority of the surface, between the crest and toe, was within remoulded, completely decomposed granite. The maximum depth of the slip surface below ground was 8 m (Fig. 4).

Shear strength

The decomposed granite forming the slope comprised silty gravelly sand, with fines contents ranging between 10% and 20%. Shear strength parameters of saturated samples were obtained from multi-stage, unconsolidated undrained triaxial tests with pore pressure measurements on 12 specimens from four boreholes outside the distressed zone of the slope. The specimens were retrieved from depths between 0·9 m and 11·0 m below ground surface. FMSW (2000) reported that the average cohesion and friction angle based on the results of multi-stage tests were 8 kPa and 38° respectively.

In conformity with our screening criteria, only the results of the first shearing stage of the tests are considered. Hence each test comprised one point in $p'-q$ space, which did not allow the estimation of cohesion and friction angle for individual tests. Figure 5, however, shows that the $p'-q$ results of the 12 specimens are consistent with the regional data.

Back-analyses of slope failure

Back-analyses of slope failures are, generally, of great value in understanding failure mechanisms and designing slope remedial measures. For the Shek Kip Mei slope, the location and geometry of the rupture surface (Fig. 4) are well defined by post-failure investigations. However, the mobilised shear strength parameters along the slip surface are uncertain, and pore water pressures at failure are unknown. So, many combinations of strength parameters and pore pressures give a factor of safety 1·0, and the value of a back-analysis based on a deterministic approach is greatly diminished.

To address the uncertainties in shear strength and pore water pressure, a probabilistic approach is used to investigate

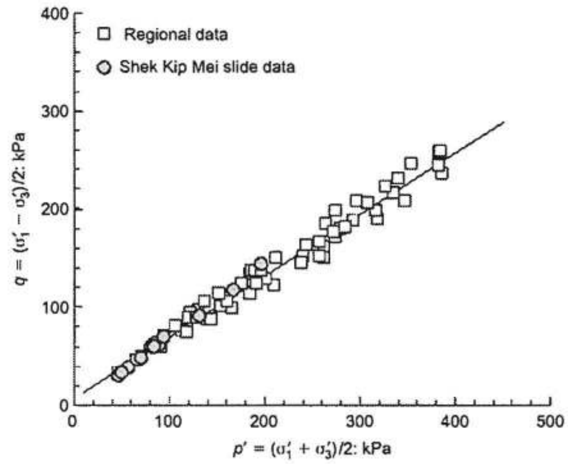

Fig. 5. $p'-q$ plot of triaxial test results of regional database, and site-specific results at Shek Kip Mei slide

the Shek Kip Mei slide. The pore pressure that triggered failure, regardless of the preceding events or processes (e.g. rainfall intensity and duration, rate of infiltration), is modelled using the pore pressure ratio, r_u. Although the pore pressure ratio is not an accurate representation of the pore pressure distribution along the slip surface, it is an index with an impact equivalent to that of the complex and unknown pore water pressure.

The probability distribution of the pore pressure ratio at failure is estimated from a probabilistic back-analysis of the failed slope using the methodology by El-Ramly *et al.* (2002a). A model of the slope geometry, stratigraphy and rupture surface (Fig. 4) is developed in an Excel spreadsheet. The Spencer method of slices (Spencer, 1967) is used in the model, with the limit equilibrium equations rearranged such that the factor of safety is integrated as a deterministic input of unity, and the pore pressure ratio as the output. Two values of the pore pressure ratio, corresponding to moment and force equilibrium equations, are computed and the spreadsheet calculations are iterated using different inclinations of the resultant interslice forces until the difference between the two pore pressure ratios is less than 1%.

The shear strength parameters of the decomposed granite are considered random variables and are assigned representative probability distributions using @Risk statistical functions. Since no probability distributions of the cohesion and friction angle could be deduced from site-specific data, regional probability distributions are used in the analysis. The friction angle is approximated by a log-normal probability distribution having the same mean and variance as the regional data (Fig. 2). This approximation reduces the simulation time. The effective cohesion is represented by the non-parametric probability distribution of the regional data (Fig. 3).

Monte Carlo simulation is performed using @Risk and the prepared spreadsheet model for 15 000 iterations. The mean pore pressure ratio at failure is 0·09 with a standard deviation of 0·12 (a coefficient of variation of 1·33). It ranges between a minimum of −0·39 (suction) to a maximum of 0·55. Figure 6 shows the histogram and the probability distribution function of the pore pressure ratio. The large coefficient of variation indicates that the uncertainty in pore pressure at failure is substantial. This is attributed to the large uncertainty in the effective cohesion, and the fact

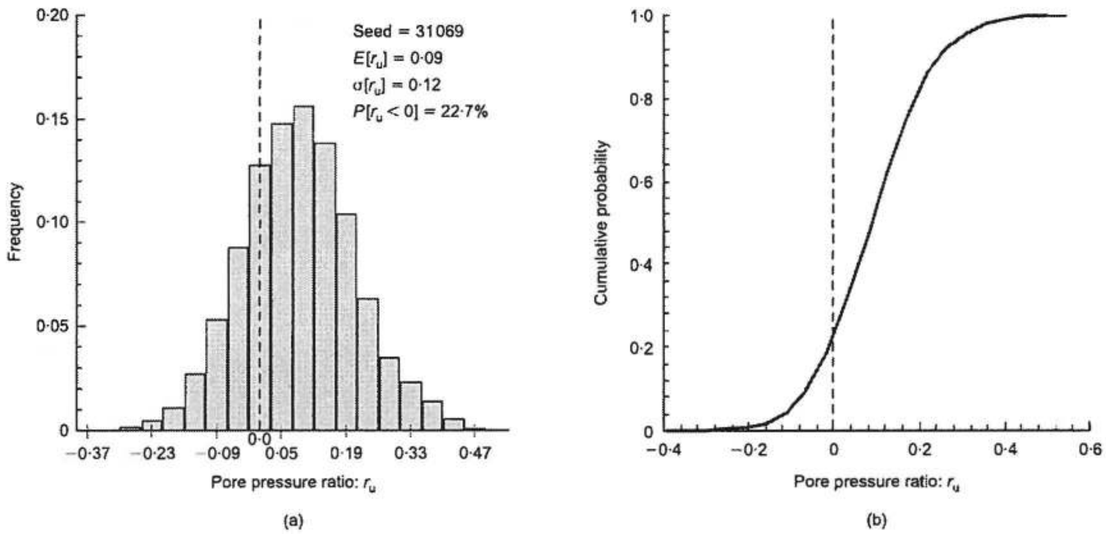

Fig. 6. (a) Histogram and (b) probability distribution function of pore pressure ratio at failure

that no variance reductions are applied to the probability distributions of strength parameters. The reason for not applying variance reductions is to directly estimate the probability distribution of the point-to-point variation in pore pressure ratio, rather than the average value over the length of the slip surface. The probability of a negative pore pressure ratio is approximately 23% (Fig. 6), implying that the slope might have failed while some suction was present in the soil mass. FMSW (2000) noted, however, that field observations and site setting made it unlikely that a significant suction was maintained in the slope at failure.

The objective of the back-analysis is to estimate the pore pressure at failure for the design of remedial measures. In practice, it is common to ignore the effect of suction in design. So a minimum threshold of zero pore pressure ratio is added to the spreadsheet, and the Monte Carlo simulation is repeated 25 times (each simulation with 15 000 iterations) to minimise the noise in the output distribution of the pore pressure ratio. Figure 7 shows the average probability distribution of the pore pressure ratio at failure based on the 25 simulations. The mean value is 0·10 with a standard deviation of 0·10.

It should be recognised that the back-calculated pore pressure ratio is positively correlated with the shear strength of the decomposed granite. The correlation arises because the back-calculated pore pressure ratio for any pair of cohesion and friction angles generated in the simulation process meets the constraint of a factor of safety of 1. In other words, if the generated cohesion and friction angles are high, the computed pore pressure ratio will be high in order to satisfy the condition of a factor of safety of 1. Similarly, if the generated c' and ϕ' are small, the pore pressure ratio will also be small. This association can be represented by the correlation coefficients $\rho(r_u, c')$ and $\rho(r_u, \phi')$ between pore pressure ratio and the average cohesion and friction angle along the slip surface. The significance of these correlation coefficients depends on the geometries of the slope and slip surface. For example, the effective normal stresses acting on a shallow slip surface in a flat slope would be small. Hence the back-calculated pore pressure is likely to be weakly correlated with the friction angle, and strongly correlated with the cohesion. On the other hand, the correlation between pore pressure ratio and friction angle would be significant for deep-seated slip surfaces. For the Shek Kip Mei slope, the correlation coefficients $\rho(r_u, c')$ and $\rho(r_u, \phi')$ are computed from the input and output data of one simulation run. They are estimated to be 0·73 and 0·63 respectively.

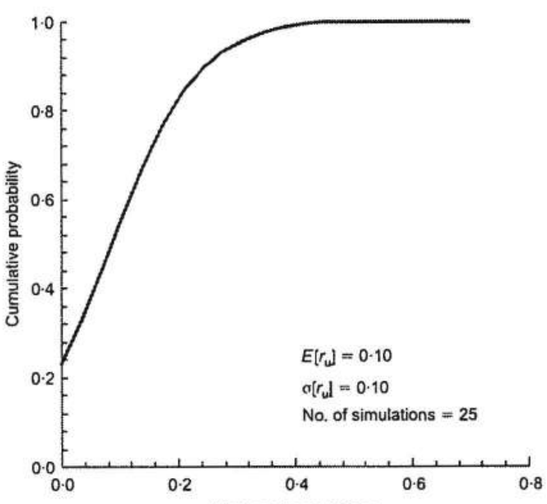

Fig. 7. Probability distribution function of pore pressure ratio at failure, truncated at minimum threshold of zero

Hypothetical slope remediation

Using the mean values of shear strength parameters and the back-calculated pore pressure ratio, the Shek Kip Mei slope is redesigned to a flatter inclination deemed safe based on a conventional deterministic approach. This remedial option is hypothetical for illustration purposes, and is not necessarily the recommended mitigation measure. Adopting a design factor of safety of 1·4 and the same configuration of the failed slope (five batters separated by berms), the modified design has an overall slope angle of 31·2°, 1·65h : 1v, compared with the 44° slope of the failed geometry. The computed factor of safety of the redesigned slope is 1·45. Given the uncertainties in shear strength and pore water pressure, the reliability of this design is unknown.

The modified slope configuration is analysed probabilistically to assess the reliability of the computed factor of safety. The probabilistic methodology (El-Ramly *et al.*, 2002a) uses a spreadsheet model of the revised slope and the Spencer method of slices. The friction angle and cohesion of the decomposed granite and the pore pressure ratio are random variables having the probability distributions in Figs 2, 3 and 7. Ideally, the correlations between pore pressure ratio and shear strength parameters should be incorporated in the probabilistic model. Accounting for these correlations is complex, however. First, the correlation coefficients estimated from the back-analysis are functions of the geometry and slip surface of the failed slope. Hence they are not directly applicable to the improved slope, whose geometry and potential failure modes are different. Second, these correlation coefficients represent the association between the average pore pressure ratio and the average cohesion and friction angle. Here, strength parameters and pore pressure ratio are modelled as random fields, which means that their values vary along the slip surface. In each simulation iteration, the friction angle, for example, is modelled using several correlated values that represent the spatial variation of the friction angle along the slip surface. Accounting for cross-correlations between input variables, in addition to the autocorrelations between values of each input variable, is a formidable task.

In analysing the stability of the improved slope, the positive correlations between pore pressure ratio and shear strength parameters are ignored. As a result, the likelihood that low values of cohesion and friction angle generated in Monte Carlo simulation are accompanied by high pore pressure ratios is greater. This latter combination of low strength and high pore pressure is what gives rise to the failure. Hence ignoring the correlation between pore pressure and shear strength would result in conservative estimates of the probability of unsatisfactory performance and reliability index.

With regard to the critical slip surface, two candidate circular surfaces were considered: the deterministic critical slip surface estimated in the conventional slope analysis, and the minimum reliability index surface found by the Hassan & Wolff (1999) algorithm. Initial probabilistic analyses indicated that the latter surface is more critical, and it was adopted in the spreadsheet model.

The probabilistic methodology requires an estimate of the autocorrelation distance, r_0, or scale of fluctuation, δ, to account for the spatial variability of input parameters. The spatial variability of residual soils is attributed to weathering processes, rather than to depositional environments as is the case with most soils. Weathering processes are random, highly variable, and independent of orientation, so the variability of residual soils is isotropic although erratic. Quantifying the spatial variability of residual soils requires data at close spacings, seldom available in practice. In the absence of such data, simplifying assumptions can be made to account for the spatial variability of granitic soils. First, the spatial variability is assumed isotropic. Second, a small autocorrelation distance is adopted to reflect the erratic nature of this material. Based on typical autocorrelation distances (El-Ramly *et al.*, 2003a), an isotropic autocorrelation distance $r_0 = 5$ m is assumed. These assumptions are consistent with Lumb's (1983) suggestion that the horizontal and vertical scales of fluctuation of residual soils are of the same order and, perhaps, in the range 1–5 m. The probabilistic analysis in the following sections is based on an isotropic autocorrelation distance of 5 m. The sensitivity of the results to the autocorrelation distance is investigated in a final section.

The spatial variability of the pore pressure ratio along the slip surface is modelled as a one-dimensional, stationary random field following the same procedures used for the shear strength parameters. An isotropic autocorrelation function characterised with an autocorrelation distance of 5 m is assumed. El-Ramly *et al.* (2003a) pointed out that the spatial variability of pore water pressure is not a characteristic soil property. Rather, it is a response to the spatial variability of the flow parameters of the soil mass: the more variable the soil is, the more erratic the pore pressure will be. This is a valid assumption for Hong Kong granitic soils where abundant random relict joints, many of which are blocked with impervious clay infill, exist. As a result, the macro-permeability of decomposed granite varies significantly over short distances leading to substantial variations in pore water pressures, as noted in the introduction section of the paper. A more rigorous assessment of the spatial variability of pore water pressure should consider rainfall intensity, rate of infiltration, regional and local flow patterns, state of stress, and time.

Trial Monte Carlo simulation performed using @Risk and the prepared spreadsheet model indicated that 32 000 iterations are required to minimise the noise in the estimated probability of unsatisfactory performance from random sampling of input parameters. Using a seed number of 31 069, the mean factor of safety is estimated to be 1·45 with a standard deviation of 0·17 (Fig. 8). The probability of unsatisfactory performance, probability of factor of safety $\leq 1{\cdot}0$, is $2{\cdot}3 \times 10^{-3}$. Because the simulation is based on random sampling of the input variables, the calculated probability of unsatisfactory performance is also a variable. Based on 25 further simulations using different seed numbers, the mean probability of unsatisfactory performance is estimated to be $2{\cdot}1 \times 10^{-3}$ with the 95% confidence interval around the mean from $2{\cdot}0 \times 10^{-3}$ to $2{\cdot}2 \times 10^{-3}$. The reliability index β, another probabilistic safety indicator, is equal to 2·61. Despite the effort to address all aspects of uncertainty in the probabilistic analysis, simplifying assumptions are made, and some sources of uncertainty may have gone undetected. For these reasons, comparing the probabilities of unsatisfactory performance of alternative mitigation measures would be of greater value than absolute probability figures.

El-Ramly *et al.* (2002b) presented a similar study of the Lodalen slide. The slope, cut in homogeneous marine clay, was redesigned to a flatter inclination (4h : 1v) deemed stable, and the stability of the slope was analysed probabilistically. The factor of safety was estimated to be 1·33, with a near-zero probability of unsatisfactory performance. The reliability index was 4·85. Comparison of the two cases highlights the limitations of the factor of safety in uncertain environments. Although the modified Shek Kip Mei slope is designed to a higher factor of safety, it has a higher probability of unsatisfactory performance and a lower reliability index than the Lodalen slope, and is, accordingly, less reliable. Such inconsistency arises from the inability of the factor of safety approach to account for uncertainties quantitatively. The uncertainties about shear strengths and pore water pressures in the residual soils of Hong Kong are much higher than those of the marine clay of the Lodalen slope. Hence it is logical to expect the same factor of safety to have different meanings in the two cases. The same level of reliability would require a higher design factor of safety for the Shek Kip Mei slope.

Estimating probability of unsatisfactory performance, in addition to the deterministic factor of safety, is an important step forward in the assessment of slope problems dominated by uncertainty. However, adequate design also requires considering failure consequences. El-Ramly *et al.* (2003b) conducted a quantitative risk assessment of the redesigned Shek Kip Mei slope using an event tree analysis. The study estimated the risk of loss of life for the residents of Block

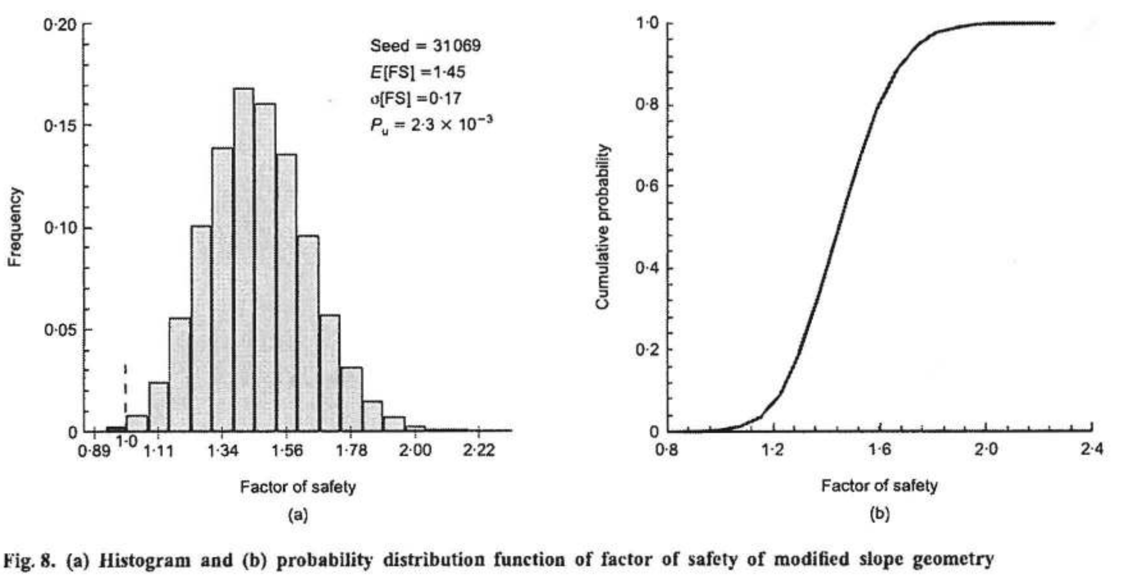

Fig. 8. (a) Histogram and (b) probability distribution function of factor of safety of modified slope geometry

36 at the foot of the slope (Fig. 4) by sliding shear failures. Although the likelihood of a large slope failure was low, the estimated risk was high owing to the potential of a large number of fatalities in the event of building collapse under the impact of slide debris.

Impact of autocorrelation distance

Our probabilistic analyses are based on an autocorrelation distance of 5 m. The sensitivity of the estimated probability of unsatisfactory performance to the value of the autocorrelation distance was examined by repeating the probabilistic analysis using autocorrelation distances of 3 m and 7 m. The computed probabilities of unsatisfactory performance were $0{\cdot}1 \times 10^{-3}$ for $r_0 = 3$ m, and $7{\cdot}7 \times 10^{-3}$ for $r_0 = 7$ m. The sensitivity of the probability of unsatisfactory performance to autocorrelation distance is attributed to two factors. First, the analysis is dominated by spatial variability of the granitic soils, rather than statistical sources of uncertainty such as sparse data or the use of empirical correlations and factors. Second, Hong Kong granitic soils vary significantly and erratically over short distances. Where the sensitivity of the probability of unsatisfactory performance to autocorrelation distance hinders decision-making, additional efforts might be required to estimate autocorrelation distances on a site- or formation-specific basis. El-Ramly *et al.* (2002b, 2003a) showed that the impact of the autocorrelation distance on the probability of unsatisfactory performance was not significant in two cases in homogeneous soils.

CONCLUDING REMARKS

The results from this study and similar investigations demonstrate the limitations of the deterministic factor of safety in environments of high uncertainty. In such environments, the reliability, and hence the significance of the computed factor of safety, are unknown, limiting the ability of the geotechnical engineer to make a rational decision on the adequacy of a slope design. The use of a combination of probabilistic and deterministic slope analyses provided a more efficient framework for the investigation and design of remedial measures for the Shek Kip Mei slide in Hong Kong. The adequacy of mitigation measures would not be complete, however, without considering failure consequences.

With regard to acceptable probabilities of unsatisfactory performance, the authors believe that probabilistic slope analyses of numerous case histories of failed and adequate slopes are the most reliable approach towards establishing consistent probabilistic design criteria. Probabilistic analyses of a tailings dyke performing adequately (El-Ramly *et al.*, 2003a) and two cut slopes redesigned on the basis of data from post-failure investigations and slope back-analyses (El-Ramly *et al.*, 2002b; and this study) indicate probabilities of unsatisfactory performance up to $2{\cdot}1 \times 10^{-3}$.

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