Probabilistic Delay Control and Road Side Unit Placement for Vehicular Ad Hoc Networks with Disrupted Connectivity

Atef Abdrabou, *Member, IEEE* and Weihua Zhuang, *Fellow, IEEE*

***Abstract*—This paper studies the multihop packet delivery delay in a low density vehicular ad hoc network (VANET). We address a disrupted vehicle-to-infrastructure communication scenario, where an end-to-end path is unlikely to exist between a vehicle and the nearest road side unit (RSU). We present an analytical framework, which takes into account the randomness of vehicle data traffic and the statistical variation of the disrupted communication channel. Our framework employs the effective bandwidth theory and its dual, the effective capacity concept, in order to obtain the maximum distance between RSUs that stochastically limits the worst case packet delivery delay to a certain bound (i.e., allows only an arbitrarily small fraction of the packets received by the farthest vehicle from the RSU to exceed a required delay bound). Our study also investigates the effect of the vehicle density, transmission range, and speed difference between vehicles on the end-to-end packet delivery delay. Extensive simulation results validate our analytical framework.**

***Index Terms*—Delay, multihop, vehicular ad hoc network, vehicle-to-infrastructure communication, disrupted connectivity.**

I. INTRODUCTION

MANY vehicles today are equipped with wireless communication functions that can facilitate vehicle-to-vehicle and vehicle-to-infrastructure communication. Increased storage capacity, computing and communications power, coupled with advances in wireless networking technology, bring a potential to enable new applications for drivers and passengers in the vehicles. Therefore, vehicular ad hoc networks (VANETs) recently have started to attract attention from many researchers in both industry and academia [1]-[6]. The Federal Communications Commission (FCC) in the United States has allocated $5.850 - 5.925$ GHz band to promote wireless communications for safe and efficient highways. This band is planned to be used in the emerging radio standard for Dedicated Short-Range Communications (DSRC) [7] [8]. The DSRC is a short-to-medium range communication service that supports an Intelligent Transportation System (ITS) with public safety and private operations for vehicle to roadside units (RSUs) and inter-vehicle communications.

Manuscript received 5 January 2010; revised 7 May 2010 and 12 July 2010. This paper was presented in part at IEEE Globecom 2009. This work was supported by a research grant from the Natural Science and Engineering Research Council (NSERC) of Canada.

A. Abdrabou is with the Department of Electrical Engineering, UAE University, Al-Ain, Abu Dhabi, 17555, UAE (e-mail: atef.abdrabou@uaeu.ac.ae.).

W. Zhuang is with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada (e-mail: wzhuang@uwaterloo.ca.).

Digital Object Identifier 10.1109/JSAC.2011.110113.

Basically, wireless connectivity and special sensors deployed in vehicles and on highways can be utilized to continuously report real-time traffic and environmental data (e.g., information about driving habits, roadway congestion, air pollution levels), and also to provide access to email, news and entertainment applications. However, for vehicular communication networks to become a reality, a number of technical challenges should be addressed.

Data traffic initiated from vehicles is expected to be random and bursty in nature. As RSUs represent gateways to the Internet and to the infrastructure of other systems such as an ITS, vehicles transmit their real-time information and Internet access requests to RSUs. RSUs send responses to the Internet queries and road information to vehicles. However, it is difficult, in terms of infrastructure cost, to cover roads with a large number of RSUs so that every vehicle on road can always be connected to at least one nearby RSU. Instead, vehicle-to-vehicle communications should be used in a multihop fashion in order to allow vehicles to connect to the out-of-transmission range RSUs, with a reasonable number of RSUs covering the road. It is difficult to maintain an end-to-end connection between a vehicle and an RSU while vehicles are moving with a high speed, specially on road with a low vehicle density. Moreover, achieving a reasonable packet transmission delay over a disrupted multihop connection between a vehicle and an RSU is a big challenge.

Our research objective in this paper is to present an analytical framework, which helps to approximately estimate the minimum number of RSUs required to cover a road segment, for a low-density VANET, with a probabilistic vehicle-to-RSU delay guarantee given that an intermittent multihop connectivity exists between vehicles and RSUs and that data traffic from vehicles is bursty. We exploit both the effective bandwidth theory and its dual, the effective capacity concept, in order to determine a maximum separation distance between adjacent RSUs that guarantees a required maximum vehicle-to-RSU data packet delivery delay with a certain pre-determined delay violation probability (based on the application needs) [9]. We also investigate, by the aid of our analytical framework, the effect of vehicle density, transmission range, and speed difference on the end-to-end packet delivery delay when vehicles are allowed to bypass each other. Our study aims at providing an insight of the influence of these parameters on the effectiveness of multihop communications in terms of the end-to-end packet delivery delay. Most of research works

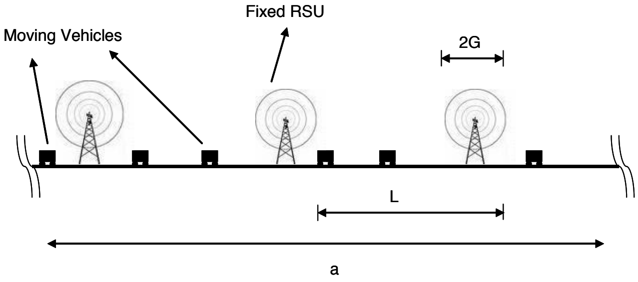

Fig. 1. An illustration of the network configuration.

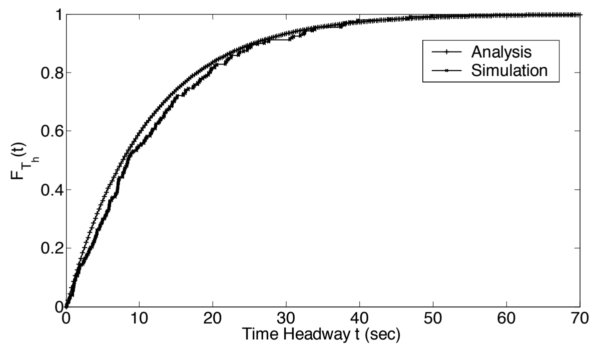

Fig. 2. The cumulative distribution function (CDF) for the time headway T_h.

in the literature that are related to disrupted connectivity in vehicular ad hoc networks focus on connectivity analysis [1] [2] [3] and average message delay evaluation [2] [4]. Wu et al. in [10] present two analytical models to study spatial propagation of information for one and two lane straight roads, without consideration of bandwidth constraints and data traffic characteristics. In [11], Yousefi et al. analyze the probability of connectivity to RSUs. The average length of a connected path from any given vehicle to an RSU is also calculated. However, no study about packet delivery delay is provided. The feasibility of information dissemination using stationary supporting units (SSUs) is investigated in [12] mainly based on computer simulations, while the vehicle-to-RSU delivery delay is not addressed.

In comparison, our work is novel in two aspects. First, it focuses on the delay analysis of vehicle-to-infrastructure packet delivery. Second, it relates packet delivery delay with random vehicle data traffic and disrupted connectivity. The rest of the paper is organized as follows. Section II describes the system model. Section III introduces the problem formulation of this research. Section IV presents the details of the proposed analytical framework. Section V provides analytical and simulation results to demonstrate the performance of of our proposed framework. Finally, Section VI concludes this research.

II. System Model

This section describes our system model with necessary assumptions in terms of the network configuration, protocol layers, and mobility model, for tractability in establishing the analytical framework.

A. Network Configuration

Consider a one-dimensional road and choose one segment of the road, which is a straight line of length a meters, as shown in Figure 1, where each two adjacent RSUs are separated by L meters. The transmission ranges of vehicles and RSUs are the same and denoted by G. Vehicles are distributed as Poisson points over the road segment. That is, given that there are k vehicles, they are independent and uniformly distributed over a initially [13]. Following the same approach as in [14], it can be proved that the proposed mobility model (described in Section II-C) approximately preserves the uniform distribution of the vehicles as the time goes by as shown in Appendix A.

We are interested in a network scenario on highways or rural areas, where the vehicle density (defined as the average number of vehicles per unit road length) is low enough to have disrupted vehicle-to-vehicle and vehicle-to-RSU connectivity. With a high vehicle density, a multihop end-to-end path can be found between a vehicle and an RSU with a high probability; however, this case is out of the scope of this research. Also, we do not consider the case where no packet relaying is possible (i.e, data packets will be carried by their originator vehicle till it meets the RSU) since wireless communication has no significant role in the packet delivery delay in such a case. This may happen either when the vehicle density is extremely low and/or the number of RSUs covering the road is fairly large. The vehicle density is assumed to be constant, i.e. over an observation period the average number of vehicles that leave the road segment under consideration is the same as the average number of vehicles that enter it.

Although an RSU can receive packets from the vehicles heading toward the RSU or moving away from it, we consider only one direction in packet transmission, as considering both directions (i) does not constitute a significant difference in the analysis, and (ii) is difficult to implement as a vehicle needs to know the location of the RSU and its own location (otherwise it may not be able to know when to switch its transmission to the RSU ahead of it). Here, we do not consider packet relaying via vehicles moving in the opposite direction. The reason is that it makes packet transmission subject to severe physical channel impairments, which are very significant due to the high relative speed between two vehicles moving in the opposite directions. In addition, the meeting time between two vehicles moving in the opposite directions may not be enough for transmitting a significant number of packets unless the available bandwidth is very large.

We assume that each vehicle empties its queue after meeting an RSU, regardless whether or not it is able to send all the packets in its queue within the meeting time. This implies that vehicle traffic load should be low enough to allow for the end-to-end delay violation probability to be satisfied. For simplicity, we assume that only one vehicle is allowed to communicate directly with the RSU at a time in a given direction, even though more than one vehicle may exist in the transmission range of the RSU. The RSU is capable of restricting the number of vehicles that can communicate

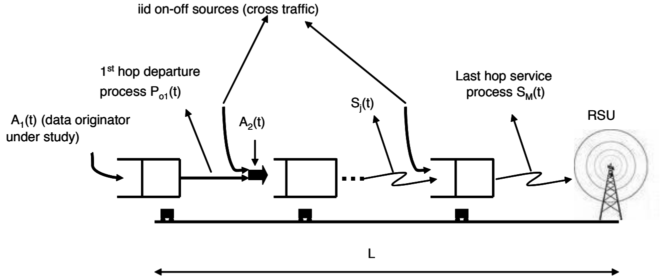

Fig. 3. A multihop connection between the furthest vehicle and the RSU with cross traffic.

directly with it to avoid increasing packet delivery delay due to packet collisions.

B. Protocol Layers

For the physical layer, consider a single channel with data rate μ in packets per second and a free space path attenuation model. For the link layer, consider the draft standard IEEE 802.11p [8] proposed to support ITS applications. The IEEE 802.11p mainly takes into account the issues related to fast mobility of vehicles when connecting to RSUs, while the main access mechanism of the IEEE 802.11 (RTS-CTS-DATA-ACK) is kept unchanged. With a low vehicle density, channel access using IEEE 802.11 will observe a small number of packet collisions. As a result, we assume a deterministic packet service time because the amount of randomness in packet service time due to the backoff procedure can be ignored with respect to packet transmission time [15].

In this research, the accuracy of our analysis is limited to sparse vehicular ad hoc networks, where average inter-vehicle distance is higher than the coverage range of an RSU, and it is unlikely to have three or more vehicles within a distance of double the transmission range. As a result, we neglect the medium access control (MAC) protocol issues in our analysis, such as interference between moving vehicles and queuing delay due to channel contention, and consider only one packet transmission occurs at a time.

C. Mobility Model

Consider only the vehicles moving on the straight line segment in one direction. The movement of a vehicle is characterized by two random variables (V, T). The first random variable V is the vehicle speed, which takes on two possible values with equal probability, namely, v_L and v_H, where $v_L < v_H$. The second random variable T is exponentially distributed with parameter λ and represents the period that the vehicle moves at a constant speed v_L or v_H. That is, a vehicle initially selects v_L (v_H) and, after T, changes to the other speed v_H (v_L). This assumption is reasonable since a vehicle driver usually tends to stay at a constant speed (specially on highways) for some time, and then changes to a higher or lower speed based on his/her will or road conditions. Normally, knowing how long in time a vehicle driver has been driving with a certain speed does not give us information (or does not affect the probability) of the amount of time that the driver will continue driving with that speed before changing to the other speed. Therefore, we use an exponential distribution (which has the memoryless property) for T in order to model the driving behavior.

At a low vehicle density condition (vehicle density not larger than 12 vehicle/mile/lane [16]), vehicles can be considered to move independently [16]. The mobility model is generic and configurable in terms of vehicle speed. For instance, by changing the rate of transition from v_L to v_H and vice versa, we change the time that a driver stays at a certain speed. Drivers change their speeds on highways based on highway design features such as level terrains [16]. They can also change their speeds based on their own will. Not all vehicles change their speeds the same way as it depends on the capability of the vehicle itself [16]. In addition, if we make v_L very close or equal to v_H, the model approaches a constant speed model, which is adopted by other researchers [10] [17]. It is reported in [16] and [18] that the time headway and the distance headway between vehicles in a low vehicle density case can be modeled by the exponential distribution. In Figure 2, we show by ns-2 simulations (using the ns-2 mobility scenario files) that our model satisfies the probability distribution for the time headway T_h. Here, the vehicle density is taken to be 0.0022 vehicle/meter and the average speed is $40m/s$. The analytical results are obtained based on the cumulative distribution function of the exponential distribution with a parameter approximated to the product of vehicle density and the average vehicle speed [18]. Note that the distance headway is already assumed to follow the exponential distribution by the vehicle spatial Poisson distribution.

In summary, based on the measurements introduced in [16], a mobility model for low density VANETs should (i) capture vehicles moving independently from one another, and (ii) support that both the time headway and distance headway are exponentially distributed. Our mobility model satisfies both characteristics.

III. Problem Formulation

Consider a multihop connection between a vehicle, B, at the edge of the road segment under study and the RSU, as in Figure 1. Suppose that the vehicle density is Γ vehicles/meter.

The problem can be illustrated by the aid of Figure 3. We assume that vehicle data traffic sources are on-off iid with exponentially distributed on and off times. The average on time is $1/\alpha_s$, average off time is $1/\beta_s$, and data rate at on time is R_s. All vehicles are supposed to send their data packets to the RSU by the help of other vehicles except the source node of the last hop. Given the number of hops, M, the arrival process $A_j(t)$ at the j^{th} hop ($j \in \{2, \ldots, M\}$), which is the number of packet arrivals within time t, depends on the departure or output process $P_{oj-1}(t)$ and the service process $S_{j-1}(t)$ of the preceding hop $j-1$. Throughout the paper, we use subscript o to indicate an output process parameter and subscript i to indicate an input process parameter. Define the j^{th} hop departure process $P_{oj}(t)$, where $j \in \{1, \ldots, M\}$, as the number of packets that leave the queue of a vehicle at the j^{th} hop within time t. Define the j^{th} hop service process $S_j(t)$, where $j \in \{1, \ldots, M\}$, as the number of packets that the wireless channel between the $(j-1)^{th}$ and j^{th} adjacent vehicles can carry within time t. Note that $S_j(t)$ equals zero at times when the two vehicles are out-of-range with respect to each other. Indeed, the channel service process is controlled mainly by mobility, and hence $S_j(t)$ are approximately the same for all $j \in \{1, \ldots, M-1\}$ except the last hop service process $S_M(t)$, since the RSU is stationary.

The service requirement in terms of delay, given the number of M hops, can be represented as

$$\Pr(D = \sum_{j=1}^{M} d_j > D_{\max}) \leq \varepsilon \tag{1}$$

which can be simplified to

$$\Pr(D = \sum_{j=1}^{M} d_j > D_{\max}) \leq \sum_{j=1}^{M} \Pr(d_j > \frac{D_{\max}}{M}) = \varepsilon \tag{2}$$

where d_j is the packet delay over the j^{th} hop, D_{max} is the allowed maximum end-to-end packet delivery delay, and ε is the maximum delay violation probability.

Although our objective is to find maximum L for a given D_{max}, our framework follows a mathematically easier approach by solving (2) to find D_{max} given L and a vehicle density. Note that the data traffic originated at each vehicle is random and bursty, and the connection between adjacent vehicles is intermittent. Therefore, packets are stored at vehicles while no connection is available. Note also that M is a random variable that depends on L, vehicle speed, Γ, λ, and G as in Section IV. For mathematical tractability, we assume that M can be represented by its average value $E[M]$[1]. We allow a small fraction (up to $\varepsilon > 0$) of data packets to arrive after D_{max} for a smaller number of required RSUs than that with $\varepsilon = 0$. In fact, a maximum L translates directly to a minimum number of RSUs that can cover a road segment to satisfy constraint (1).

IV. Analytical Framework

Our analytical framework is based on that (2) is related to the packet arrival process at each hop. The summed terms in (2) can be achieved for a certain random arrival process $A_j(t)$ (of a Markovian type) at the j^{th} hop using the effective bandwidth theory. The effective bandwidth theory indicates that the queue length and the corresponding delay at a node can be bounded exponentially for different stochastic traffic types, if an amount of bandwidth equal to the effective bandwidth of the source is provided by the channel [19].

In order to solve for D_{max}, the summed terms in (2) satisfy

$$\Pr(d_j > d_{jm}) \leq e^{-\theta_j d_{jm}} = \frac{\varepsilon}{M}, \quad \forall j = 1, \ldots, M \tag{3}$$

where d_{jm} is the delay bound at hop j that can be exceeded with a probability of at most $\frac{\varepsilon}{M}$ and θ_j is a QoS-related parameter that depends on the channel rate [19]. The effective bandwidth can be obtained using

$$\eta_{bj}(\theta_j) = \lim_{t\to\infty} \frac{1}{t}\frac{1}{\theta_j} \log E[e^{\theta_j A_j(t)}], \quad \forall \theta_j > 0. \tag{4}$$

Then, we solve for d_{jm} to obtain

$$D_{\max} = M \max_j d_{jm}. \tag{5}$$

The effective bandwidth $\eta_{bj}(\theta_j)$ is the amount of bandwidth that should at least be provided by a constant rate channel service in order to satisfy (2). However, the channel service at every hop changes randomly since it goes to zero for a random disruption time. The effective capacity concept has been developed in [20] as the dual of the effective bandwidth theory when the channel rate varies randomly. In this concept, a source with a deterministic packet arrival rate should limit its data rate to a certain maximum value in order to ensure that its delay bound (d_{jm}) is violated with a probability of at most $\frac{\varepsilon}{M}$. In order to achieve (3) in a time-varying channel with service process $S_j(t)$, the source rate (if fixed) should be limited to the channel effective capacity $\eta_{cj}(\theta_j)$ given by

$$\eta_{cj}(\theta_j) = -\lim_{t\to\infty} \frac{1}{t}\frac{1}{\theta_j} \log E[e^{-\theta_j S_j(t)}], \quad \forall \theta_j > 0. \tag{6}$$

It has been shown in [21] that, if both the traffic source rate and the channel capacity are time varying (which is our case), the effective bandwidth of the source should be equal to the effective capacity of the channel, in order to satisfy the stochastic delay bound. For a sufficiently large d_{jm}, the total delay per hop also satisfies (3) but with θ_j given by

$$\theta_j = r_j \eta_{cj}(r_j) \tag{7}$$

where r_j is the unique solution of the following equation

$$\eta_{cj}(r_j) = \eta_{bj}(r_j). \tag{8}$$

From the preceding description of the problem, we infer that, in order to find D_{max}, we need to characterize both $A_j(t)$ and $S_j(t)$ at every hop in order to solve (8), (7), (3), and (5). Indeed, the characterization of $A_j(t)$ at the j^{th} hop (for $j \in 2, \ldots, M$) requires the characterization of $S_{j-1}(t)$ and $P_{oj-1}(t)$. Due to the mathematical complexity of the problem, we make simplified approximations as described in the following.

A. Characterization of Service Process $S_j(t)$

First, we characterize the channel service process $S_M(t)$ at the last hop. Define the last hop as the hop where a vehicle either connects directly to the RSU or is the only vehicle that approaches the RSU while there is no other vehicle in the RSU coverage range. The channel service process at the last hop can be modeled by an on-off process, where the on state corresponds to $S_M(t) > 0$ and the off state corresponds to $S_M(t) = 0$. Both the on and off times follow a general distribution. We evaluate the effective capacity of the channel at the last hop using the results in [22], which indicate that an approximation to the effective bandwidth of an on-off general traffic source can be obtained by the effective bandwidth of an exponential on-off source that has the same average values, respectively for the on and off times. Since this also holds for the effective capacity for a channel, it is sufficient to calculate the average on and off channel times at the last hop.

Generally, the relation between the on period $T_{M_{on}}$ (off period $T_{M_{off}}$) at the last hop and the distance U_{on} (U_{off})

[1] In the rest of the paper, we use M to denote $E[M]$ for simplicity.

that a vehicle moves during the on time (off time) can be described by

$$T_{M_{on}} = \frac{U_{on}}{V}, \quad T_{M_{off}} = \frac{U_{off}}{V}. \tag{9}$$

The service process $S_M(t)$ can start its on or off time at a random position on the road segment. Hence, U_{on} and U_{off} are independent from the selection of V, which gives

$$\begin{aligned} E[T_{M_{on}}] &= \frac{1}{2}\left[\frac{1}{v_L} + \frac{1}{v_H}\right] E[U_{on}], \\ E[T_{M_{off}}] &= \frac{1}{2}\left[\frac{1}{v_L} + \frac{1}{v_H}\right] E[U_{off}]. \end{aligned} \tag{10}$$

The event of a vehicle moving a distance of at least u during $T_{M_{on}}$ happens with the simultaneous occurrence of two events; namely, there is no other vehicle within a distance of u from the end of the RSU coverage ahead of the vehicle, and there is at least one vehicle within a distance of $2G-u$ (the rest of the coverage area). Taking the uniform distribution of vehicle location into account, we have

$$\Pr(U_{on} > u) = \frac{\left(1-\frac{u}{a}\right)^{a\Gamma}\left[1-\left(1-\frac{2G-u}{a}\right)^{a\Gamma}\right]}{1-\left(1-\frac{2G}{a}\right)^{a\Gamma}}. \tag{11}$$

Integrating (11) from 0 to $2G$ gives

$$E[U_{on}] \cong G - \frac{2}{3}\Gamma G^2. \tag{12}$$

Similarly, we model the event of a vehicle moving a distance of at least u during $T_{M_{off}}$ by the simultaneous occurrence of two events; namely, the event of having no vehicles within a distance of $u+2G$ from the end of the coverage range of the nearest RSU ahead of the vehicle and the event of having at least one vehicle within a distance of $L-(u+2G)$.

This gives

$$\Pr(U_{off} > u) = \frac{\left[1-\left(1-\frac{L-(2G+u)}{a}\right)^{a\Gamma}\right]\left[1-\frac{2G+u}{a}\right]^{a\Gamma}}{\left[1-\left(1-\frac{L-2G}{a}\right)^{a\Gamma}\right]\left[1-\frac{2G}{a}\right]^{a\Gamma}}. \tag{13}$$

Integrating (13) from 0 to $L-2G$ leads to

$$E[U_{off}] \cong \frac{a}{a\Gamma+1}\frac{\left[\left(1-\frac{2G}{a}\right)^{a\Gamma+1}-\left(1-\frac{L}{a}\right)^{a\Gamma+1}\right]}{\left[1-\left(1-\frac{L-2G}{a}\right)^{a\Gamma}\right]\left[1-\frac{2G}{a}\right]^{a\Gamma}}. \tag{14}$$

Thus, by using (10), (12), and (14), we can determine the average on and off times of the service process $S_M(t)$ and use the results of [22] to calculate its effective capacity.

Next, we characterize the service process $S_j(t)$ for $j \in \{1,\ldots,M-1\}$. Consider two vehicles, one directly following the other. We model the process representing the relative speed between the two vehicles by a continuous time Markov chain (CTMC) with a state space $H = \{h_0, h_1, h_2\}$. State h_0 represents a negative relative speed, when the vehicle in the front moves with v_L and the vehicle behind moves with v_H. State h_1 models a zero relative speed (i.e., both vehicles move with the same speed). State h_2 represents the opposite case of state h_0 (a positive relative speed). As each vehicle keeps the same speed for an exponential time with an average of $1/\lambda$, the transition rate between any two states of the Markov process equals 2λ.

We use the CTMC model to characterize the channel service process $S_j(t)$, $j \in \{1,\ldots,M-1\}$. Actually, data packets are transfered to the RSU over a number of hops, where each hop consists of a converging epoch and a diverging epoch. A converging epoch happens when the vehicle behind starts to approach the vehicle in the front and the diverging epoch is the opposite to the converging one. In fact, the epochs represent the way that both locomotion and wireless communication can contribute to the packet delivery delay from a vehicle to an RSU. The first passage time complementary cumulative distribution function $F_{h_0h_2}(t)$ between state h_0 and state h_2 can be used to approximate the distribution of both the converging and diverging epochs. It can be obtained using the technique described in [13] as in Appendix B. This leads to

$$F_{h_0h_2}(t) \approx e^{-6\lambda t}. \tag{15}$$

The average number of hops that a packet will take to reach the nearest RSU can then be obtained as

$$M = \frac{6\lambda(L - E[U_{on}] - E[U_{off}])}{v_L + v_H}. \tag{16}$$

The average number of hops depends on the average vehicle speed and the average durations of converging and diverging epochs. As packet propagation is done by locomotion and wireless communications, packets generated after a vehicle has just passed an RSU will most likely suffer the maximum delay as this packet has to travel the largest distance (until it reaches the next RSU in its way). Data packets that travel over multiple hops arrive at RSUs before their generator vehicles become at the last hop.

A channel can be on or off during a converging epoch, as vehicle A may approach vehicle B from behind during a converging epoch but vehicle A may or may not be able to contact vehicle B before the converging epoch elapses. Similarly, a channel can be on or off during a diverging epoch as vehicle A and vehicle B may be in contact at the beginning of a diverging epoch. We approximate the channel service process $S_j(t)$, for $j \in \{1,\ldots,M-1\}$, as an on-off process with the following average on and off times[2]

$$E[T_{on}] \approx \frac{p_{on}}{6\lambda}, \quad E[T_{off}] \approx \frac{p_{off}}{6\lambda} \tag{17}$$

where p_{on} and p_{off} denote the stationary probability of the channel being on and off, respectively. Let I be an indicator random variable, where $I=1$ represents a converging epoch, $I=0$ represents a diverging epoch, and $Pr(I=1) = Pr(I=0) = 0.5$. Let C denotes the event of the channel being on or off, where $Pr(C=on) = p_{on}, Pr(C=off) = p_{off} = 1-p_{on}$. Taking into account the probability of vehicles to catch up each other and the distance between vehicles at the beginning of a converging or diverging epoch, the probabilities p_{on} and p_{off} can be obtained using the following equations (as in Appendix C)

[2]We dropped the subscript j from the average channel on time $E[T_{on}]$ and off time $E[T_{off}]$ to simplify the notations as we assume that the channel service process for all hops has the same statistics for $j \in \{1\ldots M-1\}$.

$$\Pr(C = on|I = 1) \approx \left(\frac{\Gamma}{y+\Gamma}\right)\left(e^{-2G\Gamma} + e^{-(y+\Gamma)G} - 2e^{-(y+2\Gamma)G}\right) + \left((1-e^{-G\Gamma})^2 - \frac{\Gamma(1-e^{-G\Gamma})}{y+\Gamma}\right). \tag{18}$$

$$\Pr(C = off|I = 0) \approx \left(\frac{\Gamma}{y-\Gamma}\right)\left(e^{-G\Gamma} - e^{-yG}\right)\left(1-e^{-G\Gamma}\right) + e^{-G\Gamma}. \tag{19}$$

where $\frac{1}{y} = \frac{(v_H - v_L)}{12\lambda}$ represents the average change in distance between two vehicles following each other during a converging or a diverging epoch.

B. Characterization of Departure Process $P_{oj}(t)$

In this subsection, we obtain an approximate expression of the departure process $P_{oj}(t)$ of hop j, where $j \in \{1, \ldots, M-1\}$. Note that the departure process of hop j contributes to the arrival process of the next hop. Since the on time and off time of the channel are typically much longer than the on time and off time of the traffic sources, several cycles of on and off times of the traffic sources are likely to occur during an off time or an on time of the channel. Therefore, when the channel becomes on, the departure process will also be on, with a probability approaching one (as the average off time of the channel is much longer than the average off time of a traffic source), until the channel finishes transmitting all the packets in the queue. Since the utilization factor ρ_j (the ratio of the packet arrival rate to the packet service rate) of the vehicle queue has to be less than one for the queue to be stable, there will be some periods during an on time of the channel when the data packet queue of a vehicle becomes empty. Consequently, the average on time of the departure process, $1/\alpha_{oj}$, is equal to the average time the vehicle queue stays busy during a channel on time and can be obtained as

$$\frac{1}{\alpha_{oj}} \approx \rho_j \left(E[T_{on}] + E[T_{off}]\right). \tag{20}$$

A low utilization factor approximation ($\rho_j \ll 1$) is used in (20) since a low traffic load guarantees that the vehicle at the last hop is able to transmit the packets (that it receives from other hops) to the nearest RSU during one meeting opportunity within the required delay bound and delay violation probability.

We approximate the departure process $P_{oj}(t)$ of hop j, $j \in \{1, \ldots, M-1\}$, as an exponential on-off process with an average on time of $1/\alpha_{oj}$ and average off time of $1/\beta_{oj}$, which is given by

$$\frac{1}{\beta_{oj}} \approx (1-\rho_j)\left(E[T_{on}] + E[T_{off}]\right). \tag{21}$$

C. Characterization of Arrival Process $A_j(t)$

In order to characterize $A_j(t)$, $j \in \{2, \ldots, M\}$, we use the results of [23] to approximate the superposition of the on-off exponential departure process and a local on-off exponential traffic arrival process at a certain hop with an equivalent on-off exponential arrival process. It has been shown in [23] that the superposition of two on-off sources has almost the same characteristics and effect on the node queue as an exponential on-off source, in terms of packet delay, on the long term and relatively short term as well. The parameters (average on time $1/\alpha_{ij}$, average off time $1/\beta_{ij}$, and data rate at the on time R_{ij}) characterize $A_j(t)$ and can be obtained using equivalent statistics as [19]

$$R_s u_s + \mu u_{oj-1} = R_{ij} u_{ij}, \quad \forall j \in \{2, \ldots, M\} \tag{22}$$

$$R_s^2 u_s (1-u_s) + \mu^2 u_{oj-1}(1-u_{oj-1}) = R_{ij}^2 u_{ij}(1-u_{ij}) \tag{23}$$

$$R_{ij}^2 u_{ij}(1-u_{ij}) e^{-(\alpha_{ij}+\beta_{ij})} = R_s^2 u_s (1-u_s) e^{-(\alpha_s+\beta_s)} + \mu^2 u_{oj-1}(1-u_{oj-1}) e^{-(\alpha_{oj-1}+\beta_{oj-1})} \tag{24}$$

$$u_s = \frac{\beta_s}{\alpha_s + \beta_s}, \quad u_{oj-1} = \frac{\beta_{oj-1}}{\alpha_{oj-1} + \beta_{oj-1}}, \quad u_{ij} = \frac{\beta_{ij}}{\alpha_{ij} + \beta_{ij}}. \tag{25}$$

Solving (22)-(25) leads to the characterization of $A_j(t)$, provided that α_{oj-1} and β_{oj-1} are known. We obtain α_{oj-1} and β_{oj-1} from (20), (21) and

$$\rho_j = \frac{(E[T_{on}] + E[T_{off}])(R_s u_s + \mu u_{oj-1})}{E[T_{on}]\mu}. \tag{26}$$

From (25), (20), and (21),

$$u_{oj-1} = \frac{E[T_{on}]\rho_{j-1}}{E[T_{on}] + E[T_{off}]} \tag{27}$$

which leads to

$$\rho_j = j\rho_1, \quad \forall j \in \{2, \ldots, M-1\}. \tag{28}$$

D. Solving for D_{max}

Finally, we solve (3) using the effective bandwidth of the traffic input process at each hop $j \in \{1, \ldots, M\}$, given by [24]

$$\eta_{bj}(x) = \left(\frac{R_{ij}}{2} - \frac{\beta_{ij}+\alpha_{ij}}{2x}\right) + \sqrt{\left[\frac{R_{ij}}{2} - \frac{\beta_{ij}+\alpha_{ij}}{2x}\right]^2 + \frac{\beta_{ij}R_{ij}}{x}} \tag{29}$$

and the effective capacity equation [20] [24]

$$\eta_{cj}(x) = \frac{-\max\limits_m \Lambda_m\left(\bar{Q}_j - x\Phi_j\right)}{x} \tag{30}$$

where $\bar{Q}_j$ is the transition rate matrix for the channel service process, Φ_j is a 2×2 diagonal matrix with channel data rate at on and off times (μ and 0) in its main diagonal, and $\Lambda_m(A)$ is the m^{th} eigenvalue of the matrix A. Solving (29) and (30) gives the values of d_{jm}, from which D_{max} can be obtained using (5) for a maximum violation probability of ε.

V. Simulation Results

We validate our framework using the ns-2 simulator. Table I gives the system parameter values used in the analysis and simulations unless otherwise specified. We implement the mobility model mentioned in Section II-C inside the ns-2 mobility scenario generator. Meanwhile, vehicles are allowed to bypass each other over the road segment under study. Note that the ns-2 simulation implements the IEEE 802.11 MAC protocol with no limitation on the number of vehicles that can contend for the same MAC channel.

TABLE I
SYSTEM PARAMETERS

System Parameter	Value
Packet Size	1024 Bytes
Road Segment Length (a)	$40km$
Channel Data Rate (μ)	$2Mbps$
α_s	$2.5s^{-1}$
β_s	$0.2s^{-1}$
Peak Source Rate (R_s)	200Kpbs
Vehicle Density (Γ)	0.0014 vehicle/meter
RSU & Vehicle Tx Range (G)	$300m$
λ	$\frac{1}{120}s^{-1}$

To retain a fixed vehicle density over an observation period, we assume that the vehicle that leaves the road segment at a certain speed returns to the beginning of the road segment at the same speed and spends an exponentially distributed random time before changing to the other speed. This provides stability to observation measurements based on time average.

We calculate the delay bound using the proposed analytical framework as in Section IV for a target ε of 5% and validate the result, using the simulator, by finding the delay bound leading to a measured violation probability that matches the required target. In this section, we first illustrate how we validate our analytical framework, and then we provide a study of the effect of changing system parameters such as Γ, G, and the speed difference on the packet delivery delay.

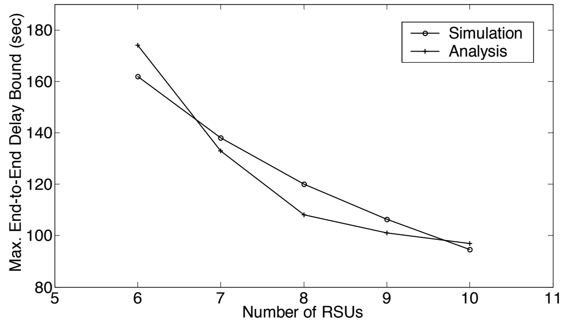

Fig. 4. Maximum end-to-end delay with the number of RSUs.

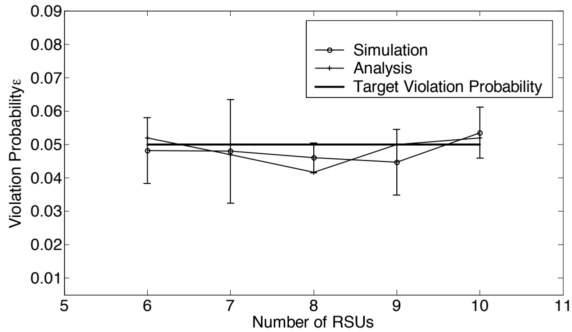

Fig. 5. Violation probability with the number of RSUs.

A. Model Validation

Figure 4 shows the maximum end-to-end delay results obtained analytically and by computer simulations with different number of RSUs covering the road segment under study. The end-to-end delay is measured from the moment that a packet is generated at the source vehicle to the moment that it is delivered to the nearest RSU. It is clear that the end-to-end delay increases rapidly as the number of RSUs covering the road segment decreases, as data packets tend to be relayed over a large number of hops to meet an RSU. Figure 5 shows the delay bound violation probability, obtained by simulations and analysis, for a 5% target violation probability. The violation probability values correpond to the maximum end-to-end delay results presented in Figure 4 for the same number of RSUs. Figures 4 and 5 indicate that our analytical framework can efficiently determine the number of RSUs to limit the maximum end-to-end delay bound probabilistically to a certain target violation probability.

Although Figure 4 shows a close match between the analytical and simulation results, it also shows an increasing trend of the difference between the two results as the number of RSUs decreases. The reasons for this trend are due to the approximations and the simplified assumptions made to make the problem mathematically tractable, especially in (2) where the end-to-end delay bound is obtained by adding up the maximum delay M times, and the usage of the average value of M.

It is worth noting that the accuracy of our analysis depends on the number of RSUs that are used to cover the road, vehicle density, transmission range, frequency of speed change, and vehicle average speed. In fact, the five parameters control the average number of hops that a packet travels from a vehicle to an RSU. It is observed that our analysis is close to simulation results when the average number of hops is less than or equal to four. When the average number of hops becomes higher, the discrepancy between the simulation and the analytical results increases and the maximum end-to-end delay also increases dramatically.

B. Delay Performance and System Parameters

We exploit our proposed analytical framework to investigate how the packet delivery delay is affected by changing some system parameters (i.e., vehicle density, transmission range, and speed difference). Our aim is to show which of these parameters plays a significant role in the RSU placement.

Figures 6 and 7 show both the analytical and computer simulation results of the maximum end-to-end packet delivery delay and the corresponding delay violation probability versus the vehicle density when there are 6 RSUs covering the road segment under study. It is evident from Figure 6 that the end-to-end delay is not sensitive to a limited change in the vehicle density[3] (within a limit that keeps the VANET sparse), which counters the intuition that packet relaying should be enhanced by increasing vehicle density. The main reason is the speed difference between vehicles, which makes two vehicles likely to catch up each other, especially when the density increases. In fact, when a vehicle V_1 approaches another vehicle V_2 in front of it, the packets previously sent from V_1 to V_2

[3]We assume that the density is kept low enough to let the mobility model closely represents the reality for a sparse VANET. If the density further increases, vehicles may move in clusters as shown in [16].

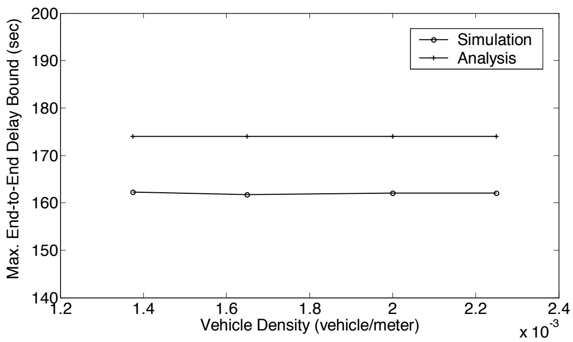

Fig. 6. End-to-end delay bound variation with the vehicle density.

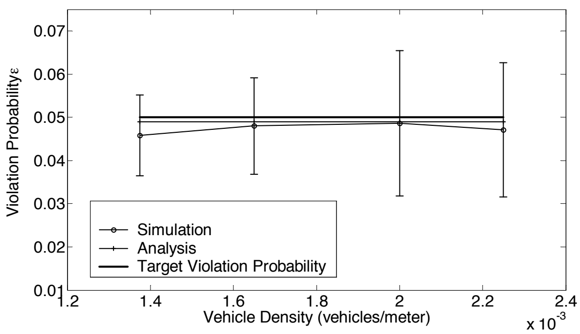

Fig. 7. Delay violation probability variation with the vehicle density.

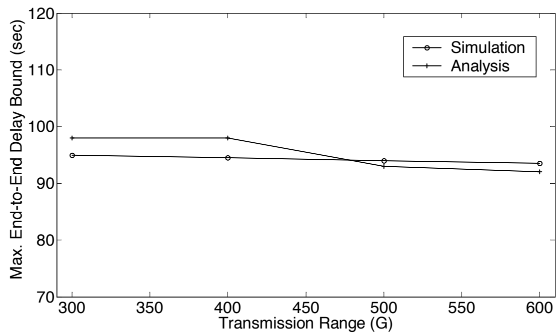

Fig. 8. End-to-end delay bound variation with the transmission range.

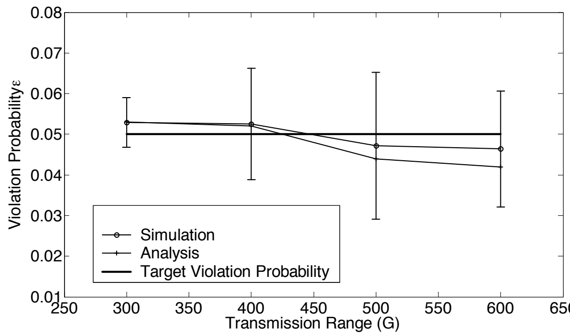

Fig. 9. Delay violation probability variation with the transmission range.

for forwarding to the RSU will be returned to the originator vehicle V_1 when it bypasses V_2. Apparently, this does not constitute a useful packet relaying in terms of packet delivery delay. As a result, the packet delivery delay does not decrease with an increase in vehicle density in such a situation.

In fact, vehicles in a low density or a sparse vehicular network (vehicle density ≤ 12 vehicle/mile/lane [16]) move independently as indicated by [16]. We assume also that vehicles can bypass one another; otherwise vehicles have to adjust speed in order to follow each other, which contradicts with the independent mobility assumption. If vehicles move with constant but different speeds, after some observation period, faster vehicles will catch up slower ones. If fast vehicles can bypass slow ones, fast vehicles can store their packets until they meet an RSU since sending data packets to slow vehicles increases packet delivery delay. It has been shown in [10] that, in sparse VANETs, if vehicles move with constant speeds selected uniformly from $[v_L, v_H]$ and vehicles can bypass each other, packet propagation speed is controlled mainly by vehicle speed but not dependent on changing vehicle density as long as the VANET is in the sparse network range. In this research, we show that, when vehicles change their speed independently (from v_L to v_H and vice-versa) and can bypass each other, the end-to-end packet delivery delay (which is directly related to packet propagation speed) is not affected by a change in the vehicle density as long as the VANET remains sparse.

Figures 8 and 9 show both the analytical and computer simulation results of the maximum end-to-end packet delivery delay and the corresponding delay violation probability with different transmission range G, $\Gamma = 0.0014$ vehicle/meter and 10 RSUs covering the road segment under study. It can be readily seen from Figure 8 that the packet delivery delay almost does not change with the limited increase in the transmission range. In general, increasing either the transmission range or vehicle density should have the same effect on packet delivery delay as both increase the connectivity probability among vehicles. However, the increase in the transmission range does not significantly alter the connectivity status of the network (i.e., the VANET stays sparse regardless of this change) as in the case of increasing vehicle density. As a result, increasing the transmission range does not lead to a smaller packet delivery delay. The slight change in the end-to-end delay as seen in Figure 8 is due to increasing the transmission range of the RSU (that equals G). Figure 9 shows that the delay violation probability values measured by simulations for different transmission range are in close match to the analytical results.

Figures 10 and 11 show both the analytical and computer simulation results of the maximum end-to-end packet delivery delay and the corresponding delay violation probability with different values of v_H while $v_L = 30m/s$, $\Gamma = 0.0014$ vehicle/meter, and 8 RSUs covering the road segment under study. Figure 10 clearly indicates that the packet delivery delay decreases as the speed difference increases. Useful packet relaying happens when a vehicle V_1 approaches a vehicle V_2 in front of it in a converging epoch and delivers its packets to V_2 without bypassing. Then, V_2 carries the packets and moves away from V_1 in a diverging epoch to approach another vehicle and so on. As a result, when the speed difference increases, the likelihood of useful packet relaying increases as V_2 moves away faster from V_1 to approach another vehicle in the front, and hence the packet delivery delay decreases.

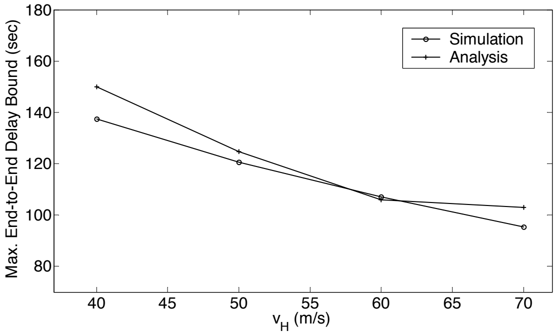

Fig. 10. End-to-end delay bound variation with the vehicle speed difference (v_H varies, $v_L = 30m/s$).

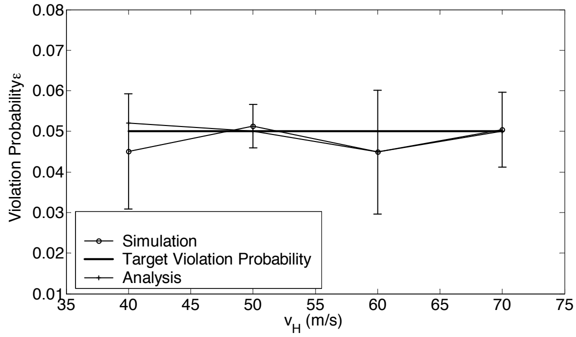

Fig. 11. Delay violation probability variation with the vehicle speed difference (v_H varies, $v_L = 30m/s$).

In summary, Figures 7, 9, and 11 indicate that all the end-to-end delay bound values used in our study satisfy the target violation probability. They also show that our analytical framework is effective in calculating the end-to-end delay bound that satisfies the target violation probability with different system parameters. Variations in vehicle density and transmission range do not have a significant impact on the end-to-end packet delivery delay for a sparse VANET when vehicles move with different speeds and are allowed to bypass one another.

VI. Conclusion

In this paper, we present an analytical framework to statistically estimate the maximum packet delivery delay from a vehicle with a random traffic source to an RSU for a low density VANET, where a data packet is being relayed via vehicle-to-vehicle communications. The framework aims at determining the minimum number of RSUs required to cover a straight road while satisfying the service requirement in terms of the transmission delay over the multiple hops. Numerical results demonstrate that the end-to-end packet delivery delay is not influenced by a variation in the vehicle density or the transmission range if vehicles are allowed to bypass one another, as long as this variation keeps the sparseness of the VANET. Simulation results validate the accuracy of the proposed framework, showing that a proper number of the RSUs can satisfy a certain delay bound probabilistically.

Appendix A

In this proof, we follow the same approach as in [14]. Consider a vehicle moves according to the proposed mobility model on a line segment of length a. The movement process of a vehicle consists of movement periods. Each movement period lasts for an exponentially distributed duration with parameter λ, during which the vehicles moves at a constant speed. A new period begins when the vehicle changes its speed. Let S and Q denote the starting and the ending locations of the vehicle in a movement period, respectively. The location S is equally likely to be anywhere in the road segment. Starting from a point at a location S, a vehicle may move to a destination at location Q either in front of or behind S (since it starts again from the beginning when it reaches the end of the road segment). Considering a long road segment, we neglect the probability that the vehicle may go over the whole segment twice or more within a movement period. The joint probability density function (PDF) $f(s,q)$ of S and Q, given a certain speed v, can be expressed as

$$f(s,q|V=v) = \begin{cases} \dfrac{\lambda\left[e^{\frac{-\lambda(q-s)}{v}} + e^{\frac{-\lambda(a+q-s)}{v}}\right]}{av\left[1-e^{\frac{-\lambda(2a-s)}{v}}\right]}, & \text{s} \leq \text{q} \leq \text{a}; \\ \dfrac{\lambda\left[e^{\frac{-\lambda(a+q-s)}{v}}\right]}{av\left[1-e^{\frac{-\lambda(2a-s)}{v}}\right]}, & 0 \leq \text{q} \leq \text{s}. \end{cases} \tag{31}$$

Let X denotes the vehicle location at any instant. We derive the PDF of X, $f_X(x)$, by calculating the cumulative distribution function (CDF) $F_X(x)$ and then differentiate it. For the i^{th} movement period, let T_i to denote the duration of this period and T_{xi} the duration that the vehicle stays within the section $[0,x]$ during this period. If we observe the mobility process of a vehicle for a sufficient number of movement periods, it can be shown that the accumulated time a vehicle stays in $[0,x]$ during the observation divided by the total observation time, given some speed v, converges to $P(X \leq x|V=v)$ [14]. This implies that

$$\Pr(X \leq x|V=v) = \frac{E[T_x|V=v]}{E[T|V=v]} = \frac{E[L_x|V=v]}{E[L|V=v]} \tag{32}$$

where L_x is distance the vehicle has moved within $[0,x]$ during a movement period, T_x is the time the vehicle stays within the section $[0,x]$, and L is the traveled distance during a movement period.

Using (31), we have

$$\begin{aligned} E[L|V{=}v] \approx & \frac{1}{a}\int_{s=0}^{a}\int_{q=s}^{a}(q-s)\frac{\lambda}{v}\left[e^{-\lambda\frac{q-s}{v}} + e^{-\lambda\frac{a+q-s}{v}}\right]dqds \\ & + \frac{1}{a}\int_{s=0}^{s=a}\int_{q=0}^{q=s}(a-s+q)\frac{\lambda}{v}e^{-\lambda\frac{a+q-s}{v}}dqds. \end{aligned} \tag{33}$$

In order to get a closed-form solution for $E[L|V=v]$, we neglect the term $e^{\frac{-\lambda(2a-s)}{v}}$ in the denominator of (31), as this term remains much smaller than one for a sufficiently large value of a and any value of s within the integration limit.

Similarly, we evaluate $E[L_x|V=v]$ by conditioning on the location S, which is equally likely to be anywhere over the road segment a, independent of x. The end location Q of a vehicle is dependent on the initial location S. A vehicle can be initially within the distance $[0,x]$ or $[x,a]$, moves for an exponential time, then reaches the location Q, $[0,x]$ or $[x,s]$ or $[s,a]$ if $S\in[x,a]$ or $[0,s]$, $[s,x]$, $[x,a]$ if $S\in[0,x]$. Thus, we obtain $E[L_x|V=v]$ as follows

$$\begin{aligned}E[L_x|V=v]\approx&\frac{1}{a}\int_{s=0}^{x}\int_{q=0}^{x}(x+q-s)\frac{\lambda}{v}\left[e^{-\lambda\frac{a+q-s}{v}}\right]dqds\\&+\frac{1}{a}\int_{s=0}^{x}\int_{q=s}^{x}(q-s)\frac{\lambda}{v}\left[e^{-\lambda\frac{q-s}{v}}\right]dqds\\&+\frac{1}{a}\int_{s=0}^{x}\int_{q=x}^{a}(x-s)\frac{\lambda}{v}\left[e^{-\lambda\frac{q-s}{v}}+e^{-\lambda\frac{a+q-s}{v}}\right]dqds\\&+\frac{1}{a}\int_{s=x}^{a}\int_{q=x}^{a}x\frac{\lambda}{v}\left[e^{-\lambda\frac{a+q-s}{v}}\right]dqds\\&+\frac{1}{a}\int_{s=x}^{a}\int_{q=0}^{x}q\frac{\lambda}{v}\left[e^{-\lambda\frac{a+q-s}{v}}\right]dqds.\end{aligned}\tag{34}$$

Using (32)-(34), we obtain (35) and hence

$$\Pr(X\le x)=\frac{1}{2}\left[\Pr(X\le x|V=v_L)+\Pr(X\le x|V=v_H)\right].\tag{36}$$

Using appropriate values for a in the order of tens of kilometers, λ in terms of hundreds of seconds, and speed in the order of tens of meters per second, we obtain $\Pr(X\le x)\approx\frac{x}{a}$. That is, $f_X(x)\approx\frac{1}{a}, x\in[0,a]$.

Appendix B

In order to obtain the first passage time for the CTMC mentioned in Section IV, we use the technique described in [13]. We construct another CTMC with a slightly different state space $\bar{H}$

$$\bar{H}=(H\backslash h_2)\cup\{z\}.\tag{37}$$

Let q_{ij} be the instantaneous transition rate from state i to state j for the original CTMC, then the transition rate for the modified CTMC is

$$\bar{q}_{ij}=\begin{cases}q_{ij}, & i,j\in(H\backslash\{h_2\}), i\neq j\\ q_{ik}, & i\in(H\backslash\{h_2\}), k=h_2, j=z\\ 0, & i=z, j\in(H\backslash\{h_2\}).\end{cases}\tag{38}$$

Let $\bar{v}_i$ denotes the leaving rate of the modified CTMC for state i defined as

$$\bar{v}_i=\begin{cases}v_i, & i\in(H\backslash h_2)\\ 0, & i=z.\end{cases}\tag{39}$$

In the modified CTMC, we use an absorbing state z with leaving rate of zero. Using the uniformization technique [13], we can obtain the transition probability $\bar{P}_{h_0z}(t)$ by taking the uniformization transition rate $v=6\lambda$. Let $\bar{P}^*_{lm}$ be the transition probability from state l to state m for the modified CTMC after uniformization, $\bar{P}_{lm}$ be the transition probability from state l to state m for the modified CTMC before uniformization, where $l,m\in\bar{H}$. Then, we have

$$\bar{P}^*_{lm}=\begin{cases}1-\frac{\bar{v}_l}{6\lambda}, & l=m\\ \frac{\bar{v}_l}{6\lambda}\bar{P}_{lm}, & l\neq m\end{cases}\tag{40}$$

Using (40), we obtain $\bar{P}_{h_0z}(t)$ as [13]

$$\bar{P}_{h_0z}(t)=\sum_{n=1}^{\infty}\bar{P}*^{n}_{h_0z}e^{-6\lambda t}\frac{(6\lambda t)^n}{n!}\approx\sum_{n=1}^{\infty}e^{-6\lambda t}\frac{(6\lambda t)^n}{n!}\tag{41}$$

where $\bar{P}*^{n}_{h_0z}$ is the n-step transition probability associated with the discrete time Markov chain with transition probabilities given by (40). Note that $\bar{P}*^{n}_{h_0z}$ approaches one for a small value of n. According to [13],

$$F_{h_0h_2}(t)=1-\bar{P}_{h_0z}(t)\approx e^{-6\lambda t}.\tag{42}$$

Due to the symmetry in the original CTMC, the diverging epoch can be derived in the same way.

Appendix C

We denote the distance between two vehicles by a random variable W. As the location of any vehicle is uniformly distributed over the road segment and under the assumption that the number of vehicles follows a Poisson distribution with density Γ, W follows an exponential distribution with parameter Γ [13]. The key idea to prove (18) and (19) is to calculate the probability of having a contact between two adjacent vehicles given $W\le G$ (the transmission range) or $W>G$ at the beginnings of a converging and diverging epochs, respectively. That is,

$$\begin{aligned}\Pr(C=on|I=1)&=\Pr(on|I=1,W\le G)\Pr(W\le G)\\&+\Pr(on|I=1,W>G)\Pr(W>G)\end{aligned}\tag{43}$$

$$\begin{aligned}\Pr(C=off|I=0)&=\Pr(off|I=0,W\le G)\Pr(W\le G)\\&+\Pr(off|I=0,W>G)\Pr(W>G).\end{aligned}\tag{44}$$

Note that $\Pr(W\le G|I=1)=\Pr(W\le G)$ as the event $[W\le G]$ and $[I=1]$ are independent. Consider two adjacent vehicles V_1 and V_2, where V_1 follows V_2. Since the speed difference between the two vehicles may become zero at times, we assume for simplicity that V_1 or V_2 will move with an average relative speed of $(\frac{v_H-v_L}{2})$ during a converging or a diverging epoch, respectively. This implies that V_1 during a converging epoch moves an exponentially distributed distance with parameter y toward V_2 since the converging and diverging epochs follow approximately an exponential distribution as illustrated in Appendix B. Similarly, V_2 moves an exponentially distributed distance with parameter y away from V_1 during a diverging epoch.

For a converging epoch, the probability of an on time of the service process of V_1 depends on the probability that V_1 approaches V_2 without passing it during the epoch. Therefore,

$$\begin{aligned}\Pr(C=on|I=1)\approx&\left(1-e^{-G\Gamma}\right)\int_0^G\left(1-e^{-yw}\right)\Gamma e^{-w\Gamma}dw\\&+e^{-G\Gamma}\int_G^\infty\left(e^{-y(w-G)}-e^{-yw}\right)\Gamma e^{-w\Gamma}dw\end{aligned}\tag{45}$$

which can be manipulated to (18).

For a diverging epoch, the probability that V_1 will observe an off time depends on the probability that V_2 will go out of the transmission range of V_1 within the epoch. It is given by

$$\begin{aligned}&\Pr(C=off|I=0)\approx\\&\quad\left(1-e^{-G\Gamma}\right)\int_0^G e^{-y(G-w)}\Gamma e^{-w\Gamma}dw+e^{-G\Gamma}\end{aligned}\tag{46}$$

which, after calculating the integration, leads to (19).

$$\Pr(X \le x | V = v) \approx \frac{-v\left(v\left(e^{-\frac{(a+x)\lambda}{v}} - e^{-\frac{\lambda a}{v}} + e^{-2\frac{\lambda a}{v}} - e^{-\frac{\lambda(2a-x)}{v}}\right) + \lambda x\left(e^{-\frac{(a+x)\lambda}{v}} - 2e^{-\frac{\lambda a}{v}} + 1 + e^{-\frac{\lambda(2a-x)}{v}} + e^{-2\frac{\lambda a}{v}}\right)\right)}{2v^2 e^{\frac{-\lambda a}{v}} + \lambda^2 a^2 e^{\frac{-\lambda a}{v}} - 2v^2 e^{\frac{-2\lambda a}{v}} - v\lambda a e^{\frac{-2\lambda a}{v}} - v\lambda a}. \quad (35)$$

REFERENCES

[1] M. Khabazian and M. Ali, "A performance modeling of connectivity in vehicular ad hoc networks," *IEEE Trans. Veh. Tech.*, vol. 57, no. 4, Jul. 2008, pp. 2440–2450.

[2] A. Agarwal, D. Starobinski, and T. Little, "Analytical model for message propagation in delay tolerant vehicular ad hoc networks," *Proc. IEEE VTC08*, May 2008, pp. 3067–3071.

[3] S. Panichpapiboon and W. Pattara-atikom, "Connectivity requirements for self-organizing traffic information," *IEEE Trans. Veh. Tech.*, vol 57, no. 6, Nov. 2008, pp. 3333–3340.

[4] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," *ACM Perform. Eval.*, vol. 62, no. 1–4, May 2005, pp. 210–228.

[5] A. Wasef and X. Shen, "EDR: Efficient decentralized revocation protocol for vehicular ad hoc networks", *IEEE Trans. Veh. Tech.*, vol. 58, no. 9, 2009, pp. 5214–5224.

[6] A. Wasef, Y. Jiang, and X. Shen,"DCS: An efficient distributed certificate service scheme for vehicular networks", *IEEE Trans. Veh. Tech.*, vol. 59, no. 2, 2010, pp. 533–549.

[7] IEEE. 5.9 GHz Dedicated Short Range Communications (DSRC), [online]. Available: http://grouper.ieee.org/groups/scc32/dsrc/.

[8] IEEE WG, *IEEE 802.11p/D2.01, Draft Amendement to Part 11: Wireless Medium Access Control (MAC) and Physical Layer (PHY) specifications: Wireless Access in Vehicular Environments*, Mar. 2007.

[9] A. Abdrabou and W. Zhuang, "On a stochastic delay bound for disrupted vehicle-to-infrastructure communication with random traffic," *Proc. IEEE Globecom'09*, DOI 10.1109/GLOCOM.2009.5425848, Nov.-Dec. 2009.

[10] H. Wu, R.M. Fujimoto, G.F. Riley, and M. Hunter, "Spatial propagation of information in vehicular networks," *IEEE Trans. Veh. Tech.*, vol. 58, Jan. 2009, pp. 420-431.

[11] S. Yousefi, E. Altman, R. Elazouzi, and M. Fathy, "Improving connectivity in vehicular ad hoc networks: An analytical study," *Computer Communications*, vol. 31, 2008, pp. 1653–1659.

[12] C. Lochert, B. Scheuermann, M. Caliskan, and M. Mauve, "The feasibility of information dissemination in vehicular ad-hoc networks," *Proc. WONS07*, Jan. 2007, pp. 92–99.

[13] H. Tijms, *A First Course in Stochastic Models*, 3rd ed. John Wiley & Sons, 2003.

[14] C. Bettstetter, G. Resta, and P. Santi, "The node distribution of the random waypoint mobility model for wireless ad hoc networks," *IEEE Trans. Mobile Computing*, vol. 2, no. 3, Sept. 2003, pp. 257–269.

[15] A. Abdrabou and W. Zhuang, "Stochastic delay guarantees and statistical call admission control for IEEE 802.11 single-hop ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 10, pp. 3972-3981, Oct. 2008.

[16] A.D. May, *Traffic flow fundamentals*,Prentice Hall, 1990.

[17] T. Little and A. Agarwal, "An information propagation scheme for VANETs," *IEEE Intell. Transportation Syst.*, Sep. 2005, pp. 155 - 160.

[18] N. Wisitpongphan, F. Bai, P. Mudalige, V. Sadekar, and O. Tonguz, "Routing in sparse vehicular ad hoc networks", *IEEE J. Sel. Areas Commun.*, vol. 25, no. 8, Oct. 2007, pp. 1538-1556.

[19] M. Schwartz, *Broadband Integrated Networks*, Prentice Hall, 1998.

[20] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, July 2003, pp. 630-643.

[21] D. Wu and R. Negi, "Effective capacity-based quality of service measures for wireless networks," *ACM Mob. Nets. and App. (MONET)*, vol. 11, Feb. 2006, pp. 91–99.

[22] K. Elsayed and H. Perros,"On the Effective Bandwidth of Arbitrary on/off Sources", *Proc. Sixth IFIP WG6.3 Conf. on Perf. of Comp. Nets.*, 1996, pp. 257 – 271.

[23] K. Sriram and W. Whitt, "Characterizing superposition arrival processes in packet multiplexers for voice and data," *IEEE J. Sel. Areas Commun.*, vol. 4, no. 6, Sep. 1986, pp. 833–846.

[24] G. Kesedis, J. Walrand and C.S.Chang, "Effective bandwidth for multiclass Markov fluids and other ATM sources," *IEEE/ACM Trans. Netw.*, vol. 1, Aug. 1993, pp. 424–428.

Atef Abdrabou (M'09) received the Ph.D. degree in 2008 from the University of Waterloo, Ontario, Canada, in electrical engineering. In 2010, he joined the Department of Electrical Engineering, UAE University, Al-Ain, Abu Dhabi, UAE, where he is an Assistant Professor. He is a co-recipient of a Best Paper Award of IEEE WCNC 2010.

Dr. Abdrabou has been awarded the National Science and Engineering Research Council (NSERC) of Canada postdoctoral fellowship in 2009. His current research interests include network resource management, QoS provisioning and information dissemination in self-organizing wireless networks.

Weihua Zhuang (M93-SM01-F'08) received the B.Sc. and M.Sc. degrees from Dalian Maritime University, China, and the Ph.D. degree from the University of New Brunswick, Canada, all in electrical engineering. Since October 1993, she has been with the Department of Electrical and Computer Engineering, University of Waterloo, Canada, where she is a Professor and a Tier I (Senior) Canada Research Chair in wireless communication networks. Her current research focuses on resource allocation and QoS provisioning in wireless networks.

Dr. Zhuang is a co-recipient of the Best Paper Awards from IEEE WCNC 2007 and 2010, IEEE ICC 2007, and the International Conference on Heterogeneous Networking for Quality, Reliability, Security and Robustness (QShine) 2007 and 2008. She is the Editor-in-Chief of IEEE Transactions on Vehicular Technology, a Fellow of IEEE, and an IEEE Communications Society Distinguished Lecturer.