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PROBABILISTIC FINITE ELEMENTS*

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In the design of critical components, such as rocket engine parts, we often find large uncertainties in material properties and loads, particularly when considering the ultimate capacity of the components. One reason for this is that the extreme conditions, such as temperature or load, are not known, nor are the material properties at extreme temperatures as well established. Therefore, a single finite element analysis of a component may be quite misleading since it gives no information on the range of responses that can be expected.

Although analysts often try to guard against this shortcoming by varying several of the parameters either arbitrarily or on the basis of their intuition, a more rational and methodical approach to dealing with this difficulty would be very useful. The probabilistic finite element method (PFEM) has been developed in response to these needs.

In PFEM [1-3], finite element methods have been efficiently combined with second-order perturbation techniques to provide an effective method for informing the designer of the range of response which are likely in a given problem. The designer must provide as input the statistical character of the input variables, such as yield strength, load magnitude, and Young's modulus, by specifying their mean values and their variances. The output then consists of the mean response and the variance in the response. Thus the designer is given a much broader picture of the predicted performance than with simply a single response curve. These methods are applicable to a wide class of problems, provided that the scale of randomness is not too large and the probabilistic density functions possess decaying tails. By incorporating the computational techniques we have developed in the past 3 years for efficiency, the probabilistic finite element methods are capable of handling large systems with many sources of uncertainties.

Sample results for an elastic-plastic ten-bar structure and an elastic-plastic plane continuum with a circular hole subject to cyclic loadings with the yield stress on the random field are depicted in Figs. 1-4. For the ten-bar structure, a 5% coefficient of variation in the yield stresss gives a 13% coefficient of variation in the displacement of node 1 and an 11% coefficient of variation in the stress of element 1. For this example, along with many others (not shown here), PFEM compares very well with the Monte Carlo Simulation (MCS) and the Hermite Gauss Quadrature (HGQ) (see Fig. 2). This is an example where a situation where a small variance in the yield strength can result in a much larger variance in the response. It should be noted that the ratios of computer time are 1 to 400 when the PFEM is compared to MCS.

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As for the elastic/plastic continuum problem, the mean displacement and stress are sinusoidal, resembling closely the forcing function (Fig. 4). The variances are close to zero until the plate begins to yield in compression. After this, the variance jumps to a higher value and remains steady until the yielding in tension begins. This phenomenon repeats every cycle.

A third numerical example for PFEM methods is a turbine blade problem. Some results for the turbine blade model shown in Fig. 5 will be presented. The blade is subjected to a random impulsive load and the yield stress is random.

A natural extension of these methods would be to consider fatigue and failure analysis. Finite element methods, such as PFEM, for analyzing fatigue and fracture in a probabilistic manner, are very scarce. The fracture related quantities such as fracture toughness, size and orientation of the cracks, are usually hard to determine exactly. These and other quantities, which govern the crack growth, can be treated by finite elements in a similar manner, although it woud be necessary to incorporate first and second order reliability methods and to embed singularities in the variational statements to correctly represent cracks. The experience obtained so far suggest that this is a logical extension of PFEM.

REFERENCES

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2. Liu, W. K., Belytschko, T. and Mani, A., "Probabilistic Finite Elements for Nonlinear Structural Dynamics," Computer Methods in Applied Mechanics and Engineering, Vol. 56, 1986, pp. 61-81.

3. Liu, W. K., Belytschko, T. and Mani, A., "Applications of Probabilistic Finite Element Methods in Elastic/Plastic Dynamics," Engineering for Industry, ASME, Vol. 109/1, pp. 2-8.

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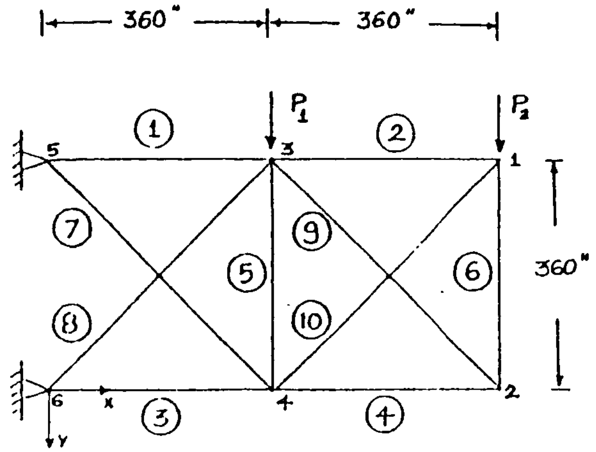
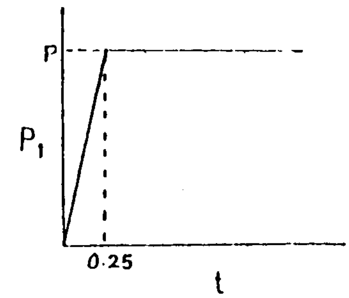




$E = 30.0 \times 10^6$

$E_T = 30.0 \times 10^4$

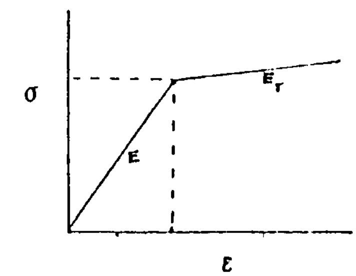
$A = 6.0$

$\rho = 0.30$

$\sigma_Y = 15000.0$

$P = 175.0 \times 10^3$

$P_2 = 0.0$



Problem Statement of a Ten-Bar Nonlinear Structure.

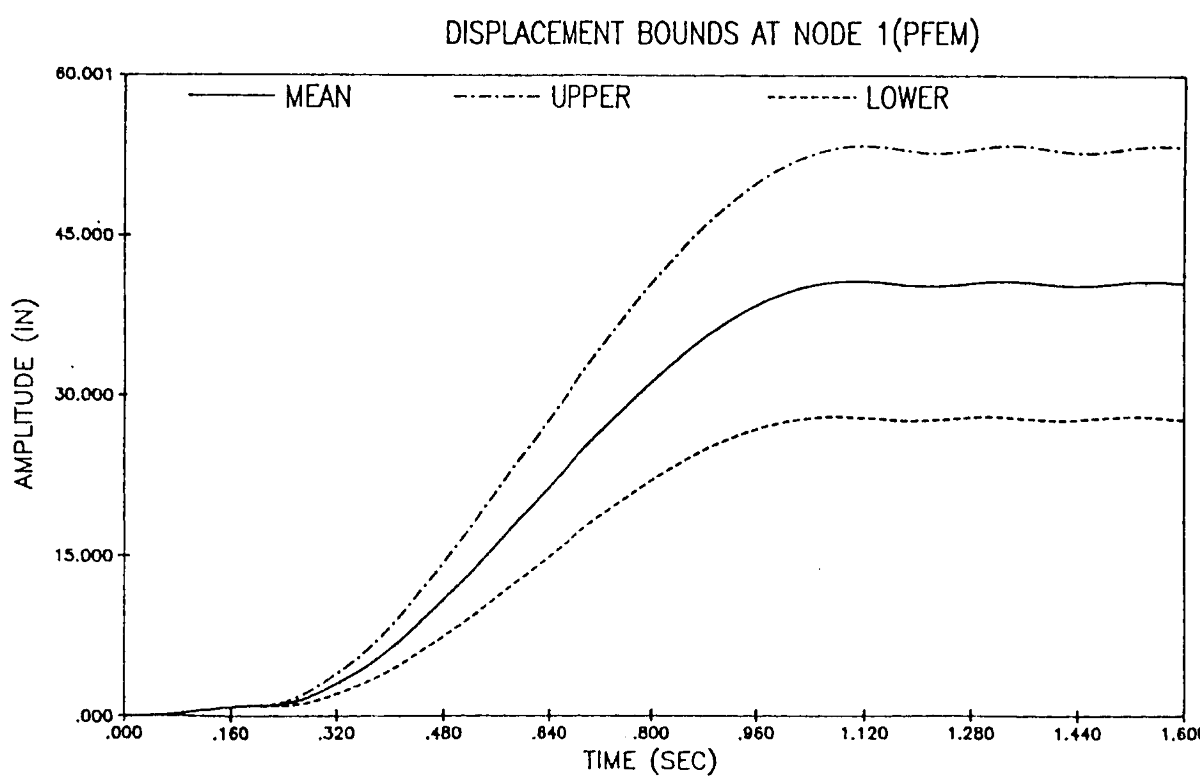
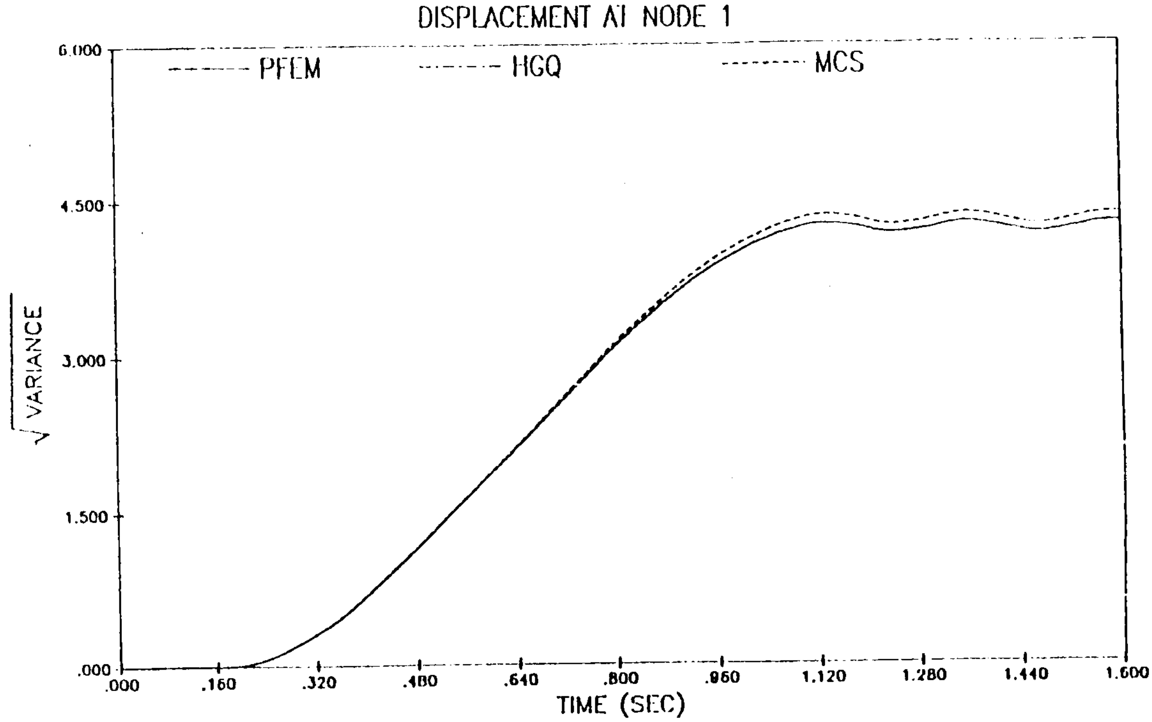
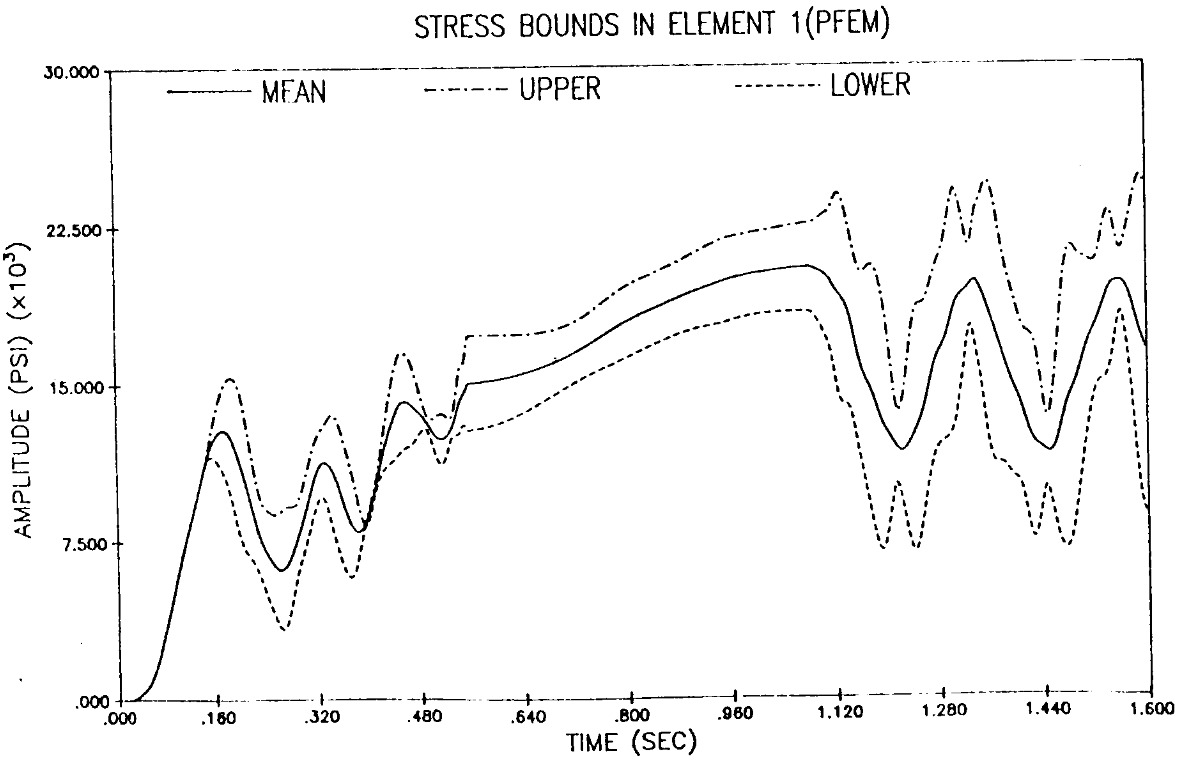


Fig. 1

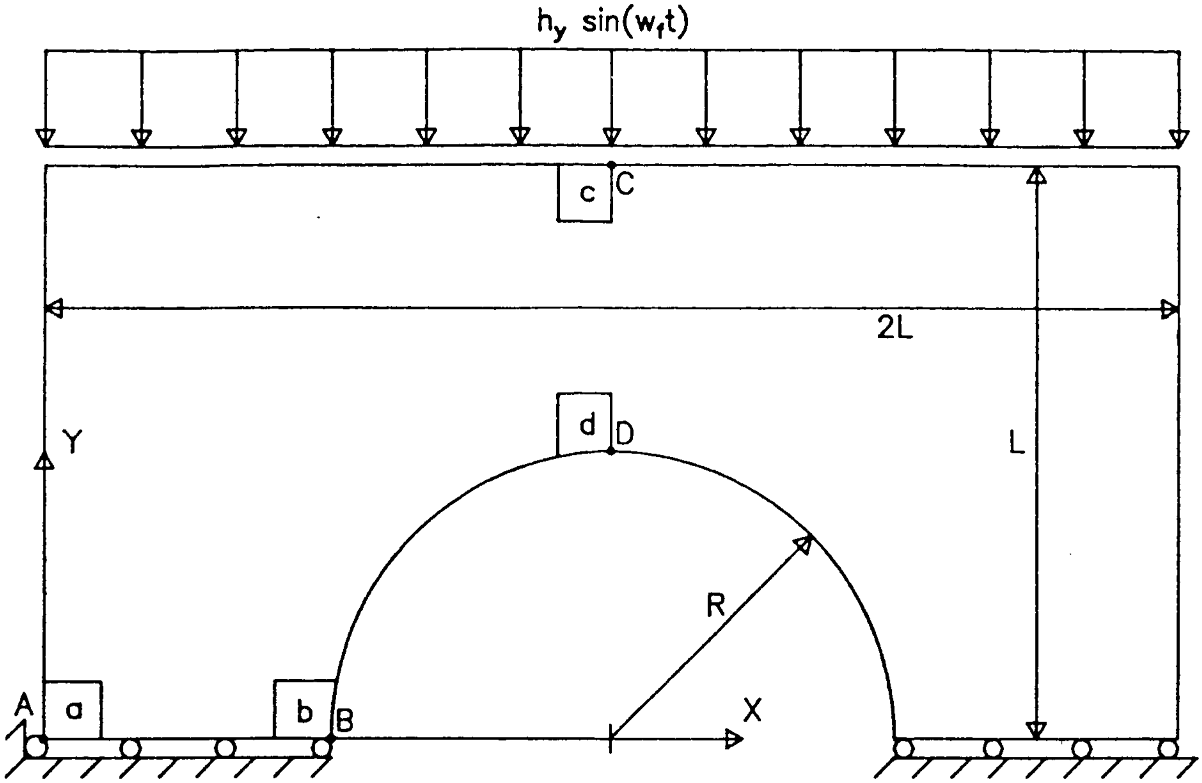


Comparison of the Variance of the y-displacement of Node 1 Using (1) Probabilistic Finite Element Method (PFEM); (2) Hermite Gauss Quadrature (HGQ); and (3) Monte Carlo Simulation (MCS).



± 3 Sigma Bounds of the Stress in Member 1 using Probabilistic Finite Element Methods (PFEM)

Fig. 2



Problem Constants

$E = 3.0 \times 10^7$ lb/in^2
Density = 0.3 lb/in^3
Thickness = 1.0 in
L = 6.0 in
R = 3.0 in
Poissons Ratio = 0.3
h_y = 2000.0 lb/in
w_f = 1500.0 rad/sec
Delta t = 1.0×10^{-4} sec
Rayleigh Damping Parameters
$e_0 = 0.0 \quad e_1 = 1.5 \times 10^{-6}$

Random Load

24 Random Variables
Coefficient of Variation = 0.1
Mean Load = 2000.0 lb/in
Spatial Correlation
$R(x_i,x_j) = \exp(-\text{abs}(x_i-x_j)/L_F)$

Mesh Data

4 Node 2D Plane Strain Continuum Element in Radial Mesh
784 Nodes, 720 Elements
Point a = Element 1
Point b = Element 15
Point c = Element 346
Point d = Element 360
Point A = Node 1
Point B = Node 16
Point C = Node 385
Point D = Node 400

Random Material

15 Random Variables
Coefficient of Variation = 0.1
Mean Youngs Mod. = 3.0×10^7 lb/in^2
Spatial Correlation
$R(x_i,x_j) = \exp(-\text{abs}(x_i-x_j)/L_E)$

Fig. 3 Problem Statement: Plain Strain Continuum with a Circular Hole.

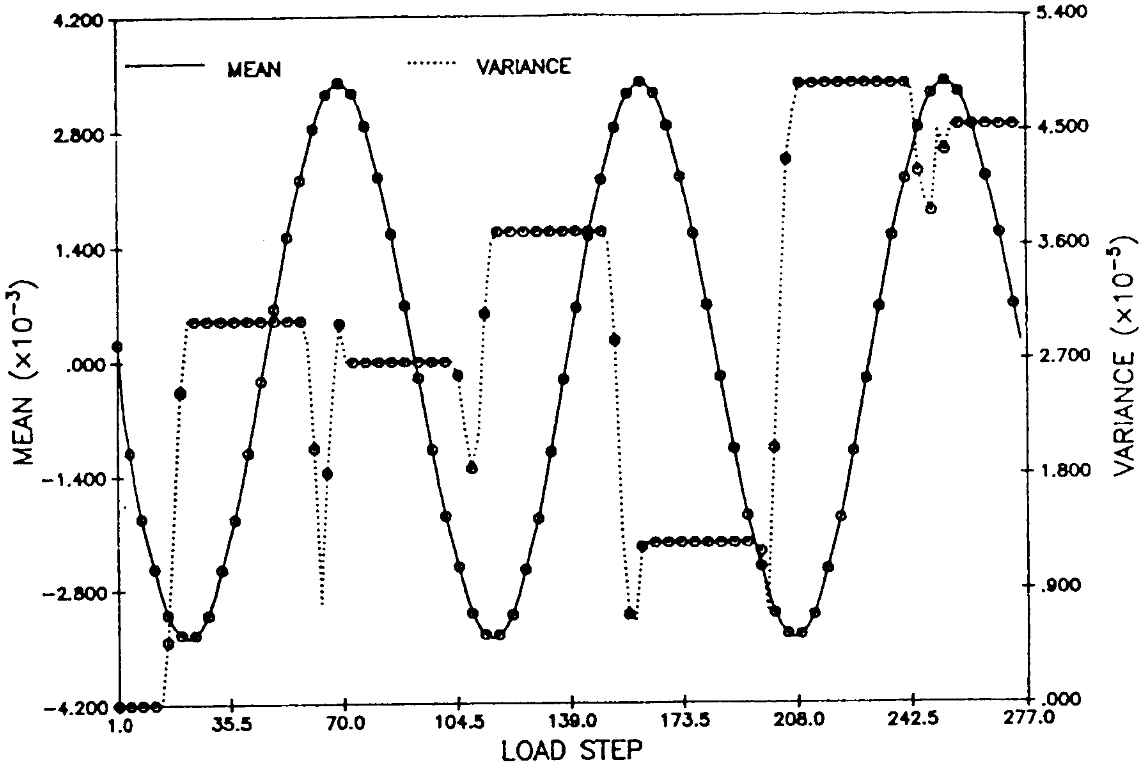


Fig. 4a Mean and Variance of Node 400 y-Displacement versus Load Steps.

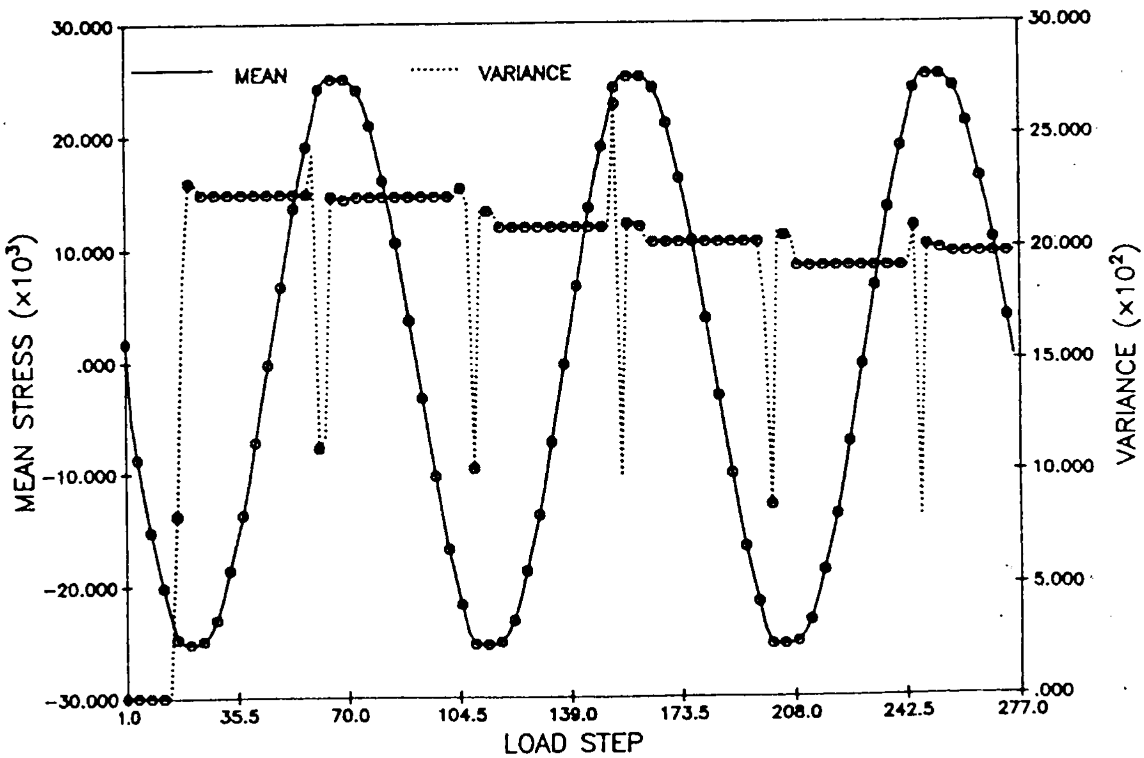


Fig. 4b Mean and Variance of Stress in Element 15 versus Load Steps.

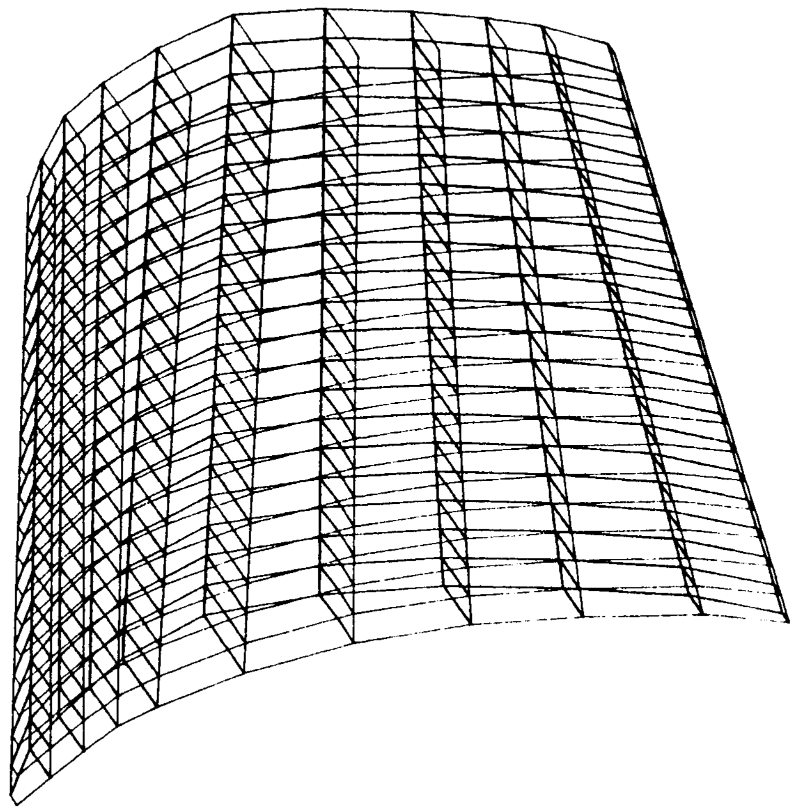


Fig. 5a

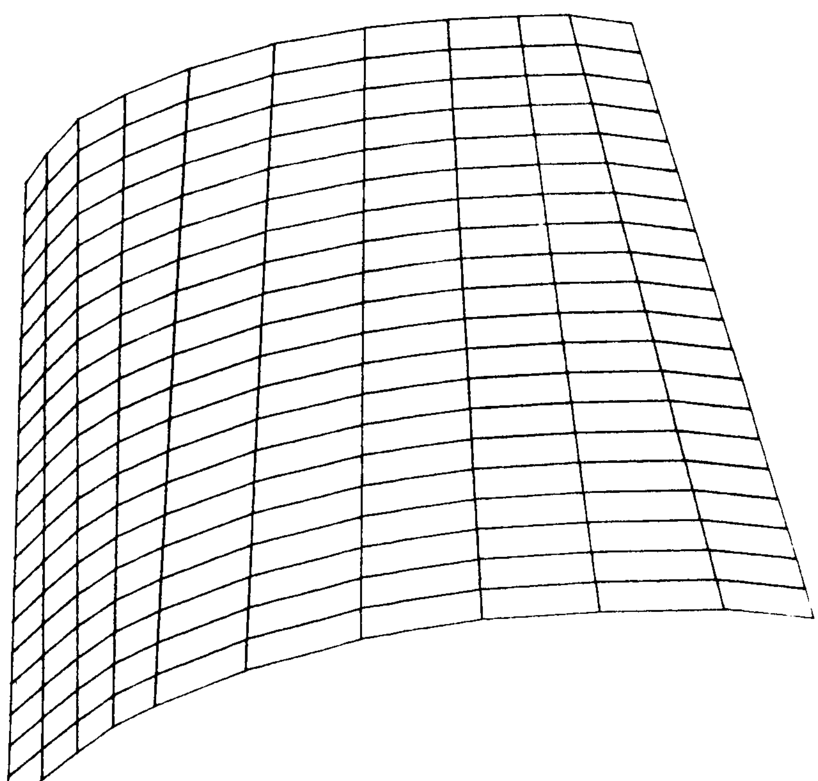


Fig. 5b