Probabilistic inductive classes of graphs

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Abstract

The idea of mathematical induction is well known for some centuries. It can be used to prove for a statement to hold or to define a certain class of objects. To define an *inductive class* of objects [2] we have to give (1) a class of *initial* objects, and (2) a list of *generating rules* that transform object(s) already in the class into an object also in the class. The inductive class consists exactly of the objects that can be obtained from the initial objects in finite number of steps using the generating rules. Eberhard [3] was the first one to define classes of graphs using an inductive definition. In graph theory the inductive definitions for several classes of graphs were proposed.

In our work we think of a graph as a “skeleton”of the network. With inductive definitions of graphs one can describe the evolution of a graph in some prescribed manners. The transitions (transformations by rules) can be viewed as implicit time steps. We extend the standard notion of inductive classes of graphs (ICGs) with probability. We consider two main possibilities: (1) adding probability to a rule selection, and (2) using probability to determine the part of a graph where the rule is applied.

We describe the restrictions and the assumptions that have to be met before applying probability to a certain ICG. Further on we look at some specific graph/network properties when applying certain probabilistic ICG, such as the change of the number of vertices and edges through time, the expected vertex degrees, etc. We show some general results for ICGs that meet the assumptions and we apply these results to some existent network models (i.e. preferential attachment model [1], feedback network model [4]).

References

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