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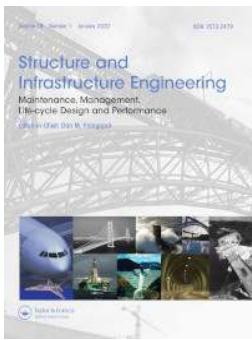
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# Probabilistic life cycle cash flow forecasting with price uncertainty following a geometric Brownian motion

M. (Martine) van den Boomen<sup>a,b</sup>, H. L. M (Hans) Bakker<sup>a</sup>, D. F. J (Daan) Schraven  and M. J. C. M (Marcel) Hertogh<sup>a</sup>

<sup>a</sup>Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands; <sup>b</sup>Research Centre for Sustainable Port Cities, Rotterdam University of Applied Sciences, Rotterdam, The Netherlands

## ABSTRACT

In the Netherlands, probabilistic life cycle cash flow forecasting for infrastructures has gained attention in the past decennium. Frequencies, volume and unit prices of life cycle activities are treated as uncertainty variables for which an expert-based triangular distribution is assumed. The current research observes the absence of time-variant variables typical for infrastructure life cycles among which price (de-)escalation. Moreover, previous research has shown that price (de-)escalation and its uncertainty should not be ignored as it may lead to over or underestimation of costs, especially for public sector organisations which use low discount rates. For that reason, the current research has searched for a more data-driven approach to include price (de-)escalation and its uncertainty by adopting a price forecasting method from the financial domain, a Geometric Brownian Motion. The uncertainty variables drift and volatility are obtained from publicly available price indices. This approach is easily included in the current practice for probabilistic cost forecasting which is demonstrated on a case study. The case study shows that ignoring price increases may lead to an underestimation of total discounted costs of 13%. From an academic perspective, the current research advocates inclusion of price uncertainty in multi-objective optimisation modelling of infrastructure life cycle activities.

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## KEYWORDS

Cash flow; forecasting; Geometric Brownian Motion; infrastructure; life cycle; probabilistic

## 1. Introduction

Cash flow forecasting is essential in construction and life cycle management of infrastructures (Bakker, Frangopol, & Tsompanakis, 2018; Mirzadeh, Butt, Toller, & Birgisson, 2014). Unfortunate examples exist in construction management and engineering, where contractors lose business because of inadequate cash flow forecasting. A recent Dutch case is the bankruptcy of a contractor with a turnover of 40 million Euros in 2018. Price increases of material and labour costs put this contractor out of business (Cobouw, 2019). Moreover, governmental agencies may underestimate future budgets and financing needs as real future costs are overlooked (Treiture et al., 2018).

Annamalaisami and Kuppuswamy (2019) conducted a meta-survey to investigate causes of cost-overruns in infrastructure projects. Seven main factors were identified: 'quantity, price, scope, resource utilisation, quality non-acceptance, delay in the construction and other external environmental factors'. Probabilistic forecasting acknowledges the uncertain nature of such factors and becomes more important when projects stretch over longer time frames where infrastructure life cycles are considered. For life cycle activities, the factors identified by Annamalaisami

and Kuppuswamy (2019) can be grouped as quantity, price, scope (activities) and timing (frequencies).

In the Netherlands, probabilistic costing has been introduced in construction and engineering since approximately 2010. From its origin, emphasis has been given to construction and project costs. National guidelines and tooling are available to assist cost engineers in estimating probabilistic project costs (CROW, 2013, 2018b). Inclusion of project uncertainties leads to probabilistic cost estimates which are generally visualised as a bandwidth of possible costs around a constant expected value. The Dutch standard follows a Monte Carlo Simulation approach well described by Wang, Chang, and El-Sheikh (2012) and Singh (2017). The Dutch standard is in line with international standards such as provided by Commonwealth of Australia (2018), UK Department for Transport (2017) and NASA (2015).

Just recently probabilistic life cycle costing has been added to these Dutch standards which were originally developed for construction and engineering projects (CROW, 2018b). The current research observes that the same principles for quantifying uncertainty of construction and project costs are applied to life cycle costs. Frequencies, quantities and unit prices are assumed to be stochastically distributed. In the absence of data, a triangular distribution is generally

CONTACT M. (Martine) van den Boomen  [m.vandenboomen@tudelft.nl](mailto:m.vandenboomen@tudelft.nl)  Faculty of Civil Engineering and Geosciences, Delft University of Technology, Building 23, room 6.71, P.O. Box 5048, 2628 CN Delft, The Netherlands

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**Table 1.** Upper and lower bounds of triangular distributions in a few Dutch publicly available infrastructure cost forecasts.

Project type	Upper/Lower bounds of unit prices	Source
Municipal roads reconstruction	−10% / +20%	(Kragten, 2019)
Municipal sewer works reconstruction	−10%/ +15%	(Kragten, 2019)
Sludge Treatment life cycle costs	± 15%	Confidential
New sanitation concept investment & life cycle costs	± 35%	(Tauw, 2016)
Investment heavy rail	± 40%	(Tauw, 2017)
Replacement wooden bicycle bridge	± 20%	(Gemeente Heemstede, 2015)

assumed as it only requires experts' estimates of minimum, mean and maximum values of each frequency, quantity and unit cost. Table 1 shows a few publicly available infrastructure cost forecasts where upper and lower bounds for unit prices are provided for the triangular distributions. Currently, limited information on cost forecasts is publicly available because of their generally confidential nature. Therefore, it is difficult to draw generic conclusions, but Table 1 indicates that unit prices for municipal works have upper and lower bounds in a range of 10% and 20% whereas innovative concepts or large investments may reach levels of  $\pm 35\%$  to 40% or more. Currently there are no standards for establishing upper and lower bounds of unit prices. In contrast to construction costs, data are even more scarce for life cycle costs. Based on professional experience, the authors observe approximately  $\pm 15\%$  as common values for estimating upper and lower bounds of unit prices of infrastructure life cycle costs.

Figure 1 visualises an example of a triangular distribution with mean costs of 100 and upper and lower bounds of  $\pm 15\%$ . This is a simplified representation for illustrative purposes. The Dutch standard for probabilistic cash flow forecasting additionally accounts for 5% and 95% confidence levels on the upper and lower bounds. Sampling from the distributions and running a Monte Carlo Analysis provides numerous cost scenarios which are discounted, counted and presented as a frequency distribution graph.

This approach, however, does not acknowledge time-variant uncertainties. Anticipating on the conclusions of the literature review, which are presented in the following section, the current research observes that especially price (de-)escalations are often overlooked. The triangular distribution accounts for uncertainty but the expected value of the mean unit price remains constant over time. Price (de-)escalations are important for adequate life cycle cost forecasting. This is especially valid for public sector organisations which use low discount rates (Treiture et al., 2018).

The aim of the current research is investigating how price (de-)escalation and its uncertainty may impact the conventional approach to probabilistic cash flow forecasting. As a first step towards more accurate probabilistic cash flow forecasting, the current research explores how currently available time-series of prices can be used to estimate uncertainty variables of a fundamental financial forecasting method. In doing so, the current research adopts a generally accepted method for price forecasting from the financial domain, a so-called Geometric Brownian Motion (GBM), which removes the necessity for choosing an arbitrary distribution for price uncertainty. The application of this method can partially remove the current subjectivity in estimating

confidence bounds. As such, the current research proposes a more data-driven approach to probabilistic life cycle cash flow forecasting and further research into price (de-)escalation and long-term price predictions.

The outline of this paper is as follows. First a literature review is presented about the current state of literature on uncertainty modelling in life cycle costing analysis and price (de-)escalation. The next section explains the GBM-based method for a more data-driven probabilistic cash flow forecasting. Public data on price (de-)escalation is investigated and used to derive uncertainty bounds. Hereafter, the method is applied to a case study and compared with the current approach to probabilistic cash flow forecasting. Results are discussed and conclusions presented.

## 2. Literature review

Probabilistic life cycle cash flow forecasting has gained attention in the past decennium both in scholarly research and in practice, but scientific results only slowly percolate to practice. An interesting and recent overview of uncertainties and uncertainty quantification approaches in scholarly research is provided by Larsson Ivanov, Honfi, Santandrea, and Strippel (2019). Although the authors focus on Life Cycle Analysis (LCA) and not on Life Cycle Cost Analysis (LCCA), the conclusions are valid for both domains. As many, the authors classify uncertainties in aleatory and epistemic uncertainties. Aleatory uncertainties refer to the randomness of uncertainty variables whereas epistemic uncertainties refer to a lack of knowledge on uncertainty variables. Moreover, the authors classify uncertainty quantification approaches in probabilistic or stochastic methods and qualitative or (semi) expert judgment-based methods. The validity of the method of choice is strongly tied to the extent of aleatory and epistemic uncertainties. In the probabilistic range, Monte Carlo Simulations are identified as the most common approach. However, the authors correctly warn that in the absence of data supporting distributions of uncertainty variables, the result of Monte Carlo Simulations should be viewed with extreme care as it suggests a level of accuracy which does not exist.

In addition, Scope, Ilg, Muench, and Guenther (2016) and Ilg, Scope, Muench, and Guenther (2017) provide an extensive literature review on uncertainty and uncertainty modelling in life cycle costing analysis. The authors identified parameter, model and scenario uncertainty and methods to address these. Price escalation is mentioned as an uncertainty factor out of 33 identified uncertainty sources. Generic guidance is given for dealing with uncertainty, however, the choice for an appropriate method remains case specific and again depends on the extent of aleatory and

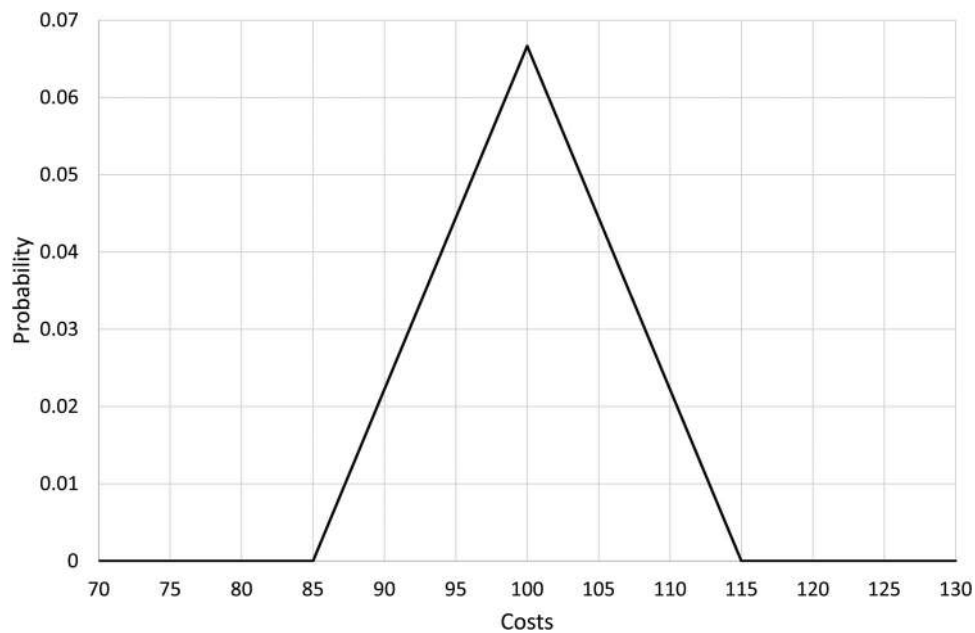


Figure 1. Example of a triangular distribution.

epistemic uncertainties. A main problem is the lack of data supporting probabilistic approaches. Still, Monte Carlo Simulation is identified as most commonly used. The authors also observe the popularity of assuming expert-judgement based triangular distributions for uncertainty variables in the absence of data. Sun and Carmichael (2018) present an equally interesting literature review on uncertainty of financial variables within infrastructure life cycle costing. Focus is put on uncertainty related to cash flows, interest rates, timing of the cash flows and LCCA duration. The authors likewise make a distinction in aleatory and epistemic uncertainties. Moreover, they identified a range of uncertainty quantification methods for each of the four identified uncertainties which could also be classified as stochastic and (semi) expert-judgement based approaches.

The current research observes that price (de-)escalation and its uncertainty did not emerge as a specific factor of interest in this dedicated literature review on financial variables, indicating that it may not be on the radar of scholarly research on LCCA. Although cash flow uncertainty is identified, including various distributions, this uncertainty is mainly attributed to increasing costs as a consequence of ageing, timing and volume related uncertainties and not price escalation. Other researchers indeed observe the lag in attention for price uncertainty in construction and engineering. Swei, Gregory, and Kirchain (2015) and Swei, Gregory, and Kirchain (2017) explicitly state that price variations for asphalt have been ignored in scholarly research. As a reason the authors put forward a lack of empirical data and as such unfamiliarity with the application of econometric forecasting models. Likewise, Ilbeigi, Castro-Lacouture, and Joukar (2017) observe a gap in knowledge in the USA on prediction of the uncertainty of asphalt prices and stress the risk this poses to proper cash flow forecasting.

Similar observations are done by Faghih Sayed Amir and Kashani (2018) who extend their conclusions from asphalt to steel and cement prices in the United States. The

volatility of prices of construction materials are identified by the authors as a main contributor to deviations in cost estimates. Younis, Rehan, Unger, Yu, and Knight (2016) investigated the impact of inflation and its uncertainty on (waste) water mains capital works and equally concluded their significance for adequate cost estimates. This research resulted in the development of a dedicated unit cost index database for these infrastructures (Rehan et al., 2016). Anastasia, Andrew, and Feargal (2018) investigated the impact of price uncertainty modelling of electricity on the profitability of offshore wind parks. As electricity prices are known to be volatile, adequate predictions are essential for investors. Also Van den Boomen, Spaan, Shang, and Wolfert (2020) observed the absence of price uncertainty in infrastructure decision making and stressed its importance, especially for public sector organisations which use low discount rates.

Prices change and are subject to uncertainty (Anastasia et al., 2018; Kowal, Conforti, Hergt, & Sager, 2018). The current research observes that data on prices are publicly available at each bureau for labour statistics and specialised agencies. However, in current probabilistic cash flow forecasting, prices are dealt with as an epistemic uncertainty and its (de-)escalation is often ignored whereas prices to some extent are aleatory uncertainties for which both data and forecasting methods exist. The question is why this data is not mined and used? Ignoring price variations and their uncertainty may lead to over or underestimation of real costs. Previous research has indicated that price fluctuations and price uncertainty may be dominant factors in probabilistic life cycle cost forecasting. Important is that estimates are reproducible and based on objective data.

### 3. Method development

An approach to data-driven probabilistic price forecasting needs data and a forecasting method. As stated by the

previous mentioned researchers, the choice of method for uncertainty quantification depends on the availability of data (Larsson Ivanov et al., 2019; Scope et al., 2016; Sun & Carmichael, 2018). Price data is generally available in the form of time-series. Each bureau for labour statistics and specialised agencies provide historic prices for construction, engineering, manufacturing and services. As such the Dutch Central Bureau of Statistics presents Producer Price Indices (PPIs) for both construction of civil engineering works (CBS, 2020b), manufacturing (CBS, 2020e), engineering services (CBS, 2020f) and labour price development for construction (CBS, 2020d). Another interesting price index database in the Netherlands is provided by CROW (2018a). This database depicts long-term historic price developments of construction materials, energy and labour and is originally meant to protect contractors against negative consequences of price fluctuations. Recalculation is done afterwards, and as such contractors are compensated for a certain amount of financial project risks. This risk mitigation regulation for construction works in the Netherlands originates from 1995 and has as a positive side effect that by now it has gained a valuable resource of historic prices for construction works.

The current research proposes to use historic price indices for future predictions. Although historic prices do not guarantee the validity of future predictions, it does give some information about expected developments. For example, past recessions and their impact on prices may reflect future recessions although underlying causes such as technological advancements and global transitions, may differ. The current research argues that long-term past prices to some extent reflect future trends which outweigh the current practice where price escalation is practically ignored, and uncertainty bounds are often based on estimated values of  $\pm 15\%$ . Long-term past prices provide additional information which is available and easily accessible.

The second ingredient is a method for price forecasting. The current research proposes a Geometric Brownian Motion (GBM) for predicting future prices and their uncertainty. A GBM is a well-known and broadly accepted statistical method originating from the financial domain (Anastasia et al., 2018; Davison, 2014; Francis & Kim, 2013). In the discussion section of this research some other financial forecasting methods are considered as well. The choice for a GBM is motivated from a pragmatic point of view. It only needs past prices to predict future prices and, as is demonstrated later on, supports the general approach to dealing with compounded inflation or deflation. Therefore the results of a GMB are easily explained in contrast to more advanced methods which often build on the principles of a GBM but regress on more variables (i.e. polynomial regression of historic data, adding regression on prediction errors, adding regression on explanatory variables). These advanced methods prove value when sufficient and accurate data are available for a specific context, as many of these methods provide unique prediction equations for unique data sets (black box). As such these advanced methods are considered less suitable for the general case, than a GBM.

A GBM describes a random walk defined by a drift and volatility. Each time step a price changes based on a constant (drift) and a random shock (volatility). Because returns on prices are compounded, a GBM takes the natural logarithm of prices when describing a random walk according to:

$$\ln(P_j) - \ln(P_{j-1}) = \mu + \sigma \varepsilon_j, \quad (1)$$

where  $\ln(P_j)$  is the natural logarithm of the price at time  $j$ ;  $\ln(P_{j-1})$  is the natural logarithm of the price at time  $j - 1$ ;  $\mu$  is a drift obtained from past prices;  $\sigma$  is a volatility obtained from past prices and  $\varepsilon_j \sim N(0, 1)$  is a shock following a normal standard distribution with a mean of 0 and a standard deviation of 1. The drift  $\mu$  and volatility  $\sigma$  are obtained from past price data. An example is provided in the appendix (Table A1). The drift and the volatility are the mean and the standard deviation of the natural logarithms of the returns, respectively. If drifts and volatilities are obtained from i.e. quarterly data, they can be annualised according to (Francis & Kim, 2013):

$$\mu_{\text{annual}} = \frac{\mu_{\text{quarter}}}{1/4} \quad (2)$$

$$\sigma_{\text{annual}} = \frac{\sigma_{\text{quarter}}}{\sqrt{1/4}} \quad (3)$$

Equation (1) can be rearranged to Equation (4) which provides a direct relationship for the next price simulation based on its previous forecast:

$$P_j = P_{j-1} \cdot \text{EXP}(\mu + \sigma \varepsilon_j) \quad (4)$$

while, if preferred, Equation (4) can be rearranged in a direct relationship between the price at time  $j$  and the price at time zero:

$$P_j = P_0 \cdot \text{EXP}((j\mu) + (\sigma(\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_j))) \quad (5)$$

When ignoring the shock, that is when  $\sigma \varepsilon_j = 0$ , Equations (4) and (5) follow the general notion of compounding inflation as by approximation  $P_{j-1} \exp(\mu) \approx P_{j-1}(1 + \mu)$ . In this relationship the left term is the continuous form and the right term the discrete form of compounding. Therefore the drift  $\mu$  can be seen as a periodic overall inflation rate. As such a GBM is not exotic. It just represents common practice when dealing with inflation but adds a shock at each time step which acknowledges price uncertainty. Equation (4) or Equation (5) with shocks can now be used to simulate future prices. This is shown in Figure 2 for the asphalt prices in the appendix. The solid black line upto year 2020 represents the fluctuating historic prices. From 2020 onwards, the solid black line represents the expected value of the future prices and the grey lines each represent a possible price path around this expected value.

In the absence of explanatory data on mechanisms explaining prices, the GBM as a first method of choice for price prediction is motivated by its closeness to the traditional way of dealing with compounded inflation or deflation. The expected value of price increases following a GBM is presented by the solid black line from 2020 onwards in Figure

2. This trend can alternatively be explained with the traditional approach for dealing with compounded inflation. In the first 20 years, from 2000 to 2020, Figure 2 shows that the price index for asphalt increases from approximately 95 to 175. The annual total inflation rate  $i$  can be derived by solving  $95(1+i)^{20} = 175$  for  $i$  and results into 3.1% per year over the past 20 years. Using this inflation rate as a predictor for the next 20 years results in an expected price index of  $175(1+0.031)^{20} = 322$  which corresponds with the expected value obtained by the application of a GBM in Figure 2. The uncertainty of this value is reflected by the possible price paths between 2000 and 2020 as shown in Figure 2.

However, as for all prediction models, there is no guarantee that past trends reflect future trends. Technology change, climate change, scarcity of fossil resources, pandemics, transition to a circular economy and energy transition will all impact future prices. In this example it is just observed that asphalt prices have on average increased by 3.1% per year over the past 20 years. Therefore, in the absence of more explanatory data on mechanisms explaining prices, the current research suggests to assume that this average annual increase will continue, while keeping in mind that many factors may alter this expected price escalation in the future.

The interesting concept about a GBM is that it reflects both price (de-)escalation and its uncertainty which fluctuates over time. Current prices have less uncertainty than future prices. The width of the cone of uncertainty reflects the volatility of prices experienced in the past. Steady historical prices will have a small cone of future uncertainty whereas strong fluctuating historical prices will have a wide cone of future uncertainty. Moreover, prices will not become negative.

Theoretically, there is no upper limit on a GBM. However, in practice this upper limit is suppressed by the volatility obtained from past price data. Hence, upper paths are extremes and have low probabilities of occurrence when running a Monte Carlo Simulation. Using GBMs to forecast prices and their uncertainty based on PPIs removes part of the current biases in choosing a distribution and uncertainty bounds. In the Netherlands, open price data are provided by CROW (2018a) and CBS Statline (CBS, 2020a). From these databases, the current research selected several categories relevant for construction and engineering and derived the drifts and volatilities for GBMs from the price indices. These generic results are presented in Table 2. The drifts and volatilities move around average values of 0.021 and 0.041 respectively, for these selected prices. This order of magnitude is representative for price development of construction and engineering in the Netherlands. From this table values will be selected for the case study of this research.

The drifts and volatilities obtained from the CROW database are slightly higher than those obtained from the CBS database. The CROW database builds on CBS data, includes additional information and is dedicated to construction and engineering. As an example, differences are seen for labour prices in Table 2. The drift and volatility of labour in the CROW database are 0.027 and 0.020, respectively. In

contrast, the drift and volatility of labour in the CBS database are 0.015 and 0.016, respectively. Differences are motivated by aggregation and potential risk mark-ups. For example, the CROW values may consider labour with flexible contracts whereas the CBS database may consider labour with permanent contracts.

GBMs can be forecasted with a Monte Carlo Simulation using their drifts and volatilities. The strength of a Monte Carlo Simulation is that it allows for combining multiple uncertainties with distinct distributions in probabilistic cash flow forecasting. A cash flow forecast is founded on multiple prices and multiple quantities each with their own uncertainty distributions. This approach to a more data-driven cash flow forecast is demonstrated in the case study but first the discount rate is discussed in the following section.

#### 4. Discounting under price (de-)escalation

Cash flow forecasts are discounted to their present values which allows for comparison between alternative scenarios. Present values may also be used to indicate the amount of capital to be reserved for future purposes. Discounting under price (de-)escalation needs correction of the discount rate. For clarity, the following explanation builds on price escalation (inflation). These expressions can be substituted with price de-escalation (deflation). If cash flows are inflated with producer price escalation (PPI or total inflation), the discount rate needs to be inflated with the long-term general inflation rate (ISO, 2008). This correction is performed according to Equation (6) (ISO, 2008; Park, 2016; Sullivan, Wicks, & Koeling, 2012):

$$r_{nom} = r_{real} + i_g + r_{real} \cdot i_g \quad (6)$$

Equation (6) expresses the relationship between the nominal or inflated discount rate and the real or uninflated discount rate. The real discount rate  $r_{real}$  is inflated with the general inflation rate  $i_g$  to arrive at the nominal or inflated discount rate  $r_{nom}$ . The long-term general inflation rate is derived from the Consumer Price Index (CPI) (CBS, 2020c; ISO, 2008; Park, 2016) and reflects a general price increase (or decrease) based on a price basket with common goods. The long-term general annual inflation rate is derived from CPI data according to (Park, 2016):

$$i_g = \left( \frac{CPI_n}{CPI_0} \right)^{1/n} - 1 \quad (7)$$

where  $CPI_n$  is the CPI in year  $n$ ,  $CPI_0$  is the CPI in a base year zero,  $n$  is the number of years between the two CPIs.

For the Netherlands the general inflation rate from 1996 to 2020 is calculated as 1.87% (CBS, 2020c). Inflating the real discount rate to a nominal discount rate as in Equation (6) results in a higher discount rate. However, because the general inflation rate is often less than the producer price inflation (Kowal, Conforti, Hergt, & Sager, 2019), this difference, also expressed as differential inflation, will mostly cause increased cash flows and therefore increased present values of discounted life cycle costs. As an illustration, the average drift for the selected categories in Table 2 is 0.021 which

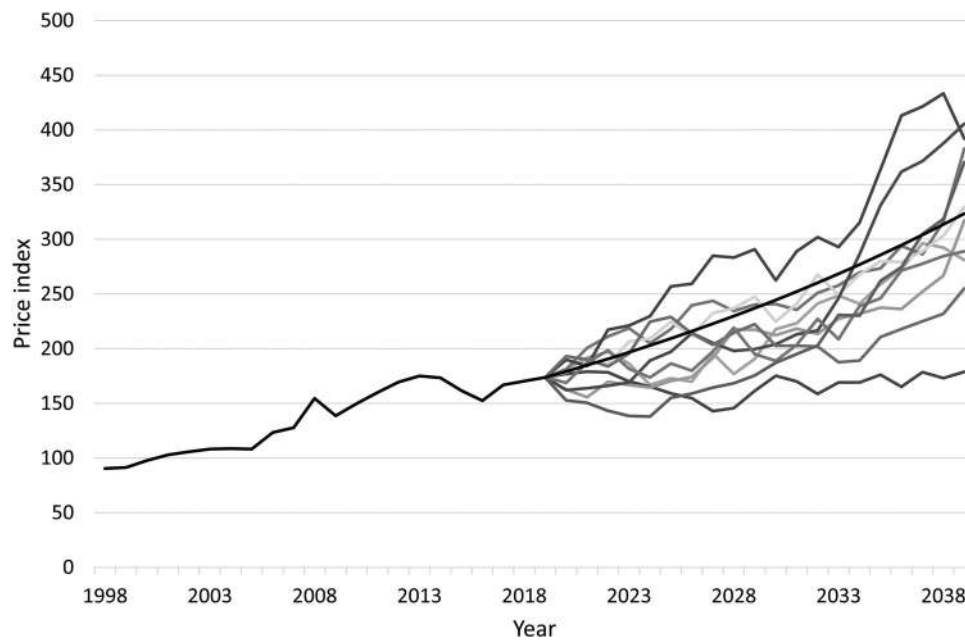


Figure 2. Predicted prices for asphalt in the Netherlands based on a GBM.

exceeds the general inflation rate of 0.0187. Therefore, these prices on average will increase at a higher rate than the general inflation.

Another point of interest is whether the general inflation rate, real interest rate and derived nominal discount rate should be treated as GBMs like the current research suggests for prices of goods and services in probabilistic cash flow forecasting. This is not advised because prices of goods and services indeed fluctuate on a short-term basis and are realistic expenditures which pose risks. In contrast, discount rates are tied to market forces but set by governmental and organisational policies and based on long-term developments and larger project portfolios. Discount rates are long-term averages which reflect a minimum attractive rate of return required by an organisation and set constant over a longer period.

In the Netherlands, a national taskforce advises the minister of Finance who decides on the discount rate for governmental public projects. The last policy dates from 2015 and prescribes a real discount rate for public infrastructure of 3% (Taskforce discount rate, 2015). Due to decreasing interest rates in the market the CPB Netherlands Bureau for Economic Policy Analysis currently advises to review the policy on the discount rate for public projects. Changes in discount rate will likely be effectuated in one or two years. The current research acknowledges that discount rates are uncertain but to a lesser extent than market prices. Discount rates are set by policy and therefore do not follow a GBM in the current application. Their impact could better be investigated with a sensitivity analysis as will be demonstrated in the case study.

## 5. Case study

The case study constructs a probabilistic life cycle forecast for a concrete bridge in the Netherlands with a design life of 100 years. The life cycle costs, excluding initial

investments as the focus of this research is on time-variant price developments, follow from a preliminary design. Life cycle activities, the design life and data are obtained from a Dutch government and proportionally indicative for many similar bridges (Table 3). First a probabilistic cash flow forecast is performed according to the Dutch standard (CROW, 2018b). Second, a probabilistic cash flow forecast is performed with the method where price uncertainty follows a GBM. Commissions and taxes are left out as the interest lies in comparison of the conventional method with the GBM-based method.

### 5.1. Data and assumptions

The probabilistic cash flow forecast contains three uncertainty variables: the frequency or corresponding timing of activities, the volume or quantities used per activity and the unit price of these volumes. The traditional method uses triangular distributions and in the absence of data, assumes uncertainty bounds of  $\pm 15\%$  of the expected values (Table 3). Sampling from the distributions results in scenarios where the timing of activities can be earlier or later; the volumes can be more or less and the prices can be more or less than the expected values. Certain activities are always performed together. The timing of these activities is sampled from the same distributions. Moreover, in this conventional approach price uncertainty is a distribution around a constant expected value.

As such, there is no price (de-)escalation taken into account and therefore life cycle costs are discounted with the real discount rate as discussed in Section 4. Because of a long-term forecast and the less volatile nature of the discount rate (Section 4), the discount rate is taken as a weighted average of the available past real discount rates during 1995–2020 and amounts 5% (Taskforce discount rate, 2015). Although the current real discount rate is lower (3%) this research assumes that the average real discount



Table 2. Drift and volatilities obtained from public price escalation data for construction and engineering in the Netherlands.

Categories <sup>a</sup>	Annualised		Interval	Period
	Drift	Volatility		
<b>PPI Civil engineering works (source CROW)</b>				
0 Labour	0.027	0.020	Monthly	1995 – 2020
3 Gasoline without VAT	0.058	0.240	Monthly	1995 – 2020
4 Electricity	0.014	0.148	Monthly	1995 – 2020
13 Concrete mortar	0.027	0.033	Monthly	1995 – 2020
17 Plastics including PVC	0.033	0.047	Monthly	1995 – 2020
19 Steel excluding reinforcing bars	0.021	0.098	Monthly	1995 – 2020
20 Road pavement bitumen	0.042	0.178	Monthly	1995 – 2020
22 Mineral asphalt mixture including fuel, ex. bitumen	0.025	0.031	Monthly	1995 – 2020
<b>PPI Construction (source CBS)</b>				
4211a Road construction; brick paving	0.021	0.015	Quarterly	1998 – 2020
4211b Road construction; asphalt paving	0.028	0.074	Quarterly	1998 – 2020
4211c Road maintenance	0.030	0.016	Quarterly	1998 – 2004
4212 Railways and underground railway	0.027	0.026	Quarterly	2000 – 2020
4213 Construction works for bridges and tunnels	0.017	0.025	Quarterly	2000 – 2020
4221 Constructions works for utility projects for fluids	0.026	0.019	Quarterly	1998 – 2020
4291 Constructions and works for water projects	0.026	0.033	Quarterly	2000 – 2020
4312 Site preparation works	0.023	0.024	Quarterly	1998 – 2020
4321 Electrical installation works	0.019	0.017	Quarterly	2000 – 2020
<b>PPI Manufacturing (source CBS)</b>				
2361 Concrete products for construction projects	0.011	0.029	Monthly	2012 – 2020
2362 Plaster products for construction purposes	0.014	0.021	Monthly	2014 – 2020
2365 Fibre cement	0.030	0.037	Monthly	2012 – 2020
251 Structural metal products	0.014	0.018	Monthly	2012 – 2020
2511 Metal structures and parts of structures	0.013	0.021	Monthly	2012 – 2020
2811 Engines & turbines, ex. aircraft, vehicle & cycle eng.	0.011	0.017	Monthly	2012 – 2020
2812 Fluid power equipment	0.021	0.030	Monthly	2012 – 2020
2813 Other pumps and compressors	0.011	0.016	Monthly	2012 – 2020
2814 Other taps and valves	0.005	0.006	Monthly	2012 – 2020
2815 Bearings, gears, gearing and driving elements	0.001	0.011	Monthly	2012 – 2020
33 Repair & installation serv. of machinery & equipment <sup>b</sup>	0.022	0.017	Monthly	2012 – 2020
<b>PI Labour (source CBS)</b>				
Labour – construction	0.015	0.016	Yearly	2001 – 2019
<b>PPI Services (source CBS)</b>				
711212 Engineering; building projects	0.021	0.042	Quarterly	2002 – 2019
711213 Engineering; power projects	0.014	0.037	Quarterly	2003 – 2019
711214 Engineering; transportation projects	0.012	0.046	Quarterly	2004 – 2019
711216 Engineering; water projects	0.011	0.042	Quarterly	2005 – 2019
711217 Engineering; manufacturing	0.022	0.025	Quarterly	2006 – 2019
711220 Project management services for construction	0.017	0.079	Quarterly	2007 – 2019

<sup>a</sup>Raw data on PPIs are obtained from CROW (2018a) and CBS (2020a)

<sup>b</sup>Raw data have been adjusted for one extreme outlier

rate over the past 25 years is more representative for the long-term forecast than the current discount rate. However, in the sensitivity analysis, the impact of various discount rates is investigated.

The probabilistic GBM-based method changes the distribution of a price variable in a GBM where drifts and volatilities are obtained from open PPI data. These drifts and volatilities for the various life cycle activities are presented in Table 4 and selected from Table 2. There is some subjectivity in selecting these values as two different databases underlie the analysis in Table 2 and the categories depicted are originally meant for construction and engineering and not dedicated to maintenance. The drift accounts for price escalation with a total inflation rate whereas the volatility accounts for its uncertainty.

As the cash flows are fully inflated when applying a GBM, discounting should be done with the nominal discount rate as explained in Section 4. The inflated discount rate (nominal discount rate) follows from the application of

Equation (7) and Equation (6) and amounts 6.96%, given a long-term real discount rate (non-inflated) of 5% and a long-term general inflation rate of 1.87% (CBS, 2020c; Taskforce discount rate, 2015). A Monte Carlo Simulation is run with 1000 simulations which provides enough basis for comparison. The Monte Carlo Simulations are performed in Excel in a few seconds and without plugins but can also be performed in Excel with plugins (i.e. @risk<sup>®</sup> or Crystalball<sup>®</sup>), in MATLAB or any other programming language.

## 5.2. Results

The results are visualised for both approaches as a frequency distribution graph of the discounted life cycle costs while running 1000 simulations (Figure 3). The left frequency distribution in Figure 3 belongs to the conventional method for probabilistic cash flow forecasting. The right frequency distribution follows from the application of a GBM to

Table 3. Life cycle activities and conventional uncertainty bounds for a new concrete bridge.

Maintenance activities	Freq. (yr)	Volume	Unit price	Frequency		Volume		Cost	
				Upper (%)	Lower (%)	Upper (%)	Lower (%)	Upper (%)	Lower (%)
Main load bearing construction – concrete deck repair	25	9,200 m2	€ 5	15	15	15	15	15	15
Replacement bearings	50	16 pc.	€ 35,000	15	15	15	15	15	15
Piles – restore concrete damages	25	410 m2	€ 18	15	15	15	15	15	15
Pavement – replacement supporting construction	20	9,200 m2	€ 35	15	15	15	15	15	15
Pavement – replacement deck	20	9,200 m2	€ 15	15	15	15	15	15	15
Replacement expansion joints	25	30 m	€ 1,250	15	15	15	15	15	15
Kerb side – restore concrete damages	10	1,600 m2	€ 18	15	15	15	15	15	15
Safety barrier – maintenance	20	1,450 m	€ 18	15	15	15	15	15	15
Safety barrier– replacement	20	1,450 m	€ 115	15	15	15	15	15	15
Railing – replacement	40	730 m	€ 300	15	15	15	15	15	15
Railing – conservation	10	730 m	€ 75	15	15	15	15	15	15
Other yearly maintenance	1	1 pc.	€ 25,000	15	15	15	15	15	15
Traffic measures	1	1 pc.	€ 4,000	15	15	15	15	15	15

predict price escalation and uncertainty while leaving the other variables and their uncertainty distributions unchanged.

It is seen in the case study that the conventional method underestimates the total life cycle costs, expressed in present values. For the case study this underestimation amounts to approximately 13%. The case study also displays an increased spread in total discounted costs for the GBM-based method. This is explained by the volatilities of the GBMs obtained from data, compared to the expert judgement's upper and lower bounds of 15%. Increasing these upper and lower bounds of the conventional method to  $\pm 50\%$  while leaving the other variables unchanged results in Figure 4. It is seen that the spread of the combined triangular distribution increases. However, the mean value of the combined triangular distributions remains unchanged because the underlying triangular distributions do not take price escalation into account.

#### Sensitivity analyses on discount rate and regression period

Two sensitivity analyses are performed: for the real discount rate and for the length of the regression period which impacts the drifts and volatilities of the GBMs. The probabilistic forecasting methods are first investigated for their sensitivity to discount rates as shown in Table 5. All discount rates pose real scenarios for public sector organisations. It is seen that lowering the long-term real discount rate to i.e. 3% will increase the current underestimation to 19% whereas increasing the long-term discount rate to 7% will lead to an underestimation of 9% for the case study.

Regressing other drifts and volatilities from the time series of the prices by changing the length of the time series will change the present values as well. Lower drifts will reduce the amount of underestimation and higher drifts will increase the amount of underestimation when comparing the conventional probabilistic forecasting method and the GBM-based method. Consequently, lower volatilities will reduce the distribution of the probabilistic GBM-based result and lower drifts will shift the mean of the probabilistic GBM-based result to the left. The volatilities and drifts in Table 4 represent observed historic fluctuations and

trends during 1995 – 2020. The case study aims to forecast life cycle cash flows over the long life cycle of this infrastructure (100 years). For that reason, the case study used the longest available historical time series of prices (25 years).

However, one could argue that old prices may not be representative for future forecasts. Moreover, it is interesting to see how the choice of the analysis period can impact results. Therefore, a sensitivity analysis is presented based on variable regression periods. For this analysis 5 periods are distinguished. The longest historic period is 25 years (1995 – 2020) followed by 20 years (2000 – 2020), 15 years (2005 – 2020), 10 years (2010 – 2020) and 5 years (2015 – 2020). The distinct variables used as input for the probabilistic calculation with GBMs are derived from the time series and presented in Tables 6 and 7.

Monte Carlo Simulations are performed with the financial variables presented in Tables 6 and 7, combined with the different activities, distributions on their timing and volumes, and the unit prices conform Table 3. The results of these simulations for scenarios 1, 2, 3 and 4 are presented in Figure 5. Scenario 0 is already displayed in Figure 3. Figure 5 displays for each scenario the present value of the life cycle cost and its distribution for the conventional and GBM-based method for probabilistic forecasting.

As expected, regression periods with lower means of unit prices (less inflation) shift the mean of the GBM-based probabilistic forecasts to the left as the future becomes cheaper. This is explained by regression on periods where fluctuating prices increase at a lower rate than the general inflation as can be seen while comparing Tables 7 and 6. The expected cheaper future is partly counterweighted by the real discount rate which shows a declining trend for the subsequent scenarios which means that the declining real discount rate contributes to the expectation of a more expensive future but to a lesser extent than the inflation contributes to a cheaper future. In contrast, the conventional probabilistic approach is not influenced by periods where prices increase or decrease as the expected value of the conventional approach does not account for price (de-)escalation. The differences seen in the conventional probabilistic forecasting method are solely explained by the

Table 4. Life cycle activities and selected uncertainty parameters for prices following a GBM.

Maintenance activities	GBM		Reference Table 1 Code (source)
	drift	volatility	
Main load bearing construction – concrete deck repair	0.017	0.025	4213 (CBS)
Replacement bearings	0.021	0.098	19 (CROW)
Piles – restore concrete damages	0.017	0.025	4213 (CBS)
Pavement – replacement supporting construction	0.028	0.074	4211b (CBS)
Pavement – replacement deck	0.028	0.074	4211b (CBS)
Replacement expansion joints	0.033	0.047	17 (CROW)
Kerb side – restore concrete damages	0.017	0.025	4213 (CBS)
Safety barrier – maintenance	0.015	0.016	Labour (CBS)
Safety barrier– replacement	0.021	0.098	19 (CROW)
Railing – replacement	0.021	0.098	19 (CROW)
Railing – conservation	0.015	0.016	Labour (CBS)
Other yearly maintenance	0.015	0.016	Labour (CBS)
Traffic measures	0.015	0.016	Labour (CBS)

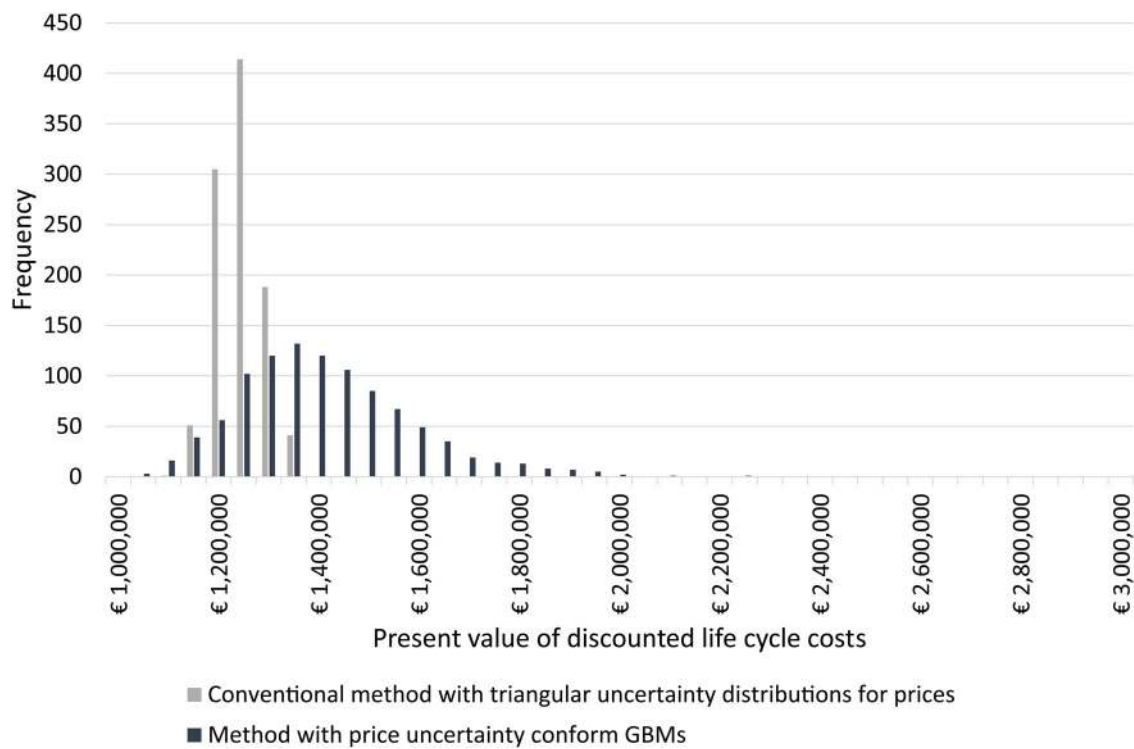


Figure 3. Probabilistic present values of the life cycle cash flows for the conventional method and the GBM-based method with input values of Table 3 and Table 4 and a real discount rate of 5%.

variability in real discount rate for each period. Lower expected discount rates make the future more expensive.

This raises an important question on what the length of a regression period for forward prediction should be? The current research displayed Figure 3 as main result but a sensitivity analysis on the regression period of historic prices reveals two scenarios (3 and 4) where the GBM-based forecasting leads to a lower present value of life cycle costs than the conventional probabilistic method. What analysis period should be used to regress on drift and volatility?

For the case study, the current research chose the longest available time series. That is because cash flows are forecasted for a life cycle of 100 years. Regressing historic data over 5 to 10 years will not capture historic trends and cyclic behaviour (seasonality) of prices for a long-term. There are for example 5-years' periods where some prices decrease but this will not mean that these prices will remain decreasing for the full life cycle. Just as the declining real discount rate will not

necessarily imply that the future real discount rate will not rise again. The historic time series show that in the Netherlands, over the long-term prices in general increase. However, if the purpose of the current research would have been the forecasting of a shorter period, for example a construction project stretching over 5 to 10 years, regression would have been done over 5 to 10 years as these recent historic prices are assumed to be more representative for short and mid-term forecasts.

The question of the length of a regression period is a very important one. The generic answer is that a regression period should sufficiently capture trends and seasonality or cyclic behaviour (Chatfield & Xing, 2019). The choice is case and purpose specific and also depends on the amount of data available. The case study is based on the data currently available and indicates that price escalation and uncertainty may lead to underestimation of costs. The additional uncertainty induced by insufficient data adds to the importance of a dedicated registration of price developments

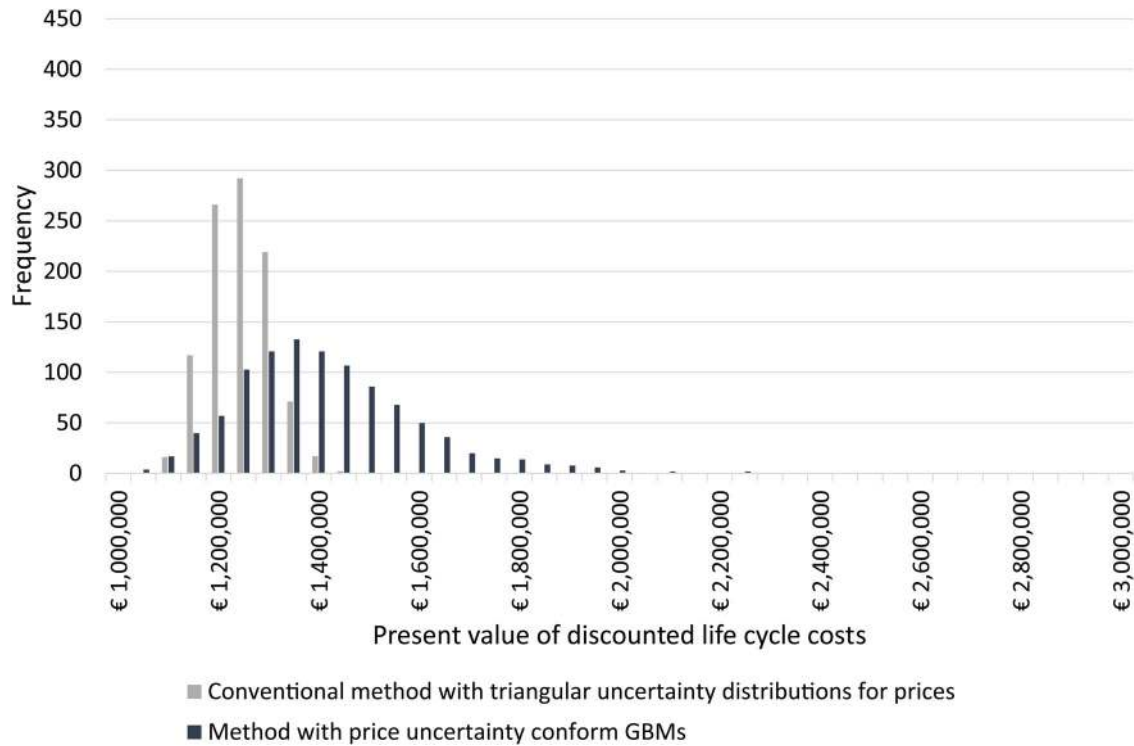


Figure 4. Probabilistic present values of the life cycle cash flows for the conventional method and the GBM-based method as in Figure 3 but with  $\pm 50\%$  upper and lower bounds for the triangular distribution of prices of the conventional method.

Table 5. Sensitivity analysis for the discount rate in the case study.

Real discount rate (general inflation rate 1.87%)	Mean of discounted life cycle costs in € millions (rounded)		Underestimation of conventional method (%)
	Conventional method	Proposed method	
1%	4.63	5.90	27
2%	3.08	3.79	23
3%	2.16	2.58	19
4%	1.59	1.84	16
5%	<b>1.22</b>	<b>1.38</b>	<b>13</b>
6%	0.96	1.07	11
7%	0.79	0.86	9

for life cycle activities and improvement of price prediction methods.

### 5.3. Verification of the probabilistic calculations

A good habit is a deterministic calculation as a reference. Because of the skewness induced by combining uncertainty distributions, a probabilistic cash flow forecast with price escalation (price escalation with uncertainty) tends to give higher present values than its deterministic counterpart (price escalation without uncertainty). However, a deterministic cash flow forecast with price escalation will generally yield higher results than a probabilistic forecast without price escalation (uncertainty but no price escalation). Therefore it is expected that a deterministic calculation for the cash flows of the case study will yield a total present value between the value of the current probabilistic method (€1.22 million without price escalation) and the GBM-based method (€1.38 million with price escalation). If this is true it is reasonably assumed that the probabilistic calculations are performed without calculation errors.

Table 6. Average general inflation rates and real discount rates for distinct time periods.

Scenario	Period	Real discount rate (%) <sup>a</sup>	General inflation rate (CPI) <sup>b</sup>
0	1995 – 2020	5.00	1.87
1	2000 – 2020	4.70	1.84
2	2005 – 2020	4.70	1.59
3	2010 – 2020	4.25	1.64
4	2015 – 2020	3.00	1.54

<sup>a</sup>Derived from Taskforce discount rate (2015)

<sup>b</sup>Derived from CBS (2020c)

Using the mathematics of a geometric series a deterministic calculation including price escalation is quickly performed. Let

$$K = \frac{(1 + \mu)}{(1 + r_{nom})}, \quad (8)$$

where  $\mu$  is the drift or total price escalation and  $r_{nom}$  is the nominal discount rate conform Equation (6). Now the present value of annual maintenance costs starting in year 1, increasing with  $\mu$  % per year and ending in year  $T_{end}$ , can be calculated with (Park, 2016):

$$P[0, T_{end}]_{annuity} = A_0(1 + \mu) \cdot \frac{1 - K^{T_{end}}}{r_{nom} - \mu} \quad (9)$$

In the case of periodic maintenance or major overhauls with an interval  $n$ , the present value over the life cycle is calculated as:

$$P[0, T_{end}]_{periodic} = M_0 \cdot \frac{K^{T_{start}} - K^{T_{end}}}{1 - K^n}. \quad (10)$$

where  $M_0$  are the major maintenance costs as it would be in base year zero;  $T_{start}$  is the year in which the first activity takes place;  $T_{end}$  is the year in which this major maintenance

Table 7. Drifts ( $\mu$ ) and volatilities ( $\sigma$ ) for the GBMs of the unit prices of the case study for distinct time periods.

Scenario	Period	17 (CROW) <sup>a</sup>		19 (CROW) <sup>a</sup>		4211b (CBS) <sup>a</sup>		4213 (CBS) <sup>a</sup>		Labour (CBS) <sup>a</sup>	
		$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$	$\mu$	$\sigma$
0	1995 – 2020	0.033	0.047	0.021	0.098	0.028	0.074	0.017 <sup>b</sup>	0.025 <sup>b</sup>	0.015 <sup>b</sup>	0.016 <sup>b</sup>
1	2000 – 2020	0.038	0.047	0.033	0.081	0.027	0.077	0.017	0.025	0.015	0.016
2	2005 – 2020	0.032	0.042	0.016	0.082	0.029	0.088	0.017	0.027	0.012	0.015
3	2010 – 2020	0.034	0.029	0.019	0.063	0.011	0.054	0.014	0.015	0.008	0.016
4	2015 – 2020	0.044	0.032	0.016	0.034	0.007	0.066	0.019	0.013	0.016	0.011

<sup>a</sup>Raw data on PPIs is obtained from CROW (2018a) and CBS (2020a)

<sup>b</sup>Values non available and assumed to be the same as 2000–2020

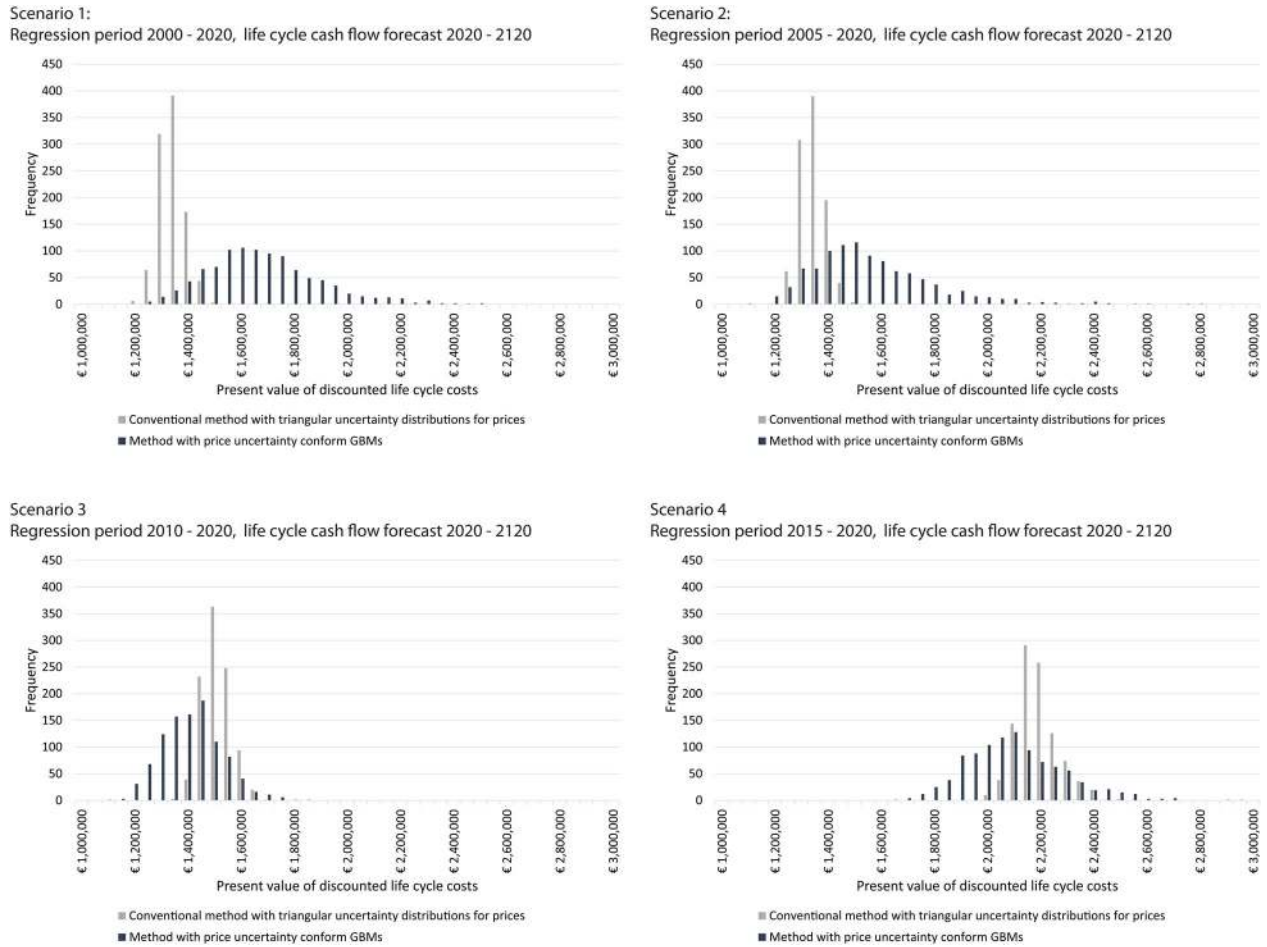


Figure 5. Sensitivity analysis on the regression period for the conventional method and GBM-based method.

activity should not occur and  $n$  is the interval between the activities. Often,  $T_{start} = n$ . If for example activities take place each 25 years over a life cycle of 100 years. Equation (10) calculates the present value of occurrence of this activity in years 25, 50 and 75. The validity of Equation (10) is verified for this example with the well-known alternative:

$$\begin{aligned}
 P[0, 100]_{periodic} = & M_0 \cdot \left( \frac{1 + \mu}{1 + r_{nom}} \right)^{25} + M_0 \cdot \left( \frac{1 + \mu}{1 + r_{nom}} \right)^{50} \\
 & + M_0 \cdot \left( \frac{1 + \mu}{1 + r_{nom}} \right)^{75}
 \end{aligned} \quad (11)$$

The deterministic calculation including price escalation but excluding uncertainty is presented in Table 8.

Equations (9) and (10) are used, as well as the data in Tables 3 and 4.

Comparison of the deterministic calculation with the probabilistic calculations shows a present value (€1.28 million) between both probabilistic values in Table 5 for a real discount rate of 5%. This is conforming to the expectation. The conventional probabilistic method does not account for price escalation. Therefore, it is expected that a deterministic calculation with price escalation yields higher expected present values of the life cycle costs. In contrast, the GBM-based probabilistic calculation including price escalation yields higher expected present values of the life cycle costs than its deterministic counterpart. These increased costs are caused by the skewness or the longer tail as can be seen in Figure 3. The deterministic calculation provides validity that the probabilistic calculations are performed properly.

## 6. Discussion

The current research proposes to include market price escalation and its uncertainty in conventional practice for probabilistic life cycle cost forecasting of infrastructures with long service lives. The method of choice for uncertainty modelling of market prices is a Geometric Brownian Motion because it follows the general notion of compounding inflation or deflation for prices, additionally accounts for uncertainty and is easily applicable in practice. The uncertainty parameters representing the drift and volatility are obtained from registered historic price indices. This approach removes some subjectivity from the current practice for probabilistic cash flow forecasting in which uncertainty distributions are based on the assumption of a triangular distribution and its upper bound, mean and lower bound are estimated based on expert-judgement. The advantage of choosing a GBM for uncertainty modelling of market prices instead of a triangular distribution is that it builds on real data and accounts for time-variability. The cone of uncertainty widens in time. Moreover, multiple GBMs for prices are easily included in the current Monte Carlo Simulation which also combines the other uncertainties among which volume and timing.

The case study demonstrates that inclusion of market price uncertainty leads to higher total discounted costs which is not surprisingly because in general market prices increase over longer time frames. Mostly PPIs exceed general inflation, therefore total discounted cost are expected to increase when long-term horizons are considered. The current practice of a triangular distribution for cash flow uncertainty does not account for price escalation. The current practice will therefore in general underestimate costs especially for public sector organisations which use low discount rates. The current study emphasises the importance of a more data-driven inclusion of price escalation and its uncertainty. This may lead to improvement of current practice for probabilistic life cycle cash flow forecasting of infrastructures. As a first step, the current research proposes a GBM-based probabilistic forecasting method. However, a GBM-based forecasting method does not provide final answers.

First, the choice for a GBM is motivated by its simplicity. It is a fundamental forecasting method for market prices obtained from the financial domain, but it does not necessarily mean that all prices behave like a GBM. Two other financial forecasting methods are for example Autoregressive Integrated Moving Average (ARIMA) and Mean-Reverting Jump-Diffusion (MRJD) processes (Anastasia et al., 2018). A GBM is part of the ARIMA family and also labelled as ARIMA(0,1,0), but it has many other family members which could be considered. Moreover, Ilbeigi et al. (2017) focus on time-series models classified as ARCH/GARCH, which account for conditional volatilities in (asphalt) prices whereas a GBM assumes a constant volatility. Khanzadi, Eshtehardian, and Mokhlespour Esfahani (2017) take it one step further and propose the application of Bayesian belief networks when both prices and explanatory variables are available. Furthermore, Kohrs, Mühlichen, Auer, and Schuhmacher (2019) build on a multi-factor price

forecasting method for volatile gas prices. The disadvantage of a GBM is that it does not take underlying cause-effect relationships for price fluctuations into account. Moreover, long-term prices are believed to convert around their mean whereas a GBM will divert, even though probabilities are small.

However, the advanced methods mentioned above need detailed data on specific prices under specific circumstances to arrive at predictions. In the absence of such data, the GBM is pragmatically the first method of choice. Public data on price developments are aggregated data from industries and sectors. If this data is the best currently available, the GBM will serve its purpose and its application can help to build up better databases over time. Instead of waiting for ideal price data and prediction methods the current research advocates a bottom-up approach and learning by doing. Moreover, even the advanced methods for price prediction will currently not be able to foresee the impact of climate change, energy transition and a circular economy on price escalations and their uncertainties.

As a direction for future research the current study proposes development of dedicated price forecasting methods for infrastructure life cycle costs under specific circumstances. Sector and location specific cost databases such as initiated by Rehan et al. (2016) for waste water pipelines in Ontario, Canada are an essential prerequisite for validation of such predictive models. Such dedicated databases can continually gather evidence about future price developments and monitor the impact of transitions, like the circular economy. However, also with the currently available price data it is interesting to compare the GBM with other time-series forecast methods as mentioned above.

A second limitation of the current research is its focus on fundamental cash flow forecasting in an early stage with limited data available. The current research is targeted at gradually improving current practice where the timing of life cycle activities is based on expert judgement and not on condition assessment. The methods for probabilistic cash flow forecasting do not optimise the timing of activities. One could argue that sophisticated methods, like multi-objective optimisation exist to optimise the timing of these activities while taking condition deterioration into account. The current case study displays 15 life cycle activities. However, (waste) water treatment installations quickly display over 600 life cycle activities. Simultaneous multi-objective optimisations of hundreds of life cycle activities would be a real challenge as such models will fall prey to state explosion.

Nevertheless, Markov Decision Process (MDP) is a method capable of optimising life cycle activities of infrastructure under uncertainty (Boucherie & Dijk, 2017; Frangopol, Dong, & Sabatino, 2017; Van den Boomen, 2020). MDP models found in infrastructure life cycle management generally incorporate transition probabilities for transferral from one condition state to another. Therefore, MDPs account for uncertainty on condition deterioration and as such the timing of i.e. maintenance works or partial replacements. Research on multi-objective optimisation models for infrastructure life cycle activities is for example

**Table 8.** Deterministic present value calculation of the life cycle cash flows with price escalation for the case study, long-term real discount rate of 5% and general inflation rate of 1.87% (costs in million Euros).

Maintenance activities	Freq.(yr)	Volume	Unit price	Drift	Present value over life cycle
Main load bearing construction – concrete deck repair	25	9,200 m2	€ 5	0.017	17,558
Replacement bearings	50	16 pc.	€ 35,000	0.021	55,249
Piles – restore concrete damages	25	410 m2	€ 18	0.017	2,817
Pavement – replacement supporting construction	20	9,200 m2	€ 35	0.028	254,570
Pavement – replacement deck	20	9,200 m2	€ 15	0.028	109,101
Replacement expansion joints	25	30 m	€ 1,250	0.033	24,866
Kerb side – restore concrete damages	10	1,600 m2	€ 18	0.017	43,044
Safety barrier – maintenance	20	1,450 m	€ 18	0.015	13,863
Safety barrier– replacement	20	1,450 m	€ 115	0.021	106,622
Railing – replacement	40	730 m	€ 300	0.021	38,193
Railing – conservation	10	730 m	€ 75	0.015	78,701
Other yearly maintenance	1	1 pc.	€ 25,000	0.015	461,883
Traffic measures	1	1 pc.	€ 4,000	0.015	73,901
				<b>Total</b>	<b>1,280,368</b>

provided by Almeida, Teixeira, and Delgado (2015); Faddoul, Raphael, and Chateaneuf (2011), D. Frangopol (2011), Oliveira, Santos, Denysiuk, Moreira, and Matos (2017) and Lin, Yuan, and Tovilla (2019). The challenge of course, is again accurate condition data to assess transition probability matrices (Adey, Hackl, & Lethanh, 2017; Lethanh, Hackl, & Adey Bryan, 2017).

The current research observes that these life cycle activities optimisation models do not take price escalations and their uncertainty into account. The current research therefore proposes to extend multi-objective optimisation models with price increases and uncertainty using financial price forecasting methods. At the same time, the current research identifies room for improvement of the GBM-based approach for probabilistic cash flow forecasting. This needs improvement of the quality of data and development of more accurate financial prediction models. Another route is taking ageing or condition deterioration into account. From a professional point of view, there is also a need for more objective predictions on increasing life cycle cash flows as a consequence of ageing of infrastructure.

## 7. Conclusions

Probabilistic cash flow forecasting supports budgeting and financing of infrastructure and has grown in interest over the past decennium. The conventional approach in the Netherlands builds on three uncertainty variables: timing of activities, volumes and costs. Generally, a triangular distribution is applied for each of the variables with user defined upper and lower bounds. In the absence of data, values of i.e.  $\pm 15\%$  from the expected values are used. The expected values remain constant over time. Sampling from the distributions and discounting the life cycle costs of the various scenarios, result in a frequency distribution of their present values. Based on such graph a decision maker can conclude on the probabilities that life cycle costs will remain between certain confidence bounds. The strength of this approach is its applicability in practice while having limited data. However, there are several limitations. First, results are subjective when the estimated probability distributions are based on expert judgement instead of data analysis. Second, time-variance is neglected in this approach which weakens

havoc on price predictions which are subject to (de)inflation.

As first step forwards to better price prediction methods, the current research proposes to model price (de-)escalations and their uncertainties with a fundamental financial forecasting method, a Geometric Brownian Motion (GBM). A GBM describes a random walk around a time-variant expected value. Numerous random walks represent a cone of uncertainty which widens further in time around increasing or decreasing expected price values. This reflects a general notion of price (de-)escalation with more uncertainty in the far future. A GBM also reflects a general notion that volatile prices experience more uncertainty than less volatile prices.

The parameters describing GBMs are derived from publicly available Producer Price Indices (PPIs) which introduces less subjectivity in the conventional approach for probabilistic cash flow forecasting. Moreover, as PPIs are often known to exceed general inflation rates, realistic costs which are currently forgotten, are included in cash flow forecasts. A case study for a concrete bridge demonstrates that ignoring price escalation may lead to an expected underestimation of discounted life cycle costs of 13%. Especially for infrastructures with long life cycles and public sector organisations using low discount rates, incorporation of price increases should not be overlooked. The method for its uncertainty modelling is easily included in the current approach for probabilistic cash flow forecasting.

Nevertheless, there are limitations. Current publicly available PPI data is based on sector averages, aggregated and targeted at construction and engineering. Not all PPI data is fit for specific infrastructure life cycle purposes such as operations and maintenance. Besides, discrepancies in price developments are observed between different databases. The current research therefore advocates to establish uniform and dedicated databases for price developments for construction, engineering, operations and maintenance of infrastructures. Second, a GBM is a fundamental price forecasting method but it does not necessarily describe the behaviour of specific prices.

As outlook for further research it is recommended to investigate price (de-)escalations for specific sectors or infrastructures and to improve or expand current price

registrations for life cycle activities. Further it is recommended to investigate how to improve price predictions for infrastructure life cycle activities, not just by better regression of historic time series but also by understanding the mechanisms which cause price fluctuations. Finally, various research demonstrates the importance of addressing price (de-)escalation in infrastructure life cycle management. Price (de-)escalation and its uncertainty should therefore be integrated in life cycle costing models based on condition deterioration and vice versa.

## Disclosure statement

No potential conflict of interest was reported by the authors.

## ORCID

D. F. J (Daan) Schraven  <http://orcid.org/0000-0003-0647-1172>

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## Appendix

**Table A1.** Example for obtaining drift and volatility from past prices.

Price indices for asphalt closed construction			
Year	PPI = P	ln(P)	ln(P <sub>t</sub> ) - ln(P <sub>t-1</sub> )
1998	90.4	4.50	
1999	91.3	4.51	0.010
2000	97.8	4.58	0.069
2001	102.9	4.63	0.051
2002	105.7	4.66	0.027
2003	108.2	4.68	0.023
2004	108.7	4.69	0.005
2005	108.2	4.68	-0.005
2006	123.4	4.82	0.131
2007	127.7	4.85	0.034
2008	154.6	5.04	0.191
2009	138.7	4.93	-0.109
2010	149.7	5.01	0.076
2011	159.9	5.07	0.066
2012	169.5	5.13	0.058
2013	175	5.16	0.032
2014	173.4	5.16	-0.009
2015	161.7	5.09	-0.070
2016	152.5	5.03	-0.059
2017	166.8	5.12	0.090
2018	170.4	5.14	0.021
2019	173.7	5.16	0.019
<b>Drift</b>	<b>Mean</b>	$\mu$	<b>0.031</b>
<b>Volatility</b>	<b>Stdev</b>	$\sigma$	<b>0.066</b>