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Probabilistic predictions of fatigue crack behaviour

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Abstract

It is shown that reasonable predictions of fatigue crack growth can be obtained using damage mechanics concepts. A model that considers the growth of a main crack as a sequence of failures of sub-elements along the crack path is used to perform computer simulations of crack propagation in centre notched specimens. Probabilistic criteria are introduced from the statistical analysis of S/N curves for fatigue life. Scattering of the data is reproduced in sub-elements ahead of the crack tip. The result is a set of simulations for fatigue crack propagation through the same track. In order to check the model, fatigue crack growth tests were carried out in grade II titanium sheet samples, using a servohidraulic MTS machine. The results of experimental curves are compared to those of simulated curves. The good agreement observed between the sets of curves shows the possibilities of damage mechanics to give reliable lifetime predictions.

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1 Introduction

Fatigue is the degradation process of materials caused by repeated loading, which results in a finite lifetime of mechanical components. One of the primary mechanisms of the phenomenon is the nucleation and further growth of small cracks [1]. The prediction of the fatigue life of components is a major problem in mechanical and materials engineering. The study of fatigue crack propagation (FCP) is aimed at structural residual life estimations. The analysis of this process usually includes the concepts of fracture mechanics in form of semi-empirical models like the well-known "Paris law" of FCP [2].

Because of a weak correlation between predictions and actual lifetime of components, some other considerations have been introduced into methods of analysis. For example, it's known that most of metallic materials have a polycrystalline microstructure of random orientation, described by various parameters, which may seriously affect crack growth. As a result, the deterministic theories of FCP could be accepted only as an approximation of the phenomenon. Thus, scattering inherent to fatigue crack growth data comes into consideration, and the process is described in terms of statistical distributions and their parameters [3-5].

Some significant restrictions to this methodology are related to the prediction of the material's behaviour in a wide range of actual loading conditions. The local approach of fracture, in the framework of damage mechanics, may point out a way to overcome such limitations. In this case, the propagation of the main crack is considered as a sequence of failures of sub-elements along the crack path [6]. Thus, proper estimations of kinetiks of crack growth could be performed using only fatigue lifetime data obtained for smooth samples (specimens without initial cracks) and the basic mechanical properties of the material [7,8].

The aim of this work is to contribute to crack propagation modelling by the introduction of statistical lifetime distribution into damage accumulation equation. The area near the crack tip is considered as a set of sub-elements, which individual cyclic loading conditions are calculated in function of the stress field related to the external loading. The model is used to perform computer simulations of FCP in centre notched specimens. At each simulation, a random number generator attributes to each sub-element a corrective factor of fatigue lifetime related to parameters of statistical distribution of S/N curve. The procedure is repeated several times, giving a set of crack growth curves through the same track. Besides, each set of random values attributed to sub-elements leads to a new crack growth history, under the same loading regime.

For demonstration purpose, an application for available experimental data for grade II titanium is developed. The results of simulations are compared to a set of experimental curves of fatigue crack growth.

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2 Modelling Approach

The modelling of fatigue crack growth by methods of continuum damage mechanics is based on three main keystones: equation of lifetime reduction as a function of the stress-strain state; analysis of stress and strain in the cracked element and the crack advance criterion. For simplicity, well-known theories were chosen in order to illustrate the modelling scheme: S/N curves for fatigue lifetime; mode I elastic stress field equations (being cut off above an appropriate value of the stress to take in account crack tip plasticity); and linear damage accumulation (known as Palmgren-Miner [9,10] rule) for local fracture criterion.

In this case, crack propagation under stationary uniaxial cyclic loading can be simulated from only one S/N curve obtained for the same minimum/maximum load ratio. An approximation of experimental data by S/N equation determines the lifetime as a function of maximum stress of loading cycle, considering an individual error for each experimental point. Now, the cracked element is discretized as a set of sub-elements corresponding to steps of crack advance. Statistical distribution of the relative errors will be reproduced in sub elements ahead of the crack tip, attributing to each one a tendency represented by a relative deviation value. Thus, a crack propagation curve is generated by considering the sequencial failures of the sub-elements due to a cyclic loading. The procedure is repeated several times using Monte Carlo method, giving a set of crack growth curves through the same track. In this way, each set of random deviation values attributed to sub-elements lead to a new history of FCP process under the same loading and geometrical conditions.

2.1 Processing of Basic Data

The basic experimental data are given in a table of "N_i *versus* S_i" points corresponding to m fatigue life tests, where S_i and N_i are the maximal stress of loading cycle and the number of cycles to fracture of sample number i, respectively ($i = 1, ..., m$). These data are described by eqn (1), where the constants A and B are calculated by the minimum squares method.

$$\log N = A + B(S) \tag{1}$$

For each experimental point the relative error x_i of life prediction is given by eqn (2):

$$x_i = \frac{\log N_i}{A + B(S_i)} - 1 \tag{2}$$

The variance of x_i is assumed to be independent on the stress level and given by eqn (3):

$$\sigma^2 = \frac{\sum_{i=1}^{m}\left(\frac{\log N_i}{A + B(S_i)} - 1\right)^2}{m} \tag{3}$$

The parameter x_i is then approximated by the standard normal distribution and discretized into q bands. The mean point of each band is given by X_k, where, $k=1, \ldots, q$. The probability of each band is given by p_k and thus we have the accumulated probability given by:

$$Pa_k = \sum_{g=1}^{k} p_g \tag{4}$$

The parameter Pa_k, which value is 1 for $k=q$, is a main indicator of local properties to be used in probabilistic simulations.

2.2 Discretization of Cracked Element

For discrete simulation of FCP from an initial crack size a_0, a linear crack path is considered and discretized in a set of sub-elements of length da, small in comparison with element dimensions. The stress-strain state of each sub-element is assumed uniform, and crack advance through sub-element is instantaneous if local fracture criterion is achieved. The random number generator allows to attribute to each sub-element a value of parameter Pa_k which identifies the corresponding band of deviations distribution and thus gives the relative error value x_i.

The maximum stress of cycle in sub-element number i after failure of j sub-elements is given in terms of Stress Intensity Factor K_I of fracture mechanics, as shown in eqn (5) and illustrated by Figure 1.

$$S_{i,j} = \frac{K_I(a_0 + jda)}{\sqrt{2\pi(i-j)da}} \tag{5}$$

2.3 Algorithm of FCP Simulation

The number of cycles to local fracture of first sub-element is estimated from S/N curve with correction by the individual error parameter x_1, resulting in:

$$n_1 = \log N_1 = \left(A + BS_{1,0}\right)\left(1 + x_1\right) \tag{6}$$

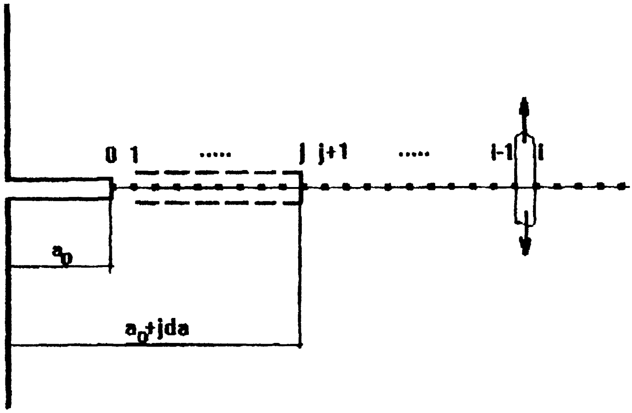

Figure 1. Crack Path, Crack Size and a Sub-Element.

The damage accumulated in the other sub-elements during this period is estimated as:

$$d_{i,1} = \frac{n_1}{\left(A + BS_{i,0}\right)\left(1 + x_i\right)}; i = 2,3,\ldots \tag{7}$$

The number of cycles to each crack increment, Δn_j, corresponds to accumulation of critical damage in next sub-element, beginning from damage already accumulated under previous increments. According to Palmgren-Miner rule, we have:

$$d_{j,j} = 1 - \sum_{l=1}^{j-1} d_{j,l} \tag{8}$$

which leads to:

$$\Delta n_j = d_{j,j}\left(A + BS_{j,j-1}\right)\left(1 + x_j\right) \tag{9}$$

During this period, the remaining sub-elements will accumulate the damage:

$$d_{i,j} = \frac{\Delta n_j}{\left(A + BS_{i,j-1}\right)\left(1 + x_i\right)} \tag{10}$$

and the failure of the next sub-element is to be predict, repeating until total failure of cracked element.

3 Experimental

In order to check the model, commercially pure titanium (grade II) sheet samples, with 1.5mmm in thickness, were tested in this work. The mechanical properties of the material are the following: Tensile strength = 487 MPa, yield strength = 332 MPa and elongation to the fracture = 25%. Basic experimental data of fatigue life, i.e., a S/N curve was obtained for minimum/maximum load ratio = 0.5 using 30 smooth specimens.

Fatigue crack growth tests were performed in a MTS machine model 810.23M. A number of 15 centre notched specimens were tested, with following dimensions: width = 50mm, notch length = 12mm. Stationary cyclic loading was applied, the maximum load of cycle was 8.0kN and the minimum was 4.0kN. Initial crack measurements were made from total crack length = 15mm. Crack size was measured using a travelling microscope. Measures were taken to the left and to the rigth from the center, and a mean value was calculated.

4 Results and Discussion

Figure 2 shows the experimental poinis and S/N curve.

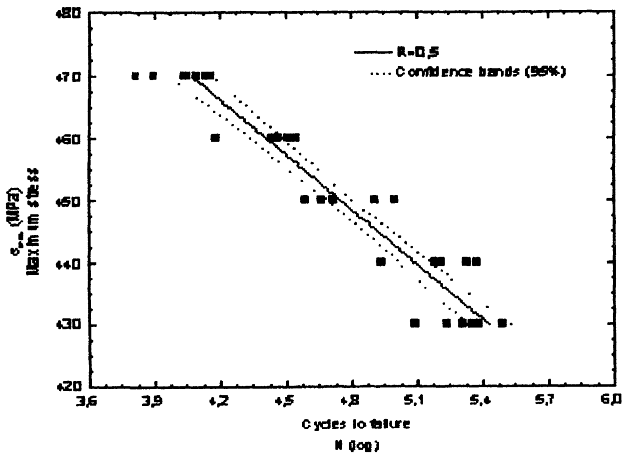

Figure 2. Fatigue Curve of CP-Ti.

These results present a typical scattering characteristic of fatigue life testing. Constants of eqn (1) were calculated and the results are: $A = 19.99708$ and $B = -0.03388$. Equations (1) and (4) were used to calculate the parameters to be used in computer simulations. The numerical code was developed in Pascal language and uses equations (5) to (10) for simulation of crack propagation. The results of 15 simulations, performed for the same conditions of the fatigue crack growth experiments, are given in plots "crack length *versus* number of cycles" as shown in Figure 3. In this case, the domain was divided into 100 sub-elements.

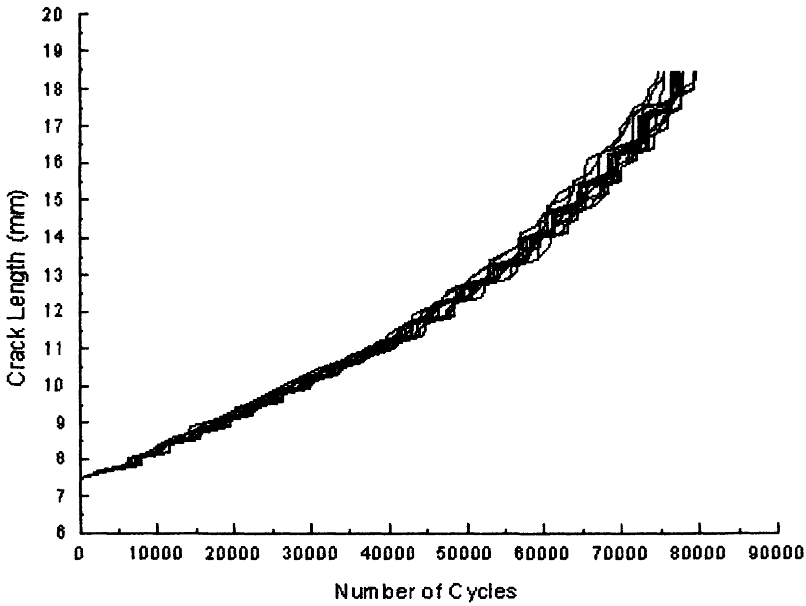

Figure 3. Fatigue Crack Growth Simulations.

For comparison, the results of fatigue crack growth tests are given in Figure 4. Analysis of these data is done in order to check the performance of the model. A first look shows that the simulated and experimental curves have similar shapes, although simulation predicts an initial crack growth faster than the tests. As the crack approximates critical crack length of 18.5mm, the increase in crack growth rate of experimets is higher than that of the model.

In Figure 5 it is shown a good approximation of experimental results, in terms of number of cycles for critical crack length, by the normal distribution. Therefore, the 95% confidence interval for the expected value is calculated from the mean of the experiments and its standard deviation, using *t-Distribution* table [11].

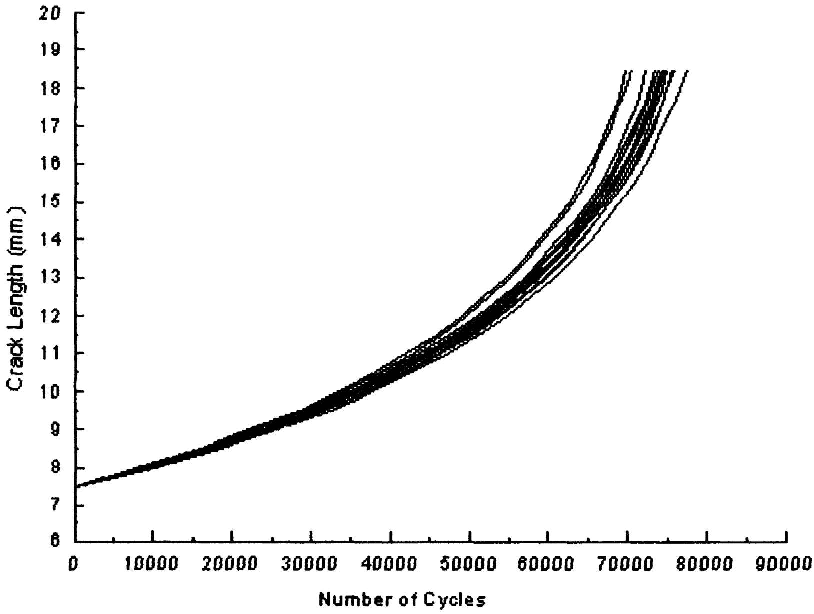

Figure 4. FCP Experimental Data.

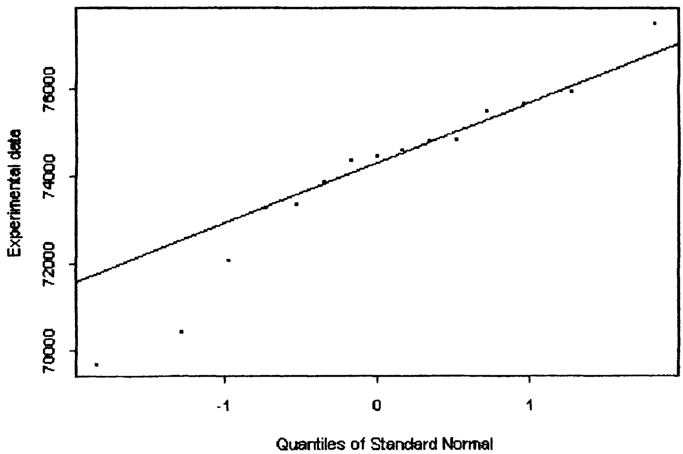

Figure 5. Approaching of Experimental Data by Normal Distribution.

Calling μ the expected value of the number of cycles to critical crack length, we have, from experimental data, $72{,}892 \leq \mu \leq 75{,}160$. The mean value calculated from simulations is $N = 77{,}093$. Obviously these quantities are statistically different. But if we take the central point of the confidence interval (*74,026*), we note the difference of only 4% between this value and the mean value of simulations. This fact shows that with some further improvements in the model it is possible to obtain very precise predictions of crack growth rate without previous knowlege of FCP data.

5 Conclusion

The modelling approach developed in this work allowed to obtain probabilistic predictions of crack propagation, with basis on the scattering of the fatigue life data for smooth elements. Experimental results on crack growth showed good agreement with those obtained from computer simulations. This shows the possibilities of damage mechanics to give reliable lifetime predictions in various condictions. Moreover, the model can be generallized for any stress and strain field, damage accumulation model as well as take in account several models of stress-strain behaviour of material, in order to give more precise results.

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References

1. Fine, M.E.: Fatigue Resistance of Metals. *Metallurgical Transactions A*, 11A, pp. 365-379, 1980.

2. Paris, P. & Erdogan, F.: A Critical Analysis of Crack Propagation Laws. *J. Basic Eng., Trans. ASME 85*, pp. 528-534, 1963.

3. Virkler, D.A., Hillberry, B.M., Goel, P.K.: The Statistical Nature of Fatigue Crack Propagation. *Transactions of the ASME*, 101, pp. 148-153, 1979.

4. Rocha, M.M. e Schuëller, G.I.: Probabilistic Requirements for Validating High-Cycle Fatigue Crack Growth Models. *Proc. of 8th Brazilian Symposium on Tubing and Pressure Vessels and 2nd Latin America Symposium on Tubing and Pressure Vessels.* Gramado, RS (Brazil), pp. 358-367, 1994 *(in English)*.

5. Shimokawa, T. & Kakuta, Y.: Application of Monte Carlo Simulation for Fractographic Analysis of Fatigue Crack Propagation. *Int. J. Fatigue*, 18 (5), pp. 321-327, 1996.

6. Lemaitre, J.: Local Approach of Fracture. *Engineering Fracture Mechanics*, 25, pp. 532-537, 1986.

7. Pastoukhov, V.A.: Mathematical and Methodological Aspects on Crack Growth Predictions from Fundamentals of Material Lifetime. *Proc. of 2nd Conference on Fracture Mechanics*, ABM, São José dos Campos, SP (Brazil), pp. 119-138, 1996 *(in Portuguese)*.

8. Castro, J.T.P. e Kenedi, P.P.: Prediction of Fatigue Crack Growth Rates from Manson-Coffin Concepts. *Brazilian Journal of Mechanical Sciences,* 17 (3), pp. 292-303, 1995 *(in Portuguese)*.

9. Leve, H.L.: Cumulative Damage Theories. In: Madayag, A.F., ed. *Metal Fatigue: Theory and Design*. John Wiley, New York, 1969.

10. Miner, M.A.: Cumulative Damage in Fatigue. *Journal of Applied Mechanics*, pp. 159-164, 1945.

11. Box, G.E.P., Hunter, W.G., Hunter, J.S.: *Statistics for Experiments: An Introduction to Design, Data Analysis and Model Building*, John Willey, New York, 1978.