Probabilistically Shaped 8-PAM Suitable for Data Centers Communication

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ABSTRACT
We study non-uniform 8-PAM signaling schemes, based on Maxwell-Boltzmann and exponential source distributions, as candidates suitable for data centers' communications. We evaluate mutual information of these schemes against corresponding uniform 8-PAM scheme, for different parameters of source distributions. The properly chosen variable parameters for different SNR-regions allows us to closely approach Shannon limit, and to significantly outperform the corresponding uniform counterpart. We demonstrate that LPDC-coded 8-PAM with exponential distribution outperforms the LDPC-coded 8-PAM with uniform distribution of the same achievable information rate by 1.8 dB, and this improvement is comparable to that obtained from mutual information calculations.
Keywords: 8-PAM, probabilistic shaping, distribution matching, exponential distribution, LDPC coding.

1. INTRODUCTION

Nowadays the requirements of transmission capacity are becoming higher and higher in data center interconnects, internet services, and many other communications. Hard decision forward error correction (FEC) code is now widely applied for reliable transmission inside the data centers, and by applying the soft-decision (SD) scheme, we can significantly improve the system performance.

Pulse-amplitudes modulation (PAM) is commonly used in short reach optical communication applications, digital TV, as well as various Ethernet applications [11]. Unfortunately, the gap to the Shannon capacity for uniformly distributed 8-PAM is huge.

In recent years, the probabilistic shaping (PS) distribution is a more attractive way of modulation compared to uniform signaling, which allows us to reduce the gap to the Shannon capacity [5-7], because with the PS distribution larger amplitudes are transmitted less often, thus enabling a transmission with a lower signal-to-noise ratio (SNR) at the same FEC overhead, when compared to the uniform distribution. Experiments [5] have confirmed the PS distribution benefits; however, different schemes with PS distribution do not outperform the corresponding uniform schemes for all SNR values.

In this paper, we study non-uniform 8-PAM for two different schemes of PS distributions, namely, the Maxwell-Boltzmann and exponential distributions. We show that both PS schemes outperform uniform 8-PAM for low and medium SNR values, but perform worse in high SNR region. To overcome this problem, we propose to use the variable source distribution parameter scheme, optimized for different SNR regions. On such a way we are able to significantly outperform corresponding uniform scheme, in terms of mutual information, for all SNR values. This scheme represents an excellent candidate for future data center applications.

2. NONUNIFORM SIGNALING SCHEMES

Figure 1 shows the system model that is similar to [2], the input data into the distribution matcher (DM) is uniformly distributed binary data stream, which is then transformed into 8 amplitudes, which are $A^n=A_1...A_n$ with a certain distribution P we want. The DM is invertible so that we can recover the data bits originally sent into the DM from the symbols with different amplitudes A^n.

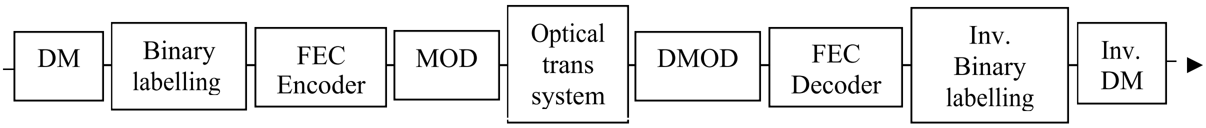

Figure 1. System model structure.

As shown in Fig. 2, we use the binary reflected Gray code (BRGC) to label the 8-PAM constellations. For the uniform distributed modulation, these 8 constellations have a probability distribution equal to 0.125 for every constellation point. For the distribution with probabilistic shaping (PS), the probabilities for different constellation points will be different, as shown in Fig. 3 [12]. Now we study two relevant source distributions, namely Maxwell-Boltzmann and exponential distributions.

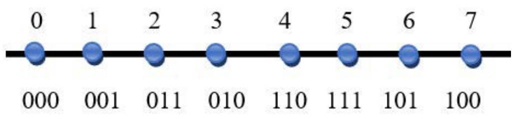

Figure 2. The BRGC for 8-PAM.

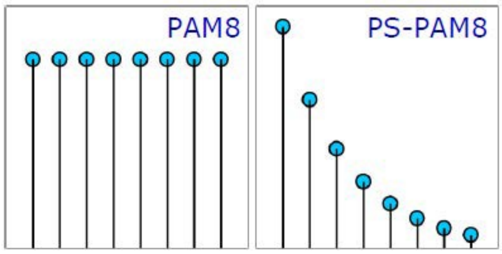

Figure 3. Probability distribution for uniform and PS distributed 8-PAM constellations.

2.1 Maxwell-Boltzmann distribution and exponential distribution

Compared to the uniform distribution, the PS distribution has different probabilities for different constellation points. For Maxwell-Boltzmann (MB) distribution introduced in [2-4], each constellation point a_i will have a transmission probability as follows:

$$P(a_i) = \exp(-\lambda \|a_i\|^2)/Z(\lambda), \qquad \lambda \geq 0 \tag{1}$$

Exponential distribution is the PS distribution, suitable for intensity modulation with direct detection (IM/DD) schemes, in which different constellation points a_i are transmitted with probabilities:

$$P(a_i) = \exp(-\lambda \|a_i\|)/Z(\lambda), \qquad \lambda \geq 0 \tag{2}$$

where the function $Z(\lambda)$ is used to normalize the probability to make sure that probabilities of occurrence of symbols sum-up to one. So the normalization function $Z(\lambda)$ is defined as:

$$Z(\lambda) = \begin{cases} \sum_i \exp(-\lambda \|a_i\|^2), & \text{for MB distribution} \\ \sum_i \exp(-\lambda \|a_i\|), & \text{for exponential distribution} \end{cases}. \tag{3}$$

From this definition, we can see that the different MB and exponential distributions can be obtained for different values of parameter λ; and when λ tends to zero, it reduces down to the uniform distribution. On the other hand, for a large enough value of λ, the probability of outer points will become sufficiently small and for a finite length of code sequences, these outer points could be never selected.

2.2 The Channel Model

We transmitted the signal through an AWGN channel with different signal-to-noise ratios, SNRs, because a memoryless AWGN channel is a fair enough approximation of the optical transmission system with both coherent optical detection, and direct detection channel dominated by thermal noise of a transimpedance amplifier that follows the photodetector. The input-output channel relation can be expressed as:

$$Y_i = X_i + W_i, \qquad i = 1, 2, \cdots, n \tag{4}$$

wherein the X_i's are transmitted nonnegative 8-PAM symbols, W_i is the Gaussian noise source with variance σ^2, and Y_i is the received signal. So, we have that the conditional probabilities for different constellation points are given by

$$q(y_i | x_i) = p_z(y_i - x_i) \tag{5}$$

To evaluate the different source distribution performances, we calculate the mutual information (MI) for different symbol SNRs, wherein the MI is defined as:

$$I(X;Y) = \sum_y \sum_x p(x,y) \log\left(p(x,y)/\left(p(x)p(y)\right)\right) \tag{6}$$

where $p(x)$ and $p(y)$ are probabilities of variables $X = x$ and $Y = y$, and $p(x, y)$ is the joint probability of X and Y.

We move our attention now to the proposed variable PS shaping scheme, suitable for data center applications, and other systems with optical direct detection, such as the passive optical networks (PONs).

3. PROPOSED NONUNIFORM SIGNALING PAM SCHEME SUITABLE FOR DATA CENTER APPLICATIONS

3.1 Optimized exponential distribution

We optimized the MI for different values of λ and the MI results for optimized exponential distribution are summarized in Fig. 4. The dashed blue curve represents the MI for uniformly distributed 8-PAM. As expected, it tends to 3 bits/symbol for high SNR values, corresponding to the entropy of uniform source distribution. The solid curves correspond to the MI of optimized PS 8-PAM for different values of λ, which are {0.09, 0.29, 0.57, 1}. From Fig. 4 we can see that the shaping gain of exponential distribution with a smaller λ will be lower, and that the mutual information improvement is also smaller than those with a larger value of the parameter λ.

By a close inspection into every exponential distribution we can see that for the SNR lower than 7 dB, the exponential distribution with $\lambda = 1$ has the highest mutual information. On the other hand, for SNRs between 7 dB and 14 dB, the exponential distribution with $\lambda = 0.57$ has the best performance. Further, for the SNR between 14 dB and 21.2 dB, we find that exponential distribution with $\lambda = 0.29$ is the best. For the SNR range between 21.1 dB and 25.9 dB, we find that the exponential distribution with λ of 0.09 is the most suitable. Finally, for the largest SNRs, we will use the uniform distribution, because the mutual information tends to the entropy of the signal.

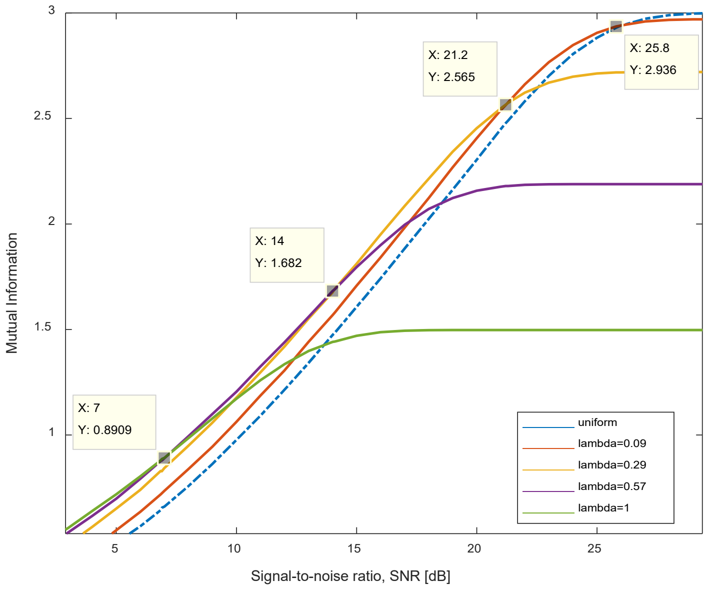

Figure 4. Mutual information vs. SNR of optimized exponential distribution of 8-PAM signal compared with uniform distribution.

From Fig. 4 we can see that the exponential distribution based 8-PAM provides a shaping gain up to 2.2dB; or in other words, the exponential distribution provides an increase in MI of 0.27 bits/symbol. As the SNR increases, the shaping gain and MI improvements become lower, and with λ=0.57 and λ=0.29, the shaping gain and MI improvements are only 1.55dB and 0.196 bits/symbol, and 0.69 dB and 0.078 bits/symbol, respectively. Clearly, by using the proposed variable parameter λ exponential distribution, for different SNR intervals, we can adapt to the time-varying channel conditions.

3.2 Combining the PS 8-PAM with LDPC coding and comparison between MB and exponential distribution

The quasi-cyclic (QC) LDPC code we use for the uniform distribution has a codeword length of n = 18488 and 6931 parity bits, which means there are 11557 information bits in a codeword. Therefore, the code rate is 0.625 and corresponding overhead is 60%. On the other hand, the QC-LDPC code we use for the PS distribution has a codeword length of n=16935 and 3385 parity check bits, which means there are 13550 information bits in a codeword. So, the code rate is 0.8 and the corresponding overhead is 25%. We perform three outer [a *posteriori* probability (APP)-LDPC decoder] iterations, and for each outer-iteration we use 20 inner (LDPC decoder) iterations.

To make sure the comparison between PS and uniform distribution is fair, we should use schemes with the same achievable information rate. For the uniform distribution, the entropy is 3 bits, and code rate is 0.625, so the achievable information rate (AIR) is:

$$C = Entropy - Rate \times (\#\ of\ bits/symbol) = 1.875\ bits/symbol \qquad (5)$$

On the other hand, for the same AIR, the entropy of the PS distribution is 2.475 bits, which corresponds to exponential distribution with $\lambda = 0.4222$, and MB distribution with $\lambda = 0.0785$.

Figure 5 shows the bit error rate (BER) vs. SNR per bit for both uncoded case and three outer and 20-inner iterations of the LDPC coded 8-PAM, for uniform, MB, and exponential distribution signaling schemes. Dashed curves represent the uncoded case, while the solid curves correspond to the LDPC-coded 8-PAM after 3 outer iterations, with blue curves corresponding to MB distribution, red curves to exponential distribution, and green curves to uniform distribution, respectively. Comparing the decoded curves with the uncoded curves, we can see that LDPC-coded 8-PAM with both uniform and PS distributions provide large coding gains. On the other hand, the LDPC-coded 8-PAM with MB distribution outperforms the corresponding LDPC-coded 8-PAM with the uniform distribution for the same AIR by 1.17 dB at BER of 10^{-5}, and coded exponential distribution outperforms the MB distribution and uniform distribution for the same AIR at BER of 10^{-5} by 0.45 dB and

1.62 dB, respectively. So, the exponential distribution performs better than MB distribution. When we look back to the MI curve shown in Fig. 4, we can see the shaping gain for PS distribution at the same AIR provides the improvement comparable to that obtained by the LDPC-coded modulation.

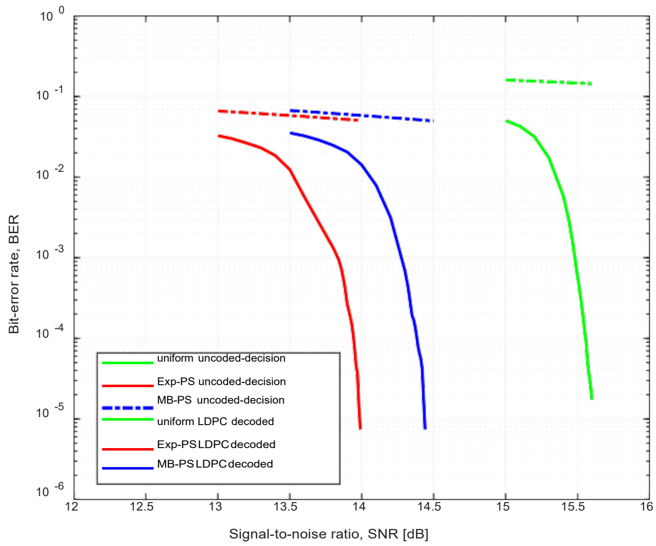

Figure 5. BER vs. SNR per bit for uniform, MB, and exponential distribution in LDPC-coded 8-PAM.

4. CONCLUSIONS

In this paper, we have studied the MI of non-uniform source distributions for 8-PAM against that of uniform 8-PAM. We have found that MI for exponential source distribution based PS for nonnegative 8-PAM outperforms the uniform 8-PAM for low and medium SNR values. We have proposed variable parameter exponential source distribution based 8-PAM transmission suitable for use in data center applications. The proposed scheme is suitable to deal with time-varying nature of optical direct detection link in data centers and for PON applications. The proposed non-uniform 8-PAM scheme has been able to closer approach the Shannon limit than any other scheme reported so far for data center and PON applications. The proposed non-uniform 8-PAM scheme has been also studied for joint use with LDPC coding. It has been shown that LDPC-coded 8-PAM with exponential distribution outperforms the LDPC-coded 8-PAM with MB distribution of the same AIR by 0.45 dB, as well as the uniform distribution of the same AIR by 1.62 dB. This improvement has been found to be consistent with the MI curves.

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