

Probability and Random Processes

*With Applications to Signal Processing
and Communications*

Edition 2

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