

Letter to the Editor

## Probability as a Physical Motive

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**Abstract:** Recent theoretical progress in nonequilibrium thermodynamics, linking the physical principle of Maximum Entropy Production (“MEP”) to the information-theoretical “MaxEnt” principle of scientific inference, together with conjectures from theoretical physics that there may be no fundamental causal laws but only probabilities for physical processes, and from evolutionary theory that biological systems expand “the adjacent possible” as rapidly as possible, all lend credence to the proposition that probability should be recognized as a fundamental physical motive. It is further proposed that spatial order and temporal order are two aspects of the same thing, and that this is the essence of the second law of thermodynamics.

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### Introduction

“Because that’s where the money is.” According to legend from America’s 1930s era of economic depression and gangsters, that was the pithy response of the notorious outlaw and jail-breaker John Dillinger to the question, “Why do you rob banks?” It’s a catchy wise crack (albeit referring to reckless and foolhardy behavior), as it boldly suggests an amusingly simple alternative view of action, or of motivation—a view perhaps not anticipated by the question as posed. It is a view which implicitly presumes possibility rather than necessity, in a figure-ground perceptual switch, from recognition of that which is prescribed by familiar rules, to recognition of that which has not been ruled out. It is a switch from explanation as prior cause (“efficient cause”) to explanation as attraction (“final cause” [1]).

The significance of what I have called a figure-ground perceptual switch must be appreciated, since the point of this letter is to suggest a sort of inverted view of “motive”, that is, what we accept as an explanation for the course of events.

It seems to be a natural habit of human cognition, when presented with a perceptual field of any sort, to distinguish some figure from some complementary ground. For example, we have all seen certain black-and-white images with which we may experience a perceptual switch as we swap what we recognize as figure with what we recognize as ground. Similarly, certain composers have created such effects musically, so that the listener may experience a perceptual switch as to which voices carry the figure melody and which stand as the background harmony.

Such considerations may seem to be of little relevance to objective science, but I suggest otherwise. For example, in mathematics, proofs by *reductio ad absurdum* are commonly employed; in this case the problem is more tractable when approached with a sort of inverted view: one supposes the logical complement of what one wishes to prove, and derives a contradiction, which implies that the complement of the (complementary) supposition must be the case. In electronics, it is customary to think of circuits in terms of applied voltages as the “background” and resulting currents as the figures of interest. Perhaps this is so because voltage sources, such as batteries, are more commonly encountered than current sources. But both analysis and design can sometimes be facilitated by a perceptual switch, thinking of currents as “ground” and resulting voltages as figures of interest. Then, for example, one speaks of a resistor “generating” a voltage, by virtue of the current flowing through it. In theoretical physics, figure-ground inversions of point of view have been fruitfully assumed with respect to the priority of particle or field, where initially a particle (by its charge or mass) was thought to produce a field, but later the idea arose that “particles” were actually effects of fields.

In general, science is like puzzle-building, proceeding as far as possible from one direction, then continuing from another direction. Impasses provide the signal, not that nature is essentially paradoxical, but that the explanatory power of the current direction of work, of the model used to explain nature, has been exhausted. I suggest that we have reached such a point with the very model of causal motives as explanations for physical processes, and that the second law of thermodynamics was the first signal, the statistical foundations of quantum mechanics the second signal. Heretofore, causality has been the figure of interest seen to be operating against a background of uncertainty; now perhaps we should take the inverted view and accept uncertainty, or extent of possibilities, as a physical motive, in the sense that it may account for physical processes with logical economy and without reference to a particular observer.

So the folk story of John Dillinger is taken as a jumping-off point for an argument in favor of an alternative model of physical motive in science, the logical antecedent of which is the second law of thermodynamics and Carnot’s seminal observation that “wherever there exists a difference of temperature, motive force can be produced” [2]). Carnot’s observation is echoed by George Spencer-Brown’s statement, on the foundations of logic: “There can be no distinction without motive, and there can be no motive unless contents are seen to differ in value” [3]. The role of distinction, or difference, in both physical and mental processes, is so implicit as to be ignored, yet it should be recognized as the most fundamental concept in our models of dynamics. Carnot could as well have said that where a difference of height exists (*e.g.* hydraulic head), motive force can be produced, or that where a

difference of wind speed exists, motive force can be produced. A generalization of Carnot's statement of the second law of thermodynamics is that wherever a difference exists, a motive force also exists, in the form of the probability that the difference will evolve toward equilibrium.

I equate differences, in the foregoing broad sense, with improbability, because there are fewer "easy" (low-energy) ways that states of difference can be realized than states of indifference can be realized. Hence "probability", as used in speaking of "probability as a physical motive", is supposed to be a measure of opportunities for change of state. In this sense probability refers more to phase-space volume than to either ratios of outcomes derived from sampling or to any observer's degree of certainty as to present or future state.

The proposal is not that this probability should somehow be sensible to a system, so as to operate as a familiar causal motive (as one might understand "pulling" in terms of the front face of a hook *pushing* on the back face of a hitch); rather the proposal is that this probability might be recognized as a motive *per se*, insofar as it describes the correlations between observed states. Given the habit of insistence on proximate, causal explanation, to assume this view may require what I have called a figure-ground perceptual switch. But such a view is no more strange than the natural expectation that ripples on the surface of a pond should spread outward.

Understanding that any difference or gradient inheres the potential to produce motive force and *vice versa* facilitates the understanding of such phenomena as, for example, the flight of albatrosses, which utilize the potential available in wind shear to stay aloft, or the formation of placer deposits, where stream flow effectively differentiates alluvium according to density. Thus distinction (difference, or gradient) and motive are reciprocally dependent, and this motive, arising from the possibility of change (or "flow") toward equilibrium, is indeed the most fundamental physical motive.

What is proposed here is not a scientific theory or hypothesis in the Popperian sense, but a geometric conceptual model of notions of spatial order and temporal order (*i.e.* evolution), inspired by such useful principles as that of Maximum Entropy Production (see, for example, [4]) and "MaxEnt" ([5, 6]), and by the idea that probability figures more fundamentally than causality in theoretical physics [7]. The evolution of life and biological diversification seem particularly persuasive that "nature loves opportunities", leading to Kauffman's conjecture that living systems "expand the dimensionality of the adjacent possible as rapidly as possible" [8]. By this it is meant that evolution proceeds in such a way as to maximize the number of directions in which life might continue to evolve—which can be interpreted as another sort of maximum-entropy principle when entropy is understood as a measure of "spread". The aim of this letter is to synthesize disparate conceptual models of dynamics and to contribute to the conceptual foundations of scientific inquiry and theory in this area.

### **Explanation in science**

One might suspect that the Dillinger character resonated with popular imagination because he represented a sort of subversive freedom at a time when most people's experience was dominated by severe economic constraints and scarcity of opportunities. But the idea suggests itself, after the "because that's where the money is" remark, that a similar figure-ground perceptual switch might often be appropriate in science, as the ultimate aim is to explain what we observe as simply and as generally as possible, not as properly as possible. It has often been observed that traditions of scientific

viewpoint define the questions that are asked as well as the form of hypotheses that are entertained. The suggestion here is that it may sometimes be possible to simplify the understanding of nature by revising one's viewpoint, perhaps even to see exceptions as rules. Such a figure-ground perceptual switch is justifiable if it leads to logical economy—fewer primitive concepts and relations accounting for more observations—regardless whether it conforms to the traditional, familiar view.

It may be just as useful to presume that nature moves toward opportunities, as it is to presume that nature is strictly compelled by causal laws. As a naturalist, one is more impressed by nature's opportunism than by its competitiveness. Over sixty years ago Schrödinger remarked that “physical laws rest on atomic statistics and are therefore only approximate” [9]. More recently Anandan has argued that “there are no fundamental causal laws but only probabilities for physical processes” [7]. Given that these probabilities are at least invariant with respect to different observers, and that they lead to correct predictions, perhaps it is appropriate to recognize these probabilities as the physical motive that explains change (*i.e.* evolution of state). With such a view, one does not expect some sort of retroactive causal motive, nor any “sense” on the part of physical systems about their possible future; rather one identifies the field of possibilities itself as the motive for change.

The opening line was intended to introduce the idea that nature has an “outlaw” character, which is not to say that nature is haphazard, ergodic, or entirely unruly. The outlaw character of nature is the ground that essentially complements its “law abiding” character, which has been the figure of interest for centuries of modern science in pursuit of mechanistic causal explanation. In Solomonoff's and Chaitin's formal developments of inductive inference based on algorithmic information theory [10, 11], a law, as a symbolic string, is essentially a summary of many “observation” strings. In this scheme, a law can be thought of as the shortest computer program capable of generating all those other strings which encode observations to which the law pertains (and it would not generate any strings representing contrary observations). Thus the correlations between observations are explicitly taken as the substance of laws. Laws, as symbolic strings, are summaries that *describe* correlations rather than *exhibiting* them. In information theory, such “decorrelation” is one of the steps of information compression. Explanation then can be considered to be information compression; always correlative but not necessarily causal, it is time-neutral. For there is no requirement that only those correlations with the proper temporal relation be admitted for summary. Even if the world were such that future events determined past events, the correlations could be summarized as laws, which might be considered as the reasons, or motives, for sequences of events. The second law of thermodynamics is of this nature: describing what we expect to observe in the evolution of systems, it does not appear to explain the “cause” that would motivate such behavior, but it summarizes correlations between observations.

### **The second law of thermodynamics, macrostate and microstate**

The laws of science describe patterns and rhythms that are observed in nature [12], as relations between objects. The statistical tendency for differences to equilibrate, for the improbable to lead to the probable, is one such recurrent, reproducible pattern, whether the interpretation be phenomenological or epistemological. I allude to the controversy regarding the interpretation of entropy objectively (for example, Denbigh & Denbigh [13]) *vs.* subjectively (for example, Jaynes [14]).

The work of Clausius, Maxwell, Boltzmann, and Gibbs in statistical mechanics was motivated by the desire to provide a mechanistic basis for understanding thermodynamics, and in particular for understanding such notions as that of entropy, in terms of the details of molecular motions. The second law appears a bit peculiar in comparison to other physical laws, in that it refers not to individual objects but to systems: collections of objects together with their relations. Moreover, as Gibbs observed, in statistical mechanics one takes a broader view of systems, by concerning oneself not simply with the evolution of given systems, but with the evolution of given *distributions* of systems [15]. Hence the idea of a *macrostate*, as an ensemble of consistent *microstates*. The microstates may differ in microscopic detail, but they are indistinguishable macroscopically in terms of the chosen system parameters (such as temperature, pressure, etc., which are typically averages).

There appears again something of a controversy regarding the matter of reconciling time-symmetric dynamics and time-asymmetric thermodynamics. The above-mentioned approach, distinguishing more or less arbitrarily between macrostate and microstate, is sometimes called “coarse-graining”; contraposed to this is the derivation of thermodynamic time asymmetry via choice of time-asymmetric equations of dynamics (the program of Prigogine and the Brussels and Austin schools), sometimes called “extended dynamics” [16]. A theorem from Lasota and Mackey [17] is supposed to render the controversy moot, but since I do not fully appreciate the mathematics, I choose here the more intuitively-tractable macrostate-microstate view to make my argument.

### *Phase Space*

Every possible state of a system can be thought of as a point in an abstract space having as many dimensions as there are degrees of freedom for the system in all its detail. For example, in a simple mechanical system comprising four objects, the degrees of freedom might be considered to be the positions and the momenta of the four objects. Each object’s position could be specified by three spatial coordinates, and likewise each object’s momentum vector could be specified by three spatial components. So there would be  $4 \cdot (3 + 3) = 24$  degrees of freedom for this mechanical system of three objects, and it could be represented by a point in an abstract space of 24 dimensions. In general the state of such a system of  $N$  objects can thus be thought of as a point in  $6N$ -dimensional space. This is the phase space of the system.

Regarding the state of a system, or of the universe as a whole, as a point in phase space, change is thought of as movement of the point in phase space. The probabilistic questions of what state the system is likely to be in, or what state it is likely to change to, become abstract spatial questions about the proximity and sizes of regions in phase space.

### *Macrostate as a Region of Phase Space*

A macrostate may be regarded as a certain region (volume) in phase space, the interior points of which are the microstates that are consistent with the parameter values that define that macrostate. In these terms, Boltzmann expressed entropy  $H$  in terms of number of microstates  $W$ :

$$S_B = k_B \ln W, \quad (1)$$

Where  $k_B$  is Boltzmann's constant. Entropy, then, is associated with a *measure* (generalized volume) of phase space [18]. Increasing entropy, being associated with increasing probability, might therefore be understood in terms of the path of a point in phase space, from one macrostate region to another macrostate region of larger volume.

The *distribution* of microstates associated with a given macrostate is expected, by the second law of thermodynamics, to evolve in such a way that its "envelope" occupies a larger and larger phase-space volume as time goes on. The initial phase-space volume of the distribution is not to be thought of as expanding, but rather as becoming more convoluted, as it were [19], like dye stirred into a liquid, so as to occupy a larger phase-space volume *in terms of the initial macrostate parameters*. An equivalent statement of the second law of thermodynamics is that observed macrostates of lesser phase-space volume (*i.e.* less probable macrostates) tend to go over into macrostates of greater phase-space volume (*i.e.* more probable macrostates).

On the other hand, time-symmetry of dynamical laws means that the formulas describing the evolution of states (microstates) are invertible; this one-to-one correspondence between successive states implies that the phase-space volume of a given distribution does not change through time (in accordance with the Liouville theorem). This seems to be the crux of controversy regarding the objective *vs.* subjective nature of entropy: if entropy is phase-space volume, and this volume doesn't change for a distribution of states evolving through time, then it would seem that there can be no objective increase in entropy. But in fact for any given macrostate parameters, the overwhelming majority of microstates consistent with those macrostate parameters evolve to microstates lying outside the originally-defined macrostate region. Why? "Because that's where the space is."

### **Prediction *vs.* retrodiction**

We are accustomed to thinking that the past is known, whereas the future is unknown. But surprisingly, if the past is not recorded, the opposite is more nearly true: it is easier to predict from a given macrostate into the future, than to retrodict from a given macrostate into the past. For if past macrostates pass into larger future macrostates (the second law), while past distributions maintain their phase-space volume into the future (the Liouville theorem), then the correspondence between macrostates from past to future must not be one-to-one; there must be convergence of macrostates forward in time, specifically toward a macrostate of equilibrium, and divergence of macrostates backward in time. Therefore macrostate prediction is easier than macrostate retrodiction: given a present macrostate, we see that any number of prior macrostates (of lesser phase-space volume) could have evolved into the present macrostate, whereas the present macrostate (along with, perhaps, any number of distinct macrostates) can be expected, by the second law, to evolve into the same posterior macrostate of larger phase-space volume. Hence macrostate prediction is easier than macrostate retrodiction insofar as the second law of thermodynamics guarantees that macrostates go over into larger-volume macrostates, even as the associated microstate *distributions* maintain their phase-space volume as they evolve and their envelopes spread out into larger macrostates. However, more will be said about this in connection with Maximum Entropy Production and probability slopes.

Given a particular *microstate* on the other hand, prediction and retrodiction are on equal footing as they employ the same time-symmetric dynamical laws; thus if one had detailed knowledge of

molecular positions and momenta after an irreversible process had occurred (such as when a glass of water falls from a table, breaks, and spills its contents), one could “run the movie backward” to reconstruct the history, the information being present implicitly in the correlations of microscopic detail. That is to say, the “randomized” microstate after the spill is actually quite unique when regarded in all its detail, even though it appears to be a member of a much larger class, in terms of macrostate parameters, than it had been before the spill. A complete description of the latter microstate would be extremely lengthy, except that certain correlations would be discovered which would allow this microstate to be summarized, in effect, as “that state resulting from the fall of a glass from a table”.

The foregoing illustrates that information entropy and thermodynamic entropy are not identical and do not change with time in the same way. Information entropy is a concept which can be applicable to an individual system, in accordance with algorithmic information theory [20]. But thermodynamic entropy implicitly applies to *distributions* of systems, as mentioned previously in connection with Gibbs’ work in statistical mechanics [15]. Equivalently, one might subdivide a single system into subsystems and observe the second law of thermodynamics operating as a “flow of entropy” [21], down a hierarchy of scale, or from velocity coordinates to spatial coordinates. One can be confident, by the second law of thermodynamics, that certain macrostate parameters of the system, being statistical averages, will change with time in a way which reflects a larger number of possible microstates consistent with the latter macrostate. But the number of parameters required to specify the macrostate need not change. So any increase in thermodynamic entropy is not reflected in any increase in information entropy of the macrostate parameters.

An equivalent expression of the second law of thermodynamics, due to Penrose, is that systems that have been separated in the past are uncorrelated. Conversely, after “randomizing” interactions, systems remain correlated. Prigogine’s insight was that “irreversibility is a flow of correlations” [22]. My interpretation of this is that correlations flow from the macrostate scale to the microstate scale, but are conserved.

Thinking solely in terms of least-biased inference, given a present state, specified by some macrostate parameters, one would make the same estimate of past macrostate as of future macrostate: one would expect an increase of entropy either way [23]. The resolution of this apparent paradox, according to Penrose, is that the past was constrained, namely by a condition of exceedingly low entropy (or high improbability) early in the history of our universe [24]. In Penrose’s view, this early state of our universe in fact explains the second law of thermodynamics, if only by taking in its place an initial state of high improbability.

### **Information, entropy, and order**

For the purpose of this paper, I would like to use the term “spatial order” to refer to the improbability associated with low entropy. I acknowledge that many authors would disagree with the identification of entropy with disorder, some for example preferring to identify entropy with “freedom” [25], but I choose the word “order” over the word “negentropy” because linguistically it better fits a property, or a disposition, than does the word “negentropy”. Order, as I propose to use the word, is a measure of disequilibrium, or of compactness in phase space, in so far as a given state can be regarded

as a member of a relatively small macrostate class. In my preferred terminology, the algorithmic information content of a highly ordered state would be relatively low.

Analogously, I would like to use the term “temporal order” to refer to the improbability of a temporal sequence. For a sequence of high temporal order, relatively few sequences would fit the “macro” description of the sequence, and its algorithmic information content would likewise be low. I conjecture that spatial order and temporal order are two aspects of the same thing, and that from any observer’s point of view, they exhibit a reciprocal relation, as temporal order induces spatial order (“The flow of energy through a system tends to organize that system” [26], the “dissipative structures” of Prigogine, etc.), and spatial order induces temporal order (The spontaneous organization of a system tends to accommodate the flow of energy through that system: MEP [27, 28]).

Gibbs refined the Boltzmann expression of entropy (equation 1), to account for the possibility that not all microstates be equally probable:

$$S_G = - k_B \sum p_i \ln p_i ,$$

where  $k_B$  is again Boltzmann’s constant, and the  $p_i$  are members of a probability distribution of microstates consistent with the macrostate [29].

Shannon, working in the field of communications theory [30], quantified the uncertainty  $U$  of a message yet to be received as:

$$U_S = - \sum p_i \log_2 p_i ,$$

where the  $p_i$  are a probability distribution of possible messages. Clearly the two equations are isomorphic, being measures of the spread of distributions of possibility, so by analogy Shannon ventured to call his measure of information “entropy”.

Notwithstanding Shannon’s own warnings regarding careless use of such terms as “information” and “entropy” [31], there is much disagreement in the literature about their relation, in spite of such insights as that of Landauer and Bennett [32] relating the two. Not to confuse *knowledge* with *information*, I accept the Shannon use of the term “information” to refer to a measure of the capacity to inform, hence uncertainty. Noting the mathematical isomorphism between Shannon’s information entropy and Gibbs’ thermodynamic entropy, some authors have attempted to construct second-law analogies between biological “information” and thermodynamic entropy [33], while others have rejected information theory and constructed their own definitions of information [34]. In fact, Wicken’s definition of “information”, based on improbability, is close to my proposed definition of “order”.

I believe that disagreements about “information” and “entropy” have to do with the question of how they change with time. The mathematical isomorphism can hardly be considered accidental, yet a strictly parallel second law need not follow. On the contrary, it is enlightening to note the opposite arrows of time, with respect to information entropy *decrease vis-à-vis* thermodynamic entropy *increase* with time: in any irreversible process of communication, information entropy must *decrease* [35]. This is so because information is a difference, a relational quantity, being a measure of the potential to inform [36].



## The arrow of time

The second law of thermodynamics is considered to be at once the most certain of physical laws and also the most perplexing or intriguing in its implications, as expressed variously by Einstein [37], Eddington [38], Prigogine [39], and many other eminent scientists and philosophers of science. Evidently its certainty is accounted for by the fact that certainty itself is, in a sense, its subject; its intrigue may be accounted for by the fact that it stands alone (almost [40]) among the laws of dynamics as the one that essentially confirms the difference between past and future and that accounts for any sensible change in the world.

A four-hundred-year tradition of scientific inquiry, expressed in equations of dynamics that are time-symmetric, and in relativity transformations that mix time with space, has perhaps led away from an intuitive personal perspective of time and toward a geometric perspective of space-time. The geometric perspective seems simpler and more objective, so that it is compelling to suppose that our intuitive perspective of time, in relation to the world *becoming* what it is, may be provincial and somewhat illusory. Yet the geometric perspective of time as a dimension forces one to confront the question of why one direction in this dimension should be different from another, or indeed, why any particular value of this dimension's coordinate (such as  $t = 0$ , or "now", in the vernacular) should be accorded any special status. (In other words, why do we have the unequivocal sense of being centered at the present, if all of space-time has a "geometric" existence?) Ilya Prigogine, who spent a lifetime studying nonequilibrium thermodynamics and developing equations of extended dynamics that break time-symmetry, was primarily motivated in his studies by an interest in these questions of time [41].

The second law of thermodynamics, or increasing entropy, is apparently not the only arrow of time, as pointed out by Popper with the example of radiating waves [42], but aside from the psychological sense of time, it seems to provide the most pervasive objective evidence of irreversibility. In "The many faces of irreversibility" [43], Denbigh argues that irreversibility (which I take to be synonymous with time-asymmetry) is a broader concept than that of entropy increase, and that its essence seems to be a "fanning out", or expansion in space, either physical or abstract. Why do things expand? "Because that's where the space is"?

## MEP and MaxEnt

The ideas of non-equilibrium thermodynamics have found application in various fields, notably biology and meteorology. Maximal or minimal principles of energy flow or entropy production in biology have been proposed by many authors including Johnstone [44], Lotka [45], Schrödinger [9], Hamilton [46], Lovelock [47], Wicken [48], Brooks & Wiley [32], Schneider & Kay [25], and Salthe [49]; Paltridge, among others, advanced the principle of Maximum Entropy Production (MEP) in meteorological modeling [27, 50].

MEP is the tendency for open systems far from equilibrium to respond to imposed differences, or gradients, by evolving to a steady state in which the production of entropy is maximized. The principle of MEP appears as a regenerative combination of the effect of difference-induced flow in organizing a system ("making a difference"), together with flow-induced organization enhancing the ability of the system to conduct the flow. If the principle is known to be valid for a system, then it can be used (like

the Le Chatelier - Braun principle of chemistry, or indeed, like the second law of thermodynamics in general) to predict averaged parameters of system behavior, without knowledge of the system's details.

The aim of the so-called "MaxEnt" information-theoretical formulation of non-equilibrium statistical mechanics due to Jaynes is to represent most accurately and fully the uncertainty about the state of a system, given the constraints of all that is known about the system. Based on the Bayesian concept of probability as a measure of ignorance, rather than on the Monte Carlo concept of probability as a sampling ratio, it is used to quantify the spread of possible microstates, given only knowledge of the macrostate. It is considered by its advocates to be a very generally applicable principle of scientific inference, to be used to arrive at a least-biased estimate of any unknown quantity, given some limited knowledge of the system under observation.

Recently the MaxEnt method has been extended by Dewar [51, 6, 52] to estimate not just most likely states, but most-likely state-change paths—*i.e.* system evolution. MEP as a thermodynamic principle describing system evolution is thereby derived, along with other well-known physical principles such as the Fluctuation Theorem, which quantifies the likelihood of so-called "irreversible" processes actually proceeding in reverse [53]. It has been almost a century since Paul and Tatiana Ehrenfest published *The Conceptual Foundations of the Statistical Approach in Mechanics* [54], which clarified certain points of disagreement; now the time seems ripe for a reconciliation of divergent views on the nature of entropy and the second law of thermodynamics. I suggest that this reconciliation might come about by the recognition of probability, not as ignorance, and not as dice, but as a physical motive.

### **Probability as a topography**

As described above, in the phase-space view of a system, its state is represented as a single point. System evolution is then a sequence of points, and it may be possible to visualize the second law of thermodynamics as the movement of the point, representing the state of the system, into ever-larger macrostate volumes. Even if "macrostate" and "entropy" have no absolute meaning until we have specified the set of parameters which define the thermodynamic state of the system [14], it is nevertheless true that, regardless of choice of macrostate parameters, according to the second law of thermodynamics we should expect this movement, on average (*i.e.* except for "fluctuations"), to proceed from smaller macrostate volumes to larger macrostate volumes.

If one then imagines macrostate volume inversely mapped to an "elevation", one obtains a surface topography on which system evolution is expected, by the second law, to proceed "downhill". This is admittedly nothing but a conceptual device, lacking mathematical rigor. But it helps draw attention to the question of why anything flows downhill, why a puddle spreads. What would the Dillinger answer be? We might say that water flowing downhill responds to the physical motive of gravitational force. But we don't know what gravity is (though to preserve the precept of local action we may say that it is curvature of space-time); in fact flow of mass in response to a gravitational gradient is just the same sort of second-law journey, downhill on the imagined phase-space topography, as is flow of heat in response to a temperature gradient, there being more opportunities for matter to be collected—greater probability for such states to be realized— than for it to remain dispersed, just as there are more ways

for a bunch of molecules of a given velocity distribution (*i.e.* average temperature) to be spatially dispersed than to be spatially segregated by velocity.

This is admittedly a difficult point because, as reflected in the Gibbs refinement of the Boltzmann formula for entropy (and analogously in Shannon's refinement of Hartley's measure of information [55]), the probability associated with a given macrostate depends not only on the number of microstates consistent with the macrostate, but also on their individual probabilities. The individual probabilities in turn are presumably functions of their energy distributions. There is of course energy in gravitational potential (collective difference in location of mass). If gravity is taken to be curvature of space-time, then the assertion that there are more opportunities for matter to be collected than for it to remain dispersed is equivalent to the statement that there are more ways for space-time to be "wrinkled" than for it to be flat.

Continuing with the conceptual device of probability topography derived from phase space, one can then visualize MEP as "flow", not just downhill, but down the steepest gradient. Of course this is what one might expect intuitively. I expect that the question of whether it holds universally amounts to the question of how probability slope depends on choice of macrostate parameters. The question of *retrodition* may also be recast in terms of finding the steepest probability gradient back —*minimum* (negative) entropy production in the reverse time direction. The work of Dewar [51, 6] puts these intuitive notions on firmer mathematical ground. More importantly, I think, the work bridges the gap between phenomenological understanding of MEP and epistemological understanding of MaxEnt, opening the door to acceptance of probability as a fundamental physical motive.

It is not the intent of this letter to discuss the relative reality of mental models, or maps, on the one hand, and the physical world, on the other. Rather the intent is to suggest the consideration of "probability"—extent of possibility in phase space—as a fundament of the physics of change and spatial differentiation in our model, rather than as a computational tool or as "metadata".

### **Opportunity and necessity**

It has been assumed that probability must be interpreted either "objectively" as a measure of ratios of outcomes, or "subjectively" as a measure of observers' ignorance [5, 52]. I claim that there is another interpretation of probability, outside this dichotomy: probability may be interpreted as a measure of possibility, or of opportunity for realization. Some have remarked that Monod's work *Le hasard et la nécessité* (Chance and Necessity) [56] was written "in the gloomiest of existential traditions" [48]. I suspect that profound questions about origins and destinations—reasons for being—were mistaken for a profound sense of estrangement. Monod's awareness of kinship with all life, and with nature, is amply apparent; if there was a sense of estrangement it might have been with the odd circumstance in which we find ourselves, caught between science's presumption of causal origins and religion's assumption of purposive destiny. How does one interpret *hasard*, or "chance"? Is the sense that of risk? of randomness? More likely it is that of fortune, or of *possibility*. This is the sense of probability as a physical motive.

## Reciprocal relations

The beginning of thermodynamics is generally taken to be Carnot's observation that a difference of temperature can produce motive force. The practical realization of this principle is of course the heat engine (the generic term for such as the steam engine and internal-combustion engine), where a difference in the form of chemical potential is first discharged via combustion in order to generate the temperature difference from which motive force can be produced. Conversely, motive force can produce a temperature difference, practically in the form of a heat pump, or refrigerator.

Over a century after Carnot, in an early foray into nonequilibrium thermodynamics, Onsager [57] explained the reciprocal relations that are observed between pairs of difference (potential) and flow, such as the *Peltier* effect (whereby an electric current develops a temperature difference) *vis-à-vis* the *Seebeck* effect (whereby a heat flow develops an electrical potential). The work of Prigogine and the Brussels and Austin schools in non-equilibrium thermodynamics carried on to explore the many implications of the reciprocal relations between differences and flows. The generative, as well as the dissipative, aspect of the second law of thermodynamics is now more generally recognized (see, for example, [58]).

All these discoveries in nonequilibrium thermodynamics lead one to venture the view that, in the big picture, "order" and its concomitant "disordering" comprise a feedback loop, a self-determining spiral cascade of differences and flows which change form but which may exhibit "strange loopiness", to borrow Hofstadter's term [59], in that they refer back to themselves by inclusion. By analogy to Onsager's specific reciprocal relations, generic "order" may behave as a sort of "auto-reciprocal" singleton, insofar as any "dissipative structure" resulting from flow also degrades, conversely generating another flow. Order (as I've presumed to use the word both spatially and temporally) is necessarily self referential in space-time, exhibiting regenerative, self-propagating (even if self-damping, "senescent") effects. Order dissipates, but dissipation orders, and this holds both spatially and temporally.

## Spatio-temporal order

I have suggested that spatial order and temporal order are two aspects of the same thing. In the geometric model of space-time, spatial order (improbability of state) and temporal order (improbability of evolution) determine each other, in the manner (if the reader will indulge a sloppy analogy) of the electric field and the magnetic field, or of a Fourier pair such as position and momentum. This "static", geometric conception of unified spatio-temporal order is an attempt to integrate the insights of special relativity [60] into the investigation of the second law of thermodynamics and the consideration of probability as a physical motive. It follows the tradition of seeking simplification by assuming invariance—requiring the proposed concept of order not to depend essentially on point of view. Hence the notion that time and space must figure similarly with regard to the structure of spatio-temporal order.

The proposed conceptual model of spatio-temporal order explicitly extends in both space and time, since the proposed definitions of spatial order and of temporal order are based on a notion of improbability of state or of evolutionary sequence, which makes sense only for some sort of

distribution or sequence that is one of a class of possibilities. A temporal cross-section of the model would be an illustration of the second law of thermodynamics, but the real attraction of the model would be in relating the spatial and temporal aspects of order, and in providing a more complete perspective on the asymmetry of time.

This spatio-temporal order must exhibit a “twist”, or “handedness”, to reflect the fact that (as far as the evolution of order is concerned) time has an “arrow”. But if the model is at all viable, and unless its mathematical description should involve non-invertible functions, it would imply that knowledge of local structure of a region (around “here” and “now”, for example) should allow extrapolation from here-now in all spatio-temporal directions, as hinted previously in the suggestion of a *minimum* entropy production principle for *retrodiction* of macrostate. One might expect that the current macrostate was arrived at by the most unlikely evolution, from the most unlikely prior macrostate, such that both are consistent with the model as a whole.

## Conclusion

The main point of this communication is to argue that probability should be considered to be a fundamental physical motive, sufficient unto itself for explaining change. Some seventy five years of work in nonequilibrium thermodynamics, revealing the constructive aspect of the second law of thermodynamics, together with relatively recent progress substantiating the connections between information theory (probability) and thermodynamics, lead to this opportunity to entertain a new conceptual basis for understanding systems and their evolution, for posing questions and formulating hypotheses.

I’ve tried to substantiate the proposed new view of probability by way of a topological model of phase-space evolution, in which entropy increases with progress “downhill”, and MEP is flow in the steepest direction. I’ve assumed a definition of “order” in terms of improbability—relatively small macrostate volume—and proposed an integration of “spatial order” and “temporal order” concepts. It is hoped that the second law of thermodynamics would then be seen as a partial description of the structure of spatio-temporal order, and that the structure of spatio-temporal order would afford further insight into time asymmetry.

Questions that must be addressed include: How, exactly, is spatial order to be quantified? How is temporal order to be quantified? How can they be made commensurable? What, then, is the effect of a Lorentz transformation on the measure of order? If the proposed probability topography of phase space can be clearly defined, how will its slope depend on the choice of macrostate parameters?

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