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Probability of large movements in financial markets

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ABSTRACT

Based on empirical financial time series, we show that the "silence-breaking" probability follows a super-universal power law: the probability of observing a large movement is *inversely proportional to the length of the on-going low-variability period*. Such a scaling law has been previously predicted theoretically [R. Kitt, J. Kalda, Physica A 353 (2005) 480], assuming that the length-distribution of the low-variability periods follows a multi-scaling power law.

1. Introduction

The power laws and scaling behaviour are present in numerous aspects of human societies and in the nature. One of the first promoters of the concept of the power law was Vilfredo Pareto (cf. Ref. [1]), who studied the wealth distribution in different societies. Further, Harvard linguistics professor George Zipf, again a representative of social sciences, observed that only few words in English language are used very often, and most of the words are used rarely. (cf. Ref. [2]). Nowadays, physicists are very used to the power laws, which, however, are sometimes somewhat counter-intuitive. Indeed, the presence of a power law means that there are some representatives of a population, which are very different from the typical members of that population. For example, as the result of a social evolution, the wealth of a single individual can qualitatively change the wealth of a large community. This is completely different from the biological evolution: e.g. the weight of a single living creature makes always only a tiny contribution to the net biomass of the corresponding (non-small) population.

The presence of a wide spectrum of power laws in finances can be ascribed to the fact that socially interacting humans form large complex systems, which are characterized by self-organized criticality [3]. This makes finances (alongside with the turbulence) a fruitful polygon for studying various aspects of scale-invariance. Indeed, various power laws have been observed in financial time series since 1960s, by B. Mandelbrot (cf. Ref. [4,5] and the references therein). In 1999, Ausloos and Ivanova also reported the multi-fractality in financial time series (cf. Ref. [6]). Around 1990s, the studies of scale-invariance in finances became more extensive, cf. Ref. [7–10], effectively creating a new branch of statistical physics — the *econophysics*. A recent overview of the progress in understanding the scaling and its universality in finances can be found in Ref. [11]. The aim of the current study is to contribute to the *understanding of the origins of universality*. Mathematically, our basic idea is very simple, nearly trivial. However, when dealing with the sources of universality, mathematically simple and robust models have better chances of describing reality, than complex and elaborate constructions; cf. the Occam's razor.

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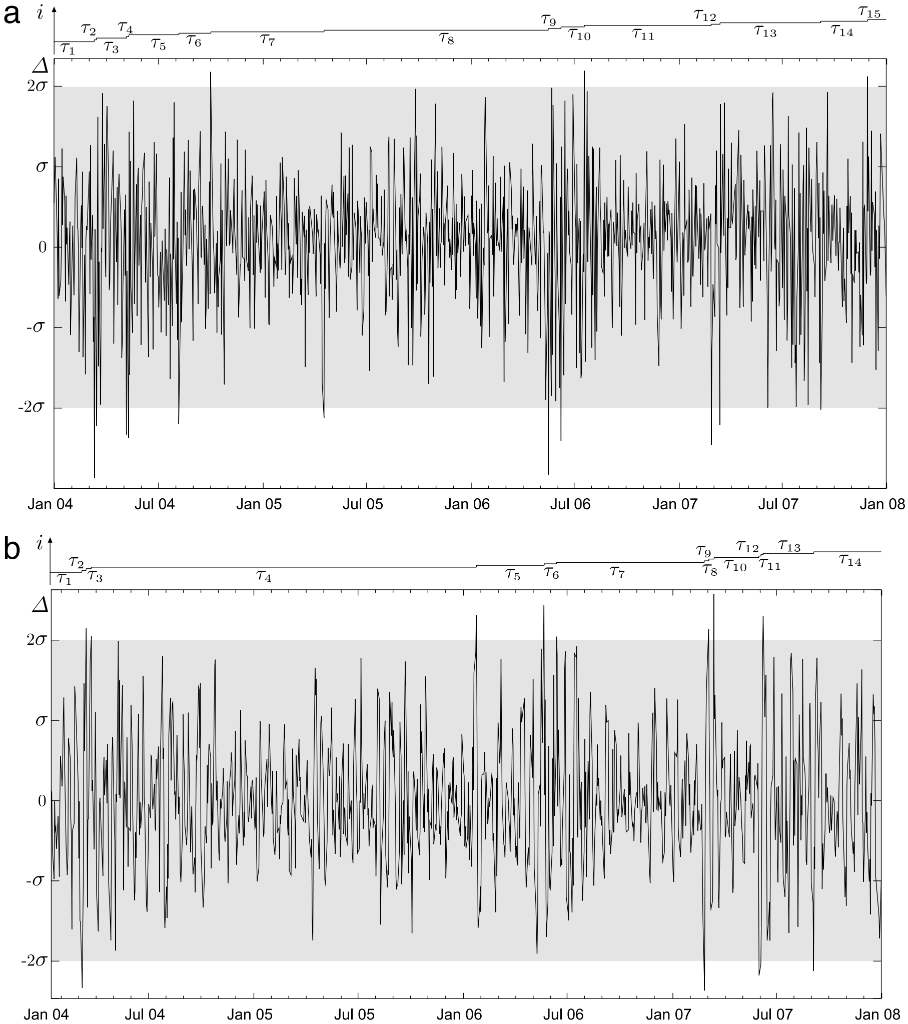

Fig. 1. Variability Δ of the DAX index and the respective low-variability periods for 4 years (2004–2007), using $w = 1$ day (a) and $w = 10$ days (b). For a pre-fixed threshold level $\delta = 2\sigma$ (where σ is the standard deviation of the signal), low-variability intervals of duration τ_i are formed as the intervals corresponding to such graph segments, which lay entirely inside the gray area. The small graphs in top share the time axis with the bottom graph, and illustrate the fragmentation of them into low-variability intervals by plotting the interval index i versus time t.

The attempt to successfully and systematically predict the direction of future movements of asset prices can be compared to the attempts of inventing the *perpetum mobile*. However, the attempt to characterize and predict the risk (or volatility) may offer significantly better results and therefore the volatility is one of the most-studied phenomena in Econophysics.

One of the most challenging features of the volatility dynamics are the intermittently appearing extreme price movements, which are often accompanied by overall increase of volatility over a certain time window. Traditionally, such a behaviour has been described by multi-fractal spectra. The multi-fractal analysis is undoubtedly a powerful tool; however, due to the involved mathematical methods, it is not well suited for practical applications of the prediction and optimisation of risks. This observation motivated us to introduce a complementary method of the length-distribution analysis of the *low-variability periods* [12–14]. The low-variability periods are defined as consequent time periods, during which the price changes of the observed asset (as compared to the local average over a sliding window of width w) remains under a pre-set threshold δ. The illustration of the method is shown in Fig. 1. This method (with certain modifications) was developed independently within different contexts, and put under extensive tests by several research groups. So, the effects of the long-term memory and clustering of extreme events in various time series (cf. Ref. [15–19]), are, in fact, closely related to (and in a certain sense covered by) the low-variability period analysis. The same applies to the studies of the time intervals τ between *volatilities* which are above a threshold q [20–25]).

Our previous studies [14,26] have shown that

- financial time series are typically characterized by multi-scaling behaviour of the low-variability periods (power law can be observed for a certain range of the parameters δ and w, and the scaling exponent depends on these parameters);