

**Probability,  
Statistics,  
and  
Reliability  
for Engineers  
and  
Scientists**  
THIRD EDITION

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