Probe Branes, Time-dependent Couplings and Thermalization in AdS/CFT

Tatsuma Nishioka

(University of Tokyo)

based on arXiv:1005.3348

with S. R. Das (Kentucky) and T. Takayanagi (IPMU)

Introduction

- Non-equilibrium systems with time-dependent gauge coupling or masses are surely interesting, and can be realized experimentally in cold atom physics

- but there are few analytical tool to investigate such a system except certain extreme limits
    - Sudden change (quench)
    - Slow change

- The AdS/CFT correspondence gives us a powerful tool to study a strongly-coupled gauge theory in terms of dual gravity even in time-dependent case

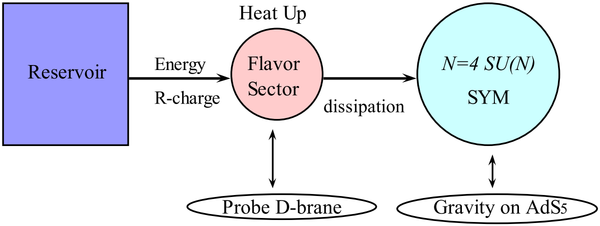
Time-dependent AdS/CFT

- One can obtain some insight for time-dependent gravitational backgrounds using dual gauge theory and vice versa
  - space-like singularity
  - non-linear fluid dynamics
  - thermalization/black hole formation

- Solving the Einstein equations is sometimes difficult in time-dependent case

- We will setup a probe time-dependent D-brane on fixed backgrounds to describe non-equilibrium steady states

Non-equilibrium system in AdS/CFT



- The dual field theory is regarded as an open system with time-dependent couplings or masses
- Bulk and the probe brane have different temperatures respectively

Plan

Part I: Stationary case

- Rotating D-brane and emergent black hole
- Dual CFT interpretation
- Thermal properties

Part II: Non-stationary case

- Quantum quench in free field theory
- Toy model: rotating D1

Part I: Stationary case

Rotating D1-brane in AdS

- Consider a probe D1-brane in $AdS_{d+2} \times S^q$ space-time:

$$ds^2 = -f(r)dt^2 + r^2 \sum_{i=1}^{d} dx_i^2 + \frac{dr^2}{f(r)} + (d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\Omega_{q-2}^2)$$

- The D1-brane extends in (t, r) direction, and is specified by

$$\varphi = \varphi(t, r) \ , \qquad \theta = \frac{\pi}{2}$$

- The DBI action is

$$S = -T_{D1} \int dudv \ f(r) \left[1 - \frac{4}{f(r)} \partial_u \varphi \partial_v \varphi \right]^{1/2}$$

where we introduced the outgoing and ingoing Eddington-Finkelstein coordinates

$$u = t - \int \frac{dr}{f(r)} \ , \qquad v = t + \int \frac{dr}{f(r)}$$

Rotating D1-brane in AdS

- The equation of motion:

$$\partial_u\partial_v\varphi + \frac{2}{L}\partial_v\varphi\partial_u\left(\frac{\partial_u\varphi\partial_v\varphi}{f(r)}\right) + \frac{2}{L}\partial_u\varphi\partial_v\left(\frac{\partial_u\varphi\partial_v\varphi}{f(r)}\right) = 0$$

$$L = 1 - \frac{4}{f(r)}\partial_u\varphi\partial_v\varphi$$

- There exist simple solutions:

$$\varphi = \varphi(u) \text{ (advanced) }, \qquad \varphi = \varphi(v) \text{ (retarded)}$$

- For the retarded solution, the induced metric is

$$ds^2_{ind} = -f(r)dudv + (\partial_v\varphi)^2 dv^2 = 2drdv - [f(r) - (\partial_v\varphi)^2]dv^2$$

- This is a two-dimensional AdS Vaidya black hole with an apparent horizon at $f(r) = (\partial_v\varphi)^2$!

Emergence of black hole on rotating D-brane

- In Poincare coordinate $f(r) = r^2$, if we take $\varphi(v) = \omega v$ (rotating in the internal sphere), the induced metric becomes

$$ds^2 = -(r^2 - \omega^2)d\tau^2 + \frac{dr^2}{r^2 - \omega^2}$$
$$\tau \equiv t - \frac{1}{r} - \frac{1}{2\omega}\log\frac{r-\omega}{r+\omega}$$

- This is a AdS black hole with the Hawking temperature

$$T_H = \frac{\omega}{2\pi}$$

- Notice that the bulk of AdS is still at zero temperature under a probe approximation

Dual CFT

- If we focus on $AdS_5 \times S^5$, our system is dual to $\mathcal{N}=4$ SYM coupled to an monopole
- The $\mathcal{N}=4$ SYM is at zero temperature, while the monopole is at finite temperature T_H
- The rotation along φ direction induces the time-dependent coupling in dual $\mathcal{N}=2$ theory

$$\int dt \left[\bar{Q} \ \left[\mathrm{Re}(\Phi_3 e^{i\omega t})\right]^2 \ Q + \tilde{Q} \ \left[\mathrm{Re}(\Phi_3 e^{i\omega t})\right]^2 \ \bar{\tilde{Q}}\right]$$

where Φ_3 is an adjoint scalar field in $\mathcal{N}=4$ SYM and $(Q, \tilde{Q})$ are the hypermultiplets coming from D1-D3 open strings

Charge/Energy dissipation

- To obtain the energy flux, consider the energy-momentum tensor in the bulk (not on the brane!)
- The rotating D1-brane gives non-zero contribution to them

$$\sqrt{-g}\ T^t{}_t = T_{D1}\frac{1 + r^2\varphi'^2}{\sqrt{1 + r^2\varphi'^2 - \dot{\varphi}^2/r^2}} = T_{D1}\left(1 + \frac{\omega^2}{r^2}\right)$$

$$\sqrt{-g}\ T^r{}_t = T_{D1}\frac{r^2\dot{\varphi}\varphi'}{\sqrt{1 + r^2\varphi'^2 - \dot{\varphi}^2/r^2}} = T_{D1}\omega^2$$

$$\sqrt{-g}\ T^r{}_r = -T_{D1}\frac{1 - \dot{\varphi}^2/r^2}{\sqrt{1 + r^2\varphi'^2 - \dot{\varphi}^2/r^2}} = -T_{D1}\left(1 - \frac{\omega^2}{r^2}\right)$$

Charge/Energy dissipation

- The time evolution of the total energy

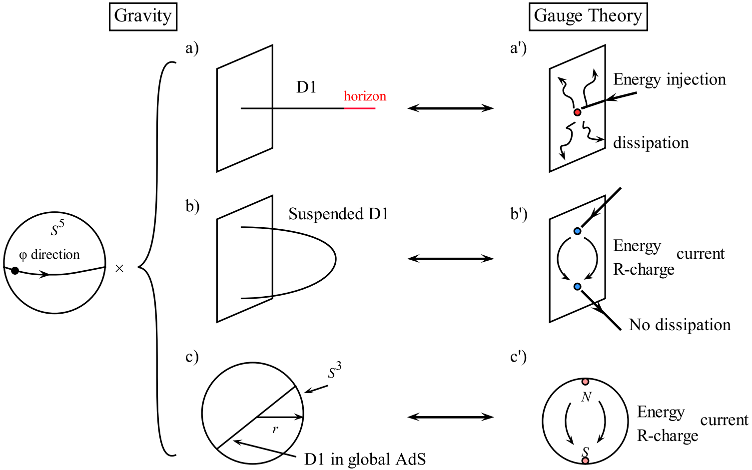
$$\frac{dE}{dt} = \frac{d}{dt}\int dr\sqrt{-g}T^t{}_t = \sqrt{-g}\ T^r_t\ \Big|_{r=0}^{r=\infty} = T_{D1}\omega^2 - T_{D1}\omega^2 = 0$$

- Therefore we find the energy flow $\frac{dE}{dt} = T_{D1}\omega^2$ from the boundary to the horizon
- The R-charge flow also can be calculated as

$$\left.\frac{dQ_R}{dt}\right|_{r=0} = -T_{D1}\frac{r^2\varphi'}{\sqrt{1+r^2\varphi'^2-\dot{\varphi}^2/r^2}}\Big|_{r=0} = -T_{D1}\omega$$

where $Q_R = \frac{\delta S}{\delta\dot{\varphi}} = T_{D1}\int dr\frac{\omega}{r^2}$

Schematics for rotating solutions



Rotating D7-branes in $AdS_5 \times S^5$

- So far we have assumed that the rotating D-brane is point-like in the sphere
- Now we consider a probe D7-brane wrapped on S^3 in S^5
- This configuration is dual to the $\mathcal{N} = 2$ SYM with flavor hypermultiplets
- The mass of the hypermultiplet will be time-dependent in this case

Rotating D7-branes in $AdS_5 \times S^5$

- The D7-brane is extending in the whole AdS_5 spacetime and S^3 in S^5 with a rotation in φ direction

- We assume

$$\theta = \theta(r) \ , \quad \varphi = \omega t + g(r)$$

- The solutions to the equation of motion for DBI action will be

$$g'(r) = 0$$

or

$$g'(r) = \frac{1}{r^2 \sin\theta} \sqrt{\frac{(r^2 - \omega^2 \sin^2\theta)(1 + r^2\theta'^2)}{(A^2 r^8 \cos^6\theta \sin^2\theta - 1)}}$$

Solution 1: $g'(r) = 0$

- The equation of motion for θ is

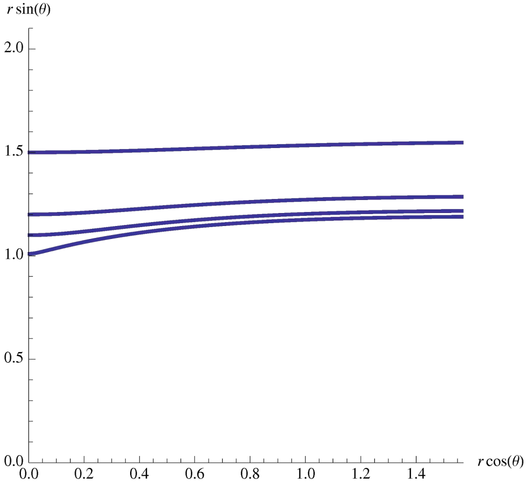
$$3r^3\cos^2\theta\sin\theta\sqrt{L} - r^3\cos^4\theta\sin\theta\frac{r^2g'^2 - \frac{\omega^2}{r^2} - \omega^2\theta'^2}{\sqrt{L}} + \partial_r\left[r^3\cos^3\theta\frac{(r^2-\omega^2\sin^2\theta)\theta'}{\sqrt{L}}\right] = 0$$

- We can solve this numerically by imposing a boundary condition

$$r\sin\theta \to m \ \text{ at } r = \infty\ , \qquad r'(\theta = \frac{\pi}{2}) = 0$$

where m is the mass of hypermultiplet

Solution 1: $g'(r) = 0$



Solution 2: $g'(r) \neq 0$

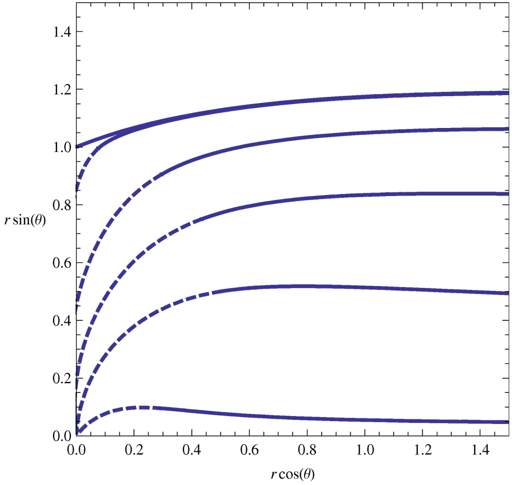
- To avoid a singularity, we require that

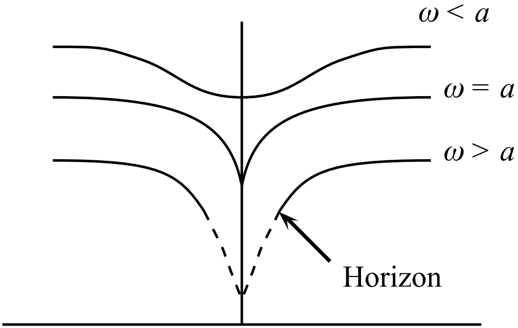
$$a^2 = \omega^2 \sin^2 \alpha \ , \qquad A^2 a^8 \cos^6 \alpha \sin^2 \alpha = 1$$

at some point $(r = a, \theta = \alpha)$

- This point becomes the horizon on the rotating D7-brane

Solution 2: $g'(r) \neq 0$





- As we change the angular velocity ω (equivalently the mass m), the shape of D7 is changed
- In the dual CFT, the mass of the hypermultiplet becomes time-dependent $m(t) = e^{i\omega t} m$

Part II: Non-stationary case

Toy model for quench

- We constructed time-dependent solution of D1-brane with the induced metric

$$ds^2 = 2drdv - (r^2 - \varphi'(v)^2)dv^2$$

- To describe a quantum quench we take ($v \equiv t - 1/r$)

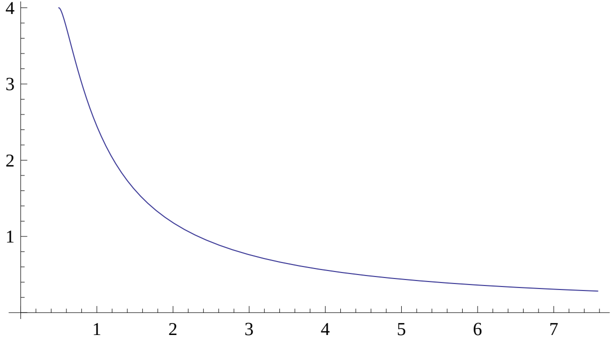
$$\varphi(v) = \varphi_0(1 + \tanh kv)$$

- If we take k very large, it approaches a step function
- The apparent horizon is located at

$$r = \varphi'(v) = \frac{k\varphi_0}{\cosh^2 kv}$$

- In this model, the relation between the temperature and the boundary time is roughly

$$T_H(t) = \varphi'(v) \ , \qquad t = v + \frac{2}{\varphi'(v)}$$



- Thermalization actually occurs!

Quantum quench in free field theory

- Consider free field theory

$$H = \int d^d k \left(\frac{1}{2}\pi_k \pi_{-k} + \frac{1}{2}\omega_k^2 \phi_k \phi_{-k} \right)$$

- If we change the mass at $t = 0$, the correlation function becomes

$$\begin{aligned} &C_{\beta_0}(k; t_1, t_2) \equiv T\big\langle \phi_k(t_1)\phi_{-k}(t_2)\big\rangle \\ &= \frac{1}{2\omega_k} e^{-i\omega_k |t_1 - t_2|} + \left[\frac{\omega_{0k}}{4} \left(\frac{1}{\omega_{0k}^2} + \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0k}}{2} - \frac{1}{2\omega_k} \right] \cos \omega_k (t_1 - t_2) \\ &\quad + \frac{\omega_{0k}}{4} \left(\frac{1}{\omega_{0k}^2} - \frac{1}{\omega_k^2} \right) \coth \frac{\beta_0 \omega_{0k}}{2} \cos \omega_k (t_1 + t_2) \end{aligned}$$

- The temperature depends on the momentum

$$\beta_{eff}(k) = \frac{1}{\omega_k} \log \frac{(\omega_k - \omega_{0k})^2 + e^{\beta_0 \omega_{0k}} (\omega_k + \omega_{0k})^2}{(\omega_k + \omega_{0k})^2 + e^{\beta_0 \omega_{0k}} (\omega_k - \omega_{0k})^2}$$

Summary

- We construct non-equilibrium system using a probe brane
- Also we construct a toy model for quantum quench