A discriminating probe of gravity at cosmological scales

Pengjie Zhang,[1, 2] Michele Liguori,[3] Rachel Bean,[4] and Scott Dodelson[5, 6]

[1]*Shanghai Astronomical Observatory, Chinese Academy of Science, 80 Nandan Road, Shanghai, China, 200030*
[2]*Joint Institute for Galaxy and Cosmology (JOINGC) of SHAO and USTC*
[3]*Department of Applied Mathematics and Theoretical Physics,*
Centre for Mathematical Sciences, University of Cambridge,
Wilberforce Road, Cambridge, CB3 0WA, United Kingdom
[4]*Department of Astronomy, Cornell University, Ithaca, NY 14853*
[5]*Center for Particle Astrophysics, Fermi National Accelerator Laboratory, Batavia, IL 60510-0500*
[6]*Department of Astronomy & Astrophysics, The University of Chicago, Chicago, IL 60637-1433*[*]

The standard cosmological model is based on general relativity and includes dark matter and dark energy. An important prediction of this model is a fixed relationship between the gravitational potentials responsible for gravitational lensing and the matter overdensity. Alternative theories of gravity often make different predictions for this relationship. We propose a set of measurements which can test the lensing/matter relationship, thereby distinguishing between dark energy/matter models and models in which gravity differs from general relativity. Planned optical, infrared and radio galaxy and lensing surveys will be able to measure E_G, an observational quantity whose expectation value is equal to the ratio of the Laplacian of the Newtonian potentials to the peculiar velocity divergence, to percent accuracy. We show that this will easily separate alternatives such as ΛCDM, DGP, TeVeS and $f(R)$ gravity.

PACS numbers:

Introduction.— Predictions based on general relativity plus the Standard Model of particle physics are at odds with a variety of independent astronomical observations on galactic and cosmological scales. This failure has led to modifications in particle physics. By introducing dark matter and dark energy, cosmologists have been able to account for a wide range of observations, from the overall expansion of the universe to the large scale structure of the early and late universe [1]. Alternatively, attempts have been made to modify general relativity at galactic [2] or cosmological scales [3, 4]. A fundamental question then arises: *Can the two sets of modifications be distinguished from one another?*

The answer is "No" if only the zero order expansion of the universe is considered. By allowing the dark energy equation of state $w_{\rm DE}$ to be a free function, the expansion history $H(z)$ produced by any modified gravity can be mimicked exactly. Fortunately, structure formation in modified gravities in general differs [5, 6, 7, 8, 9, 10, 11, 12] from that in general relativity. The difference we focus on here is the relationship between gravitational potentials responsible for gravitational lensing and the matter overdensity. Lensing is sensitive to $\nabla^2(\phi - \psi)$ along the line of sight where ϕ and ψ are the two potentials in the perturbed Friedman-Robertson-Walker metric: $ds^2 = (1 + 2\psi)dt^2 - a^2(1 + 2\phi)d\mathbf{x}^2$ and a is the scale factor. In standard general relativity (GR), in the absence of anisotropic stresses, $\phi = -\psi$, so lensing is sensitive to $\nabla^2\phi$. The Poisson equation algebraically relates $\nabla^2\phi$ to the fractional overdensity δ, so lensing is essentially determined by δ along the line of sight. This is a prediction of the standard, GR-based theory that is generally not obeyed by alternate theories of gravity.

Testing this prediction is non-trivial. Astronomers often use the galaxy overdensity as a probe of the underlying matter overdensity, but the two are not exactly equal. Here we propose a test of this prediction which is relatively insensitive to the problem of galaxy bias. The basic idea is simple:

- Extract the matter overdensity at a given redshift by measuring the velocity field. Matter conservation relates velocities to the overdensities. The measurement of the velocity field can be accomplished by studying the anisotropy of the galaxy power spectrum in redshift space.

- Extract the lensing signal at this redshift by cross-correlating these galaxies and lensing maps reconstructed from background galaxies.

More quantitatively, the galaxy-velocity cross power spectrum $P_{g\theta} \equiv -\langle\delta_g(\mathbf{k})\theta(-\mathbf{k})\rangle$ can be inferred from redshift distortions in a galaxy distribution. Here, $\theta \equiv \nabla \cdot \mathbf{v}/H(z)$ and $\mathbf{v}$ is the comoving peculiar velocity. In the linear regime, matter conservation relates θ to δ by $\theta = -\dot{\delta}/H = -\beta\delta$, where $\beta \equiv d\ln D/d\ln a$ and D is the linear density growth factor. So, $P_{g\theta} = \beta P_{g\delta}$, satisfying the first goal above. Cross correlating the same galaxies with lensing maps constructed from galaxies at higher redshifts, $P_{\nabla^2(\phi-\psi)g}$ can be measured. The ratio of these two cross-spectra is therefore is a direct probe

of $\nabla^2(\phi - \psi)/(\beta\delta)$. It does not depend on galaxy bias or on the initial matter fluctuations, at least in the linear regime. Modifications in gravity will in general leave signatures in either β and/or the Poisson equation.

Galaxy-Velocity Cross-correlation.— A galaxy's peculiar motion shifts its apparent radial position from x_z to $x_z^s = x_z + v_z/H(z)$ in redshift space, where v_z is the comoving radial peculiar velocity. The coherent velocity component changes the galaxy number overdensity from δ_g to $\delta_g^s \simeq \delta_g - \nabla_z v_z/H(z)$. The stochastic velocity component mixes different scales and damps the power spectrum on small scales. The redshift space galaxy power spectrum therefore has the general form

$$P_g^s(\mathbf{k}) = \left[P_g(k) + 2u^2 P_{g\theta}(k) + u^4 P_\theta(k) \right] F\left(\frac{k^2 u^2 \sigma_v^2}{H^2(z)} \right) \quad (1)$$

where $u = k_\parallel/k$ is the cosine of the angle of the $\mathbf{k}$ vector with respect to radial direction; P_g, $P_{g\theta}$, P_θ are the real space galaxy power spectra of galaxies, galaxy-θ and θ, respectively; σ_v is the 1D velocity dispersion; and $F(x)$ is a smoothing function, normalized to unity at $x = 0$, determined by the velocity probability distribution. This simple formula has passed tests in simulations on scales where $\delta \lesssim 1$ [13]. The derivation of Eq. (1) is quite general, so it should be applicable even when gravity is modified.

The distinctive dependence of P_g^s on u allows for simultaneous determination of P_g, $P_{g\theta}$ and P_θ [14]. The parameters we want to determine are the band powers of $P_{g\theta}(k)$ [1] defined such that $P(k) = P_\alpha$ if $k_\alpha \leq k < k_{\alpha+1}$, where $k_1 < k_2 < \cdots < k_\alpha < \cdots$. We denote $P_\alpha^{(1)}$ as the band power of $P_{g\theta}$. For a $\mathbf{k}_i$ in each k bin, we have a measurement of P_g^s, which we denote as P_i. The unbiased minimum variance estimator of $P_\alpha^{(1)}$ is $\hat{P} = \sum W_i P_i$, where $W_i = \frac{F_i}{2\sigma_i^2}(\lambda_1 + \lambda_2 u_i^2 + \lambda_3 u_i^4)$. Here, $F_i \equiv F(k u_i \sigma_v/H)$, σ_i^2 is the variance of P_i and the three Lagrange multipliers λ_α ($\alpha = 1, 2, 3$) is determined by

$$\lambda = (0, \frac{1}{2}, 0) \cdot \mathbf{A}^{-1} ; \ A_{mn} = \sum_i u_i^{2(m+n-2)} \frac{F_i^2}{2\sigma_i^2} . \quad (2)$$

Galaxy-galaxy lensing.— Weak lensing is sensitive to the convergence κ, the projected gravitational potential along the line of sight:

$$\kappa = \frac{1}{2} \int_0^{\chi_s} \nabla^2(\phi - \psi) W(\chi, \chi_s) d\chi . \quad (3)$$

Here, W is the lensing kernel. For a flat universe, χ, χ_s are the comoving angular diameter distance to the lens and source, respectively. Eq. 3 is a pure geometric result and can be applied to any modified gravity models where photons follow null geodesics.

A standard method to recover the redshift information is by the lensing-galaxy cross correlation. For galaxies in the redshift range $[z_1, z_2]$, the resulting cross correlation power spectrum under the Limber's approximation is

$$
\begin{aligned}
C_{\kappa g}(l) &= \left(4 \int_{\chi_1}^{\chi_2} n_g(\chi) d\chi \right)^{-1} \qquad (4) \\
&\times \int_{\chi_1}^{\chi_2} W(\chi, \chi_s) n_g(\chi) P_{\nabla^2(\phi-\psi)g}(\frac{l}{\chi}, z)\chi^{-2} d\chi \\
&\simeq \frac{W(\bar{\chi}, \chi_s)}{4 l \Delta\chi} \int_{l/\chi_2}^{l/\chi_1} P_{\nabla^2(\phi-\psi)g}(k, \bar{z}) dk \\
&= \sum_\alpha f_\alpha(l) P_\alpha^{(2)} .
\end{aligned}
$$

Here, $\chi_{1,2}$ are the comoving angular diameter distance to redshift $z_{1,2}$ and $\bar{\chi}$ is the mean distance. The band power $P_\alpha^{(2)}$ of $P_{\nabla^2(\phi-\psi)g}$ is defined at the same k range as $P_\alpha^{(1)}$. In practice, we measure the band power $C_{\kappa g}(l, \Delta l)$. The weighting $f_\alpha(l, \Delta l)$ is defined correspondingly. For each l, only a fraction of α having $f_\alpha(l, \Delta l) \neq 0$ contribute.

A discriminating probe of gravity.— With the above measurements, one can construct an estimator

$$\hat{E}_G = \frac{C_{\kappa g}(l, \Delta l)}{3 H_0^2 a^{-1} \sum_\alpha f_\alpha(l, \Delta l) P_\alpha^{(1)}} , \quad (5)$$

whose expectation value is

$$\langle \hat{E}_G \rangle = \left[\frac{\nabla^2(\phi - \psi)}{-3 H_0^2 a^{-1} \theta} \right]_{k=\frac{l}{\bar{\chi}}, \bar{z}} = \left[\frac{\nabla^2(\phi - \psi)}{3 H_0^2 a^{-1} \beta \delta} \right]_{k=\frac{l}{\bar{\chi}}, \bar{z}} . \quad (6)$$

The fractional error on $\hat{E}_G$ is

$$\frac{\langle \Delta E_G^2 \rangle}{E_G^2} \simeq \frac{\Delta C^2}{C_{\kappa g}^2} + \frac{\sum_\alpha f_\alpha^2 \Delta P_\alpha^{(1),2}}{(\sum_\alpha f_\alpha P_\alpha^{(1)})^2} , \quad (7)$$

where $\Delta C^2 = [C_{\kappa g}^2 + (C_\kappa + C_\kappa^N)(C_g + C_g^N)]/(2 l \Delta l f_{\text{sky}})$. Here, C_κ, C_κ^N, C_g, C_g^N are the power spectra of weak lensing convergence, weak lensing shot noise, galaxy and galaxy shot noise, respectively, and f_{sky} is the fractional sky coverage. Errors on E_G at any two adjacent bins are correlated, since they always share some same k modes. However, by requiring $l_\alpha/\chi_1 = l_{\alpha+1}/\chi_2$, where $l_1 < l_2 < \cdots < l_\alpha < \cdots$ and $k_\alpha = l_\alpha/\chi_2$, E_G measurement at each l bin only involves two k bins and thus only errors in adjacent bins are correlated.

We choose spectroscopic surveys AS2, ADEPT and SKA as targets of redshift distortion measurements, and LSST and SKA as targets of lensing map reconstruction. SKA lensing maps can be constructed through cosmic
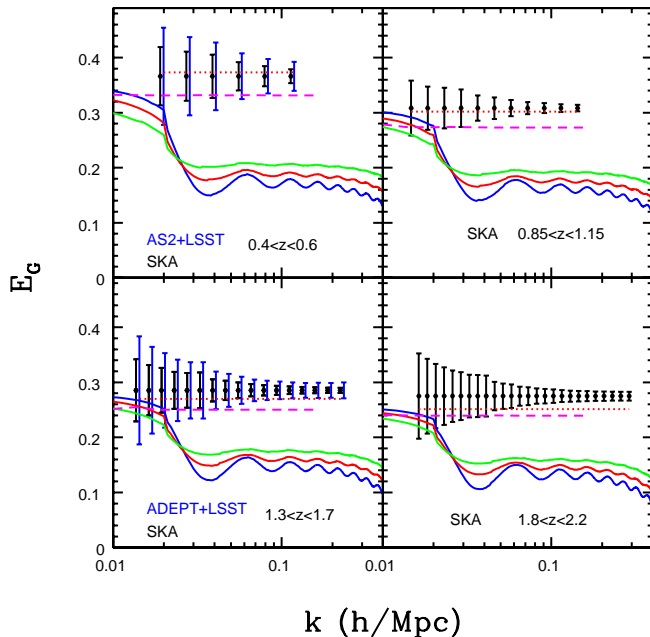
---

[1] The determination of $\mathbf{k}$ relies on the angular diameter distance $D_A(z)$ and $H(z)$. Since both quantities can be measured by other methods with high accuracy, we will assume D_A and $H(z)$ to be known. We further approximate $\sigma_v = 300$ km/s, since we restrict our analysis to the linear regime where $k^2\sigma_v^2/H^2 \ll 1$ and $F(k^2 u^2 \sigma_v^2/H^2) \simeq 1$. The error caused by this approximation is at the 1% level and negligible.

FIG. 1: E_G as a smoking gun of gravity. We only show those k modes well in the linear regime ($\Delta_m^2(k,z) \leq 0.2$) in four redshift bins. For clarity, we shift the error bars of AS2+LSST and ADEPT+LSST slightly rightward. Irregularities in the error-bars are caused by irregularities in the available discrete $\mathbf{k}$ modes of redshift distortion. Dotted lines are the results of a flat DGP model with $\Omega_0 = 0.2$. Dashed lines are for $f(R) = -\lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$ with $\lambda_2 = 100$. They have roughly the same expansion history as the fiducial cosmology at $z < 2$. Solid lines with wiggles are for TeVeS with $K_B = 0.08, 0.09, 0.1$, where the lines with most significant wiggles have $K_B = 0.1$.

magnification utilizing its unique flux dependence, with S/N comparable to that of LSST through cosmic shear [15]. Survey specifications are summarized in TABLE I. The fiducial cosmology adopted is the ΛCDM cosmology, with the WMAP best fit parameters $\Omega_0 = 0.26, \Omega_\Lambda = 1 - \Omega_0, h = 0.72, \sigma_8 = 0.77$ and $n_s = 1$. The result is shown in figure 1. We find that, errors in $C_{\kappa g}$ measurements are in general much larger than errors in $P_{g\theta}$ measurements. At $k < 0.1 h/\mathrm{Mpc}$, cosmic variance in $C_{\kappa g}$ measurements in general dominates the E_G error budget, resulting in decreasing error-bars toward larger k. This makes f_{sky} and the lensing source redshifts the two most relevant survey parameters for E_G error estimation.

We restrict our discussion to sub-horizon scale perturbations and express equations hereafter in the Fourier form. Four independent linear equations are required to solve for four perturbation variables δ, θ, ψ and ϕ. The mass-energy conservation provides two: $\dot{\delta} + H\theta = 0$ and $\dot{H}\theta + 2H^2\theta - k^2\psi/a^2 = 0$. For at least ΛCDM, quintessence-CDM, DGP and $f(R)$ gravity, the other two

TABLE I: Summary of target surveys.

	redshift	area/deg^2	N_{gal}	band
AS2[a]	z< 0.8	10,000	$\sim 1.5 \times 10^6$	optical
ADEPT[b]	$1 < z < 2$	28,700	$\sim 10^8$	infrared
SKA[c]	$z \lesssim 5$	$22,000$ (assumed)	several$\times 10^9$	radio
LSST[d]	$z \lesssim 3.5$	10,000	several$\times 10^9$	optical

[a]Private communication with Daniel Eisenstein
[b]http://www7.nationalacademies.org/ssb/BE_Nov_2006_bennett.pdf
[c]http://www.skatelescope.org/
[d]http://www.lsst.org

takes the general form

$$\phi = -\eta(k,a)\psi ,$$
$$k^2(\phi - \psi) = 3H_0^2\Omega_0 a^{-1}\delta G_{\mathrm{eff}}(k,a) . \qquad (8)$$

Here Ω_0 is the cosmological matter density in unit of the critical density $\rho_c \equiv 3H_0^2/8\pi G$. MOND has extra scalar and vector perturbations and does not follow the general form of Eq. 8 [6, 7].

(1) **ΛCDM**: $\eta = 1$ and $G_{\mathrm{eff}} = 1$. Dynamical dark energy will have large-scale fluctuations [16], but, for models with large sound speed and negligible anisotropic stress, such as quintessence, these are negligible at sub-horizon scales and Eq. 8 still holds.

(2) **Flat DGP**: $\eta = [1 - 1/3\beta_{\mathrm{DGP}}]/[1 + 1/3\beta_{\mathrm{DGP}}]$ and $G_{\mathrm{eff}} = 1$ [9], where $\beta_{\mathrm{DGP}} = 1 - 2r_c H(1 + \dot{H}/3H^2) < 0$ and $r_c = H_0/(1 - \Omega_0)$. Ω_0 differs from that of ΛCDM, in order to mimic $H(z)$ of ΛCDM.

(3) $f(R)$ **gravity**: in the sub-horizon limit, $G_{\mathrm{eff}} = (1 + f_R)^{-1}$ [11] and $\eta = 1$ [12], with $f_R \equiv df/dR|_B$ where B denotes the FRW background. This falls naturally out of a conformal transformation of the expression for E_G in the Einstein frame into the Jordan frame, noting that Einstein frame scalar field fluctuations are negligible on subhorizon scales [12]. We numerically solve the full perturbation equations in the Einstein frame since it is computationally simpler [12] and then conformally transform to the Jordan frame, which we choose as the physical frame, evaluating β such that $E_G = \Omega_0/(1 + f_R)\beta$. In the limit that $f_R \to 0$, e.g. for $f(R) \sim \lambda_1 H_0^2 \exp(-R/\lambda_2 H_0^2)$ [11] with $\lambda_1 \ll \lambda_2$, the evolution is observationally equivalent to ΛCDM. For modes that entered the horizon prior to matter-radiation equality, as we consider here, β, and therefore E_G, is scale invariant for IR modifications to gravity, with $f_R > 0$.[2] The scale independence of D and β also holds in ΛCDM, Quintessence-CDM, DGP. An observed scale-independent deviation in E_G from ΛCDM could signify a special class of modified gravity, as shown in Fig. 1.

___

[2] Scales larger than the horizon at matter-radiation equality are suppressed [12] and, if measurable, would have a scale dependent increase in the value of E_G in comparison to the small scale value.

(4) **TeVeS/MOND**. Besides the gravitational metric, TeVeS [2] contains a scalar and a vector field. These new fields act as sources for the gravitational potential ϕ in the modified Poisson equation and can change the evolution of cosmological perturbations with respect to standard gravity [6, 7]. We considered a TeVeS model with $\Omega_b = 0.05$, $\Omega_\nu = 0.17$, $\Omega_\Lambda = 0.78$ and we adopted a choice of the TeVeS parameters that produces a significant enhancement of the growth factor. The TeVeS E_G is significantly different from the standard E_G (Fig. 1).[3] It exhibits scale dependence with accompanying baryons acoustic wiggles. Both features are due to the vector field fluctuations, which *play a significant role* in structure formation [7]. These fluctuations decrease toward small scales and cause the scale dependency of E_G. We also checked that they affect the final shape of the acoustic oscillations of the other components significantly. As a result, oscillations in ϕ, ψ and δ do not cancel out perfectly in TeVeS when we take the ratio, thus producing the wiggles in E_G.

For the four gravity models investigated, differences in E_G are much larger than observational statistical uncertainties. Planned surveys are promising to detect percent level deviation from GR and should distinguish these modified gravity models unambiguously.

At large scales, gravity is the only force determining the acceleration of galaxies and dark matter particles. So we assumed no galaxy velocity bias. As statistical errors in E_G measurements reach the 1% level (Fig. 1), several other systematics may become non-negligible. One is the accuracy of the redshift distortion formula (Eq. 1), which may be problematic for those modes with large u, even at very linear scales [13]. A remedy is to exclude them when extracting $P_{g\theta}$, at the expense of statistical accuracy. As discussed before, accuracy of E_G measurement is dominated by accuracy of $P_{\nabla^2(\phi-\psi)g}$ measurements and is thus less affected. A less severe one is the nonlinear evolution, which becomes non-negligible at $\Delta_m^2 \gtrsim 0.1$. In general relativity, nonlinear corrections to density and velocity differ (Fig. 12, [17]). A direct consequence is that E_G develops a dependence on the matter power spectrum. Similar effects in modified gravity models are expected. This can be corrected by high order perturbation calculations, which should work well where $\Delta_m^2 \lesssim 0.2$.

We thank Daniel Eisenstein, Bhuvnesh Jain and

---

[1] See, e.g. D. Spergel, et al. 2006, astro-ph/0603449; M. Tegmark, et al. 2006, astro-ph/0608632; A. Riess et al. 2006, astro-ph/0611572;

[2] M. Milgrom. ApJ, 207, 371 (1983); Jacob D. Bekenstein, Phys.Rev. D70 (2004) 083509

[3] Gia Dvali, Gregory Gabadadze, Massimo Porrati. Phys.Lett. B485 (2000) 208; C. Deffayet, PLB, 502, 199 (2001);

[4] Sean M. Carroll, Vikram Duvvuri, Mark Trodden, Michael S. Turner. Phys.Rev. D70 (2004) 043528 ; Sean M. Carroll, Antonio De Felice, Vikram Duvvuri, Damien A. Easson, Mark Trodden, Michael S. Turner. Phys.Rev. D71 (2005) 063513;

[5] M. White, C.S. Kochanek, 2001, ApJ, 560, 539; Akihito Shirata, Tetsuya Shiromizu, Naoki Yoshida, Yasushi Suto, Phys.Rev. D71 (2005) 064030; C. Sealfon, L. Verde & R. Jimenez, Phys.Rev. D71(2005) 083004; H. Stabenau, B. Jain, Phys.Rev. D74(2006)084007

[6] C. Skordis, D. F. Mota, P. G. Ferreira, C. Boehm. Phys.Rev.Lett. 96 (2006) 011301 ; Constantinos Skordis, Phys.Rev. D74 (2006) 103513

[7] Scott Dodelson, Michele Liguori. 2006, Phys.Rev.Lett. 97 (2006) 231301

[8] Arthur Lue, Roman Scoccimarro, Glenn Starkman. Phys.Rev. D69 (2004) 124015; L. Knox, Y.-S. Song, J.A. Tyson. 2005, astro-ph/0503644; Mustapha Ishak, Amol Upadhye, David N. Spergel. Phys.Rev. D74 (2006) 043513

[9] Kazuya Koyama, Roy Maartens. JCAP 0601 (2006) 016

[10] Tomi Koivisto, Hannu Kurki-Suonio. Class.Quant.Grav. 23 (2006) 2355-2369; Tomi Koivisto. Phys.Rev. D73 (2006) 083517; B. Li, M.-C. Chu. Phys.Rev. D74 (2006) 104010; Yong-Seon Song, Wayne Hu, Ignacy Sawicki. 2006, astro-ph/0610532; B. Li & J. Barrow. 2007, gr-qc/0701111

[11] Pengjie Zhang. Phys.Rev. D73 (2006) 123504

[12] Rachel Bean, David Bernat, Levon Pogosian, Alessandra Silvestri, Mark Trodden. Phys.Rev. D75 (2007) 064020

[13] Roman Scoccimarro. Phys.Rev. D70 (2004) 083007

[14] Max Tegmark, Andrew J. S. Hamilton, Yongzhong Xu. Mon.Not.Roy.Astron.Soc. 335 (2002) 887-908

[15] P. Zhang & U.L. Pen 2005, Physical Review Letters, 95, 241302

[16] J. Weller & A.M. Lewis, Mon.Not.Roy.Astron.Soc. 346 (2003) 987; R. Bean & O. Dore, 2004, PRD, 69, 083503

[17] F. Bernardeau, S. Colombi, E. Gaztanaga, R. Scoccimarro. Phys.Rept. 367 (2002) 1-248; astro-ph/0112551

---

[3] To simplify the numerical treatment of the TeVeS perturbations equations while retaining a good qualitative description of all the significant physical effects at the same time, we introduced several approximations. Namely we assumed instantaneous recombination and employed the tight coupling approximation between baryons and photons at all scales before decoupling; moreover we evolved perturbations in the massive neutrino component in a simplified way by switching off neutrinos perturbations when they were below the free steaming scale and treating them as a fluid above the free streaming scale.