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PROBLEMS OF HEAVY ELECTRIC TRACTION.

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A study of the general subject of electric traction for the suburban lines of present steam roads discloses some interesting features which do not necessarily arise as important factors in the selection of equipment for rapid transit lines in cities or the usual interurban lines. Furthermore, the many variables that enter into the problem lead to a serious question as to the necessity of the elaborate methods of determining train and motor characteristics which have been put forth in the recent literature on heavy electric traction. The advantage of simpler and shorter methods becomes very apparent.

Work now in progress for the Long Island Railroad Company may be taken as typical of projects of this character. It is the purpose of this paper to place before the INSTITUTE some of the questions brought out in the engineering of this work and to describe more particularly some of the simple things that were done to facilitate decision regarding motor equipments for cars.

The Long Island Railroad system has at present two passenger terminals in the City of New York, one at Atlantic Avenue and Flatbush Avenue in the Borough of Brooklyn and one at the Thirty-fourth Street Ferry in Long Island City, Borough of Queens. When the tunnels of the Pennsylvania, New York, and Long Island Railroad Company are completed there will be a third terminal for the Long Island Railroad passenger traffic in the Borough of Manhattan. From the present terminals there branches out an extensive system of through and suburban lines, over which is maintained a train service, both express and local, reaching all parts of the Island. In addition to the suburban service into these terminals there is an interchange of traffic with the Brooklyn Rapid Transit Company at two points, and a connection is to be established between the subway system of the Long Island Railroad in Atlantic Avenue and the extension of the Rapid Transit Subway now being constructed between Brooklyn and the south end of Manhattan. The accompanying map shows the location of the western lines of the Long Island Railroad and the new connections which are to be made.

Extensive plans for the electrification of the suburban service of this road have been under consideration for some years. These plans provide for the immediate adoption of electric traction on the lines emanating from Flatbush Avenue and for a progressive extension of electric traction over all of the strictly suburban routes, as rapidly as the conditions shall justify. On some of these routes the traffic is of a purely suburban nature, providing facilities for out-of-town residents to reach Manhattan and Brooklyn. On other sections the important traffic is an excursion movement to and from New York's great playgrounds at the beaches. For the convenience of the suburban residents, a frequent service in comparatively short trains is to be provided, but for the excursion service comparatively long trains are necessary. As a result, the number of cars per train in this system will vary from one to eight or possibly ten. The initial service, limited to a maximum of six cars per train, results in 52 types of runs varying in number of cars per train and distance between stops. On some sections the lines are fairly free from grades and curves, but on others the grades and curves are heavier than would be supposed from a general knowledge of the contour of Long Island. These few statements will indicate in a general way the complex nature of the problem involved in the selection of a suitable electrical system for such a service and in the determination of types and sizes of apparatus to be used. In the working out of large enterprises important details are frequently settled by quite simple processes of reasoning. Even in a complex problem like the one under consideration, some main points may be decided in this way.

(a) The number of cars per train varies from one to six, as stated. Eight- or ten-car trains may be desirable at times. Such variations in train length cannot be provided for economically with electrical locomotives. Motor-cars must, therefore, be used, and for all except trains of one or two cars, more than one motor car per train is necessary. The multiple-unit system of motor-car operation is, therefore, best suited to this service.

(b) The greatest flexibility for make-up of trains is obtained by making all cars motor-cars.

(c) On the other hand the first cost of equipment and the cost of maintenance and inspection is least if the motor selected is of the largest size practicable, and the number of equipments required is thus made a minimum.

(d) For a miscellaneous service of the character contemplated all trains, both local and express, should preferably be provided with equipment having the same speed characteristics, so that all motor cars are available for all classes of service. (Trains may operate as express in one direction and as local in the other.)

(e) For express runs of this suburban service, averaging not over five miles between stops, a moderately high speed, say 50 to 55 miles per hour, is found by experience in steam practice to be most suitable. For these runs, therefore, it is essential that the electric trains shall be able to approximate these speeds.

(f) Future development of the service, such as increase in number of stops and decrease in running time, will mean heavier work for the motors. Therefore, reasonable reserve must be provided in the equipment, and consideration must be given to equipment which will make possible a convenient increase in number of motors per train.

Size of Motors:

The largest motor in general use for motor-car trains is of 200 h.p. nominal rating, two of these being about the limit of motor capacity that can be placed on a truck with 33- or 36-inch wheels and a reasonably short wheel-base. On the basis of condition (c), this should be the motor adopted, provided the train combinations can be effected satisfactorily with motors of this size. Condition (f) is also complied with by selecting the largest motor practicable, as the number of motors per train in the initial service will then be a minimum.

Gear Ratio:

To meet condition (e), a gear ratio must be selected which permits the operation of trains at high speeds without damage to the equipment. On the other hand, the best starting conditions and least heating effect are obtained with high gear ratio. The gear ratio should, therefore, be as high as practicable and still permit of the maximum train speeds necessary for the express schedules. Provision must also be made for extra speeds which may occur on down grades. From observation of the existing service it was concluded that it is safe to assume 60 miles per hour as a maximum.

Train Weights:

The motor cars of the service are to be new steel cars similar to those adopted for the Rapid Transit Subway. For the determination of motor characteristics, trailer cars were assumed to have the same size and weight of bodies. The weights assumed were as follows:

	Motor Car.	Trailer.
Seated Load....................	81 000 lb.	60 400 lb.
Standing Load..................	88 000 lb.	66 000 lb.

These assumed weights are in practical agreement with the actual weights of the subway equipment

The 200-h.p. motor was found to be the proper size for the service of the subway; but the schedule conditions are so different that this in itself does not signify that the same motors are suitable for the Long Island Railroad conditions where the distance between stops for most of the local runs is greater than that of the express runs of the subway, and the express runs are longer in proportion. More or less of the usual calculating must, therefore, be done to reach a decision as to whether the characteristics of the standard motor of this size are suitable for this service or whether a smaller size could be used to advantage.

In Volume 19 of the Transactions of the A. I. E. E. and in the technical press of the last two or three years, there are a number of interesting papers giving theoretical and practical information regarding electric motors for railway service. Of these papers that of Messrs. Arnold and Potter on Acceleration Tests gives some very interesting results of actual records of speed-time tests of steam and electric trains of different weights, illustrated by diagrams which admit of close comparison between theoretical and practical results. The scholarly and somewhat bewildering papers by Mr. Mailloux and by Dr. Hutchinson are of interest as representing the academic side of some of the problems involved; and Mr. Gotshall contributes some very interesting illustrations of theoretical speed-time curves derived for a specific case. The subjects of train resistance, acceleration rates, and rating of motors have been reported on at length and each investigator has viewed them from a different standpoint. The effect of the inertia of rotating parts of a train, previously neglected, has also been given its full share of importance. With all this information at hand, however, there are still some very pertinent questions which arise in the mind of an engineer approaching the solution of such a problem:

Who is right?

How much of this tedious calculating is necessary?

Whether used to overcome mechanical friction, head resistance, gravity, or inertia, how much power must be delivered to the motor cars to move a train from one terminus to the other?

How close will practice work out to theory?

What allowance must be made for local limitations on the possible speed of train equipments?

Some of the things that were done to obtain satisfactory answers to these questions may be of interest to the members of the INSTITUTE.

I. PLOTTED AND TEST RUNS COMPARED:

The power required to overcome inertia and gravity may be calculated with exactness. Train resistance, however, made up of head resistance, mechanical friction, and skin friction, may be estimated only by empirical formulas, based on practical tests. Such formulas have been derived and published by various engineers. Owing to differences in character of equipment and road-bed, nature of the train service, design of car bodies and trucks, methods of test, etc., there are material differences in the conclusions reached. For ordinary electric car service on urban and suburban lines, where the number of stops is frequent and the maximum speed is low, the train resistance is relatively unimportant as compared with the effect of inertia and grades, but as the speed increases the train resistance becomes a more important matter. The lack of agreement of these formulas when applied to the conditions of the L. I. R. R. are illustrated in Fig. 1. The Davis and the Baldwin Locomotive Works' formulas are extremes and they therefore afford a sufficient illustration of the divergence of authorities on this subject. These formulas are expressed as follows:

$$\text{Baldwin Locomotive Works, } R = 3 + \frac{V}{6.}$$

$$\text{W. J. Davis,} \quad R = 4 + 0.13\,V + \frac{.004\,A\,V^2}{T}\,[1 + 0.1\,(n-1)]$$

R = Train resistance in lb. per ton of 2000 lb.
V = Velocity in miles per hour.
A = Cross sectional area of car in sq. ft.
T = Weight of train in tons of 2000 lb.
n = Number of cars per train.

The formula marked "Smith" is one suggested by one of the authors and has been found to give results close to actual practice as will be explained in the following paragraphs. This formula has the following expression:

$$R = 3 + 0.167\,V + 0.0025\,\frac{A}{T}\,V^2.$$

COMPARATIVE TRAIN PERFORMANCES COMPUTED WITH DIFFERENT RESISTANCE FORMULAS.

2-car train; weight, 70.6 tons; 2 Westinghouse No. 86 motors; 19:63 gear; 36-in. wheel; run, 1.67 miles, level; 540 volts.

Formula.	Time Seconds. Total	Time Seconds. Power on	Per cent. Time Power on	Speed miles per hr	kw-hr	Watt-hr. per ton Mile.
Davis	210	187	89	28.6	7.51	63.6
Smith	210	136	65	28.6	5.76	48.8
Baldwin Locomotive Works	210	103	49	28.6	4.66	39.5

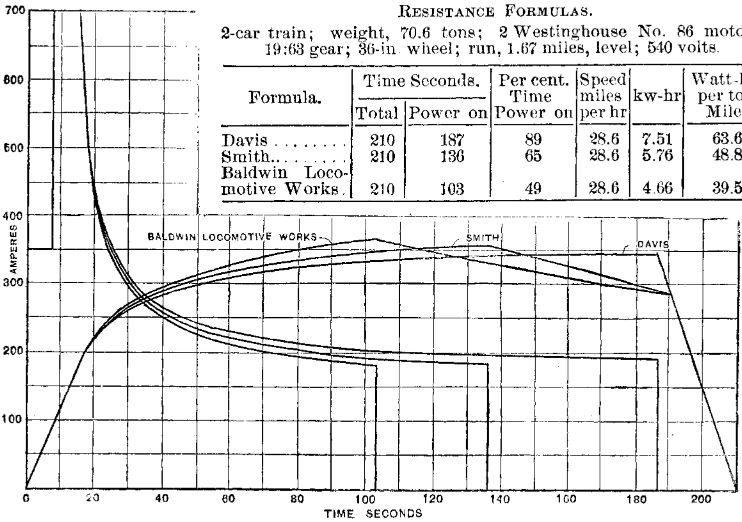


FIG. 1.

The speed-time curves in Fig. 1 are derived for a two-car train with cars of weight and dimensions similar to those of the Rapid Transit Railway in Manhattan. The length of run is an average for the total Long Island suburban service. It will be noted that the power consumption (watt-hours per ton mile) varies from 39.5 to 63.6, the figure according to Davis' formula being 61 per cent. greater than the Baldwin figure. Moreover if the conditions were such that the average speed of 28.6 miles an hour between stations had to be maintained, the speed-time curve of Davis would be impracticable because of no allowance

TABLE 1.

AVERAGE SPEED AND WATT-HOURS PER TON MILE

Comparison of Theoretical Results with Actual Tests.

G. R. G. H. & M. RY.

(Schedule speed the same.)

Two Westinghouse No. 50-C Motors. Weight of train 29.5 tons.

Test No.	Davis		Smith		Test	
	Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile
1	27.9	98.	27.9	79.2	27.9	88.2
2	32.7	99.2	32.7	81.6	32.7	81
3	30.4	85.7	30.4	75.0	30.4	86.2
4	30.4	115.1	30.6	111.5	30.6	110.

D. Y. A. A. & J. RY.

(Schedule speed the same.)

Four Westinghouse No. 76 Motors. Weight of train 32 tons.

Test No.	Davis		Smith		Test	
	Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile
5	28.3	99.	28.3	94.	28.3	99.
6	31.5	102.	31.5	96,	31.5	105.
7	33.4	95.	33.4	89.	33.4	97.

ARNOLD & POTTER TESTS.

(Power to be cut off at same distance from start, if possible. Schedule speed not the same.)

8 G. E. No. 55 Motors.

Test No.	Weight Train Tons	Davis		Smith		Test	
		Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile	Miles per hour	Watt-hours per ton mile
8	228.5	26.6	87.	27.6	72.	27.1	79.4
9	201.5	27.7	98.5	28.4	84.5	28.4	82.
10	175.5	29.2	102.	30.9	84.	29.9	86.9
11	148.5	29.6	111.	30.6	89.5	30.6	93.4

in coasting to provide for possible delays, etc. Therefore, whereas the Baldwin formula indicates a satisfactory equipment working with very low power consumption, the Davis formula for the same equipment shows relatively a very high power consumption and an impracticable equipment. The necessity of careful investigation of this matter in connection with our problem is apparent.

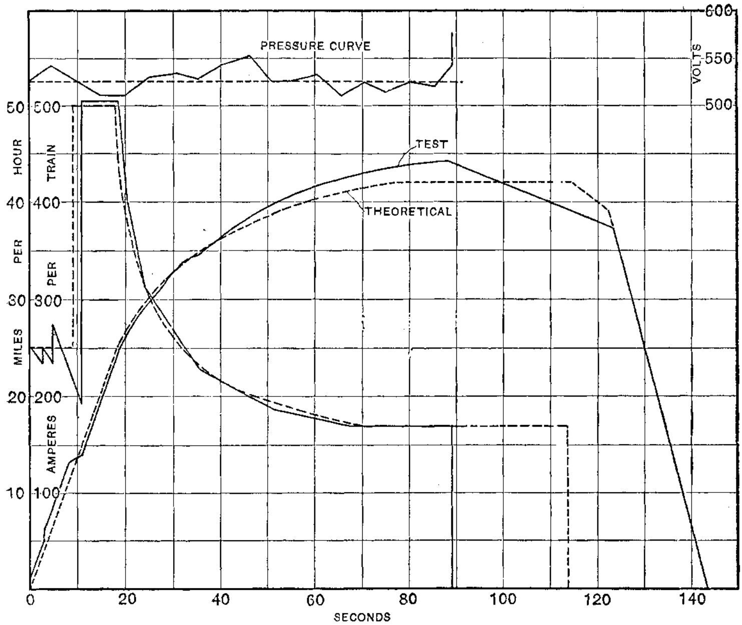


FIG. 2.

Test No. 2—G. R. G. H & M. Ry.

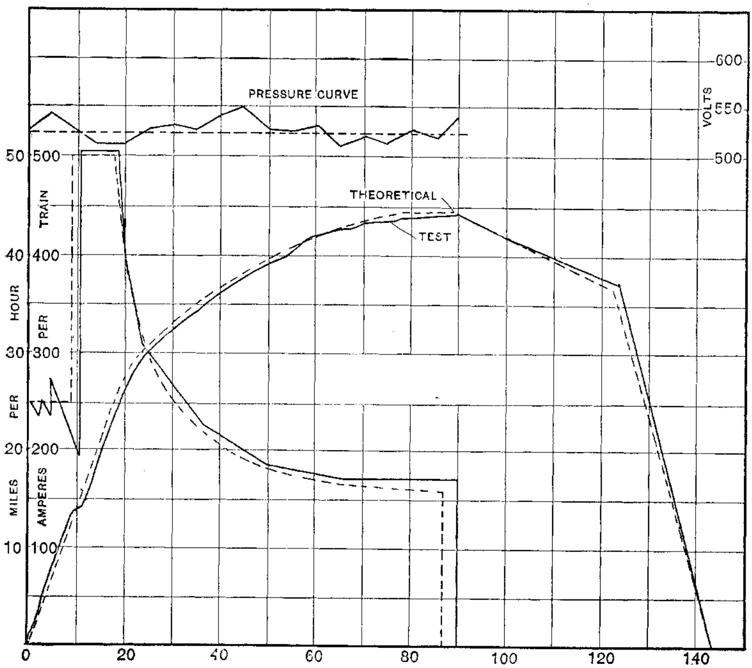
	Results. Plotted.	Test.
Time, seconds	144	144
Distance, miles	1.31	1.31
Schedule speed miles per hr	32.7	32.7
Kilowatt-hours	3.83	3.13
Watt-hr. per ton mile	99.2	81

Single car; weight 29.5 tons; 2 Westinghouse No. 50 C. Motors. *Davis* formula used in plotting run. Note — Theoretical speed curves are corrected for variations in grade and pressure.

APPLICATION OF FORMULAS:

The essential points which we wish to determine are the schedule speeds which can be obtained with given equipments, the power that will be required to obtain these speeds, and the heating effect on the motors. The value of train resistance is best determined at full speed, or while coasting; but except in the case of long runs most of the power is consumed in accelerating. By comparing actual test runs with plotted runs from start to stop, we get some check on the train resistance formulas, but, what is of more immediate importance, we get a verification of the accuracy of all the elements entering into our calculations.

To effect a comparison of theoretical results with actual tests,



FIG, 3.

Test No. 2—G. R. G. H. & M. Ry.

	Results.	
	Plotted.	*Test.*
Time, seconds............	144	144
Distance, miles...........	1.31	1.31
Schedule speed miles per hr.	32.7	32.7
Kilowatt-hours...........	3.16	3.13
Watt hr. per ton mile......	81.6	81

Single car; weight 29.5 tons; 2 Westinghouse No. 50 C. motors. *Smith* formula used in plotting run. Note — Theoretical speed curves are corrected for variations in grade and pressure.

some special tests were made on the Grand Rapids, Grand Haven and Muskegon Railway, and on the Detroit, Ypsilanti, Ann Arbor & Jackson Railway in Michigan. These tests were necessarily confined to trains of one car each. The tests reported by Messrs. Arnold and Potter, made at Schenectady, were on trains of greater lengths. To supplement the single-car data, theoretical runs of trains of identical composition and equipment were plotted for comparison with some of these Schenectady tests. The various comparisons made are summarized in Table 1, which, therefore, shows the comparison of theoretical and actual results for trains of from one to eight cars per train.

On the G. R. G. H. & M. road the equipment consists of two

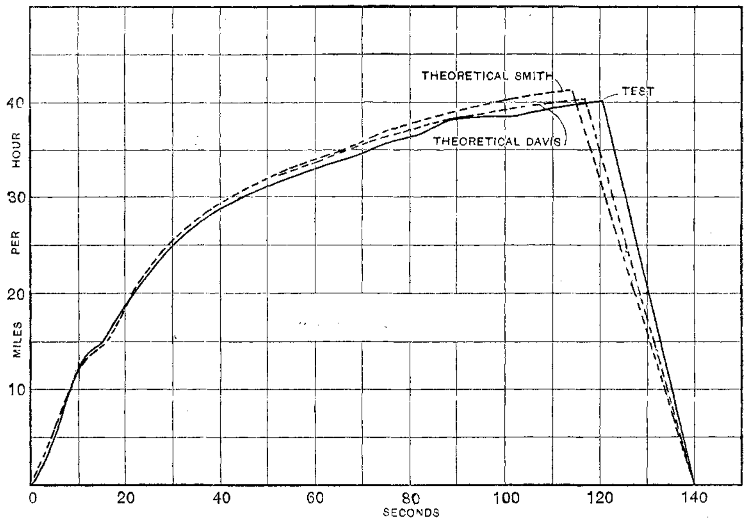


FIG. 4.

Test No. 5—D. Y. A. A. & J. Ry.

Results.	*Davis.*	*Test.*	*Smith.*
Time, seconds	140	140	140
Distance, miles	1.1	1.1	1.1
Schedule speed miles per hr.	28.3	28.3	28.3
Kilowatt-hours	3.49	3.49	3.33
Watt-hr. per ton mile	99	99	94

Single car; weight 32 tons; 4 Westinghouse No. 76 motors. *Davis* and *Smith* formulas used in plotting runs.

Note—Power cut off at same distance from start. Average speed between stops maintained constant by varying braking rate.

Theoretical speed curves are corrected for variations in grade and pressure.

150-h.p. motors mounted on a heavy Baldwin truck with Gibbs suspension. The unequipped truck is also of the Baldwin make and the entire equipment is of the highest standard. On the D. Y. A. A. & J. road the equipment consists of four 75-h.p. motors mounted on rather light trucks. Both roads have first-class roadbeds, well ballasted, and laid with 70-lb. rails of standard A. S. C. E. section. The tests were made on sections of track practically straight and level. A chronograph was used to determine the speed at every revolution of the wheels, and the current and pressure were read by observers simultaneously at five-second intervals. From these tests speed-time and current-time curves were plotted. In making these tests the instructions were to accelerate and brake at as high a rate as practicable; therefore, the curves obtained may be considered as representing the limit obtainable in practice with the existing conditions.

For comparison with these actual results, theoretical runs were plotted in which the length of run was made the same, the average speed was made the same where possible, and the average rate of braking was assumed to be the same as in the actual experiments. The curves were corrected for pressure and grade so as to conform as closely as possible to the conditions prevailing during the test. The observed current readings were plotted and also the assumed theoretical values of the current corresponding to calculated speeds. The watt-hours per ton-mile were calculated from the observed tests either from wattmeter readings or by multiplying together the simultaneous voltmeter and ammeter readings. In the case of calculated runs, the value was obtained by integration, proper allowance being made for variation in pressure. The results of all the various tests are shown in Table 1.

The characteristics of the test curves and the comparison with plotted runs on the G. R. G. H. & M. Ry. are illustrated by Figs. 2 and 3. The differences resulting from the use of the Davis or the Smith formula are also indicated.

The D. Y. A. A. & J. Ry. tests are also summarized in Table 1. The character of the results obtained is illustrated in Fig. 4. In these tests there was no coasting and the conditions were such as to cause a considerable drop in pressure during the first part of acceleration. This may have caused sufficient error in the readings to account for the lack of agreement between tests and the calculations based on the Smith formula. The power curves were plotted and the watt-hours per ton-mile were computed therefrom as in the other cases, but the current curves have not been reproduced in this Fig. on account of the difficulty of plotting them all without confusion.

The comparison between actual results and theoretical runs in the case of the Arnold and Potter tests was effected in the same manner as for the single car tests. See Table 1 and

Figs. 5, 6, 7, and 8. It is at once evident from the comparative curves and the table of results that either of the two formulas for train resistance is reasonably close to the actual conditions. The exhibits when analyzed lead to the following conclusions as to the relative applicability of the two formulas:

The acceleration curve from the Davis formula conforms more

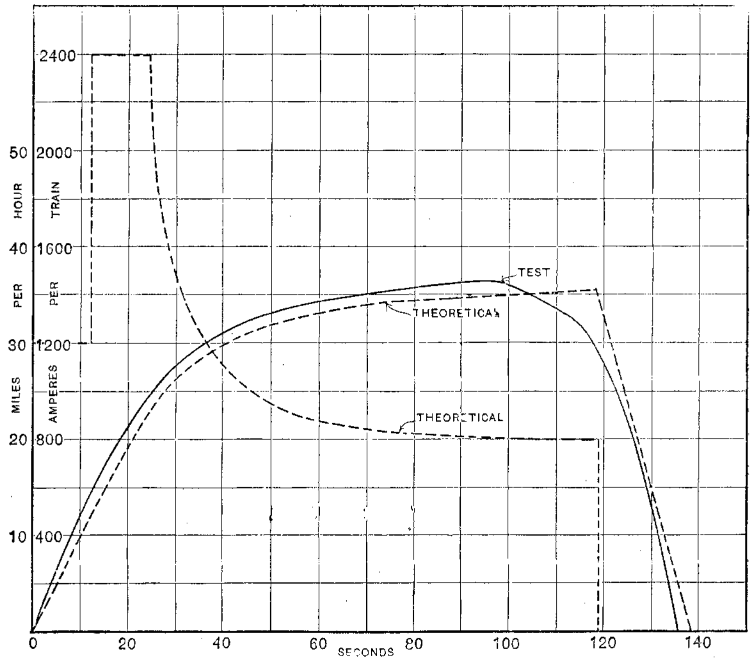


FIG. 5.

Test No. 8—Arnold & Potter. Run No. 1.

Results.	*Plotted.*	*Test.*	8 cars; weight 228.5 tons; 8 G.E. No. 55.
Time, seconds	138	135.5	*Davis* formula used in plotting run.
Distance, miles	1.02	1.02	Head wind, 15 miles per hr.
Schedule speed miles per hr.	26.6	27.1	Average pressure, 570 volts.
Kilowatt-hours	20.3	18.5	Starting current, 300 amperes per motor.
Watt-hr. per ton mile	87	79.4	by wattmeter.

closely to the tests for the D. Y. A. A. & J. car, but for the G. R. G. H. & M. car and for the Arnold and Potter trains it falls below the actual curve, in some cases to such an extent that the theoretical schedule speed is below the actual even with coasting eliminated. The Smith formula gives accelerations practically coincident with the G. R. G. H. & M. tests and somewhat higher than the actual for the other tests, the difference in any case not being sufficient to be misleading in the results as to schedule speed. This difference may easily be accounted for by difference in pressure or inaccuracies in observation of instruments. In the case of the D. Y. A. A. & J. car, the type of trucks or the drops in pressure may either of

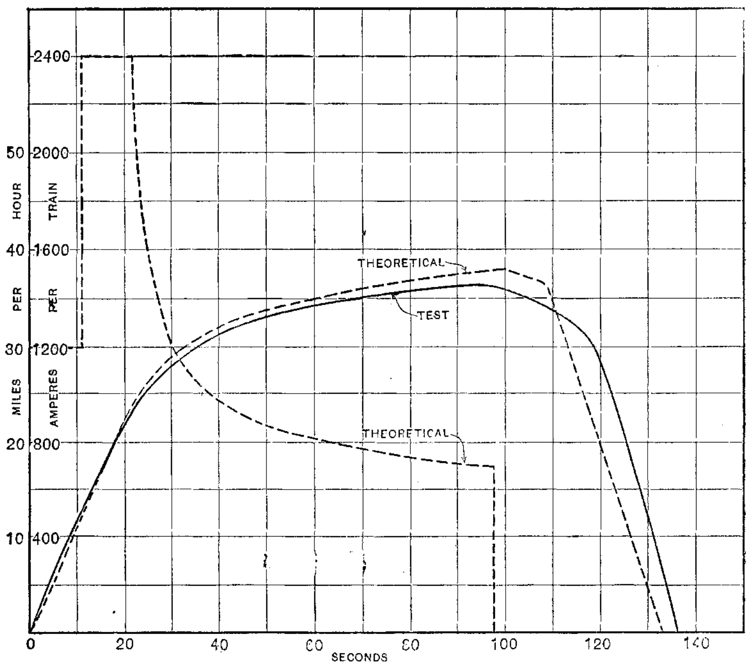


FIG. 6.

Test No. 8—Arnold & Potter. Run No. 1.

Results.	*Plotted.*	*Test.*	
			8 cars; weight 228.5 tons; 8 G.E. No. 55.
Time, seconds	133	135.5	*Smith* formula used in plotting run.
Distance, miles....	1.02	1.02	Head wind, 15 miles per hr.
Schedule speed			Average pressure, 570 volts.
miles per hr.....	27.6	27.1	Starting current, 300 amperes per
Kilowatt-hours....	16.7	18.5	motor.
Watt-hr. per ton			
mile............	72	79.4	by wattmeter.

them have caused the reduced acceleration. In the case of the Arnold and Potter tests the most decisive point in the comparison is the fact that whereas the use of the Smith formula permitted of cutting off the power at the same distance from the start as in the test, the Davis formula resulted in the theoretical runs in which the schedule speeds were lower than the tests even when the power was applied up to the point where braking began. In other words, if the Davis formula be correct the trains could not have made the runs in the time that they actually did make without an excessive braking rate. The conclusion, therefore, is that the Smith formula most nearly represents the actual train resistance for all of these tests.

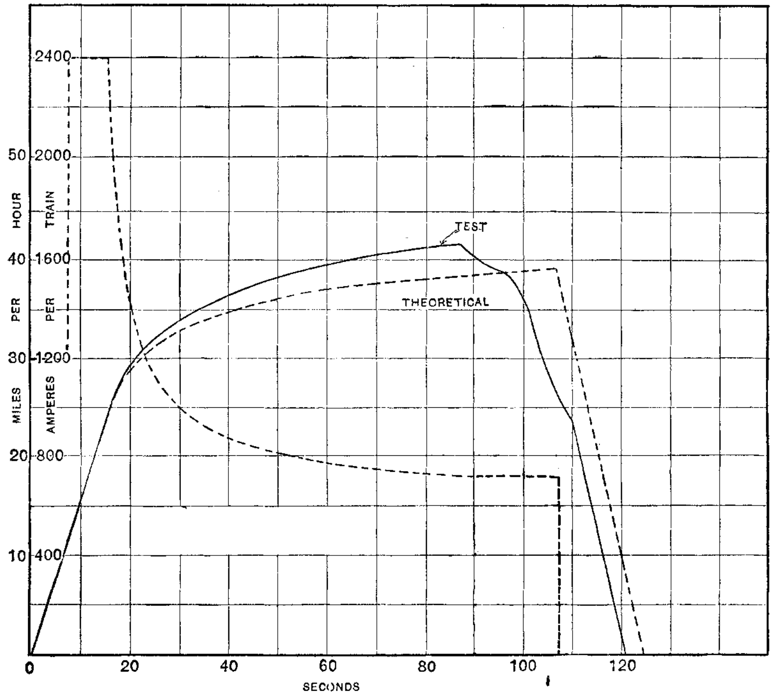


FIG. 7.

Test No. 11—Arnold & Potter. Run No. 7.

Results.	Plotted.	Test.	
			5 cars; weight 148.5 tons; 8 G.E. No. 55 motors.
Time, seconds	124	120	*Davis* formula used in plotting run.
Distance, miles	1.02	1.02	Head wind, 15 miles per hr.
Schedule speed miles per hr.	29.6	30.6	Average pressure, 570 volts. Starting current, 300 amperes per motor.
Kilowatt-hours	16.7	14.3	
Watt-hr. per ton mile	111	93.4 by wattmeter.	

With reference to power required, the actual results appear to fall between the results obtained with either of the two formulas. The calculations based on the Smith formula are closer to the actual results in most cases. For the G. R. G. H. & M. car, where the acceleration curves agree, the watt-hours per ton mile are also very close, and in the Arnold and Potter tests even with a reduced schedule speed the watt-hours per ton-mile are considerably greater for the Davis formula than for the observed tests. The conclusion reached is that the Smith formula affords a better basis for calculating the average speeds,

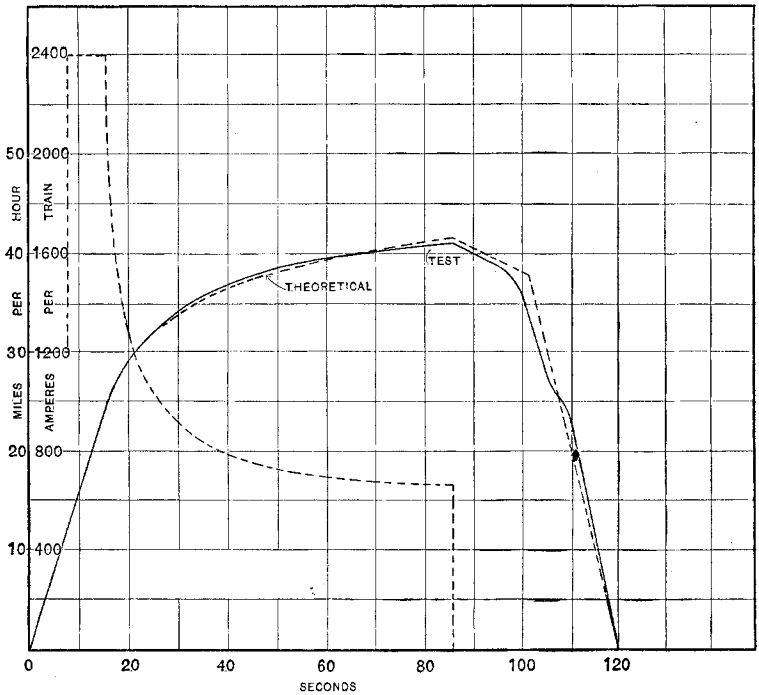


FIG. 8.

Test No. 11—Arnold & Potter. Run No. 7.

	Results.		
	Plotted.	*Test.*	5 cars; weight 148.5 tons; 8 G.E. No. 55.
Time, seconds.....	120	120	*Smith* formula used in plotting run.
Distance, miles....	1.02	1.02	Head wind, 15 miles per hr.
Schedule speed			Average pressure, 570 volts.
miles per hr.....	30.6	30.6	Starting current, 300 amperes per
Kilowatt-hours....	13.5	14.3	motor.
Watt-hr. per ton			
mile...........	89.5	93.4 by wattmeter.	

and is sufficiently close for the determination of power required. Furthermore the comparative results with these various train weights and types of motor, give us a confidence in all the assumptions made and in the customary use of the motor curves and other data furnished by the manufacturers.

Test of Equipment Similar to that Proposed for the Long Island Railroad.

Through the courtesy of the Interborough Rapid Transit Company and Westinghouse Electric & Manufacturing Company it is possible to present the record of a special test which

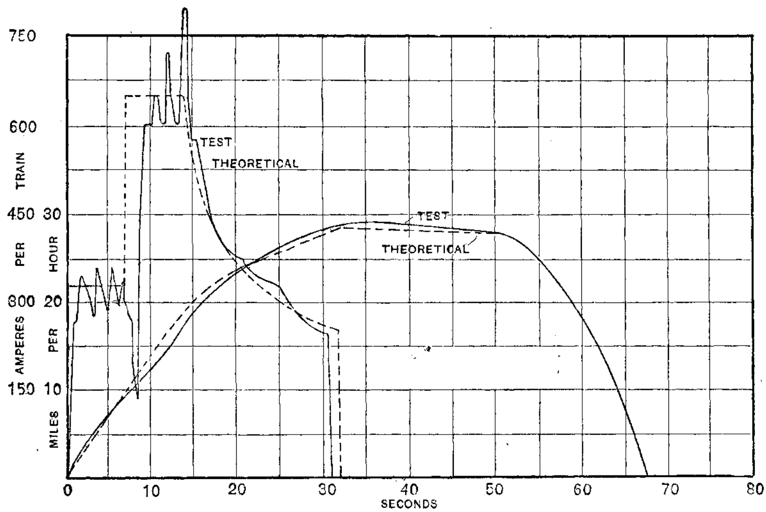


Fig. 9.

Comparison of Test and Theoretical Train Curve Using Actual Braking Curve.

Test made on 9th avenue line of Manhattan Ry. Co., Jan. 30, 1903.

Pressure 580 volts; 1 motor car; 1 trailer; weight 62.5 tons; distance 2065 ft., 93d Street to 104th Street; 2 Westinghouse No. 86 motors; gear 19:63; 33-in. wheels.

	Results.	
	Test.	*Theoretical.*
Average speed without stop	20.9	20.9
Schedule speed with 15 sec. stop	17.05	17.05
Kilowatt-hours per run	1.93	2.13
Kilowatt-hours per car mile	2.475	2.72
Watt-hours per ton mile	79.3	87.2
Average kilowatts per train	84.3	92.9
Sq. root mean sq. current per motor for 85 sec.	149.0	157.5
Starting current per motor	294.	325.

Smith formula used in plotting run.

has a direct bearing on this subject, because the type of motor equipment used in the test is the same electrically as that recommended for the L. I. R.R. project. On January 30, 1903, a test was made on the Ninth Avenue line of the Manhattan Elevated Railway, of the sample motors submitted by the Westinghouse Electric & Manufacturing Company as representing the motors which that company was to furnish to the Interborough Rapid Transit Company. In these tests the train weight corresponded closely with the weight per motor of the equipment proposed for the L. I. R.R. and the motors were of the same size and general characteristics. The length of the test run was too short to permit the cars to reach maximum

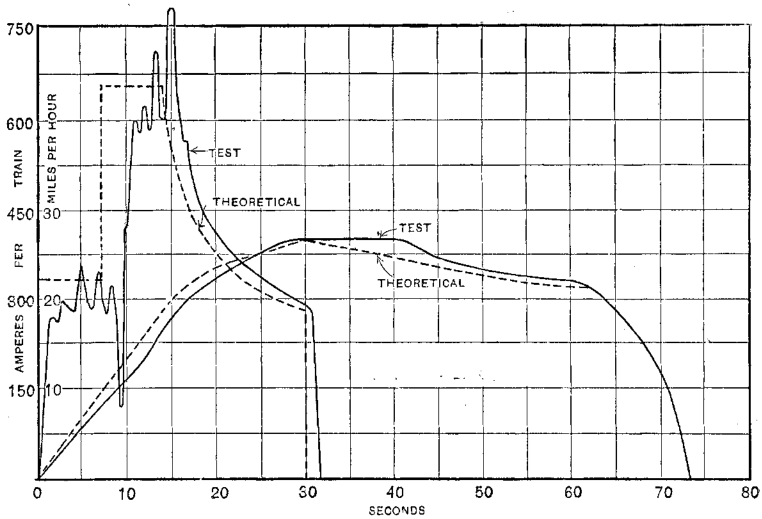


FIG. 10.

COMPARISON OF TEST AND THEORETICAL TRAIN CURVE USING ACTUAL BRAKING CURVE.

Test made on 9th avenue line of Manhattan Ry. Co., Jan. 30, 1903.

Pressure 580 volts; 1 motor car; 1 trailer; weight 62.5 tons; distance 2090 ft., 104th Street to 93d Street; 2 Westinghouse No. 86 motors; gear 19:63; 33-in. wheel.

	Results.	
	Test.	*Theoretical.*
Average speed without stop	19.32	19.32
Schedule speed with 15 sec. stop	16.10	16.10
Kilowatt-hours per run	1.91	2.06
Kilowatt-hours per car mile	2.415	2.65
Watt-hours per ton mile	77.0	83.0
Average kilowatts per train	77.3	82.5
Sq. root mean sq. current per motor for 85 sec.	145.2	157.0
Starting current per motor	300.0	325.0

Smith formula used in plotting run.

speed; as the value of train resistance is best determined at maximum speed, this test cannot by itself be considered an absolute check on calculated curves.

When viewed in the light of tests of other equipment on longer runs, however, it affords a good indication as to how close this specific equipment will conform to the theoretical basis of our determinations. The results of this test are given in Figs. 9 and 10. Fig. 9 gives the results of a north-bound run between 93d Street and 104th Street, and Fig. 10 gives results for a south-bound run on the same section of road. Actual results are indicated by the solid lines, and for comparison there are given on the same Figs., theoretical curves calculated for the corresponding conditions, the train resistance formula and the methods used in the calculations being the same as for the other exhibits hereof and for all L. I. R.R. determinations. In these tests the braking rate was not constant but increased as the speed decreased, and for comparison the same braking curve was assumed for the theoretical as for the test runs.

The results in these comparative curves are tabulated under the respective Figs. The theoretical run was plotted to obtain the same average speed as the test. This involved assuming the power applied for a longer period than the test in one case and a shorter period in the other. In both cases, however, it will be noted that the power requirements (watt-hours per ton-mile) and the square root of mean square current are greater for the theoretical run than for the test. Furthermore, it will be noted that the average starting current per motor was less in the test than in the theoretical run. The conclusions derived from this exhibit are as follows:

(a) The motor control did not permit of the rapid application of power assumed in the calculations, time being lost in going from series to parallel. This, however, did not affect the results adversely.

(b) At the maximum speed when current was on, namely, 26 to 28 miles per hour, the theoretical curve shows practically the same energy consumption as the test. This indicates that the train resistance was practically as calculated. For corresponding speed on the motor curve, the power required according to theory is in both cases slightly higher than the power required according to test; this indicates that the Smith formula gives a train resistance which is slightly higher than the actual resistance for this equipment. The Davis formula, giving higher values than Smith would give results still further from the actual conditions.

(c) The calculations for energy consumption and square root of mean square current give results slightly higher than the test.

(d) The theoretical methods, therefore, give values which are slightly on the safe side for short runs.

(e) The tests do not afford direct evidence of the accuracy of the theoretical methods when applied to longer runs with higher speeds, but the indications are that for such runs the theoretical values will also be on the safe side and close enough for all practical purposes.

II. Use of Typical Run Curves:

As previously stated, the proposed electrical schedule for the Long Island Railroad provides for a large number of combinations of weights of trains and lengths of run. It is obvious that a very great amount of time would be consumed

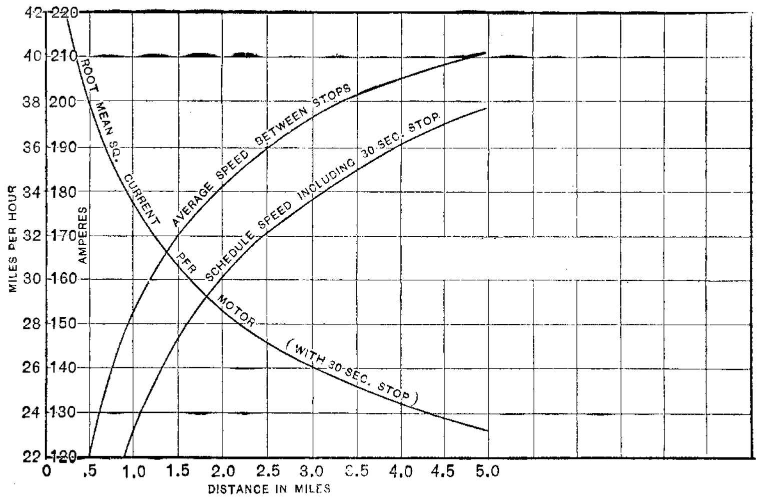


Fig. 11.

General Performance Curve.

3-car train for L. I. R.R.; two 44-ton motor cars; one 33-ton trailer car; total weight 121 tons; four No. 86 Westinghouse motors; 23:59 gear ratio; 36-in. wheel; 540 volts; 1.5 miles per hr. per sec. braking; no coasting.

if an attempt were made to plot each of these runs and to make the determinations of schedule speed, power consumption, and heating effect for the various types and sizes of motors which might be used. To avoid this expenditure of time it was necessary to adopt certain approximate methods, and one method which was used after verification by a practical comparison, is what may be called the "Typical Run Curve."

Disregarding grades and curves, a single run for a distance equal to the average length between stops will represent the

TABLE 2.

ROCKAWAY PARK DIVISION.

77-ton Train. Two Westinghouse No. 86 Motors. 30-second stop. No Coasting

Run.	Length of Run	Square root of mean square current. Plotted Run East	Plotted Run West	Average Plotted Both Ways	Typical Run.
Long Island City					
Bushwick Junction	3.92	154.5	137.3	145.9	147.9
Fresh Pond Junction	.46	223.9	227.0	225.4	226.8
Glendale Junction	1.61	178.2	179.0	178.6	179.3
Brooklyn Hills	.65	241.7	220.5	231.1	215.0
Woodhaven Junction	.97	186.5	207.0	196.7	200.2
Ozone Park	.30	227.0	274.0	250.5	240.0
Aqueduct	1.34	176.0	192.0	184.0	186.8
Ramblersville	.73	201.5	210.5	206.0	210.0
Howards	.76	209.5	207.0	208.2	209.3
Goose Creek	.93	201.7	200.5	201.1	201.8
The Raunt	.96	200.5	209.8	205.1	200.6
Broad Channel	.83	205.7	203.5	203.1	205.7
Beach Channel	.74	210.5	208.5	209.5	209.9
Hammel	.47	225.0	231.0	228.0	226.0
Holland	.39	229.5	210.5	220.0	232.0
Seaside	.49	227.5	224.5	226.0	229.4
Rockaway Park	.75	209.5	209.6	209.5	209.6
Average for round trip	.938	199.0	201.9	200.45	201.2

TABLE 3.

NORTH SIDE DIVISION

77-Ton Train. Two Westinghouse No. 86 Motors. 30-second stops. No Coasting.

Run.	Length of Run	Square root of mean square current. Plotted Run East	Plotted Run West	Average Plotted Both Ways	Typical Run.
Long Island City					
Woodside	3.04	165.5	150.1	157.8	156.
Winfield	1.00	194.0	202.7	198.3	199.
Elmhurst	.81	210.5	202.0	206.2	207.
Corona	.92	215.0	210.0	212.5	202.
Whitestone Junction	1.20	179.8	195.8	187.8	191.5
Flushing, Main Street	1.98	209.9	195.0	202.4	171.5
Murray Hill	.90	237.0	195.5	216.2	203.
Broadway	.54	223.9	218.0	220.5	221.
Auburndale	.78	224.0	226.0	225.0	208.
Bayside	.97	208.5	202.0	205.2	200.
Douglaston	1.26	174.0	186.0	180.0	180.3
Little Neck	.55	233.0	225.0	229.0	220.5
Great Neck	1.24	199.3	165.5	182.4	190.
Manhasset	1.53	202.8	202.0	202.4	181.3
Port Washington	2.66	168.0	158.0	163.0	160.5
Average sq. root mean sq. for whole run	1.20	198.0	191.5	195.0	191 0
Average for round trip		24.2	24.9	24.55	24.

conditions pertaining to a through run; by plotting results for such average runs of different lengths, curves are obtained which will give exact results for the actual conditions assumed, and approximate results for the varying distances between stops and on a succession of light grades and curves. Fig. 11 illustrates the character of the curves obtained in this way.

The principal question involved as to the propriety of such a typical run curve is whether the result so obtained (by considering the track straight and level and the distance between all stops the same) will agree closely enough with the average of results obtained from plotting the time-speed curves of an entire round trip, taking the grades and curves into account and placing the stops exactly. It would seem reasonable that the effects of grades in one direction should balance those in the opposite direction when calculating the general results of a run plotted for a round trip, when the time of a single run is not greater than one hour, as in this case. If the results of train runs plotted complete be compared with typical run curve results, and if this comparison is generally favorable under varying conditions of grade and length of run, it is safe to assume that typical run curves can be used in place of plotted runs for purposes of general estimate and comparison of different types of equipment when operated under similar conditions. The comparisons have, therefore, been made for a given equipment by making plotted runs in opposite directions over the same grades on certain routes of the Long Island R.R. both on the comparatively level runs of the Manhattan Beach and Rockaway Beach divisions and on the more hilly runs from Long Island City to Port Washington. Tables of two of these runs are appended hereto (Tables 2 and 3), which give values for the square root of mean square current for individual runs between stops and the average for round trips. It is to be noted in passing that these particular tables represent motor performances that proved too heavy for the motors and had to be abandoned. They, however, illustrate the point under discussion.

It is possible to make similar comparisons for kilowatt-hours per car-mile and watt-hours per ton-mile, but it is safe to assume that a check made on the comparative determinations for square root of mean square current will indicate the correctness of the assumptions for the determination of the other requirements.

In general it will be noted that the differences between typical run results and the plotted runs, even in the case of the North

Side Division, on which conditions of grade are most severe, do not exceed five per cent. for the square root of mean square current and about four per cent. for speed. This would seem to be sufficiently close for practical purposes in making estimates for given equipment operated over runs of varying lengths where grades are so comparatively light as in the case of the Long Island R.R.

A verification of the accuracy of this method for power determinations in service on an interurban road of the normal

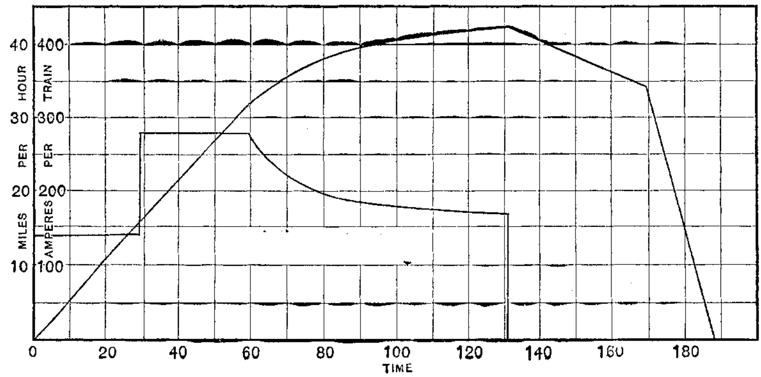


Fig. 12.

Average Performance in Service, of G. R. G. H. & M. Ry. Car.

Two Westinghouse 50 C. motors; 20:51 gear ratio; 0.55 miles per hr. per sec. initial acc.; 525 volts; 140 amperes per motor starting current; weight 29.5 tons; average grade, level.

	Results. Theoretical.	Observed average. G. R. G. H. & M. Ry.
Distance, miles	1.54	1.54
Time, seconds	188	188
Schedule speed miles per hr	29.5	29.5
kilowatt-hours	3.84	3.65
Watt-hr. per ton mile	84.5	80.3
Root mean sq. current, amperes	95.6	

Smith Formula.

amount of curvature in a comparatively level country is obtained by plotting an average run for the G. R. G. H. & M. road, Fig. 12 shows a run plotted for the same distance, time, initial acceleration, and percentage of time coasting and braking as averaged on a round trip service run of a single car of the G. R. G. H. & M. Ry. outside of terminal cities. The tabulated data under this Fig. shows that the watt-hours per ton-mile were within six per cent. of the same value for the theoretical average as for the observed average of an actual test.

The foregoing examples demonstrate that for preliminary determinations of motor characteristics, reasonably close results are obtained by assuming a typical run, except possibly where very unusual conditions exist. No claim is made that the method is new; in fact it is quite generally used. The data given simply show that under ordinary circumstances it is reasonable to use this short method and thus dispense with a large amount of tedious calculating.

III. Schedule Speed Requirements and Limitations:

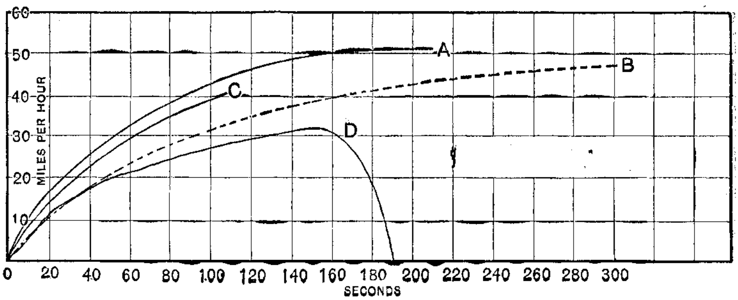


Fig. 13.

Time-Speed Curve of 3-Car Steam Train.

A.—From best locomotive performance reduced to level. Weight of train, including locomotive No. 59, 171.9 tons disregarding light live load. Weight of locomotive, including tender, 88.5 tons. Weight on drivers, 37.5 tons.

Time-Speed Curve of 5-Car Steam Train.

B.—From best locomotive performance reduced to level. Weight of train, including locomotive No. 64, 232.6 tons disregarding light live load.

Time-Speed Curve of 3-Car Steam Train.

C.—From actual run made on a level. Weight of train, including locomotive No. 72, 175.1 tons, disregarding light live load.

Time-Speed Curve of 5-Car Steam Train.

D.—From actual run made on a level. Weight of train, including locomotive, No. 64, 232.6 tons disregarding light live load.

The operating conditions of a large steam railroad introduce limitations which cut down the possible speed of the motive power equipment. Many of these are conditions which do not relate to the nature of the equipment and over which the locomotive engineer has no control; for instance, curves, road crossings, junction points, yard limitations, meeting points on single track lines, time to handle baggage and express matter, etc. There are delays at stations and delays during runs, some of which occur regularly, but many of which are intermittent. Because of the complicated service these delays are

TABLE 4.

LONG ISLAND RAILROAD.

LONG ISLAND CITY TO VALLEY STREAM VIA FAR ROCKAWAY. EASTBOUND.

Length of Run, 22.97 miles.

Train Number.	1287	1303	1321	Average
Locomotive number	72	34		
Locomotive weight	179,000	154,500		
Number of cars	3	4	5	4
Total weight of train (tons)	175.1	186.55		
Schedule time of run	56 :00	55 :00	58 :00	56 :00
Actual time of run	52 :30	74 :20	59 .00	62 :00
Schedule number of stops	9	10	12	10
Actual number of stops	10	11	12	11
Maximum acceleration	.6	.7	.65	.65
Grade on which made	-1.00	-0.08	-0.08	-0.38
Maximum retardation	1.6	1.8	1.9	1.76
Grade on which made	Level	Level	Level	Level
Average acceleration	.47	.46	.35	.426
Average retardation	1.04	1.06	1.22	1.11
Maximum speed	56.5	52.5	50.	53.
Average length of run	2.08	1.91	1.77	1.92
Average time of run	4 :03	4 :48	4 :10	4 20
Average maximum speed	39.9	34.8	33.5	36.1
Length of longest run	7.77			
Time of longest run	11 :20			
Average time of stops	0 :49	1 :55	0 :25	0 :63
Average speed between stops	30.8	23.9	25.5	26.
Possible average speed from curve	34.5	33.7	32.7	33.7
Ratio average to possible average speed	89.4%	71.%	76.%	79.6%

TABLE 5.

LONG ISLAND RAILROAD.

WHITESTONE DIVISION. WESTBOUND.

Length of run, 11.75 miles.

Train Number.	346	360	318	Average
Locomotive number	59	35	20	
Locomotive weight	177,000	154,500	139,500	
Number of cars	3	3	5	4
Weight of train (tons)	171.9	162.85	217.85	184.2
Schedule time of run	28 :00	28 :00	31 :00	29 :00
Actual time of run	28 :34	27 :34	35 :15	30 :27
Schedule number of stops	7	7	7	7
Actual number of stops	7	7	7	7
Maximum acceleration	.9	.95	.5	.78
Grade on which made	-0.65	-0.65	-0.75	-0.68
Maximum retardation	1.8	2.0	1.9	1.9
Grade on which made	1.07	0.69	1.28	1.01
Average acceleration	.44	.35	.29	.36
Average retardation	1.43	1.5	1.2	1.38
Maximum speed	52	50	44.5	48 8
Average length of run	1.47	1.47	1.47	1.47
Average time of run	3 .08	3 :14	3 :55	3 :26
Average maximum speed	41	38.3	33.5	37.6
Length longest run			1.63	
Time longest run			3 :47	
Average time at stops	0 :29	0 :15	0 :33	0 :26
Average speed between stops	28.1	27.3	22.5	25.9
Possible average speed from curve	30.8	30.8	30.8	30.8
Ratio actual to possible average speed	91.3%	88.6%	73%	84.2%

greater on the suburban lines of a steam road than usually occur in interurban or elevated electric service. It is, therefore, evident that in the electrical operation of such suburban service the loading of the motors will differ materially from the loading in an ideal run which might be made with the conditions removed.

After considering various methods of providing for these conditions it was decided to make a series of tests of the present steam equipment to determine the speed time characteristics of the steam trains and the degree to which these characteristics are affected by the local conditions. To this end a test car was equipped with a speed-recording device and runs were made over various routes with this car attached to regular trains. Complete records were obtained of the movement of each train throughout a round trip, and observations made of local conditions which affect speed. These records were carefully analyzed and the results tabulated. In some instances the engineers were requested to make the best time possible; in other cases they were not advised that a test was in progress.

In summarizing these test runs comparisons were made between the average speed between stops as actually attained and the possible average speed if best acceleration had been made for each run. The character of record obtained is illustrated in Tables 4 and 5. It was found that the ratio of the actual speed to the possible speed without local limitations is from 71 to 91.3 per cent. Even when the engineers were requested to do their best they were able in only one or two instances to exceed 90 per cent. of the speed which appeared from the test to be possible. On runs 335 and 346 of the Whitestone Division in which locomotive No. 59 made the best single record, the average speed for the run was also nearest to the maximum limit.

From these records were selected the four best single accelerations. Fig. 13 shows the speed-time characteristics of these accelerations. Curve D illustrates the general character of record made by the instrument in these tests. This run, having been made on a level, conforms exactly with the record obtained. Curves A and B were obtained on grades, and the actual record has been corrected to illustrate the performance of these trains when reduced to a level. In each case the average acceleration over the run was less than the characteristic shows, but these curves illustrate the maximum that the steam equipment was able to do. The composition of trains and their estimated weights are indicated under Fig. The live load on these runs was so light that it was disregarded. On Fig. 14 will be found two curves (E and F) of actual runs on grades, compared with run A of the Fig. 13. According to curve E, a five-car train on a down grade was accelerated at a rate far below the best performance and even the three-car train with locomotive No. 35 (curve F) was not accelerated on a down grade with a rate any better than the best single performance on a level.

It will be noted that the curve *A* was made by a locomotive

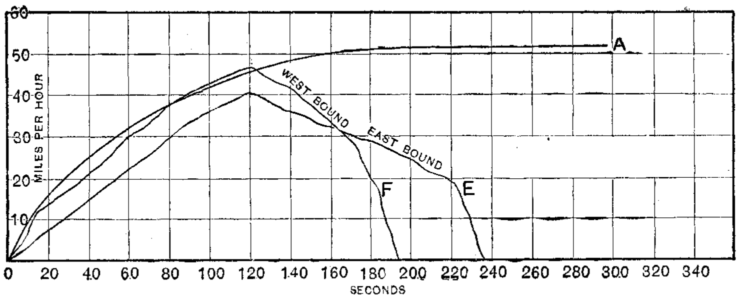


FIG. 14.

TIME-SPEED CURVE OF 3-CAR STEAM TRAIN.
A.—Taken from curve sheet No. 13.

TIME-SPEED CURVE OF 5-CAR STEAM TRAIN.
E.—From College Point to Whitestone, made by 5-car steam train. Weight, including locomotive No. 18, 276.1 tons disregarding small live load. Acceleration made on average grade of –0.78 per cent.

TIME-SPEED CURVE OF 3-CAR STEAM TRAIN.
F.—From Whitestone to College Point, made by 3-car steam train. Weight, including locomotive No. 35, 162.85 tons, disregarding small live load. Acceleration made on average grade of –0.74 per cent.

whose weight was 51.5 per cent. of the total weight of train. The officials of the railroad were of the opinion that service equivalent to that which could be performed by this locomotive, No. 59, with only three cars would be sufficient for the electric equipment. The best performance of this locomotive was, therefore, taken as the criterion by which to work in determining the proper characteristics for the electric equipment. The condition laid down was that the electrical equipment, operating a train with standing load in the cars, should be able to make the same average speed between stops as this particular steam equipment.

TABLE 6.

DATA FOR SUBURBAN RUNS. L. I. R.R.

OMITTING RUNS WHERE NUMBER OF STOPS IS SAME AS HEREIN BUT TIME AT STOPS IS LESS.

Run.	Distance miles.	Time minutes.	Train sheet schedule speed entire run.	Inter-mediate No. of stops per trip.	Time out for stops.	Average Length of Run miles.	Time of average Run minutes.	A. Average speed between stops miles pr hr	B. Limit av. speed best steam train miles pr hr	Ratio of $\frac{A.}{B}$ steam.	C. Limit average speed electric train 23:59 gear ratio.	Ratio $\frac{A}{C}$ Electric.
Long Island City. Port Washington.	18.18	41.5	26.3	7	4.5	2.27	4.62	29.5	36.0	81.9	35.5	83.2
		43.0	25.4	8	5.0	2.02	4.22	28.7	34.0	84.4	34.0	84.5
Long Island City. Whitestone Landing.	11.75	31.0	22.8	7	3.5	1.47	3.44	25.6	30.8	83.2	31.8	80.5
Long Island City. Rockaway Park.	16.3	39.25	24.9	5	2.5	2.72	6.12	26.7	37.5	71.2	36.5	73.2
		39.0	25.1	6	2.75	2.33	5.18	27.0	36.0	75.0	35.5	76.1
		40.0	24.4	7	3.5	2.04	4.57	26.8	34.0	78.8	34.0	78.8
		41.0	23.8	8	4.25	1.81	4.08	26.6	33.0	78.8	33.5	79.5
		43.0	22.8	10	4.0	1.48	3.55	25.0	30.8	81.2	31.8	78.7
		43.75	22.3	11	4.5	1.36	3.27	25.0	30.0	83.3	31.0	80.7
		47.0	20.8	14	5.5	1.09	2.76	23.7	27.0	88.0	29.0	81.8
Long Island City. Valley Stream.	22.97	59.5	23.1	11	7.5	1.91	4.33	26.4	33.7	78.4	33.7	78.3
Long Island City. Manhattan Beach.	16.1	37.0	26.1	3	3.0	4.03	8.5	28.5	41.0	69.5	39.0	73.0
		38.0	25.4	4	2.5	3.22	7.1	27.2	39.0	69.8	37.5	72.6
		40.0	24.2	5	3.75	2.68	5.87	27.4	37.5	73.2	36.5	75.0
		44.0	22.0	11	6.5	1.34	3.12	25.8	29.8	86.5	31.3	82.5
Flatbush. Rockaway Park.	15.88	39.5	24.5	7	4.0	1.98	4.56	26.0	34.0	76.5	34.0	76.5
		44.75	21.5	11	5.0	1.32	3.31	23.9	29.5	81.2	31.0	77.2
		45.5	20.9	12	5.75	1.22	3.06	23.9	28.5	84.0	30.0	79.6
		47.0	20.2	15	6.5	0.99	2.53	23.5	26.5	88.6	28.5	82.5
		48.0	19.8	16	6.75	0.93	2.43	23.0	26.0	88.5	28.0	82.2
Flatbush. Jamaica.	9.63	19.0	30.4	1	1.0	4.81	9.0	32.0	42.5	75.3	40.0	80.0
		23.0	25.1	5	3.0	1.61	3.33	29.0	31.5	92.1	32.5	89.2
Flatbush. Valley Stream.	22.55	60.5	22.4	13	9.0	1.61	3.68	26.1	31.8	82.0	32.0	82.4

Taking curve *A* as a basis, determinations were made of what may be termed the "speed-limit" for steam trains. This is a curve indicating the average speed in miles per hour between stops with the rate of acceleration corresponding to curve *A*, without coasting, and with a braking rate of 1.5 miles per hour per second. On Fig. 15 there will be found a characteristic speed-time curve of this steam train making runs of from one mile to three miles. From this curve the speed-limit curve No. 16 was plotted. Table No. 6 shows how this speed-limit of steam trains compares with the average speed between stops required to make various typical runs of the proposed suburban service. It will be noted that the average speed of the schedule runs is from 71.2 to 92 per cent.

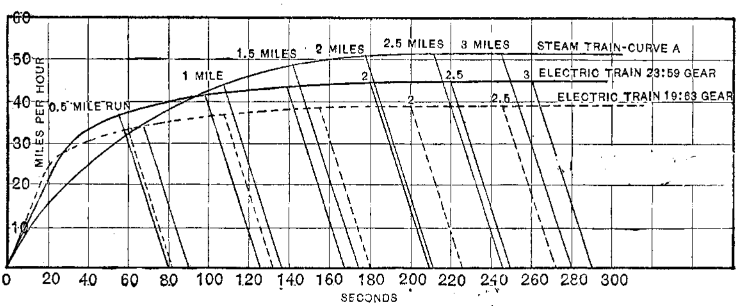


FIG. 15.

TIME-SPEED CURVE OF STEAM TRAIN.
Curve *A* of sheet No. 13.

TIME-SPEED CURVES OF PROPOSED ELECTRIC TRAINS.
Best possible performance on level grade.
Weight of train, 121 tons, with standing load.
(All 3-car trains.)

of the speed limit, with about 80 per cent. as a mean figure.

SCHEDULE CONDITIONS OF PROPOSED ELECTRIC SERVICE:

Obviously, to make the same time, the speed characteristics of the electric equipment selected for this service must be such that the schedule can be maintained with similar speed reserve, unless the limiting conditions are removed. Some of these conditions are being removed and others probably will be. Elimination of grade crossings at roads and junction points, handling of baggage and express matter by special trains, improvement of yard facilities, etc., will make it possible to operate trains at speeds closer to the limit of the equipment. On the other hand, as the population along the road increases the tendency will be to increase the number of stops with as little increase as possible in the running time; therefore, it was decided not to make any material allowance for improved conditions in the initial service. The probability of such improvement, however, suggests another precaution to be taken in the selection of equipment; namely, the provision of a suitable reserve in capacity of motors to carry the greater loads which will result from running at speeds closer to the limit.

On Fig. 15 are time-speed curves of a three-car electric train with two motor cars and one trailer, with standing load in the cars, which curves may be compared with the corresponding curve of the steam train on the same Fig. It

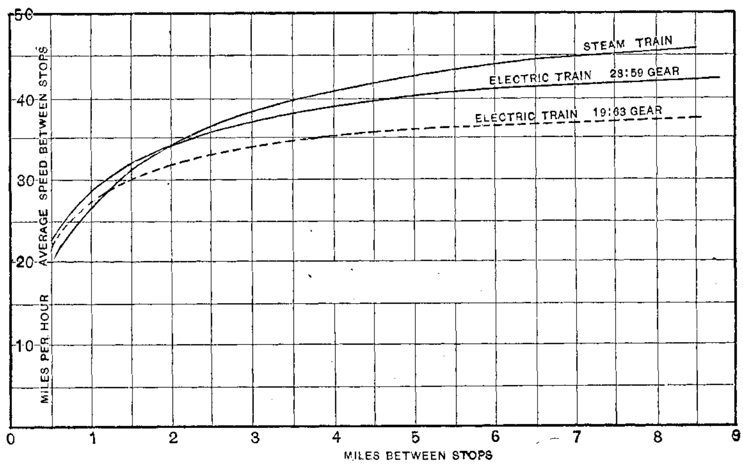


FIG. 16.

SPEED LIMIT CURVES FOR 3-CAR TRAINS.

Showing best possible speed between stops over various lengths of run, without coasting.

Steam train based on curve *A* for acceleration.

Electric train with standing load weighs 121 tons.

will be noted that the electric equipment with the lower gear ratio has a maximum speed on straight level track of approximately 45 miles per hour, and with the higher gear ratio, 39 miles per hour; whereas the limit of the steam train on the level is 52 miles per hour. For a run of one mile or less between stops, however, the electric train with either gear ratio makes better time than the steam train. For a 1.5-mile run the electric train with higher gear ratio falls behind the steam train, and at slightly over two miles the electric train with the lower gear ratio falls behind.

The comparative speeds of these trains are still better illustrated on the speed-limit curves of Fig. 16. The general average length of run between stops for the suburban service is about $1\frac{2}{3}$ miles. It is evident from the speed-limit curves on curve sheet 16 that a three-car electric train with two motor cars and gear ratio 23:59 will average slightly better as to speed than the three-car steam train, but that the same equipment with higher gear ratio falls considerably below the speed-limit of the steam train. From this exhibit it is ob-

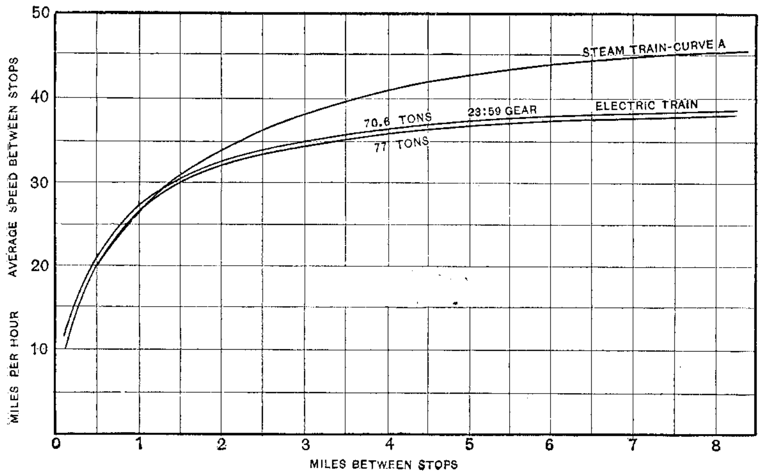


FIG. 17.

SPEED LIMIT CURVES FOR MOTOR CAR AND TRAILER.
Compared with steam train.
Electric train weighs 70.6 tons, seated load.
Electric train weighs 77 tons, standing load.
Steam train based on curve *A* for acceleration.

vious that schedule requirements dictate that no gear ratio higher than 23:59 should be adopted.

It is also evident from these speed-limit curves that if 23:59 is the lowest gear ratio practicable as determined by limitations, the three-car train unit will have to be made up of two motor cars and one trailer, in order that the requisite speeds may be reached. The weight per motor for this equipment is 30.3 tons, which on this basis of speed requirements is obviously close to the limit. This is also made evident by the speed-limit curves of Fig. 17. In this case a two-car train is made up of one motor car and one trailer. It will be noted that such an electric train, with standing load, falls below the speed-limit of the steam train at slightly over one mile between stops. As regards speed requirements, therefore, the two-car train with motor car and trailer has not sufficient motor capacity, and except in cases where speed limitations are slight, two-car trains will have to be made up of two motor cars in order to make the schedule. On this Fig. 17, the corresponding curve for train with seated load has been plotted to indicate the difference in schedule speed possible during the light traffic hours of the day, as compared with the hour of heavy load morning and evening. It will be noted that in the case of heavy equipment of this character the changes in weight of live load do not materially affect the speed characteristics of the equipment.

Attention was called to the column of Table 6 which shows the ratio between the average speed of steam train between stops on different runs according to schedule, and the speed-limit possible with the best steam train in each case. Another column of this table gives the ratio of average speed to the speed-limit of the three-car electric train for the same runs. It will be noted that the reserve speed in the electrical equipment corresponds closely with the reserve in the steam equipment throughout the list. It, therefore, seems evident that this three-car train unit fulfils the requirements laid down as to speed, and can be depended upon to make the required schedule with standing load in the cars, allowing for all the delays contemplated in the present schedule for steam equipment.

It is obvious that trains made up into combinations of motor-cars and trailers that give less weight per motor will be able to make better speeds than indicated for this three-car train. Furthermore, as the number of cars per train increases, the relative effect of head resistance decreases. For these longer trains it is, therefore, possible that a greater proportion of trailer cars may be used in some instances without reducing the speed-limit to such an extent as to prevent the train from making schedule time.

Conclusion.

It is not the purpose of this paper to follow out all the technical processes by which final conclusions were deduced as to size and characteristics of motor equipments for the project referred to. Sufficient illustrations have been given to show:

First, that a study of the operating conditions of even a complex project quickly narrows the selection of motors down to one or two sizes, and that these conditions also largely determine the speed characteristics of the equipment; *secondly*, methods of computation are now developed to a point of sufficient accuracy to be considered reliable for purposes of selecting equipment; *thirdly*, short methods may be used with discrimination, at least for preliminary determinations.

With reference to the use of short methods, a remark may be made in closing regarding the advantage to be gained by deriving a set of general formulas, applicable to all sizes of motors. On first consideration, such a problem as herein described makes such data seem almost indispensable. When we find that only one or two sizes of motor need be considered, however, the advantage of so general a method becomes less apparent. On the other hand, with a little ingenuity, graphical or mechanical methods may be devised for plotting characteristic curves, which make it possible to determine the values of speed, current, etc., throughout a complete run with surprising rapidity. At the present state of development, therefore, it appears that the most satisfactory results may be obtained by employing the characteristics of a specific equipment rather than by spending time in deriving a set of hypothetical characteristics in which many important variables are neglected.