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## PROCESS CAPABILITY INDICES FOR GEOMETRICAL PRODUCT SPECIFICATION - CASE STUDY OF PROCESS CAPABILITY INDEX WITH SKEWED DISTRIBUTION OF GEOMETRICAL PRODUCT SPECIFICATION

**Abstract:** Generally, the geometrical characteristics are calculated in a composed way, as the normal distribution cannot be assumed in many cases. Various methods have been proposed to determine how the process capability index should be calculated in such cases. In this paper, using the data from the machining process, several methods presented in the ISO 22614 series and those presented thus far in Japan are examined and compared. Additionally, this paper analyzes what methods should be used in a certain case depending on the purpose and the status of the process (start-up or improvement of the process, assurance of the post-process).

**Keywords:** Machining process, Distribution, Coordinate value, PCA, ISO 22514

### 1. Introduction

This paper addresses the Process Capability Index (hereafter referred to as Cp or Cpk) of Geometrical Product Specification (GPS) accuracy in the machining process (ISO 1101:2017). Not only a single-dimension parameter such as length or diameter but also geometrical characteristics based on two- or three-dimensional parameters such as concentricity or perpendicularity are required for the machining process. In general, GPS is defined during the process design to meet the scope of the assembled product and is controlled its shape during machining precisely. Therefore, Cp calculation may also be required.

GPS of items often does not follow the normal distribution. It is a composite value calculated from coordinates, or by the

difference between the maximum and minimum value based on their definition. Many such distributions are skewed by a long tail, and the tolerance is regulated by the upper specification limit only. However, errors in manufacturing process may occur because the upper performance index of Cpk is calculated sometimes without considering the skewness of GPS distribution. Several methods for Cp calculation when GPS does not follow the normal distribution have been presented in the ISO 22514 series (ISO 22514-6:2013, ISO 22514-1:2014, ISO 22514-4:2016). Furthermore, various methods have been discussed in Japan.

This paper deals with various methods and ideas for calculating Cp of GPS of items, which are compared using data of an actual machining process. Advantages, drawbacks, and limitations of each method are

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considered.

After reviewing previous studies and international standards in Chapter 2 and explaining the characteristics of the process in Chapter 3, Cp values are calculated by various methods in Chapter 4. Further, Chapter 5 discusses how Cp should be calculated and evaluated in a certain case, accounting for the purpose of the product, its machining conditions, and post-process assurance for the customers.

It is essential to review actual shape or coordinate values of a product before calculating the GPS value to enhance the machining process. Owing to the recent progress in process automatization, a large amount of factual data can be obtained automatically. In addition, we discuss a possibility of using these data both in the Cp determination and in process improvement.

## **2. Known methods for Cp calculation of GPS**

Various methods have been proposed to calculate Cp in case of GPS skewed distribution. This paper covers five of them.

### **2.1. Approximate by some distribution that fits the obtained data**

In this method, the data obtained from the process are fit by some distribution subsequently. The ISO 22514 series regulates the method for calculation of Cp values by percentile after fitting the data by some distribution. In particular, ISO 22514-4 (Annex C) proposes applying the Lognormal, Rayleigh, and Weibull distributions to cases of non-normal distributions. The method for correcting the Cp value using the third and fourth moments is also disclosed.

As a normal distribution can be assumed in the case of lognormal distribution, the defect rate corresponding to the obtained Cp value may be estimated (e.g., if  $C_p = 1.0$ , the estimated defect rate is approximately

0.27%). For other distributions, however, the 0.135 percentile point (the correspondence between Cp and defect rate when a normal distribution is assumed) should be found according to the ISO 22514 methods, where Cp is 1.0 for the upper and lower sides, respectively, and this relation does not exist for other values of Cp.

### **2.2. Adjust Cp with a correction coefficient**

The method comprises calculation of Cp using a general method for the data obtained from the process and its multiplication by some correction coefficient. The details are disclosed in Munechika (1986).

Note that the method in Chapter 2.1 considers only the relationship with the defect rate at  $C_p = 1.0$ , whereas the present method involves the same for cases other than  $C_p = 1.0$ .

### **2.3. Evaluate the shape and coordinates before calculating GPS**

GPS value is calculated from the measured coordinates or difference of maximum/minimum values of the feature error. For example, in the case of position or concentricity, a GPS value can be decomposed into original X and Y coordinates to evaluate Cp. The advantage of this method is that the normal distribution can be assumed because they are non-composite value. This method is proposed by Izaki et.al (2002) and Nishina (2009).

If the data decomposed into coordinate values follow the normal distribution, Cp value corresponds to the defect rate.

### **2.4. Evaluate using multivariate normal distribution**

The method for calculating Cp by multivariate normal distribution of coordinate values that assume mainly position tolerance is presented in ISO 22514-6. The method estimates Cp by defining percentile boundaries of the two-dimensional

normal distribution on the coordinate plot. If the data can be assumed as multivariate normal distribution, Cp value corresponds to the defect rate.

**2.5. Evaluate using principal component analysis**

This method proposed by Izaki et al (2002) uses the principal component analysis (PCA)

to set new evaluation axis that maximizes the variation difference from the coordinate axis. This method, however, does not consider the relation between Cp and a defect rate.

The methods described above are briefly summarized in Table 1.

Further, each method is examined using actual process data, and its advantages and issues are discussed.

**Table 1.** Overview of the methods examined in this paper

Chapter	Method	Means	Correspondence with defect rate
4.1	Approximate by some distribution	Assume Lognormal distribution in this case study	Yes (if Lognormal distribution can be assumed)
4.2	Adjust Cp value with coefficient	Calculate the coefficient	Yes
4.3	Evaluate the coordinate values	Decompose X and Y coordinate	Yes (if normal distribution can be assumed)
4.4	Multivariate normal distribution	Assume bivariate normal distribution	
4.5	Multivariate normal distribution	Apply principal component analysis	No

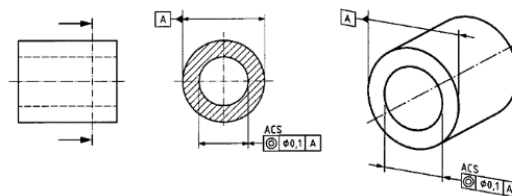
**3. Data of this case study**

**3.1. Overview of the data**

The data used in this study are concentricity values of N = 446 pieces of the same product after a metal machining measured during two weeks on the basis of X and Y coordinates.

**3.2. The reason of considering GPS using coordinate values**

This subsection addresses the relationship between concentricity and coordinate values. The concentricity tolerance is defined in ISO 1101:2017. For example, in the case of an item displayed in Figure 1, “the extracted center of the inner circle in any cross-section shall be within a circle of diameter 0.1, concentric with datum A defined in the same cross-section.”



**Figure 1.** Explanation figure in ISO 1101:2017 (17.14 Concentricity and coaxiality specification)

The concentricity is calculated from the coordinates by the formula (1), where  $(x_0, y_0)$  is the center of datum.

$$\text{Concentricity}D = 2 \times \sqrt{(x - x_0)^2 + (y - y_0)^2} \tag{1}$$

The method of evaluating geometric characteristics by coordinate values, as described in Chapters 2.3 and 2.4, is applied (Figure 2). For example, assume that the concentricity tolerance is  $\varnothing 20 \mu\text{m}$  ( $\varnothing 0.02 \text{ mm}$ ). Hence, as one can see in Figure 2, the permissible tolerance of both the coordinates is  $\pm 10 \mu\text{m}$ . Additionally,  $(\pm 10 \mu\text{m}, \pm 0 \mu\text{m})$  seems acceptable, but it is not reasonable to target this value in the actual machining process. The control range in the actual process must be within  $(\pm 7 \mu\text{m}, \pm 7 \mu\text{m})$  at worst. If the coordinate values are in the hatched area of Figure 2, they are fortunately within the tolerance in the actual process.

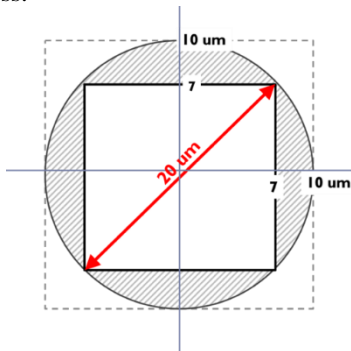


Figure 2. Concentricity tolerance and the target in the actual machining process

In actual processes, operators input to the machine the target coordinates not a GPS value such as concentricity or position. Therefore, it is reasonable that  $C_p$  is evaluated for the decomposed X and Y coordinates.

#### 4. Calculating PCI value: comparison by several methods

This section addresses  $C_p$  values of the concentricity calculated using the various

methods described in Chapter 2. The data obtained from the process can be plotted in coordinates as shown in Figure 3. By the analogy to Figure 2, the concentricity tolerance ( $\varnothing 20 \mu\text{m}$ ) is shown by the circle, whereas the range of process control ( $\pm 7 \mu\text{m}$ ,  $\pm 7 \mu\text{m}$ ) is shown by the square. The concentricity histogram of these data is shown in Figure 4.

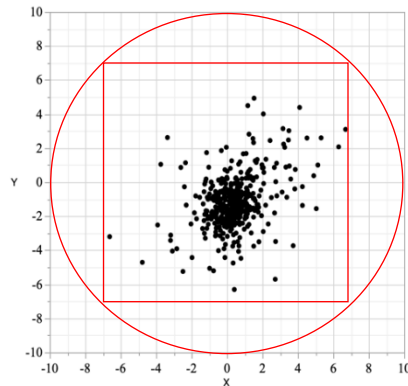


Figure 3. Plotting of the concentricity on coordinate

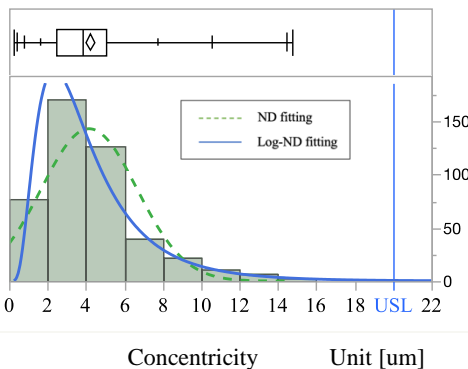


Figure 4. Histogram of the concentricity

Although the distribution is evidently not normal, a common error is to assume a normal distribution and calculate the mean and standard deviations by equation (2) without considering the skewness of the distribution.

(Wrong example)

Mean value: 4.17  $\mu\text{m}$

Standard deviation: 2.49  $\mu\text{m}$

Upper criteria: 20.0  $\mu\text{m}$

Then,

$$C_{pkU} = \frac{(20.0 - 4.17)}{3 \times 2.49} = 2.12$$

( $C_{pkU}$  means that Cp value has only upper criterion.)

(2)

According to the obtained CpkU, the process capability (Nagata, Y., & Munechika, M. (2011) seems sufficient, but the skewness of the distribution or the shape of the long tail are not considered, which would lead to a wrong interpretation of the process (Izaki et al., 2002; Kotz, & Lovelace, 1998; Munechika, 1986; Taam et al., 1993).

Further, we apply each of the various methods described above to determine correct Cp values.

#### 4.1. Approximate by some distribution that fits the obtained data

To consider the skewness of the distribution found in the process data, a distribution other than the normal distribution may be applied. Here  $Cpk = 0.93$  is calculated using the lognormal distribution and formula (3)–(5), see descriptions in Annexes C.3.1 and C.3.2 of ISO 22514-4.

First, the logarithm of each data value was taken, and its mean and standard deviations were found.

$$\hat{\mu} = \bar{X} = \frac{1}{N} \sum_{i=1}^N (\log_e x_i) \tag{3}$$

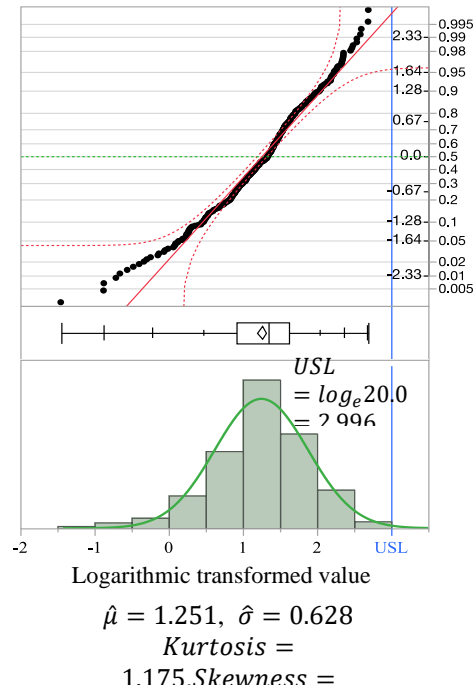
$$\hat{\sigma} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (\log_e x_i - \bar{X})^2} \tag{4}$$

As the result of above, the mean value  $\hat{\mu}$  and the standard deviation  $\hat{\sigma}$  after logarithmic transformation are (1.251, 0.628), respectively.

As the  $USL = \log_e 20.0$ , then,

$$C_{pk} = \frac{(\log_e 20.0 - \hat{\mu})}{3 \times \hat{\sigma}} = 0.93 \tag{5}$$

The histograms of the log-transformed values and their plots on lognormal probability plot are illustrated in Figure 5.



**Figure 5.** Logarithmic histogram of concentricity and fitting of lognormal distribution

Comparing Figure 4 and 5, it seems quite rare for the concentricity to exceed USL (20  $\mu\text{m}$ ) in Figure 4, whereas the logarithmic histogram in Figure 5 looks like the USL might be exceeded any time. In this case, the lognormal distribution may not fit well to the data at the distribution tail.

#### 4.2. Adjust Cp value with a correction coefficient

Secondly, the method that adjusts Cp with the correction coefficient  $\lambda$  was applied according to formula (6)–(10).

As a result, the Cp value was  $Cp\lambda = Cp \times \lambda = 1.198$ .

Cp calculation before correction:

$$C_{pkU} = \frac{(USL - \hat{\mu})}{3 \times \hat{\sigma}} \tag{6}$$

$$\hat{\mu} = \sum x_i / N \tag{7}$$

$$\hat{\sigma} = \sqrt{\sum (x_i - \hat{\mu})^2 / (N - 1)} \tag{8}$$

Then, the correction coefficient  $\lambda$  is determined as follows:

$$\lambda = \frac{\sqrt{k_3^2 + 18Cp \cdot k_3 + 9} - 3}{3Cp \cdot k_3} \tag{9}$$

Where

$$k_3 = \frac{\sqrt{N} \sum (x_i - \hat{\mu})^3}{\{\sum (x_i - \hat{\mu})^2\}^{1.5}} \quad (k_3 \neq 0) \tag{10}$$

In this case study,  $\lambda = 0.564$ , then,  $Cp_p = Cp_k \times \lambda = 2.12 \times 0.564 = 1.196$ .

### 4.3. Evaluate the shape and coordinates before calculating GPS

The concentricity values were decomposed into X and Y coordinates, and each Cp value was calculated assuming the normal distribution. Histograms of the coordinate values are shown in Figure 6 and 7, respectively. The upper and lower limits are defined here as the control range of the machining target in the actual process (+/- 7  $\mu\text{m}$ , +/- 7  $\mu\text{m}$ ).

Calculation of Cp :

$$Cp_X = \frac{7 - (-7)}{6 * 1.40} = 1.66 \tag{11}$$

$$CpkU_X = \frac{0.007 - (0.41 \times 10^{-3})}{3 * 1.40 \times 10^{-3}} = 1.57 \tag{12}$$

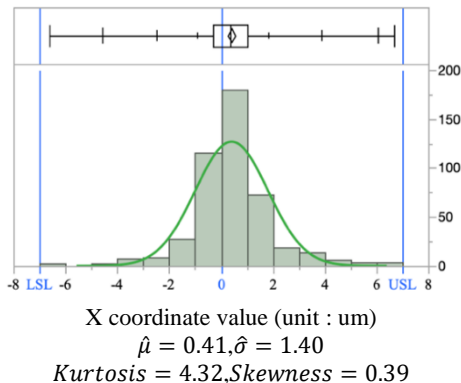


Figure 6. Histogram of X coordinate

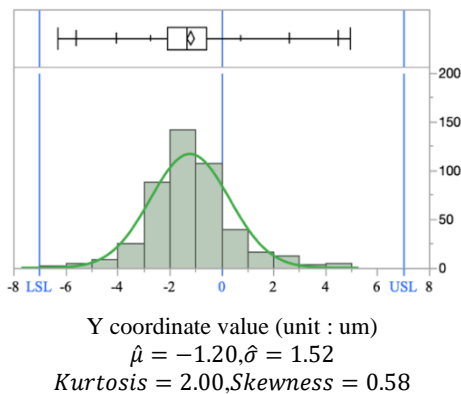


Figure 7. Histogram of Y coordinate

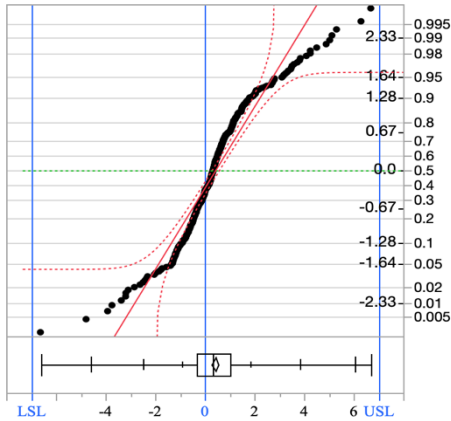
Calculation of Cp :

$$Cp_Y = \frac{7 - (-7)}{6 * 1.52} = 1.53 \tag{13}$$

$$CpkU_Y = \frac{7 - (-1.20)}{3 * 1.52} = 1.27 \tag{14}$$

If the general formula for Cp calculation is applied, the Cp values seem like that the process capability is sufficient. However, as there are some data around LSL and USL, it is questionable whether the process capability is as good as the calculated Cp. In addition, the kurtosis is particularly high in the X-coordinates, and it is not appropriate

for a simple normal distribution. The results of plotting the X-coordinates on a normal probability plot are shown in Figure 8.



**Figure 8.** Plotting on normal probability paper

Generally, in the machining process, the coordinate command value is corrected toward the target coordinate values, and it is quite possible that the kurtosis will be high in long-term machining data. Therefore, the Cp values corrected for kurtosis and skewness were also calculated using the method described in Annex B of ISO 22514-4.

[ X coordinate value ]

From kurtosis = 4.32 and skewness = 0.39, the following correction factors are obtained using the reference table in Annex B.

99.865 percentile = 4.50

0.135 percentile = 3.34

50.0 percentile = 0.033

The corrected Cp value is calculated using these figures.

$$Cp'_X = \frac{USL - LSL}{\hat{X}_{99.865\%} - \hat{X}_{0.135\%}} = \frac{7 - (-7)}{(\hat{\mu} + \hat{\sigma} \times 4.50) - (\hat{\mu} - \hat{\sigma} \times 3.34)} = 1.27 \tag{15}$$

whereas CpkU for the USL that considers the

bias of the mean value from the target is as follows:

$$CpkU'_X = \frac{USL - \hat{X}_{50\%}}{\hat{X}_{99.865\%} - \hat{X}_{50\%}} = \frac{0.007 - (\hat{\mu} + \hat{\sigma} \times 0.033)}{(\hat{\mu} + \hat{\sigma} \times 4.50) - (\hat{\mu} + \hat{\sigma} \times 0.033)} = 1.04 \tag{16}$$

[ Y coordinate value ]

From kurtosis = 2.00 and skewness = 0.58, the following correction factors are obtained using the reference table in Annex B.

99.865 percentile = 4.35

0.135 percentile = 3.00

50.0 percentile = 0.06

The corrected Cp value is calculated using these figures.

$$Cp'_Y = \frac{USL - LSL}{\hat{X}_{99.865\%} - \hat{X}_{0.135\%}} = 1.25 \tag{17}$$

$$CpkL'_Y = \frac{\hat{X}_{50\%} - LSL}{\hat{X}_{99.865\%} - \hat{X}_{50\%}} = 0.90 \tag{18}$$

Table 2 summarizes the results obtained by several methods. There are remarkable differences depending on the method.

#### 4.4. Evaluate using multivariate normal distribution

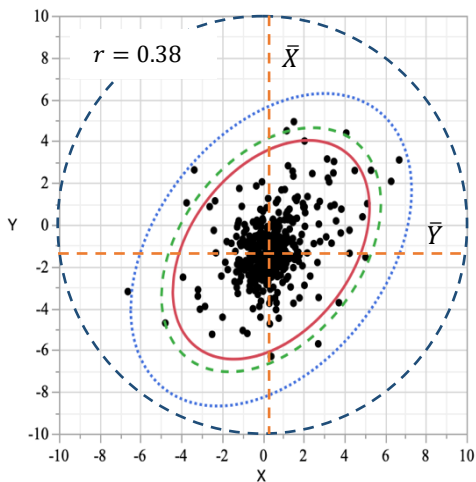
In ISO 22514-6, the multivariate normal distribution mainly assuming positional tolerance is used for coordinate values. This method plots the data on a two-dimensional coordinate plot and estimates Cp by fitting a probability ellipse (in particular, a probability ellipse of 99.73% corresponding to  $\pm 3\sigma$ ) assuming a two-dimensional normal distribution. Application of this method to this case study is illustrated in Figure 9. The ellipses represent, starting from the center, the probability dispersion  $3\sigma$  (= 99.73%),  $4\sigma$ , and  $5\sigma$  from the mean.

**Table 2.** Summary of calculated Cp / Cpk values

Method	Chapter	Cp / Cpk	Concentricity	X coordinate	Ycoordinate
Assume normal distribution (No correction)	4.3	Cp	-	1.66	1.53
	4	Cpk	2.12 (wrong)	1.57	1.27
Assume lognormal distribution	4.1	Cpk	0.93	-	-
Adjust Cp value with coefficient	4.2	Cp	1.20 <sup>(a)</sup>	1.27 <sup>(b)</sup>	1.25 <sup>(b)</sup>
	4.3	Cpk	-	1.04 <sup>(b)</sup>	0.90 <sup>(b)</sup>

(a) Determined in Chapter 4.2 (b) Determined in Chapter 4.3

According to the ISO 22514-6 method, Cpk should be calculated instead of the Cp, because the mean value is out of the target (origin) value. As 99.73% of the probability ellipses are completely within the acceptable range, and there is no data outside the acceptable range, it can be concluded that a Cpk value is at least greater than 1.



**Figure 9.** Example of fitting the probability ellipses of two-dimensional normal distribution at the coordinate plot (solid line: 99.73% range) (The horizontal and vertical dashed lines indicate the mean values of X and Y, respectively)

In this case study, nine of 446 pieces are plotted outside the 4σ range, in contrast to the theory that 99.73% of the data should be within the ± 3σ range. Probably, a two-dimensional normal distribution cannot be assumed because the kurtosis of the data is too high.

This method does not provide accurate calculation of a Cpk value. According to ISO 22514-6, 7.2.3, Cpk is calculated as follows (19):

$$C_{pk} = \frac{1}{3} \Phi^{-1} \left( \frac{P + 1}{2} \right) \tag{19}$$

where  $\Phi^{-1}$  is reciprocal of the normal probability density function, and P is probability of falling into the tolerance ellipse.

Here P is the probability of exceeding the tolerance range. Its value can be estimated from the observed number of defects, or it can be estimated theoretically on the basis of the obtained data. However, as in this case the probability of exceeding the tolerance range is around zero, it is difficult to calculate Cp or Cpk with enough accuracy.

The ISO also suggests a method of Cp calculation as the ratio of the tolerance range area to the variation range area of the process. It should be noted, however, that the ellipse area does not necessarily correspond



directly to the Cp value of the geometric property. This parameter cannot be used as a general indicator, as noted in Annex C.1 of the ISO.

Despite the method according to ISO 22614-6, which is presented in Figure 10 is not suitable for accurate Cp determination, it is useful in the actual process control because of the following reasons. In the case of machining using a typical tool, the two coordinate variables are not independent of each other. As the movement of one spindle of the machine tool determines the coordinate on a workpiece, it is unlikely that X and Y coordinates are completely independent and uncorrelated. It would be more natural for them to have a certain degree of correlation. If so, showing the normal variation range as a multidimensional probability ellipse is reasonable. A point appearance out of the range may alert the workers on some abnormality of the process.

**4.5. Evaluate using principal component analysis**

PCA is performed on multivariate coordinate values to derive the axis with the largest variation (the first principal axis). Then, Cp or Cpk is calculated as the distance on the axis to the tolerance range (ex. circle range of the concentricity). In this method, Cp is obtained without bounding to the X and Y-axes. Based on the present process data, bivariate data of X and Y coordinates are obtained in this case study. PCA is performed starting from the covariance matrix and calculation of eigenvectors, eigenvalues, and contribution rates (Tables 3, 4).

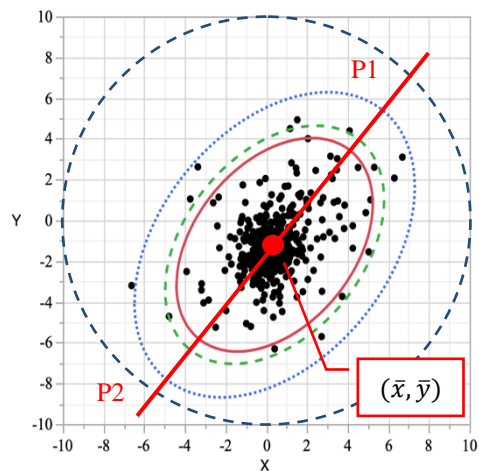
**Table 3.** Eigenvectors on PCA

	1 <sup>st</sup> principal axis	2 <sup>nd</sup> principal axis
X coordinate	0.62853	0.77778
Y coordinate	0.77778	-0.62853

**Table 4.** Eigenvalues and Contribution rates on PCA

	1 <sup>st</sup> principal axis	2 <sup>nd</sup> principal axis
Eigenvalues	2.9733	1.3109
Contribution rates	69.4 %	30.6%

The first principal axis passes through the point where the slope = 0.7778/0.62853 and the mean of X and Y. Then, the equation of the first principal axis in the coordinates is derived, and its intersections (P1 and P2 in Figure 10) with the circle of the tolerance range (radius 0.01) is found. As the standard deviation corresponds to the square root of the eigenvalue, Cp is derived by the following formula by the distance from the median point to the intersections with the circle.



P1 (7.088, 7.054)  
P2 (-5.416, -8.406)

**Figure 10.** Intersection of the first principal axis and the tolerance circle

$$Cp = \frac{\text{Distance from the median point to intersection}}{3 \times \sqrt{\text{Eigenvalue}}} \tag{20}$$

In this case, the distance from the median point to the intersection P2 is shorter;

therefore, Cpk for the distance to P2 is calculated as follows:

$$Cpk = \frac{9.267}{3 \times \sqrt{2.9733}} = 1.792 \quad (21)$$

The same calculation was applied to the second principal component axis, and the minimum Cpk calculated using PCA was the same as above.

This method should be used when a certain degree of correlation between the variables is confirmed. It should not be used when the direction of the first principal axis fluctuates in unstable processes. This method does not consider the correlation between Cp and the defect rate. Therefore, it cannot be directly compared with the methods in Sections 4.1–4.4. It should be used only as an indicator of process robustness in the direction of the greatest variation.

## 5. Discussion

### 5.1. Summary on Cp calculation methods

As summarized in Table 2, there is a number of methods for calculating Cp of geometric characteristics, which may vary depending on the method. If Cp is evaluated as a geometric characteristic (concentricity in our case), correction of the distribution should be applied, or other distributions such as lognormal distribution should be used. Moreover, it is necessary to verify whether normal or other distributions can be applied for the obtained data by calculating basic statistical parameters including kurtosis and skewness and plotting on probability paper.

Items with the same geometrical characteristic may show different distribution shapes depending on the process or equipment used. Understanding the distribution shape using histograms and statistical parameters is an essential preliminary step in Cp calculation.

### 5.2. Utilization of coordinate values for the analysis of geometric characteristics

For engineers and workers who manage the process on site, it is considered that histograms of the coordinate values and coordinate plots are simple indicators for intuitive control of the process and its improvement.

In this case study, Cpk < 1.0 calculated after its decomposing into coordinate values and correcting the distribution considering the strong kurtosis. In the beginning of mass production, the process with Cp below 1.0 is unacceptable and must be improved using histograms and coordinate plots to ensure that the Cp value is not below 1.0.

For example, in the considered case, there is a deviation from the target center (origin), especially along the Y coordinate. Regarding the geometrical characteristics, the deviation from the center is determined as  $\sqrt{x^2 + y^2}$ , so the deviation impact is larger than that in one-dimensional case (if the X and Y coordinate deviate each by 1 from the origin, the deviation as a geometric characteristic is  $\sqrt{2}$  times larger). Using our actual data, it can be inferred that there is always a certain inclination of the spindle against the workpiece, and even if the X coordinate can be adjusted near the origin, the Y value will deviate from the target value by the amount of spindle inclination. Therefore, correcting the spindle inclination results in the improvement of the geometric characteristics value. Such improvement may be achieved from analysis of histograms and coordinate plots, rather than just on the basis of the concentricity value.

### 5.3. Purpose and status of Cp evaluation

The Cp calculation process for geometrical characteristics can be distinguished into two main types. The first one is based on the data distribution with some correction, whereas

the second one is the process interpretation based on coordinate plots with an inferential calculation. They are illustrated in Chapters 4.1–4.3, and 4.4–4.5, respectively. These two types may be used depending on the process status and the purpose of the calculation and evaluation.

Cp values should be calculated in the following situations:

- (1) In the beginning start-up of the process: decision on its acceptance or rejection, decision and approval for the transition to mass production;
- (2) In the initial period of mass production: understanding evaluation of the process robustness and improvement activities to be performed;
- (3) After mass production: Process assurance for post-processings and/or customers.

There are many situations where Cp determination is required to evaluate the equipment appropriateness in the beginning of a process and whether a process is acceptable. In such cases, it is better to use coordinates rather than geometrical characteristics for accurate identification of process improvement trends. The same applies to the following initial period.

Then, at the mass production and further stages, explaining to the customer the way of determination by coordinate values seems unreasonable. In general, only geometrical characteristic values are specified in drawings and not the required coordinate values. Therefore, it would be more realistic to calculate Cp for the value of the GPS. However, as we demonstrated above, corrections to the distribution and the assumption of other than normal distribution must be considered, and a technically meaningful determination of Cp must be performed on the basis of distribution characteristics.

In summary, coordinates should be used in the process control and optimization, whereas geometric characteristics are

preferable for the post-processing and/or customers (Ogawa et.al, 2018).

## 6. Conclusion

Based on the performed case study, the following conclusions can be drawn.

- (1) Cp values for geometrical characteristics cannot be calculated to indicate the actual state of the process unless some correction is performed by analyzing the shape of the distribution.
- (2) Even if the geometrical characteristics are decomposed into coordinate values, they do not always follow the normal distribution. In particular, even third and fourth moments such as kurtosis and skewness should be considered depending on the process characteristics.
- (3) While ISO 22514-6 provides intuitively clear graphical representation to process practitioners, it is difficult to calculate the precise Cp value by this method.
- (4) PCA may also be used if there is a correlation between some technical factors of the process. Because there is no relation with the defect rate, simple comparison with the values obtained by other methods is impossible. This method should be used when a deeper analysis of the process is required.

A number of methods for Cp calculation have been devised developed to calculate the Cp value. In the Statistical Process Control Reference Manual by AIAG, the following description is given:

- There is no single index that can be universally applied to all processes.
- There is no single process that can be completely represented by a single index.

"Any inferences made from the calculated indices should be based on a proper interpretation of the data used to calculate the indices."

The present case study illustrates the above points in practice in a concrete manner. The

Cp values are werenot only evaluated on the basis of on the basis of the calculated values, data interpretation and choosingbut also on the basis of the interpretation of the data and the appropriate method ufor their

calculationsed to calculate them.

**Acknowledgment:** This work was supported by JSPS KAKENHI Grant Number JP17K01253.

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