PROCESS IDENTIFICATION AND PID CONTROL

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To our wives and children

Contents

Preface

This book focuses on the basics of process control, process identification, PID controllers and autotuning. Our objective is to enable students and engineers who are not familiar with these topics to understand the basic concepts of feedback control, process identification, autotuning and design of real feedback controllers (especially PID controllers).

Parts One and Two are aimed at undergraduate students who have not taken any courses on process control. Parts Three and Four are appropriate for graduate students and control engineers who want to design real feedback controllers or perform research on process identification and autotuning. Parts One and Two introduce the basics of process control and dynamics, the analysis tools (Bode plot, Nyquist plot) to characterize the dynamics of the process, PID controllers and tuning, and advanced control strategies that have been widely used in industry. Also, simple simulation techniques required for practical controller designs and research on process identification and autotuning are also included. Part Three provides useful process identification methods actually used in industry. It includes several important identification algorithms to obtain frequency models or continuous-time/discrete-time transfer function models from the measured process input and output data sets. Part Four introduces various relay feedback methods to activate the process effectively for process identification and controller autotuning.

We have tried to include as many examples as possible. In particular, the readers can use the numerical examples and the MATLAB R codes with slight modifications to solve actual problems in their processes or research. The codes (MATLAB Rm-files) and real-time virtual processes for the simulations and practices are available from the Wiley website at www.wiley.com/go/swsung. The codes will be useful to those who want to understand the actual implementation techniques for control, process identification and autotuning. Also, the readers can design their own controllers, implement them and confirm the performances in real time using real-time virtual processes. Also, the problem-solving ability of students can be enhanced by performing a controller design project on the basis of the virtual process. We welcome the comments of students and instructors to improve the book and the materials for lectures and simulations. Please visit our other website at http://pse.knu.ac.kr for comments and questions about this book or process systems engineering. We hope this book is useful to you.

We wish to express special thanks to the students at KNU who provided the simulation results and detailed reviews: Cheol Ho Je, Chun Ho Jeon and Yu Jin Cheon. We acknowledge

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Part One

Basics of Process Dynamics

Part One introduces the basics of process dynamics which are appropriate for an undergraduate course. Chapter 1 defines linear processes and discusses how to represent linear processes in a mathematical way. Chapter 2 introduces several simulation and numerical analysis techniques required to simulate/design process controllers. Chapter 3 discusses the dynamic behaviors of linear processes and provides several analysis tools to characterize the dynamics of the control system.

1

Mathematical Representations of Linear Processes

1.1 Introduction to Process Control and Identification

The basic concepts and terms of process control and identification are first introduced.

1.1.1 Process Control

Process control consists of manipulating variables, controlled variables and processes. The manipulating variables and the controlled variables usually correspond to the process inputs and the process outputs respectively. The objective of process control is to make the process outputs (controlled variables) behave in a desired way by adjusting the process inputs (manipulating variables). Consider the temperature control system in Figure 1.1.

The SCR unit is to provide electrical power to the heating coil, which is proportional to the voltage $u(t)$. The temperature is measured by the thermocouple sensor. The objective of the temperature control system in Figure 1.1 is to drive the temperature $y(t)$ to the desired value by adjusting $u(t)$. So, $u(t)$ and $y(t)$ are the process input (manipulating variable) and the process output (controlled variable) respectively. The role of the feedback controller is to determine $u(t)$ appropriately on the basis of the measured $y(t)$ to achieve the control objective.

Example 1.1
Consider the control system in Figure 1.2. It consists of two tanks, a control valve, a DP cell and a controller. The DP cell and the control valve are to measure the liquid level of the last tank and adjust the inlet flow rate respectively. The objective of the control system is to drive the liquid level of the last tank to a desired value. In this case, the manipulating variable is the inlet flow rate and the controlled variable is the level of the last tank.

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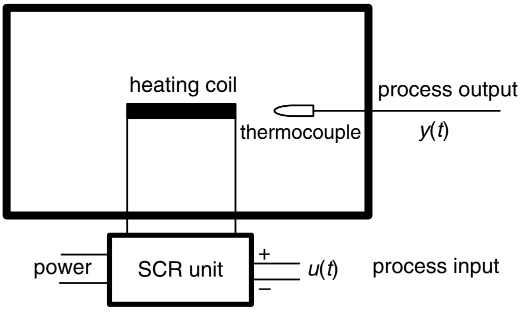

Figure 1.1 Temperature control system.

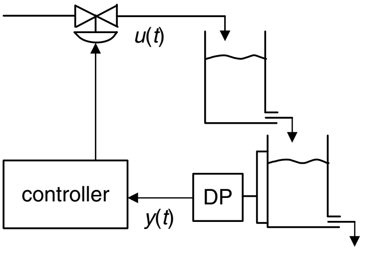

Figure 1.2 Level control system.

1.1.2 Process Identification

Process identification is the obtaining of a model of which the role is to predict the behavior of the process output for a given process input. The models are in the form of differential equations or frequency data sets (which will be explained later). From the energy balance equation for the temperature control system in Figure 1.1, the model of the following simple differential equation form can be derived:

$$\tau \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = ku(t) + b \tag{1.1}$$

where τ, k and b are known constants determined by the heat capacity, mass, amplification coefficient, heat transfer coefficient, area and ambient temperature. This is a simple example of process identification. The behavior of $y(t)$ can be predicted by solving the differential equation for a given $u(t)$. In this book, how to obtain the model from historical data of the process input and the process output will be treated without considering physical principles such as material balance, energy balance and chemical reactions. This kind of model is called a “black-box model.”

Example 1.2
Assume that the black-box model structure for a given process has the following form:

$$\tau \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = ku(t) \tag{1.2}$$

And assume that $y(t) = 1 - \exp(-2t)$ is obtained from an experiment when $u(t) = 1$ is applied to the process. Then, it is straightforward to estimate the model parameters of τ and k from the experiment. Replace $y(t)$ and $u(t)$ in (1.2) by $y(t) = 1 - \exp(-2t)$ and $u(t) = 1$. Then, (1.2) becomes $(2\tau - 1)\exp(-2t) + 1 = k$. So, $\tau = 0.5$ and $k = 1$ is obtained. This is a simple example of parameter estimation. The determination of the model structure and the parameter estimation are the core parts of process identification.

Example 1.3
Assume that the black-box model structure for a given process has the following form:

$$\tau^2 \frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\tau \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = ku(t) \tag{1.3}$$

And assume that $y(t) = 0.5\sin(t - \pi/2)$ is obtained from an experiment when $u(t) = \sin(t)$ is applied to the process. Estimate the model parameters τ and k from the experiment.

Solution Replace $y(t)$ and $u(t)$ in (1.3) by $y(t) = 0.5\sin(t - \pi/2)$ and $u(t) = \sin(t)$. Then, (1.3) becomes

$$-0.5\tau^2 \sin(t - \pi/2) + \tau\cos(t - \pi/2) + 0.5\sin(t - \pi/2) = k\sin(t)$$

which can be rewritten as $(0.5\tau^2 - 0.5)\cos(t) + \tau\sin(t) = k\sin(t)$ because $\sin(t - \pi/2) = -\cos(t)$ and $\cos(t - \pi/2) = \sin(t)$. So, $\tau = 1.0$ and $k = 1$ is obtained.

1.1.3 Steady State

When all the derivatives of the process input and process output are zero, this is called the steady state. For example, the process (1.4) will be (1.5) at steady state:

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = \frac{\mathrm{d}u(t)}{\mathrm{d}t} + 2u(t) + 2 \tag{1.4}$$

$$\frac{\mathrm{d}^2 y_{\mathrm{ss}}(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y_{\mathrm{ss}}(t)}{\mathrm{d}t} + y_{\mathrm{ss}}(t) = \frac{\mathrm{d}u_{\mathrm{ss}}(t)}{\mathrm{d}t} + 2u_{\mathrm{ss}}(t) + 2 \rightarrow y_{\mathrm{ss}}(t) = 2u_{\mathrm{ss}}(t) + 2 \tag{1.5}$$

where the subscript 'ss' denotes steady state. As shown in (1.5), all the derivatives go to zeroes at steady state. On the other hand, a cyclic steady state means that the process output and input are periodic signals.

Example 1.4
Consider the process input $u(t)$ and the process output $y(t)$ in Figure 1.3. It can be seen that the process is in steady state after $t = 8$.

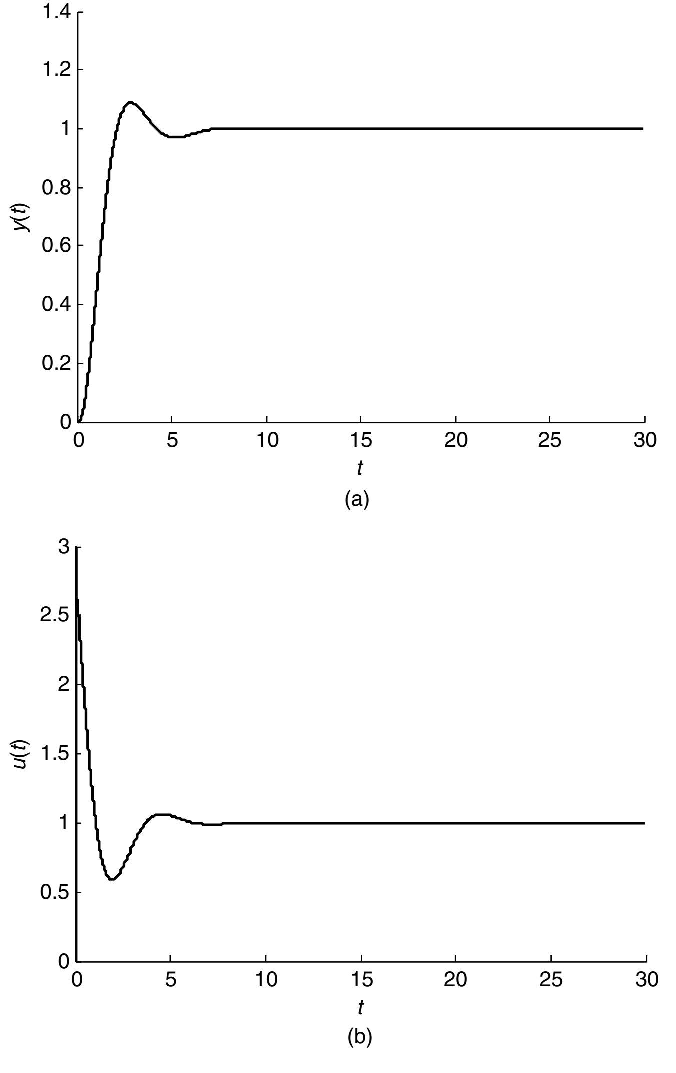

Figure 1.3 The process output and the process input of a control system.

Example 1.5
Obtain $y(t)$ for $u(t) = 2.0$ at steady state for the following process:

$$0.2\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt}(0.1 + 0.05u(t)) + y(t) = \frac{du(t)}{dt} + \sqrt{u(t)} \tag{1.6}$$

Because all the derivatives are zero at steady state, $y_{ss}(t) = \sqrt{u_{ss}(t)}$. So, $y_{ss}(t) = \sqrt{2.0}$ for $u_{ss}(t) = 2.0$ at steady state.

Example 1.6
Obtain $y(t)$ for $y_s(t) = 1.0$ at steady state for the following process:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = 0.1\frac{du(t)}{dt} + u(t) \tag{1.7}$$

$$u(t) = 1.5(y_s(t) - y(t)) + 0.5\frac{d(y_s(t) - y(t))}{dt} \tag{1.8}$$

Because all the derivatives in (1.7) are zero at steady state, $y_{ss}(t) = u_{ss}(t)$ and $u_{ss} = 1.5\ (y_{s,ss} - y_{ss})$ are obtained from (1.7) and (1.8). So, $y_{ss}(t) = 1.5/2.5$ at steady state.

Example 1.7
Consider the process input $u(t)$ and the process output $y(t)$ in Figure 1.4. It can be seen that the process is in cyclic steady state after about $t = 15$ because $u(t)$ and $y(t)$ are periodic after $t = 15$.

1.1.4 Deviation Variables

The deviation variable $\bar{x}(t)$ is the difference between the original variable $x(t)$ and a reference value x_{ref}. That is, $\bar{x}(t) = x(t) - x_{ref}$. So, it represents how far the original variable deviates from the reference value. The deviation variables for the process output and process input can be defined like $\bar{y}(t) = y(t) - y_{ref}$ and $\bar{u}(t) = u(t) - u_{ref}$ respectively. Here, y_{ref} and u_{ref} are usually the process output and the process input at steady state if there is no special notice. Note, y_{ref} is automatically fixed for the given u_{ref} at steady state. For example, the process (1.4) can be rewritten using deviation variables by subtracting (1.5) from (1.4):

$$\frac{d^2\bar{y}(t)}{dt^2} + 2\frac{d\bar{y}(t)}{dt} + \bar{y}(t) = \frac{d\bar{u}(t)}{dt} + 2\bar{u}(t) \tag{1.9}$$

$$\bar{y}(t) = y(t) - y_{ss}, \quad \bar{u}(t) = u(t) - u_{ss} \tag{1.10}$$

where $\bar{u}(t)$ and $\bar{y}(t)$ are deviation variables. u_{ss} and y_{ss} are the reference values for $u(t)$ and $y(t)$ respectively. Here, u_{ss} and y_{ss} should satisfy (1.5). So, y_{ss} is automatically fixed for the given u_{ss} at steady state.

Example 1.8
Rewrite the following process with deviation variables when the reference value for the process input $u(t)$ is chosen as 2.0.

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) + 1 = 2\frac{du(t)}{dt} + 3u(t) \tag{1.11}$$

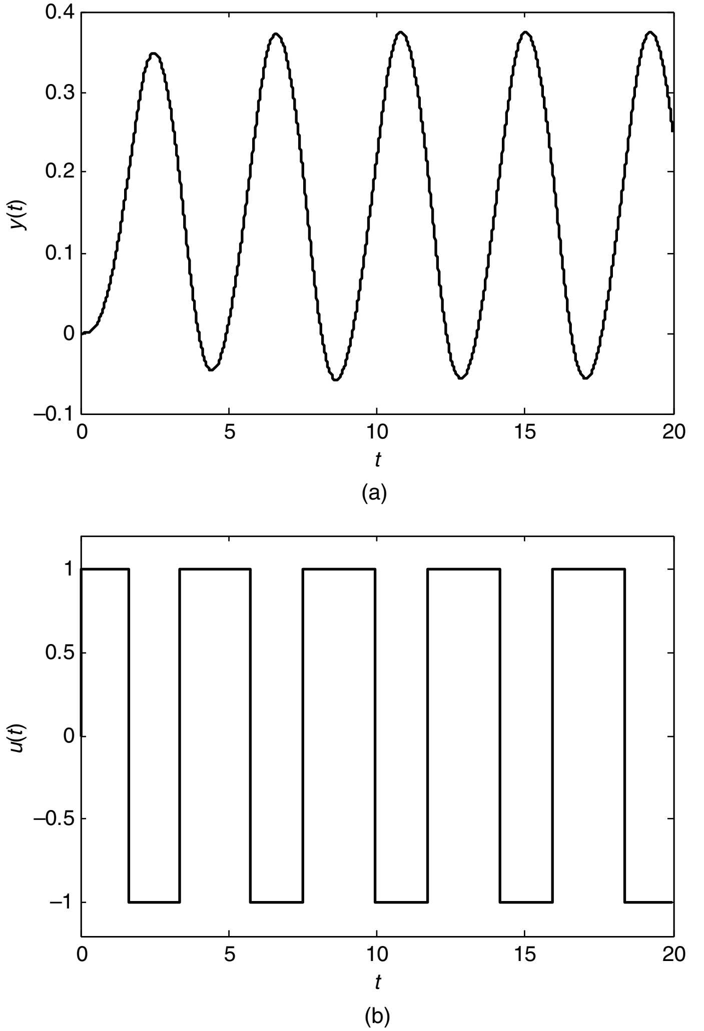

Figure 1.4 The process output and the process input of a relay feedback system.

Solution First, apply the steady-state assumption to (1.11):

$$\frac{\mathrm{d}^3 y_{ss}(t)}{\mathrm{d}t^3}+3\frac{\mathrm{d}^2 y_{ss}(t)}{\mathrm{d}t^2}+3\frac{\mathrm{d}y_{ss}(t)}{\mathrm{d}t}+y_{ss}(t)+1=2\frac{\mathrm{d}u_{ss}(t)}{\mathrm{d}t}+3u_{ss}(t) \tag{1.12}$$

By subtracting (1.12) from (1.11), the following process described by the deviation variables is obtained:

$$\frac{\mathrm{d}^3 \bar{y}(t)}{\mathrm{d}t^3}+3\frac{\mathrm{d}^2 \bar{y}(t)}{\mathrm{d}t^2}+3\frac{\mathrm{d}\bar{y}(t)}{\mathrm{d}t}+\bar{y}(t)=2\frac{\mathrm{d}\bar{u}(t)}{\mathrm{d}t}+3\bar{u}(t) \tag{1.13}$$

$$\bar{y}(t)=y(t)-y_{ss}, \quad \bar{u}(t)=u(t)-u_{ss} \tag{1.14}$$

Here, $u_{ss} = 2.0$. From (1.11), it is known that $y_{ss} = 5.0$ for $u_{ss} = 2.0$ by applying the steady-state assumption. So, the deviation variables (1.14) should be

$$\bar{y}(t) = y(t) - 5.0, \quad \bar{u}(t) = u(t) - 2.0 \tag{1.15}$$

Example 1.9
Rewrite the following process with deviation variables when the reference value for the process input $u(t)$ is chosen as 2.0:

$$\frac{d^3y(t)}{dt^3} + 3\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) + 1 = 2\frac{du(t-0.5)}{dt} + 3u(t-0.5) \tag{1.16}$$

First, apply the steady-state assumption to (1.16):

$$\frac{d^3y_{ss}(t)}{dt^3} + 3\frac{d^2y_{ss}(t)}{dt^2} + 3\frac{dy_{ss}(t)}{dt} + y_{ss}(t) + 1 = 2\frac{du_{ss}(t-0.5)}{dt} + 3u_{ss}(t-0.5) \tag{1.17}$$

By subtracting (1.17) from (1.16) the following process described by the deviation variables is obtained:

$$\frac{d^3\bar{y}(t)}{dt^3} + 3\frac{d^2\bar{y}(t)}{dt^2} + 3\frac{d\bar{y}(t)}{dt} + \bar{y}(t) = 2\frac{d\bar{u}(t-0.5)}{dt} + 3\bar{u}(t-0.5) \tag{1.18}$$

$$\bar{y}(t) = y(t) - y_{ss}, \quad \bar{u}(t) = u(t) - u_{ss} \tag{1.19}$$

From (1.16)$y_{ss} = 5.0$ is obtained for $u_{ss} = 2.0$ because $u_{ss}(t) = u_{ss}(t-0.5) = 2.0$ at steady state. So, the deviation variables (1.19) should be

$$\bar{y}(t) = y(t) - 5.0, \quad \bar{u}(t) = u(t) - 2.0 \tag{1.20}$$

1.2 Properties of Linear Processes

Linear processes are defined and several important properties of linear processes are discussed.

1.2.1 Linear Process

When the dynamics of a process can be described by a linear combination of derivatives ($d^jy(t)/dt^j$, $d^ju(t)/dt^j$, $j = 0, 1, 2, \ldots$) of the process output $y(t)$ and the process input $u(t)$ and a constant, it is a linear process. If the coefficients are time invariant (constants), then it is the time-invariant linear process. If the coefficients are time variant, then it is the time-variant linear process. For example, (1.4) is a linear process. But, the following processes are nonlinear:

$$3\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} + y(t) = \frac{du(t)}{dt}u(t) + 4u(t) \tag{1.21}$$

$$\frac{dy(t)}{dt} + y(t) = 4\sqrt{u(t)} \tag{1.22}$$

Equations (1.21) and (1.22) are nonlinear because of the $(\mathrm{d}u(t)/\mathrm{d}t)u(t)$ and $\sqrt{u(t)}$ terms respectively.

Example 1.10
Consider the following process:

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = u(t-0.5) \tag{1.23}$$

Here, it should be noted that

$$u(t-0.5) = u(t) + \sum_{i=1}^{\infty} \frac{(-0.5)^i}{i!} \frac{\mathrm{d}^i u(t)}{\mathrm{d}t^i}$$

(which will be discussed later). So, (1.23) is a time-invariant linear process. That is, linear processes can include time delays.

Example 1.11
Consider the following process:

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = u(t-0.5) \tag{1.24}$$

$$u(t) = 0.5\int_0^t (1-y(\tau))\mathrm{d}\tau + 0.1\frac{\mathrm{d}(1-y(t))}{\mathrm{d}t} \tag{1.25}$$

In Example 1.11, it is revealed that the time delay does not change the linearity. Also, by differentiating (1.24) and (1.25), the integral in (1.25) then disappears. So, (1.24) and (1.25) is a time-invariant linear process.

Example 1.12
Consider the process

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = u(t-0.5) \tag{1.26}$$

$$u(t) = 2(y_s(t) - y(t)) \tag{1.27}$$

From (1.26) and (1.27), the following process is obtained:

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = 2[y_s(t-0.5) - y(t-0.5)] \tag{1.28}$$

So, the process (1.28) of which the input and output are $y_s(t)$ and $y(t)$ is a time-invariant linear process.

Example 1.13
Consider the process

$$\frac{d^2y(t)}{dt^2} + (2+0.1t)\frac{dy(t)}{dt} + (1-0.05t)y(t) = (2+0.3t)u(t) \tag{1.29}$$

in which the coefficients are time variant. Thus, this is a time-variant linear process.

1.2.2 Superposition Rule

Suppose that the process input is a linear combination of several signals. Then, the process output is the linear combination of the respective process outputs for the several signals if the process is linear. For example, the process output $y(t)$ for the process input $u(t) = u_1(t) + 0.3u_2(t) + 1.3u_3(t)$ can be obtained without a plant test from the available information that the process is linear and the process outputs $y_1(t)$, $y_2(t)$ and $y_3(t)$ are the responses of the process to the process inputs $u_1(t)$, $u_2(t)$ and $u_3(t)$ respectively. That is, it is clear that the process output is $y(t) = y_1(t) + 0.3y_2(t) + 1.3y_3(t)$ for the given process input $u(t)$ by the superposition rule (Figure 1.5).

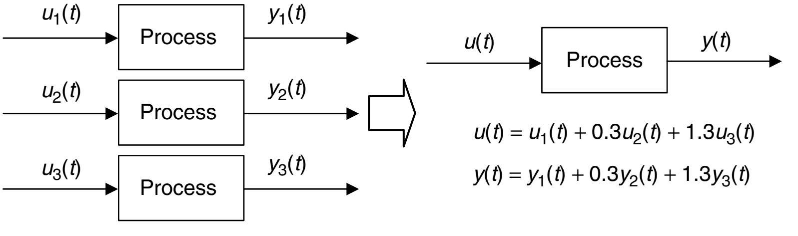

Figure 1.5 Superposition principle.

Therefore, if the pairs $(u_1(t), y_1(t))$, $(u_2(t), y_2(t))$, $(u_3(t), y_3(t))$, ... for the given linear process are known, then $y(t)$ can be easily calculated corresponding to any $u(t)$ of a linear combination $u_1(t)$, $u_2(t)$, $u_3(t)$, ...

Example 1.14
Obtain the process output $y(0.0)$, $y(0.1)$, $y(0.2)$, $y(0.3)$ of a linear process for the following process input $u(t)$:

$$u(t) = 2 \quad \text{for } t \geq 0, \quad u(t) = 0 \quad \text{for } t < 0 \tag{1.30}$$

The available information is that the responses of the process to the process input $u_1(t) = 1$ for $t \geq 0$, $u_1(t) = 0$ for $t < 0$ are $y_1(0.0) = 0.0$, $y_1(0.1) = 0.01$, $y_1(0.2) = 0.02$ and $y_1(0.3) = 0.04$.

Solution Note that $u(t) = 2u_1(t)$. Then, $y(t) = 2y_1(t)$ by the superposition rule. So, $y(0.0) = 0.0$, $y(0.1) = 0.02$, $y(0.2) = 0.04$ and $y(0.3) = 0.08$ are obtained.

Example 1.15

Obtain the process output $y(0.0)$, $y(0.1)$, $y(0.2)$, $y(0.3)$ of a linear process for the following process input $u(t)$:

$$u(t) = 1 \quad \text{for } t \geq 0.1, \quad u(t) = 2 \quad \text{for } 0 \leq t < 0.1, \quad u(t) = 0 \quad \text{for } t < 0 \tag{1.31}$$

The available information is that the responses of the process to the process input $u_1(t) = 1$ for $t \geq 0$, $u_1(t) = 0$ for $t < 0$ are $y_1(0.0) = 0.0$, $y_1(0.1) = 0.01$, $y_1(0.2) = 0.02$ and $y_1(0.3) = 0.04$, and the responses for the process input $u_2(t) = 1$ for $t \geq 0.1$, $u_2(t) = 0$ for $t < 0.1$ are $y_2(0.0) = 0.0$, $y_2(0.1) = 0.0$, $y_2(0.2) = 0.01$ and $y_2(0.3) = 0.02$.

Solution Note that $u(t) = 2u_1(t) - u_2(t)$. Then, $y(t) = 2y_1(t) - y_2(t)$ by the superposition rule. So, $y(0.0) = 0.0$, $y(0.1) = 0.02$, $y(0.2) = 0.03$ and $y(0.3) = 0.06$ are obtained.

Example 1.16

Obtain the process output $y(0.0)$, $y(0.1)$, $y(0.2)$, $y(0.3)$ of a linear time-invariant process for the following process input $u(t)$:

$$u(t) = 0 \quad \text{for } t \geq 0.1, \quad u(t) = 1 \quad \text{for } 0 \leq t < 0.1, \quad u(t) = 0 \quad \text{for } t < 0 \tag{1.32}$$

The available information is that the responses of the process to the process input $u_1(t) = 1$ for $t \geq 0$, $u_1(t) = 0$ for $t < 0$ are $y_1(-0.1) = 0.0$, $y_1(0.0) = 0.0$, $y_1(0.1) = 0.01$, $y_1(0.2) = 0.02$ and $y_1(0.3) = 0.04$.

Solution Note that $u(t) = u_1(t) - u_1(t - 0.1)$. Then, $y(t) = y_1(t) - y_1(t - 0.1)$ by the superposition rule. So, $y(0.0) = 0.0$, $y(0.1) = 0.01$, $y(0.2) = 0.01$ and $y(0.3) = 0.02$ are obtained. This example demonstrates how to obtain the impulse responses from the step responses.

Example 1.17

Obtain the process output $y(0.0)$, $y(0.1)$, $y(0.2)$, $y(0.3)$ of a linear time-invariant process for the following process input $u(t)$:

$$u(t) = 3 \quad \text{for } 0.2 \leq t, \quad u(t) = 4 \quad \text{for } 0.1 \leq t < 0.2, \quad u(t) = 2$$
$$\text{for } 0 \leq t < 0.1, \quad u(t) = 0 \quad \text{for } t < 0 \tag{1.33}$$

The available information is that the responses of the process to the process input $u_1(t) = 0$ for $t \geq 0.1$, $u_1(t) = 1$ for $0 \leq t < 0.1$, $u_1(t) = 0$ for $t < 0$ are $y_1(-0.2) = 0.0$, $y_1(-0.1) = 0.0$, $y_1(0.0) = 0.0$, $y_1(0.1) = 0.01$, $y_1(0.2) = 0.03$ and $y_1(0.3) = 0.02$.

Solution Note that $u(t) = 3u_1(t - 0.2) + 4u_1(t - 0.1) + 2u_1(t)$. Then, $y(t) = 3y_1(t - 0.2) + 4y_1(t - 0.1) + 2y_1(t)$ by the superposition rule. So, $y(0.0) = 0.0$, $y(0.1) = 0.02$, $y(0.2) = 0.10$ and $y(0.3) = 0.19$ are obtained. This example demonstrates how to calculate the process output from the impulse responses of the process. This kind of model is called an “impulse response model.”

Example 1.18

Obtain the process output $y(t)$ of a linear process for the process input $u(t) = 3\sin(t) + 2\sin(3t)$. The available information is that the responses of the process to the process input $u_1(t) = \sin(t)$ are $y_1(t) = 0.3\sin(t-0.1)$ and the responses of the process for the process input $u_2(t) = \sin(3t)$ are $y_2(t) = 0.1\sin(3t-0.2)$.

Solution Note that $u(t) = 3u_1(t) + 2u_2(t)$. Then, $y(t) = 3y_1(t) + 2y_2(t)$ by the superposition rule. So, $y(t) = 0.9\sin(t-0.1) + 0.2\sin(3t-0.2)$ is obtained.

Example 1.19

Obtain the process output $y(t)$ of a linear time-invariant process for the process input $u(t) = \sin(t)$. The available information is that the responses of the process to the process input $u_1(t) = 0.4\sin(t-0.1) + 0.2\sin(3t-0.2)$ are $y_1(t) = 0.3\sin(t-0.2) + 0.1\sin(3t-0.4)$.

Solution $y(t) = 0.3\sin(t-0.2)$ is obtained for $u(t) = 0.4\sin(t-0.1)$ and, equivalently, $y(t) = 3\sin(t-0.1)/4$ for $u(t) = \sin(t)$ from the given information and the superposition rule. Also, $y(t) = \sin(3t-0.2)/2$ is the response to the process input $u(t) = \sin(3t)$.

1.2.3 Linearization

It is notable that many nonlinear processes can be approximated effectively by linearized models. Linearization is the process of obtaining a linear model to approximate the nonlinear model. Taylor series are frequently used for linearization. Theoretically, a nonlinear function $f(u)$ can be represented by the following Taylor series:

$$f(u) = f(u_0) + \left.\frac{\mathrm{d}f}{\mathrm{d}u}\right|_{u=u_0}(u-u_0) + \frac{1}{2!}\left.\frac{\mathrm{d}^2 f}{\mathrm{d}u^2}\right|_{u=u_0}(u-u_0)^2 + \frac{1}{3!}\left.\frac{\mathrm{d}^3 f}{\mathrm{d}u^3}\right|_{u=u_0}(u-u_0)^3 + \cdots \quad (1.34)$$

The following approximation of (1.34) to (1.35) is called linearization at $u = u_0$:

$$f(u) \approx f(u_0) + \left.\frac{\mathrm{d}f}{\mathrm{d}u}\right|_{u=u_0}(u-u_0) \quad (1.35)$$

For example, the straight line in Figure 1.6 corresponds to (1.35), which is close to (1.34) around $u = u_0$.

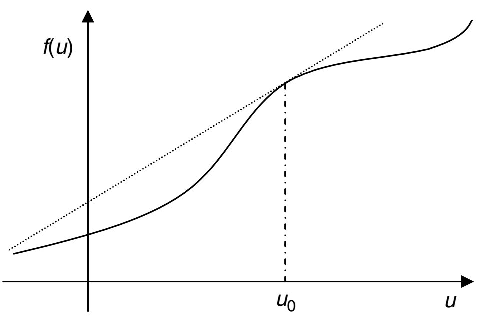

Figure 1.6 Linearization of $f(u)$ at $u = u_0$.

Equation (1.22) can be approximated by the Taylor series at $u(t) = u_0 = 1$ as follows:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = 4\sqrt{u(t)} \approx 4\sqrt{u_0} + 4\frac{1}{2}(u_0)^{-1/2}(u(t) - u_0) \tag{1.36}$$

Equation (1.36) can be rewritten to the following linearized process:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) \approx 2\sqrt{u_0} + 2(u_0)^{-1/2}u(t) \tag{1.37}$$

Equation (1.37) can be also described by the deviation variables:

$$\frac{\mathrm{d}\bar{y}(t)}{\mathrm{d}t} + \bar{y}(t) = 2(u_0)^{-1/2}\bar{u}(t), \quad \bar{y}(t) = y(t) - y_{\mathrm{ss}}, \quad \bar{u}(t) = u(t) - u_{\mathrm{ss}} \tag{1.38}$$

Now, the linearized process (1.38) is obtained for the nonlinear process (1.22).

The Taylor series approximation can be also applied to multivariable nonlinear functions such as $f(u_1, u_2)$ as follows:

$$f(u_1, u_2) \approx f(u_{1,0}, u_{2,0}) + \left.\frac{\partial f}{\partial u_1}\right|_{u_1=u_{1,0}, u_2=u_{2,0}} (u_1 - u_{1,0}) + \left.\frac{\partial f}{\partial u_2}\right|_{u_1=u_{1,0}, u_2=u_{2,0}} (u_2 - u_{2,0}) \tag{1.39}$$

Similarly, the Taylor series approximation can be applied to multivariable functions of which the number of the variables is bigger than 2 in a straightforward manner.

Example 1.20

Obtain the linearized process around $u(t) = u_0 = 2$ for the following nonlinear process, and express it with the deviation variables:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y^{1.5}(t) = u^3(t) \tag{1.40}$$

Solution Equation (1.40) becomes $y^{1.5}(t) = u^3(t)$ at steady state. So, the value of the process output $y(t)$ for $u(t) = u_0 = 2$ is $y_0 = 2^{3/1.5}$. We obtain $y^{1.5}(t) \approx 8 + 3(y(t) - 2^{3/1.5})$ and $u^3(t) \approx 8 + 12(u(t) - 2)$ by the Taylor series approximation. Then, the linearized process is

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 8 + 3(y(t) - 2^{3/1.5}) = 8 + 12(u(t) - 2) \tag{1.41}$$

Equation (1.41) is valid for the steady state. That is, the following equation is valid:

$$\frac{\mathrm{d}y_0(t)}{\mathrm{d}t} + 8 + 3(y_0(t) - 2^{3/1.5}) = 8 + 12(u_0(t) - 2) \tag{1.42}$$

So, the following linearized process represented by the deviation variables is obtained by subtracting (1.42) from (1.41):

$$\frac{\mathrm{d}\bar{y}(t)}{\mathrm{d}t} + 3\bar{y}(t) = 12\bar{u}(t) \tag{1.43}$$

$$\bar{y}(t) = y(t) - 2^{3/1.5}, \quad \bar{u}(t) = u(t) - 2 \tag{1.44}$$

Example 1.21

Obtain the linearized process around $u(t) = u_0 = 2$ for the following nonlinear process, and express it with the deviation variables:

$$\frac{dy(t)}{dt} + y(t)(1 + 0.1u^2(t)) = u^3(t) \tag{1.45}$$

Solution Equation (1.45) becomes $y(t)(1 + 0.1u^2(t)) = u^3(t)$ at steady state. So, the value of the process output $y(t)$ for $u(t) = u_0 = 2$ is $y_0 = 8/1.4$. The following equation is obtained by the Taylor series approximation for the multivariable function $y(t)(1 + 0.1u^2(t))$:

$$\begin{aligned} y(t)(1 + 0.1u^2(t)) &\approx y_0(1 + 0.1u_0^2) + (1 + 0.1u_0^2)(y(t) - y_0) + 0.2y_0u_0(u(t) - u_0) \\ &= 8 + 1.4\left(y(t) - \frac{8}{1.4}\right) + \frac{3.2}{1.4}(u(t) - 2) \end{aligned} \tag{1.46}$$

and $u^3(t) \approx 8 + 12(u(t) - 2)$ by the Taylor series approximation. Then, the linearized process is as follows:

$$\frac{dy(t)}{dt} + 8 + 1.4\left(y(t) - \frac{8}{1.4}\right) + \frac{3.2}{1.4}(u(t) - 2) = 8 + 12(u(t) - 2) \tag{1.47}$$

Equation (1.46) is valid for the steady state. That is, the following equation is valid:

$$\frac{dy_0(t)}{dt} + 8 + 1.4\left(y_0(t) - \frac{8}{1.4}\right) + \frac{3.2}{1.4}(u_0(t) - 2) = 8 + 12(u_0(t) - 2) \tag{1.48}$$

So, the following linearized process represented by the deviation variables is obtained by subtracting (1.48) from (1.47):

$$\frac{d\bar{y}(t)}{dt} + 1.4\bar{y}(t) = \left(12 - \frac{3.2}{1.4}\right)\bar{u}(t) \tag{1.49}$$

$$\bar{y}(t) = y(t) - 8/1.4, \quad \bar{u}(t) = u(t) - 2 \tag{1.50}$$

Example 1.22

Obtain the linearized process around $u(t) = u_0 = 2$ for the following nonlinear process and express it with the deviation variables:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) + 0.01\frac{dy(t)}{dt}u(t)y(t) = u(t) \tag{1.51}$$

Solution Equation (1.51) becomes $y(t) = u(t)$ at steady state. So, the value of the process output $y(t)$ for $u(t) = u_0 = 2$ is $y_0 = 2$ and the value of $dy(t)/dt$ at steady state is zero. So, the linearization should be done around $u_0 = 2$, $y_0 = 2$ and $(dy(t)/dt)_0 = 0$. The following equation is obtained by the Taylor series approximation for the multivariable function

$0.01(\mathrm{d}y(t)/\mathrm{d}t)u(t)y(t)$. Here, $\mathrm{d}y(t)/\mathrm{d}t$ should be considered one of the variables of the nonlinear function $0.01(\mathrm{d}y(t)/\mathrm{d}t)u(t)y(t)$. Also, $\partial(0.01(\mathrm{d}y(t)/\mathrm{d}t)u(t)y(t))/\partial u(t)=0$ and $\partial(0.01(\mathrm{d}y(t)/\mathrm{d}t)u(t)y(t))/\partial y(t)=0$ at steady state should be used.

$$0.01\frac{\mathrm{d}y(t)}{\mathrm{d}t}u(t)y(t) \approx 0.01u_0y_0\left(\frac{\mathrm{d}y(t)}{\mathrm{d}t}-0\right) = 0.04\frac{\mathrm{d}y(t)}{\mathrm{d}t} \tag{1.52}$$

Then, the linearized process is as follows:

$$\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}+2.04\frac{\mathrm{d}y(t)}{\mathrm{d}t}+y(t)=u(t) \tag{1.53}$$

Equation (1.53) is valid for the steady state. That is, the following equation is valid:

$$\frac{\mathrm{d}^2y_0(t)}{\mathrm{d}t^2}+2.04\frac{\mathrm{d}y_0(t)}{\mathrm{d}t}+y_0(t)=u_0(t) \tag{1.54}$$

So, the following linearized process represented by the deviation variables is obtained by subtracting (1.54) from (1.53):

$$\frac{\mathrm{d}^2\bar{y}(t)}{\mathrm{d}t^2}+2.04\frac{\mathrm{d}\bar{y}(t)}{\mathrm{d}t}+\bar{y}(t)=\bar{u}(t) \tag{1.55}$$

$$\bar{y}(t)=y(t)-2, \quad \bar{u}(t)=u(t)-2 \tag{1.56}$$

1.3 Laplace Transform

The Laplace transform plays an important role in analyzing/designing the control system. In this section, the definition of the Laplace transform is introduced. Also how to obtain the Laplace transforms for various functions and how to solve differential equations using the Laplace transform are explained.

1.3.1 Laplace Transforms, Inverse Laplace Transforms

The Laplace transform of $f(t)$ is defined as

$$L\{f(t)\}=f(s)=\int_0^{\infty}\exp(-st)f(t)\,\mathrm{d}t \tag{1.57}$$

where s is a complex variable. $f(s)$ or $L\{f(t)\}$ denotes the Laplace transform of $f(t)$. Note that $f(s)$ is a function of s because it is the integral of $\exp(-st)f(t)$ from $t=0$ to $t=\infty$, which means that the variable t disappears.

The inverse Laplace transform restores the original function $f(t)$ from the Laplace transform of $f(t)$:

$$L^{-1}\{f(s)\}=f(t) \tag{1.58}$$