Procurement when Price and Quality Matter*

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Abstract

A buyer seeks to procure a good characterized by its price and its quality from suppliers who have private information about their cost structure (fixed cost and marginal cost of providing quality). We characterize the buyer's optimal buying mechanism. We then use the optimal mechanism as a theoretical and numerical benchmark to study simpler buying procedures such as scoring auctions and bargaining. Scoring auctions can extract a significant proportion of the buyer's strategic surplus (the difference between the expected revenue from the optimal mechanism and the efficient auction). Bargaining does less well and often does worse than the efficient auction.

Keywords: optimal auction, multi-attribute auction, multidimensional screening, scoring auction, negotiation, bargaining, procurement.

JEL Codes: D44, D82, C78, L24, L22

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1 Introduction

Procurement rarely involves considerations based solely on price. Instead, concerns about the quality of the good or service provided are often important to the final decision. In this paper, we consider how a buyer who cares about quality should structure his purchasing process when suppliers compete for a single procurement contract. We ask two questions: First, what does the optimal procurement mechanism look like? And, second, how well do simpler, empirically relevant, mechanisms perform relative to a benchmark constructed using the optimal and efficient mechanisms?

When suppliers' private information about their costs can be captured by a one-dimensional parameter, the answer to the first question is well known (Laffont and Tirole, 1987, and Che, 1993). In addition, Che (1993) provides a partial answer to the second question by showing that a scoring auction implements the optimal mechanism. In a scoring auction, the buyer announces the way he will rank different offers, that is, the scoring rule; suppliers submit an offer on all dimensions of the product, and the contract is awarded to the supplier who submitted the offer with the highest score according to the scoring rule. This paper extends the analysis of the first question to environments with multidimensional private information and answers the second question more exhaustively and for several alternative procedures.

Conducting procurement when factors in addition to price matter involves moving toward what practitioners often refer to as 'complex procurement.' Large-scale defense acquisitions are an extreme example of complex procurement − the product involves many dimensions, with varying degrees of contractibility, and renegotiation of the contract is expected at many stages. At the other extreme is the acquisition of basic stationary by a corporation − a pencil is a simple thing and price is the only factor that matters for the buyer. The mechanisms used in practice to conduct complex procurement leverage, in varying degrees, the potential competition among suppliers and the scope for flexibility in product design. When evaluating the performance of simple mechanisms, we consider two popular examples: a scoring auction and bargaining. We define bargaining as occurring when the buyer negotiates with potential suppliers one at a time (that is, negotiations with a supplier must irrevocably break down before another supplier is approached), whereas in an auction the buyer can play suppliers off against one another. Bargaining tends to allow more flexibility in terms of product design than a scoring auction, at the cost of lower competition. Our results, in answer to the second question posed at the start of this paper, suggest that scoring auctions do better than bargaining and that they often yield a performance close to that of the optimal mechanism. That is, leveraging competition among suppliers leads to prices that more than compensate for the lower flexibility in product design.

The two distinguishing features of our model are that suppliers' private information about their cost structure is multidimensional and that quality is contractible and endogenously determined as part of the procurement process. The U.S. State Highway Authorities' procurement for highway repair jobs illustrates these aspects of the contracting environment.[1] For high-density traffic areas, these agencies

[1] See, for instance, Arizona Department of Transport (2002), Bajari and Lewis (2009) and Herbsman et al. (1995).

care about the cost of the job and the time required for completion. A contractor may be able to speed up the job by hiring extra labor, by using some equipment more intensively, or by shifting some resources from other jobs. Hence, suppliers' quality (here, the time they need to complete the job) is not fixed, but endogenous, with increased quality incurring a higher cost. Moreover, this marginal cost of quality is likely to vary across potential contractors in a way that is not observable to their competitors. Therefore, it represents one dimension of private information. However, there are other sources of unobserved cost heterogeneity. These include the contractors' material costs, existing contractual obligations and organizational structure, which combine to determine the fixed cost of undertaking a job at any quality level. Thus, private information is likely to be better captured by a multidimensional parameter.

We first derive the optimal procurement mechanism in a model where each potential supplier has private information about two components of her cost structure: her fixed cost and her marginal cost of providing quality. Costs on each dimension can be high or low, and we allow for any pattern of correlation between a supplier's fixed cost and her marginal cost. Across suppliers, costs are independently distributed. The buyer's objective is to maximize his expected utility subject to the suppliers' participation and incentive compatibility constraints.

The optimal procurement mechanism differs significantly from its counterpart in one-dimensional environments. It depends finely on the exact parameters of the problem, including the number of suppliers. Moreover, unlike its one-dimensional counterpart, it is not amenable to implementation by a simple-looking auction format. The source of these discrepancies can be traced back to the well-known endogeneity of the direction in which incentive compatibility constraints bind in multidimensional screening problems.

The fragility of the intuitions gained from one-dimensional models is endemic in research on multidimensional screening and it can leave the economist interested in the application of mechanism design on unsure footing. In this paper, we take a new approach, using the characterization of the optimal mechanism to construct a meaningful benchmark to investigate the performance of practical and simpler buying procedures. In doing so, we suggest one way in which results from the multidimensional screening literature can be used to constructively advance our understanding of mechanisms used in practice.

This benchmark role plays out at two levels. At the theoretical level, we can compare the allocation (probabilities of getting the contract and qualities delivered) of the optimal mechanism and that of any other mechanism of interest to understand their advantages and disadvantages.

At a numerical level, the characterization of the optimal mechanism contributes to solving what is essentially a free-parameter problem when interpreting numerical comparisons. At first glance, there are at least two candidates for benchmarking numerical simulations of the performance of simple mechanisms: the optimal mechanism and the efficient mechanism. Unfortunately, neither of these candidate benchmarks is useful on its own. To illustrate, suppose that, for some set of parameters, the optimal mechanism generates an expected utility for the buyer of 2, while the mechanism of interest returns 1. This looks like a 50% decrease in revenue. However, by adding 9 to the buyer's utility function, we could well generate expected utilities of 11 and 10, respectively. Now the decrease looks like only 9%. We create a benchmark that is immune to the distortions in the previous example by looking at the difference between the expected utility generated from the optimal mechanism and the expected utility from the efficient (buyer-optimal) auction. This difference is the surplus available to a strategic buyer. We use this measure of 'strategic surplus' as our benchmark against which to evaluate second-best mechanisms. Suppose that the efficient mechanism returns an expected utility of 0 (or 9). This allows us to conclude that the mechanism of interest captures 50% of the rents available from being a strategic buyer. This benchmark is both economically meaningful and free from the influence of (positive) affine transformations of the utility function. The characterization of the optimal mechanism is crucial to constructing this benchmark.

We apply this new approach to evaluate the performance of scoring auctions and bargaining. Our motivation for looking at these procedures is twofold. First, Asker and Cantillon (2008) have shown that scoring auctions dominate price-only auctions, beauty contests and menu auctions. Thus, scoring auctions are an obvious candidate for a simple second-best procedure. Second, buyers often adopt a less structured form of negotiation when quality matters and our model of bargaining bounds many models of negotiation in the literature. In drawing the distinction between an auction and a bargaining process, we define bargaining as a procedure in which a buyer approaches suppliers sequentially and cannot return to a supplier once negotiation break down. This means suppliers do not compete directly against each other, in contrast to auctions.

We characterize the allocations that can be implemented by a scoring auction (Theorem 2) and derive the optimal bargaining mechanism when a buyer bargains with a single supplier (Theorem 3) or several suppliers sequentially (Theorem 4). By construction, both procedures under-perform relative to the optimal mechanism. The comparison with the allocation generated by the optimal mechanism highlights several characteristics of these alternatives. First, scoring auctions can replicate the allocation probabilities of the optimal mechanism in many cases. Where scoring auctions fall short of the optimal mechanism is in their inflexibility in terms of qualities. Second, the efficient mechanism can be implemented by a scoring auction. Thus, scoring auctions can always do weakly better than the efficient mechanism. Third, bargaining is inherently inefficient and can never replicate the allocation probabilities of the optimal mechanism. However, we identify two classes of environments where they can do better than the efficient mechanism thanks to the distortion in production or in allocation probabilities that they generate.

We further investigate these questions numerically by evaluating the proportion of the strategic surplus that these simpler procedures capture, across a wide range of environments. To do this, we compute an upper bound to the expected utility from these procedures by deriving the optimal scoring auction and the optimal bargaining mechanism. We find that the optimal scoring auction does very well and, on average, captures more than two thirds of the strategic surplus. By contrast, bargaining does very badly and often even worse than the efficient auction, except when the fixed and marginal costs are highly correlated, or when there is little uncertainty about suppliers' fixed costs. Because these two classes of environments are near one-dimensional environments, it seems safe to claim that an efficient auction generally dominates bargaining when private information is multidimensional.

To further explore the reasons behind the poor performance of bargaining and the strong performance of scoring auctions, we extend our bargaining model to allow the buyer to make an offer to one supplier after negotiations have failed with all suppliers. This recall feature introduces several new effects including a potential loss of commitment power by the buyer and, as a result, multiple equilibria. If we focus on the buyer-optimal sequential equilibrium, we find that the possibility of recall greatly improves the expected utility of the buyer We interpret this result as underscoring the value of any competition among suppliers in procurement settings.

Related literature. This paper is related to the literatures on procurement and multidimensional screening. The literature on procurement is organized around several themes, including the question of how to take factors other than price into account in the procurement process (Laffont and Tirole, 1987; Che, 1993; Branco, 1997; Ganuza and Pechlivanos, 2000; Rezende, 2008; de Frutos and Pechlivanos, 2004), the impact of the potential non-contractibility of quality (Klein and Leffler, 1981; Taylor, 1993; Manelli and Vincent, 1995; Morand and Thomas, 2002; Che and Gale, 2003), and the impact of moral hazard and renegotiation (Bajari and Tadelis, 2001; Bajari, McMillan and Tadelis, 2008). See Che (2006) for an overview.

Our paper fits squarely into the first group and we abstract from the other issues. Our contribution to this literature is twofold. First, we extend prior analyses of optimal procurement to the richer environment where private information is multidimensional. Laffont and Tirole (1987) and Che (1993) characterize the optimal buying mechanism when private information is one-dimensional (the marginal cost of providing quality). Under some regularity conditions, the optimal buying scheme distorts the quality provided by the suppliers downwards, relative to their first best levels. The optimal level of distortion is independent of the number of suppliers, a property known as the "separation between screening and selection" (Laffont and Tirole, 1987). In addition, except for the presence of a reserve price, the contract is always allocated efficiently. Finally, Che shows that a scoring auction with a scoring rule that is linear in price implements the optimal scheme. Our analysis shows that these results depend heavily on the assumption of one-dimensional signals: none of these properties are robust when we move to a multidimensional setting. Second, we evaluate existing buying procedures against the benchmark constructed using the optimal and efficient mechanisms. Other papers compare the performance of different procedures: Dasgupta and Spulber (1989), Che (1993) and Chen-Ritzo et al. (2005) compare the scoring auction, which turns out to be optimal in their setting, with price-only auctions; Asker and Cantillon (2008) compare the scoring auction with price-only auctions, beauty contests, and menu auctions; Manelli and Vincent (1995), Bulow and Klemperer (1996) and (2009) compare (two different models of) negotiation with auctions. Except for Asker and Cantillon (2008), these papers are restricted to one-dimensional private information. Moreover, our paper goes beyond simply ranking procedures, first, by providing a quantitative assessment of the difference in expected utility and, second, by identifying environments where these alternative procedures are likely to perform well.

This paper also contributes to the literature in multidimensional screening. Rochet and Stole (2003) present a survey of the contracting applications of multidimensional screening. Auction applications include the optimal multi-unit auction problems studied by Armstrong (2000), Avery and Henderschott (2000) and Malakhov and Vohra (2009), and the optimal auction with externalities studied by Jehiel et al. (1999). Unlike contracting environments, our problem involves a resource constraint because the contract can be allocated to only one supplier. Unlike multi-unit auction environments, quality in our problem introduces some non-linearity. Hence, none of the existing characterization results applies to our problem and the method we use to solve for the solution is somewhat different from the methods used in these papers (even if the underlying principle is the same).

Through our emphasis on second-best mechanisms, our work echoes the research agenda laid out in Wilson (1993) of identifying simple and robust second-best mechanisms. Our contribution here is in leveraging the characterization of the optimal mechanism to analyze second-best candidates in auction environments with multidimensional private information.

2 Model

We consider a buyer who wants to buy an indivisible good for which there are N potential suppliers. The good is characterized by its price, p, and its quality, q.

Preferences. The buyer values the good (p, q) at $v(q) - p$, where $v_q > 0$ (we assume that $v_q(0) = \infty$ and $\lim_{q \to \infty} v_q(q) = 0$ to ensure an interior solution) and $v_{qq} < 0$. Supplier i's profit from selling good (p, q) is given by $p - \theta_1^i - \theta_2^i q$, where $\theta_1^i \in \{\underline{\theta}_1, \overline{\theta}_1\}$ and $\theta_2^i \in \{\underline{\theta}_2, \overline{\theta}_2\}$ ($\underline{\theta}_1 < \overline{\theta}_1$ and $0 < \underline{\theta}_2 < \overline{\theta}_2$.). For future reference, let $\Delta\theta_1 = \overline{\theta}_1 - \underline{\theta}_1$ and $\Delta\theta_2 = \overline{\theta}_2 - \underline{\theta}_2$. Given the binomial support of θ_1 and θ_2, there are four supplier types: $(\overline{\theta}_1, \overline{\theta}_2)$, $(\underline{\theta}_1, \overline{\theta}_2)$, $(\overline{\theta}_1, \underline{\theta}_2)$, $(\underline{\theta}_1, \underline{\theta}_2)$, which we denote, for brevity, hH, lH, hL and lL. We will sometimes use $(\theta_{1k}, \theta_{2k})$ to denote supplier type k. For example, $(\theta_{1lH}, \theta_{2lH}) = (\underline{\theta}_1, \overline{\theta}_2)$. Note that the buyer and the suppliers are risk neutral.

Social welfare. Let $W_k(q) = v(q) - \theta_{1k} - \theta_{2k} q$, the social welfare associated with giving the contract to type k with quality q. Define $W_k^{FB} = \max_q W_k(q)$. Given the single crossing condition, $q_{lH}^{FB} = q_{hH}^{FB} < q_{hL}^{FB} = q_{lL}^{FB}$ (to save on notation, we will use the short-hand notation $\overline{q}$ and $\underline{q}$ to describe the first-best levels of qualities, $\overline{q} < \underline{q}$).

Our assumptions, thus far, yield an incomplete ordering of types in terms of the first-best levels of welfare they generate. To simplify the analysis, we restrict attention to the case where $W_{lH}^{FB} < W_{hL}^{FB}$. The natural ordering of types is, thus, $lL \succ hL \succ lH \succ hH$. Importantly, the assumption that $W_{lH}^{FB} < W_{hL}^{FB}$ implies $\Delta\theta_1 - \Delta\theta_2 \underline{q} < 0$, while the sign of $\Delta\theta_1 - \Delta\theta_2 \overline{q}$ remains ambiguous.[2] This

[2] $\Delta\theta_1 - \Delta\theta_2 \underline{q} < 0$ follows from noting that $v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 \underline{q} = W_{hL}^{FB} > W_{lH}^{FB} > v(\underline{q}) - \underline{\theta}_1 - \overline{\theta}_2 \underline{q}$

assumption on the ordering of first best welfares does not affect the method we use (in particular, Theorems 2, 3 and 4 do not need this assumption). It mainly reduces the number of cases we need to consider when we characterize the optimal mechanism (Theorem 1). Under this assumption, having a low marginal cost for delivering a higher-quality product is more important than having a low fixed cost, at least in the first-best solution. This case includes, as a limit, the case where firms differ only in their marginal cost parameter, which has been studied by Laffont and Tirole (1987), Che (1993) and Branco (1997).

Information. Preferences are common knowledge among suppliers and the buyer, with the exception of suppliers' types, (θ_1^i, θ_2^i), $i = 1, ..., N$, which are privately observed by each supplier. Types are independently and symmetrically distributed across suppliers, in the sense that the probability of supplier i being of some type is independent of other suppliers' types, but the ex-ante distribution of types is the same for all suppliers. Thus, we can write the probability of each type as $\alpha_k > 0$, $k \in \{hH, lH, hL, lL\}$. We do not put any restriction on the α_k's, except for the fact that they need to sum to one. Any pattern of correlation between a supplier's fixed cost and her marginal cost is allowed.

Note: The 2-by-2 discrete type space considered here is a concession to the practical difficulties of optimal screening problems in multidimensional environments. Two alternative approaches have been used in the literature. Armstrong (1996) and Rochet and Choné (1998) solve a nonlinear pricing problem in a continuous type-space by placing sufficient restrictions on the distribution of types to pin down ex-ante the direction in which incentive compatibility constraints bind. Another alternative route is to assume highly multidimensional private information and leverage a law of large numbers to reduce this multidimensional information into something that converges to one-dimensional private information (Armstrong, 1999b). An advantage of the 2-by-2 setting is that it does not put restrictions on the direction of binding IC constraints. This allows us to explore all the economic richness that multidimensional information introduces. In our conclusions, we discuss the applicability of our results to richer informational environments.

3 Characterization of the Optimal Mechanism

The buyer's problem is to find a mechanism that maximizes his expected utility from the procurement process. For simplicity, we assume that the buyer buys with probability one (that is, we assume non-exclusion).[3] A direct revelation mechanism in this setting is a mapping from the announcements of all suppliers, $\{\theta_1^i, \theta_2^i\}_{i=1}^N$, into probabilities of getting the contract, the level of quality to deliver and a money transfer.

Given that the buyer's preference over quality levels is strictly concave, there is no loss of generality

[3] Unlike environments with continuous multidimensional types (e.g. Armstrong, 1996), the assumption of non-exclusion is not particularly restrictive in discrete type environments. It is easy to find parameter values such that all virtual welfares in the solution are positive, making non-exclusion optimal (this can be seen in expression (6), below).

in restricting attention to quality levels that are only a function of suppliers' types. Let q_k denote the quality level to be delivered by a type k supplier. This, together with suppliers' risk neutrality, implies that suppliers' payoffs and thus, behavior, depend only on their *expected* probabilities of winning and their *expected* payment. Let x_k be the probability of winning the contract conditional on being type k, and let m_k be the expected payment she receives. Finally, let U_k denote type k's equilibrium expected utility. We have: $U_k = m_k - x_k(\theta_{1k} + \theta_{2k} q_k)$.

With these simplifications and notation, the buyer's expected utility from the mechanism is given by

$$F(x_k, q_k, U_k) = N \sum_{k \in \{hH, lH, hL, lL\}} \alpha_k (x_k W_k(q_k) - U_k) \tag{1}$$

The buyer seeks to maximize this expression over contracts (x_k, q_k, U_k), subject to suppliers' incentive compatibility (IC) constraints:

$$U_k \geq U_j + x_j(\theta_{1j} - \theta_{1k}) + x_j q_j (\theta_{2j} - \theta_{2k}) \qquad \text{for all } k, j \in \{hH, lH, hL, lL\}, \tag{2}$$

individual rationality (IR) constraints:

$$U_k \geq 0 \qquad \text{for all } k \in \{hH, lH, hL, lL\}, \tag{3}$$

and subject to the feasibility constraint that the probability of awarding the contract to a subset of the types is always less than or equal to the probability of such types in the population:

$$N \sum_{k \in K} \alpha_k x_k \leq 1 - (1 - \sum_{k \in K} \alpha_k)^N \text{ for all subsets } K \text{ of } \{hH, lH, hL, lL\} \tag{4}$$

Finally, non-exclusion imposes that

$$N \sum_{k \in \{hH, lH, hL, lL\}} \alpha_k x_k = 1 \tag{5}$$

Border (1991) guarantees that the feasibility constraint is both necessary and sufficient for the expected probabilities x_k to be derived from a real allocation mechanism. This ensures that the solution to the maximization problem of (1) subject to (2), (3), (4) and (5) is implementable.

The buyer's problem has four individual rationality constraints, 12 incentive compatibility constraints and 15 feasibility constraints. We can simplify them somewhat with the following lemmas:

Lemma 1: *Consider the feasibility constraints (4), and define an n-type constraint as a feasibility constraint with the relevant subset K having n elements. The following statements hold:*

i. At most, one 1-type constraint binds; at most, one 2-type constraint binds; and, at most, one 3-type constraint binds.

ii. These binding constraints are nested, in the sense that the type in the binding 1-type constraint must belong to the binding 2-type constraint, and so on.

The proof of Lemma 1 is in Appendix A. The intuition is as follows. Suppose that, at the solution, the contract is allocated according to the following order of priority: $lL \succ lH \succ hL \succ hH$, i.e. give the contract to a type lL if there is one, otherwise to a type lH if there is one, and so on. This means that the ex-ante probability that a lL type gets the contract is the probability that there is at least one type lL supplier among the N suppliers, i.e., $N\alpha_{lL}x_{lL} = 1-(1-\alpha_{lL})^N$. Thus the 1-type constraint binds for lL. It cannot bind for any other types because a binding constraint for another type would imply that *that* type has priority over all other types in the allocation, a contradiction. Next, $lL \succ lH \succ hL \succ hH$ also means that the contract is allocated to a type lL or lH whenever there is one among the N suppliers. This means that the ex-ante probability of a type lL or lH winning, $N(\alpha_{lL}x_{lL} + \alpha_{lH}x_{lH})$, is the probability that there is at least one of these types among the suppliers, $1-(1-\alpha_{lL}-\alpha_{lH})^N$. Thus, the 2-type constraint binds for $\{lL, lH\}$, showing that the binding constraints are indeed nested. Statement (i) of Lemma 1 suggests that it could be the case that, say, no 1-type constraint binds. This will be the case, for instance, if the order of priority is $lL \sim lH \succ hL \succ hH$, that is, lL and lH have priority over all the other types, but if there are a lL type and a lH type, the buyer allocates the contract among them randomly. In this case, no 1-type constraint binds. Finally, note that the suppliers' expected probabilities are weakly aligned with their order of priority in the sense that, if $k \succ j$, then $x_k > x_j$ but if $k \sim j$, then $x_k \gtreqqless x_j$.

For future reference, denote the winning probabilities resulting from the efficient allocation ($lL \succ hL \succ lH \succ hH$) by x_k^{FB}, $k \in \{hH, lH, hL, lL\}$. Denote the winning probabilities for type lH and hL resulting from the allocation according to order of priority $lL \succ lH \succ hL \succ hH$ by $x_{lH}^{\max}$ and $x_{hL}^{\min}$.

Standard manipulation of the incentive compatibility constraints and the individual rationality constraints allows us to order the probabilities of winning in a limited way.

Lemma 2: *At any solution,* $x_{lH} \geq x_{hH}$, $x_{lL} \geq x_{hL}$ *and* $U_{hH} = 0$

The key difficulty we face in characterizing the solution to the buyer's problem is in identifying the set of binding constraints at the optimum together with the associated partition of the parameter space. Our approach is to start with the buyer-optimal efficient mechanism. The buyer-optimal efficient mechanism is the mechanism that implements the efficient allocation in the way most favorable to the buyer. Efficiency requires that qualities are set such that $q_{lL} = q_{hL} = \underline{q}$ and $q_{lH} = q_{hH} = \overline{q}$, and that the probabilities are set equal to the first-best probabilities, i.e., $x_k = x_k^{FB}$ for all k. Efficiency does not pin down payments to suppliers when private information is discrete. The buyer-optimal efficient mechanism (which we will simply refer to in the remainder as "the efficient mechanism") sets payments to maximize the buyer's expected utility while satisfying all incentive compatibility constraints. In practice, only two sets of IC constraints bind at the efficient mechanism, as the next lemma establishes:

Lemma 3: *When* $\Delta\theta_1 > \Delta\theta_2\overline{q}$, $IC_{lH,hH}$, $IC_{hL,hH}$ *and* $IC_{lL,hL}$ *bind in the efficient mechanism. When* $\Delta\theta_1 < \Delta\theta_2\overline{q}$ $IC_{lH,hH}$, $IC_{hL,lH}$ *and* $IC_{lL,hL}$ *bind (see Figure 1).*

Insert Figure 1 Here

The proof of Lemma 3 can be found in Appendix A. From this starting point, we progressively adjust the qualities (the q's) until the buyer's utility is maximized conditional on the x's or until a new IC constraint binds. If no new IC constraint binds we optimize over the x's and turn our attention back to the q's. If a new IC constraint binds, we adjust the x's and q's under the additional constraint imposed by the new binding IC constraint. In this way, we progressively reach a point where there is no scope for improvement through either changing the x's or the q's. At this point, we will have reached the global maximum as guaranteed by the next lemma. Moreover, this approach ensures that we cover the entire parameter space since our starting points cover the whole space. The sketch of proof of Theorem 1 illustrates this approach in more detail.

Lemma 4: *The first order conditions of the Lagrangian of the maximization problem (1) subject to (2), (3), (4) and (5) are necessary and sufficient for a global maximum.*

The proof of Lemma 4 is in Appendix A. It allows us to prove the main result of this section:

Theorem 1: Characterization of the optimal buying mechanism
Define $q_{hH}^2 = \arg\max_q\{W_{hH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}} q\Delta\theta_2\}$ *and* $q_{lH}^2 = \arg\max_q\{W_{lH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{lH}} q\Delta\theta_2\}$.[4]
Part I: When $\Delta\theta_1 - \Delta\theta_2\overline{q} \geq 0$, *the probabilities of winning and quality levels in the optimal mechanism are as given in Table 1.*
Part II: When $\Delta\theta_1 - \Delta\theta_2\overline{q} < 0$, *the probabilities of winning and quality levels in the optimal mechanism are as given in Table 2.*

Insert Tables 1&2 Here

Sketch of Proof*:* The full proof of Theorem 1 is very long (18 pages). Here, we provide only a proof for solutions 1.1.a and 1.1.b to illustrate our approach to deriving the full characterization. The reader is referred to the online appendix for the full proof.[5]

Consider the efficient auction. Let $U_{k,j}$ be the expected utility of a type k pretending she is of type j. To ensure incentive compatibility, while minimizing suppliers' rents, suppliers' expected utilities in the efficient auction must be set such that $U_k = \max_{j\neq k} U_{k,j}$ and $U_{hH} = 0$.
From Lemma 3, we need to consider only two cases. If $\Delta\theta_1 - \Delta\theta_2\overline{q} \geq 0$, the per-supplier buyer's expected utility in the efficient auction, $\sum_k \alpha_k[x_k^{FB} W_k(q_k) - U_k]$, is given by:

$$\alpha_{lH}x_{lH}^{FB}W_{lH}(q_{lH}) - \alpha_{lH}x_{hH}^{FB}\Delta\theta_1 + \alpha_{hH}x_{hH}^{FB}W_{hH}(q_{hH}) + \alpha_{hL}x_{hL}^{FB}W_{hL}(q_{hL}) - \alpha_{hL}x_{hH}^{FB}q_{hH}\Delta\theta_2$$
$$+\alpha_{lL}x_{lL}^{FB}W_{lL}(q_{lL}) - \alpha_{lL}x_{hL}^{FB}\Delta\theta_1 - \alpha_{lL}x_{hH}^{FB}q_{hH}\Delta\theta_2$$

(where all qualities are initially equal to the first-best qualities) or, to highlight the virtual welfare

[4] We use the superscript '2' to differentiate this quality from the first best quality.
[5] All online materials are avaliable at http://pages.stern.nyu.edu/~jasker/ and http://www.ecares.org/ecantillon.htm

generated by each supplier:

$$\alpha_{lH}x_{lH}^{FB}W_{lH}(q_{lH})+\alpha_{hH}x_{hH}^{FB}\left[W_{hH}(q_{hH})-\frac{\alpha_{lH}}{\alpha_{hH}}\Delta\theta_1-\frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}}q_{hH}\Delta\theta_2\right] \tag{6}$$
$$+\alpha_{hL}x_{hL}^{FB}\left[W_{hL}(q_{hL})-\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1\right]+\alpha_{lL}x_{lL}^{FB}W_{lL}(q_{lL})$$

The rents of suppliers lL and hL depend positively on q_{hH}, and the buyer can increase his expected utility by decreasing q_{hH}, ideally until

$$q_{hH}^2=\arg\max_q\{W_{hH}(q)-\frac{\alpha_{lH}}{\alpha_{hH}}\Delta\theta_1-\frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}}q\Delta\theta_2\}$$

Suppose that no new IC constraint binds in the process. (This will be the case if $x_{lH}^{FB}[\Delta\theta_1-\Delta\theta_2\overline{q}]>x_{hH}^{FB}[\Delta\theta_1-\Delta\theta_2 q_{hH}^2]$). Now consider again (6), where we set $q_{hH}=q_{hH}^2$, $q_{lH}=\overline{q}$, and $q_{hL}=q_{lL}=\underline{q}$. There is no further scope for improvement by distorting qualities. Furthermore, the virtual welfare of lL is clearly the largest of all, so that it is optimal to set $x_{lL}=x_{lL}^{FB}$. However, the relative ranking of the virtual welfare of lH and hL is unclear. If $W_{hL}(\underline{q})-\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1>W_{lH}(\overline{q})$, the virtual welfare generated by supplier hL remains larger than that of lH, so the optimal allocation is the first-best allocation. This is solution 1.1.a.

Suppose, instead, that the virtual welfare associated with lH is larger than that associated with hL, itself larger than the virtual welfare associated with hH (formally, and referring to (6), $W_{lH}(\overline{q})>W_{hL}(\underline{q})-\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1\geq W_{hH}(q_{hH}^2)-\frac{\alpha_{lH}}{\alpha_{hH}}\Delta\theta_1-\frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}}q_{hH}^2\Delta\theta_2$). In this case, the buyer would rather give the contract to supplier lH than to supplier hL, i.e., he would like to change the order of priority in the allocation. Increasing x_{lH} while decreasing x_{hL} concurrently (keeping $\alpha_{lH}x_{lH}+\alpha_{hL}x_{hL}+\alpha_{lL}x_{lL}^{FB}$ constant) does not initially affect any of the virtual welfare, and it increases the buyer's expected utility. This process continues until either a new IC constraint binds or we have reached the feasibility constraint for $x_{lH}:N\left(\alpha_{lH}x_{lH}^{\max}+\alpha_{lL}x_{lL}^{FB}\right)=1-(\alpha_{hL}+\alpha_{hH})^N$. Suppose that we reach $x_{lH}=x_{lH}^{\max}$ before any new IC constraint binds. The qualities and probabilities are then all optimized given the binding constraints and lemma 4 guarantees that we have reached the global maximum. This corresponds to solution 1.1.b. Solution 1.1.c. arises if a new IC constraint binds in the process. Solutions 1.1.d. and 1.1.e. arise when the ordering of virtual social welfares is such that type lH is preferred to type hH, which, in turn, is preferred to type hL. **End of the sketch of the proof.**

Tables 1 and 2 present the main features of the solution. The second column describes the probabilities of winning, and the last four columns describe the qualities at the solution (an interval means that the optimal level of quality lies in this interval). For instance, Solution 1.2.b has $x_{lL}=x_{lL}^{FB}$, which is greater than $x_{hL}(<x_{hL}^{FB})$. This is, in turn, greater than $x_{lH}\left(>x_{lH}^{FB}\right)$ and $x_{hH}=x_{hH}^{FB}$. Both q_{lL} and q_{hL} are at the first best-levels, $q_{hH}\in(q_{hH}^2,\overline{q})$ and $q_{lH}\in(q_{lH}^2,\overline{q})$. Both are distorted below the first-best level. The conditions that define each solution depend on the resulting binding constraints and virtual welfares, as summarized in Figures 2 and 3. The value of the objective function and the value of the control variables at the solution are continuous in the parameters of the model.

Insert Figure 2 Here

The following patterns emerge from the tables. First, because the conditions delimiting the different cases depend on the probabilities of winning, the solution depends on the number of suppliers, as well as on the usual parameters of the environment (distributions of types and cost structure). The dependence of the optimal scheme on the number of suppliers is typical of multidimensional environments where the binding IC constraints are endogenous (Palfrey, 1983, Armstrong, 2000, Avery and Hendershott, 2000). No such effect is present in one-dimensional environments (Laffont and Tirole, 1987).

Second, there is some downward distortion in the quality provided by the high marginal-cost suppliers.[6] The quality provided by the low marginal-cost suppliers is never distorted.

Third, probabilities of winning are also often distorted relative to the efficient auction. Specifically, the probabilities of winning of the high marginal-cost suppliers are sometimes distorted upwards, whereas the probability of winning of low marginal-cost supplier hL is sometimes distorted downwards. This is because the expected surplus of hL can, in turn, affect the information rents of the lL type (while lH's expected surplus may not). As a result, reducing x_{hL} (and hence increasing x_{lH}) decreases the information rents of lL. Note that the allocation of supplier lL is never distorted.

Insert Figure 3 Here

Putting these last two aspects together − productive and allocative distortions − we find no systematic "bias against quality" in the two-dimensional model, unlike in the one-dimensional model (Laffont and Tirole, 1987 and Che, 1993). While the economic conclusions differ, the underlying economic motivation is the same: reducing suppliers' rents. The qualities of the high marginal-cost types are distorted downwards to reduce the low marginal-cost supplier's benefit from imitating them. As illustrated in Figures 3 and 4, all binding constraints between suppliers with different marginal costs are from the low marginal-cost supplier to the high marginal-cost supplier so this "trick" is effective. This is also the case in the one-dimensional model where the distortion of high-cost types' quality lowers the informational rents of the low cost types. Similarly, the reason why supplier hL's probability of winning is sometimes below her first-best level is to reduce supplier lL's rent. In each case, the optimal level of distortion balances a trade-off between the costs in terms of lost social welfare and the benefits in terms of reduced rents.

Lastly, we note that the solution approach differs from that taken in the rest of the literature. The optimal multi-unit auction problems studied in Armstrong (2000), Avery and Henderschott (2000),

[6] The finding that quality is only distorted downward is specific to the case $W_{lH}^{FB} < W_{hL}^{FB}$. If $W_{lH}^{FB} > W_{hL}^{FB}$, it is possible to generate an optimal mechanism in which q_{hL} is distorted above first best. This appears to be the only significant qualitative distinction between the $W_{lH}^{FB} < W_{hL}^{FB}$ and $W_{lH}^{FB} > W_{hL}^{FB}$ cases.

Malakhov and Vohra (2009), and Manelli and Vincent (2007) are linear programming problems. Candidates for a solution in a linear programming problem are extreme points. The standard solution technique is to characterize the parameter space over which these extreme points are, indeed, solutions. Our auction problem is not a linear programming problem but, instead, a concave programming problem. This is reflected in the solution: both the value of the objective function and the value of the control variables are continuous in the parameters of the model.

Multidimensional screening models that give rise to concave programming problems are studied by Dana (1993), Armstrong and Rochet (1999), and Armstrong (1999a), or Laffont, Maskin and Rochet (1987) and Rochet and Stole (2002) for continuous-type analogues. The standard solution technique used in these papers is to posit a set of binding constraints and characterize the parameter space over which the first-order conditions are satisfied given these binding constraints.

Our problem differs from those considered in these papers in two respects. First, we have many more constraints: on top of the standard four individual rationality and 12 incentive compatibility constraints that these problems have in their discrete form, the auction aspect of our problem adds 15 feasibility constraints. Moreover, because suppliers' utility function takes the form $m_k - x_k(\theta_1^i + \theta_2^i q_k)$ where x_k and q_k interact as a multiplier of θ_2^i, the two instruments at the disposal of the buyer do not perform a symmetric role as in the models in Dana (1993) and Armstrong and Rochet (1999). The consequences are twofold. First, the number of "solutions" − i.e., configurations of binding constraints at the optimum − is larger. This is seen in Tables 1 and 2 (Armstrong and Rochet (1999) have, at most, six solutions to consider). Second, it is harder to reduce *a priori* the number of constraints that are likely to bind. By seeking incremental improvements from the efficient mechanism, our constructive approach to the characterization of the solution guarantees that we cover the entire parameter space.

4 Scoring Auctions

In practice, implementation of the optimal mechanism requires overcoming at least two significant challenges. First, implementation requires precise knowledge of the environment. Second, in most instances, implementation of the optimal mechanism using a simple, easily explained mechanism is not possible. This limits the extent to which the mechanism can be explained to market participants at low cost and also limits the buyer's ability to administer procurement at low cost (since administration would require, for example, highly skilled staff).

These challenges suggest that, for practical purposes, second-best solutions that are simple and perform well in a variety of settings are likely to be more useful. Commonly used procedures are obvious candidates. They include scoring auctions, price-only auctions with minimum quality standards, beauty contests, menu auctions where suppliers can submit several price-quality offers, and bargaining. Asker and Cantillon (2008) have shown that scoring auctions yield a higher expected utility to the buyer than a price-only auction with minimum standards or a beauty contest, and that they dominate menu auctions when a second price or an ascending format is used. Hence, our contenders for second-best procedures are scoring auctions and bargaining. We analyze scoring auctions in this section and bargaining in the next two sections.

In a scoring auction, the buyer announces a scoring rule that is linear in price, $S(p,q) = \widetilde{v}(q) - p$ (with $\widetilde{v}_q \geq 0$, $\widetilde{v}_{qq} \leq 0$ and $\max \widetilde{v}(q) - \theta_2^i q$ admitting a single interior solution), suppliers submit price-quality bids (p,q), and the winner is the supplier whose bid generates the highest score according to the scoring rule.[7] The winner's resulting obligation depends on the auction format. In a first-score scoring auction, the winner must deliver a quality level at a price that matches the score of his bid. In a second-score scoring auction, the winner must deliver a quality level at a price that matches the second-highest score submitted. Scoring auctions are increasingly used in public and private procurement and are supported by several procurement software packages (see, Asker and Cantillon, 2008 for examples and references).

4.1 Theoretical properties

Scoring auctions put some additional structure on suppliers' bidding behavior. First, given a scoring rule $\widetilde{v}(q) - p$, suppliers choose their bids to maximize the score they generate given their profit target, π, i.e., they solve $\max_{(p,q)}\{\widetilde{v}(q) - p\}$ subject to $p - \theta_1^i - \theta_2^i q = \pi$. Substituting for p inside the maximizer yields

$$\max_q \{\widetilde{v}(q) - \theta_1^i - \theta_2^i q - \pi\} \tag{7}$$

A property of the solution is that q is independent of π, the profit target, and of θ_1^i, the fixed cost. Second, a standard incentive compatibility argument establishes that the ordering of suppliers' winning probabilities must correspond to their ability to generate a higher score (intuitively, define $\max_q\{\widetilde{v}(q) - \theta_1^i - \theta_2^i q\}$ as the supplier's type). Thus, a scoring auction will implement a particular allocation if two conditions hold:

1. [production constraint] Given the scoring rule, suppliers maximize (7) by choosing the level of quality assigned by the allocation.

2. [ranking constraint] The ranking of $\max_q\{\widetilde{v}(q) - \theta_1^i - \theta_2^i q\}$ and, thus, the ranking of the scores are consistent with the assigned probabilities of winning.

The next Theorem characterizes the set of allocations that can be implemented by a scoring auction.

Theorem 2: *An allocation $\{(x_k, q_k)\}_k$ can be implemented with a scoring auction if and only if (1) $q_{lH} = q_{hH}$, $q_{hL} = q_{lL}$ with $q_{lH} = q_{hH} < q_{hL} = q_{lL}$, (2) $\alpha_{lH}x_{lH} + \alpha_{hL}x_{hL} = \alpha_{lH}x_{lH}^{FB} + \alpha_{hL}x_{hL}^{FB}$, $x_{hH} = x_{hH}^{FB}$ and $x_{lL} = x_{lL}^{FB}$, (3) $\Delta\theta_1 - \Delta\theta_2 q_{hL} \leq 0$ when $x_{hL} > x_{hL}^{\min}$ and (4) $\Delta\theta_1 - \Delta\theta_2 q_{lH} \geq 0$ whenever the allocation is such that $x_{lH} > x_{lH}^{FB}$.*

[7] Asker and Cantillon (2008) refer to this auction format as a quasilinear scoring auction to emphasize the linearity of the scoring rule in p.

Theorem 2 clarifies the constraints that a scoring auction places on the possible allocations. Its proof can be found in Appendix A. The first condition says that two suppliers with the same marginal cost of quality must be providing the same level of quality. Moreover, suppliers with a lower marginal cost of quality must deliver higher levels of quality. These two properties follow from the structure of (7). The second condition says that, at equilibrium, type lL must win over any other type, and that type hH must lose against any other type. The reason is that type lL generates the highest value for $\max_q\{\widetilde{v}(q) - \theta_1^i - \theta_2^i q\}$ for any scoring rule and that type hH generates the lowest such value. The third and fourth conditions follow from the combination of the production constraint and the ranking constraint. Finally, to prove the sufficiency part of the claim, we construct a scoring rule that implements the allocation under conditions (1) through (4).

An immediate consequence of Theorem 2 is that the efficient allocation can be implemented by a scoring auction. Such a scoring auction has a scoring rule that corresponds to the buyer's preferences and uses for example a second-score format.[8]

Theorem 2 also clarifies why scoring auctions cannot, in general, implement the optimal solution. First, q_{hH} and q_{lH} differ generically in the optimal mechanism. Moreover, the optimal mechanism requires $x_{hH} > x_{hH}^{FB}$ in several cases. This said, scoring auctions have two potential advantages over the efficient auction. First, they allow for distortion in production. Second, they allow some distortion in allocation probabilities in the same direction as the optimal mechanism. The next section investigates these properties numerically.

4.2 Computational Results

Having identified the constraints that scoring auctions place on allocations, we now turn to the question of their relative performance. We interpret the difference between the buyer's expected utility from using the optimal mechanism and from using the (buyer-optimal) efficient auction as the surplus available to a strategic buyer. We ask to what extent scoring auctions can capture this surplus.

To answer this question, we first compute an upper bound to the expected utility that scoring auctions generate by adding the constraints of Theorem 2 onto the initial problem and solving the resulting program numerically. The resulting expected utility is then compared with the expected utility from the efficient auction and the expected utility from the optimal mechanism.

Insert Figure 4 Here

Figure 4 shows the results for an environment where $v(q) = 3\sqrt{q}$, $\underline{\theta}_1 = \underline{\theta}_2 = 1$, $\overline{\theta}_2 = 2$, $N = 2$ and $\alpha_k = 0.25$. The value of $\Delta\theta_1$ varies along the x-axis. The expected utility from the optimal mechanism

[8] For the scoring auction to generate as much utility to the buyer as possible, type-specific down-payments must be included. These down-payments are an artifact of the discrete type space, where allocations only partially pin down payments. They maintain incentive compatibility and increase the buyer's utility. Fudenberg and Tirole (1991) provide details.

lies above that from the optimal scoring auction, which, in turn, dominates that from the efficient auction. As the value of $\Delta\theta_1$ increases, the expected utility decreases in all three mechanisms. This is to be expected. When $\Delta\theta_1$ increases, the maximum level of welfare decreases because suppliers' fixed costs increase. Moreover, fixed costs become relatively more important as a source of adverse selection. The kink at $\Delta\theta_1 = 0.5625$ corresponds to the point when the binding incentive compatibility constraint for type hL in the efficient mechanism switches from $\text{IC}_{hL,lH}$ to $\text{IC}_{hL,hH}$ (thus $\Delta\theta_1 = \Delta\theta_2\overline{q}$). The resulting increase in the weight of $\Delta\theta_1$ in the buyer's expected utility explains the kink.

As $\Delta\theta_1$ tends to 0, the source of adverse selection reduces to one dimension, the marginal cost. In this case, Che (1993) has shown that a scoring auction implements the optimal mechanism. The reason why the expected utility from the optimal scoring auction does not converge to the expected utility of the optimal mechanism in our graph is that there is some discontinuity in the optimal scoring auction at $\Delta\theta_1 = 0$. As long as $\Delta\theta_1 > 0$, scoring auctions impose that $x_{lH} > x_{hH}$ (Theorem 2). This leaves some informational rent to lH and increases the rents of hL and lL relative to the case where $x_{lH} = x_{hH}$. When $\Delta\theta_1 = 0$, suppliers lH and hH are essentially the same. The optimal scoring auction will, thus, set $x_{lH} = x_{hH}$ and leave no rent to supplier lH.

We replicate this simulation exercise for a range of environments by varying the values for the α_k's and some of the other parameters of the model. Table 3 reports the results. The third column reports the average percentage of the strategic surplus captured over the full range of values that $\Delta\theta_1$ can take. The fourth column reports the maximum percentage of the surplus that the optimal scoring auction captures together with the corresponding value of $\Delta\theta_1$. The fifth column does the same for the worst relative performance of the optimal scoring auction. Finally, columns 6 and 7 report the percentage of $\Delta\theta_1$ values for which the performance of the optimal scoring auction is greater than 80% (column 6) or within ten percentage points of its worst performance (column 7). For the core set of experiments (experiments 1 through 22), $v(q) = aq^b$ with $a = 3$ and $b = 0.5$, $\underline{\theta}_1 = \underline{\theta}_2 = 1$, $\overline{\theta}_2 = 2$, $N = 2$. The bottom part of the table considers other values for a, b, $\underline{\theta}_2$ and $\overline{\theta}_2$. (We keep $N = 2$ in all our experiments because this is where the actual choice of mechanisms is likely to matter most). Figure 5 shows the relative performance of the optimal scoring auction as $\Delta\theta_1$ changes for selected probability configurations.

Insert Table 3 Here, Insert Figure 5 Here

The results are as follows. First, in every experiment, there exists a value of $\Delta\theta_1$ for which the optimal scoring auction does as well as the optimal mechanism. Second, the point at which this occurs seems somewhat persistent across environments. Third, the optimal scoring auction captures, on average, more than two thirds of the strategic surplus, even though this proportion can dip down to 20% for some values of $\Delta\theta_1$ in some environments. Fourth, the optimal scoring auction does worst when the fixed cost and the marginal cost are negatively correlated. We now investigate each of these points in more detail.

In every experiment, there exists a value of $\Delta\theta_1$ for which the optimal scoring auction does as well as the optimal mechanism. Given Theorem 2, this must happen at parameter values such that there are binding incentive compatibility constraints directed to both lH and hH from low marginal-cost suppliers in the optimal scheme (otherwise, there is no chance that the qualities provided by suppliers lH and hH are the same in the optimal scheme). Inspection of Tables 1 and 2 suggests that the only candidates consistent with implementation with a scoring auction are solutions 1.1.c., 1.2.a, 1.2.b or 1.2.c. (Recall that scoring auctions require $x_{hH} = x_{hH}^{FB}$). Closer inspection of the numerical solution suggests that the maximum performance of the optimal scoring auction happens when the optimal mechanism corresponds to either solutions 1.2.a, or 1.2.c.

An inspection of Figure 5 and the results in Table 3 suggests that the optimum is reached at a similar region in each set of simulations (in particular, this point is always less than 0.5625, the point at which the binding incentive compatibility constraint for type hL in the efficient mechanism switches from $\mathrm{IC}_{hL,lH}$ to $\mathrm{IC}_{hL,hH}$). This raises the question of why there and not elsewhere? The top right panel in Figure 5 suggests that that maximum level of expected utility can be reached for values of $\Delta\theta_1$ to the right of the dip in revenue. To investigate this, we ran a set of experiments with the probabilities $(40, 40, 10, 10)$ and $(45, 45, 5, 5)$: experiments 13 and 20. In this setting, the optimal mechanism corresponds to solution 1.1.d in Table 1. In experiment 13, at $\Delta\theta_1 = 0.93375$, the optimal scoring auction captures 99.03% of the available strategic surplus, whereas in experiment 20, at $\Delta\theta_1 = 1.02375$, this is raised to 99.89%. While it appears that the scoring auction does well in these regions, it falls short of the optimal mechanism because of the restriction that $x_{hH} = x_{hH}^{FB}$ in the scoring auction.

It has been noted that the point at which the optimal scoring auction does as well as the optimal scheme is constant across experiments 1-7 and 14-19. After further inspection, this is an artifact of the common symmetric structure of these parameter settings. These settings are such that $\alpha_{lL} + \alpha_{hL} = \alpha_{hH} + \alpha_{lH}$. A comparison with experiments 8 and 21 illustrates this point: In experiment 6, the probabilities are such that $\alpha_{lL} = \alpha_{hL} > \alpha_{lH} = \alpha_{hH}$, resulting in a move in the location of the optimum; experiment 21 makes a similar perturbation, with the additional shock that $\alpha_{lH} < \alpha_{hH}$.

The optimal scoring auction does very well overall. It captures, on average, more than two thirds of the surplus, and in 12 of the 19 core experiments reported in Table 3, it captures more than 80% of the surplus for the majority of the values $\Delta\theta_1$ can take. This excellent performance seems due to the relative flexibility that scoring auctions leave in terms of allocation.

Table 3 and Figure 5 also indicate that scoring auctions perform less well in some environments. This poor performance tends to happen around the point at which there is a kink in the expected utility of the efficient auction. This coincides with the point at which both incentive compatibility constraints out of type hL are close to binding in the efficient mechanism (one must bind, and the other is 'close' to binding). As a result, those IC constraints leave little scope for rent extraction before they bind. Given that the scoring auction is less flexible in the face of these constraints than the optimal mechanism, it is not surprising that its relative performance suffers.

Similarly, a negative correlation between the marginal cost and the fixed costs decreases the performance of the optimal scoring auction (see experiments 5, 6 and 7). Intuitively, a negative correlation moves the environment further from the one-dimensional environment for which scoring auctions are known to do well (Che, 1993). The weight of types lH and hL is large in the total expected utility of the buyer, and so the gains from distorting quality tend to make q_{hH} far from being first-best. In light of this, it is noteworthy that the scoring auction does not always perform strongly when types are positively correlated. In experiments 2 and 3, where types are increasingly correlated, the scoring auction appears to be doing increasingly well. However, in experiment 4, this trend does not continue. What is happening here is that the extra flexibility in the optimal mechanism is able to exploit the environment as it moves toward the one-dimensional case far sooner than the scoring auction. The relative performance of the scoring auction in experiment 4 reflects a reconfiguration of the optimal mechanism in the face of the changing environment, rather than any significant change in the scoring auction itself.

Lastly, because the performance of the optimal scoring auction only gives us a bound on the performance of scoring auctions more generally, we also investigate the performance of non-optimal scoring auctions. Specifically, we consider scoring rules that correspond to the true preferences of the buyer except that they place an arbitrary lower value on quality. For the case of $S(p,q) = 0.95v(q) - p$ for example and across the core experiments explored in table 3, we find that this naïve scoring auction captures approximately 60% of the strategic surplus if it leads to a distortion in allocations relative to the efficient mechanism, whereas the ranking between the naïve scoring auction and the efficient auction is unclear if both lead to the same allocations (and only differ in the induced qualities).

5 Bargaining

We now turn to bargaining. Our goal in this section is to illustrate the potential costs and benefits of bargaining in the presence of quality concerns. A first difficulty that we face is in deciding what bargaining encompasses. Indeed, there are many ways to model bargaining between a buyer and one or several suppliers. Existing bargaining models include alternating offers between a buyer and a supplier (e.g. Rubinstein, 1982, Ausubel and Deneckere, 1989), repeated buyer offers to a single supplier (e.g. Fudenberg and Tirole, 1983) and models where a buyer faces several potential suppliers (e.g. Manelli and Vincent, 1995, De Fraja and Muthoo, 2000). In this section, we call bargaining any procedure where suppliers are not put in direct competition with one another. Intuitively, once we allow the buyer to go back and forth between suppliers in search for the best bargain, we essentially have an auction. This dividing line is, of course, arbitrary.[9] We explore the possibility of recalling a supplier after the breakdown of discussions in the next section.

[9] For a different view, see Bulow and Klemperer (1996) and (2009).

In line with the rest of the paper, we adopt a mechanism design approach (this is in the spirit of Myerson and Satterthwaite, 1983, Riley and Zeckhauser, 1983, Wang, 1998). Our results then provide an *upper* bound to what bargaining can achieve in the environments we consider, *independently* of the specific form bargaining takes. When available, we identify a specific procedure that implements the optimal bargaining mechanism. To ensure comparability with the results in the previous sections, we assume that the buyer buys for sure and that there is no time discounting.[10]

We answer the question in two steps. Each step corresponds to a different view of bargaining. First, we consider a buyer who bargains with a single supplier. We find that a menu of two take-it-or-leave-it offers implements the optimal bargaining mechanism. Second, we consider a buyer who bargains with multiple suppliers sequentially. The idea here is that the buyer is free to haggle with each supplier as much as he wants but once negotiation breaks down, he goes to another supplier and never returns. A sequence of take-it-or-leave-it offers implements the optimal sequential bargaining mechanism. As in the previous section, we first highlight the theoretical properties of the optimal bargaining mechanism before turning to numerical simulations to get a sense of magnitudes.

5.1 Theoretical properties

5.1.1 One buyer - one supplier

Given the agents' risk neutrality and the convexity of buyer's preference over quality, we can summarize any bargaining procedure between a buyer and a supplier by a quality level and an expected payment. Let (p_k, q_k) denote the expected payment and the quality level provided by type k in the optimal direct mechanism. Lemma 5 shows that the price and the quality provided by suppliers in the optimal mechanism are only a function of their marginal costs. The intuition is that contracts of the form (p, q) are unable to screen over suppliers' fixed cost.

Lemma 5: *In the optimal direct mechanism,* $(p_{lL}, q_{lL}) = (p_{hL}, q_{hL})$ *and* $(p_{lH}, q_{lH}) = (p_{hH}, q_{hH})$.

Proof: Consider the outcome for the low marginal cost types. Incentive compatibility requires:

$$\begin{aligned} \mathrm{IC}_{lL,hL} &: \; p_{lL} - \underline{\theta}_1 - \underline{\theta}_2 q_{lL} \geq p_{hL} - \underline{\theta}_1 - \underline{\theta}_2 q_{hL} \\ \mathrm{IC}_{hL,lL} &: \; p_{hL} - \overline{\theta}_1 - \underline{\theta}_2 q_{hL} \geq p_{lL} - \overline{\theta}_1 - \underline{\theta}_2 q_{lL} \end{aligned}$$

Thus, $p_{lL} - \underline{\theta}_2 q_{lL} = p_{hL} - \underline{\theta}_2 q_{hL}$, and the outcomes lie on an isoprofit locus for suppliers hL and lL, $\{(p, q) : p_{lL} - \underline{\theta}_2 q_{lL} = p_{hL} - \underline{\theta}_2 q_{hL}\}$. Since the buyer has strictly convex preferences, there is a unique contract on this locus that maximizes his utility. Q.E.D.

From now on, let (p_L, q_L) and (p_H, q_H) denote the outcome for the low marginal cost types and the high marginal cost types respectively. The optimal direct mechanism solves:

$$\max_{(p_L,q_L),(p_H,q_H)} (\alpha_{lH} + \alpha_{hH})(v(q_H) - p_H) + (\alpha_{hL} + \alpha_{lL})(v(q_L) - p_L)$$

[10] As for the optimal mechanism, the requirement that the buyer buy for sure can be perfectly consistent with optimality if the buyer values the good sufficiently highly. The no discount feature rules out time as a screening device.

subject to suppliers' IR and IC constraints. Following standard arguments, supplier hH's IR constraint and the downward IC constraint bind:

$$p_H^1 = \overline{\theta}_1 + \overline{\theta}_2 q_H \tag{8}$$

$$p_L^1 = p_H^1 - \underline{\theta}_2 q_H + \underline{\theta}_2 q_L \tag{9}$$

Substituting for p_H^1 and p_L^1 into the objective function yields:

$$\max_{q_H, q_L} (\alpha_{lH} + \alpha_{hH})(v(q_H) - \overline{\theta}_1 - \overline{\theta}_2 q_H) + (\alpha_{hL} + \alpha_{lL})(v(q_L) - \overline{\theta}_1 - \underline{\theta}_2 q_L - \Delta\theta_2 q_H)$$

The solution is given by:

$$q_L^1 = \underline{q} \tag{10}$$

$$q_H^1 = \arg\max_q \left\{ v(q) - \overline{\theta}_2 q - \frac{(\alpha_{hL} + \alpha_{lL})}{(\alpha_{lH} + \alpha_{hH})} q \Delta\theta_2 \right\} \tag{11}$$

We have thus proved:

Theorem 3: The buyer's maximum expected utility from bargaining with a single supplier is given by:

$$V_1 = (\alpha_{hL} + \alpha_{lL})(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 \underline{q}) + (\alpha_{lH} + \alpha_{hH}) \max_q \left(v(q) - \overline{\theta}_1 - \overline{\theta}_2 q - \frac{(\alpha_{hL} + \alpha_{lL})}{(\alpha_{lH} + \alpha_{hH})} q \Delta\theta_2 \right)$$

Clearly, the optimal one-buyer one-supplier bargaining mechanism can be implemented by a menu of two take-it-or-leave-it offers, $\{(p_H^1, q_H^1), (p_L^1, q_L^1)\}$, given by (8)-(11). Suppliers accept the offer as long as it meets their individual rationality constraint.

5.1.2 Sequential bargaining

Suppose now that the buyer can bargain with several suppliers in any way he wants, as long as he does so sequentially. If negotiation with one supplier breaks down and he switches to another supplier, he cannot return to the initial supplier. The optimal sequential mechanism in this environment solves a dynamic programming problem. The buyer approaches suppliers one at a time and offers them a menu of optimal screening contracts that take into account the number of remaining suppliers.

Let V_n describe the continuation value from the optimal sequential mechanism when n suppliers remain. Clearly, V_1 coincides with the buyer's expected utility from the optimal one-buyer one-supplier bargaining mechanism. When the buyer faces more than one suppliers, exclusion is optimal, and the probability of negotiation breakdown (and thus of moving to another supplier) is strictly positive. The next lemma shows this and that the buyer's expected utility increases in the number of suppliers he faces.

Lemma 6: *Let K_n be the set of supplier types for whom the buyer's offers, when n buyers remain, are acceptable. The buyer's expected utility from the optimal sequential mechanism increases with the* *number of suppliers. In the optimal sequential mechanism, exclusion is optimal as soon as $N > 1$ and $|K_n| \leq |K_{n-1}|$ for all $n > 1$.*

Proof: The maximum surplus the buyer can extract from a supplier is $v(\underline{q}) - \underline{\theta}_1 - \underline{\theta}_2\underline{q}$, the maximum surplus generated by supplier lL. Since $\alpha_{lL} < 1$, $V_n < v(\underline{q}) - \underline{\theta}_1 - \underline{\theta}_2\underline{q}$ for all n. Whenever the buyer faces $n > 1$ remaining suppliers, an available strategy is to offer the contract (p, q), where $p = \underline{\theta}_1 + \underline{\theta}_2\underline{q}$ and $q = \underline{q}$, that is only accepted by supplier lL. Thus $V_n \geq \alpha_{lL}(v(\underline{q}) - \underline{\theta}_1 - \underline{\theta}_2\underline{q}) + (1 - \alpha_{lL})V_{n-1} > V_{n-1}$. This also shows that exclusion is optimal.
Let x be the buyer's current expected payoff when n suppliers remain and $K_n = \{lL\}$, i.e. $V_n = x + (1 - \alpha_{lL})V_{n-1}$. Similarly, let y be the current expected payoff when $K_n = \{lL, lH\}$. The buyer prefers to make offers only acceptable to $K_n = \{lL\}$ rather to $K_n = \{lL,\ lH\}$ if $x + \alpha_{lH}V_{n-1} > y$. Since V_n is increasing, $K_n = \{lL\}$ preferred to $K_n = \{lL,\ lH\}$ when n suppliers remain, implies that $K_{n'} = \{lL\}$ preferred to $K_{n'} = \{lL,\ lH\}$ for all $n' > n$. A similar argument establishes that if $K_n = \{lL,\ hL\}$ is preferred to $K_n = \{lL, lH, hL\}$, it is also preferred for $n' > n$. The same argument also applies when we replace $K_n = \{lL,\ lH\}$ by $K_n = \{lL,\ hL\}$. The claim follows. QED.

Theorem 4: *The outcome in the optimal sequential mechanism is a function of the number of remaining suppliers. When only one supplier remains, the outcome is described by (8)-(11). When $n > 1$ suppliers remain, the outcome takes the form (p_H^n, q_H^n), (p_L^n, q_L^n) together with the set of supplier types for whom these contracts are acceptable. This menu of contracts is the one that yields the largest continuation value among the four described in the following table:*

Insert Table 4 Here

Proof: See Appendix A.

As before, the buyer can implement the optimal sequential mechanism with a sequence of menus of take-it-or-leave-it offers. These offers take the form given in Theorem 4. In the unique sequential equilibrium, suppliers accept the best offer that is acceptable to them.

Theorem 4 suggests that the optimal sequential mechanism has at least two potential advantages. First, it can distort production. Second, it can distort the probabilities of winning. For example, a first-period offer that is acceptable only to suppliers lL and lH distorts the probabilities that lH wins, x_{lH}, upwards and distorts x_{hL} downwards relative to the probabilities in the efficient auction, as is sometimes required in the optimal mechanism. However, this comes at the cost of a distortion in the probabilities of allocating the contract to types lL and hH. Indeed, it is easy to check that $x_{lL} < x_{lL}^{FB}$, unless the optimal offer in all rounds but the last is acceptable only to type lL (and recall from Theorem 1 that $x_{lL} = x_{lL}^{FB}$ always in the optimal mechanism). In addition, $x_{hH} > x_{hH}^{FB}$ in all cases except if $K_n = \{lL, lH, hL\}$ for all $n > 1$.

These costs and benefits of the optimal sequential mechanism are best illustrated for the case of two suppliers. To do this, we rewrite the expected utility from the efficient auction as $\sum \Pr_k VW_k$, where

$\Pr_k$ is the probability that the mechanism allocates the contract to a type k supplier and VW_k is the associated virtual welfare. Table 5 summarizes the values that these variables take in the efficient auction (using Lemma 3):

Insert Table 5 Here

Similarly, the expected utility from the optimal sequential procedure can be written as $\sum \widetilde{\Pr}_k \widetilde{VW}_k$ where $\widetilde{\Pr}_k$ is the probability that the optimal sequential mechanism allocates the contract to supplier k and $\widetilde{VW}_k$ is the "resulting" virtual welfare.[11] The idea, then, is to compare the $\Pr_k$'s with the $\widetilde{\Pr}_k$'s and the VW_k's with the $\widetilde{VW}_k$'s. The first example illustrates the advantage of being able to distort qualities.

Example 1: The optimal sequential mechanism always captures a positive proportion of the strategic surplus when $\Delta\theta_1$ is sufficiently small. We prove this by showing that $\sum \Pr_k VW_k < \sum \widetilde{\Pr}_k \widetilde{VW}_k$. When $\Delta\theta_1$ is small, the main source of adverse selection is marginal cost, and suppliers lL and hL, and lH and hH, respectively, are very much alike. Consider the strategy that consists of making an offer that is acceptable only to suppliers lL and hL in the first period. Using theorems 3 and 4, the resulting expected utility is given by:

$$\begin{aligned} V_2 &= (\alpha_{hL}+\alpha_{lL})W_{hL}^{FB} + (\alpha_{lH}+\alpha_{hH})V_1 \\ &= \alpha_{lL}(2-\alpha_{hL}-\alpha_{lL})W_{lL}^{FB} \\ &\quad +\alpha_{hL}(2-\alpha_{hL}-\alpha_{lL})\left(W_{hL}^{FB} - \frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1\right) \\ &\quad +(\alpha_{lH}+\alpha_{hH})^2\left(W_{hH}(q_H^1) - \frac{(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})}\Delta\theta_2 q_H^1\right) \end{aligned}$$

where we have grouped types lH and hH. Comparing this expression with the second column of Table 6 suggests that $VW_{lL} = \widetilde{VW}_{lL}$ and $VW_{hL} = \widetilde{VW}_{hL}$. Moreover, $\Pr_{hL} + \Pr_{lL} = 1-(1-\alpha_{hL}-\alpha_{lL})^2 = \widetilde{\Pr}_{hL} + \widetilde{\Pr}_{lL} = (\alpha_{hL}+\alpha_{lL})(2-\alpha_{hL}-\alpha_{lL})$. Thus, when $\Delta\theta_1$ is very small, $\Pr_{lL} VW_{lL} + \Pr_{hL} VW_{hL} \simeq \widetilde{\Pr}_{lL}\widetilde{VW}_{lL} + \widetilde{\Pr}_{hL}\widetilde{VW}_{hL}$ since $VW_{hL} \simeq VW_{lL}$. Turning to the utility contribution of types lH and hH in the efficient auction, we get, using Table 6 and after some simplifications:

$$\begin{aligned} &\alpha_{lH}(\alpha_{lH}+2\alpha_{hH})W_{lH}^{FB} + \alpha_{hH}^2 W_{hH}^{FB} - (\alpha_{lH}+2\alpha_{hH})(\alpha_{hL}+\alpha_{lL})\Delta\theta_2\overline{q} \\ &+((\alpha_{lH}+\alpha_{hH})(\alpha_{hL}+\alpha_{lL}) - \alpha_{lH}\alpha_{hH})\Delta\theta_1 \\ = &\ (\alpha_{lH}+\alpha_{hH})^2\left[W_{hH}^{FB} - \frac{(\alpha_{lH}+2\alpha_{hH})(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})^2}\Delta\theta_2\overline{q}\right] \\ &+(\alpha_{lH}+\alpha_{hH})(1-\alpha_{hH})\Delta\theta_1 \end{aligned}$$

The first term of this expression is strictly less than $(\alpha_{lH}+\alpha_{hH})^2\left(W_{hH}(q_H^1) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{lH}+\alpha_{hH}}\Delta\theta_2 q_H^1\right)$ given the way q_H^1 is constructed (optimal level of distortion) and the fact that $\frac{(\alpha_{lH}+2\alpha_{hH})(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})^2} >$

[11] We write "resulting" because the virtual welfare associated with a given type is not uniquely pinned down in the sequential mechanism. We exploit this flexibility in the remaining discussion.

$\frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{lH}+\alpha_{hH}}$. The second term becomes negligible as $\Delta\theta_1$ decreases. Thus, when $\Delta\theta_1$ is small enough, the optimal sequential procedure dominates the efficient auction because it is able to distort qualities. **End of example 1.**

Another way to view example 1 is to note that as $\Delta\theta_1$ converges to zero, the environment converges essentially to the "standard" one-dimensional environment, where the optimal mechanism is such that lL and hL win over hH and lH and qualities are distorted. The sequential mechanism replicates these features when $K_2 = \{lL, hL\}$. In fact, the expected utility from the optimal sequential mechanism converges to the expected utility from the optimal mechanism as $\Delta\theta_1$ converges to zero. (Table 6 below provides evidence on this).

In related research, Wang (1998) shows that a menu of take-it-or-leave-it offers implements the optimal mechanism in a multi-period one-supplier setting with $\Delta\theta_1 = 0$ and arbitrary discount rates. This is due to the fact that when $\underline{\theta}_1 = \overline{\theta}_1$, time is not needed to screen over types. The buyer only needs to screen over the variable costs (θ_2's) and quality is a superior instrument to do this. Similarly, in our setting, as $\Delta\theta_1 \to 0$ competition between suppliers becomes relatively less important for screening in the optimal mechanism, allowing bargaining to perform comparatively well.

The next example illustrates the advantage provided by the ability to distort allocation probabilities:

Example 2: For large values of $\Delta\theta_1$, the optimal sequential mechanism can capture a positive fraction of the strategic surplus. Again, we prove this by comparing $\sum \Pr_k VW_k$ and $\sum \widetilde{\Pr}_k \widetilde{VW}_k$. Consider the period 1 strategy that offers a contract to types lL and lH only. The resulting expected utility for the buyer is given by:

$$\begin{aligned}
V_2 &= \alpha_{lL}(W_{lL}^{FB} - \Delta\theta_2 q_H^{**}) + \alpha_{lH} W_{lH}(q_H^{**}) + (1-\alpha_{lH}-\alpha_{lL})V_1 \\
&= \alpha_{lL}(2-\alpha_{lH}-\alpha_{lL})W_{lL}^{FB} \\
&\quad +\alpha_{hL}(1-\alpha_{lH}-\alpha_{lL})(W_{hL}^{FB} - \frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1) \\
&\quad +\alpha_{lH}(2-\alpha_{lH}-\alpha_{lL})\left(\frac{1}{(2-\alpha_{lH}-\alpha_{lL})}W_{lH}(q_H^{**}) + \frac{(1-\alpha_{lH}-\alpha_{lL})}{(2-\alpha_{lH}-\alpha_{lL})}W_{lH}(q_H^1)\right) \\
&\quad +\alpha_{hH}(1-\alpha_{lH}-\alpha_{lL})\left(W_{hH}(q_H^1) - \frac{\alpha_{lH}}{\alpha_{hH}}\Delta\theta_1 - \frac{(\alpha_{hL}+\alpha_{lL})}{\alpha_{hH}}\Delta\theta_2 q_H^1 - \frac{\alpha_{lL}}{\alpha_{hH}(\alpha_{hH}+\alpha_{hL})}\Delta\theta_2 q_H^{**}\right) \\
&= \sum_k \widetilde{\Pr}_k \widetilde{VW}_k
\end{aligned}$$

Comparing this with the probabilities and the levels of virtual welfare in Table 5, it is clear that $VW_{lL} = \widetilde{VW}_{lL}$, $VW_{hL} = \widetilde{VW}_{hL}$, $VW_{lH} > \widetilde{VW}_{lH}$ and $VW_{hH} \lesseqgtr \widetilde{VW}_{hH}$. Moreover, the sequential procedure essentially places lH in front of hL in the order of priority in the allocation, resulting in the following ordering of probabilities: $\Pr_{lL} > \widetilde{\Pr}_{lL}$, $\Pr_{hL} >> \widetilde{\Pr}_{hL}$, $\Pr_{lH} << \widetilde{\Pr}_{lH}$ and $\Pr_{hH} < \widetilde{\Pr}_{hH}$. When $\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1$ is large enough, $VW_{hL} << VW_{lH}$, $\widetilde{VW}_{lH}$. Thus, this allocation can increase expected utility. **End of example 2**

An example of environment where the effects described in examples 1 and 2 arise is the following: $v(q) = 3\sqrt{q}$, $\underline{\theta}_1 = \underline{\theta}_2$, $\overline{\theta}_2 = 2$, $\alpha_{lL} = \alpha_{hH} = 0.35$, $\alpha_{lH} = \alpha_{hL} = 0.15$.

5.2 Computational results

We now explore the performance of bargaining numerically. Because sequential bargaining always does better than bargaining with a single supplier, we focus on sequential bargaining with two suppliers. Table 6 reports the proportion of the strategic surplus, i.e. the difference between the expected utility from the optimal mechanism and the expected utility from the efficient auction, that the optimal sequential mechanism captures with two suppliers. For the simulations, we assume that $v(q) = 3\sqrt{q}$, $\underline{\theta}_1 = \underline{\theta}_2 = 1$, and $\overline{\theta}_2 = 2$. $\overline{\theta}_1$ takes values between 1 to 2.125, which corresponds to the maximum value permitted by the assumption that $W_{lH}^{FB} < W_{hL}^{FB}$. Negative values indicate that the optimal sequential mechanism does worse than the efficient auction.

Insert Table 6 Here

On average (i.e., across all possible values of $\Delta\theta_1$), the optimal sequential mechanism does worse, and often much worse, than the efficient auction. Hence, it necessarily does worse than the optimal mechanism and a scoring auction. The poor performance of sequential mechanisms is confirmed by the small fraction of values for $\Delta\theta_1$ where the optimal sequential mechanism captures at least 80% of the strategic surplus (second-to-last column) and where it does better than the efficient auction (last column).

There are two exceptions to the poor performance of the optimal sequential mechanism. First, and as suggested by example 1, the optimal sequential mechanism does very well and, in fact, as well as the optimal mechanism when $\Delta\theta_1 = 0$ (fourth column in the table). Second, the optimal sequential mechanism does better overall when there is strong positive correlation between types (experiment 4 in the table). The reason is related to example 2 above: when costs are highly correlated, $\frac{\alpha_{lL}}{\alpha_{hL}}$ is high, and the virtual welfare associated with type hL tends to be lower than the virtual welfare associated with lH. Thus, a contract acceptable only to types lH and lL in the first period reverses the order of priority of types hL and lH and can increase expected utility. Note, however, that this is not the end of the story. Indeed, the optimal sequential mechanism does poorly in experiments 17 through 19, even though the ratio $\frac{\alpha_{lL}}{\alpha_{hL}}$ is high there too. The reason is that a first-period offer acceptable to types lL and lH also increases the probability that a type lH wins over a type lL. Experiments 17 through 19 illustrate that this is particularly costly in terms of expected utility when $\alpha_{lH} > \alpha_{lL}$.

6 Recall

Suppose now that the buyer can go back and forth between suppliers at no cost.[12] The optimal mechanism in this case corresponds to the optimal mechanism derived in section 3 and it provides an upper bound to what the buyer can achieve. Intuitively, allowing the buyer to come back and

[12] De Fraja and Muthoo (2000) consider a bargaining game between a seller and two potential buyers where the seller can go back and forth between the two buyers. Switching involves a cost.

forth between suppliers blurs the distinction between bargaining and auctions. In fact, if bargaining involves multiple suppliers and no restriction on negotiation with one and the other, auctions can be seen as special cases of bargaining protocols.

Because this upper bound does not provide much insight about the new effects at play when we allow the buyer to go back and forth between suppliers, we explore these new effects in a simple stylized model with two suppliers and one recall stage. In stage 1, the buyer makes a take-it-or-leave-it offer to supplier 1. If supplier 1 rejects the buyer's offer, the buyer makes a take-it-or-leave-it offer to supplier 2 in stage 2. Finally, if supplier 2 rejects the offer, the buyer can make one last offer to either supplier (recall stage). The supplier to whom the offer is made accepts or rejects and this ends the game. Offers are observed by all.

Relative to the optimal sequential mechanism, two new effects arise. First, recall increases the competition between the two suppliers and thus makes rent extraction easier. Second, recall reduces the buyer's commitment power because, with positive probability at equilibrium, he comes back to the same supplier with a more attractive offer. Suppliers take this into account: they only accept an offer if it yields a higher payoff than their expected payoff from a recall offer.

In the remainder of this section, we discuss the key theoretical features of the equilibrium in this simple game and complement them with results from numerical experiments using the same parameters as earlier. The purpose is to illustrate these two new effects, not to fully characterize the equilibrium. We use sequential equilibrium as the equilibrium concept (Kreps and Wilson, 1982).

In any candidate equilibrium, the buyer recalls the supplier with the highest updated probability of having low marginal costs. Let μ^i_{lL}, μ^i_{lH}, μ^i_{hL}, μ^i_{hH} denote the updated probabilities about supplier i's type at the beginning of the recall stage, and let $\pi^i = \mu^i_{lL} + \mu^i_{hL}$, the updated probability that supplier i has low marginal costs. The offer the buyer makes to the recalled supplier depends on his updated beliefs about the supplier's type. Its derivation follows the steps in section 5.1.1 with the updated beliefs.[13] We denote the resulting offer by $(p^{\text{recall}}_L(\pi^i), q^{\text{recall}}_L(\pi^i))$, $(p^{\text{recall}}_H(\pi^i), q^{\text{recall}}_H(\pi^i))$. We show in Appendix B that the buyer's expected utility from recalling supplier i is increasing in π^i. Thus, if $\pi^1 > \pi^2$, the buyer prefers to recall supplier 1. If $\pi^1 < \pi^2$, he prefers to recall supplier 2. He is indifferent otherwise.

This means that we have essentially three categories of equilibrium paths to consider: paths where the buyer recalls supplier 1 for sure if he reaches the recall stage (i.e. if supplier 1 rejected his offer in stage 1 and supplier 2 rejected his offer in stage 2), paths where he recalls supplier 2 for sure, and paths where he mixes at the recall stage. All three categories of paths arise in equilibrium play in the numerical experiments (we show in Appendix B that on-equilibrium play where the buyer recalls supplier 2 for sure yields the same expected utility for the buyer as the optimal sequential mechanism).

In stages 1 and 2, the equilibrium specifies the beliefs, the set of supplier types for whom the offer is acceptable, the menu of optimal screening contracts and the suppliers' decision rule. The logic for

[13] We can show that $\mu^i_{hH} > 0$ holds in all equilibria.

deriving the optimal screening contracts is similar to that in the proof of Theorem 4. In particular, there are six possible offers to consider in stage 1: offers that are only acceptable to type lL, offers that are only acceptable to the low marginal cost types (lL and hL), offers that are only acceptable to low fixed cost types (lL and lH), offers that are acceptable to all but type hH, offers that are acceptable to all types and offers that are acceptable to none.

The difference with sequential bargaining is that suppliers' individual rationality constraints are now endogenous and depend on their expectations about the recall stage. To illustrate, consider any equilibrium and let δ be the probability that supplier 2 declines the buyer's offer in stage 2 and σ the probability that the buyer recalls supplier 1. The offer in the recall stage acts as an outside option for supplier 1 when he considers his stage 1 offer. Specifically, type lL will accept offer (p_L, q_L) in stage 1 if and only if

$$p_L - \underline{\theta}_1 - \underline{\theta}_2 q_L \geq \delta\sigma(\underbrace{\overline{\theta}_1 + \Delta\theta_2 q_H^{\text{recall}}(\pi^1) + \underline{\theta}_2 q_L^{\text{recall}}(\pi^1)}_{p_L^{\text{recall}}(\pi^1)} - \underline{\theta}_1 - \underline{\theta}_2 q_L^{\text{recall}}(\pi^1))$$

which yields the following endogenous individual rationality constraint:

$$\text{IR}_{lL} : p_L \geq \underline{\theta}_1 + \underline{\theta}_2 q_L + \underbrace{\delta\sigma(\Delta\theta_1 + \Delta\theta_2 q_H^{\text{recall}}(\pi^1))}_{\text{rent}} \tag{12}$$

Repeating the exercise for other types yields, at stage 1,

$$\text{IR}_{hL} \; : \; p_L \geq \overline{\theta}_1 + \underline{\theta}_2 q_L + \delta\sigma\Delta\theta_2 q_H^{\text{recall}}(\pi^1) \tag{13}$$

$$\text{IR}_{lH} \; : \; p_H \geq \underline{\theta}_1 + \overline{\theta}_2 q_H + \delta\sigma\Delta\theta_1 \tag{14}$$

$$\text{IR}_{hH} \; : \; p_H \geq \overline{\theta}_1 + \overline{\theta}_2 q_H \tag{15}$$

Compared to the earlier bargaining model, in which supplier 1's outside option is zero, recall clearly increases supplier 1's bargaining power in stage 1. On the other hand, exclusion of supplier 2 in stage 2 is now possible. This reduces supplier 2's bargaining power.

Another way to look at the effect of recall is that it introduces an additional instrument to screen over fixed costs. In sequential bargaining, exclusion is the only way for the buyer to screen over supplier 1's fixed costs. In the presence of recall, the buyer can screen supplier 1's fixed costs over stages because $\overline{\theta}_1 > \underline{\theta}_1 + \delta\sigma\Delta\theta_1$ (compare (12) and (13), and (14) and (15)).[14]

We now discuss the second effect introduced by recall: the potential reduction of commitment power. The Coase conjecture has described how repeated interactions with potential suppliers can hurt a buyer because of the impossibility for the buyer to commit not to buy from a supplier (Coase, 1972). This commitment problem is not an issue in our case because we impose that the buyer buys anyway, even when there is no recall. Instead, commitment issues arise in two subtle ways in our setting: first,

[14] There is an analogy with bargaining over multiple periods of time when there is discounting. Here, $\delta\sigma$ plays the role of an endogenous time discount.

through the inability of the buyer to commit to a quality level for the high marginal cost types, q_H^{recall}, in the recall stage, and second, through the inability of the buyer to commit to recalling supplier 1 with a given probably if he is indifferent between the two suppliers in the recall stage.

Let us first consider the loss of commitment power due to the fact that the buyer cannot commit to q_H^{recall} in stage 1. Suppose IR_{lL} or IR_{hL} bind in stage 1. Let Pr_1 and Pr_2 be the probability of a trade in stage 1 and stage 2 respectively, and let f_1, f_2 be the expected stage payoffs for the buyer in stages 1 and 2. Finally, let V^{recall} be the buyer's expected utility in the recall stage. The buyer's expected utility from the recall game is given by $\text{Pr}_1 f_1 + \text{Pr}_2 f_2 + (1 - \text{Pr}_1 - \text{Pr}_2)V^{\text{recall}}$. Because IR_{lL} or IR_{hL} bind in stage 1, f_1 is a function of q_H^{recall} (see (13) and (14)). Yet, because q_H^{recall} is chosen in the recall stage, it is chosen to maximize V^{recall} and not $\text{Pr}_1 f_1 + \text{Pr}_2 f_2 + (1 - \text{Pr}_1 - \text{Pr}_2)V^{\text{recall}}$. In other words, recall can sometimes involve suboptimal choices for q_H^{recall} due to the inability of the buyer to commit.

The second commitment problem arises from the fact that the buyer cannot commit ex-ante to a specific behavior in the event that he is indifferent at the recall stage. This leads to multiple equilibria in the recall game, some of which are inferior from the buyer's perspective. To see this, suppose the buyer makes an offer that is only acceptable to type lL in stage 1. At the beginning of stage 2, his continuation payoff depends on his recall strategy. Clearly, if his stage 2 offer is such that $\pi^1 \neq \pi^2$, then the buyer should recall the supplier for whom π^i is highest. If $\pi^1 = \pi^2$ however (which implies his stage 2 offer is only acceptable to type lL), his continuation payoff is increasing in σ, the probability of recalling supplier 1, because this reduces supplier 2's reservation value. Because any σ is consistent with sequential equilibrium, there may exist a value of σ, σ^*, such that if $\sigma > \sigma^*$, making an offer in stage 2 to type lL only is optimal, whereas another strategy is better if $\sigma < \sigma^*$. This leads to the possibility of multiple equilibria. Of course, if the buyer could commit, he would commit to a value of σ that selects the better equilibrium. In practice however, he cannot commit since he is indifferent ex-post between recalling supplier 1 and supplier 2.

Because of the possibility of multiple equilibria, we evaluate bargaining with recall in two steps.[15] In a first step, we focus on the sequential equilibrium that generates the highest expected utility for the buyer when there are multiple equilibria. Table 7 reports the percentage of the strategic surplus that this equilibrium captures for the same parameters as Table 6. Thus Table 7 serves to illustrate, from the buyer's point-of-view, the best-case interaction between the increased competition among suppliers and the potential lack of commitment regarding qualities that recall introduces. It shows that the competition effect dominates: relative to the optimal sequential mechanism, the buyer is able to capture a much larger fraction of the strategic surplus (comparison between Table 6 and Table 7). This dominance is uniform for all values of $\Delta\theta_1$. On average, bargaining with recall continues to perform worse than the optimal scoring auction (comparison between Table 3 and Table 7). It can do better for some values of $\Delta\theta_1$ however. The last column of Table 7 shows the percentage of values for $\Delta\theta_1$ for which bargaining with recall does better.

[15] Code for computing equilibrium and accompanying explanatory text are available online.

Insert Table 7 Here

In a second step, we focus on the parameters for which multiple equilibria exist. Table 8 shows that equilibrium multiplicity is pervasive in the recall game. When multiple equilibria exist, the difference in buyer expected utility between the best and the worst equilibria can represent a large fraction of the strategic surplus (last two columns). Conditioning on multiple equilibria existing, the third column gives the percentage of $\Delta\theta_1$ for which the buyer is worse off in the worst equilibrium of bargaining with recall than in the efficient mechanism. In seven of the fifteen experiments with multiple equilibria, the efficient mechanism does better in the majority of cases. In two cases, experiments 4 and 14 the worst equilibria are never worse than the efficient mechanism. Experiments 14 through 16 are particularly interesting as they suggest that as high fixed cost types become more likely, multiple equilibria problems may become less important.

Insert Table 8 Here

Taken together the results for the recall game yield two messages. First, even introducing limited direct competition among suppliers (through the recall stage) can yield great benefits. This echoes and reinforces the message from the comparison between sequential bargaining and the optimal scoring auction on the benefit of competition. Second, commitment about allocation decisions is a key issue and the lack of commitment on this dimension can unravel many of the benefits of competition. Lack of commitment about qualities, however, appears less problematic.

7 Concluding remarks

In this paper, we have asked how a buyer should optimally structure his buying process when suppliers' private information is multidimensional and quality is contractible, and how well commonly used procedures such as scoring auctions and bargaining perform. We have answered the second question by combining a theoretical analysis of the restrictions that such simpler procedures impose on allocations with numerical analyses of their performance.

Our main results are that scoring auctions do well and that bargaining does poorly. Our interpretation of both sets of results, combined with our analysis of how recall changes those results, is that utility maximization is more about "getting allocation probabilities right" than about distorting qualities. This is the main reason why scoring auctions do well and bargaining does so poorly. This is also why the buyer's loss of commitment regarding allocation in the recall game is more costly than his loss of commitment regarding qualities.

Because the scoring auction's "right kind of flexibility in terms of allocation probabilities" and the generic misallocation of contracts in bargaining are intrinsic features of these procedures and do not depend on the number of suppliers, we are confident that the bottom line of our numerical results extends to more than two suppliers. (Of course, as the number of suppliers goes to infinity, it is straightforward to show that the expected utility from all procedures converges to the same value, which is W_D^{FB} - full extraction).

An *a priori* restrictive assumption in our analysis is the binary structure of private information, and it is worthwhile to comment on it here. First, we note that the main results concerning the optimal mechanism (such as the facts that it depends on the number of suppliers, that it involves both productive and allocative inefficiencies, and that suppliers with the same marginal cost for quality generically supply different quality levels) are all driven by the endogeneity of the binding incentive compatibility constraints. For this reason, we expect them to hold in more general environments. Second, the generic misallocation in bargaining can only become worse in richer informational environments, whereas scoring auctions continue to allocate the contract efficiently, conditional on the announced scoring rule. Thus, under the (reasonable) conjecture that optimal allocation is the first-order issue in these complex procurement settings, then the dominance of scoring auctions is likely to extend to these richer environments.

The poor performance of bargaining in our paper is in stark contrast with its popularity among practitioners in complex procurement settings. This might be due to procurement managers' intrinsic preference for procedures over which they have control or to objective factors not modelled in this paper, such as unknown preferences, non-contractibility, renegotiation, or other moral hazard-type issues. Those objective factors may provide a rationale for using bargaining over scoring auctions in complex procurement. Studying their impact on the performance of bargaining is a venue for further research

Alternatively, what is referred to in common speech as bargaining may incorporate elements of competition between suppliers, however loosely structured this competition may be. If this is true, then the results from the recall game suggest that even introducing a little competition may dramatically improve the performance of bargaining-like procedures, perhaps justifying the ubiquity of what is loosely termed negotiation or bargaining.

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Appendix A

Lemma 1: *Consider the feasibility constraints*

$$N\sum_{k\in K}\alpha_k x_k \leq 1-(1-\sum_{k\in K}\alpha_k)^N \text{ for all subsets } K \text{ of } \{hH, lH, hL, lL\}$$

and define an n-type constraint as a feasibility constraint with the relevant subset K having n elements. The following statements hold:

i. At most one 1-type constraint binds, at most one 2-type constraint binds and at most one 3-type constraint binds.

ii. These binding constraints are nested, in the sense that the type in the binding 1-type constraint must belong to the binding 2-type constraint, and so on.

Proof of Lemma 1: The claim relies on the fact that the function $f(t) = t^N$ for $N \geq 2$ is strictly convex. There are two generic cases to rule out: two constraints binding with no type in common, and two non nested constraints binding with some type in common.

Case 1: No overlap. Suppose, towards a contradiction, that the constraint for lH and the constraint for $\{hH,\ hL\}$ bind. Then, from (4), $N\left(\alpha_{lH}x_{lH}+\alpha_{hH}x_{hH}+\alpha_{hL}x_{hL}\right) = 2-(1-\alpha_{lH})^N-(1-\alpha_{hH}-\alpha_{hL})^N > 1-(1-\alpha_{lH}-\alpha_{hH}-\alpha_{hL})^N$ since $1+(1-\alpha_{lH}-\alpha_{hH}-\alpha_{hL}) = (1-\alpha_{lH})+(1-\alpha_{hH}-\alpha_{hL})$ and $(1-\alpha_{lH})$ and $(1-\alpha_{hH}-\alpha_{hL})$ lie in $(1-\alpha_{lH}-\alpha_{hH}-\alpha_{hL}, 1)$. That is, (4) is violated for $\{lH, hH, hL\}$. All cases with no overlap are proved in this way.

Case 2: Some overlap. Suppose, towards a contradiction that the constraint for $\{lH,\ hH\}$, and that for $\{hH,\ hL\}$ is binding. Since (4) holds for hH, this means that

$$\begin{aligned} N\left(\alpha_{lH}x_{lH}+\alpha_{hH}x_{hH}+\alpha_{hL}x_{hL}\right) &\geq 1-(1-\alpha_{lH}-\alpha_{hH})^N-(1-\alpha_{hH}-\alpha_{hL})^N+(1-\alpha_{hH})^N \\ &> 1-(1-\alpha_{lH}-\alpha_{hH}-\alpha_{hL})^N \text{ by convexity} \end{aligned}$$

This contradicts (4) for $\{lH, hH, hL\}$. All cases with some overlap are proved in this way.

This proves that binding constraints are nested and that they cannot be more than one constraint of a type to bind. Q.E.D.

Lemma 3 (Binding constraints in the efficient mechanism): *When $\Delta\theta_1 > \Delta\theta_2\overline{q}$, $IC_{lH,hH}$, $IC_{hL,hH}$ and $IC_{lL,hL}$ bind in the efficient auction. When $\Delta\theta_1 < \Delta\theta_2\overline{q}$ $IC_{lH,hH}$, $IC_{hL,lH}$ and $IC_{lL,hL}$ bind (see Figure 1).*

Proof of Lemma 3: Let $U_{k,j}$ be the expected utility of a type k pretending she is of type j. To satisfy incentive compatibility, while minimizing suppliers' rents, suppliers' expected utilities must be set such that $U_k = \max_{j\neq k} U_{k,j}$. Let $U_{hH} = 0$ (we can check *ex post* that this will satisfy supplier hH's incentive compatibility constraints).

Claim 1: $U_{hL} = \max\{U_{hL,hH}, U_{hL,lH}\}$

Proof of claim 1: We simply need to show that $U_{hL,lL} < U_{hL,hH}$ or $U_{hL,lL} < U_{hL,lH}$. By definition, $U_{hL,lL} = U_{lL} - x^{FB}_{lL}\Delta\theta_1$. If $U_{lL} = U_{lL,hL}$, we get $U_{hL,lL} = U_{hL} + (x^{FB}_{hL} - x^{FB}_{lL})\Delta\theta_1 < U_{hL}$. If instead,

$U_{lL} = U_{lL,lH}$, we have $U_{hL,lL} = U_{lH} + x_{lH}^{FB}\Delta\theta_2\overline{q} - x_{lL}^{FB}\Delta\theta_1 < U_{lH} + x_{lH}^{FB}\Delta\theta_2\overline{q} - x_{lH}^{FB}\Delta\theta_1 = U_{hL,lH} \leq U_{hL}$ (we can rule out $U_{lL} = U_{lL,hH}$ since it is dominated for supplier lL).

Claim 2: $U_{lH} = U_{lH,hH} = x_{hH}^{FB}\Delta\theta_1$.
Proof of claim 2: We first show that $U_{lH,hH} > U_{lH,hL}$. By definition, $U_{lH,hL} = U_{hL} + x_{hL}^{FB}\Delta\theta_1 - x_{hL}^{FB}\underline{q}\Delta\theta_2$ where $U_{hL} = \max\{U_{hL,hH}, U_{hL,lH}\}$ by claim 1. We consider each case in turn.
(a) If $U_{hL} = U_{hL,hH}$, then $U_{lH,hL} = x_{hH}^{FB}\Delta\theta_2\overline{q} + x_{hL}^{FB}\Delta\theta_1 - x_{hL}^{FB}\underline{q}\Delta\theta_2$ and $U_{lH,hH} > U_{lH,hL}$ if and only if $x_{hL}^{FB}(\Delta\theta_1 - \Delta\theta_2\underline{q}) - x_{hH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q}) < 0$. The first term is negative since, by assumption, $W_{lH}(\overline{q}) < W_{hL}(\underline{q})$. The second term may be positive or negative, but even when it is negative, $x_{hL}^{FB}(\Delta\theta_1 - \Delta\theta_2\underline{q}) < x_{hH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q}) < 0$ since $\underline{q} > \overline{q}$ and $x_{hH}^{Fb} < x_{hL}^{FB}$.
(b) If $U_{hL} = U_{hL,lH}$, then $U_{lH,hL} = U_{lH} - x_{lH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q}) + x_{hL}^{FB}(\Delta\theta_1 - \Delta\theta_2\underline{q}) > U_{lH}$ because $-x_{lH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q}) + x_{hL}^{FB}(\Delta\theta_1 - \Delta\theta_2\underline{q}) < 0$ by a similar argument as in point (b).
We next show that $U_{lH,hH} > U_{lH,lL} = U_{lL} - x_{lL}^{FB}\Delta\theta_2\underline{q}$. When $U_{lL} = U_{lL,lH}$, $U_{lH,lL} = U_{lH} + x_{lH}^{FB}\Delta\theta_2\overline{q} - x_{lL}^{FB}\Delta\theta_2\underline{q} < U_{lH}$. When $U_{lL} = U_{lL,hL}$, $U_{lH,lL} = U_{hL} + x_{hL}^{FB}\Delta\theta_1 - x_{lL}^{FB}\Delta\theta_2\underline{q} < U_{lH,hL} = U_{hL} + x_{hL}^{FB}\Delta\theta_1 - x_{hL}^{FB}\Delta\theta_2\underline{q}$. We conclude that $U_{lH} = U_{lH,hH}$.

Claim 3: $U_{lL} = U_{lL,hL}$.
Proof of claim 3: When $U_{hL} = U_{hL,lH}$, $U_{lL,hL} = x_{hL}^{FB}\Delta\theta_1 - x_{lH}^{FB}\Delta\theta_1 + x_{lH}^{FB}\overline{q}\Delta\theta_2 + x_{hH}^{FB}\Delta\theta_1 > U_{lL,lH} = x_{lH}^{FB}\overline{q}\Delta\theta_2 + x_{hH}^{FB}\Delta\theta_1$ since $x_{hL}^{FB} > x_{lH}^{FB}$. When $U_{hL} = U_{hL,hH}$, $U_{lL,hL} = x_{hL}^{FB}\Delta\theta_1 + x_{hH}^{FB}\overline{q}\Delta\theta_2$. Given that $U_{lL,lH} = x_{lH}^{FB}\overline{q}\Delta\theta_2 + x_{hH}^{FB}\Delta\theta_1$, $U_{lL,hL} > U_{lL,lH}$ if and only if $x_{hL}^{FB}\Delta\theta_1 - x_{lH}^{FB}\Delta\theta_2\overline{q} > x_{hH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q})$, which is automatically satisfied when $\Delta\theta_1 - \Delta\theta_2\overline{q} > 0$ (indeed, $x_{hL}^{FB}\Delta\theta_1 - x_{lH}^{FB}\Delta\theta_2\overline{q} > x_{lH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q})$ $> x_{hH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q})$), the only time when $U_{hL} = U_{hL,hH}$.

This leads us to:

$$\begin{aligned}
U_{lH} &= U_{lH,hH} = x_{hH}^{FB}\Delta\theta_1 \\
U_{hH} &= 0 \\
U_{hL} &= \max\{U_{hL,hH}, U_{hL,lH}\} = \max\{x_{hH}^{FB}\overline{q}\Delta\theta_2, -x_{lH}^{FB}(\Delta\theta_1 - \Delta\theta_2\overline{q}) + x_{hH}^{FB}\Delta\theta_1\} \\
U_{lL} &= U_{lL,hL} = x_{hL}^{FB}\Delta\theta_1 + U_{hL}
\end{aligned}$$

In practice, this generates two cases depending on the sign of $\Delta\theta_1 - \Delta_2\overline{q}$. When $\Delta\theta_1 - \Delta_2\overline{q} > 0$, $U_{hL,hH} > U_{hL,lH}$. When $\Delta\theta_1 - \Delta_2\overline{q} < 0$, $U_{hL,lH} > U_{hL,hH}$. Q.E.D.

Lemma 4: *The first order conditions of the Lagrangian of the maximization problem (1) subject to (2), (3), (4) and (5) are necessary and sufficient for a global maximum.*

Proof of Lemma 4: Consider the following change of variables: $z_{1k} = x_k$, $z_{2k} = x_k q_k$. Let

$\widetilde{F}(z_{1k}, z_{2k}, U_k) = N\sum_k \alpha_k(z_{1k}W_k(\frac{z_{2k}}{z_{1k}}) - U_k)$. The problem becomes:

$$\begin{aligned}
&\max_{z_{1k},z_{2k},U_k} \widetilde{F}(z_{1k}, z_{2k}, U_k) \qquad \text{s.t.} \\
U_k &\geq U_j + z_{1j}(\theta_{1j} - \theta_{1k}) + z_{2j}(\theta_{2j} - \theta_{2k}) \qquad \text{for all } k, j \in \{hH, lH, hL, lL\} \\
U_k &\geq 0 \qquad \text{for all } k \in \{hH, lH, hL, lL\} \\
N\sum_{k\in K} \alpha_k z_{1k} &\leq 1 - (1 - \sum_{k\in K} \alpha_k)^N \text{ for all subsets } K \text{ of } \{hH, lH, hL, lL\} \\
N \sum_{k\in\{hH,lH,hL,lL\}} \alpha_k z_{1k} &= 1
\end{aligned}$$

The constraints are linear in the control variables so the constraint qualification holds and the objective function is concave.[16] The first order conditions of the resulting Lagrangian are thus necessary and sufficient for a global maximum. To prove that the first order conditions of the original problem are also necessary and sufficient, we need to check that the first order conditions of the two problems are equivalent. To see this, let $G(x_k, q_k, U_k)$ gather all constraint terms of the Lagrangian of the original problem, and let $\widetilde{G}(z_{1k}, z_{2k}, U_k)$ gather the constraint terms of the Lagrangian of the transformed problem. We must show that (x_k^*, q_k^*, U_k^*) solves the first order conditions of $\max_{x_k,q_k,U_k} F(x_k, q_k, U_k) + G(x_k, q_k, U_k)$ if and only if $(x_k^*, x_k^* q_k^*, U_k^*)$ solves the first order conditions of $\max_{z_{1k},z_{2k},U_k} \widetilde{F}(z_{1k}, z_{2k}, U_k) + \widetilde{G}(z_{1k}, z_{2k}, U_k)$. The first order conditions with respect to U_k are identical. The first order condition with respect to q_k, $F_{q_k}(x_k^*, q_k^*, U_k^*) + G_{q_k}(x_k^*, q_k^*, U_k^*) = 0$, takes the form

$$N\alpha_k x_k^* W_k'(q_k^*) - \sum \lambda_l x_k^*(\theta_{2k} - \theta_{2l}) = 0$$

(where λ_l are the Lagrangian multipliers of the constraints). This is equivalent to the first order conditions of the transformed problem with respect to z_{2k},

$$N\alpha_k W_k'(\frac{z_{2k}}{z_{1k}}) - \sum \lambda_l(\theta_{2k} - \theta_{2l}) = 0 \tag{16}$$

as long as $x_k^* > 0$ for all k, a consequence of the non exclusion condition (5). Finally, the first order condition with respect to x_k, $F_{x_k}(x_k^*, q_k^*, U_k^*) + G_{x_k}(x_k^*, q_k^*, U_k^*) = 0$ takes the form:

$$N\alpha_k W_k(q_k^*) - \sum \lambda_l[(\theta_{1k} - \theta_{1l}) + q_k^*(\theta_{2j} - \theta_{2l})] - N \sum_{K \text{ st } k\in K} \gamma_K \alpha_k = 0 \tag{17}$$

The first order condition of the transformed problem takes the form:

$$N\alpha_k W_k(\frac{z_{2k}}{z_{1k}}) - N\alpha_k \frac{z_{2k}}{z_{1k}} W_k'(\frac{z_{2k}}{z_{1k}}) - \sum \lambda_l(\theta_{1j} - \theta_{1l}) - N \sum_{K \text{ st } k\in K} \gamma_K \alpha_k = 0$$

This is equivalent to (17) as soon as (16) holds. QED

[16] The hessian is block diagonal with each block given by $\begin{bmatrix} \alpha_k \frac{z_{2k}^2}{z_{1k}^3} W'' & -\alpha_k \frac{z_{2k}}{z_{1k}^2} W'' & 0 \\ -\alpha_k \frac{z_{2k}}{z_{1k}^2} W'' & \alpha_k \frac{W''}{z_{1k}} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Theorem 2: *An allocation can be implemented with a scoring auction if and only if (1) $q_{lH} = q_{hH}$, $q_{hL} = q_{lL}$ with $q_{lH} = q_{hH} < q_{hL} = q_{lL}$, (2) $\alpha_{lH}x_{lH} + \alpha_{hL}x_{hL} = \alpha_{lH}x_{lH}^{FB} + \alpha_{hL}x_{hL}^{FB}$, $x_{hH} = x_{hH}^{FB}$ and $x_{lL} = x_{lL}^{FB}$, (3) $\Delta\theta_1 - \Delta\theta_2 q_{hL} \leq 0$ when $x_{hL} > x_{hL}^{\min}$ and (4) $\Delta\theta_1 - \Delta\theta_2 q_{lH} \geq 0$ whenever the allocation is such that $x_{lH} > x_{lH}^{FB}$.*

Proof of Theorem 2: Let $S_k(q) = \widetilde{v}(q) - \theta_{1k} - \theta_{2k}q$. We first prove the necessary conditions. Recall from the discussion in the main text that, in a scoring auction, suppliers select their offers to maximize the score they generate, given their profit target, $\{\widetilde{v}(q) - \theta_1^i - \theta_2^i q - \pi\}$. The solution only depends on suppliers' marginal cost, which establishes condition (1) given that $\theta_{2lH} = \theta_{2hH} > \theta_{2hL} = \theta_{2lL}$. Condition (2) follows from the fact that lL can always generate a strictly higher score than either lH and hL for all choices of the scoring rule $\widetilde{v}(.)$. Similarly, both lH and hL can always generate a strictly higher score than hH so they must win against a hH type.
When $x_{hL} > x_{hL}^{\min}$, $S_{hL}(q_{hL}) \geq S_{lH}(q_{lH})$, or else lH should have priority over hL in the allocation. This implies that

$$\begin{aligned} \widetilde{v}(q_{hL}) - \overline{\theta}_1 - \underline{\theta}_2 q_{hL} &\geq \widetilde{v}(q_{lH}) - \underline{\theta}_1 - \overline{\theta}_2 q_{lH}, \text{ that is,} \\ \Delta\theta_1 - \Delta\theta_2 q_{hL} &\leq \widetilde{v}(q_{hL}) - \widetilde{v}(q_{lH}) - \overline{\theta}_2(q_{hL} - q_{lH}) \end{aligned}$$

In addition, incentive compatibility requires that lH generates a higher score by choosing q_{lH} than q_{hL}, i.e.

$$\widetilde{v}(q_{hL}) - \widetilde{v}(q_{lH}) - \overline{\theta}_2(q_{hL} - q_{lH}) \leq 0$$

Combining both inequalities yields condition (3). Similarly, when $x_{lH} > x_{lH}^{FB}$, $S_{lH}(q_{lH}) \geq S_{hL}(q_{hL})$, else hL should have priority in the allocation. This implies $\Delta\theta_1 - \Delta\theta_2 q_{lH} + \underline{\theta}_2(q_{hL} - q_{lH}) + \widetilde{v}(q_{lH}) - \widetilde{v}(q_{hL}) \geq 0$. In addition, hL must be generating a higher score by choosing q_{hL} over q_{lH}, i.e. $\underline{\theta}_2(q_{hL} - q_{lH}) + \widetilde{v}(q_{lH}) - \widetilde{v}(q_{hL}) \leq 0$. Combining both inequalities yields condition (4).
To prove sufficiency, we construct a scoring rule that implements the intended allocation in a second-score auction (in a second-score auction, it is a dominant strategy to submit bids generating scores $S_k(q_k) = \max_q\{\widetilde{v}(q) - \theta_{1k} - \theta_{2k}q\}$). Consider

$$\widetilde{v}(q) = v(q)1_{\{q \leq q_{lH}\}} + v(q_{lH})1_{\{q > q_{lH}\}} + \epsilon 1_{\{q \geq q_{hL}\}}$$

For this scoring auction to implement the outcome, two conditions must be satisfied. First, suppliers must be choosing the assigned qualities when they maximize their scores. Second, the ranking of the scores must (weakly) correspond to the assigned ranking of types in the allocation.
Given the shape of this scoring rule, the two relevant choices are q_{lH} and q_{hL}. lH prefers q_{lH} to q_{hL} if and only if $v(q_{lH}) - \underline{\theta}_1 - \overline{\theta}_2 q_{lH} \geq v(q_{lH}) + \varepsilon - \underline{\theta}_1 - \overline{\theta}_2 q_{hL}$ i.e. $\varepsilon \leq \overline{\theta}_2(q_{hL} - q_{lH})$ (hH's preferences yield the same condition). hL prefers q_{hL} to q_{lH} if and only if $v(q_{lH}) + \varepsilon - \overline{\theta}_1 - \underline{\theta}_2 q_{hL} \geq v(q_{lH}) - \overline{\theta}_1 - \underline{\theta}_2 q_{lH}$, i.e. $\varepsilon \geq \underline{\theta}_2(q_{hL} - q_{lH})$ (lL's preferences yield the same condition). Hence, suppliers choose their assigned qualities if ε satisfies the following inequalities:

$$\underline{\theta}_2(q_{hL} - q_{lH}) \leq \varepsilon \leq \overline{\theta}_2(q_{hL} - q_{lH}), \tag{18}$$

which is possible by condition (1). Next, hL generates a higher score if and only if $S_{hL}(q_{hL}) = v(q_{lH}) + \varepsilon - \overline{\theta}_1 - \underline{\theta}_2 q_{hL} \geq S_{lH}(q_{lH}) = v(q_{lH}) - \underline{\theta}_1 - \overline{\theta}_2 q_{lH}$ i.e.

$$\varepsilon \geq \Delta\theta_1 - \overline{\theta}_2 q_{lH} + \underline{\theta}_2 q_{hL} = \Delta\theta_1 - \Delta\theta_2 q_{hL} + \overline{\theta}_2 (q_{hL} - q_{lH}) \tag{19}$$

lH generates a higher score otherwise. Inequalities (18) and (19) are always compatible if $\Delta\theta_1 - \Delta\theta_2 q_{hL} \leq 0$ holds. When the solution is such that $x_{lH} > x_{lH}^{FB}$, we need $S_{lH}(q_{lH}) \geq S_{hL}(q_{hL})$:

$$\varepsilon \leq \Delta\theta_1 - \overline{\theta}_2 q_{lH} + \underline{\theta}_2 q_{hL} = \Delta\theta_1 - \Delta\theta_2 q_{lH} + \underline{\theta}_2 (q_{hL} - q_{lH})$$

instead. It is compatible with (18) if $\Delta\theta_1 - \Delta\theta_2 q_{lH} \geq 0$, which is guaranteed by condition (4). Q.E.D.

Theorem 4: *The outcome in the optimal sequential mechanism is a function of the number of remaining suppliers. When only one supplier remains, the outcome is described by (8)-(11). When $n > 1$ suppliers remain, the outcome is a menu of optimal screening contracts of the form (p_H^n, q_H^n), (p_L^n, q_L^n) together with the set of supplier types for whom these contracts are acceptable. This menu of contracts is the one that yields the largest continuation value among the four described in the following table:*

Table 4: Contracts in the optimal sequential procedure

K_n	Offers (p_H^n, q_H^n) and (p_L^n, q_L^n)	V_n (continuation value)
lL	$(\underline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$	$\alpha_{lL}W_{lL}^{FB}+(1-\alpha_{lL})V_{n-1}$
lL,hL	$(\overline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$	$(\alpha_{lL}+\alpha_{hL})W_{hL}^{FB}+(\alpha_{lH}+\alpha_{hH})V_{n-1}$
lL,lH,hL	<u>If $\Delta\theta_1-\Delta\theta_2\overline{q}>0$</u>: $(\underline{\theta}_1+\overline{\theta}_2\overline{q},\overline{q})$ and $(\overline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$ <u>If $\Delta\theta_1-\Delta\theta_2\overline{q}\leq 0$</u>: $(\underline{\theta}_1+\overline{\theta}_2 q_H^*, q_H^*)$ and $(\underline{\theta}_1+\underline{\theta}_2\underline{q}+\Delta\theta_2 q_H^*,\underline{q})$, with $q_H^*=\max\left\{\frac{\Delta\theta_1}{\Delta\theta_2}, \arg\max_q \{v(q)-\overline{\theta}_2 q-\frac{(\alpha_{lL}+\alpha_{hL})}{\alpha_{lH}}\Delta\theta_2 q\}\right\}$	$(\alpha_{lL}+\alpha_{hL})W_{hL}^{FB}+\alpha_{lH}W_{lH}^{FB}$ $+\alpha_{hH}V_{n-1}$ $(\alpha_{lL}+\alpha_{hL})(W_{hL}^{FB}-\Delta\theta_2 q_H^*)$ $\alpha_{lH}W_{lH}(q_H^*)+\alpha_{hH}V_{n-1}$
lL,lH	$(\underline{\theta}_1+\overline{\theta}_2 q_H^{**}, q_H^*)$, $(\underline{\theta}_1+\underline{\theta}_2\underline{q}+\Delta\theta_2 q_H^{**},\underline{q})$ with $q_H^{**}=\arg\max\{v(q)-\overline{\theta}_2 q-\frac{\alpha_{lL}}{\alpha_{lH}}\Delta\theta_2 q\}$ Solution possible only if $\Delta\theta_1-\Delta\theta_2 q_H^{**}\geq 0$	$\alpha_{lL}(W_{lL}^{FB}-\Delta\theta_2 q_H^{**})+$ $\alpha_{lH}W_{lH}(q_H^{**})+(\alpha_{hL}+\alpha_{hH})V_{n-1}$

(The first column in the table indicates the set of supplier types who will accept the buyer's offer when n suppliers remain, and the third column indicates the buyer's continuation value, V_n).

Proof: In the last period, the result follows from the derivation in the one-buyer one-supplier case. Let K_n be the set of supplier types for whom the buyer's offer is acceptable when n suppliers remain. Given suppliers' cost structure, $K_n \in \{\{lL\}, \{lL, lH\}, \{lL, hL\}, \{lL, lH, hL\}, \{lH, hH, hL, lL\}\}$. Lemma 6 rules out $K_n = \{lH, hH, hL, lL\}$. We examine the optimal outcome for the other three inclusion sets. By lemma 5, we can restrict attention to outcome of the form (p_H, q_H), (p_L, q_L). The optimal outcome when $K_n = \{lL\}$ is trivial.

<u>$K_n = \{hL, lL\}$</u>: Only one outcome is offered in this case: $(\overline{\theta}_1 + \underline{\theta}_2\underline{q},\ \underline{q})$. It satisfies the IR constraint of type hL and the IC constraint of type lL. Type lH (and a fortiori type hH) is excluded because $p = \overline{\theta}_1 + \underline{\theta}_2\underline{q} < \underline{\theta}_1 + \overline{\theta}_2\underline{q}$ by assumption.

$\underline{K_n = \{lH, hL, lL\}}$: The optimal direct mechanism solves $\max_{(p_L,q_L),(p_H,q_H)}\{(\alpha_{hL}+\alpha_{lL})(v(q_L)-p_L)+\alpha_{lH}(v(q_H)-p_H)\}$ subject to type lH's IR constraint, $p_H - \underline{\theta}_1 - \overline{\theta}_2 q_H \geq 0$, type hL's IR constraint, $p_L - \overline{\theta}_1 - \underline{\theta}_2 q_L \geq 0$ and the IC constraint that low marginal types don't select the contract intended to the high marginal types, $p_L - \underline{\theta}_2 q_L \geq p_H - \underline{\theta}_2 q_H$. Clearly, lH's IR constraint binds so that the two remaining constraints can be expressed as:

$$\begin{aligned} \mathrm{IR}_{hL} &: \; p_L \geq \overline{\theta}_1 + \underline{\theta}_2 q_L \\ \mathrm{IC} &: \; p_L \geq \underline{\theta}_1 + \underline{\theta}_2 q_L + \Delta\theta_2 q_H \end{aligned}$$

If $\Delta\theta_1 - \Delta\theta_2 \overline{q} > 0$, IR_{hL} binds and IC is slack at the optimum. The buyer's expected utility is given by

$$(\alpha_{lL}+\alpha_{hL})\left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 \underline{q}\right) + \alpha_{lH}\left(v(\overline{q}) - \underline{\theta}_1 - \overline{\theta}_2 \overline{q}\right)$$

If $\Delta\theta_1 - \Delta\theta_2 \overline{q} \leq 0$, the IC constraint binds and IR_{hL} may or may not bind depending on whether $q_H^* = \arg\max_{q_H}\{(\alpha_{hL}+\alpha_{lL})(v(\underline{q})-\underline{\theta}_1-\underline{\theta}_2\underline{q}-\Delta\theta_2 q_H)+\alpha_{lH}(v(q_H)-\underline{\theta}_1-\overline{\theta}_2 q_H)\}$ satisfies the condition that $\Delta\theta_1 - \Delta\theta_2 q_H^* \leq 0$. The resulting buyer's expected utility is given by:

$$(\alpha_{lL}+\alpha_{hL})\left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 \underline{q} - \Delta\theta_2 q_H^*\right) + \alpha_{lH}\left(v(q_H^*) - \underline{\theta}_1 - \overline{\theta}_2 q_H^*\right)$$

$\underline{K_n = \{lH, lL\}}$: lH's IR constraint binds, $p_H = \underline{\theta}_1 + \overline{\theta}_2 q_H$ and lL's IC constraint binds, $p_L = \underline{\theta}_1 + \underline{\theta}_2 q_L + \Delta\theta_2 q_H$. The optimal direct mechanism solves $\max_{q_L,q_H}\{\alpha_{lH}(v(q_H) - \underline{\theta}_1 - \overline{\theta}_2 q_H) + \alpha_{lL}(v(q_L) - \underline{\theta}_1 - \underline{\theta}_2 q_L - \Delta\theta_2 q_H)$, thus $q_L = \underline{q}$ and $q_H^{**} = \arg\max\{v(q) - \overline{\theta}_2 q - \frac{\alpha_{lL}}{\alpha_{lH}}\Delta\theta_2 q\} > q_H^*$. For this solution to be feasible we need in addition that hL is indeed excluded, i.e. that $p_L - \overline{\theta}_1 - \underline{\theta}_2 q_L \leq 0$, i.e. $\Delta\theta_1 - \Delta\theta_2 q_H^{**} \geq 0$. The buyer's expected utility is given by

$$\alpha_{lL}\left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 \underline{q} - \Delta\theta_2 q_H^{**}\right) + \alpha_{lH}\left(v(q_H^{**}) - \underline{\theta}_1 - \overline{\theta}_2 q_H^{**}\right)$$

Q.E.D.

Appendix B: Supporting results for the recall section

Lemma B1: Suppose $\mu^i_{hH} > 0$ and let $V(\pi^i)$ denote the buyer's expected utility from the recall stage when $\mu^i_{lL} + \mu^i_{hL} = \pi^i$. Then $V(\pi^i)$ is increasing in π^i.

Proof: From the derivation in section 5.1.1 and as long as $\pi^i \in (0,1)$, the buyer's expected utility from the recall stage is equal to:

$$V(\pi^i) = \arg\max_{q_H} \left\{(1-\pi^i)\left(v(q_H) - \overline{\theta}_1 - \overline{\theta}_2 q_H\right) + \pi^i \left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2\underline{q} - \Delta\theta_2 q_H\right)\right\}$$

By the envelope theorem,

$$\begin{aligned}
\frac{d}{d\pi^i}V(\pi^i) &= -\left(v(q_H) - \overline{\theta}_1 - \overline{\theta}_2 q_H\right) + \left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2\underline{q} - \Delta\theta_2 q_H\right) \\
&= -\left(v(q_H) - \overline{\theta}_1 - \underline{\theta}_2 q_H\right) + \left(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2\underline{q}\right) \\
&> 0 \text{ by the definition of } \underline{q}. \text{ QED}
\end{aligned}$$

Lemma B2: All sequential equilibria of the recall game in which the buyer recalls supplier 2 for sure on the equilibrium path, yield the same expected utility to the buyer as the optimal sequential bargaining mechanism.

Proof: In the proof, we take as given that the buyer will recall supplier 2 for sure if supplier 1 and supplier 2 both reject his offer in stages 1 and 2 respectively and focus on the subgame starting in stage 2 after a rejection by supplier 1. In the recall stage, sequential rationality requires that the buyer makes the offers $(p^{\text{recall}}_L(\pi^2), q^{\text{recall}}_L(\pi^2)$, $(p^{\text{recall}}_H(\pi^2), q^{\text{recall}}_H(\pi^2))$ where $\pi^2 = \mu_{lL} + \mu_{hL}$ correspond to his updated beliefs about supplier 2. The proof proceeds through three claims.

Claim 1: In stage 2, the IR constraints of the low marginal costs suppliers are the same and the IR constraints of the high marginal costs suppliers are the same.

Proof of claim 1: Suppose supplier 2 expects that the buyer will have beliefs μ^2_{lL}, μ^2_{lH},.... in the recall stage (and $\pi^2 = \mu^2_{lL} + \mu^2_{hL}$). Thus he expects the buyer to make the offer $(p^{\text{recall}}_L(\pi^2)$, $q^{\text{recall}}_L(\pi^2))$, $(p^{\text{recall}}_H(\pi^2), q^{\text{recall}}_H(\pi^2))$. The recall stage offer acts as supplier 2's outside option when he decides whether to accept the offer in stage 2. Type lH will accept offer (p_H, q_H) in stage 2 if and only if

$$p_H - \underline{\theta}_1 - \overline{\theta}_2 q_H \geq \underbrace{\overline{\theta}_1 + \overline{\theta}_2 q^{\text{recall}}_H(\pi^2)}_{p^{\text{recall}}_H(\pi^2)} - \underline{\theta}_1 - \overline{\theta}_2 q^{\text{recall}}_H(\pi^2)$$

which yields $p_H \geq \overline{\theta}_1 + \overline{\theta}_2 q_H$ as the IR constraint for type lH. Repeating this exercise for type hH we have:

$$p_H - \overline{\theta}_1 - \overline{\theta}_2 q_H \geq \underbrace{\overline{\theta}_1 + \overline{\theta}_2 q^{\text{recall}}_H(\pi^2)}_{p^{\text{recall}}_H(\pi^2)} - \overline{\theta}_1 - \overline{\theta}_2 q^{\text{recall}}_H(\pi^2)$$

which also simplifies to $p_H \geq \overline{\theta}_1 + \overline{\theta}_2 q_H$. This proves that types hH and lH have the same IR constraint. The proof for the second part of the claim is identical.

Claim 2: Taking as given that the buyer recalls supplier 2 for sure in the recall stage, there exists a continuum of sequential equilibria in the subgame starting at stage 2 where the buyer makes the same offer in stage 2 and in the recall stage. The continuation value at stage 2 in these equilibria is equal to $V_1(\alpha_{hL}+\alpha_{lL})$, the expected value from the optimal one-buyer one-supplier bargaining mechanism defined in theorem 3.

Proof of claim 2: Consider the following strategies by the buyer and supplier 2. In stage 2, the buyer makes the supplier the offers (p_H^2, q_H^2), (p_L^2, q_L^2) defined in (8)-(11) which correspond to the optimal take-it-or-leave-it offers that implement the optimal one-buyer one-supplier bargaining mechanism. In the recall stage, the buyer makes the offers $(p_L^{\text{recall}}(\pi^2), q_L^{\text{recall}}(\pi^2)$, $(p_H^{\text{recall}}(\pi^2), q_H^{\text{recall}}(\pi^2))$ where $\pi^2 = \mu_{lL}^2 + \mu_{hL}^2$ correspond to his updated beliefs about supplier 2. If supplier 2 prefers the stage 2 offer over the recall stage offer, he accepts the stage 2 offer. He accepts the recall offer if the recall offer is preferred. If indifferent, the type α_k supplier accepts the offer in stage with probability λ_k such that $\frac{(\lambda_{lL}\alpha_{lL}+\lambda_{hL}\alpha_{hL})}{(\lambda_{lH}\alpha_{lH}+\lambda_{hH}\alpha_{hH})} = \frac{\alpha_{lL}+\alpha_{hL}}{\alpha_{lH}+\alpha_{hH}}$. If all types, when indifferent, accept with certainty, then beliefs are such that $\pi^2 \geq \alpha_{lL}+\alpha_{hL}$. Supplier 2 accepts the offer in the recall stage.
Clearly, supplier 2's strategy is optimal and so is the buyer's recall strategy. If supplier 2 sometimes accept in the recall stage, these strategies lead to identical offers in both stages given the requirement that $\frac{(\lambda_{lL}\alpha_{lL}+\lambda_{hL}\alpha_{hL})}{(\lambda_{lH}\alpha_{lH}+\lambda_{hH}\alpha_{hH})} = \frac{\alpha_{lL}+\alpha_{hL}}{\alpha_{lH}+\alpha_{hH}}$. If all types accept in stage 2, $q_H^2 \geq q_H^{\text{recall}}(\pi^2)$ given the off-the-equilibrium beliefs $\pi^2 \geq \alpha_{lL}+\alpha_{hL}$ if $\lambda_k = 0$ for all k. In both cases, the buyer's expected payoff is equal to $V_1 = (\alpha_{hL}+\alpha_{lL})(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2 q) + (\alpha_{lH}+\alpha_{hH})\max_q \left(v(q) - \overline{\theta}_1 - \overline{\theta}_2 q - \frac{(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})} q\Delta\theta_2\right)$.
The final step is to argue that it is optimal in stage 2 for the buyer to offer the one-shot take-it-or-leave-it offer defined in (8)-(11). This is the case because this offer yields the same expected utility as the optimal one-buyer one-supplier bargaining mechanism.

Claim 3: There is no other sequential equilibrium in the subgame starting in stage 2.
Proof of claim 3:
By claim 1, the stage 2 offers are either only acceptable to the low marginal cost types, only the high marginal cost types, to no types or to all types. We consider these cases in turn:
<u>Case 1</u>: Stage 2 offers are acceptable to all types.
Since the optimal offer made by the buyer is unique conditional on the strategies of supplier 2 described in claim 2, we consider whether other equilibria exist in which supplier 2 mixes differently when indifferent (yielding $\pi^2 \neq \alpha_{lL}+\alpha_{hL}$) or where off-equilibrium beliefs are such that $\pi^2 < \alpha_{lL}+\alpha_{hL}$. Let $(\widetilde{p}_H^2, \widetilde{q}_H^2)$, $(\widetilde{p}_L^2, \widetilde{q}_L^2)$ denote the stage 2 offers of the buyer in such hypothetical alternate equilibrium.
(i) Equilibria where all types accept in stage 2 and off-equilibrium beliefs are $\pi^2 < \alpha_{lL}+\alpha_{hL}$: In such an equilibrium, $q_H^{\text{recall}}(\pi^2) > q_H^{\text{recall}}(\alpha_{lL}+\alpha_{hL})$. This recall offer acts as a constraint on $(\widetilde{p}_H^2, \widetilde{q}_H^2)$, $(\widetilde{p}_L^2, \widetilde{q}_L^2)$ leading to an expected utility to the buyer equal to $(\alpha_{lL}+\alpha_{hL})(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2\underline{q})$ $+ (\alpha_{lH}+\alpha_{hH})\left(v(q_H^{\text{recall}}(\pi^2)) - \overline{\theta}_1 - \overline{\theta}_2 q - \frac{(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})} q_H^{\text{recall}}(\pi^2)\Delta\theta_2\right) < (\alpha_{hL}+\alpha_{lL})(v(\underline{q}) - \overline{\theta}_1 - \underline{\theta}_2\underline{q})$ $+(\alpha_{lH}+\alpha_{hH})\max_q \left(v(q) - \overline{\theta}_1 - \overline{\theta}_2 q - \frac{(\alpha_{hL}+\alpha_{lL})}{(\alpha_{lH}+\alpha_{hH})} q\Delta\theta_2\right)$. The buyer is better off not offering an acceptable contract in stage 2 and waiting until the recall stage to offer the optimal one-shot contract

(note that, by lemma B1, recall when $\pi^2 < \alpha_{lL} + \alpha_{hL}$ implies recall of supplier two in this alternate strategy since $\widetilde{\pi}^2 = \alpha_{lL} + \alpha_{hL} > \pi^2$). Thus, there cannot be such equilibria.
(ii) Equilibria where some types mix in such a way that $\pi^2 \neq \alpha_{lL} + \alpha_{hL}$.
Consider first equilibrium candidates such that $\pi^2 > \alpha_{lL} + \alpha_{hL}$. Some low marginal cost types must be rejecting the stage 2 offer to wait for the recall offer and because, by assumption, the stage 2 offer is acceptable for them, it must be that they are indifferent. Thus $q_H^{\text{recall}}(\pi^2) = \widetilde{q}_H^2$ must hold. Consider the buyer's resulting expected utility:

$$
\begin{aligned}
&(\lambda_{lH}\alpha_{lH} + \lambda_{hH}\alpha_{hH})\left(v(\widetilde{q}_H^2) - \overline{\theta}_1 - \overline{\theta}_2\widetilde{q}_H^2\right) \\
&+(\lambda_{lL}\alpha_{lL} + \lambda_{hL}\alpha_{hL})\left(v(\widetilde{q}_L^2) - \overline{\theta}_1 - \underline{\theta}_2 q_L^2 - \Delta\theta_2\widetilde{q}_H^2\right) \\
&+((1-\lambda_{lH})\alpha_{lH} + (1-\lambda_{hH})\alpha_{hH})\left(v(q_H^{\text{recall}}(\pi^2)) - \overline{\theta}_1 - \overline{\theta}_2 q_H^{\text{recall}}(\pi^2)\right) + \\
&((1-\lambda_{lL})\alpha_{lL} + (1-\lambda_{hL})\alpha_{hL})\left(v(q_L) - \overline{\theta}_1 - \Delta\theta_2 q_H^{\text{recall}}(\pi^2) - \underline{\theta}_2 q_L\right)
\end{aligned}
$$

$q_H^{\text{recall}}(\pi^2) = \widetilde{q}_H^2$ is optimal if $\frac{(\lambda_{lL}\alpha_{lL}+\lambda_{hL}\alpha_{hL})}{(\lambda_{lH}\alpha_{lH}+\lambda_{hH}\alpha_{hH})} = \frac{((1-\lambda_{lL})\alpha_{lL}+(1-\lambda_{hL})\alpha_{hL})}{((1-\lambda_{lH})\alpha_{lH}+(1-\lambda_{hH})\alpha_{hH})}$ or, in other words, if $\pi^2 = \alpha_{lL} + \alpha_{hL}$, a contradiction.
Consider next equilibrium candidates such that $\pi^2 < \alpha_{lL} + \alpha_{hL}$. If some low marginal cost types wait until the recall stage then $q_H^{\text{recall}}(\pi^2) = \widetilde{q}_H^2$ must hold and we reach a contradiction as before. If not, so that $\pi^2 = 0$, $q_H^{\text{recall}}(\pi^2) = \overline{q}$ and the individual rationality constraint of low marginal cost types require that $\widetilde{q}_H^2 \geq \overline{q}$. The buyer's resulting expected utility is strictly less than V_1, his expected utility of the one-shot game if he does not make any offer in stage 2 and makes the optimal one-shot offer in the recall game.
<u>Case 2</u>: Stage 2 offer only acceptable to the low (or high) marginal cost types.
This class of potential equilibria can be ignored because by case 1 above and claim 2, the buyer can obtain the same expected utility as the optimal bargaining mechanism by playing according to the sequential equilibrium strategies described in claim 2.
<u>Case 3</u>: Stage 2 offer not acceptable to any types.
Trivially, the expected utility from making this offer in stage two reaches the bound set by the optimal bargaining mechanism since the certainty of recall means that the offers made in the recall stage will be those corresponding to the optimal bargaining mechanism.

This is the final step in establishing lemma 2B. QED

Table 1: Probabilities of winning and quality levels when $\Delta\theta_1 - \Delta\theta_2\overline{q} \geq 0$

Solution	Probabilities of Winning	q_{lL}	q_{hL}	q_{lH}	q_{hH}
Condition: $x_{hH}^{FB}[\Delta\theta_1 - \Delta\theta_2 q_{hH}^2] \leq x_{lH}^{FB}[\Delta\theta_1 - \Delta\theta_2\overline{q}]$					
1.1.a	$x_k = x_k^{FB}$	$\underline{q}$	$\underline{q}$	$\overline{q}$	q_{hH}^2
1.1.b	$x_{lL} = x_{lL}^{FB} > x_{lH} = x_{lH}^{\max} > x_{hL} = x_{hL}^{\min} > x_{hH} = x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$\overline{q}$	q_{hH}^2
1.1.c	$x_{lL} = x_{lL}^{FB} > x_{lH}^{\max} \geq x_{lH} > x_{hL} \geq x_{hL}^{\min} > x_{hH} = x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ \overline{q})$	$(q_{hH}^2, \overline{q})$
1.1.d	$x_{lL} = x_{lL}^{FB} > x_{lH} = x_{lH}^{\max} > x_{hL}^{\min} = x_{hL} > x_{hH} > x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ \overline{q})$	$(q_{hH}^2, \overline{q})$
1.1.e	$x_{lL} = x_{lL}^{FB} > x_{lH} > x_{hL} = x_{hH} > x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ q_{hH})$	$(q_{hH}^2, \overline{q})$
Condition: $x_{hH}^{FB}[\Delta\theta_1 - \Delta\theta_2 q_{hH}^2] > x_{lH}^{FB}[\Delta\theta_1 - \Delta\theta_2\overline{q}]$					
1.2.a*	$x_k = x_k^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ \overline{q})$	$(q_{hH}^2, \overline{q})$
1.2.b*	$x_{lL} = x_{lL}^{FB} > x_{hL}^{FB} > x_{hL} > x_{lH} > x_{lH}^{FB} > x_{hH} = x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ \overline{q})$	$(q_{hH}^2, \overline{q})$
1.2.c*	$x_{lL} = x_{lL}^{FB} > x_{hL}^{FB} > x_{hL} = x_{lH} > x_{lH}^{FB} > x_{hH} = x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ \overline{q})$	$(q_{hH}^2, \overline{q})$
Other relevant solutions are 1.1.b, 1.1.c, 1.1.d and 1.1.e					

Additional conditions as well as exact values for the variables in the individual solutions are available in a separate appendix posted online. Recall, $q_{hH}^2 = \arg\max_q\{W_{hH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}} q\Delta\theta_2\}$ and $q_{lH}^2 = \arg\max_q\{W_{lH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{lH}} q\Delta\theta_2\}$. The notation $(q_{lH}^2, \overline{q})$ in the q_{lH} column means $q_{lH} \in (q_{lH}^2, \overline{q})$, (similarly for the q_{hH} column).

* Under the condition that $\Delta\theta_1 - \Delta\theta_2\overline{q} \geq 0$ we can tighten the bound on q_{hH} so that $q_{hH} \in (q_{hH}^2, q_{lH})$

Table 2: Probabilities of winning and quality levels when $\Delta\theta_1 - \Delta\theta_2\overline{q} < 0$

Solution	Probabilities of Winning (x's)	q_{lL}	q_{hL}	q_{lH}	q_{hH}
Condition: $x_{hH}^{FB}[\Delta\theta_1 - \Delta\theta_2\overline{q}] \geq x_{lH}^{FB}[\Delta\theta_1 - \Delta\theta_2 q_{lH}^2]$					
2.1.a	$x_k = x_k^{FB}$	$\underline{q}$	$\underline{q}$	q_{lH}^2	$\overline{q}$
2.1.b	$x_{lL} = x_{lL}^{FB} > x_{hL}^{FB} > x_{hL} = x_{lH} > x_{lH}^{FB} > x_{hH} = x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	q_{lH}^2	$\overline{q}$
2.1.c	$x_{lL} = x_{lL}^{FB} > x_{hL} = x_{hL}^{FB} > x_{lH}^{FB} > x_{lH} > x_{hH} > x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	(q_{lH}^2, q_{hH})	$(q_{hH}^*,\ \overline{q})$
2.1.d	$x_{lL} = x_{lL}^{FB} > x_{hL}^{FB} > x_{lH} = x_{hL} > x_{lH}^{FB} > x_{hH} > x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ q_{hH})$	$(q_{hH}^*,\ \overline{q})$
2.1.e	$x_{lL} = x_{lL}^{FB} > x_{lH} > x_{hL} = x_{hH} > x_{hH}^{FB}$	$\underline{q}$	$\underline{q}$	$(q_{lH}^2,\ q_{hH})$	$(q_{hH}^*,\ \overline{q})$
Condition: $x_{hH}^{FB}[\Delta\theta_1 - \Delta\theta_2\overline{q}] < x_{lH}^{FB}[\Delta\theta_1 - \Delta\theta_2 q_{lH}^2]$					
The relevant solutions are 1.2.a, 1.2.b, 1.2.c, 2.1.c, 2.1.d and 2.1.e					

Additional conditions as well as exact expressions for the variables in the individual solutions are available in a separate appendix posted online. Recall, $q_{hH}^2 = \arg\max_q\{W_{hH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}} q\Delta\theta_2\}$ and $q_{lH}^2 = \arg\max_q\{W_{lH}(q) - \frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{lH}} q\Delta\theta_2\}$. The notation $(q_{lH}^2, \overline{q})$ in the q_{lH} column means $q_{lH} \in (q_{lH}^2,\ \overline{q})$, (similarly for the q_{hH} column).

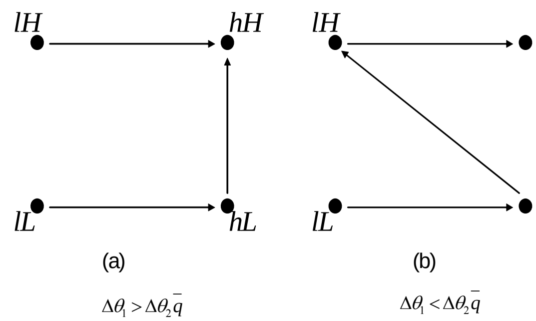


Figure 1: binding constraints in the efficient auction

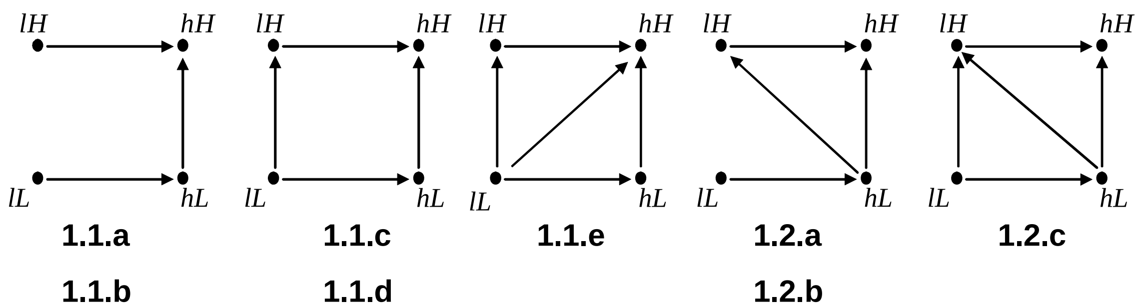


Figure 2: Binding IC constraints at the solution when $\Delta\theta_1 - \Delta\theta_2\overline{q} \geq 0$

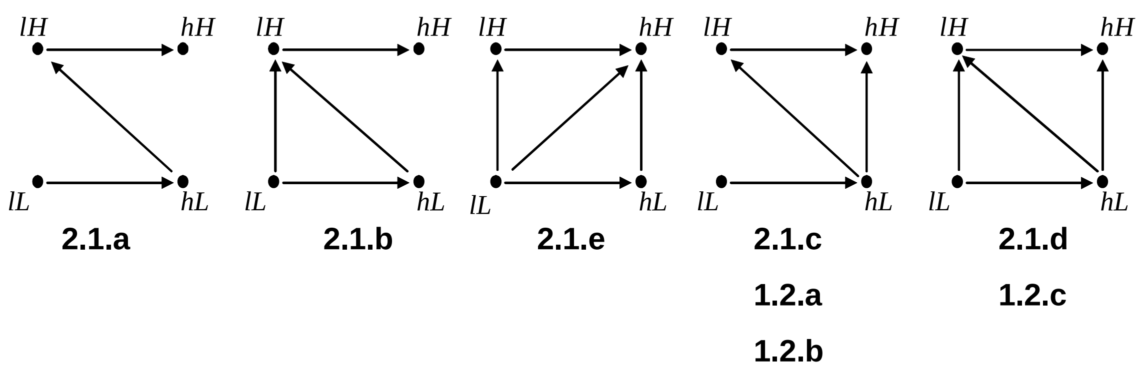
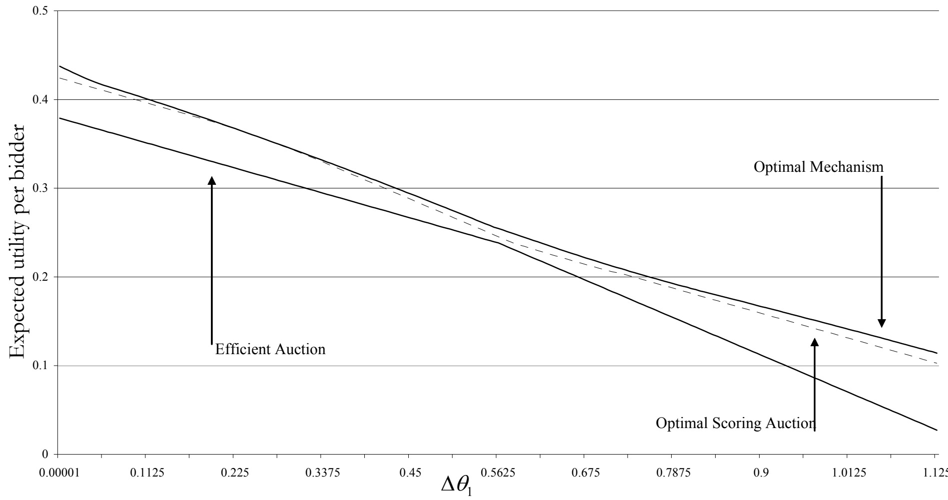
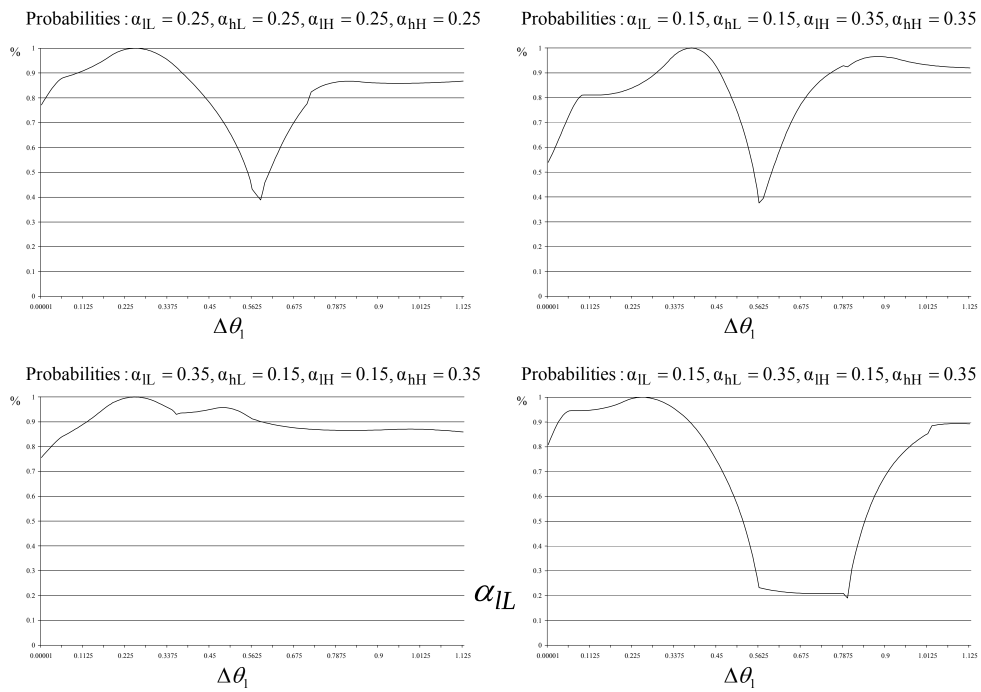


Figure 3: Binding IC constraints at the solution when $\Delta\theta_1 - \Delta\theta_2\overline{q} < 0$



: Figure 4: Expected utility from different mechanisms: $N = 2$, $\alpha_{lL} = \alpha_{hL} = \alpha_{lH} = \alpha_{hH} = 0.25$



: Figure 5: Strategic surplus captured by the optimal scoring auction ($N = 2$)

Table 3: Percentage of the strategic surplus captured by the optimal scoring auction

	Probabilities				Average	Max	$\Delta\theta_1$ at	Min	$\Delta\theta_1$ at	% of $\Delta\theta_1$'s such that percentage is:	
	α_{lH}	α_{hH}	α_{hL}	α_{lL}			maximum		minimum	>80% of opt. mech.	< minimum +10%
						Core Parameter Values					
1	25	25	25	25	82.5	100	0.248	38.9	0.585	74.1	4.5
2	20	30	20	30	87.4	100	0.248	73.9	0.574	86.6	19.4
3	15	35	15	35	90.1	100	0.248	75.6	0.000	97.5	7.0
4	10	40	10	40	82.3	100	0.248	59.3	1.124	56.7	25.8
5	30	20	30	20	76.1	100	0.248	28.4	0.686	62.2	13.4
6	35	15	35	15	68.5	100	0.248	19.1	0.799	52.7	22.4
7	40	10	40	10	60.4	100	0.248	12.7	0.810	45.7	31.8
8	20	20	30	30	81.2	100	0.180	42.3	0.596	68.2	5.0
9	15	15	35	35	79.6	100	0.124	44.9	0.630	47.3	7.0
10	10	10	40	40	78.6	100	0.068	44.2	0.698	40.8	6.5
11	30	30	20	20	83.5	100	0.315	37.2	0.574	74.1	3.5
12	35	35	15	15	83.9	100	0.383	37.6	0.563	75.6	3.5
13	40	40	10	10	83.6	100	0.450	35.8	0.563	60.2	4.5
14	20	30	30	20	77.9	100	0.248	46.5	0.709	44.8	14.9
15	15	35	35	15	77.8	100	0.248	55.8	0.833	41.3	26.9
16	10	40	40	10	82.1	100	0.248	65.0	0.968	45.5	35.6
17	30	20	20	30	89.2	100	0.248	67.0	0.518	83.6	13.4
18	35	15	15	35	94.8	100	0.248	80.9	0.000	100	20.4
19	40	10	10	40	98.0	100	0.248	84.4	0.000	100	8.9
						Extensions					
20	45	45	5	5	82.6	100	0.506	20.9	0.000	50.2	2.0
21	15	25	30	30	81.7	100	0.153	53.5	0.038	51.2	7.5
22	16	23	41	20	75.2	100	0.180	46.3	0.821	39.3	13.4
						Robustness: $a = 1$					
23	25	25	25	25	82.5	100	0.028	38.9	0.065	74.1	4.5
24	15	35	15	35	90.1	100	0.028	75.6	0.000	97.5	7.0
25	35	15	35	15	68.5	100	0.028	19.1	0.088	52.7	22.4
						Robustness: $b = 0.7$					
26	25	25	25	25	0.737	100	0.285	0.429	2.078	49.3	24.9
						Robustness: $\Delta\theta_2 = 2$					
27	25	25	25	25	0.751	100	0.180	0.409	0.765	53.2	19.9
						Robustness: $\underline{\theta}_1 = 2$					
28	25	25	25	25	0.827	100	0.248	0.389	0.585	74.6	4.0

Notes: Each experiment sets the probabilities of each type (ordered $\alpha_{lH}, \alpha_{hH}, \alpha_{hL}, \alpha_{lL}$), then computes the expected utility for the optimal mechanism, the efficient auction and the optimal scoring auction for 201 values of $\Delta\theta_1$ covering the full range of the parameter values allowed by the model.

Table 4: Contracts in the optimal sequential mechanism

K_n	Offers (p_H^n, q_H^n) and (p_L^n, q_L^n)	V_n (continuation value)
lL	$(\underline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$	$\alpha_{lL}W_{lL}^{FB}+(1-\alpha_{lL})V_{n-1}$
lL,hL	$(\overline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$	$(\alpha_{lL}+\alpha_{hL})W_{hL}^{FB}+(\alpha_{lH}+\alpha_{hH})V_{n-1}$
lL,lH,hL	<u>If $\Delta\theta_1-\Delta\theta_2\overline{q}>0$:</u>	$(\alpha_{lL}+\alpha_{hL})W_{hL}^{FB}+\alpha_{lH}W_{lH}^{FB}$
	$(\underline{\theta}_1+\overline{\theta}_2\overline{q},\overline{q})$ and $(\overline{\theta}_1+\underline{\theta}_2\underline{q},\underline{q})$	$+\alpha_{hH}V_{n-1}$
	<u>If $\Delta\theta_1-\Delta\theta_2\overline{q}\leq 0$:</u>	
	$(\underline{\theta}_1+\overline{\theta}_2 q_H^*, q_H^*)$ and $(\underline{\theta}_1+\underline{\theta}_2\underline{q}+\Delta\theta_2 q_H^*,\underline{q})$, with	$(\alpha_{lL}+\alpha_{hL})(W_{hL}^{FB}-\Delta\theta_2 q_H^*)$
	$q_H^*=\max\left\{\frac{\Delta\theta_1}{\Delta\theta_2}, \arg\max_q \left\{v(q)-\overline{\theta}_2 q-\frac{(\alpha_{lL}+\alpha_{hL})}{\alpha_{lH}}\Delta\theta_2 q\right\}\right\}$	$\alpha_{lH}W_{lH}(q_H^*)+\alpha_{hH}V_{n-1}$
lL,lH	$(\underline{\theta}_1+\overline{\theta}_2 q_H^{**}, q_H^*)$, $(\underline{\theta}_1+\underline{\theta}_2\underline{q}+\Delta\theta_2 q_1^{**},\underline{q})$	$\alpha_{lL}(W_{lL}^{FB}-\Delta\theta_2 q_H^{**})+$
	with $q_H^{**}=\arg\max\left\{v(q)-\overline{\theta}_2 q-\frac{\alpha_{lL}}{\alpha_{lH}}\Delta\theta_2 q\right\}$	$\alpha_{lH}W_{lH}(q_1^{**})+(\alpha_{hL}+\alpha_{hH})V_{n-1}$
	Solution possible only if $\Delta\theta_1-\Delta\theta_2 q_H^{**}\geq 0$	

(The first column in the table indicates the set of supplier types who will accept the buyer's offer when n suppliers remain).

Table 5: Virtual welfares and probabilities in the efficient auction, by type

Type	Virtual welfares		Probabilities
	$\Delta\theta_1-\Delta\theta_2\overline{q}>0$	$\Delta\theta_1-\Delta\theta_2\overline{q}<0$	$\Pr_k=N\alpha_k x_k^{FB}$
$lL:$	W_{lL}^{FB}	W_{lL}^{FB}	$1-(1-\alpha_{lL})^2$
$hL:$	$W_{hL}^{FB}-\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1$	$W_{hL}^{FB}-\frac{\alpha_{lL}}{\alpha_{hL}}\Delta\theta_1$	$(1-\alpha_{lL})^2-(1-\alpha_{lL}-\alpha_{hL})^2$
$lH:$	W_{lH}^{FB}	$W_{lH}^{FB}+\frac{\alpha_{lL}+\alpha_{hL}}{\alpha_{lH}}\Delta\theta_1-\frac{\alpha_{lL}+\alpha_{hL}}{\alpha_{lH}}\Delta\theta_2\overline{q}$	$(1-\alpha_{lL}-\alpha_{hL})^2-(1-\alpha_{lL}-\alpha_{hL}-\alpha_{lH})^2$
$hH:$	$W_{hH}^{FB}-\frac{\alpha_{lH}}{\alpha_{hH}}\Delta\theta_1-\frac{\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}}\overline{q}$	$W_{hH}^{FB}-\frac{\alpha_{lH}+\alpha_{hL}+\alpha_{lL}}{\alpha_{hH}}\Delta\theta_1$	α_{hH}^2

Table 6: Percentage of the strategic surplus captured by the optimal sequential procedure*

	Probabilities				Average	Max	$\Delta\theta_1$ at max	Min	$\Delta\theta_1$ at min	% $\Delta\theta_1$ s.t. >80%	% $\Delta\theta_1$ s.t. >0%
1	25	25	25	25	-152.7	100	0	-504.4	0.5625	6.9	20.8
2	20	30	20	30	-78.2	100	0	-279.6	0.5625	7.9	23.8
3	15	35	15	35	-7.2	100	0	-83.6	0.5625	7.9	35.6
4	10	40	10	40	55.9	100	0	39.2	0.5625	8.9	100
5	30	20	30	20	-238.7	100	0	-663.9	0.66375	5.9	18.8
6	35	15	35	15	-365.0	100	0	-942.2	0.7875	5.0	15.8
7	40	10	40	10	-591.8	100	0	-1,446.7	0.9	5.0	13.9
8	20	20	30	30	-126.4	100	0	-404.0	0.5625	5.9	22.8
9	15	15	35	35	-107.6	100	0	-356.2	0.5625	5.9	21.8
10	10	10	40	40	-95.5	100	0	-284.4	0.5625	5.0	18.8
11	30	30	20	20	-198.6	100	0	-746.5	0.5625	6.9	18.8
12	35	35	15	15	-280.2	100	0	-1.302.9	0.5625	6.9	14.9
13	40	40	10	10	-394.1	100	0	-1,961.4	0.5625	6.0	9.9
14	20	30	30	20	-147.2	100	0	-384.1	0.66375	7.9	25.7
15	15	35	35	15	-108.2	100	0	-276.6	0.7875	10.9	32.7
16	10	40	40	10	-46.1	100	0	-164.7	0.9	15.8	40.6
17	30	20	20	30	-129.3	100	0	-411.4	0.5625	5.9	16.8
18	35	15	15	35	-93.9	100	0	-244.8	0.5625	5.0	13.9
19	40	10	10	40	-61.4	100	0	-142.0	0.5625	4.0	13.9

*Each experiment sets the value of the α_k's (ordered $\alpha_{lH}, \alpha_{hH}, \alpha_{hL}, \alpha_{lL}$) and computes the expected utility from the optimal mechanism, the optimal sequential auction and the efficient auction for all the values for $\Delta\theta_1$ allowed by the model. Each experiment samples 101 equally spaced values for $\Delta\theta_1$

Table 7: Percentage of the strategic surplus captured by the buyer in the best equilibrium of the recall game*

	Probabilities				Average	Max	$\Delta\theta_1$ at max	Min	$\Delta\theta_1$ at min	% $\Delta\theta_1$ s.t. >80%	% $\Delta\theta_1$ s.t. >0%	% $\Delta\theta_1$ s.t. > scoring
1	25	25	25	25	58.7	100	0	21.4	0.3038	6.9	100	16.8
2	20	30	20	30	63.4	100	0	23.7	0.2813	10.9	100	9.9
3	15	35	15	35	63.9	100	0	43.8	0.2588	21.8	100	6.9
4	10	40	10	40	62.6	100	0	45.1	0.6300	14.9	100	7.9
5	30	20	30	20	46.8	100	0	0.4	0.5625	6.9	100	16.8
6	35	15	35	15	26.5	100	0	-109.9	0.5625	5.9	74.3	14.9
7	40	10	40	10	2.4	100	0	-168.0	0.6750	5.9	66.3	9.9
8	20	20	30	30	59.9	100	0	7.7	03488	7.9	100	17.8
9	15	15	35	35	62.8	100	0	11.4	0.4275	9.9	100	20.8
10	10	10	40	40	64.6	100	0	0.0	56.25	17.8	99.0	18.8
11	30	30	20	20	58.3	100	0	38.8	0.2925	6.9	100	15.8
12	35	35	15	15	57.5	100	0	38.5	0.3600	6.9	100	14.9
13	40	40	10	10	57.0	100	0	37.3	0.4163	6.9	100	12.9
14	20	30	30	20	59.6	100	0	27.9	0.3150	9.9	100	21.8
15	15	35	35	15	64.4	100	0	37.6	0.5625	13.9	100	29.7
16	10	40	40	10	73.0	100	0	48.3	0.5625	30.7	100	38.6
17	30	20	20	30	60.1	100	0	18.3	0.2813	5.9	100	13.9
18	35	15	15	35	62.5	100	0	14.8	0.2588	20.8	100	3.0
19	40	10	10	40	65.1	100	0	10.2	0.2363	27.7	100	3.0

*Each experiment sets the value of the α_k's (ordered $\alpha_{lH}, \alpha_{hH}, \alpha_{hL}, \alpha_{lL}$) and computes the expected utility from the optimal mechanism, the best equilibrium of the game with recall and the efficient auction for all the values for $\Delta\theta_1$ allowed by the model. Each experiment samples 101 equally spaced values for $\Delta\theta_1$.

Table 8: Performance of the least favorable equilibria in the recall game*

	Probabilities				% of $\Delta\theta_1$ w/ multiple equilibria	Conditional on multiple equilibria: % of $\Delta\theta_1$ s.t. utility < efficient mech.	Mean difference: best and worst**	Max difference: best and worst
1	25	25	25	25	12.9	7.7	15.1	26.8
2	20	30	20	30	0.0	-	-	-
3	15	35	15	35	44.6	77.8	70.1	116.2
4	10	40	10	40	34.7	0.0	16.2	27.4
5	30	20	30	20	21.8	31.8	30.0	66.5
6	35	15	35	15	37.6	68.4	53.8	125.7
7	40	10	40	10	56.4	80.7	111.7	209.8
8	20	20	30	30	0.0	-	-	-
9	15	15	35	35	17.8	100.0	141.8	163.7
10	10	10	40	40	49.5	100.0	196.2	352.5
11	30	30	20	20	16.8	17.6	26.2	64.5
12	35	35	15	15	24.8	28.0	38.8	111.5
13	40	40	10	10	32.7	57.6	64.4	197.0
14	20	30	30	20	6.9	0.0	4.6	8.0
15	15	35	35	15	0.0	-	-	-
16	10	40	40	10	0.0	-	-	-
17	30	20	20	30	16.8	29.4	23.3	46.5
18	35	15	15	35	17.8	50.0	37.0	69.7
19	40	10	10	40	17.8	61.1	55.4	99.1

*Each experiment sets the value of the α_k's (ordered $\alpha_{lH}, \alpha_{hH}, \alpha_{hL}, \alpha_{lL}$) and computes the expected utility from the optimal mechanism, the optimal sequential auction and the efficient auction for all the values for $\Delta\theta_1$ allowed by the model. Each experiment samples 101 equally spaced values for $\Delta\theta_1$. ** "Mean difference: best and worst" describes the difference in the percentage of strategic surplus captured by the best and worst equilibrium of the recall game. The mean is over values of $\Delta\theta_1$ conditional on the existence of multiple equilibria.