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PRODUCT AND PRICE COMPETITION IN A DUOPOLY

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This paper examines the role of consumer preferences, costs, and price competition in determining the competitive product strategy of a firm. In the model studied here, there are two identical firms competing on product quality and price. They face consumers who prefer a higher quality product to a lower quality product, but differ in how much they are willing to pay for quality. The consumers can also choose a substitute if they don't like the product-price offerings of the two firms. For the firms, a higher quality product costs more to produce than a lower quality product. The paper shows that the equilibrium strategy for each firm should be to differentiate its product from its competitor, with the firm choosing the higher quality choosing the higher margin as well. This differentiation, however, is not efficient—that is, it is possible to choose two other products and offer them at prices that cover their marginal cost, and still satisfy consumers' "needs" better in the aggregate. A monopolist, by contrast, would differentiate his product line efficiently. This suggests that cannibalization has different effects on product strategy than competition. The paper also shows that if one firm enters the market first, then it can defend itself from later entrants, and gain a first-mover advantage, by preempting the most desirable product position.
(Product Strategy; Pricing Strategy; Game Theory)

1. Introduction

The idea that a firm must distinguish its products from its competitors is a pervasive one in marketing. Shapiro, Dolan and Quelch (1985, p. 452), in their recent textbook, describe it as follows: "In choosing a position for a new product, management must match an appropriate package of benefits, clearly differentiated from competitive products on important dimensions, with a specific target segment whose needs are not fully satisfied by existing products." Porter (1980) calls the same thing a focus strategy.

The purpose of this paper is to explore the theoretical underpinnings of this prescription. The need for such an exploration becomes evident when one sees how easy it is to overturn the prescription. Consider Figure 1, a typical two-dimensional perceptual map of consumers' ideal-points and firms' positions. The ideal-points are distributed uniformly all over the map. Imagine a firm, called B, located at b. What position should we recommend for a new firm A entering this market? If we follow the "fill the holes in the market" prescription, then we would probably ask firm A to locate in the center of the quadrant south-west of b, at a. But this strategy doesn't maximize A's market-share. A can do better by moving along the line ab and positioning itself right next to B (but not quite at b)! That way, A assures itself of nearly all consumers whose ideal-points lie

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south-west of b. Of course, if B could still change its position, it wouldn't like to stay at b; B would jump over A and position itself right next to A on the line ab. This jockeying for position would only end when both firms are at the "center" of the market, sharing it equally. Political scientists have used this sort of argument to explain the similarity of candidates' positions in elections; cf. Downs (1957), Riker and Ordeshook (1973, Chapters 12 and 13).

Prices and costs play no role in the example above. But in reality they do. If a firm chooses a position too close to its competitors, then the ensuing price competition might be too intense. But if it chooses a position too far from its competitors, then the manufacturing costs may be too high and the market share too small. Another reality is the presence of substitutes outside the product class. In the example, we implicitly assumed that consumers stay with a firm as long as it is the best of the available alternatives (in *that* product class). Thus, a consumer located in the south-west corner of the market stayed with firm A as it moved from a toward b. What if consumers drop out of the market if all the available choices are "too far from them"? (In the political context, this would correspond to people not voting at all because they feel all candidates' positions are too far from theirs.) Would moving toward a competitor still be optimal?

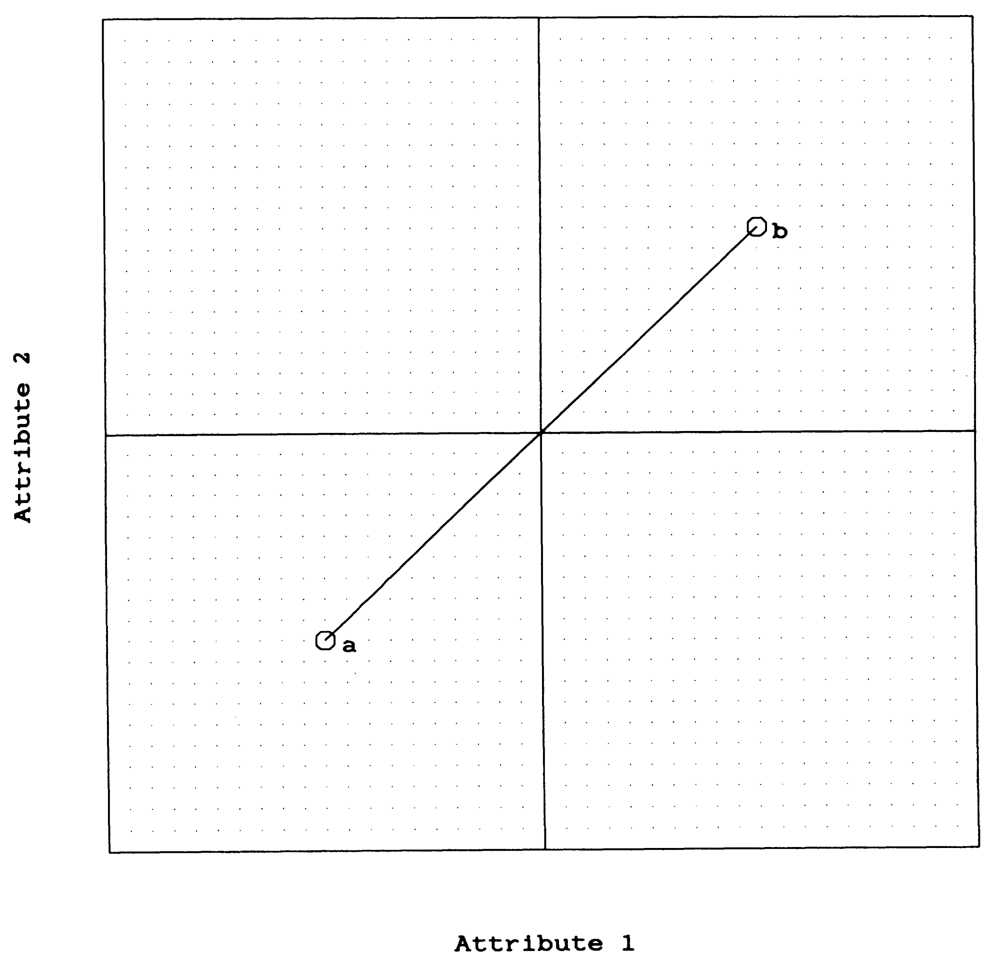

FIGURE 1

This paper examines the issues raised above in a simple model of two identical firms competing on a single product attribute, and price. We call this attribute quality because consumers are assumed to prefer more of it to less. Consumers differ in their willingness to pay for quality and are perfectly informed about the product and price offerings of the two firms. The latter assumption means, in particular, that we don't distinguish between a product's perceptual position and its "physical" position. On the firm side, we assume that a higher quality product costs more to produce than a lower quality product.

Two different modes of product competition are analyzed. In the first simultaneous-choice model, each firm is assumed to choose its product quality simultaneously with the other firm. That is, neither firm knows the other's product quality when choosing its own. This corresponds to the situation in industries where there is no product leader. In the second model, we assume that the two firms choose their products sequentially. First one firm—the leader—chooses its product and announces it. Then the second firm, having observed the first firm's product quality, chooses its own. This would be the situation in an industry where there is sequential entry: the second entrant gets to see the first entrant's product before choosing his own. In either model, the strategic problem for each firm is to choose a product quality and a price to maximize its profits, recognizing that the other firm is doing the same.

Our major results are as follows. First, in both the simultaneous-product-choice model and the sequential-product-choice model, the equilibrium strategy of each firm should be to differentiate its product from the other firm, with the firm choosing the higher quality choosing the higher margin as well. Second, these equilibrium product selections of the two firms are not efficient. That is, it is possible to choose two other products and offer them at prices that cover their marginal cost, and still satisfy consumers' "needs" better in the aggregate. This result is interesting because a *monopolist* under the same circumstances as the two firms would choose his two products efficiently. This suggests that cannibalization within a firm's product line has different effects on product strategy than competition between firms. The reason for the difference must be that whereas the monopolist can internalize the competition between his two products, the two firms—because they make their product and price decisions independently—can't. Third, in connection with the sequential-entry model of competition, we show that: (1) the leader benefits from anticipating the arrival of a later entrant, (2) this benefit is realized by choosing a product different from what he would have chosen if he had not anticipated a later entrant, and (3) there is a first-mover advantage because of the leader's ability to preempt a product position. Result #2 shows in particular that a product strategy designed to have a *defensive* role will be different from a strategy which doesn't have such a role.[1]

Besides the above results, perhaps the main contribution of this paper is the following insight. In a situation where a firm has to trade-off consumer preferences, costs, price competition with another firm, and competition with a substitute, we learn that its equilibrium product strategy must reconcile two opposing forces. One force draws it closer to its competitor: *Each* firm wants to go after the product position that is most favorable in terms of consumer preferences, manufacturing costs, and competition with a substitute—the monopolist's best position. The other force moves it away from its

[1] The concept of *defensive* product strategy embodied in this statement is different from the one that Hauser and Shugan (1983) propose. Hauser and Shugan view defensive strategy as the reaction of an incumbent faced with new competition. I view defensive strategy as defensive *preparations*. The incumbent knows that new competition is coming in and he prepares for it by choosing a product that "crowds" the new entrant. Then, when the new entrant does come in, he doesn't have to react. Obviously, defensive preparations can only be made on marketing variables which can be committed to. Product design, because it is costly to change, is such a variable.

competitor: If the firms were positioned too close to each other, then consumers would choose between them on the basis of price; that would create the incentive to compete on price, and the net result would be low profits for both firms. In §7 I discuss how this insight can be used to explain a number of results in the literature.

My model and methodology follow in the tradition of Hotelling (1929) and the numerous other works that he inspired. In Hotelling's model two firms compete on store location and price. (Store location is the product.) Hotelling argues that the equilibrium strategy for each firm is to choose a location at the center of the market—this is his famous "Principle of Minimum Differentiation."[2] The argument is that for any location of one firm, the other firm has an incentive to move toward its opponent in order to expand the territory under its exclusive control. Recently, d'Aspremont, Gabszewicz and Thisse (1979) have shown that Hotelling's argument is flawed. Hotelling did not consider the possibility of firms choosing prices that take away all of their opponent's market share. If such undercutting strategies are allowed, then a (pure strategy) price equilibrium doesn't exist in Hotelling's model when the two firms are close to each other. So nothing can be said about whether a firm gains or loses by choosing a product position close to its competitor's. When d'Aspremont, Gabszewicz and Thisse modify Hotelling's model to enable a price equilibrium to exist at all product positions, they find that each firm's equilibrium product strategy is to locate at the ends of the market, maximally differentiated from its competitor.

My model differs from Hotelling and d'Aspremont et al. in several aspects. First, whereas these authors assume consumers' ideal points are heterogeneous under equal prices for all products, I assume homogeneous ideal points under equal prices. In my model, quality is the dimension on which the firms can differentiate themselves, and *ceteris paribus*, every consumer prefers a higher quality to a lower quality. So every consumer has the same ideal point: infinite quality. Second, Hotelling and d'Aspremont et al. assume each product costs the same to produce, but I assume a higher quality product costs more to produce than a lower quality product. This allows me to capture the trade-off between the benefits of moving away from a competitor with the costs of doing so.

Shaked and Sutton's (1982) consumer model is virtually identical to mine, but they also assume that each product costs the same to produce. So their analysis turns out very different from mine. In their model, being the higher quality firm is always better than being the lower quality firm, regardless of how close the competitor is, regardless of how high the quality is. Their equilibrium, therefore, has one firm choosing the highest feasible quality and the other firm choosing a quality below that. (If there were no upper bound on quality in their model, then an equilibrium wouldn't exist.) On the other hand, in my model it is *not* always better to have the higher quality. If one firm chooses a high quality and the lower-quality competitor gets too close to it, then the first firm would prefer to be the lower quality supplier. This is so not only because the first firm would like to get away from the competitor—which, after all, could be done by choosing a still higher quality—but also because it doesn't want to choose a quality that costs too much (relative to what consumers are willing to pay for it).[3] Another difference between this paper and Shaked and Sutton's (and Prescott and Visscher 1977 and the others cited above) is that I allow for competition from a passive substitute—a substitute whose quality and price are fixed. This allows me to give consumers the option of

[2] Recall that my Figure 1 example does the same thing. But the crucial difference is that Hotelling makes his argument *with* price competition and profit maximization whereas in my example market share was being maximized and there was no price competition.

[3] For example, if car company A offers a car with gas-mileage 50 mpg, then car company B can differentiate itself from A by building a 70 mpg car or a 30 mpg car. B may find it more profitable to go with 30 mpg, because, *ceteris paribus*, it costs too much to build a 70 mpg car.

not buying from any of the two firms and to see what effect this has on the firms' equilibrium strategies. In my equilibria, therefore, the two firms do not serve the whole market; a part of the market buys the substitute.

In the marketing literature, strategic competition in products has not been modeled until recently. For example, in Kuehn and Day's (1962) early paper, the firm's motivation to differentiate its product from competitors' is solely the market share advantages differentiation brings: instead of sharing the market with numerous competitors, you get an entire segment for yourself. Price competition plays no role in the story. Furthermore, Kuehn and Day do not consider competitors' reactions—theirs is not an equilibrium approach. More recent examples include Karnani (1983), Rao and Bass (1985) and Eliashberg and Jeuland (1986). Karnani studies the competition in supply quantities with product positions exogenously fixed; Rao and Bass study dynamic pricing strategies in a homogeneous products model; and Eliashberg and Jeuland study dynamic pricing strategies in a heterogeneous products model. Rao (1977) models product choices explicitly, but he too doesn't study the product equilibrium. Hauser (1988) is an exception. He models strategic competition in products and prices much as I do, but his consumer model is more similar to Hotelling's model than mine. This may not seem so at first glance since Hauser's model has two attributes and Hotelling's model only one, but Hauser imposes the restriction that the feasible products must lie on the circumference of a circle, in the first quadrant, and with this restriction the two attributes collapse into one, and every consumer type has a distinct ideal product on this attribute under equal prices. In my model, as mentioned before, all consumers have the same ideal product under equal prices. Two other differences between our models are worth mentioning. Hauser, like Shaked and Sutton (1982), assumes that each product costs the same to produce and that there is no substitute. So he obtains an equilibrium with maximum differentiation, covering the whole market. I don't.

The rest of the paper is organized as follows. In the next section, the model is set up and the nature of market segments characterized. In §3, two benchmark solutions are developed for later comparison with the various equilibria. One benchmark is the efficient solution and the other benchmark is the monopolist's solution. In the efficient solution, we are looking at the two products that maximize consumer surplus under marginal-cost pricing. These are the products that best satisfy consumers' needs in the aggregate, while covering costs. On the other hand, the monopolist's solution is the set of two products that maximizes a single firm's profits. I begin the discussion of competitive product positioning in §4 with the price equilibrium. This price equilibrium applies to both models of product competition considered in this paper. In §5, I compute the product equilibrium in the first of these models, with both firms choosing their products simultaneously. In §6 I compute the product equilibrium with sequential entry. §7 concludes the paper. Proofs not in the text are in an appendix.

2. The Model

In this section I set up the environment in which I will pursue my investigations. The objective is to keep things simple so that we can focus on the issues that interest us and keep the analysis tractable. The following assumptions define this environment:

1. There are two firms, indexed 1 and 2, and they each choose a product from the interval $[0, \infty)$. This one-dimensional representation of the feasible set of products means that these firms are *competing* on one product attribute. I emphasize "competing" because this assumption doesn't restrict the number of attributes the firms' products can have. The firms' products can have any number of attributes, but they are assumed to be setting only one here.

2. A consumer of type-t is willing to pay up to ts for a unit of product s. Thus: (1) all consumers prefer more of the attribute to less, (2) a higher type of consumer is willing to

pay more for the same product than a lower type, and (3) the consumer's type is her *marginal* willingness to pay for increments of the attribute. The consumer's marginal willingness to pay—her type—can be interpreted as the consumer's "importance weight" for the attribute. It will be our "segmentation base" here. Because *all* consumers prefer more of the attribute to less, we will call this attribute "quality." But this is not the only interpretation possible: If the product class is cars, then gas-mileage is such an attribute; if the product class is express mail services, then delivery speed is such an attribute.

3. Consumer types are distributed uniformly on $[a, b]$, $0 < a < b$. This assumption says, among other things, that consumers are heterogeneous in this market. Earlier we noted that consumer types differ in their marginal willingness to pay for quality. Putting the two together, we get heterogeneity in consumers' marginal willingness to pay, and that is necessary for product differentiation. As for why we also assume that consumer types are distributed *uniformly*, the reason is we would like to remove non-uniformity of the consumer preference distribution as a possible explanation of product positioning. In my model, as we will see later, every consumer type has a distinct ideal product under marginal-cost pricing—when all feasible products are sold at their marginal cost. Then, because consumer types are distributed uniformly, these ideal products are also distributed uniformly. So if a firm chooses to produce a certain quality, it is not because more consumers have that quality as their ideal product than any other. Contrast this with the situation in Kuehn and Day (1962) where "pockets" of consumer ideal-points determine the products firms choose.

4. Consumers can observe the product qualities and prices available before they decide to buy. If they do decide to buy, they buy one unit of the product that gives them the largest consumer surplus—the difference between what they are willing to pay and what they are asked to pay. If the maximum surplus obtainable from the two firms is less than zero, then they will not buy any of the products. This option of not buying anything is represented as the choice of a "substitute" of quality zero and price zero: $s_0 = 0$, $p_0 = 0$.[4]

5. Each firm's marginal cost of supplying a product of quality s is αs^2 ($\alpha > 0$), regardless of the quantity supplied. There are no fixed costs. The assumption of identical cost functions rules out a trivial explanation of product differentiation, namely technological differences between the firms. The quadratic functional form for the marginal costs is the most tractable way of capturing a property that is crucial to my model, namely, marginal cost increases with quality, and at a faster rate than any consumer's willingness to pay. Because of this property, for each consumer type t, there exists a quality $s^0(t)$ such that $ts^0(t) - \alpha[s^0(t)]^2 = 0$. Thus, neither firm would want to sell a quality greater than $s^0(t)$ to t; indeed, $s^0(b)$ is the absolute upper limit on quality sold to anyone. If marginal costs didn't increase faster than some consumer's willingness to pay, then it would be optimal to supply every consumer with infinitely high quality—unless an arbitrary upper bound on quality is imposed as in Shaked and Sutton (1982) and Hauser (1988). The assumption of zero fixed costs is relatively innocuous since fixed costs have no effect on the equilibrium provided each firm's equilibrium contribution margin exceeds the fixed cost.

In what follows, I will call intervals of consumer types, such as $[t_1, t_2]$, segments. Also, firm 1's product will be denoted by s_1, its price by p_1, firm 2's product by s_2, and its price by p_2. My first task is to characterize the nature of each firm's market. Firm i's (i

[4] The designation of "buying nothing" as a substitute of quality zero is essentially a normalization. The substitute doesn't have to be a product of zero quality. For example, if the product class in question is automobiles, a consumer's choice of the "buying nothing" option may actually mean the choice of a bicycle. By representing this choice as the choice of a car of quality zero, what we are really doing is measuring a consumer's surplus from a car relative to her (maximum) surplus from a bicycle.

$= 1, 2$) market, M_i, is defined as the set of consumer types who get greater surplus from its product-price offering than from the other firm's product-price offering or the substitute. That is,

$$M_i \overset{\text{def}}{=} \{t \in [a, b] : ts_i - p_i \geq ts_j - p_j \text{ for } j = 0, 1, 2\} \qquad \text{for} \qquad i = 1, 2.$$

Proposition 1 below says that each firm's market is an interval of types, a market segment. So it is not possible to have, say, consumers t and t' choosing a firm's product, but not consumer $(t + t')/2$. Furthermore, Proposition 1 asserts that the boundaries of a firm's market segment are the consumer types who are indifferent between the firm and its neighbors (provided these neighbors have a market). Finally, firms' market segments stack up in the order of the firms' qualities: a firm with a higher quality product has a "higher" segment.

PROPOSITION 1. *Let (s_1, p_1) and (s_2, p_2) be the product and price offerings of the two firms with $0 \leq s_1 \leq s_2$. Then i's market ($i = 1, 2$) can be characterized as:*

1. $s_i \leq s_j$ *and* $p_i > p_j \Rightarrow M_i = \varnothing$,
2. $s_i < s_j$, $t \in M_i$, $t' \in M_j \Rightarrow t \leq t'$,
3. *If $M_i \neq \varnothing$, then $M_i = [t_i, t_{i+1}]$, where t_i is either a or the type of consumer indifferent between i and the next lower quality product with a nonempty market and t_{i+1} is either b or the type of consumer indifferent between i and the next higher quality product with a nonempty market.*

As a result of Proposition 1 we can now specify each firm's market segment. Suppose $s_1 < s_2$. Then, if each firm's market is nonempty, $M_1 = [t_1, t_2]$ and $M_2 = [t_2, b]$, where $t_1 = \max\{p_1/s_1, a\}$ and $t_2 = (p_2 - p_1)/(s_2 - s_1)$. Thus, firm 1's market share, m_1, is $(t_2 - t_1)/(b - a)$ and firm 2's market share is $m_2 = (b - t_2)/(b - a)$. Here $(p_2 - p_1)/(s_2 - s_1)$ is the consumer type that is indifferent between (s_1, p_1) and (s_2, p_2) and p_1/s_1, if it is not less than a, is the consumer type indifferent between (s_1, p_1) and (s_0, p_0). If $p_1/s_1 < a$, then $t_1 = a$, and the substitute's market share is zero; in this market-covered case, the duopolists face no competition from the substitute. Since we are interested in studying the effect of competition from a substitute on the duopolists' behavior, we shall be interested in the market-not-covered case only. We will assume that the market is sufficiently diverse—i.e., b/a is sufficiently large—so that the duopolists find it in their interest to leave some market share to the substitute. As we will see in the next section, $b > 5a$ is a sufficient condition for the firms not to cover the market.

3. The Efficient Solution and the Monopolist's Solution

I now set up two benchmark configurations of products with which I will later compare the duopolists' equilibrium product choices. One benchmark is the efficient solution and the other benchmark is the monopolist's solution. We are interested in the efficient solution because it tells us what products we should be choosing if our goal was to maximize total consumer value while covering costs. We are interested in the monopolist's solution because we would like to see how a single profit-maximizing entity's choice of two products differs from two profit-maximizing firms' choice of one product each. In the former, price-discrimination and cannibalization considerations apply; in the latter, competitive considerations apply.

The Efficient Solution

We can start by defining the efficient product for a *given* consumer type. The efficient product for consumer t is the product that maximizes the total surplus from serving her—the product that maximizes the difference between her willingness to pay, ts, and marginal cost, αs^2. (Total surplus is equal to $ts - \alpha s^2$ because for any price p, consumer's surplus is $ts - p$ and producer's surplus is $p - \alpha s^2$.) It is the product that maximizes consumer t's surplus under marginal-cost pricing—the product consumer t would choose if *all* feasible products were sold at their marginal cost. We are interested in this product because it represents the trade-off between "giving the consumer what she wants" and the cost of doing so. Denote type t's efficient product by $s^*(t)$. It is easy to see that $s^*(t) = t/2\alpha$. (Just differentiate $ts - \alpha s^2$ with respect to s and solve the resulting first-order condition.) Since $s^*(t)$ is increasing in t, a consumer's efficient total surplus, $ts^*(t) - \alpha[s^*(t)]^2$, is also increasing in the consumer's type. If each type of consumer could be offered a distinct product, then the set of products that maximizes aggregate total surplus is the interval $[a/2\alpha, b/2\alpha]$.

Suppose, however, that only one product could be offered for the entire market. In this case the efficient solution in the market-not-covered case will be the product-price combination (s, p) that maximizes the aggregate total surplus $\int_{t_1}^{b} (ts - \alpha s^2)$ where $t_1 = p/s \geq a$ is the lowest type of consumer served under (s, p). Notice that even though a given consumer's total surplus is unaffected by price, the aggregate total surplus is: the higher the price, the smaller the market served, and hence the smaller the aggregate total surplus. Maximizing the aggregate total surplus with respect to p yields $p = \alpha s^2$—thus the product should be priced at marginal cost—and then maximizing with respect to s yields $b/3\alpha$. The lower boundary of the market served is $b/3$ and that is greater than a if and only if $b \geq 3a$. Thus, when $b \geq 3a$, the one-product market-not-covered efficient solution is to sell $b/3\alpha$ at marginal cost. When $b < 3a$, the best single efficient product is $(b + a)/4\alpha$ and it covers the market when sold at marginal cost. (The two solutions are continuous at $b = 3a$.)

Since our industry consists of two products, we are primarily interested in the efficient solution with two products. This is the solution to

$$\max_{\substack{s_1, s_2 \\ p_1, p_2}} \sum_{i=1}^{2} \int_{t_i}^{t_{i+1}} (ts_i - \alpha s_i^2)dt, \tag{3.1}$$

where $s_1 < s_2$, $t_3 = b$, and $t_1 = p_1/s_1$ (provided $p_1/s_1 \geq a$) and $t_2 = (p_2 - p_1)/(s_2 - s_1)$ are the segment boundaries of product 1. Again, maximizing with respect to (p_1, p_2) yields marginal-cost pricing as the efficient pricing rule. And then maximizing with respect to (s_1, s_2) yields the following solution:[5]

$$s_1 = b/5\alpha, \qquad s_2 = 2b/5\alpha,$$
$$p_1 = b^2/25\alpha, \qquad p_2 = 4b^2/25\alpha,$$
$$m_1 = 2b/5(b - a), \qquad m_2 = 2b/5(b - a),$$
$$\text{Total surplus} = 0.08b^3/\alpha.$$

This solution is feasible in the sense of the market not being covered if and only if $b \geq 5a$. If $b < 5a$, then it is efficient to cover the market and the solution then is $s_1 = (b + 3a)/8\alpha$, $s_2 = (3b + a)/8\alpha$. (The two solutions are continuous at $b = 5a$.)

The efficient solution with two products must balance three considerations: "giving every consumer what she wants," the marginal cost of quality, and serving as many consumers as possible. These trade-offs encompass two notions of efficiency: product efficiency and coverage efficiency. The first refers to *how well* we serve the consumers we choose to serve. For example, if we choose to serve consumer type t, then we should provide her with $s^*(t)$ to maximize product efficiency because that is the product that maximizes t's consumer surplus under marginal-cost pricing. Coverage efficiency refers

[5] The first-order conditions have two "roots," the one in the text, and $s_1 = b/3\alpha$, $s_2 = 2b/3\alpha$. But the latter root is not a solution because it has $t_2 = b$, implying zero market share for the higher quality product.

to *how many* consumers we choose to serve; to maximize coverage efficiency we should serve as many consumers as possible. If there had been no constraint on the number of products which could be offered, the efficient solution would have been the *interval* $[s^*(a), s^*(b)]$ and both types of efficiency would have been maximized. Every consumer would have been served—maximizing coverage efficiency—and every served consumer would have received her efficient product—maximizing product efficiency. But once we introduce the two-products constraint, the efficient solution is forced to trade-off between coverage efficiency and product efficiency when the market is diverse (when $b \geq 5a$). Only two types of consumers get their efficient product—the types $2b/5$ and $4b/5$ (remember type t's efficient product is $t/2\alpha$)—and the segment $[a, b/5]$ is left unserved. The first effect is not surprising: with two products you cannot possibly serve a continuum of consumer types efficiently. But why does the efficient solution also leave the segment $[a, b/5]$ unserved when $b \geq 5a$ (and only when $b \geq 5a$)? The reason is, the "higher" types provide more total surplus when served efficiently than the "lower" types. (Recall the earlier comment that $ts^*(t) - \alpha[s^*(t)]^2$ is increasing in t.) And there are more "higher" types when b is large than when it is small. So eventually, as b goes beyond $5a$, aggregate efficiency is increased by serving the "higher" types better, even if it means not serving some "lower" types at all.

The Monopolist's Solution

The monopolist's one-product market-not-covered solution is to sell $b/3\alpha$ at price $2b^2/9\alpha$; cf. Moorthy (1984). This solution is feasible if and only if $b \geq 3a/2$. The monopolist's one-product market covered solution is to sell $a/2\alpha$ at price $a^2/2\alpha$. (The two solutions are continuous at $b = 3a/2$.) With two products, price-discrimination considerations come into play, and with that comes the problem of cannibalization. The two-products monopoly problem with the market not covered can be stated as follows:

$$\max_{\substack{s_1,s_2\\ p_1,p_2}} \sum_{i=1}^{2} (t_{i+1} - t_i)(p_i - \alpha s_i^2)$$

where $t_3 \equiv b$, $t_2 = (p_2 - p_1)/(s_2 - s_1)$ and $t_1 = p_1/s_1$ (if $p_1/s_1 \geq a$). The solution is:

$$\begin{aligned} s_1 &= b/5\alpha, & s_2 &= 2b/5\alpha,\\ p_1 &= 3b^2/25\alpha, & p_2 &= 7b^2/25\alpha,\\ m_1 &= b/5(b-a), & m_2 &= b/5(b-a),\\ p_1 - \alpha s_1^2 &= 2b^2/25\alpha, & p_2 - \alpha s_2^2 &= 3b^2/25\alpha, \end{aligned}$$

$$\text{Consumer surplus} = 0.02b^3/\alpha.$$

This solution is feasible (in the sense of the market not being covered) if and only if $b \geq 5a/3$ because $t_1 = 3b/5 \geq a$ if and only if $b \geq 5a/3$. If $b < 5a/3$, then we must set $t_1 = a$ in the maximization problem, and then the solution is $s_1 = (3a - b)/4\alpha$, $s_2 = (a + b)/4\alpha$. (The two solutions are continuous at $b = 5a/3$.) Note that the transition from the market-covered solution to the market-not-covered solution happens sooner—at a lower b level—in the monopolist's solution than in the efficient solution. This is because price levels are higher when a firm is maximizing profits than when it is maximizing total surplus. In the monopolist's solution, prices exceed marginal cost; in the efficient solution, prices equal marginal cost. Thus, as b increases, s_1 and s_2 increase—as in the efficient solution—but the concomitant increase in prices makes p_1 exceed as_1 sooner than in the efficient solution. Another way to put this is to say that the monopolist's solution favors product efficiency over coverage efficiency even more so than the efficient solution. This is because the lower consumers do not give the monopolist much profit (see below) *and* serving them entails keeping the lower-quality product's price below as_1, reducing his ability to extract surplus from the more lucrative higher segments. Thus, even when maximum efficiency requires incomplete coverage—when $b \geq 5a$—the "efficient" segment to leave, $[a, b/5]$, is a subset of what the monopolist chooses to leave, $[a, 3b/5]$.

The monopoly and efficient solutions (in the market-not-covered case) share some properties, but there are some differences as well:

1. In both solutions, a doesn't appear in the prices or the products. This is because consumers in the segment $[a, b/5]$ choose the substitute, not s_1 or s_2.

2. In the efficient solution, the lower quality product is the efficient product of the median consumer in that product's market segment and the higher quality product is the efficient product of the median consumer in the higher quality product's market segment. That is, $s_1 = s^*(2b/5)$ where $2b/5$ is the median consumer in s_1's market segment, and $s_2 = s^*(4b/5)$ where $4b/5$ is the median consumer in s_2's market segment.

3. The monopolist's products are positioned exactly where the efficient products are. Contrast this with the market-covered case (when $b < 5a/3$), when the monopolist's products are further apart than the efficient ones. The reason for the difference is that in the market-not-covered case, cannibalization-minimization considerations must be traded off against increased competition with the substitute. (In the market-covered case, only cannibalization-minimization considerations apply.) If the lower-quality product were too close to the substitute, then competition with the substitute (which is priced at zero) would send prices down throughout the product line. These two considerations exactly offset each other in our model, and the monopolist ends up placing his products efficiently when he doesn't cover the market.[6]

4. Despite the above, the monopolist's solution is less efficient than the efficient solution because its coverage efficiency is lower.

5. In the efficient solution, even though each product's market segment is of the same size, because efficient total surplus increases with consumer type, the segment served by the higher quality product produces greater total surplus than the segment served by the lower quality product. In the context of the monopoly solution this leads to the result that the profit on the higher quality product is higher than the profit on the lower quality product. Thus, the common intuition that higher quality products carry higher margins and generate greater profits is seen as a straightforward consequence of the fact that higher quality products serve higher consumer types, and higher types bring with them higher efficient total surplus. Put another way, the monopolist's higher-quality product does better than his lower-quality product because the former is closer to the best *single* product location for the monopolist—$b/3\alpha$—than the latter. The best single product location for the monopolist is the place on the product spectrum that best reconciles consumer preferences for quality, the marginal cost of quality, and competition with the substitute. To the extent that a product is closer to this coveted product location than another, it does better.

6. As b increases—as the average marginal willingness to pay for quality increases in the market—the efficient products increase in quality, and correspondingly, the monopolist's products also increase in quality. In the monopoly solution this leads to higher margins and profit contributions for both products; in the efficient solution it leads to higher total surplus for both products.

7. As α increases, both products decrease in quality and they get closer to each other. This is not surprising because as α increases, the marginal cost function rises, and its

[6] This exact balance between the effects of cannibalization and the effects of competition with the substitute is an artifact of the linear and uniformly distributed preferences in my model. In Moorthy (1984) I have shown that the monopolist's products can be higher or lower in quality than the efficient ones if preferences are nonlinear or if the distribution of preferences is nonuniform.

slope increases. In the monopoly solution this leads to lower margins and profits for both products; in the efficient solution it leads to lower total surplus for both products.

Having set up two benchmark solutions in which there is only one decision-maker, I now proceed to investigate the effect of competition between two decision-makers.

4. The Price Equilibrium

I begin the analysis of competitive product positioning by computing the price equilibrium. This price equilibrium will apply to the product equilibrium in §5, where both firms choose their products simultaneously, and to the product equilibrium in §6, where one firm chooses its product before the other firm. But before I get into the computation of the price equilibrium, I want to explain the equilibrium concepts being used.

Suppose we are in the simultaneous-product-choice model. In this model, the competition between the firms occurs in two stages. In the first stage, each firm chooses a product quality, simultaneously with the other firm, and fixes it. In the second stage, each firm, having observed the other's product, chooses a price for its product, simultaneously with the other firm. The simultaneous choice of product quality means that there is no product leader in this market—neither firm knows the other's product quality when choosing its own. Similarly, the simultaneous choice of price signifies the absence of a price leader—neither firm is aware of the other's price when choosing its price. But why do we have two stages—why are prices being chosen *after* the products? Because product choices are more permanent than price choices—price competition often takes place under conditions where products cannot be changed. Also, the two-stage modeling enables the existence of a (pure-strategy) equilibrium, when none would exist if products and prices were chosen simultaneously; cf. Stokey (1980).

Given this structure of moves, how do we define an equilibrium here? We do it in two steps, starting with the price equilibrium. The Nash equilibrium in prices is simply a price for firm 1 and a price for firm 2 such that neither firm wishes to choose a different price unilaterally; cf. Moorthy (1985). Necessarily, this price equilibrium will be a function of the products chosen in the first stage. What products will the firms choose? Each firm's product choice depends on (1) what it thinks the other firm's product will be, (2) what price it expects to charge as a function of its product and the other firm's product, and (3) what it expects the other's price function to be. The natural candidate for these price expectations, and the one supported by the subgame-perfectness criterion from game theory (Moorthy 1985), is the price equilibrium described above.[7] In other words, when choosing its product, a firm must say: "If I choose product s_1 and firm 2 chooses product s_2, then the prices we charge later will be the prices in the price equilibrium corresponding to s_1 and s_2." A (subgame-perfect) product equilibrium, then, is a product for firm 1 and a product for firm 2 such that neither firm would choose a different product unilaterally, recognizing that the profitability of all product selections will be determined on the basis of the price equilibrium that follows.

In the sequential product choice model in §6 the price equilibrium continues to be defined as above. The only thing that changes is the product equilibrium. One firm chooses its product *before* the other firm here. The firm which chooses second—the second entrant—*reacts* to the first entrant's product, so his product strategy is a function of the first entrant's product. A (subgame-perfect) product equilibrium in this

[7] Essentially, the subgame-perfectness criterion says that an equilibrium in the first stage should not be sustained by second-stage strategies that are not themselves in equilibrium in the second-stage game. First-stage equilibria which do not satisfy the subgame-perfectness criterion are being sustained by spurious second-stage threats—threats which the competitors will not find in their self-interest to carry out once they are in the second stage.

model is therefore a product for the first entrant and a product *function* for the second entrant such that the first entrant would not choose a different product unilaterally and the second entrant would not choose a different product function unilaterally, recognizing that the profitability of all product selections will be determined on the basis of the price equilibrium that follows.

It is obvious from the definitions above that I must first compute the price equilibrium for given product positions and only then compute the product equilibrium. Let s_1 be firm 1's product and s_2 be firm 2's product. (At this point I am not concerned about how these products were chosen. For the price equilibrium, only the product positions matter, not the manner in which they were chosen.) If $s_1 = s_2$ and $p_1 \leq p_2$, then the lower boundary of firm 1's market segment will be p_1/s_1 if p_1/s_1 is greater than a; cf. Proposition 1. Hence, for $s_1 = s_2$, firm 1's market share will be $[b - (p_1/s_1)]/(b - a)$ if $as_1 \leq p_1 < p_2 \leq bs_1$, $(1/2)[b - (p_1/s_1)]/(b - a)$ if $as_1 \leq p_1 = p_2 \leq bs_1$, and zero if $p_1 > p_2$. Its profit function, $\Pi(p_1, p_2; s_1 = s_2)$, will be:[8]

$$\begin{cases} [b - (p_1/s_1)](p_1 - \alpha s_1^2), & \text{if} \quad as_1 \leq p_1 < p_2 \leq bs_1, \\ (1/2)[b - (p_1/s_1)](p_1 - \alpha s_1^2), & \text{if} \quad as_1 \leq p_1 = p_2 \leq bs_1, \\ 0, & \text{if} \quad p_1 > p_2. \end{cases}$$

Firm 2's profit function is identical to firm 1's. Note that the profit function is discontinuous along the ray $p_1 = p_2$ when $s_1 = s_2$ and $p_2 \leq bs_2$. This discontinuity is a consequence of the assumption that consumers have perfect information about the product qualities and prices of the two firms. With such perfect information, if two firms have the same quality, then consumers choose between them only on the basis of price. If a firm raises its price above its competitor's by even a small amount, it can go from half the total market to no market.

Now let us consider the case $s_1 \neq s_2$. When $s_1 < s_2$, the lower boundary of firm 1's market segment is p_1/s_1 and its upper boundary is $(p_2 - p_1)/(s_2 - s_1)$ if $b \geq (p_2 - p_1)/(s_2 - s_1) \geq p_1/s_1 \geq a$; when $s_1 > s_2$, the lower boundary of firm 1's market segment is $(p_1 - p_2)/(s_1 - s_2)$ and its upper boundary is b if $b \geq (p_1 - p_2)/(s_1 - s_2) \geq p_2/s_2 \geq a$; cf. Proposition 1. Similarly for firm 2. Therefore firm 1's (and firm 2's) profit function when $s_1 \neq s_2$, $\Pi(p_1, p_2; s_1 \neq s_2)$, is:[9]

$$\begin{cases} \left(\dfrac{p_2 - p_1}{s_2 - s_1} - \dfrac{p_1}{s_1}\right)(p_1 - \alpha s_1^2), & \text{if } s_1 < s_2 \text{ and } a \leq p_1/s_1 \leq (p_2 - p_1)/(s_2 - s_1) \leq b, \\ \left(b - \dfrac{p_1 - p_2}{s_1 - s_2}\right)(p_1 - \alpha s_2^2), & \text{if } s_1 > s_2 \text{ and } a \leq p_2/s_2 \leq (p_1 - p_2)/(s_1 - s_2) \leq b. \end{cases}$$

Note that the profit function is: (1) continuous and differentiable in (p_1, p_2) everywhere in the region described, (2) increasing in p_2, and (3) strictly concave in p_1.

Now we compute the price equilibrium. The case $s_1 = s_2$ is straightforward.

PROPOSITION 2. *When both firms have the same product, the only price equilibrium is for each firm to price at marginal cost. Thus, if both firms choose the same product, both firms make zero profits in the price equilibrium.*

So when $s_1 = s_2$, the only price equilibrium is $(\alpha s_1^2, \alpha s_1^2)$. For this price equilibrium to not cover the market and for it to generate non-negative market shares for the two firms, we must have $as_1 \leq \alpha s_1^2 \leq bs_1$, that is, $s_1 = s_2 \in [s^0(a), s^0(b)]$.

[8] We do not give the entire profit function here, only the region where the market is not covered by the two firms and where firm 1's price does not exceed bs_1.

[9] The profit function only covers the region where both firms have nonnegative market shares and the market is not covered.

Now consider the case $s_1 < s_2$. For $i = 1, 2$, firm i's maximization problem, when the market is not covered, is:

$$\max_{p_i} (t_{i+1} - t_i)(p_i - \alpha s_i^2), \qquad \text{subject to: } p_1 \geq a s_1,$$

where t_1, the lower market boundary of firm 1, is p_1/s_1, t_2, the upper (lower) market boundary of firm 1 (firm 2) is $(p_2 - p_1)/(s_2 - s_1)$ if $t_2 \in [t_1, b]$, and t_3, the upper market boundary of firm 2, is b. The first-order conditions for a price equilibrium with $a \leq t_1 \leq t_2 \leq b$ are given by:

$$p_1 - \alpha s_1^2 = \left(\frac{p_2 - p_1}{s_2 - s_1} - \frac{p_1}{s_1}\right)\left(\frac{s_1}{s_2}\right)(s_2 - s_1), \tag{4.1}$$

$$p_2 - \alpha s_2^2 = \left(b - \frac{p_2 - p_1}{s_2 - s_1}\right)(s_2 - s_1). \tag{4.2}$$

The condition (4.1) arises from firm 1's maximization problem; condition (4.2) arises from firm 2's maximization problem. Each firm conjectures the other's price and chooses its price to maximize its profits. Conditions (4.1) and (4.2) are nothing but statements of the tradeoffs between margin and market share that each firm must make in order to maximize profits. Rearranging the terms in (4.1) and (4.2) we see that each firm's best price is the average of its marginal cost and the price that gives all of the market to its opponent (assuming the latter price is greater than marginal cost):[10]

$$p_1 = ((s_1/s_2)p_2 + \alpha s_1^2)/2, \tag{4.3}$$

$$p_2 = (p_1 + b(s_2 - s_1) + \alpha s_2^2)/2. \tag{4.4}$$

Strategic price competition between the firms has not entered the picture yet. Effectively, each firm is taking the other firm as a fixed-price substitute.[11] For example, (4.3) can be thought of as giving the best price of a *monopolist* located at s_1 and facing the fixed-price substitute (s_0, p_0) at one end and (s_2, p_2) at the other. Similarly, (4.4) can be thought of as giving the best price of a monopolist located at s_2 and facing the fixed-price substitute (s_1, p_1) at its lower end (but no substitute at its upper end). To go from this passive view of competition to strategic competition we need to solve (4.3) and (4.4) simultaneously for the (candidate) price equilibrium. *It is this solution process that introduces strategic considerations into the competition.* Instead of thinking of the competitor as a fixed-price substitute, each firm is considering the competitor's best response to its price, and then checking whether its price is still optimal given the competitor's response. The candidate price equilibrium is:

$$p_1 - \alpha s_1^2 = \left[\frac{s_1(s_2 - s_1)}{(4s_2 - s_1)}\right][b + \alpha(s_2 - s_1)], \tag{4.5}$$

$$p_2 - \alpha s_2^2 = \left[\frac{2s_2(s_2 - s_1)}{(4s_2 - s_1)}\right][b - \alpha(s_2 + s_1/2)]. \tag{4.6}$$

This candidate equilibrium satisfies the condition that if firm 2 chooses its price according to (4.6), then firm 1 will not deviate from the price given by (4.5), and similarly if firm 1 chooses according to (4.5), then firm 2 won't deviate from the price in (4.6). But two more conditions need to be satisfied before we can call this a price equilibrium:

1. The market is not covered, i.e., $p_1/s_1 \geq a$. This translates to

$$(b - a)(s_2 - s_1) - (as_2 - \alpha s_2^2) - 2s_2(a - \alpha s_1) \geq 0. \tag{4.7}$$

[10] In the language of "best-response functions," (4.3) is firm 1's best-response function and (4.4) is firm 2's best-response function. Note that when one firm raises its price, the other's best response is to raise its price as well. The two firms are strategic complements; cf. Bulow, Geanakoplos and Klemperer (1985).

[11] This is in addition to the substitute (s_0, p_0) already in the model.

2. The two firms' market shares, which are proportional to $(p_2 - p_1)/(s_2 - s_1) - p_1/s_1$ and $b - (p_2 - p_1)/(s_2 - s_1)$, are nonnegative, i.e., $p_1/s_1 \leq (p_2 - p_1)/(s_2 - s_1) \leq b$. It turns out that (4.5) and (4.6) always satisfy $p_1/s_1 < (p_2 - p_1)/(s_2 - s_1)$. The remaining part of this condition translates to:[12]

$$s_2 + s_1/2 \leq s^0(b). \tag{4.8}$$

We shall call the region covered by conditions (4.7) and (4.8) R. That is,

$$R \stackrel{\text{def}}{=} \{(s_1 < s_2) : (4.7) \text{ and } (4.8) \text{ are satisfied}\}.$$

R is nonempty. For example, when $b = 3a$, R is as in Figure 2.

The price equilibrium in R is given by (4.5) and (4.6). And the entire price equilibrium when both firms are in the market and the market is not covered is given by:[13]

$$p_1 - \alpha s_1^2 = \begin{cases} \left[\dfrac{s_1(s_2 - s_1)}{(4s_2 - s_1)}\right][b + \alpha(s_2 - s_1)], & \text{if} \quad s_1 < s_2, \\ 0, & \text{if} \quad s_1 = s_2, \\ \left[\dfrac{2s_1(s_1 - s_2)}{(4s_1 - s_2)}\right][b - \alpha(s_1 + s_2/2)], & \text{if} \quad s_1 > s_2. \end{cases}$$

Both firms' equilibrium price strategies are the same. When firm 1 is the lower-quality firm its (price equilibrium) margin is given by (4.5) and when it is the higher-quality firm its (price equilibrium) margin is given by (4.6) with s_2 and s_1 interchanged. And when $s_1 = s_2$, the equilibrium margin is zero. Figure 3a shows how firm 1's equilibrium margin changes with s_1 when $s_2 = 0.4b/\alpha$. When $s_1 = 0$, the equilibrium margin is zero because no consumer is willing to pay anything for a product of zero quality; when $s_1 = s^0(b) - s_2/2$, it is zero because now the quality is too high; when $s_1 = s_2$, the equilibrium margin is zero because of price competition.

The firms' equilibrium market shares are:

$$m_1 = \begin{cases} \left(\dfrac{s_2}{4s_2 - s_1}\right)\left(\dfrac{b + \alpha(s_2 - s_1)}{b - a}\right), & \text{if} \quad s_1 < s_2, \\ (1/2)\left(\dfrac{b - \alpha s_1}{b - a}\right), & \text{if} \quad s_1 = s_2, \\ \left(\dfrac{2s_1}{4s_1 - s_2}\right)\left(\dfrac{b - \alpha(s_1 + s_2/2)}{b - a}\right), & \text{if} \quad s_1 > s_2. \end{cases}$$

Firm 1's equilibrium market share is proportional to $[s_2/(s_1(s_2 - s_1))](p_1 - \alpha s_1^2)$ when it is the lower quality firm, it is proportional to $[1/(s_1 - s_2)](p_1 - \alpha s_1^2)$ when firm 1 is the higher quality firm, and it is proportional to $(1/2)(b - \alpha s_1)$ when $s_1 = s_2$. Figure 3b shows how m_1 changes with s_1 when $s_2 = 0.4b/\alpha$.

[12] This condition is different from the similar condition when $s_1 = s_2$. Then, both firms' equilibrium market shares were positive if and only if $s_2 < s^0(b)$. But now if $s_2 > s^0(b) - s_1/2$, then $m_2 = 0$. Why the difference? The reason is, when $s_1 < s_2$, firm 2's equilibrium price is greater than what it would be if $s_1 = s_2$—in the former case it is greater than marginal cost, in the latter case it is equal to marginal cost. So firm 2's equilibrium price exceeds bs_2 sooner—i.e., at a lower quality level—than it would if firm 2's product were identical to firm 1's.

[13] For the record, the price equilibrium when both firms are in the market and the market is covered is: $p_1 = [(b - 2a)(s_2 - s_1) + 2\alpha s_1^2 + \alpha s_2^2]/3$, $p_2 = [(2b - a)(s_2 - s_1) + 2\alpha s_2^2 + \alpha s_1^2]/3$.

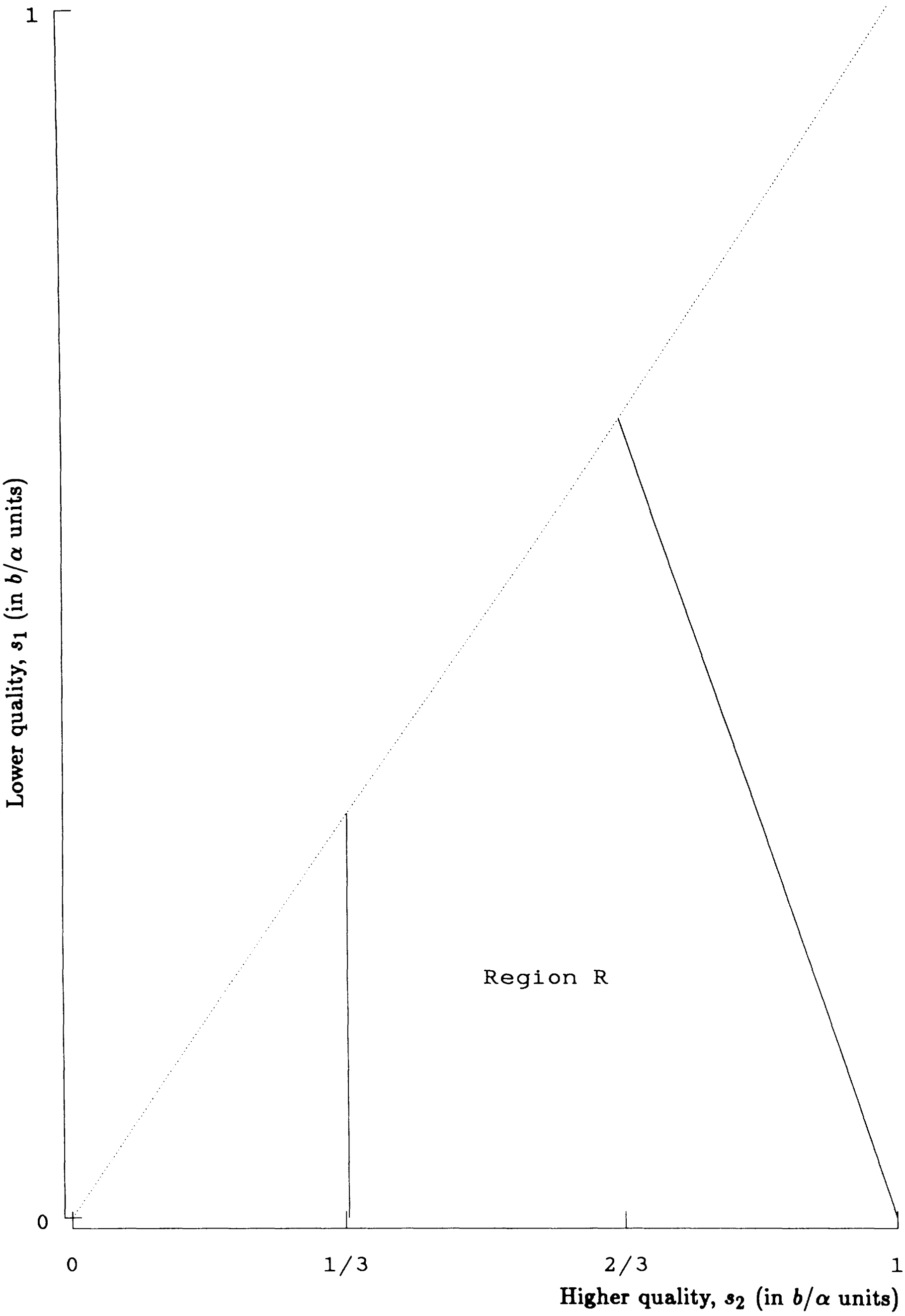

FIGURE 2. The Feasible Product Region R for $b = 3a$.

Finally, each firm's equilibrium profits are:

$$\Pi(s_1, s_2) = \begin{cases} s_1 s_2(s_2 - s_1)\left[\dfrac{b + \alpha(s_2 - s_1)}{4s_2 - s_1}\right]^2, & \text{if} \quad s_1 < s_2, \\ 0, & \text{if} \quad s_1 = s_2, \\ (s_1 - s_2)\left[\dfrac{2bs_1 - \alpha s_1^2 - s_1(s_1 + s_2)}{(4s_1 - s_2)}\right]^2, & \text{if} \quad s_1 > s_2. \end{cases} \tag{4.9}$$

Figure 3c shows firm 1's profit function for $s_2 = 0.4b/\alpha$. The profit function is bimodal. When s_2 is "large"—as it is in Figure 3c—firm 1 prefers the lower mode, but we shall see in the next section that when s_2 is "small" firm 1 prefers the higher mode.

We close this section by noting the following properties of the price equilibrium:

1. It is unique. So there is no ambiguity about the profits each firm can expect in the price equilibrium.

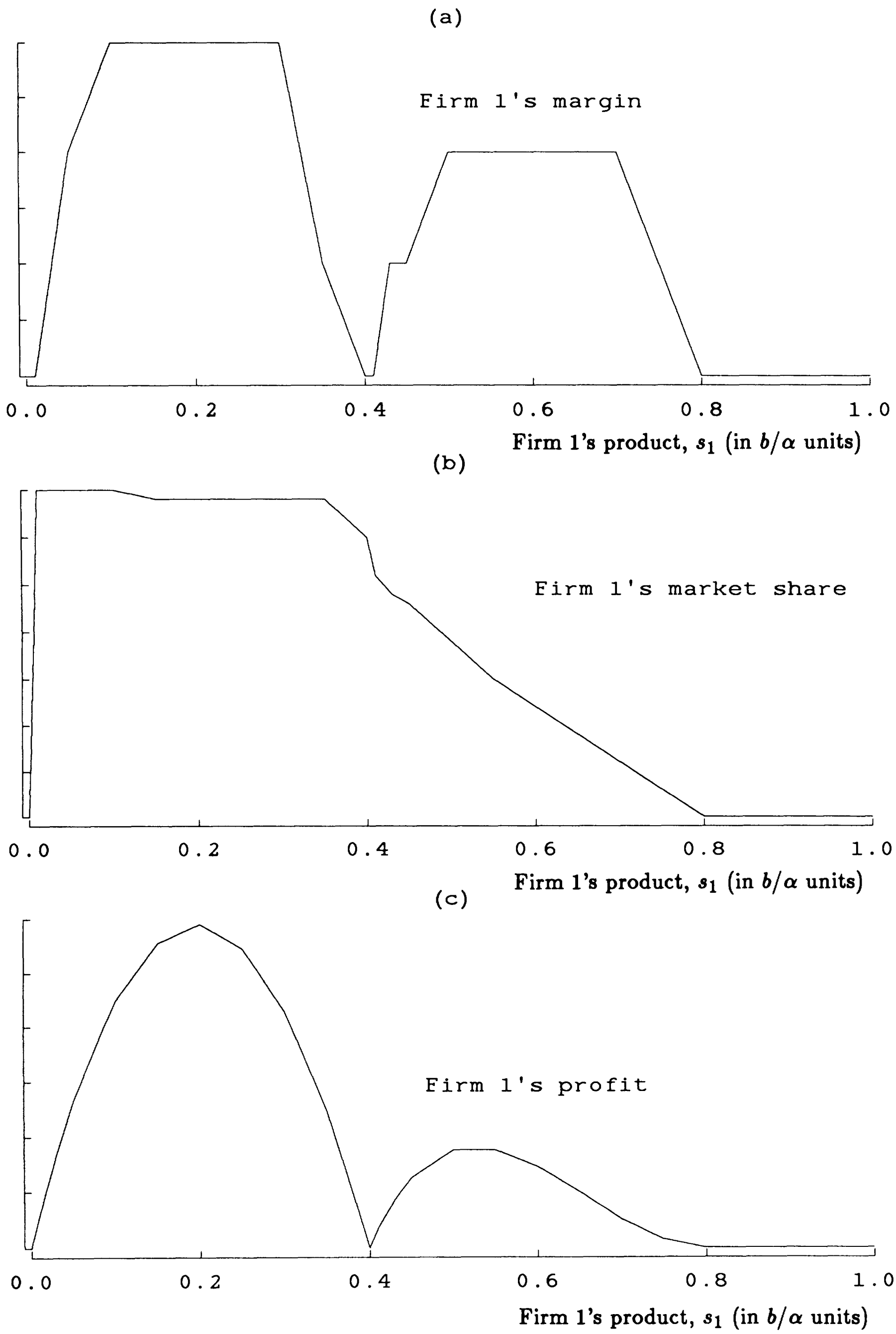

FIGURE 3. The Price Equilibrium with $b = 3a$, $s_2 = 0.4b/\alpha$.

2. For $(s_1, s_2) \in R$, each firm's market share is $2s_2/(4s_2 - s_1)$ times the market share it can guarantee for itself—the market share when it prices at marginal cost and its opponent responds optimally. This fraction, $2s_2/(4s_2 - s_1)$, is a measure of the market share each firm is willing to sacrifice in equilibrium in order to obtain a positive margin.

3. The equilibrium margins are positive, but tend to zero as $s_1 \rightarrow s_2$. Therefore, the equilibrium is continuous at $s_1 = s_2$.

5. The Product Equilibrium with Simultaneous Choices

In the previous section we computed the market-not-covered price equilibrium for given product positions. In this section we compute the product equilibrium, when the two firms choose their products simultaneously anticipating the price equilibrium. From Figure 3 it is obvious that neither firm will ever choose a product of zero quality or a product of quality greater than or equal to $s^0(b)$. It is also obvious that neither firm would choose the same product quality as its opponent. These observations we state as a proposition.

PROPOSITION 3. *Regardless of where the other firm is located, each firm's best product strategy is to choose a distinct product from the interval* $(0, s^0(b))$.

Now we compute the product equilibrium. We do this in two steps. First we *fix* the firms' product *ordering*—we don't allow them to "jump" over each other—and examine the product choices of the lower-quality firm and the higher-quality firm. This gives us a *local* product equilibrium, (s_l^*, s_h^*)—local equilibrium in the sense that if, say firm 2, chooses s_h^*, then firm 1 would find it optimal to choose s_l^* under the constraint that its quality must be lower than firm 2's and, similarly, if firm 1 chooses s_l^*, then firm 2 would find it optimal to choose s_h^* under the constraint that firm 2 must be the higher quality firm. (Needless to say, every product equilibrium must also be a local product equilibrium.) In the second step we remove the product ordering constraint and verify that neither firm wishes to change its position in the ordering unilaterally. That is, if (say) firm 1 chooses s_h^* then firm 2 would choose s_l^* even though it could have chosen a quality greater than firm 2's and, similarly, if firm 2 chooses s_l^*, then firm 1 would choose s_h^* even if it could have chosen a quality lower than firm 2's.

The lower-quality firm's optimal product satisfies the following first-order condition (obtained by differentiating the first part of the profit function (4.9) with respect to s_1):

$$(s_h - 2s_l)(4s_h - s_l)(b + \alpha(s_h - s_l)) + 2s_l(s_h - s_l)(b - 3\alpha s_h) = 0. \tag{5.1}$$

The higher-quality firm's optimal product satisfies the following first-order condition (obtained by differentiating the last part of (4.9) with respect to s_1):

$$s_h(4s_h - s_l)(b - \alpha(s_h + s_l/2)) + (s_h - s_l)(\alpha s_l^2 - 2s_l(b - 2\alpha s_h) - 8\alpha s_h^2) = 0. \tag{5.2}$$

For any s_h, (5.1) gives the lower-quality firm's best-response function, $s_l(\cdot)$, if $(s_l(s_h), s_h) \in R$ and for any s_l, (5.2) gives the higher-quality firm's best-response function, $s_h(\cdot)$, if $(s_l, s_h(s_l)) \in R$. Figure 4 shows these best-response functions. As the lower-quality firm "moves up" in quality, the higher-quality firm moves up in quality, and as the higher-quality firm moves up in quality, the lower-quality firm moves up as well (remember that at this point neither firm can "jump" over the other). A local product equilibrium is a pair of products (s_l^*, s_h^*) such that $s_l(s_h^*) = s_l^*$ and $s_h(s_l^*) = s_h^*$. In other words, $s_l^* = s_l(s_h^*) = s_h^{-1}(s_h^*)$ where s_h^{-1} is the inverse of the higher-quality firm's best-response function (for example, $s_h^{-1}(s_h^*)$ is the lower-quality firm's product for which the higher-quality firm's best-response is s_h^*). Figure 4 shows $s_l(\cdot)$ and $s_h^{-1}(\cdot)$ and their intersection, the local product equilibrium.

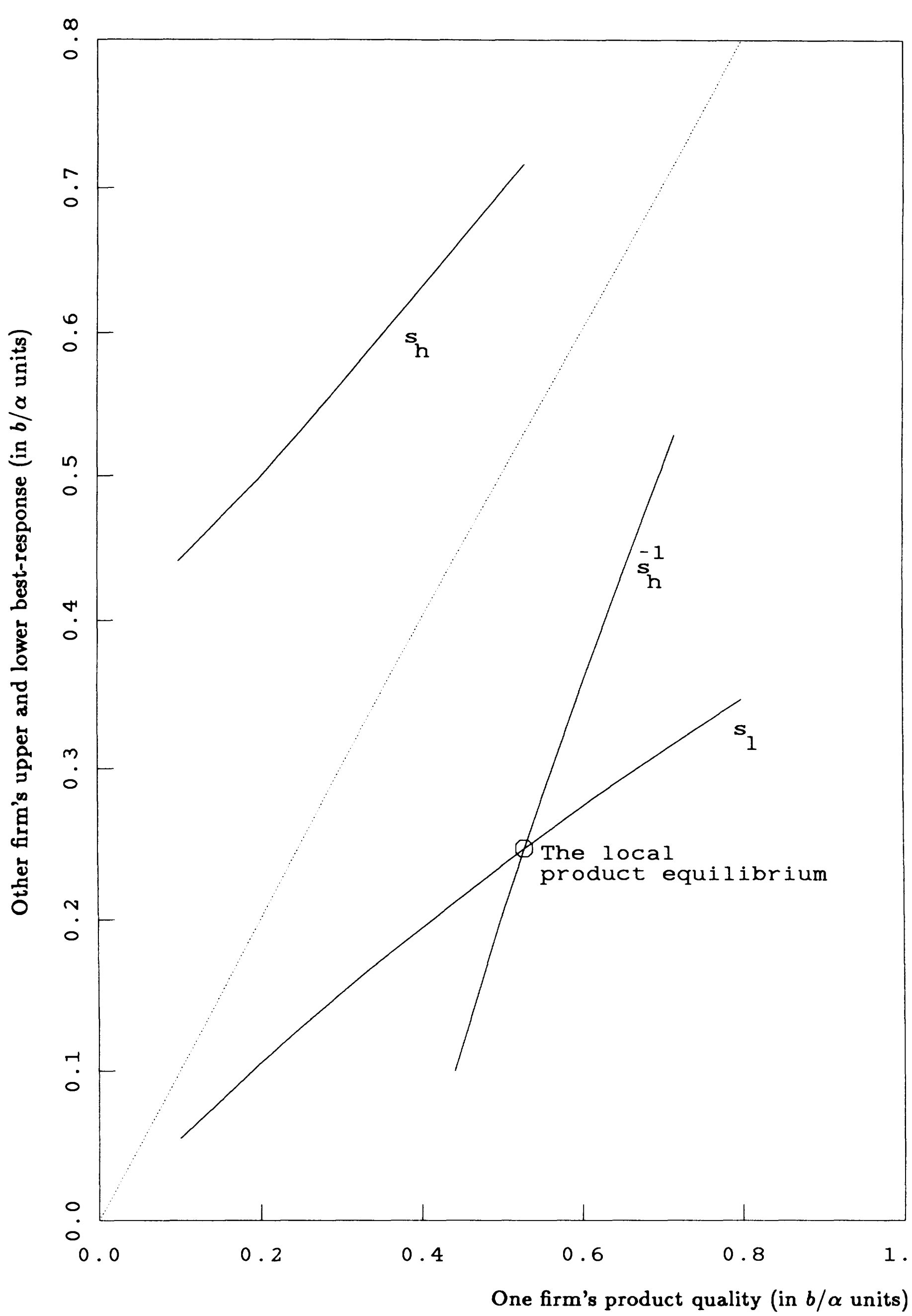

FIGURE 4. The Upper and Lower Best-Response Functions.

The local product equilibrium, which is approximately $(0.2474b/\alpha, 0.5288b/\alpha)$, will be a global product equilibrium—a subgame perfect Nash equilibrium—if the following conditions are satisfied:

1. The market is not covered under the resulting price equilibrium and both firms have nonnegative market shares. That is, the local product equilibrium is in the region R.

2. Neither firm wishes to change its position in the quality ordering unilaterally. That is, if firm 2 has the quality $0.5288b/\alpha$, then firm 1 doesn't want to choose an even higher quality (in preference to $0.2474b/\alpha$) and if firm 1 has the quality $0.2474b/\alpha$, then

firm 2 doesn't want to choose an even lower quality (in preference to $0.5288b/\alpha$). (Of course, the same applies with the identities of the firms interchanged.)

The first condition is satisfied if $b \geq 2.2705a$ approximately. To see whether the second condition is satisfied we examine the firms' *global* best-response function.

If firm 1 is the lower-quality firm then its best-response function is given by $s_l(\cdot)$ whereas when it is the higher-quality firm, then its best-response function is given by $s_h(\cdot)$. Therefore, its *global* best-response function, $s_1(\cdot)$, is given by

$$s_1(s_2) = \begin{cases} s_l(s_2), & \text{if} \quad \Pi(s_l(s_2), s_2) > \Pi(s_h(s_2), s_2), \\ \{s_l(s_2), s_h(s_2)\}, & \text{if} \quad \Pi(s_l(s_2), s_2) = \Pi(s_h(s_2), s_2), \\ s_h(s_2), & \text{if} \quad \Pi(s_l(s_2), s_2) < \Pi(s_h(s_2), s_2). \end{cases}$$

Similarly for firm 2. Figure 5 shows either firm's global best-response function. For $s_2 < 0.2908b/\alpha$, firm 1 prefers to be the higher-quality firm, whereas, when $s_2 > 0.2908b/\alpha$, firm 1 prefers to be the lower-quality firm. For $s_2 = 0.2908b/\alpha$, firm 1 is indifferent between being the higher-quality firm or the lower-quality firm—its profits from $s_l(0.2908b/\alpha)$ and $s_h(0.2908b/\alpha)$ are the same. These results must be contrasted with those in Shaked and Sutton (1982). There, being the higher quality product is always better, regardless of how close the competitor is. The reason for the difference is that in my model marginal costs increase with quality whereas in Shaked and Sutton's model marginal costs are constant in quality. Thus, in my model, when the competitor's quality is "high," choosing a still higher quality would mean high marginal costs (relative to what consumers are willing to pay), high prices, low market shares, and hence low profits. On the other hand, if the competitor's product quality is "low," then being the lower-quality firm entails positioning yourself very close to the competitor or the substitute, and that leads to severe price competition and low profits.

Since $s_l^* < 0.2908b/\alpha$ and $s_h^* > 0.2908b/\alpha$, the local product equilibrium is indeed a global product equilibrium for $b \geq 2.2705a$ (approximately). There are two product equilibria, then, for $b \geq 2.2705a$: $s_1 \approx 0.2474b/\alpha$, $s_2 \approx 0.5288b/\alpha$ and $s_1 \approx 0.5288b/\alpha$, $s_2 \approx 0.2474b/\alpha$.[14] The two equilibria are identical except for the ordering of the firms' products. This reflects the complete symmetry between the two firms in our model.

PROPOSITION 4. *If $b \geq 2.2705a$, then there exist two product equilibria with the market not covered. In one equilibrium firm* 1 *chooses the product* $0.2474b/\alpha$ *and firm* 2 *chooses the product* $0.5288b/\alpha$. *In the other equilibrium firm* 1 *chooses the product* $0.5288b/\alpha$ *and firm* 2 *chooses the product* $0.2474b/\alpha$.[15]

Designating the lower quality firm by the subscript l and the higher quality firm by the subscript h, we can write the equilibrium margins, market shares, and profits of the two firms as follows:

$$p_l \approx 0.109b^2/\alpha, \qquad p_h \approx 0.335b^2/\alpha,$$

$$p_l - \alpha s_l^2 \approx 0.0478b^2/\alpha, \qquad p_h - \alpha s_h^2 \approx 0.0554b^2/\alpha,$$

$$m_l \approx 0.3628b/(b-a), \qquad m_h \approx 0.1968b/(b-a),$$

$$\Pi_l \approx 0.0173b^3/\alpha, \qquad \Pi_h \approx 0.0109b^3/\alpha,$$

$$\text{Consumer surplus} \approx 0.0442b^3/\alpha.$$

[14] In addition to these pure strategy equilibria there is also the following mixed-strategy equilibrium: each firm randomizes between $0.1477b/\alpha$ and $0.5574b/\alpha$ with the probabilities 0.49 and 0.51. The number $0.1477b/\alpha$ is nothing but $s_l(0.2908b/\alpha)$ and $0.5574b/\alpha$ is nothing but $s_h(0.2908b/\alpha)$, and 0.49 and 0.51 are the probabilities which make the "other" firm indifferent between $0.1477b/\alpha$ and $0.5574b/\alpha$.

[15] For the record, the product equilibrium with the market covered—when $b < 9a/5$—is $s_l = (5a - b)/8\alpha$ and $s_h = (5b - a)/8\alpha$.

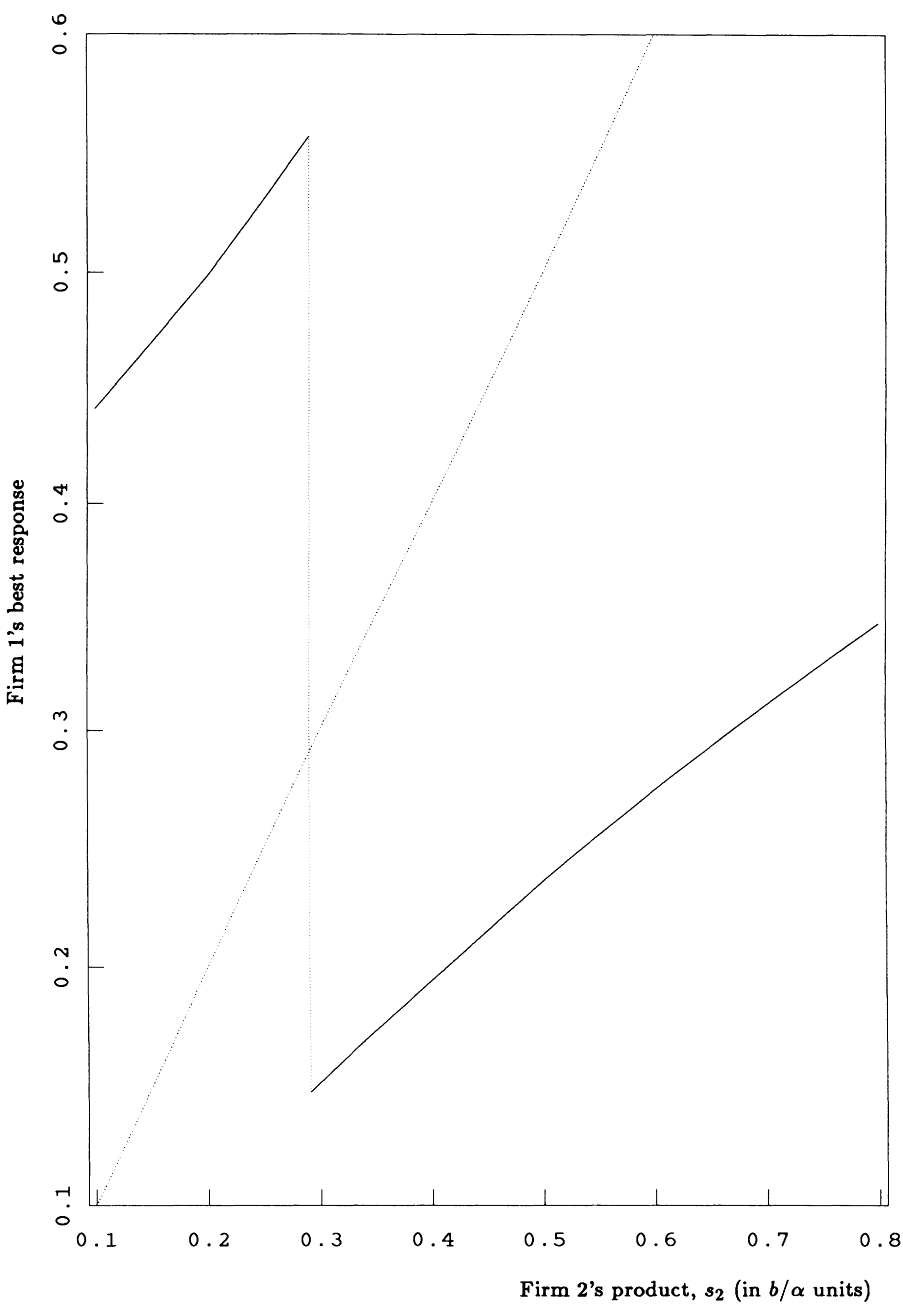

FIGURE 5. The Global Best-Response Function.

Note the following properties of the product equilibrium:

1. The product equilibrium is not efficient. The efficient product locations are $0.2b/\alpha$ and $0.4b/\alpha$, so the equilibrium products are too high in quality. Not only is product efficiency compromised as a result, coverage efficiency is also compromised. The market left uncovered in the product equilibrium is $[a, 0.4404b]$, which is larger than the market left uncovered in the efficient solution, but smaller than the market left uncovered in the monopoly solution. The product equilibrium's coverage efficiency is

higher than the monopolist's coverage efficiency because the margins are lower in the product equilibrium than in the monopolist's solution. These lower margins also lead to greater consumer surplus in the product equilibrium. The reason the products are not placed efficiently in the product equilibrium is that both firms find the price competition excessive at the efficient locations. If the upper firm were to place its product at $0.4b/\alpha$, then Figure 5 tells us that the lower firm would choose a quality *less* than $0.2b/\alpha$, and if the lower firm were at $0.2b/\alpha$, then the upper firm would choose a quality *greater* than $0.4b/\alpha$.

2. The lower firm's equilibrium product is the efficient product of consumer type $0.4948b$, and consumer type $0.4948b$ belongs in the lower firm's market segment. But the upper firm's equilibrium product is *greater* than $b/2\alpha$, the efficient product of type b. That is, the upper firm's product does not meet the preferences of *any* consumer efficiently. Why? Because, by locating outside the efficient interval, the upper firm reduces the price competition with the lower firm, raising both firms' margins. But what about market share? Would any consumers choose an inefficient product in preference to an efficient product? Yes, if both products are sold at positive margins. For example, suppose $s_1 = s^*(t)$ for some $t \in (a, b)$ and $s_2 > s^*(b)$. Then, as long as $ts_2 > \alpha s_2^2$, i.e., $s_2 < s^0(t)$, it is possible to find two prices $p_1 > \alpha s_1^2$ and $p_2 > \alpha s_2^2$ such that $ts_2 - p_2 > ts_1 - p_1$.

3. The lower firm makes greater profits than the upper firm. This result is to be contrasted with our earlier observations that in the efficient solution the higher quality product generates more total surplus than the lower quality product and in the monopolist's solution the higher quality product is more profitable than the lower quality product. The reason for the difference is that in the product equilibrium the upper firm has moved further away from the best single monopoly position—$b/3\alpha$—than has the lower firm. (The distance between $0.5288b/\alpha$ and $b/3\alpha$ is greater than the distance between $0.2474b/\alpha$ and $b/3\alpha$.) In contrast, in the monopoly solution, the higher quality is closer to $b/3\alpha$ than the lower quality. Because of this property, if one firm got first shot at choosing the equilibrium it would play, then it will choose the equilibrium where it is the lower-quality firm. *This* "first-mover" advantage is worth $0.0064b^3/\alpha$ ($=0.0173b^3/\alpha - 0.0109b^3/\alpha$) in profits.

4. As b increases, both products increase in quality, but the gap between them also increases. The increase in the firms' qualities is due to the increase in average willingness to pay and the increase in the distance between the products is due to the increase in $b - a$.

5. As α increases, both products decrease in quality, the gap between them decreases, and profits decrease.

6. Sequential Entry

We now examine the case where one firm chooses its product *before* the other firm. Price choices are still simultaneous, as in §4. In other words, first one firm chooses its product and fixes it; the other firm observes the first firm's product and then chooses its product; finally, each firm chooses its price, not knowing its competitor's price. This model of competition is similar to the ones in Prescott and Visscher (1977), Rao (1977), and Lane (1980), and it depicts the way competition evolves in most industries. First one firm enters, and then the competition comes in. There is a period in which the first entrant enjoys monopoly status, but that period is not being modeled here explicitly because our focus is on the firms' product choices, not the timing of entry. We particularly want to understand how product choice is affected by the first entrant *anticipating* the reactions of future entrants. What are the rewards to having foresight? Is there a first-mover advantage?

Suppose firm 1 enters first but it doesn't anticipate the later entry of firm 2. Given that firm 1 thinks it is going to be the only firm in the industry, it is going to choose the best single product for a monopolist: $b/3\alpha$. Later, firm 2 enters, finds firm 1 located at $b/3\alpha$, and chooses $b/6\alpha$, the best response to $b/3\alpha$ per Figure 5.[16] From the profit function computed earlier it is easy to calculate that firm 1's profits will be $b^3/54\alpha$ and firm 2's profits will be $b^3/108\alpha$.

Now suppose firm 1 anticipates the later entry of firm 2 and that the eventual price equilibrium will not cover the market. It must then reason as follows: "If I choose a product with $s_1 < 0.2908b/\alpha$, then firm 2 will choose $s_h(s_1)$ and become the higher quality firm. On the other hand if I choose $s_1 > 0.2908b/\alpha$, then firm 2 will choose $s_l(s_1)$ and become the lower quality firm. With $s_1 = 0.2908b/\alpha$, firm 2 is indifferent between $s_h(s_1)$ and $s_l(s_1)$, but I may assume that he will make the choice that is better for me."[17] Firm 1's profit function is therefore:

$$\Pi_1 = \begin{cases} \Pi(s_1, s_h(s_1)), & \text{if} \quad s_1 < 0.2908b/\alpha, \\ \max\{\Pi(s_1, s_h(s_1)), \Pi(s_1, s_l(s_1))\}, & \text{if} \quad s_1 = 0.2908b/\alpha, \\ \Pi(s_1, s_l(s_1)), & \text{if} \quad s_1 > 0.2908b/\alpha. \end{cases}$$

Figure 6 depicts firm 1's profit function. There is a discontinuity at $s_1 = 0.2908b/\alpha$—the first entrant's profits jump up as he goes from being the lower-quality firm to being the higher-quality firm. Firm 1's profits are maximized at $s_1 = 0.2908b/\alpha$ (with firm 2 responding with $s_2 = 0.1477b/\alpha$)!

So with sequential entry, the equilibrium is:

$$\begin{aligned} s_1 &\approx 0.2908b/\alpha, & s_2 &\approx 0.1477b/\alpha, \\ p_1 &\approx 0.1367b^2/\alpha, & p_2 &\approx 0.0456b^2/\alpha, \\ p_1 - \alpha s_1^2 &\approx 0.0521b^2/\alpha, & p_2 - \alpha s_2^2 &\approx 0.0238b^2/\alpha, \\ m_1 &\approx 0.3639b, & m_2 &\approx 0.3273b, \\ \Pi_1 &\approx 0.0189b^3/\alpha, & \Pi_2 &\approx 0.0078b^3/\alpha, \end{aligned}$$

$$\text{Consumer surplus} \approx 0.0448b^3/\alpha.$$

Note that $t_1 \approx 0.3088b$ in this equilibrium, so this equilibrium is feasible in the sense of not covering the market if and only if $b \geq 3.2383a$ (approximately). Thus we get

PROPOSITION 5. *For $b \geq 3.2383a$, there exists an equilibrium with the market not covered in the sequential entry game. In this equilibrium, the first entrant chooses the product $0.2908b/\alpha$ and the second entrant chooses the product $0.1477b/\alpha$.*

The first entrant's profits are higher when he chooses his equilibrium strategy than when he chooses the monopoly position myopically. The value of foresight about future entry is $0.0004b^3/\alpha$ ($=0.0189b^3/\alpha - b^3/54\alpha$). The first entrant's equilibrium

[16] At this point Hauser and Shugan (1983) would argue that firm 1 should "defend" itself by choosing the best-response to $b/6\alpha$. But we won't be considering that possibility here because we assume that product changes are prohibitively expensive. Moreover, as we shall see presently, the question of responding to a new entrant's product doesn't arise if the incumbent has already chosen his product position anticipating the entrant's product strategy.

[17] Some readers may object to my presumption of benevolence on the part of the later entrant when firm 1 chooses $0.2908b/\alpha$. But such a presumption is not really necessary to my argument. The first entrant can guarantee a choice in his favor if he chooses a product arbitrarily close to $0.2908b/\alpha$. For example, if $s_h(0.2908b/\alpha)$ is better for the first entrant than $s_l(0.2908b/\alpha)$, then he can choose $0.2908b/\alpha - \epsilon$, inducing $s_h(0.2908b/\alpha - \epsilon)$ from the second entrant and assuring himself a profit arbitrarily close to what he would have got with $s_1 = 0.2908b/\alpha$, $s_2 = s_h(0.2908b/\alpha)$.

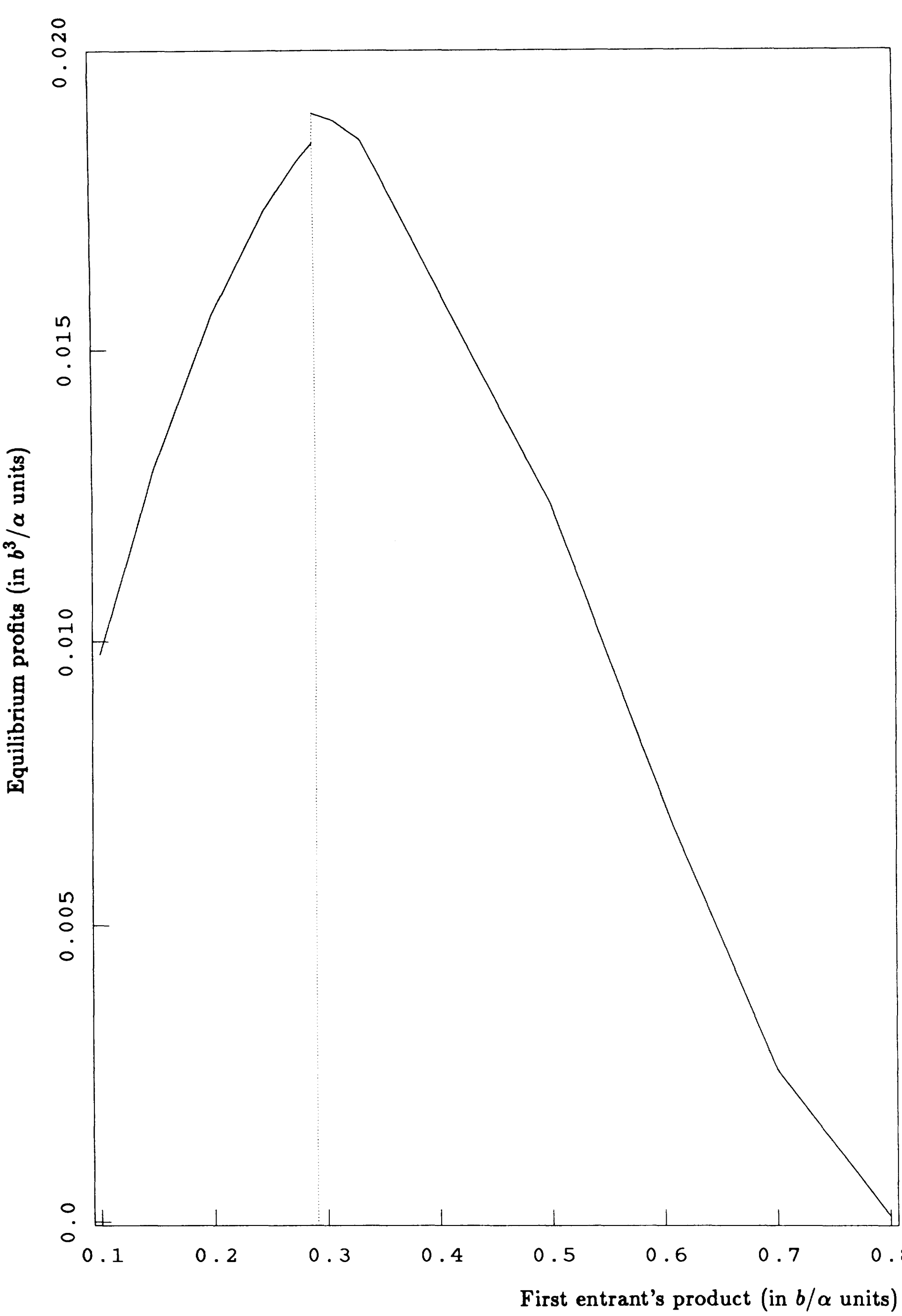

FIGURE 6. The First Entrant's Profit Function.

profits are also higher than the second entrant's equilibrium profits (by $0.0111b^3/\alpha$). So there is a first-mover advantage. This advantage comes from the first entrant's ability to *preempt* a favorable product location (close to the monopolist's best single product location) and *control* the second entrant's product choice. The ability to control the other firm's product choice is not there in the simultaneous-moves equilibrium analyzed earlier. Thus, firm 1's equilibrium profits are higher when it is the first entrant than when it is the lower firm in the simultaneous-moves equilibrium of §5. In §5 we argued that if firm 1 got first shot at choosing which of the two simultaneous-product-choice equilibria it would play, then it would choose the equilibrium where it was the lower quality firm, and *this* "first-mover" advantage was worth $0.0064b^3/\alpha$. The first-mover advantage that we are talking about here is worth more: $0.0111b^3/\alpha$. Other differences between the sequential-moves equilibrium and the simultaneous-moves equilibrium include:

1. The higher quality firm makes more profits than the lower quality firm in the sequential-moves equilibrium whereas it is the opposite in the simultaneous-moves equilibrium.
2. Coverage efficiency is higher in the sequential-moves equilibrium: the market left uncovered is $[a, 0.3088b)$ which is a subset of the market left uncovered in the simultaneous-moves equilibrium. This is because the lower quality is lower in the sequential-entry equilibrium.
3. Both firms' products are within the efficient interval in the sequential-moves equilibrium whereas in the simultaneous-moves equilibrium only the lower-quality product is in the efficient interval.
4. The consumers are better off in the sequential-moves equilibrium than in the simultaneous-moves equilibrium.

7. Conclusion

The intuitive view of competitive product strategy as the "filling of holes in the market" has been refined in this paper in the context of two firms competing on product quality and price. The paper makes explicit the role of consumer preferences, firms' costs, and price competition in determining a firm's equilibrium product strategy.

The main contribution of the paper is the insight that a firm's equilibrium product strategy is the result of two opposing forces, one bringing the firms closer, the other moving them apart. A firm chooses a product similar to its competitor because both firms seek the product location which is most desirable in terms of consumers' willingness to pay and costs—the location a single-product monopolist would have chosen. A firm differentiates its product from its competitor because product differentiation weakens price competition and raises profits. This insight helps explain my results and the results in the literature. In my model, both forces operate, so each firm's equilibrium strategy is to choose a product different from its competitor's product, but straddling the monopolist's best single product. In Shaked and Sutton's (1982) model, also, both forces operate. Quality is costless in their model, so the most desirable single product location for a monopolist is the highest feasible quality. But both firms cannot be at this location (because of price competition), so one firm ends up choosing the highest feasible quality and the other firm a quality less than that. In d'Aspremont, Gabszewicz, and Thisse (1979) and Hauser (1988)—which are finite ideal-point models—products are again identical in terms of manufacturing costs. But now *every* product location is equally desirable for a single-product monopolist because consumers' ideal-points are uniformly distributed and there is no competition from a substitute. So, effectively, only the first force operates, leading to maximally differentiated products.

The other contribution of this paper is that it shows how the first entrant in a market can use his product to defend himself against future entrants. If he does defend himself this way, the first entrant can also get a first-mover advantage. The concept of defensive product strategy used here is different from the one in Hauser and Shugan (1983). In this paper, the first entrant chooses his product *anticipating* that a second entrant will come in, see his product, and react to it. Then, when the second entrant does come in—at the location the first entrant expected him to—the first entrant is already well positioned to handle the new competition.[18] In other words, I view defensive product strategy as "*defensive preparations*." Hauser and Shugan view defense as a *reaction* to attack. Obviously, defensive strategy in the sense of defensive preparations makes sense only if the preparations are such that *the attacker can take them as given*—otherwise the attacker will simply ignore those preparations, attack wherever he wants to, and force the defender to respond to his attacks. Thus product design—because it is costly to change—is a good marketing variable to make defensive preparations on, price is not. The first-mover advantage in §6 comes from the first entrant's ability to *preempt* a desirable product location and force the second entrant to respond to it.

Readers may wonder what this work has to say about whether a high quality product or a low quality product is the better competitive strategy. Looking at the question from a purely decision-theoretic point of view—with the competitor's product fixed—the answer is that it depends on where the competitor's product is located. If the competitor's product is of "low" quality, then it is better to be the high-quality supplier, but if the competitor's product is of "high" quality, then it is better to be the low-quality supplier. In equilibrium, though, we get mixed answers: When the firms choose products simultaneously, it is better to be the lower-quality supplier, but when one firm chooses its product before the other firm (i.e., in the sequential-entry model), then it is better to be the higher-quality supplier.

To understand these results consider the equilibrium locations and margins of the two firms vis-a-vis the monopolist's best products and margins (cf. Figure 7). As expected, margins increase with product quality. Also, as noted before, the monopolist's product selections are efficient but the equilibrium products are not. In the simultaneous-product-choice model, where the firms are completely symmetric, both firms' equilibrium qualities are higher than the monopolist's corresponding qualities, but the margins are smaller. Price competition reduces margins more than cannibalization. So the lower quality firm seeks to get as close as possible to the position which optimizes the trade-off between consumers' willingness to pay and costs—$b/3\alpha$, the monopolist's best single product—knowing that up to a point the higher quality firm can be driven further up. This suggests that the lower quality firm has more power in determining the higher quality firm's location in this model than vice-versa. The reason for this is the competition the lower-quality firm faces from the substitute, which is transferred to the higher quality firm via the lower quality firm's margin. The higher quality firm realizes that the lower quality firm cannot be pushed too close to the substitute without reducing its margin drastically. In the sequential-product-choice model, however, the first entrant has all the power. He can preempt a position and control the second entrant's location. The choices he faces are between choosing a position very near the monopolist's best single product and having the second entrant shield him from the substitute —even though this increases the price competition between them—versus choosing a position not as close to the monopolist's best single product and facing the substitute directly. He goes for the first option.

There are many directions in which this work can be extended. One direction is to implement the analysis here for managerial purposes. In any implementation, some further analysis will be required to see how certain practical features of the real-world environment—the number of consumer types is finite as opposed to infinite, the number of feasible products is finite instead of infinite—affect the product equilibria identified here. Ultimately, what is desired is a computational procedure for the product equilibria given any distribution of consumer reservation prices. (Dobson and Kalish 1988 do this for the monopoly case.) As far as measuring the reservation prices is concerned, a good first step would be to continue to assume that the reservation price

[18] Robinson (1988) reports evidence from the PIMS data base which is consistent with this view.

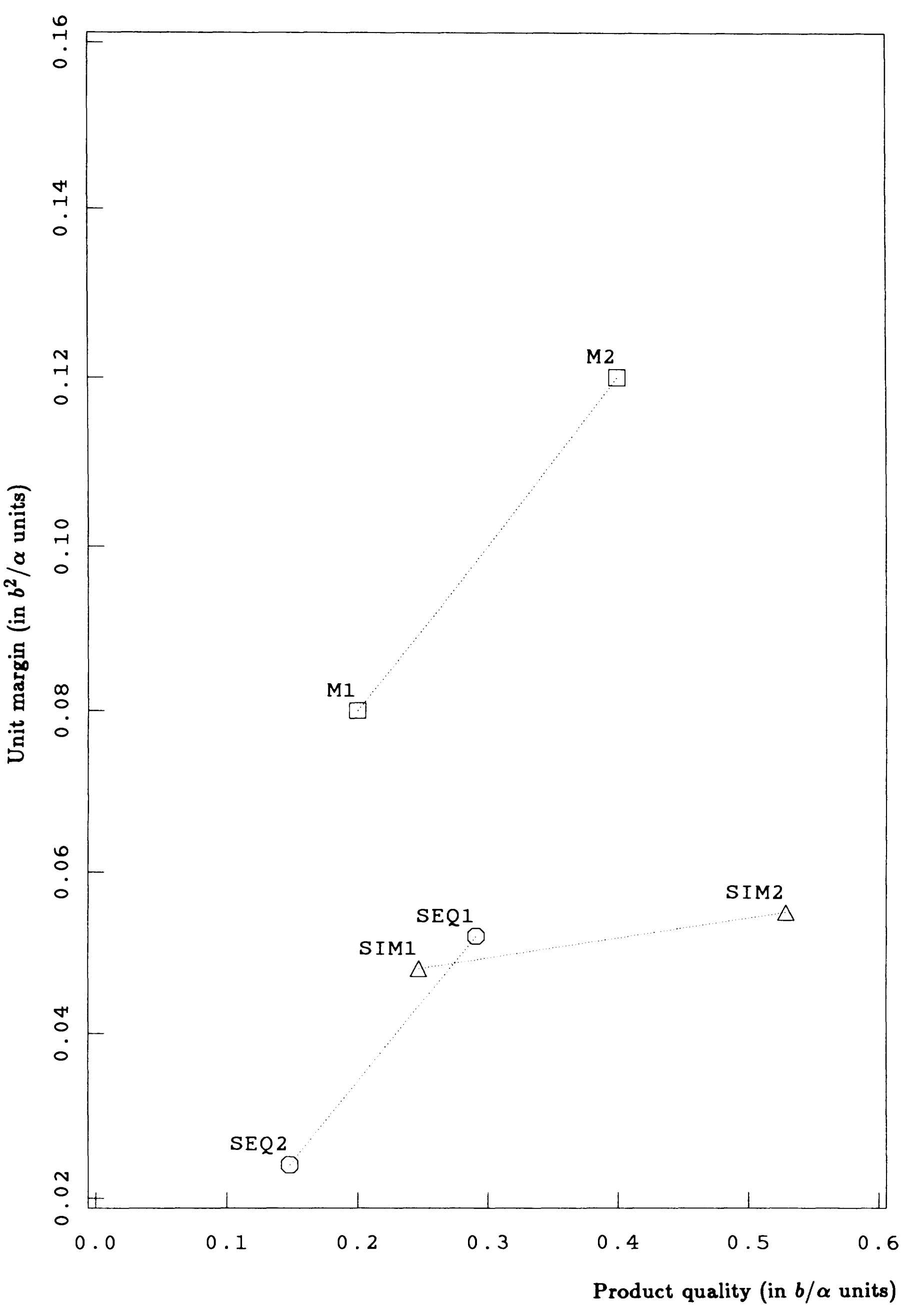

FIGURE 7. The Monopoly Solution and the Two Equilibria.

functions are linear in quality (as in §2), so that we can use conjoint analysis to measure them. Price will be the "other" attribute here and the resulting importance weight for quality will be the consumer's type. Interpolation to quality levels other than the ones used in the estimation task can be done using the methods of Pekelman and Sen (1979). Alternatively, if the feasible set of products for the two firms is finite, then the conjoint analysis need measure the reservation prices only for the quality levels that are feasible.

Theoretically, perhaps the most urgent task is to see how far our results generalize. The generalization can be in the specification of consumer preferences and firms' costs or it can be in the number of firms. In the case of no price competition, Eaton and

Lipsey (1975) have shown that there is no equilibrium with three firms (because no firm wants to be the "interior" firm). But we anticipate that an equilibrium will exist in our set-up. Again, the coveted position is the monopolist's best position, but if one firm takes it (or is close to it) the other firms may not want it for fear of price competition. So we are likely to get an equilibrium with firms strung out along the whole product spectrum. The interesting questions are, how does the product and price equilibrium change as the number of firms increases? Does the equilibrium configuration of products converge to the efficient set of products?[19] Another extension of this work would be to go from one product per firm to multiple products per firm. Some preliminary work has already been done on this; see Brander and Eaton (1984), Wang (1985), and Moorthy (1988). A central issue here is the pattern of competition across product lines—do firms tend to specialize at one or the other end of the product spectrum or do they try to provide a product for "every taste" (with each firm's products intermingled among its competitors)? Going back to one product per firm, it would be interesting to look at two-attribute (and by extension multiattribute) competition to see how firms compete across attributes. Carpenter (1986) and Hauser (1988) have already done preliminary work on this, but as I pointed out earlier, Hauser's model is essentially a one-attribute model because of production restrictions on how the two attributes can covary. If there are no production restrictions on how the two attributes can covary, how would the variation in relative preference for the two attributes across the population affect product and price competition?

Finally, the sequential-entry model of §6 should be the source of much future work. In that model, dynamics were not explicitly modeled; my attention was solely on the sequence of moves. Rao and Rutenberg (1979) have shown how important timing can be in capacity expansion decisions. In the context of product choice, too, timing would be important, especially when there is uncertainty about market preferences. The first entrant has to weigh the risk of judging market preferences incorrectly and the possibility of attracting other competitors (if he is successful), with the benefits of product preemption, brand loyalty, and experience curve advantages. But the later entrants, having observed the success or failure of the incumbent, can read the market better. Can there be an equilibrium here with some firms choosing to enter early and others choosing to enter late? Chatterjee and Sugita (1987) suggest that there can be.[20]

[19] I have shown elsewhere that if firms compete on quality and quantity, then this convergence does happen. See Moorthy (1985).

[20] This paper was received in August 1983 and has been with the author 37 months for 3 revisions.

Appendix

PROOF OF PROPOSITION 1. The first part of the proposition is obvious. To prove the second part assume $s_i < s_j$, $t \in M_i$, $t' \in M_j$, and $t > t'$. Then, because $t' \in M_j$, $t'(s_j - s_i) \geq p_j - p_i$. But this implies $t(s_j - s_i) > p_j - p_i$, contradicting the initial assumption that $t \in M_i$. As for the third part of the proposition, we first show that each product's market is a connected segment. Suppose $t, t' \in M_1$ with $t' > t$. If $s_1 = s_2$, then $p_1 = p_2$ (by the first part of the proposition) and necessarily $(t + t')/2 \in M_1$. If $s_1 < s_2$, then $[(t + t')/2](s_1 - s_2) > t'(s_1 - s_2) \geq p_1 - p_2$ and $[(t + t')/2](s_1 - s_0) \geq t(s_1 - s_0) \geq p_1 - p_0$, so $(t + t')/2$ must be in M_1. Hence product 1's market is connected. Similarly for product 2. Finally, suppose $M_0 = [a, t'_1]$, $M_1 = [t_1, t_2]$, $t_1 > a$, and t_1 strictly prefers s_1 to s_0. By the second part of the proposition, $t_1 \geq t'_1$. If $t_1 = t'_1$, then obviously t_1 is indifferent between s_1 and s_0, contradicting our initial supposition. So suppose $t_1 > t'_1$. Then $t_1(s_1 - s_0) > p_1 - p_0$ by assumption and $t'_1(s_1 - s_0) \leq p_1 - p_0$ because $t'_1 \in M_0$. So there exists a $t \in [t'_1, t_1)$ such that $t(s_1 - s_0) = p_1 - p_0$. For this t it is also true that $t(s_1 - s_2) \geq p_1 - p_2$ (because $t < t_1$ and $t_1(s_1 - s_2) \geq p_1 - p_2$). Therefore $t \in M_1$, contradicting our assumption that t_1 is the lower boundary of s_1's market. The rest of the proofs follow similarly. ■

PROOF OF PROPOSITION 2. If one firm prices at marginal cost, then the other firm does not gain anything by pricing above marginal cost because its market share then will be zero. And it does not gain anything by pricing below marginal cost either because then it will lose money on each of the units it sells. Thus both firms pricing at marginal cost is an equilibrium. But is pricing at marginal cost the *only* equilibrium? Suppose there is an equilibrium with either firm pricing above marginal cost—say firm 1—and suppose (without loss of generality) that $\Pi_1 \geq \Pi_2$ in this equilibrium. For this equilibrium to make sense it must be that $m_1 > 0$. Then firm 2 can price just below firm 1 and capture all of firm 1's market share, and achieve a monopoly position in the market. Such a deviation will increase firm 2's profits. So there cannot be an equilibrium with either firm pricing above marginal cost. ■

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