Product and Process Innovation in a Growth Model of Firm Selection*

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Abstract

Recent empirical evidence based on firm-level data emphasizes firm heterogeneity in innovation activities and the different effects of product and process innovations on firm productivity- and aggregate growth. To match this evidence, this paper develops an endogenous growth model with two sources of firm heterogeneity: production efficiency and product quality. Both attributes evolve through firms' innovation choices and permanent shocks. Growth is driven by innovation, idiosyncratic improvements, firms selection and entrants imitation. A calibration based on the Spanish manufacturing sector shows that only 8.13% of aggregate growth can be attributed to net entry and 73.81% to innovation. Instead distinguishing the growth impact of quality upgrading from the one of cost reduction shows that growth in quality explains almost 30% of total output growth. Compared to single attribute models of firm heterogeneity, the model provides a complete characterization of firms' innovation choices explaining the partition of firms along different innovation strategies and generating consistent firm size distributions.

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1 Introduction

Globalization and the rise of new technologies have challenged firms' abilities in developing innovation strategies to face increasing market competition. Innovation has become a fundamental source of firm survival and growth.[1] The literature has widely analyzed the relationship between innovation and economic growth.[2] However, little attention has been paid to the relationship between firm heterogeneity and innovation activities and even less to the relationship between firm heterogeneity and different innovation strategies as well as to their impact on firms' competitiveness and productivity growth. The channel between firm growth and aggregate growth is still comparatively unexplored. Understanding the determinants of firms' innovation strategies and the mechanism of resource reallocation through which they impact on aggregate growth is therefore crucial and can also contribute to enhance the effectiveness of policies aimed at fostering economic growth and welfare.

This need comes together with an increasing availability of data at the firm-level which distinguish between process and product innovation.[3] These data

[1]For instance, on a panel of Dutch firms Cefis and Marsili (2005) find that the expected longevity of innovative firms is 11% higher than non-innovative firms while Doraszelski and Jaumandreu (2008) using a Spanish panel estimate that the sole contribution of firms that perform R&D explains between 45% and 85% of productivity growth in the industry with intermediate or high innovation activity. Moreover, Bartelsman and Doms (2000) report evidence of a self-reinforcing mechanism between productivity and innovation. Profitable firms have a higher propensity to innovate and innovation is positively related with productivity and productivity growth.

[2]Few examples are Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990).

[3]The European Commission has developed a program aimed at studying the innovation systems of the States member of the European Union with the scope of promoting innovation and growth. The core of the program is based on firm-level surveys (Community Innovation Surveys) which ask detailed questions about the innovation investments of firms distinguishing between process and product innovations. In particular, process innovation occurs when firms introduce some significant modification of the productive process as the introduction of new machines or the introduction of new methods of organization, while product innovation occurs when firms report a new or improved good. This information is then merged with structural and macroeconomic data drawn from OECD surveys. Additionally, some European Countries carry out nation-specific surveys. For instance, in Spain there is the *Encuestas Sobre Estrategias Empresariales* that is issued every three years. The same analysis becomes more difficult with American

have stimulated a series of empirical studies which highlight three main pieces of evidence: innovations are *heterogeneous*, *asymmetric*, and *complementary.*

Firstly, innovation are *heterogeneous* in the sense that some firms do not innovate, some firms specialize in process innovation, others in product innovation and some in both types of innovations. Thus, firms have different incentives to invest either in product or process innovation. Table 1 shows the share of firms across the different innovation strategies for four European countries.[4] Huergo and Jaumandreu (2004) finds in a sample of Spanish firms in the manufacturing sector that half of the firms never innovate, 30% undertake either process or product innovation and 20% of the firms undergo both types of innovations. Similar statistics are also available for Germany and Great Britain (Harrison et. al. (2008)) and the Netherlands (Cefis and Marsili (2005)).

Table 1: Heterogeneity in Innovation Strategies

Country	**Share of Innovative Firms**			
	No Innovation	**Process**	**Product**	**Process and Product**
Spain	55.4%	12.2%	12.4%	20%
Germany	41%	10.2%	21%	27.4%
Great Britain	60.5%	11%	14.2%	14.3%
Netherlands	36.6%	5.8%	18.8%	42.7%

Secondly, the innovation strategies are *asymmetric.* Parisi et. al. (2006) estimate on an Italian panel that process innovation increases productivity by 14% and product innovation by 4% over a three year period. As expected, innovating firms are characterized by a productivity distribution that stochastically dominates the productivity distribution of non-innovators. But in the case of product innovation the distribution becomes more skewed to the right. Huergo and Jaumandreu

data where innovation is measured as patents and therefore the two innovations cannot be distinguished. However, for a concise summary Klette and Kortum (2004) report a list of stylized facts concerning firm R&D, innovation, and productivity.

[4]It should be noticed that the data sets are not homogeneous. Hence table 1 does not allow comparisons across countries but only the ability to observe the stated heterogeneity in the innovation choices.

(2004) show similar results for Spain and highlight a relation betwen firm size and type of innovation. Small firms are more likely to undertake product innovation while large firms are more likely to undertake process innovation.

Thirdly, innovations are *complements*. Process innovation is more frequent than product innovation, while the probability of introducing a product innovation is higher for firms that also introduce a process innovation in the same period. However process innovation does not necessarily imply product innovation.[5] Firms innovate on their existing products, aiming at increasing product differentiation and hence prices, in the hope of exploiting consumers' willingness to pay for a higher quality good. Instead process innovation increases the firms' production efficiency. This leads to higher firm productivity, lower prices and a larger scale of production.[6] Complementarity between process and product innovation then arises: product innovation allows new product designs but these new designs become profitable only when they are affordable for the consumers.

Entry and exit play an important role in explaining the reallocation of resources from less productive firms to more productive firms and therefore growth.[7] In addition, Huergo and Jaumadreu (2004) show that exit is associated with a lower level of pre-exit innovations, while entrants present a high probability of innovation.

Existing growth literature cannot explain all these pieces of evidence as it treats quality upgradings and cost reduction innovations as interchangeable. Moreover, the literature on heterogeneous firms is usually based only on one factor of heterogeneity, either cost efficiency or the ability of producing quality. In these models a single attribute monotonically predicts firms' revenue, competitiveness, and innovation. This characteristic then implies a threshold firm size above which all firms innovate and below none do and hence predictions not in line with the empirical results.

Hence, motivated by the discrepancy between the existing theoretical literature and the empirical evidence, this paper proposes a new framework able to explain

[5] See Miravate and Pernias (2004) on data for the ceramic tile industry in Spain, Martinez-Ros (1999) for Spanish manufacturing firms and Parisi et. al. (2006) for Italy.

[6] See Smolny (1998) for an empirical study on the effects of process and product innovation on the prices charged by German firms.

[7] Foster et. al. (2001) on data from the US manufacturing sector find that more than 25% of the growth between 1997 and 1998 was due to net entry. However, Bartelsman et. al. (2004) find that in Europe the contribution of net entry is comparatively low than in US.

and quantitatively replicate the empirical regularities discussed. It analyzes the effects of cost reduction (process) and quality improving (product) innovations on firm dynamics, productivity- and aggregate growth, highlighting the importance of product quality in the growth process. For this purpose, I develop a general equilibrium model with endogenous process and product innovation. The industry dynamics are taken from Hopenhayn (1992) using monopolistic competition as in Melitz (2003). Firms produce differentiated goods and are heterogeneous in their production efficiency and in their product quality. The evolution of both efficiency and quality is given by an idiosyncratic permanent component and by an endogenous component proportional to the optimal investment decision taken by the firm. Product innovation increases firms product quality while process innovation increases firm production efficiency. In each period non profitable incumbents exit the industry, and are replaced by new firms. Entrants imitate the average incumbent as in Gabler and Licandro (2005) and Luttmer (2007) and on average they are more productive than exiting firms increasing the average productivity of the industry. Hence, growth arises due to firms' innovation and firms' self-selection and is sustained endogenously by entrants' imitation.

The model is calibrated to match the Spanish manufacturing sector for which there is a large availability of firm-level data and related empirical studies on both firm dynamics and innovation dimensions. Besides matching closely the data, the model generates moments and a firm size distribution consistent with the empirical evidence. The interplay between the two sources of firm heterogeneity and costly innovation results in a non-monotonic relation between firm size and innovation strategies. Small firms undertake product innovation, medium firms both process and product innovation while large firms specialize mainly in process innovation. Moreover, it emphasizes the importance of the reallocation of resources among incumbents and innovators as the main source of growth. In fact, firms' turnover explains only 8.13% of aggregate growth and when innovation is banned output growth declines by 3.1 percentage points. Another interesting prediction that can be empirically tested is the contribution of the growth in production efficiency and product quality in explaining productivity growth. The model predicts that efficiency growth plays the major role explaining 69.8% of output growth. Additionally, this model contributes to the literature that tries to understand why firm heterogenity is persistent endogenizing the evolution of firm technology.

In this model the relationship between firm size and innovative strategies is more articulate in explaining why different firms choose optimally different innovation strategies. Additionally, comparing industries that differ for innovation costs or for entry barriers allows for a better understanding of the growth rate composition and how it is affected by changes in the industry structure. Hence this model provide a suitable framework for the analysis of policy implications aimed at fostering growth.

1.1 Related Literature

This paper attempts to link the literature on firm dynamics and endogenous growth theory by explicitly modeling different types of firm-level innovations. As in the seminal models of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), innovation is firm-specific and it is motivated by the appropriation of revenues associated with a successful R&D investment. In Romer (1990) growth is driven by two elements. The first one is the invention of new inputs which make the production of the final good sector more efficient. In this sense and from the point of view of the final good firm it can be seen as process innovation. The second one is knowledge spillovers from past R&D: the higher the stock of knowledge, the easier the invention of new varieties. In this paper there is a similar spillover, which is the imperfect imitation of incumbent firms by entrants. Grossman and Helpman (1991) introduce growth through quality improving innovation of existing products. However, in their model, different qualities are perceived as perfect substitutes and hence the representative consumer buys only the cheapest variety (adjusted by quality). Instead, in my model each variety is perceived as different by the consumer and higher quality varieties give higher utility. In Aghion and Howitt (1992) growth is based on the idea of Schumpeterian creative destruction in which new innovations replace the previous ones driving the incumbent monopolist out of the industry. The creative destruction mechanism is not far from the idea of firm selection. Successful firms grow and drive out of the market unsuccessful ones. Based on these general features my work adds firm heterogeneity, permanent idiosyncratic shocks that hit both production efficiency and product quality, and endogenous investment choices made by incumbent firms. These new elements endogenously link aggregate growth with firm-specific growth and hence with the mechanism of resource reallocation from non-innovators to innovators and from exiting to active firms. The resulting distribution of firm size is consistent with the data.

The idea of firm selection was already present in Jovanovic (1982). He introduces the first model with firm-specific stochastic productivities with unknown mean but known variance. As time goes by firms learn their productivity and the inefficient firms exit. As firms learn their productivity the effects of selection on firms evolution dies out and eventually the industry converges to a stationary equilibrium without entry and exit. For this reason, this paper takes the industry structure from Hopenhayn (1992), who develops a partial dynamics stochastic heterogeneous firms' model which generates a stationary equilibrium with entry and exit that is capable of studying the effects of structural changes in the industry on the distribution of firm size and age. Hopenhayn and Rogerson (1993) analyze the general equilibrium of the Hopenhayn model focusing on the process of labor reallocation. Both papers study the stationary equilibrium in which each firm is hit by shocks characterized by a stationary AR(1) process. However, both papers focus only on firm productivity growth between cohorts and disregard the effects on aggregate growth.

The link between the process of resource reallocation due to selection at the firm level and economic growth is studied in Gabler and Licandro (2005) and in Luttmer (2007). In both papers firm technology is hit by permanent shocks which together with firm selection and entrant imitation generates endogenous growth. The resulting stationary distribution is a consequence of the knowledge spillover that links the distribution of entrants productivities to the distribution of incumbents productivities. This assumption is necessary to generate endogenous growth. In fact without imitation, as incumbent firms become more productive through selection, the incentives to enter the industry diminish and eventually vanish. In the end no new firms enter into the industry and the equilibrium is characterized by the absence of entry and exit similarly as Jovanovic (1982). Gabler and Licandro (2005) model a competitive equilibrium with heterogeneous firms using both labor and capital as inputs. When calibrating their model on US data they show that selection and imitation account for a fifth of productivity growth. This represents a lower bound. Luttmer (2007) instead considers a monopolistic competition market in which each firm produces a different variety and it is subjected to shocks to both productivity and demand. Calibrating his model to US data he finds that half of output growth can be attributed to selection and imitation. This can be seen as an upper bound.

This paper attempts to extend Gabler and Licandro (2005) and Luttmer (2007) by considering alongside their models the role of innovation in linking firm level growth to aggregate growth. Modeling endogenously firm innovation investments in both firm efficiency and product quality can help to distinguish the differing contributions of selection and imitation versus innovation in process and product when explaining economic growth.

The other papers that shed light on the relationship between innovation, firm heterogeneity and the role of resource reallocation of the growth process are Klette and Kortum (2004) and Lenz and Mortensen (2008). The former, building on Grossman and Helpman (1991), introduces firms that exogenously differ in the profits earned by selling their own products. Endogenous growth is then generated through innovation investments aimed at increasing the number of goods produced by each firm and firms adjust the production lines in response to their own and competitors' investment in R&D. However they posit permanent exogenous differences across firm profitability and hence across the size of the innovative step. This simplification results in a distribution of innovative firms that have the same volatility as the distribution of the firms that do not innovate. This model, defining innovation as an endogenous drift into the stochastic evolution of firm productivity and quality, can account for the differing variances of the distribution of innovators and non-innovators. Lenz and Mortensen (2008) relate to Klette and Kortum (2004) introducing heterogeneity in the expected productivity of the new variety produced. But as in both models the engine of growth is a mechanism of creative destruction on the numbers of goods existing in the economy at a given point in time, they can analyze only one channel of innovation.

More recently, Atkeson and Burstein (2007) address the relation between the decision of heterogeneous firms to innovate and engage in international trade by introducing two types of stochastic innovation activities. Though their model abstracts from endogenous growth, they define as process innovation the decision to increase the stock of firm-specific factors that then translates in higher profits opportunities. This is analogous to process innovation defined in this model. They define as product innovation the creation of a new firm and hence a new product.

This is the analogous to firm entry discussed in this model. In fact, this model defines differently from them as product innovation the decision of firms to improve the quality of an exiting variety. Moreover, the jump in the efficiency and/or quality scale are, in this paper, proportional to the research intensity.

Finally two other papers of note, Melitz (2003) and Hallak and Sivadasan (2008). Melitz (2003) proposes a static model with heterogeneous firms in which the exposure to international trade increases firm selection and generates a partition among firms such that the more productive firms are the ones who gain access to foreign markets. Hallak and Sivadasan (2008), building on Melitz (2003), introduce a partial and static equilibrium model in which firms differ in two attributes: labor efficiency and ability to produce high quality varieties. Under the assumption of minimum quality requirements they study how openness affects firm distribution. In their model as in Melitz (2003) the partition of firms between domestic producers and exporters is generated by the presence of a fixed cost to enter the foreign market. Here the same mechanism is used to generate the partition of firms among the different innovation strategies. However, the firm partition and the effects on the size distribution of firms is not the result of a one-shot change but it is the result of the combination of permanent shocks on both states and inter-temporal innovation decisions.

2 The Model

This section develops a general equilibrium model in discrete time with infinite horizon.

2.1 Consumer Problem

The representative consumer maximizes his utility choosing consumption and supplying labor inelastically at the wage rate w. Its lifetime utility is assumed to take the following form:

$$U = \sum_{t=0}^{\infty} \beta^t \ln(U_t) \tag{1}$$

where $\beta < 1$ is the discount factor and t is the time index. In every period the consumer faces the problem of maximizing his current consumption across a continuum of differentiated products indexed by $i \in I$ where I is a measure of the available varieties in the economy. Specifically, the preferences are represented by an augmented Dixit-Stiglitz utility function with constant elasticity of substitution between any two goods $\sigma = 1/(1-\alpha) > 1$ with $\alpha \in (0,1)$. Hence, the utility function at time t is:

$$U_t = \left(\int_{i \in I} (q_t(i) x_t(i))^{\alpha} \mathrm{d}\mathrm{i} \right)^{\frac{1}{\alpha}}. \tag{2}$$

where $x(i)$ is the quantity of variety $i \in I$ and $q(i)$ is the corresponding quality. This utility function is augmented to account for quality variation across products and quality acts as an utility shifter: for a given price the consumer prefers products with high quality rather than products with low quality.

The per period budget constraint is $E_t = \int_{i \in I} p_t(i) x_t(i) di$ where E_t is total expenditure at time t and $p_t(i)$ is the price of variety $i \in I$ at time t. Solving the intra-temporal consumer problem yields the demand for each variety $i \in I$:

$$x_t(i) = \left(\frac{P_t q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} X_t = \left(\frac{P_t^{\alpha} q_t^{\alpha}(i)}{p_t(i)} \right)^{\frac{1}{1-\alpha}} E_t \tag{3}$$

with:

$$P_t = \left(\int_{i \in I} \left(\frac{p_t(i)}{q_t(i)} \right)^{\frac{\alpha}{\alpha-1}} \mathrm{d}i \right)^{\frac{\alpha-1}{\alpha}} \quad \text{and} \quad X_t = U_t. \tag{4}$$

P_t is the price quality index at time t of all the bundle of varieties consumed and X_t is the aggregate set of varieties consumed.

Finally, the optimal inter-temporal allocation of consumption yields the standard Euler equation:

$$\frac{X_{t+1}}{X_t} = \beta(1 + r_t). \tag{5}$$

where r_t is the return on asset holding.

2.2 Firms

This section outlines a dynamic two factors heterogeneous firm model. The first source of heterogeneity is production *efficiency*, $a(i) \in \mathbb{R}_{++}$, which increases the marginal productivity of labor, as in the seminal paper of Hopenhayn (1992), and the second source is *quality* of the firm's variety, $q(i) \in \mathbb{R}_{++} \setminus (0,1)$, which decreases the marginal productivity of labor. In this respect, a higher quality variety has a higher variable cost. Firms are distributed over productivity and quality. $\widetilde{\mu}(a, q) = \mu(a, q)I$ is the measure of firm with state (a, q) at time t, where I is the number of firms in the industry and $\mu(a, q)$ is a density function. It is assumed that each firm produces only one variety so that the index i identifies both the firm and the corresponding variety produced by that firm and I represents both the set of varieties and the mass of incumbent firms active in the industry. The following definition are used, A is the set of all production efficiencies, Q is the set of all product qualities, and $\Omega \equiv A \times Q$ is the state space.

2.2.1 Production Decision

After paying a fixed operational cost, c_f, expressed in terms of labor, active firms receive their new technology level, (a, q). Firms produce and price their own products under the assumption of monopolistic competition. As in Hallak and Sivadasan (2008), the production function is assumed to be linear in labor, n, which is the unique input, increasing in firm efficiency, a, and decreasing in firm product quality, q. That is, $x_t(i) = a_t(i)q_t(i)^{-\eta}n_t(i)$ with $\eta \in (0, 1)$. The parameter η introduces asymmetry between firm efficiency and product quality and measures the difficulties in producing a higher quality variety: the higher η, the more difficult and costly it becomes to produce a high quality product. This particular functional form is justified by empirical evidence: it generates a price distribution consistent with the estimates of Smolny (1998) and moreover complementarity between process and product innovation is obtained.

The profit maximization problem, faced by each firm, is:

$$\pi_t(a(i), q(i)) = \max_{p(i)} p_t(i)x_t(i) - w_t n_t(i) - w_t c_f \tag{6}$$

where w_t is the wage rate at time t common to all firms. The first order condition with respect to price yields the optimal pricing rule:

$$p_t(a(i), q(i)) = \frac{w_t q_t^{\eta}(i)}{\alpha a_t(i)}. \tag{7}$$

$1/\alpha$ is the constant mark-up associated with the CES demand function. In contrast to the standard models with a single factor of firm heterogeneity, firms' prices depend on both firms' efficiency and quality. Consistent with both the theoretical predictions and the empirical estimates, the price schedule is increasing in product quality and decreasing in efficiency.[8] As in Melitz (2003) the nominal wage is normalized to one. Using the monopolistic price to solve for the optimal demand for each variety yields:

$$x_t(a(i), q(i)) = \left(\frac{\alpha a_t(i) P_t^{\alpha}}{q_t(i)^{\eta-\alpha}} \right)^{\frac{1}{1-\alpha}} E_t. \tag{8}$$

Firm output is an increasing function of both the aggregates and of the efficiency level of firms. The relationship between product quality and output is ambiguous and depends on the comparison between α, related to consumer preferences, and η, coming from firm production function. If $\eta > \alpha$ then firm output is decreasing in the product quality: high quality varieties are characterized by a relatively lower market share. In this case, the positive effect of quality on consumer utility is completely offset by the related high market price. The opposite is true when $\alpha > \eta$.

The optimal labor demand is given by:

$$n_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta} \right)^{\frac{\alpha}{1-\alpha}} \left(\alpha P_t^{\alpha} \right)^{\frac{1}{1-\alpha}} E_t. \tag{9}$$

Labor input is an increasing function of both firms' state variables. Consequently, firms with more advanced technology demand more labor input. Finally, the net per period profit of firm i is given by:

$$\pi_t(a(i), q(i)) = \left(a_t(i) q_t(i)^{1-\eta} \alpha \right)^{\frac{\alpha}{1-\alpha}} (1-\alpha) P_t^{\frac{\alpha}{1-\alpha}} E_t - c_f. \tag{10}$$

Although product quality has an ambiguous effect on the optimal output of firms, profits are increasing in both labor efficiency and product quality. This provides incentives for firms to improve endogenously their position in the technology distribution via firms' innovation policies. In this respect, the model predicts that a change in efficiency impacts more a firm's profit than a change in quality.

[8] Smolny (1998), studying a panel of West German firms in the manufacturing sector in the period 1980-1992, estimates that product innovation increases the probability and the frequency of positive net prices increases by more than 18% while process innovation does not reveal a conclusive effect on firm pricing strategies. However, he clearly estimates that process innovations increases the probability of employment and especially output increases. Making increases in output and employment without a lower price is difficult. Hence the effects on output and employment support the relevance of price effects and of the complementarity between the two forms of innovation.

The different effects of firm efficiency and quality on the monopolistic price, on the output, and on the profits provide a suitable framework in which to study the interplay among different innovation choices taken by a firm and their effects on a firm's competitiveness.[9]

2.2.2 Innovation Decision

Firms receive idiosyncratic permanent shocks on both states. That is, firms' log efficiency and log quality follow a random walk. This is a way of capturing the role of firm-specific characteristics and the persistence of firm productivity which is established in the empirical literature.[10] Besides the exogenous random walks, firms can endogenously affect the evolution of their states through private innovation activities. In line with the terminology used in the surveys at the firm-level, this paper identify two different types of innovation: *process innovation* and *product innovation*. Process innovation refers to the decision of firms to invest labor, with the aim of lowering firm production costs, while product innovation refers to the decision of firms to direct labor investment at increasing the quality of the varieties produced.

According to the theoretical growth literature, the benefits derived by firms' innovation investments are proportional to the amount of resources spent. In particular, innovation introduces an endogenous drift in the random walk processes which reflects the amount of variable labor that firms optimally invest in R&D. The innovation choice is history dependent as today investment in process or product innovation results in tomorrow higher firm production efficiency and/or product quality. In addition, firms have to pay also a fixed cost of innovation, c_a and c_q, for process and product innovation, respectively. This is a way of capturing the costs necessary to set up an R&D department, to conduct market analysis and technically it determines the partition of firms among different innovation

[9] An innovation in product, aimed at increasing product quality, results in a higher market price for the given variety and, for appropriate parameters, in a contraction of the market quota. This then determines an incentive to invest also in process innovation and hence to increase firm efficiency. That in turn leads to a lower market price and to an unambiguous larger market share.

[10] For instance, the idiosyncratic shocks can capture factors as absorption techniques, managerial ability, gain and losses due to the change in the labor composition and so on.

strategies. Depending on the firms' technology state, some firms decide to innovate either in process or in product or in both types of innovation. In whichever form innovation comes, it represents a first source of endogenous growth since it shifts the bivariate firms' distribution to the right.

Specifically, log efficiency is assumed to evolve according to:

$$\log a_{t+1} = \begin{cases} \log a_t + \varepsilon^a_{t+1} & \text{when } \; z_t = 0 \\ \log a_t + \lambda^a \log z_t(a,q) + \varepsilon^{az}_{t+1} & \text{otherwise } \; . \end{cases} \tag{11}$$

Shocks are firm-specific and distributed as $\varepsilon^a_{t+1} \sim N(0,\sigma^2_a)$, $\varepsilon^{az}_{t+1} \sim N(0,\sigma^2_{az})$ where σ^2_a is the variance of the random walk when innovation does not occur and σ^2_{az} is the variance of the process when innovation takes place. $z_t(a,q) > 0$ is the labor that a firm with states (a,q) decide optimally to invest in process innovation. $\lambda^a > 0$ is a parameter that, together with the log form of the innovation drift, scales the effects of innovation. The log functional form chosen for the innovation drift is important as together with firm selection assure a bounded growth and hence the existence of a stationary distribution. Similarly log quality evolves as:

$$\log q_{t+1} = \begin{cases} \log q_t + \varepsilon^q_{t+1} & \text{when } \; l_t = 0 \\ \log q_t + \lambda^q \log l_t(a,q) + \varepsilon^{ql}_{t+1} & \text{otherwise } \; . \end{cases} \tag{12}$$

Again $\varepsilon^q_{t+1} \sim N(0,\sigma^2_q)$, $\varepsilon^{ql}_{t+1} \sim N(0,\sigma^2_{ql})$ where σ^2_q and σ^2_{ql} are the two variances without and with innovation. $l_t(a,q)$ is the variable labor devoted to product innovation and $\lambda^q > 0$ is the related scale parameter. The means of the efficiency and quality shocks are normalized to zero eliminating exogenous sources of growth. In fact, abstracting from innovation and firm selection, in expectation firms do not grow.

The random component ε is independent both across firms and over time. Moreover, the two processes, efficiency and quality, are independent.[11] Define the density function of a_{t+1} conditional on a_t as $f(a_{t+1}|a_t)$, and the density functions of q_{t+1} conditional on q_t as $p(q_{t+1}|q_t)$. The transition of the two state variables depends on the firms' innovation decisions and the idiosyncratic shocks. Considering jointly the two transition functions, $\Phi : \Omega \to \Omega$ can be defined as the joint

[11]This simplification does not affect qualitatively the model predictions, but it has the advantage to narrow the set of parameters to calibrate since it is possible to ignore the covariances of the two processes.

transition function, which moves firms' quality and efficiency states. The corresponding transition probability function is defined as $\phi : \Omega \times \Omega \rightarrow [0, 1]$, which gives the probability of going from state (a, q) to state (a', q'). The transition probability takes different forms depending on the innovation decisions and on the exit decision defined below. If the two processes are independent then $\phi(\cdot) = f(\cdot)p(\cdot)$.

2.2.3 Firm Value Function

Incumbent firms face a dynamic optimization problem of maximizing their expected value. Once abstracted from the innovation decision this is a particularly simple problem since it is a sequence of static optimizations. With the innovation scheme, current investments in innovation affect the transition probabilities and thus the value of future technology. This generates a dynamic interplay between firm technology and the innovative position taken by the firm. This is summarized by the following value function:

$$v(a, q) = \max\{v^P(a, q), v^A(a, q), v^{AQ}(a, q), v^Q(a, q)\}. \tag{13}$$

The max operator indicates that in each period firms face different discrete choices which depend on the current level of production efficiency and product quality. $v^P(a, q)$ is the value when no innovation investments occurred, $v^A(a, q)$ when a firm produces and innovates in process, $v^{AQ}(a, q)$ when both process and product innovation are undertaken and $v^Q(a, q)$ when a firm specializes only in product innovation.

Using $J = \{P, A, Q, AQ\}$ and defining with prime the next period variables, the Belman equation for each choice is given by:

$$v^J(a, q) = \max_p \left\{ \pi^J(a, q) + \frac{1}{1 + r} \max\left\{ \int_\Omega v(a', q')\phi(a', q'|a, q)\mathrm{d}a'\mathrm{d}q', 0 \right\} \right\}. \tag{14}$$

where $\pi^P(a, q)$ is given by equation (11), $\pi^A(a, q) = \pi(a, q) - z(a, q) - c_a$, $\pi^{AQ}(a, q) = \pi(a, q) - (z(a, q) + l(a, q)) - c_a - c_q$, and $\pi^Q(a, q) = \pi(a, q) - l(a, q) - c_q$.

These value functions characterize a partition of firms among the different decisions (only produce or produce and innovate, and in the latter case if process, or product or both at the same time) which depends on the relation between the technological state of each firm and the fixed costs. In fact, given the specific position of a firm inside the bivariate distribution of technology, the fixed costs of innovation generate different firms decisions consistently with equation (14). Two sources of firm heterogeneity implies that the thresholds, characterizing the border among the different innovation strategies, are given by infinite combinations of (a,q) couples. For this reason, it becomes convenient to express the reservation values in terms of efficiency as a function of quality, $a(q)$ and to obtain *cutoff functions* rather than cutoff values as in one factor heterogeneous firm models. For given $q \in Q$ it is possible to define the following cutoff functions: $a_A(q)$ delimits the area in which process innovation is optimal, $a_Q(q)$ delimits the area in which product innovation is optimal, and $a_{AQ}(q)$ delimits the area in which both innovations are chosen by the firms.[12] Appendix A provides a formal definition of these cutoff functions.

The cutoff functions are decreasing in q and hence also less efficient firms but characterized by a product with high quality may innovate. Notice that firm profits, $\pi(a, q)$, are increasing in both efficiency and quality generating the incentives to innovate which are slowed down by the log form in which the innovation drift is modeled. Abstracting from the discontinuity in the value function due to the fixed costs of innovation, the more advanced the firm technology, the higher the innovation investment but the lower the benefit due to the diminishing returns of innovation.

2.2.4 The Exit Decision

Firms exit the industry after a bad technological draw such that the expected value of continuing is lower than the exit value which has been normalized to zero.[13] Since firm value is increasing in both states the exit reservation value is decreasing in both of them. Again a cutoff function $a_x(q)$ can be defined such that:

$$E[v(a'(q), q')|(a_x(q), q)] = 0. \tag{15}$$

[12] It is equivalent to express product quality as a function of efficiency, $q(a)$. Using a specific formulation for the cutoff function does not affect the implications of the model.

[13] Notice that exit is triggered by the assumption of fixed operational costs, c_f, paid by active firms in each period. Without fixed operational costs, firms hit by bad shocks instead of exiting the market could temporary shut down their production and just wait for better periods when positive shocks hit their technology and then start again producing.

For each quality level, there is a maximum efficiency level such that below this maximum firm value is negative and therefore firms find optimally to exit the industry. Interestingly, the cutoff function $a_x(q)$ is decreasing in quality: for given efficiency firms with a high quality product can survive longer in the market when hit by a bad efficiency shock.

Firms innovation decisions, exit and the law of motion of (a, q) define the transition function $\Phi_{xI} : A \setminus A_x \times Q \to (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ where the support of efficiency is partitioned into the exit support, A_x, the production support, A_P, the process innovation support, A_A, the product innovation support, A_Q, and the process and product innovation support, A_{AQ}. These partitions differ across different elements of Q.[14] The corresponding transition probability of going from state $(a, q) \in (A_p \cup A_A \cup A_Q \cup A_{AQ}) \times Q$ to $(a', q') \in (A_p \cup A_A \cup A_Q \cup A_{AQ} \cup A_x) \times Q$ is given by a function $\phi_{xI}(\cdot)$.

2.2.5 Firms Entry

Every period there is a mass of potential entrants in the industry which are *a priori* identical. To enter firms have to pay a sunk entry cost, c_e, expressed in terms of labor. This cost can be interpreted as an irreversible investment into setting up the production facilities. After paying the initial cost, firms draw their initial a and q from a common bivariate density function, $\gamma(a, q)$. The associated distribution is denoted by $\Gamma(a, q)$ and has support in $\mathbb{R}_+ \times \mathbb{R}_+$. Define $\overline{\gamma}_e$ the mean of the joint distribution and σ^2_{ea} and σ^2_{eq} the variances of the entrants efficiency and quality processes.[15] Moreover, as in Gabler and Licandro (2005) and Luttmer (2007) I assume that entrants are on average less productive than successful incumbent and that they imitate them. In particular, the mean of the entrant distribution is a constant fraction $\psi_e \in (0, 1)$ of the mean of the joint distribution of incumbents defined as $\overline{\mu}$. That is, $\overline{\gamma}_e = \psi_e \overline{\mu}$. This knowledge spillover, that goes from incumbent firms to entrants, is the only externality of the model and combined with firm selection and innovation generates endogenous growth.[16]

[14] Appendix A defines mathematically these supports.

[15] The covariance is zero given the current assumption of independence between the evolution of the two states.

[16] Eeckhout and Jovanovic (2002) used a wider mechanisms of knowledge spillover in which all firms and not only entering firms, can imperfectly imitate the whole population of firms.

In equilibrium the free entry condition holds: potential entrants enter until the expected value of entry is equal to the entry cost:

$$v^e(a,q) = \int_{\Omega_e} v(a,q) d\Gamma(a,q) = c_e, \tag{16}$$

M_t is the mass of firms that enter in the industry at time t. At the stationary equilibrium also a stability condition holds: the mass of new entrants exactly replaces the mass of unsuccessful incumbents who are hit by a bad shock and exit the market. That is, $M' = \int_0^{a_x(q)} \int_Q I\mu(a,q)$.

2.3 Cross Sectional Distribution and Aggregates

All firms' choices and the processes for the idiosyncratic shocks yield the low of motion of firms distribution across efficiencies and qualities, $\mu(a,q)$. That is:

$$\begin{aligned} I'\mu'(a',q') = I\Bigg(&\int_{A_P}\int_Q \mu(a,q)\phi(a',q'|a,q)\mathrm{d}q\mathrm{d}a + \\ &\int_{A_A}\int_Q \mu(a,q)\phi(a',q'|a,q,z)\mathrm{d}q\mathrm{d}a + \int_{A_{AQ}}\int_Q \mu(a,q)\phi(a',q'|a,q,z,l)\mathrm{d}q\mathrm{d}a + \\ &\int_{A_Q}\int_Q \mu(a,q)\phi(a',q'|a,q,l)\mathrm{d}q\mathrm{d}a\Bigg) + M'\gamma(a',q') \end{aligned} \tag{17}$$

Tomorrow density is given by the contribution of all surviving firms (the domain of the integrals is restricted to surviving firms only) and of entrants. The contribution of new firms is represented by the last term of (17). The first integral represents the share of surviving firms that only produce and do not innovate, the second integral shows the contribution of the firms that successfully produce and invest in process innovation. The third one instead represents the firms that produce and undertake both types of innovation and finally the forth one highlights the share of producers that specialize in product innovation only.[17]

[17] Since the industry is populated by a continuum of firms and only independent idiosyncratic shocks occur the aggregate distribution evolves deterministically. As a consequence, though the identity of any firms i associated with a couple (a,q) is not determined, their aggregate measure is deterministic. For the same reason the other aggregate variables evolve deterministically.

To summarize the information about the average firm efficiency and product quality, a weighted mean of firm technology can be introduced. That is:

$$\overline{\mu} = \left(\int_{a_x(q)} \int_Q \left(a q^{1-\eta} \right)^{\frac{\alpha}{1-\alpha}} \mu(a,q) \mathrm{d}q \mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}}. \tag{18}$$

Notice that $aq^{1-\eta}$ is an index of firm level technology that maps one to one to firms' profits and size. Differing from Melitz (2003), this weighted mean not only depends on two states, efficiency and quality, but also the weights reflect the relative quality adjusted output shares of firms with different technology levels rather than the simple output shares. Moreover, the weighted mean can be also seen as the aggregate technology incorporating all the information contained in $\mu(a,q)$. In fact, it has the property that the aggregate variables can be expressed as a function of only $\overline{\mu}$ disregarding the technology distribution, $\mu(a,q)$.[18]

2.4 Equilibrium Definition

In equilibrium the representative consumer maximizes its utility, firms maximize their discounted expected profit and markets clear. The stationary equilibrium of this economy is a sequences of prices $\{p_t\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$, real numbers $\{I_t\}_{t=0}^{\infty}$, $\{M_t\}_{t=0}^{\infty}$, $\{X_t\}_{t=0}^{\infty}$ functions $n(a,q;\mu)$, $z(a,q;\mu)$, $l(a,q;\mu)$, $v(a,q;\mu)$, cutoff functions $a_x(q)$, $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ and a sequence of probability density function $\{\mu_t\}_{t=0}^{\infty}$ such that:

- the representative consumer chooses asset holding and consumption optimally so that to satisfy the Euler Equation (5),
- all active firms maximize their profits choosing a price that satisfies (7) and employment and innovation policies that satisfy $n(a,q;\mu)$, $z(a,q;\mu)$, and $l(a,q;\mu)$ yielding the value function $v(a,q)$ as specified by equation (13) and its components,
- innovation is optimal such that the cutoff functions $a_A(q)$, $a_{AQ}(q)$, and $a_Q(q)$ satisfy the previous conditions,

[18]See Appendix B for more details.

- exit is optimal such that $a_x(q)$ is given by equation (15) and firms exit if $a(q) < a_x(q)$,
- entry is optimal: firms enter until equation (16) and the aggregate stability condition are satisfied,
- the number of active firms I adjusts till the labor market clears: $L^P + L^I + Ic_f + M'c_e$.[19]
- the stationary distribution of firms evolves accordingly to (17) given μ_0, I, M and the cutoff values,
- the stability condition, $M' = \int_0^{a_x(q)} \int_Q I\mu(a,q)$, holds.

In equilibrium a_x, a_A, a_{AQ}, a_Q, I and M are such that the sequence of firms distribution is consistent with the law of motion generated by the entry and exit rules.[20]

3 Endogenous Growth

3.1 Balanced Growth Path

In general, on the Balanced Growth Path output, consumption, real wage, prices and the aggregate technology grow at a constant rate, the bivariate distribution of efficiency and quality shifts to the right by constant steps, its shape is time invariant, and the interest rate, the aggregate expenditure, the aggregate profit, the profit and the labor demand distributions, the number of firms, the firm turnover rate, and the other characteristics of the firms' distribution are constant.

Define g as the average growth rate of firm productivity, $\overline{\mu}$. It is given by a combination of the growth rate of the efficiency state, denoted by g_a, and of the growth rate of the product quality state, indicated by g_q. Intuitively, growth arises because in every period the log of the joint aggregate technology shifts to the right

[19]Where $L^P = \int_A \int_Q n(a,q)I\mu(a,q)\mathrm{d}q\mathrm{d}a$ is the production labor and $L^I = \int_A \int_Q (l(a,q) + z(a,q))I\mu(a,q)\mathrm{d}q\mathrm{d}a + I\int_{A_A} \int_Q \mu(a,q)c_a \mathrm{d}q\mathrm{d}a + I\int_{A_Q} \int_Q \mu(a,q)c_r \mathrm{d}q\mathrm{d}a + I\int_{A_{AQ}} \int_Q \mu(a,q)(c_a + c_r)\mathrm{d}q\mathrm{d}a$ is the innovation labor considering both the variable and fixed costs.

[20]Hopenhayn (1992)'s paper proves the existence of equilibrium for similar economies.

by a factor g, meaning that the average efficiency and the average product quality of the industry grow. Defining the growth factors of firm efficiency and product quality by $G_A = \frac{a_{t+1}}{a_t} = 1 + g_a$ and $G_Q = \frac{q_{t+1}}{q_t} = 1 + g_q$, the Balanced Growth Path can be found as follows. From the labor market clearing condition, given the assumption of a constant labor supply, N_s, also the number of incumbent firms, I, and the number of entrants, M, have to be constant as well as the share of labor allocated to production and innovation.[21] Aggregate expenditure, E, has to be equal to the aggregate labor income, N_s, given the wage normalization. This in turn implies that E is constant and hence also Π has to be constant. The profit distribution, equation (10), shows that $\pi(a, q)$ has to be constant because of constant fixed operational costs. Given a constant expenditure, profits are constant only if $aq^{1-\eta}P$ is constant. For positive growth rate of the technology, the previous condition holds if the price index growth factor is inversely related to the average technology growth factor, $G_P = (G_A G_Q^{1-\eta})^{-1}$. In other words, as the industry grows and the average technology advances, the price index diminishes. With the same reasoning also the distribution of manufacturing labor, equation (9), is time invariant, which together with the labor market clearing condition implies that also the distributions of the labor hired for the innovation activities, $z(a, q)$ and $l(a, q)$, are constant. From the consumer problem $E = PX$, which holds only if the aggregate consumption X grows at a constant factor $(G_A G_Q^{1-\eta})$. This results in a constant interest rate as shown by the Euler equation, $r = (1 + g)\beta - 1$. The price distribution, $p(a, q)$, decreases at a factor equal to $\frac{G_Q^{\eta}}{G_a}$ which is lower than the growth rate of the price index. This is a consequence of the fact that the price index is adjusted to consider the growth in the product quality. Finally, $x(a, q)$ grows at a factor of $\frac{G_A}{G_Q^{\eta}}$.

A Balanced Growth Path equilibrium exists if there is a g_a and a g_q consistent with the stationary equilibrium. To find these growth rates and to characterize the equilibrium itself and the stationary firms' distribution it is necessary to transform the model such that all the variables are constant along the Balanced Growth Path. Hence, all growing variables need to be divided by the corresponding growth factor, $\widetilde{s} = s/G_s^t$ and the stochastic processes in efficiency and quality need to be detrended by the respective growth rates, $\log \widetilde{a}_t = \log a_t - g_a t$ and $\log \widetilde{q}_t = \log q_t - g_q t$,

[21] If there was population growth then the number of varieties, and the number of entrant firms would grow at the same rate as population grows.

where "$\sim$" denotes the stationarized variables. In expected terms both average firm efficiency and average quality increase and thus in expectation in every period each firm falls back relative to the distribution. This transformation affects also the transition functions and hence log efficiency and log quality, in the stationarized economy, which evolve according to:

$$\log \widetilde{a}_{t+1} = \begin{cases} \log \widetilde{a}_t - g_a + \varepsilon^a_{t+1} \\ \log \widetilde{a}_t - g_a + \lambda^a \log \widetilde{z}_t + \varepsilon^{az}_{t+1} \end{cases} \tag{19}$$

$$\log \widetilde{q}_{t+1} = \begin{cases} \log \widetilde{q}_t - g_q + \varepsilon^q_{t+1} \\ \log \widetilde{q}_t - g_q + \lambda^q \log \widetilde{l}_t + \varepsilon^{ql}_{t+1}. \end{cases} \tag{20}$$

These negative trends together with decreasing return in innovation determine a finite expected lifetime for any level of technology (a, q). Any successful firm which performs innovation will not be an innovator forever but eventually it will exit the market, leading to a finite expectation and to a finite variance of the incumbent firm distribution and hence assuring the existence of a stationary distribution in the de-trended economy.

The previous discussion leads to the following proposition:

Proposition 1: *Given G_a and G_q growth factors of firms efficiency and quality the economy admits a Balanced Growth Path along which the mean of the joint distribution of incumbent firms and of entrant firms and the aggregate consumption grow at a rate $G_a G_q^{1-\eta}$, the price index decreases at a rate $G_a G_q^{1-\eta}$, the output distribution grows at a rate G_a/G_q^η, the price distribution grows at a rate G_q/G_a^η and the number of firms, the number of entrants, the aggregate expenditure, the aggregate profits, the profit distribution, and the labor distributions are constant.*

3.2 Growth Rate Determinants

Firms' *Selection* and *Innovation* drive endogenous growth which is then sustained by entrants' *Imitation*. Firm selection results from the assumption of a random walk process for both the evolution of labor efficiency and product quality together with firm exit. Considering only a cohort of firms and abstracting from the endogenous drift introduced by innovations, in the growing economy the random walk processes are characterized by constant expectations and by variances of the distribution of those firms that increase over time. However, among the given firms the ones with low efficiency and low quality exit the industry truncating the joint distribution from below. This implies that the distribution can spread only towards higher level of efficiency and quality resulting in a higher average productivity of the remaining firms in the cohort.

Firms' innovation reinforces growth. For a given set of innovative firms also the productivity and quality expectations increase over time and they depend on the initial states and on the sequences of innovation investments. In fact, after every successful innovation the average technology shifts upwards due to the endogenous drifts generating growth. However, innovation has decreasing returns through the log form in which the innovation drift is modeled. For this reason the resource reallocation effect from non-innovators to innovators is controlled by the selection effect and the result is that growth is reinforced but still bounded. As a result the average productivity of innovators grows slower than the exit cutoff. Consequently, as time goes by firms keep exiting the industry and the distribution shrinks.

Hence, entrants' imitation is needed to sustain growth and assure the existence of a stationary distribution with entry and exit. In equilibrium the mass of entrants has to be equal to the mass of firms exiting the market. However entrants are on average more productive than exiting firms otherwise they would not find optimal to enter the market. Since exiting firms are replaced by entrants with on average better efficiency and quality levels, the resulting firm distribution moves every period upwards towards higher technological levels.[22] Notice that innovation affects growth also allowing for better imitation.

When innovation occurs the efficiency and quality processes have also higher variances of the stochastic component. This increases the probability of a bad shock hitting the innovative firms and the dispersion of the innovator distribution against the distribution of non-innovators and exiting firms. On the one hand, selection results in a higher average technology for innovators because relatively bad firms fall among the pool of non-innovators resulting in a scenario where only relatively low cost and high quality firms keep innovating. On the other hand, the

[22]Randomness and innovation are important to emphasize the fundamental role of reallocation of resources in the growth process. Growth could still be generated without selection and innovation assuming that the joint mean of the entrants distribution shifts every period exogenously by g. However in this way growth would just result from entry and exit.

pool of non-innovators becomes larger, implying a higher weight to the distribution of non-innovators which has a lower average technology. The final effect of higher variances of the innovation random walks on the mean of the joint distribution is ambiguous. However, calibrating the model to match the Spanish data shows that the positive effect of innovation always outweighs the negative effect.

3.3 Growth Rate Decomposition

On the Balanced Growth Path the growth rate of aggregate and average consumption is the same and can be rewritten and approximated (the derivations are in the Appendix) as:

$$g \approx \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big) \right] \mathrm{d}q\mathrm{d}a \right\}, \tag{21}$$

where $\bar{X}$ is the average consumption, $\hat{x}(a,q) = qx(a,q)$ is the firm's quality weighted output, Φ_{xI} is the transition function with the exit and innovation rules and M/I is the entry/exit equilibrium rate. The first difference into the squared bracket represents the growth contribution of selection and innovation. That is, the difference between the quality-output weighted average productivity of surviving firms (both innovators and non innovators) and the one of the previous period incumbents. The more significant the innovation investment is, the larger $\Phi_{xI}\mu$ and the tougher selection is, the smaller $(1 - M/I)\mu$. Hence, both more innovation and tougher selection promotes growth. The second difference instead represents the contribution of entrants' imitation. The easier or cheaper the imitation mechanism (the smaller the distance between the entrants' and incumbents' distributions) the larger the contribution of entrants to the aggregate growth. Adopting the terminology introduced by Poschke (2008), μ can be divided into μ_{con}, continuing firms, and μ_{exit}, exiting firms. This allows for a further disaggregation of the aggregate growth rate:

$$\begin{aligned} g \;\approx\; & \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi\mu_{con}(a,q) - \mu_{con}(a,q)\right] \mathrm{d}q\mathrm{d}a + \right. \\ & + \left. \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\frac{M}{I}\gamma(a,q) - \mu_{exit}(a,q) \right] \right\}. \end{aligned} \tag{22}$$

The first integral catches the share of growth due to firms' innovation activities and due to the idiosyncratic shocks hitting surviving firms' level technology.[23] The second integral instead represents the share of growth due to net entry. It is clear that the selection of inefficient firms exiting the market and the imitation of new entrants generate positive growth only if entrants are on average more productive than exiting firms. This condition holds in the stationary equilibrium with positive entry. Furthermore, splitting the density of continuing firms between the densities of firms that only produce, μ_p, and of firms that innovate and produce, μ_i, the first integral in equation (22) can be further disaggregated in:

$$\begin{aligned}&\int_A \int_Q \hat{x}(a,q)^\alpha \left[\Phi\mu_{con}(a,q) - \mu_{con}(a,q)\right] \mathrm{d}q\mathrm{d}a = \\ &\int_A \int_Q \hat{x}(a,q)^\alpha \left[(\Phi\mu_p(a,q) - \mu_p(a,q)) + (\Phi\mu_i(a,q) - \mu_i(a,q))\right] \mathrm{d}q\mathrm{d}a. \end{aligned} \quad (23)$$

Among surviving firms it is now possible to calculate the share of growth that is due to only firms' experimentation based on the random walk processes without drift and the share of growth due to both experimentation and firms' innovation. The numerical analysis of the model will then quantify the share of growth due to net entry, innovation together with experimentation, and firms' experimentation.

The innovation investments of firms affect aggregate growth both directly and indirectly through a better imitation. In fact, innovation results in a higher joint mean of the incumbents' distribution and hence on entrants that can draw their initial technology from a distribution that stochastically dominates the distribution of entrants in an economy without innovation. Given that $\bar{\mu}$ is the key variable in the imitation process, the contribution of innovation on a better imitation can be assessed rewriting $\bar{\mu}$ as:

$$\bar{\mu} = \left(\int_{A_P} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_p(a,q) \mathrm{d}a\mathrm{d}a + \int_{A_I} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} \mu_i(a,q) \mathrm{d}q\mathrm{d}a \right)^{\frac{1-\alpha}{\alpha}} \quad (24)$$

[23]Without weighting the firm distribution by the share of quality weighted output the resulting expected growth rate of the average technology of continuing firms would be zero. However, given that the optimal consumption is a convex function of the technology index $aq^{1-\eta}$, by Jensen inequality, the average growth rate of the output weighted technology is positive.

and using the following equation:

$$1 = \frac{1}{\bar{\mu}^{\frac{\alpha}{1-\alpha}}} \left(\int_{A_P} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_p(a,q) \mathrm{d}q \mathrm{d}a + \int_{A_I} \int_Q (aq^{1-\eta})^{\frac{\alpha}{1-\alpha}} u_i(a,q) \mathrm{d}q \mathrm{d}a \right), \tag{25}$$

where A_P is the support of surviving firms that produce but do not innovate while $A_I = A_A \cup A_Q \cup A_{AQ}$ is the support of firms that produce and innovate. The second integral captures the contribution of innovation in determining the joint mean of the incumbent firms. It is clear that the larger this term is, the higher the indirect growth contribution of innovation via a better imitation.

4 Numerical Analysis

The algorithm, used to solve the model in the stationary equilibrium, is explained in Appendix D.

4.1 Calibration

Sixteen parameters, linked to firm dynamics characteristics, firms specific innovation behavior and the general economic environment, need to be chosen. Since all of them interact with each other to determine the stationary equilibrium only the discount factor, β, the preference parameter, α, and the imitation parameter, ψ_e are chosen *a priori*. The others are jointly calibrated to match the Spanish manufacturing sector.[24] In detail, β is set equal to 0.95 to analyze a yearly time span. Accordingly to Ghironi and Melitz (2003), α is set equal to 0.73, so that the price mark-up charged by the monopolistic firm is of 36% over the marginal cost.[25] ψ_e, relating the mean of the entrants distribution with the mean of the

[24] The Spanish economy has been empirically widely studied in both the dimensions object of this paper: the new dimension related to firm innovation behavior and the traditional dimension related to firm dynamics. Hence, from the Spanish data it is possible to obtain enough information to calibrate successfully the model. Similar studies are available also for other European countries (Bartelsman et al. (2004), Bartelsman et al. (2003) for OECD countries; Cefis and Marsili (2005) for the Netherlands, Smolny (2003) and Fritsch and Meschede (2001) for Germany).

[25] This high mark-up could be seen at odds with the macro literature that delivers a standard mark-up of around 20% over the marginal/average cost. In this model, a higher mark-up is

incumbents, is a key parameter in determining growth. For this reason it is set individually to match its empirical counterpart. That is, ψ_e is chosen such that the average size of entrants is 38% of the size of incumbent firms as estimated by Gracia and Puente (2006).

Twelve parameters are calibrated using a genetic algorithm as described by Dorsey and Mayer (1995).[26] These are: the ratio among the fixed costs, c_e/c_f, c_a/c_f, and c_q/c_f, the quality parameter η, the four variances of the incumbent random walks σ_a, σ_{az}, σ_q, and σ_{ql}, the two variances of the entrant random walks, σ_{ea} and σ_{eq}, and finally the two parameters that scale the innovation drifts into the stochastic processes, λ_a and λ_q. These parameters jointly determine the shape, the truncation functions of the stationary distribution of firms, and the partition of firms among the different innovation strategies. They are calibrated, using as targets, static and dynamic empirical moments that are informative and related to the main objective of the paper. It is possible to distinguish between two sets of targets.

Firstly, I use moments related to the literature on firm dynamics. These are firms' survival rates after two and five years upon entry, firms' yearly turnover rate, the job creation rate due to entry, the fraction of firms below average productivity, and the productivity spread, which calibrate the six variances of the model and the size of entrants with respect to exiting firms which gives information about the entry cost. Accordingly to Garcia and Puente (2006), the two and five year survival rates for Spanish manufacturing firms are estimated to be 82% and 58%, respectively.[27] They report also a yearly firm turnover rate of 9% and a job creation rate due to entry equal to 3%.[28] Garcia and Puente (2006), estimate that entrants

justified by the presence of the fixed costs. In fact, given the free entry condition, firms on average break even. Hence on average, firms price at the average cost leading to reasonaby high mark-ups over the average cost.

[26]The object of the algorithm is to jointly calibrate the parameters in order to minimize the mean relative squared deviation of twelve model moments with respect to the corresponding moments in the data. Since the problem is highly non-linear, the minimization can be characterized by many local minima and the genetic algorithm used has the nice feature to increase the probability of choosing the global minimum.

[27]Those numbers are aligned to the one reported by other developed countries as UK, Germany and Nederland (Bartelsman et al. (2003)).

[28]Firms' turnover is computed as the sum of the number of entering and exiting firms over the total number of firms while job creation rate is computed as the total amount of labor employed

firms are 23% bigger than exiting firms in terms of employment. Bartelsman et al. (2004) estimate that the fraction of Spanish firms below average productivity is equal to 83%, highlighting a right skewed firm size distribution. The last moment is the productivity spread between the 85^{th} and 15^{th} percentile which is estimated to be between 3 and 4.

A second set of moments are instead taken from the empirical literature on firm innovation. The targets used are the share of Spanish manufacturing firms performing process innovation, product innovation and the share of firms that do not innovate and the intensity of the innovation investments in process and product, respectively. In the scope of this paper these are relevant moments that help to calibrate the fixed cost of process and product innovation, η, λ_a, and λ_q. Harrison et al. (2008) working on data derived from the CIS report that 12.2% of Spanish firms in the manufacturing sector declared process innovation between 1998 and 2000, while 12.4% declare product innovation and more than half of the firms do not innovate in the time span considered. This numbers are very close to the one published by the National Statistics Institute (www.ine.es) using the ESEE. The innovation intensity, computed as the ratio between the aggregate investment in innovation and the aggregate sales, in the 1998 is of 1.71%, process innovation intensity accounts for 1.26% while product innovation intensity accounts for the remaining 0.44%.[29]

Finally, the last parameter to calibrate is the growth rate of the economy, g. In fact, the aim of this paper is to provide a model able to disentangle the contribution of efficiency and quality improvements in explaining the economy growth rate and not to test the ability of the model in matching the aggregate growth rate. For this reason g is set equal to 0.042 accordingly to the European Innovation Scoreboard (2001) and represents the labor productivity growth measured in terms of value added per worker as average over the nineties.

Table 2 shows the values assigned to the parameters characterizing the economy. The fixed costs are expressed in relation to the average employment devoted

by entering firms in a year divided by the total employment in the same year.

[29] The European Innovation Scoreboard 2001 reports an innovation intensity for the Spanish manufacturing sector in the 1998 of 2.4% of aggregate sales. This number has been computed on the basis of the CIS which includes also external $R\&D$ investments. This can explain the different numbers between the Euroean Commission survey and the INE statistics.

Table 2: Calibration

Parameter	Value	Description
Calibrated Parameters		
c_e	142.28%	Entry cost, % of average firm size
c_f	3.85%	Fixed cost, % of average firm size
c_a	31.96%	Process innovation cost, % of average firm size
c_q	16.29%	Product innovation cost, % of average firm size
η	0.74	Quality parameter
σ_a	0.15	Variance of efficiency shock
σ_{az}	0.9	Variance of efficiency shock with innovation
σ_q	0.32	Variance of quality shock
σ_{ql}	1.2	Variance of quality shock with innovation
σ_{ea}	0.40	Variance of efficiency distribution of entrants
σ_{eq}	0.48	Variance of quality distribution of entrants
λ_a	0.083	Scale coefficient for process innovation
λ_q	0.025	Scale coefficient for product innovation
Parametrization		
β	0.95	Discount factor
α	0.73	Preference parameter
θ	0.38	Relative entrant mean

to production. As expected the entry cost, which represents a sunk entry investment, is the highest. Reasonable values are attributed to the fixed cost of both process and product innovation. The parameter associated with the difficulty to produce high quality, η, is just above α.[30] When new firms enter the market there is high uncertainty on their profitability, and the probability of surviving the market competition is low. However, the growth rate of surviving young firms is on

[30]Bils and Klenov (2001) estimate quality Engel curves for 66 durable goods in US using data on consumers expenditures. They find that the weighted average slope of the quality Engel curve is of 0.76. This number is very closed to the calibrated η of this model.

Table 3: Empirical Targets and Model Statistics

Targets	**Data**	**Model**
Targets for Calibration		
Share process innovation	12.2%	13.4%
Share no innovation	55.4%	60.92%
Share product innovation	12.4%	11.1%
Product innovation intensity	0.44%	0.5%
Process innovation intensity	1.26%	1.29%
2 year survival rate	0.8	0.74
5 year survival rate	0.58	0.6
Firm turnover rate	0.09	0.086
Firm below average productivity	0.83	0.78
Job creation due to entry	0.03	0.02
Size entrants wrt exiting firms	1.23	1.31
Productivity spread	[2, 3]	2.48
Targets for Parametrization		
Entrant size/incumbent size	0.38	0.38
Mark-up over marginal cost	0.37	0.37
Growth rate of labor productivity	0.042	0.042

average higher than the growth rate of incumbents. This fragility is represented by a variance of the entrants distribution that is higher than the variance of the random walk process associated with a and q when firms only produce.[31] Innovation also increases uncertainty. This is reflected by higher variances of the corresponding random walk processes. In particular, a very high variance is associated with product innovation.[32]

[31] For OECD countries the higher uncertainty faced by entering firms is documented by Bartelsman et. al. (2004).

[32] The higher uncertainty of product innovation is, for instance, documented by Parisi et. al.

Table 3 reports the empirical targets used and the corresponding model moments. Despite the large number of parameters to calibrate, the model statistics match closely the data in both sets of targets. Hence, the innovation choices of firms, the shape of the distribution, its dynamic characteristics, and entrants' behavior seem to reproduce accurately the Spanish manufacturing sector.

4.2 The Role of Innovation

After setting g equal to 4.2%, the model predicts an annual growth rate of firms' production efficiency, g_a, of 2.93% and of product quality, g_q, of 4.64%. Using that $g \approx g_a + (1-\eta)g_q$, 69.8% of the aggregate growth is due to the growth in firms' level efficiency and that only 29.81% is due to the growth in product quality.[33] Though these figures represents the growth in efficiency and quality due to both innovation and randomness, they confirm a higher impact of efficeny in explaing growth accordingly the estimates reported by Huergo and Jamandreu (2004).

Equations (22) and (23) are used to distinguish the effect of innovation and firm experimentation, selection, and imitation in determining the aggregate growth rate. The model predicts that 8.63% of the growth is due to entry (10.61%) and exit (−1.98%) and the remaining 91.37% is due to both experimentation and innovation of the firms that remain active in the industry. Hence, incumbent firms represent the main source of growth in the Spanish manufacturing sector.[34] Decomposing further the growth contribution of incumbents in the contribution of non-innovators and innovators helps to asses the important role played by innovative firms in determining the aggregate growth rate. In fact, the growth con-

(2006).

[33]In equilibrium $(1+g) = (1+g_a)(1+g_q)^{(1-\eta)}$ holds. Approximating it using a logarithmic transformation yields $g \approx g_a + (1-\eta)g_q$.

[34]Farina and Ruano (2004) estimate that the within firm growth accounts for 58% of the aggregate Spanish productivity growth while net entry accounts between 5% and 10% and the remaining part is due to reallocation of resources between contractiong and expanding incumbents. This numbers are in line with Bartelsman et. al. (2004). Their general finding is that the role of entry and exit in explaining productivity growth is marginal compared with US. Foster et. al. (2001) find that in the U.S. Census Manufactures, more than a quarter of the increase in aggregate productivity between 1978 and 1997 was due to entry and exit. Moreover, Lenz and Mortensen (2008) estimating their model on a panel of Danish firms find that entry and exit of firms can account for 20% of the aggregate growth while within firm growth account for 55%.

tribution of non-innovators is negative (-8.34% of the 91.37%). These firms are characterized by a low level of technology and are destined to exit the market after a series of bad shocks. The high likelihood of receiveing a bad shock and the firm's powerlesseness to escape exit explains their negative contribution to growth. This negative effect is more than compensated by the growth contribution of innovative firms that develops to be the leading force of aggregate growth. However, it should be noticed that the growth derived by innovators is a combined effect of the within firm growth, of the reallocation of resources between incumbents and of tougher selection.

More insights on the importance of innovation can be obtained simulating an economy with the same parameters values in which innovation is shut down and growth is generated by only selection and imitation. In this example the share of aggregate growth due to g_a is fixed to 69.8% given the previous results and the aggregate growth rate, g is now determined endogenously. In the absence of innovation the growth rate is 1.1% falling of 3.1 percentage points. This confirm the fundamental role of innovation in explaing productivity growth in the Spanish manufacturing sector.[35]

Additionally, innovative firms have a higher weighted mean of their technology index than non-innovators. This implies that innovation increases the weighted mean of the technology distribution of active firms, that is used as reference by the entering firms. Hence innovation also means better imitation and therefore higher growth. Applying equation (25), it is possible to conclude that 84.31% of the joint mean is due to the average technology level reached by the innovative firms.

4.3 Firms Partition and Cutoff Functions

Figure 1 displays how the two attributes of firm heterogeneity together with the fixed operational and innovation costs determine the partition of firms between those exiting and remaining, and among process innovators, product innovators, and both types of innovators or non-innovators. Hence, it illustrates the equi-

[35]The growth reduction is accompanied by a lower turnover rate equal to only 1.57% showing how innovation increases also market selection. Using equation (22) the growth contribution of net entry reaches 12.1% confirming the importance of within firm growth.

librium cutoff functions and the combinations of efficiency (x-axis) and quality (y-axis) for which the different choices faced by firms are optimal. The firm distribution over the two dimensions of technology (Figure 2, left) is right skewed in both states as the largest mass of firms is concentrated in the bottom-left corner. This information complements the partition of firms and strengthens the subsequent interpretation.

The first area on the left represents the firms with production efficiency and product quality lower than $a_x(q)$ which optimally exit the market. These area represent about 9% of the total mass of firms given by the sum of incumbents and of entrants. The exit cutoff function is the border between the exit region and the region where firms remain active and only produce. Due to the trade-off between quality and efficiency this cutoff function is decreasing in quality: relatively high cost firms can survive longer in the market when the quality of their variety is high. In the second region, for slightly higher level of efficiency and quality, firms are sufficiently profitable to stay in the market but not enough to innovate, $v(a, q) = v^P(a, q)$. These are firms with relatively high level of cost but with all the possible levels of quality. In fact, product quality has a lower impact on firm profitability than production efficiency.

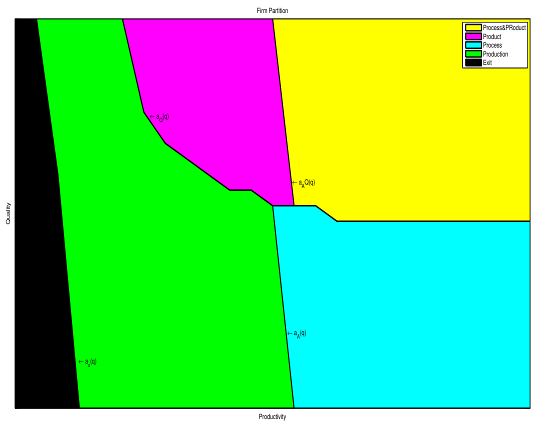

Figure 1: Firms Partition

Moving along the efficiency dimension, for relatively small level of quality, it is optimal for firms to pay c_a and undertake process innovation while for relatively high level of quality it is optimal to pay c_q and undertake product innovation. This is the result of the interplay between the fixed costs of innovation and the convexity of the profit function in a. The higher the efficiency level reached by the firm the higher the gain in terms of profitability resulting from a marginal reduction of the production cost. This explains why it is optimal for firms to innovate in process when their efficiency has already reached a minimum level. The same is true for the quality dimension, though the profit function is concave in q. However this disadvantage is compensated by the lower fixed cost of product innovation. The last region is represented by firms with high efficiency and high quality that optimally innovate in both process and product.

Table 4: Conditional Probabilities

	Exit	**No Innovation**	**Process**	**Product**	**Both**
No Innovation	5.1%	87.84%	0.84%	5.6%	0.21%
Process	0	4.5%	75.9%	0.95%	18.65%
Product	0	34.65%	1.22%	51.84%	12.3%
Both	0	1.83%	33.26%	3.3%	61.61%

Table 4 shows the equilibrium conditional probabilities of switching actions after a one-year period given the current decision of incumbent firms.[36] The first column lists the current action of the firms and the rows give the transition probabilities of each future decision. Due to the persistence of the random walk process a high probability is attached to the repetition of the current action.[37] Interestingly, consistent with the Spanish empirical evidence shown by Huergo and Jaumandreu (2004), this persistence appears less strong in the case of product innovators: 34% of product innovators today will not innovate tomorrow while 15% will switch to process innovation, both alone and with product innovation, and only 51% will repeat an innovation in product quality. The relative low persistence in quality enhancing innovation is due to the high variance associated with this decision. A

[36]This information is contained in the optimal transition function T_{XI} and the derivations are in the Appendix.

[37]This can be read as persistent firms productivity which is documented by the empirical literature in the case of Spain by Garcia et. al. (2008).

high variance implies that the probability of receiving a bad shock is high as well as the probability of switching to a differnt strategy. Empirical evidence emphasises that exit is associated with a low level of pre-exit innovation (Huergo and Jamandreu (2004) for evidence on Spanish firms). This model predicts that an incumbent firm exits the market with 5% of probabilty only if in the current year no innovation has been introduced. This also implies that an innovative firm, before exiting the market, has to receive a bad shock and become a non-innovator.

4.4 Firms Distribution

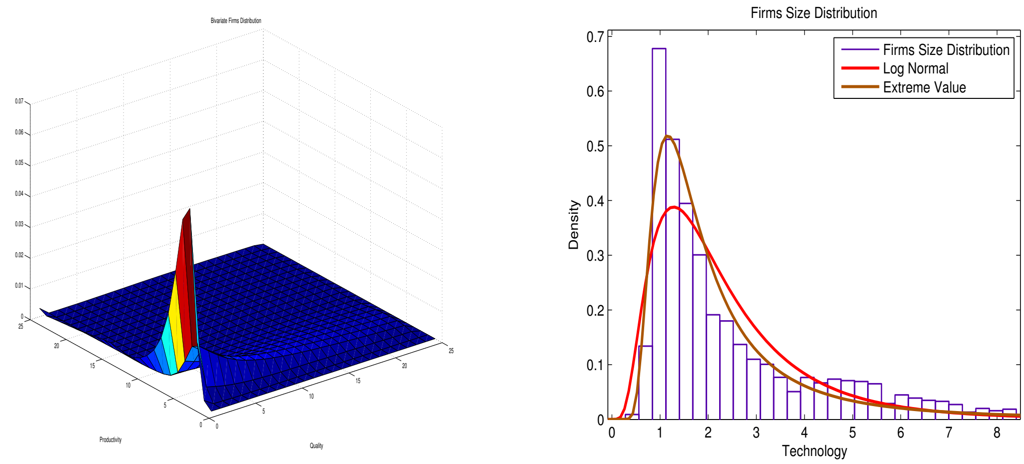

Figure 2: Bivariate and Univariate Firms Distribution

The equilibrium distribution of firms is determined endogenously and it is shaped by the static and dynamic decisions of incumbent firms together with entrants imitation. Figure 2, left panel, shows the bivariate firms distribution over the two attributes of firm heterogeneity. However, empirical studies are not able to distinguish these two dimensions and hence Figure 2, right panel, displays the corresponding univariate firm size distribution over a technological index that summarizes the information contained in a and q. That is, $aq^{1-\eta}$. Notice that this is the equivalent of the employment distribution of firms which is observed in the data. The univariate firm distribution looks right skewed and hence with a right

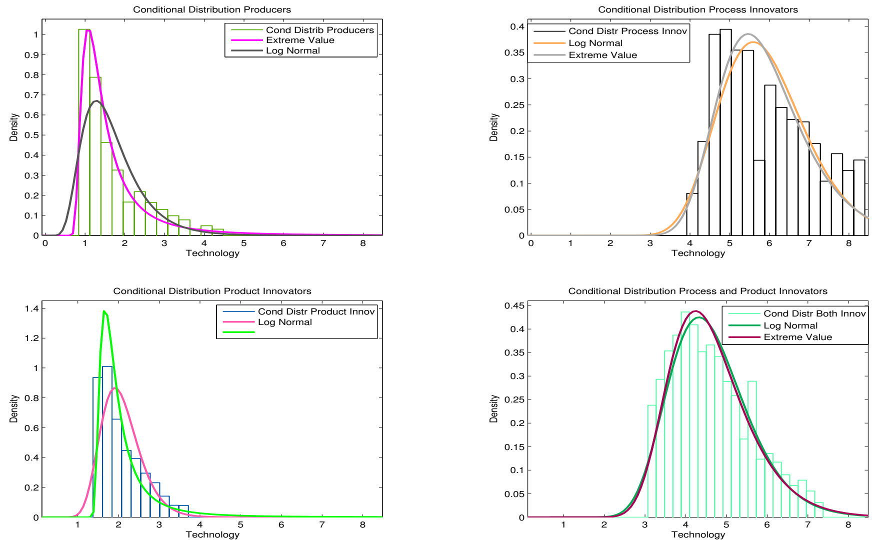

Figure 3: Conditional Firms Size Distributions

thick tail (the moments of the distribution are reported in Table 5).[38] In fact, a log-normal distribution fits the date well. However, empirically there is not much information about the moments of the size distribution of the manufacturing firms in the Spanish economy but in general it is possible to conclude that it is right skewed.[39]

The conditional distribution of firms that only produce and do not innovate is concentrated at lower levels of the technological index $aq^{1-\eta}$ than the conditional distributions of innovators (Figure 3 and Table 5). Consistently with the empirical evidence (see Doraszelski and Jaumandreu (2007)) innovative firms have a higher labor productivity and are bigger than firms that do not innovate. The comparison among innovators is more interestingly: on average small firms do product innovation, medium and large firms do both product and process innovation and

[38]The underlying distribution used to compute the skewness in Table 5 is a log-normal distribution.

[39]See Doraszelski and Jaumandreu (2007) and Garcia and Puente (2006) for Spanish firms. Cabral and Mata (2003) estimate that the distribution of Portuguese firms converge to a log-normal distribution.

large firms do process innovation.[40] Finally, the conditional distribution of product innovators is more right skewed than the distribution of firms that do process innovation or do not innovate. Also this last feature is confirmed by empirical estimations of the firm size distribution in the Spanish manufacturing sector.

Table 5: Descriptive Statistics of Firms Distributions

	Mean	**Variance**	**Coef. of Variation**	**Skewness**
Size Distribution	2.41	3.05	0.72	0.95
Cond. on Process Innov.	5.9	1.26	0.19	0.89
Cond. on Product Innov.	2.08	0.24	0.23	2.32
Cond. on Both Innov.	4.63	0.98	0.21	1.1
Cond. on No innovation	1.67	3.05	0.44	0.95

5 Comparative Statics

This section analyzes how changes in the key parameters of the model, which characterize the industry structure, affect the process of labor reallocation among firms and hence the equilibrium growth rates of the economy. In particular, changes in the innovation costs, c_a and c_q, as well as changes in the entry cost, c_e, are analyzed. Both types of costs are directly linked to growth: changes in c_a and c_q bring changes in the composition of the pool of innovative firms and changes in c_e affect the imitation process of entrants firms. High entry cost are seen as barrier to enter the industry and they are often regarded as a protection of incumbent firms and hence as a stimulus to innovation. On the other hand, high innovation costs are seen as detrimental of innovation. Hence, it becomes important to understand how the economy responds to changes in these key parameters in order to design policy recommendations aimed at fostering growth.

[40]Huergo and Jaumandreu (2004) find that innovation is systematically related to size: large firms have a higher probability of innovating but this size advantage reduces in the case of product innovation.

Using the quantitative results of Section 4.3 let fix the fraction of growth explained by the growth in efficiency to 69.8% and determine edogenously the aggregate growth rate.

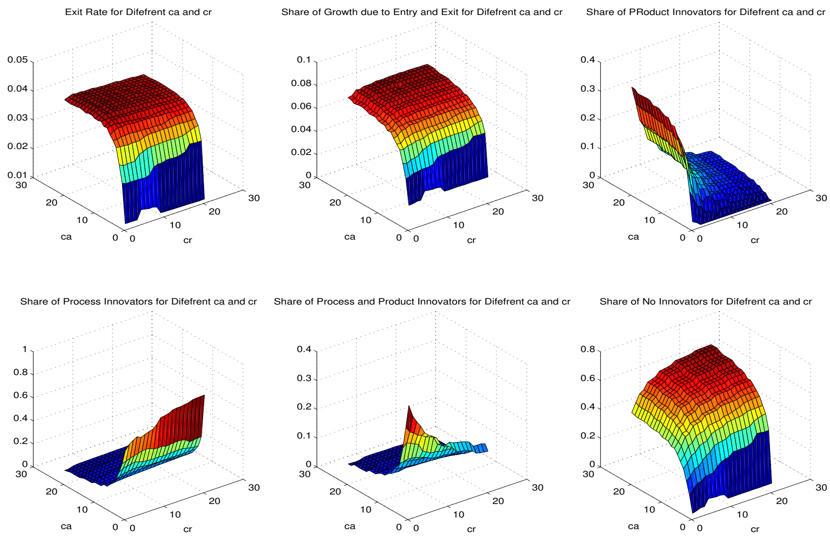

Figure 4: Comparative statics for different c_a and c_q

Figure 5, left panel, plots the equilibrium growth rate for different values of the fixed costs of innovation: on the x-axis the cost of doing product innovation, c_q, while on the y-axis the cost of doing process innovation, c_a. As both the innovation fixed costs decline two opposite effects arise. On the one hand, innovation becomes cheaper and more firms find it profitable. Hence the pool of innovative firms increases and this affects positively and directly the growth rate of the economy (Figure 4). This positive effect is then reinforced by an indirect effect. If the mass of innovators is larger, more firms will pay the fixed costs. This sustains the demand of labor and hence the wage rate, thus assuring a strong selection. On the other hand, if the innovation costs are reduced, less labor is demanded by the individual innovative firm. Consequently, the demand of labor by an innovative firm declines and hence the real wage declines to satisfy the labor market clearing condition. A lower wage translates into a weaker selection and hence in a lower effect on the economy growth rate. The final response of the growth rate to the

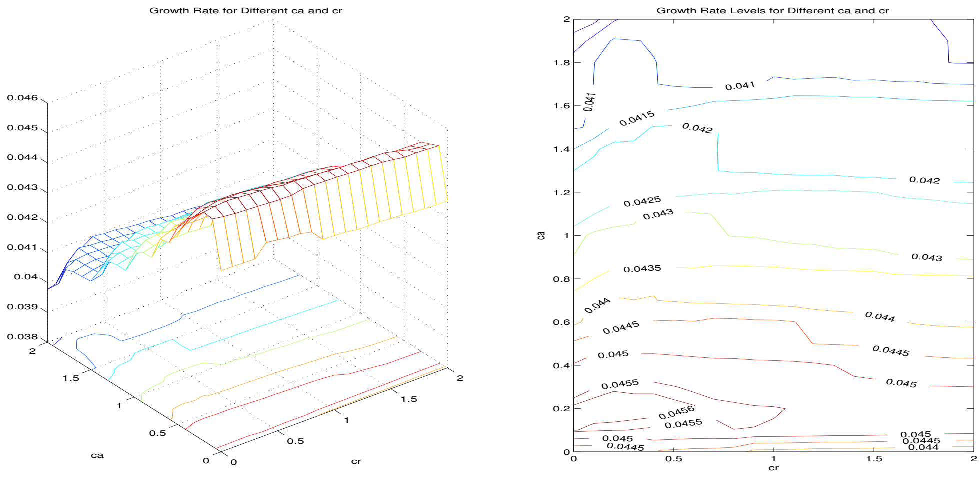

Figure 5: g for different c_a and c_q

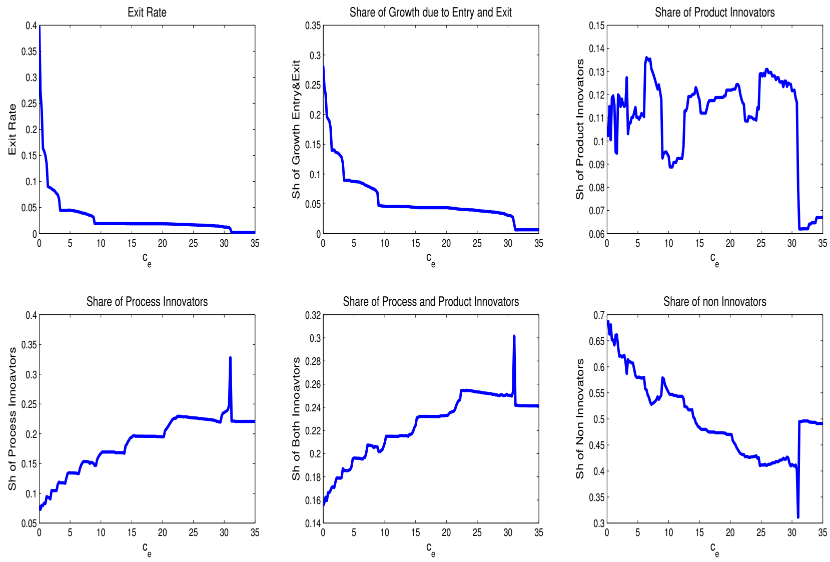

Figure 6: Comparative Statics for different c_e

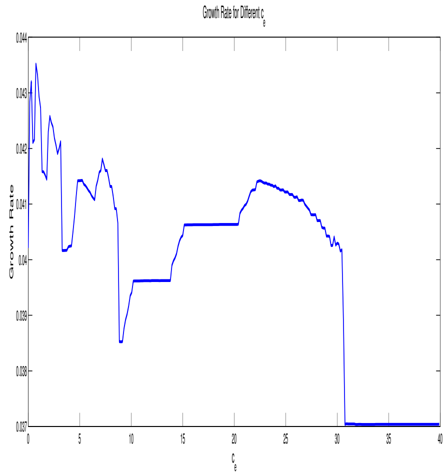

Figure 7: g for different c_e

changes in the innovation costs results from the combination of these two effects. Generally, the positive effect prevails. The lower the innovation costs, the higher the growth rate. This holds true for all the values of the fixed cost of undertaking product innovation but only for high and intermediate value of the fixed cost of doing process innovation. The maximum growth rate is obtained for $c_q = 0$ but small and positive c_a, showing that for very low levels of c_a the negative effect offsets the positive one. Additionally, the economy growth rate is more sensitive to changes in c_a than to changes in c_q. Hence, a policy aimed at promoting only growth would be more successful when used to address an increase in process innovation.

When entry cost are low, imitation is cheap (Figure 6), and many firms enter and exit the market, which results in a high growth rate (Figure 7). As the entry cost increases firm selection and imitation become weaker and the growth rate declines. However higher c_e leads to a higher expected value of entrants which in turn imply that the discounted expected profits of incumbents need to be higher. Hence, progressively the mass of innovative firms increases and this generates an inversion in the direction of the growth rate. However, as the entry barrier increases further the industry becomes more and more concentrated and the number of innovators slightly declines. Thought few firms enter the industry they drain a lot of labor increasing the wage rate and hence innovation becomes more costly.[41]

6 Final Remarks

This paper proposes an endogenous growth model with heterogeneous firms where firms differ in two dimensions: production efficiency and product quality. Both dimensions are subject to idiosyncratic permanent shocks but firms can affect endogenously their evolution through process, product or both types of innovations. Growth arises due to incumbent firms' innovation and selection and is sustained by entrants' imitation. Selection eliminates the inefficient firms from the market, thereby increasing the average productivity of incumbents. Innovation amplifies this not only increasing directly the average technology of firms but also increasing selection. Entrants imitate the average incumbent and are, on average, more productive than exiting firms. The result is that the firm distribution shifts upwards, generating growth.

The economy is calibrated to the Spanish manufacturing sector and closely matches static and dynamic moments related to the firm distribution and new moments related to the innovation behavior of firms. Hence, the model provides an accurate representation of the Spanish economy and an explanation of the heterogeneity in the innovation activities among firms. Improvements in production efficiency explain 69.8% of the output growth while quality upgrading contributes only for the remaining 30.2%. Moreover, decomposing the aggregate growth in the contribution of firm turnover and innovation and experimantation by incumbents shows that net entry contributes only marginally. In fact, more than 90% of growth is due to within and between firms growth and when innovation is banned output growth declines of almost 74%. Innovation is also necessary to survive market competion: only non-innovative firms exit the industry. An unanswered question is to identify which type of innovation, between process and product innovation, allows for a greater period of firms' longevity.

The endogenous firm size distribution is right skewed and approximated well

[41] Notice that when the entry cost is very high the industry is characterized by the absence of entering and exiting firms. This generates the irregularities in the pictures. However, the discussion of the properties of this scenario are not in the object of this paper.

by a log-normal distribution. The conditional distributions of innovators are consistent with the data: innovators are larger than non-innovators and in the case of product innovators also more right skewed. Additionally, small firms do product innovation, intermediate firms do both product and process innovation and large firms do process innovation. Hence, there is a non-monotonic relation between firm size and innovation though firm size is still an indicator of the type of innovation undertaken by firms. The industry growth rate reacts positively to reductions in the innovation costs, however the model predicts that its maximum is reached for a positive but small cost of process innovation. Though entry barriers protect and stimulates innovation, growth is maximized for relatively low entry costs which are accompanied by a more dynamic industry with a high turnover. As the industry becomes more concentrated, the aggregate share of innovators increases however growth is impacted less strongly.

These considerations leads to attractive policy recommendations aimed at fostering growth and welfare. The next step is therefore to compute the optimal allocation and design innovation policies that can implement the first best in the decentralized economy.

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Appendix

A Partitions and Innovation Cutoff Functions

Define $A_x = \{(a,q) : a \in A, q \in Q : a(q) < a_x(q)\}$ the exit support, $A_P = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^P(a,q)\}$ the production support, $A_A = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^A(a,q)\}$ the process innovation support, $A_Q = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^Q(a,q)\}$ the product innovation support and $A_{AQ} = \{(a,q) : a \in A, q \in Q \wedge v(a,q) = v^{AQ}(a,q)\}$ the process and product innovation support.

Let $B = \{(a+\epsilon, q+\epsilon)\}$ for $|\epsilon| > 0$ arbitrarily small. The innovation cutoff function are defined as $a_A = \{(a,q) : (a,q) \in A_A \wedge (A_P \cup A_Q \cup A_{AQ}) \setminus A_A \neq \emptyset\}$, $a_Q = \{(a,q) : (a,q) \in A_Q \wedge (A_P \cup A_A \cup A_{AQ}) \setminus A_Q \neq \emptyset\}$ and $a_{AQ} = \{(a,q) : (a,q) \in A_{AQ} \wedge (A_P \cup A_A \cup A_Q) \setminus A_{AQ} \neq \emptyset\}$.

B Aggregate Variables

Using the information contained in equation (19), the price index, the aggregate consumption, and the aggregate profits can be rewritten as:

$$P = \left(\int_{a_x(q)} \int_Q \left(\frac{p(a,q)}{q(a,q)} \right)^{\frac{\alpha}{\alpha-1}} I\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{\alpha-1}{\alpha}} = I^{\frac{\alpha-1}{\alpha}} p(\overline{\mu}), \tag{26}$$

$$X = \left(\int_{a_x(q)} \int_Q \big(qx(a,q)\big)^{\alpha} I\mu(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1}{\alpha}} = I^{\frac{1}{\alpha}} x(\overline{\mu}). \tag{27}$$

$$\Pi = \left(\int_{a_x(q)} \int_Q \pi(a,q) I\mu(a,q)\mathrm{d}q\mathrm{d}a \right) = I\pi(\overline{\mu}). \tag{28}$$

C Growth Rate Disaggregation

On the Balanced Growth Path, given that the number of firms is constant, the growth factor of aggregate (X) and average ($\bar{X}$) consumption coincides:

$$G = \frac{X'}{X} = \frac{\bar{X}'}{\bar{X}}. \tag{29}$$

Defining the firm's quality weighted output with $\hat{x}(a,q)$, the growth factor can be rewritten as:

$$G = \frac{\left(\int_{a_x(q)} \int_Q \hat{x}(a,q)^{\alpha} \mu'(a,q)\mathrm{d}q\mathrm{d}a \right)^{\frac{1}{\alpha}}}{\bar{X}}. \tag{30}$$

Rewrite μ' using its law of motion yields:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^{\alpha} \big(\Phi_{xI}\mu(a,q) + \frac{M}{I}\gamma(a,q)\big)\mathrm{d}q\mathrm{d}a}{\bar{X}^{\alpha}} \right)^{\frac{1}{\alpha}}, \tag{31}$$

where Φ_{xI} is the optimal transition function with the exit and innovation rules. Adding and subtracting $\bar{X}^{\alpha} = \int_{a_x(q)} \int_Q \hat{x}(a,q)^{\alpha}((1-M/I)\mu(a,q) + M/I\mu(a,q))$ to the numerator and rearranging the equation gives:

$$G = \left(\frac{\int_A \int_Q \hat{x}(a,q)^{\alpha} \big(\Phi_{xI}\mu(a,q) - \big(1-\frac{M}{I}\big)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big)\big)\mathrm{d}q\mathrm{d}a}{\bar{X}^{\alpha}} + 1 \right)^{\frac{1}{\alpha}}. \tag{32}$$

The last step to obtain the growth rate decomposition consists in taking the logarithm of both terms of the equation and approximating them using the rule $\ln(G) \approx g$, given that g is a small number. This results in:

$$g \approx \frac{1}{\alpha \bar{X}^{\alpha}} \left\{ \int_A \int_Q \hat{x}(a,q)^{\alpha} \left[\Phi_{xI}\mu(a,q) - \left(1 - \frac{M}{I}\right)\mu(a,q) + \frac{M}{I}\big(\gamma(a,q) - \mu(a,q)\big) \right] \right\}, \tag{33}$$

which is equation (29) in the main body of the paper.

D Algorithm

The state space $A \times Q$ is discretized. The grid chosen is of 30 points for each state yielding 900 technology combinations, (a,q).[42] Firms' value function is computed through value function iteration. The unknown variables are the growth rates g_a and g_q, which combines in the growth rate of the aggregate technology g, and the aggregate expenditure and price index summarized by $k = P^{\frac{\alpha}{1-\alpha}}E$. The growth rate of labor productivity, g, is fixed exogenously. For given g_a, $g_q = (G/G_a)^{\frac{1}{1-\eta}} - 1$, and k compute the stationary profit $\widetilde{\pi}(a,q;g_a,k)$ and then the firm value function $\widetilde{v}(a,q;g_a,k)$.[43] While iterating the value function, the optimal policies for the investment in process and product innovation, $\widetilde{z}(a,q;g_a,k)$ and $\widetilde{l}(a,q;g_a,k)$, are computed and the random walk processes, that govern the transition of firm productivity and product quality, are approximated using the method explained by Tauchen (1987). This step is time consuming since each firm's problem has to be solved via first order conditions for each single couple of states, (a,q), till convergence is reached. Once the value function is approximated the algorithm computes the cutoff functions $a_x(q;g_a,k)$, $a_A(q;g_a,k)$, $a_Q(a;g_a,k)$, and $a_{AQ}(q;g_a,k)$. Then the transition matrix Φ_{xI} is computed. This is the final transi-

[42]The choice of 30 grid points for each state is due to the fact that the algorithm is computationally heavy given the presence of two states and the endogenization of the dynamic choice of the innovation investment. Increasing the grid size would improve the precision of the calibration but would not affect qualitatively the results. On the other hand, the technology combination (a,q) available to firms would increase quadratically in the grid size and the code would eventually become unfeasible. Hence, given that the results are not qualitatively affected by the grid size, a quality and productivity grid of 30 points is a reasonable restriction.

[43]Notice that all the variables depend on both g_a and g_q. However for notational convenience g_q is omitted since it is a function of g_a.

tion matrix which takes into account the exit and the innovation decisions. After guessing an initial distribution for entrant firms and normalizing its initial joint mean to zero, the expected value of entry is computed. The free entry condition is used to pin down the equilibrium value of k resulting from the first iteration of the algorithm. Using the equilibrium k, the firm value, the cutoff functions, and the transition matrix can be found for given initial g_a. The bivariate firm distribution is then determined using the formula for the ergodic distribution $\widetilde{\mu} = (I - \Phi_{xI})^{-1}\Gamma$ as proved by Hopenhayn (1992). The algorithm is closed using the condition on the mean of the entrant distribution, $\overline{\gamma}_e = \psi_e \overline{\mu}$, and pinning down the equilibrium growth rate, g_a, that satisfies this equation. Once g_a is determined, g_q is determined as well. All these steps are repeated until all conditions are jointly satisfied and convergence is reached.

E Conditional Probabilities

The final transition function $T_{XI}(a', q'|a, q)$ contains all the information to compute the probability that tomorrow a firm will optimally decide to do action $Y \in A'$ given that today it choses action $X \in A$ where A' ={Exit, Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations} and A ={Not to Innovate, Do Process Innovation, Do Product Innovation, Do Both Innovations}. Weighting these probabilities by the firm density in each state allows to calculate the fraction of firms that today chose action X and tomorrow will switch to action Y. Simplify the notation and define a vector of states, s, of all the possible combinations of a and q couples. Indicating with "$\prime$" the next period variables the conditional probabilities are computed as follows

$$P(Y|X) = \frac{1}{\int_{s:A=X} \mu(s)\mathrm{d}s} \int_{s':A'=Y} \int_{s:A=X} \phi(s'|s)\mu(s)\mathrm{d}s\mathrm{d}s'. \tag{34}$$