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Product-Mix Alternatives:

Flood Control, Electric Power, and Irrigation

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by

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The public policy background

One of the perennial debating topics within the economics profession is that of water resource management. The debate usually centers about the fact that major water projects provide certain benefits which are consumed directly by the individual user (electric power and irrigation), and others where the benefits must necessarily be provided on a collective basis to all inhabitants of the river basin (flood control and malaria control).** Within a market

** Navigation and recreation benefits form an intermediate category between public and private goods. As long as the waterways and artificial lakes remain uncrowded, there is zero marginal social cost involved in more intensive utilization. But when congestion sets in, marginal social costs become positive.

economy, the one category would be termed "private" and the other "public" goods. [14]

A multi-purpose project will ordinarily make it possible to provide both private and public benefits at a lower total cost than if undertaken singly. In a market economy, therefore, the policy debate usually takes the form of how high the charges to the private user ought to be. One of the well-understood rules of this discussion is that the private power advocates and the budget balancers do everything possible to overstate the amount of joint costs allocated against power and irrigation, and that the public power and irrigation advocates do just the reverse.***

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*** For a review of various rules-of-thumb proposed to allocate costs that are essentially unallocable, see [4, Ch. IX, and 13, Part Two].

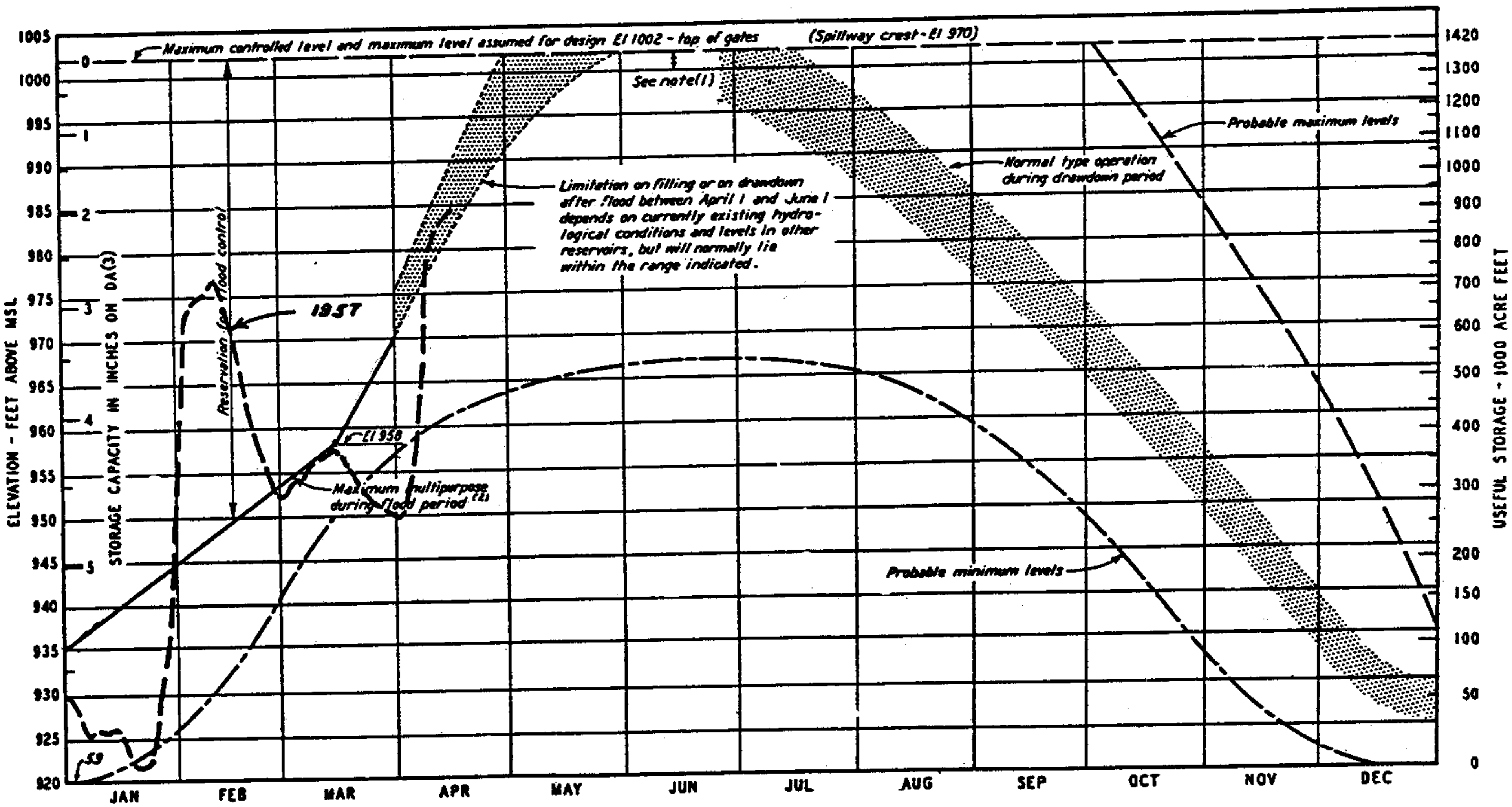
When the MIT-trained economist intervenes in this frequently unedifying dialogue, he is likely to come up with some such prescription as the following: It is idle to attempt to allocate essentially joint costs -- and particularly idle in the case of a project that has already been constructed. What is really at issue is the possibility of shifting the product-mix, accepting a little less of one benefit in order to gain more of another. In principle, it should be possible to construct a product transformation curve, and to trace out the set of alternative benefits that are available from the project. User charges ought then be set in such a way as to steer the users and the water managers toward Samuelson's "bliss point": one at which the marginal rate of physical transformation between products will be identical with the marginal rate of substitution from the viewpoint of some overall social welfare function. [14].

This paper has a much more modest goal than the determination of such a "bliss point." Instead, it is concerned with an intermediate objective: quantitative determination of the set of transformation opportunities between a single class of public benefits (flood control) and several classes of private benefits (power and irrigation). With improved knowledge of these transformation opportunities, it should at least be possible to lay to rest the myth that the U.S. Government is engaged in selling hydroelectric power as a minor byproduct of its flood control activities.

From the viewpoint of economic efficiency, it is fortunate that, despite local political pressures, our water resource managers are not compelled to give absolute priority to flood control. TVA, for example, takes a calculated risk that Tennessee floods are most likely to occur during just three months of the year, January through March. During these months -- but not at other times of the year -- TVA reserves a large fraction of its storage capacity for flood protection. (See Figure 1 for the operation of the Douglas dam and reservoir.)

Figure 1

Source: [5]



DOUGLAS OPERATION – 1957 FLOOD

If the reservoir contents exceed the predetermined flood control line during the January-March period, the reservoir is drawn down as rapidly as the channel capacity near Chattanooga (the critical point) will permit. If the system were really operated for maximum flood control, such reservoirs as Douglas would be maintained at minimum levels at all times of the year -- just to guard against the possibly catastrophic flood that might occur once in every 500 years outside the January-March interval. With such an operation for maximum flood protection -- i.e., low flood control levels at all times -- there would be a serious loss in the system's hydroelectric potential.

In order to evaluate the worth of any reservoir capacity sacrificed in order to obtain flood control benefits, it seems essential to construct a model that takes account of the fact that water inflows are subject to random, as well as to seasonal influences. (One is likely to get a distorted impression of the worth of storage capacity for hydroelectric or irrigation purposes by concentrating upon a single value out of a probability distribution -- whether that single value be the mean or the minimum recorded flow. See Appendix.) The numerical example to be described here is a purely hypothetical one. It is believed, however, that a similar evaluation of the worth of storage capacity could be worked out for systems even as complex as the TVA. Such estimates seem essential in exploring whether present levels of flood protection are best achieved through current operating procedures, or whether it would pay to construct additional local protective works (levees, floodways, and channel improvements), and correspondingly increase the amount of reservoir capacity available for hydroelectric and/or irrigation water storage. Such knowledge would also be vital in a still more ambitious kind of optimization: determining whether it would pay to increase or to decrease present levels of flood protection, and whether or not to push ahead with flood plain rezoning.

Basic assumptions

The model employed here resembles that of J.D.C. Little for the analysis of hydroelectric power production.* [10] Unlike Little's study, this one is

* Other papers which have directly influenced this one are those by Brudenell and Gilbreath [2], by Elliott and Engstrom [5], by Gessford and Karlin [7], by Kirchmayer and McDaniel [8], and by Koopmans [9]. Perhaps most of all, I have been influenced by informal conversations with Robert Dorfman and Peter Watermeyer on the work of the Harvard Water Resources Project.

not exclusively concerned with power, but rather concentrates upon the trade-offs between flood control, hydroelectricity, and irrigation. There are several technical differences from Little's study. This one deals with an infinite- rather than a finite-horizon situation. An infinite-horizon model seems particularly appropriate when there is a possibility that water may be carried over from one year's dry season into the following year's wet season, and when there is therefore no obvious cutoff date at the end of the dry season. For this reason, it was decided to employ a linear programming method for Markov process optimization [11b], rather than the functional equation technique of dynamic programming.**

** The linear programming technique possessed one further practical advantage. No special-purpose computer code had to be written.

The present model is based upon one crucial simplification: analysis of a single reservoir, rather than a multi-reservoir system.*** Such a

*** The objection to multi-reservoir systems is based wholly upon computational costs. If it takes m equations in a linear programming model to describe the possible states within a single-reservoir system, then with n reservoirs, it will take m^n equations. Even with m as small as 10, a three-reservoir system would require $10^3 = 1{,}000$ equations!

representation would probably be intolerable from the viewpoint of one who is attempting to construct detailed dispatching and operating rules for a reservoir network, but might nevertheless provide an acceptable starting-point for initial design purposes or for the overall analysis of flood control versus power and irrigation benefits.*

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* Despite the fact that the TVA included 37 hydro plants, Brudenell and Gilbreath did find it useful to analyze the system in terms of aggregate water storage, measured in billions of kilowatt-hours. [2, Figures 8 and 11]

The yearly cycle of operations is regarded as subdivided into T discrete periods, the index t being used for generic identification purposes. ($t = 1, 2, \ldots, T$) It is assumed that at the beginning of each period, the water managers know not only the initial reservoir level but also the water inflows to occur during the entire period.** The managers' problem is viewed

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** In some preliminary work, an alternative assumption was tried out: that the water managers begin each period with knowledge of the current reservoir level only, and without any advance information on the period's inflows. With this type of information structure, the decision variable was taken to be the initial discharge rate, a rate that might have to be modified later on within the period if the reservoir contents reached their upper or lower permissible limits. This formulation proved to be fairly awkward.

It was Peter Watermeyer who pointed out to me certain advantages in the present formulation, regarding the current period's inflows as known, rather than as random variables. P.A.P. Moran has also employed this approach. [12, p. 40]

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as one of constructing a sequential decision rule: selecting the quantity j to be discharged during the current period as a function of the current water availability i (initial reservoir contents plus known inflows for the period), knowing that each future period's discharge decision will be made subsequent to learning <u>that</u> period's inflows, currently known only in the form of a probability distribution.

This idealization ignores an obvious hydrological feature: serial correlation between the random components experienced during successive periods.*

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* The model could conceivably be refined to take account of serial correlation, but only at considerable computational expense. Each period's state variable would then have to be defined as a pair of numbers: initial reservoir level and the previous period's inflow.

On the other hand, it does take account of the fact that there are many river systems in which inflows during the near future can be predicted with a good degree of accuracy through knowledge of soil moisture conditions and/or snow pack. The time scale for this model must be selected in such a way as to ensure a high degree of predictability of inflows within the individual period, and yet a low degree of serial correlation between the random component of the current period's inflows and of the subsequent one's. These conflicting requirements could probably be satisfied by working with time intervals somewhere between two weeks and three months in length.**

** The time scale for peak periods of flood flow is likely to be a single week or less. Our model is compatible with a situation in which it is feasible to make a reasonably accurate forecast of total inflows within, say, the coming month, and yet infeasible to make a good single-valued forecast of peak flood flows within that month.

For simplicity in exposition, two features are omitted from the initial formulation, but are taken up in subsequent sections: (1) the effect of reservoir head-height upon the energy yield per unit of discharge; and (2) the interactions between hydroelectric and irrigation uses of water. For the time being, we will proceed as though the variations in head-height were negligible, and as though hydroelectricity represented the only significant use for the water released.

Algebraic formulation

The notation to be employed here resembles that employed in an earlier paper. [11b] Throughout, it is understood that there is a repetitive annual cycle, each cycle involving a total of T periods. I.e., the probability distribution of inflows during period $T + 1$ coincides with that for period 1; and so on.

$i(t)$ = availability of water during period $\underline{t}$ (initial reservoir contents plus inflows during period t)

$j(t)$ = volume of water to be discharged during period $\underline{t}$; water management problem consists of choosing $j(t)$ as a function of $i(t)$

$\therefore$ $i(t) - j(t)$ = reservoir contents at the end of period $\underline{t}$

$n(t)$ = water inflows during period $\underline{t}$

$\therefore$ $i(t) = i(t-1) - j(t-1) + n(t)$

$K(t)$ = reservoir capacity available for hydroelectric storage at the end of period t ; a known parameter

$p_{n(t)}$ = probability of inflow of $\underline{n}$ units during period t ; a known parameter

$c_{i(t),j(t)}$ = expected benefits from hydroelectric power generated during period $\underline{t}$ with an availability level of $i(t)$ and a discharge of $j(t)$; a known parameter

$x_{i(t),j(t)}$ = joint probability of starting period $\underline{t}$ with water availability $\underline{i}$, and of discharging $\underline{j}$ units of water during the period; an unknown set of probabilities to be determined via linear programming

In order for each period's ending reservoir contents to be nonnegative, and to lie within the capacity limits for that period, the only combinations of $i(t)$ and $j(t)$ to be considered are those for which:

$$0 \leq i(t) - j(t) \leq K(t) \qquad (1)$$

For there to be a finite number of equations in our linear programming model, it will be necessary to place some fairly light restrictions upon the quantities $i(t)$, $j(t)$, and $n(t)$. First, they must be nonnegative integers. And second, there must be lower and upper limits upon each period's inflows so that, in conjunction with the limits in (1) upon the previous period's ending reservoir levels, there will be predetermined lower and upper limits upon the water availability of water for each season. These lower and upper limits will be known, respectively, as $i_{min}(t)$ and $i_{max}(t)$.

The linear programming model is phrased in terms of choosing the unknowns $x_{i(t),j(t)}$ in such a way as to maximize expected average annual benefits subject to certain constraints of statistical equilibrium:

Maximize

$$\sum c_{i(t),j(t)} \, x_{i(t),j(t)} \qquad (2)$$

Subject to:

$$\sum_{\substack{i(t-1)-j(t-1)+n(t) \\ = i(t)}} p_{n(t)} \, x_{i(t-1),j(t-1)} = \sum_{j(t)} x_{i(t),j(t)} \qquad (3)$$

$$i(t) = i_{min}(t), \ldots i_{max}(t); \text{ all } \underline{t}$$

$$\sum_{i(t),j(t)} x_{i(t),j(t)} = 1 \qquad t = 1, \ldots, T \qquad (4)$$

$$x_{i(t),j(t)} \geq 0 \qquad \text{all } i(t), j(t) \qquad (5)$$

Restraints (4) and (5) spell out conditions that any joint probabilities $x_{i(t),j(t)}$ must satisfy; i.e., that they be nonnegative, and that those relating to each individual period must add up to unity. Condition (3) goes further. It ensures that the probability distribution of each period's initial availability of water $i(t)$ will be consistent with the probability distribution of the preceding period's ending reservoir level, together with the current period's inflows. That is, for each possible value that can be taken on by $i(t)$, the right-hand side represents the marginal total probability of $i(t)$, calculated by summing up over all discharge quantities for that availability level and period of year. The left-hand side of each equation in (3) also measures the probability of $i(t)$ -- summing over all possible combinations of the preceding period's ending reservoir level and the current period's inflows that will result in availability level $i(t)$. Statistical equilibrium can be ensured if and only if there is an equality between the right- and left-hand sides of (3).

Because of the fact that for each $\underline{t}$, $\Sigma p_{n(t)} = 1$, and because of the normalizing conditions (4), there are altogether $\underline{T}$ redundant equations in (3), one for each period $\underline{t}$. These redundant conditions may be omitted from any numerical calculations, for they will be satisfied automatically. It does not matter which equations are regarded as the redundant ones -- provided that just one equation is deleted for each period of the year.

Hydrological data inputs

In order to illustrate the use of this water storage programming model, a small-scale numerical example will be worked out. To emphasize the fact that the specific numbers are purely fictitious, we shall refer to the water managers as the HVA (the Hypothetical Valley Authority).

The HVA controls a reservoir with a total useable capacity of 2 million acre-feet for flood control and hydroelectric storage. In order to cut down on the number of equations in the resulting linear program, all inflows and outflows will be measured in units of one million acre-feet.* With these

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* All inflows will be taken as net of evaporation and seepage losses.

units, there are just three alternative levels of the reservoir to be considered: 0, 1, and 2.

Just three periods within each annual cycle will be distinguished here: (W) a wet season, with substantial danger from floods; (F) a filling season, with little danger from floods; and (D) a dry season. The probability distributions of inflow during each of these periods are hydrological data, the parameters $p_{n(t)}$. According to Table 1, these probability distributions imply mean inflows of 4, 2 and 1 units of water, respectively, during the three seasons; and a mean of 7 units' worth of annual inflows. A year's total flow may range anywhere between the lower limit of 4 units (three successive low inflows) and the upper limit of 10 units (three successive high inflows). Since serial correlation is neglected, the frequency of, say, the driest possible year may be calculated as the product of the three independent probabilities of low inflows:

probability of a year with only 4 units of inflow $= p_{3(W)}\, p_{1(F)}\, p_{0(D)}$

$$= (.3)\ (.3)\ (.3) = .027$$

Table 1 also indicates $i_{min}(t)$ and $i_{max}(t)$, the minimum and maximum levels of water availability consistent with these inflow distributions and with the reservoir limits. In the case of period W, for example, the minimum possible inflow is 3, and the minimum initial reservoir level is 0. Adding these two lower limits together, we see that $i_{min}(W) = 3$. Similarly:

$$i_{max}(W) = \text{maximum reservoir contents at end of previous season} + \text{maximum inflow during current season}$$

$$= K(D) + \max n(W)$$

$$= 2 + 5$$

$$= 7$$

Table 1. Probability Distributions of Water Inflow

$n(t)$, inflow to reservoir during period $\underline{t}$ (millions of acre-feet)	wet season, period W $p_{n(W)}$	filling season, period F $p_{n(F)}$	dry season, period D $p_{n(D)}$
0	0	0	.3
1	0	.3	.4
2	0	.4	.3
3	.3	.3	0
4	.4	0	0
5	.3	0	0
mean inflow during period t ; $\Sigma n(t)p_{n(t)}$	4.0	2.0	1.0
$i_{min}(t)$, minimum water availability (millions of acre-feet)	3	1	0
$i_{max}(t)$, maximum water availability (millions of acre-feet)	7	5	4

These preliminary calculations establish in advance that the water availability during season W may range anywhere between 3 and 7; during season F, between 1 and 5; and during season D, between 0 and 4. This means that five water levels are possible during each of the three seasons, and that there are altogether 15 equations of statistical equilibrium of the same type as (3) above. However, because of the normalizing conditions contained in (4), three of these restrictions are redundant. It will be convenient to regard the equations relating to $i_{min}(t)$ as the redundant ones, and to omit them explicitly from the constraint set.

Once we have the probability coefficients $p_n(t)$, and also $i_{min}(t)$ and $i_{max}(t)$, we are ready to write down constraints (3) and (4) in linear programming form. Table 2 contains the complete 15-row matrix of detached coefficients. In constructing this table, the right-hand side of equation group (3) above has been subtracted from the left.

The number of activities to be included within the optimization will depend upon the amount of reservoir capacity to be reserved for flood control purposes. Suppose, for example, that all two units of reservoir capacity are to be reserved for flood control purposes from the very beginning to the end of the wet season. This amount of flood storage can only be guaranteed by excluding from the optimization any activities associated with positive reservoir contents at the end of either the wet season W, or the dry season D. This means excluding the following combinations of $i(t)$ and $j(t)$:

ending reservoir levels, wet season	$i(W) - j(W) = 1$	activities #6-10
	$i(W) - j(W) = 2$	activities #11-15
ending reservoir levels, dry season	$i(D) - j(D) = 1$	activities #35-38
	$i(D) - j(D) = 2$	activities #39-41

Table 2. Detached Coefficients Matrix

t = W

		i(W) - j(W) = 0					i(W) - j(W) = 1					i(W) - j(W) = 2				
i(t),j(t)		3,3	4,4	5,5	6,6	7,7	3,2	4,3	5,4	6,5	7,6	3,1	4,2	5,3	6,4	7,5
column identification number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	i(t)															
	4(W)		-1.0					-1.0					-1.0			
	5(W)			-1.0					-1.0					-1.0		
	6(W)				-1.0					-1.0					-1.0	
	7(W)					-1.0					-1.0					-1.0
equation group (3)	2(F)	.4	.4	.4	.4	.4	.3	.3	.3	.3	.3					
	3(F)	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.3	.3	.3	.3	.3
	4(F)						.3	.3	.3	.3	.3	.4	.4	.4	.4	.4
	5(F)											.3	.3	.3	.3	.3
	1(D)															
	2(D)															
	3(D)															
	4(D)															
equation group (4)	t															
	W	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	F															
	D															
payoff coefficients, $c_{i(t),j(t)}$																
neglecting head-height		7.2	7.8	8.4	9.0	9.6	6.0	7.2	7.8	8.4	9.0	3.0	6.0	7.2	7.8	8.4
including head-height		6.0	6.8	7.4	7.8	8.2	6.0	7.2	7.8	8.4	9.0	4.0	6.8	7.8	8.6	9.4

Table 2. (Continued)

t = F

		i(F) - j(F) = 0					i(F) - j(F) = 1					i(F) - j(F) = 2			
i(t),j(t)		1,1	2,2	3,3	4,4	5,5	1,0	2,1	3,2	4,3	5,4	2,0	3,1	4,2	5,3
column identification number		16	17	18	19	20	21	22	23	24	25	26	27	28	29
	i(t)														
	4(W)														
	5(W)														
	6(W)														
	7(W)														
equation group (3)	2(F)		-1.0					-1.0				-1.0			
	3(F)			-1.0					-1.0				-1.0		
	4(F)				-1.0					-1.0				-1.0	
	5(F)					-1.0					-1.0				-1.0
	1(D)	.4	.4	.4	.4	.4	.3	.3	.3	.3	.3				
	2(D)	.3	.3	.3	.3	.3	.4	.4	.4	.4	.4	.3	.3	.3	.3
	3(D)						.3	.3	.3	.3	.3	.4	.4	.4	.4
	4(D)											.3	.3	.3	.3
equation group (4)	t														
	W														
	F	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	D														
payoff coefficients, $c_{i(t),j(t)}$															
neglecting head-height		3.0	4.2	4.8	5.4	6.0	0	3.0	4.2	4.8	5.4	0	3.0	4.2	4.8
including head-height		2.0	3.4	4.2	4.6	5.0	0	3.0	4.2	4.8	5.4	0	3.4	4.6	5.4

Table 2. (Concluded)

t = D

	i(D) - j(D) = 0					i(D) - j(D) = 1				i(D) - j(D) = 2			Constant terms
i(t),j(t)	0,0	1,1	2,2	3,3	4,4	1,0	2,1	3,2	4,3	2,0	3,1	4,2	
column identification number	30	31	32	33	34	35	36	37	38	39	40	41	
i(t)													
4(W)	.4	.4	.4	.4	.4	.3	.3	.3	.3				0
5(W)	.3	.3	.3	.3	.3	.4	.4	.4	.4	.3	.3	.3	0
6(W)						.3	.3	.3	.3	.4	.4	.4	0
7(W)										.3	.3	.3	0
equation group (3) 2(F)													0
3(F)													0
4(F)													0
5(F)													0
1(D)		-1.0				-1.0							0
2(D)			-1.0				-1.0			-1.0			0
3(D)				-1.0				-1.0			-1.0		0
4(D)					-1.0				-1.0			-1.0	0
equation group (4) t													
W													1.0
F													1.0
D	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
payoff coefficients, $c_{i(t),j(t)}$													
neglecting head-height	0	3.0	6.0	7.2	8.4	0	3.0	6.0	7.2	0	3.0	6.0	
including head-height	0	2.0	4.0	6.0	6.8	0	3.0	6.0	7.2	0	4.0	6.8	

If, however, only one unit of reservoir capacity is to be reserved for flood control, activities #6-10 and #35-38 may be included within the optimization. And with no capacity reserved for flood control, activities #11-15 and #39-41 may also be included. The optimal solution for a high level of flood protection always satisfies constraints (3), (4) and (5), and therefore provides a convenient starting-point for the simplex procedure at a lower level of protection. By partitioning activites in this way, and by using each optimal basis as the starting-point for the simplex procedure at the next lower protection level, it is possible to explore the effects of flood storage restrictions without rerunning the entire problem.

Electric power data

In order to complete the statement of the linear programming problem indicated above by conditions (2)-(5), we will need to construct the payoff coefficients $c_{i(t),j(t)}$. These coefficients measure the net social benefits to be derived during period t from starting with an availability level $i(t)$ and discharging $j(t)$ units of water. The managerial problem consists of choosing a discharge policy so as to maximize expected annual benefits -- taking account of the fact that it may be necessary to forego benefits during one season in order to increase them during another. Time discounting of benefits is neglected here, but could readily be incorporated through a similar type of model proposed by F. d'Epenoux [6].

All of HVA's hydro power demands are interruptible, but at a cost: $3.00 per acre-foot of energy deficit. (This cost might correspond to the unit loss incurred during dry years by such interruptible customers as electrochemical producers.) The minimum energy demand to be supplied by the hydro system -- if water is available -- varies seasonally as in Table 3. Since the head-height factor is being ignored, all energy requirements may be measured in terms of the standard volumetric unit: one million acre-feet of water. Note that minimum total annual demands lie well below the mean of 7 units of inflow, but that they nevertheless exceed the dry year's inflow of 4 units.

Table 3.

season	minimum demands for hydroelectric energy (millions of acre-feet)
W	2
F	1
D	2
annual total	5

Any hydro discharge in excess of the minimum demands may be employed in reducing the power output to be supplied by the HVA's thermal plants. During seasons W and F, the oldest of these thermal plants can supply the energy equivalent of up to 1 million acre-feet of water at a fuel cost equivalent to $1.20 per acre-foot of energy. During season D, these older thermal units can supply the energy equivalent of up to 2 million acre-feet of water -- also at a fuel cost equivalent to $1.20 per acre-foot. If any hydro power is available beyond what is employed to cut down on thermal generation in these older plants, the remaining power will be utilized to reduce thermal generation costs at the system's newer plants -- saving only $.60 per acre-foot of energy equivalent.

To summarize: Marginal benefits from hydro generation drop with the quantity released. There is an initial block of power valued at $3.00 per acre-foot; a second block at $1.20; and a third block of "dump" power valued at only $.60 per acre-foot, the fuel cost of the HVA's newer thermal plants.* These three valuation categories result in the curve of total benefits

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* If either turbine capacities or else total system demands were limitational, this would establish an upper bound upon the output of "dump" power, and would imply the existence of a fourth value block: zero for any water discharged over the spillways rather than through the turbines.

One other point about these value categories. In order to meet the daily and weekly fluctuations in demand within each period, it will ordinarily be economical to keep the thermal plants operating at a fairly constant level, and to employ the hydro units for "peaking" purposes. (See Bernholtz and Graham [1].) Here we presuppose that this type of short-run scheduling problem has already been solved, and that the results of the suboptimization are incorporated within the aggregate discharge-benefit relationships of Figure 2.

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versus discharge shown for each of the seasons in Figure 2. During the wet period, for example, each of the first 2 units of water discharge brings added benefits of $3.0 millions, a total of $6.0 millions for 2 units discharged. The third unit of water is employed to replace steam generation valued at $1.20 per acre-foot, resulting in total benefits of $7.2 millions for 3 units discharged. And all units of water beyond the third are valued at "dump" rates, $.60 per acre-foot. These discharge-benefit curves are translated directly into the first set of coefficients $c_{i(t),j(t)}$ shown in Table 2, those constructed ignoring any correction for head-height effects. For example, the coefficient $c_{i(W),2(W)}$ is 6.0 for all values of $i(W)$.

Optimal discharge policies

The results of the linear programming calculation are summarized in the product-mix curve of Figure 3.** The higher the level of storage capacity

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** I am indebted to Bernard Dzielinski, IBM Research Center, Yorktown Heights, New York, for running these simplex calculations; also to John Colley and to Emmanuel Uren for preliminary calculations at the Yale University Computing Center.

Figure 2. Power benefits versus water release

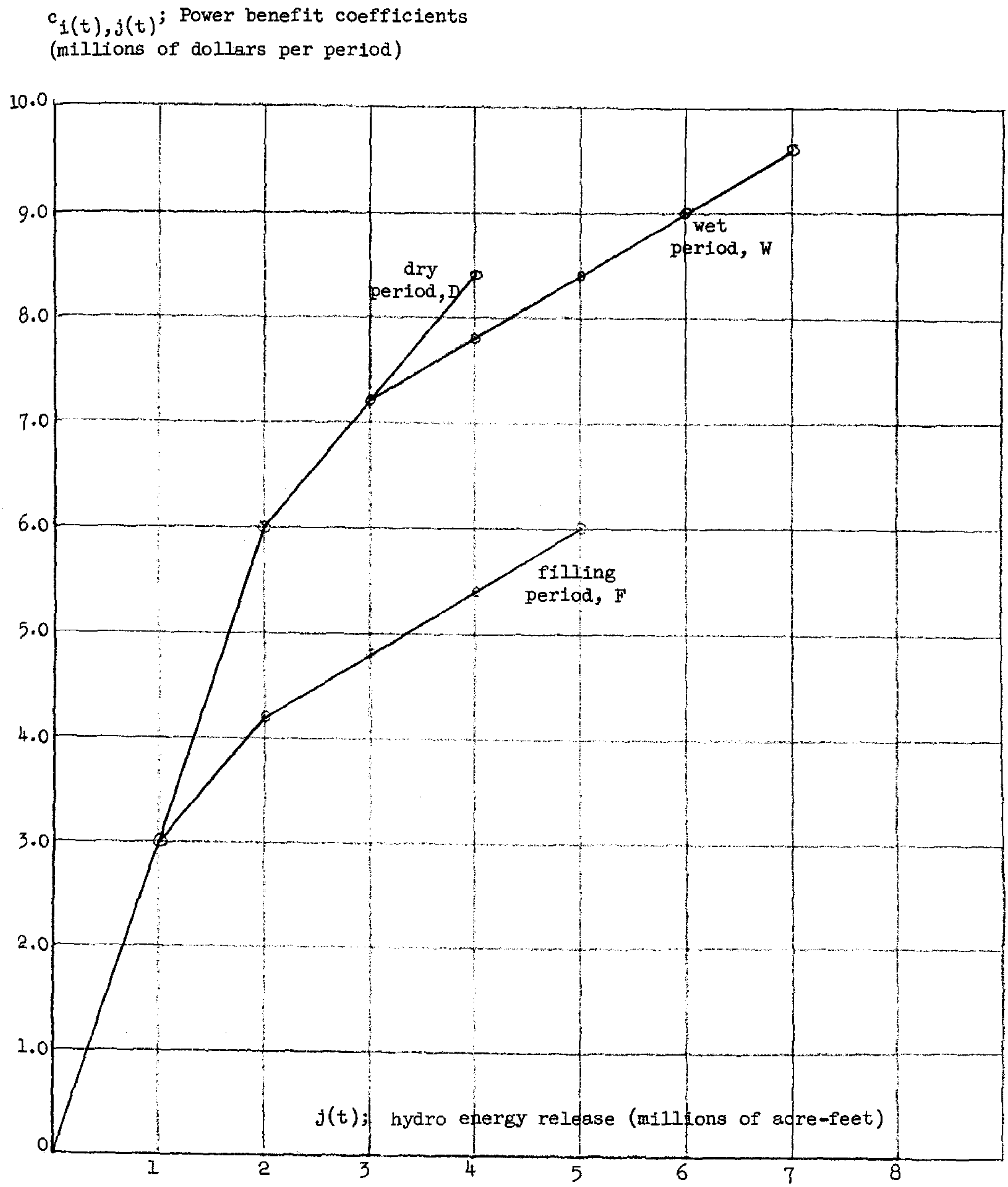
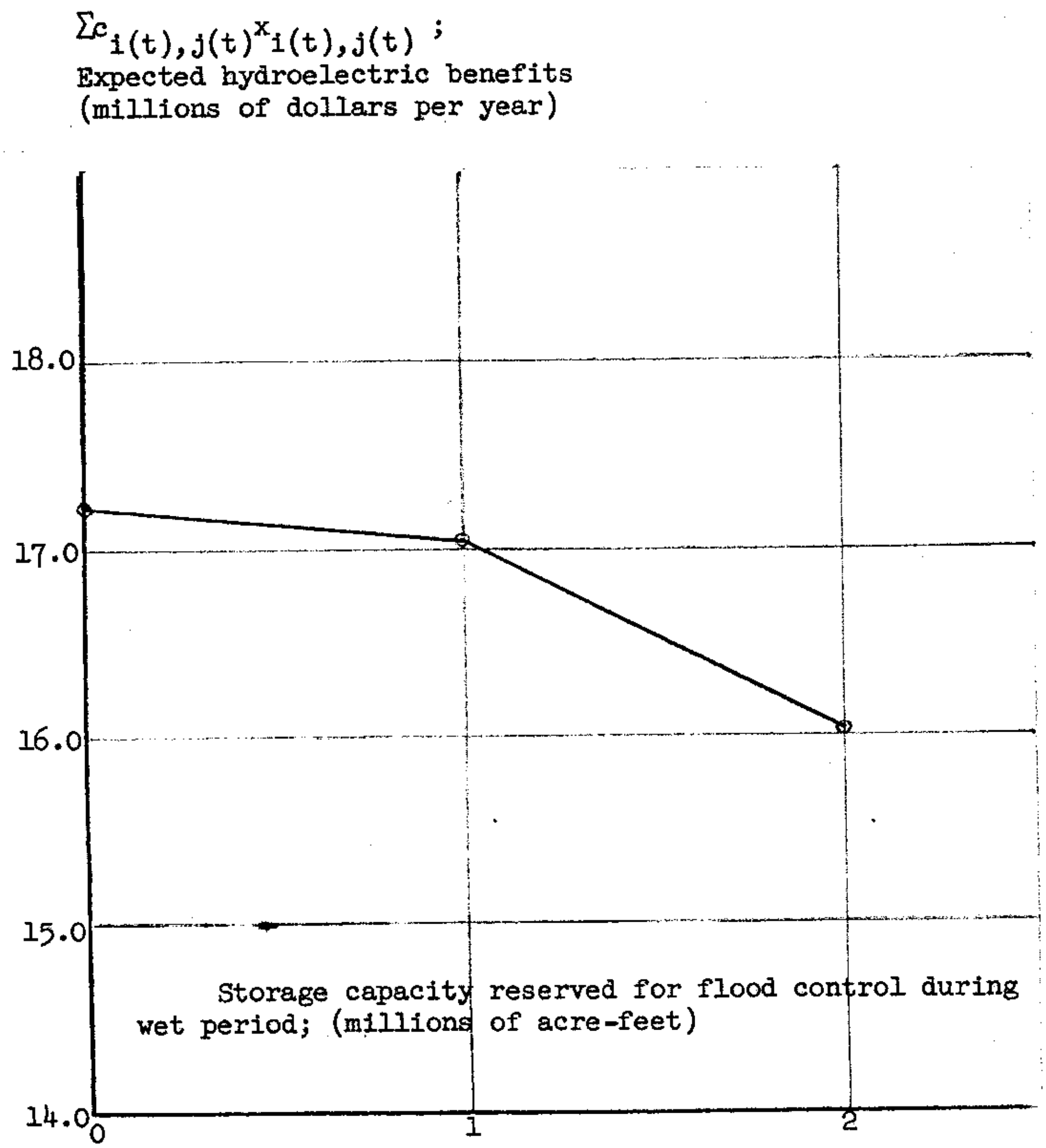


Figure 3. Product-mix alternatives: flood control and electric power benefits; neglecting correction for head-height effects



reserved for flood protection during the wet season, the lower become the expected hydroelectric benefits. Note that this product-mix relationship is highly nonlinear. The first million acre-feet reserved for flood protection result in a drop in average annual hydro benefits from $17.233 to $17.058 millions -- a difference of less than $200,000 a year. The second million acre-feet of flood storage leads, however, to a loss of over a million dollars annually.

Now for some of the details behind these overall results. (See Table 4.) For example, in the case of zero storage capacity reserved for flood protection, Table 4 lists the optimal activity levels for the filling period as follows:

$$x_{2(F),1(F)} = .210$$
$$x_{3(F),1(F)} = .370$$
$$x_{4(F),2(F)} = .330$$
$$x_{5(F),3(F)} = .090$$

This means that in following an optimal discharge policy, it will turn out that 21% of the time, the initial availability of water during season F will be 2 million acre-feet; that 37% of the time, it will be 3 million acre-feet, etc. Furthermore, these results imply the following discharge rule for season F: Whenever the water available is only 2 units, satisfy the minimum interruptible demand of 1 unit for that season. At higher availability levels, discharge just enough so that the reservoir will be filled to its 2-unit capacity at the beginning of the dry season.

Note that for each level of the state variable $i(t)$ there is at most one positive-valued joint probability $x_{i(t),j(t)}$. Despite the fact that this model permits the water manager to employ a randomized strategy -- sometimes discharging one amount and sometimes another at a given availability level -- it never pays to exercise this option. See Wagner's note on this point. [16]

Table 4. Optimal activity levels;
no correction for head-height effects

Storage capacity reserved for flood control during wet period		0		1		2	
Activities automatically excluded from simplex calculation		none		#11-15, #39-41		#6-15, #35-41	
period t	i(t)	j(t)	$x_{i(t),j(t)}$	j(t)	$x_{i(t),j(t)}$	j(t)	$x_{i(t),j(t)}$
W	3	2	.300	2	.300	3	.300
	4	3	.400	3	.400	4	.400
	5	3	.300	4	.300	5	.300
	6	-	-	-	-	-	-
	7	-	-	-	-	-	-
F	1	-	-	-	-	1	.300
	2	1	.210	1	.300	1	.400
	3	1	.370	1	.400	1	.300
	4	2	.330	2	.300	-	-
	5	3	.090	-	-	-	-
D	0	-	-	-	-	0	.090
	1	1	.063	1	.090	1	.240
	2	2	.321	2	.330	2	.340
	3	3	.379	3	.370	3	.240
	4	4	.237	4	.210	4	.090
Expected hydroelectric benefits, $\Sigma c_{i(t),j(t)} x_{i(t),j(t)}$ (millions of dollars per year)		17.233		17.058		16.044	

The optimal discharge decision rules implied by Table 4 are presented graphically in Figures 4-6, one set of rules for each of the three seasons. These curves do not always have a simple shape. The optimal discharge is not always a linear function of the initial availability of water. Incidentally, the general shape of these curves is consistent with that obtained by Gessford and Karlin in a somewhat different situation [7].

Figure 6 illustrates one qualitative feature of this solution. It pays to end every dry season with an empty reservoir. If we could be assured that this feature always held true, there would be no advantage in analyzing an infinite-horizon model. The end of the dry season would furnish a completely satisfactory cutoff date for a one-year, finite-horizon dynamic program. We shall see, however, that in the very next set of calculations, taking account of head-height effects, it does occasionally pay to carry over water from the dry to the wet season, and that therefore no specific time of year provides a really satisfactory cutoff point.

One final comment: In any feasible basis, the activity levels $x_{i(t),j(t)}$ may be employed to provide a check-sum upon the numerical analysis. Since the mean <u>annual</u> inflow of 7 units of water must equal the mean outflow, $\sum_{i(t),j(t)} j(t)\, x_{i(t),j(t)} = 7$. The reader may verify that this condition is satisfied for each of the three optimal solutions presented here.

<u>Implicit prices</u>

Epenoux has shown that a direct interpretation may be attached to the dual variables of equation group (3) in this type of model. [6] These dual variables represent the limit of a certain <u>difference</u> between discounted benefits over an infinite time horizon following an optimal discharge rule -- the limit as the interest rate approaches zero. The implicit price for equation $i(t)$ represents

Figure 4. Optimal discharge policies; wet period, W.

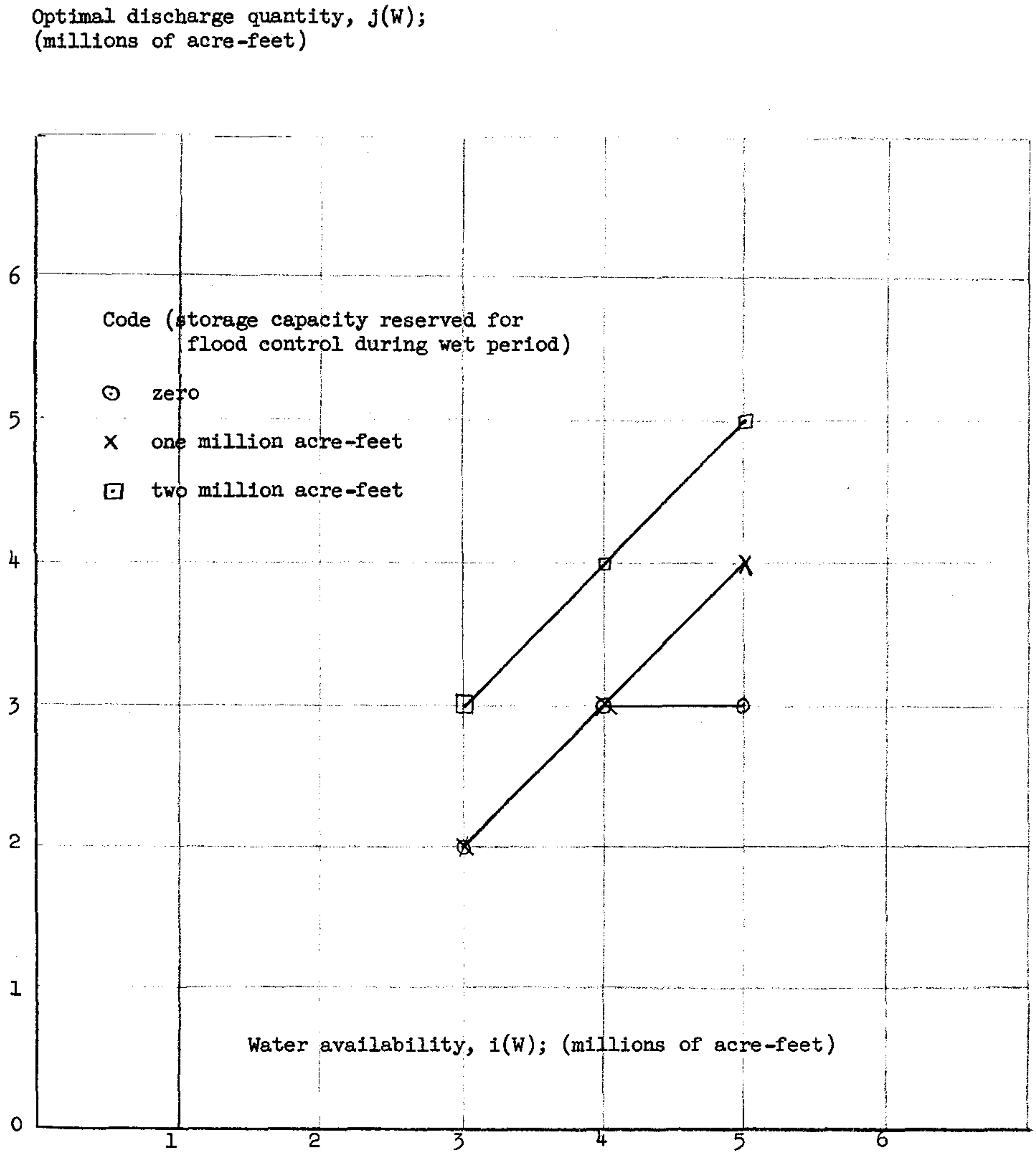


Figure 5. Optimal discharge policies; filling period, F.

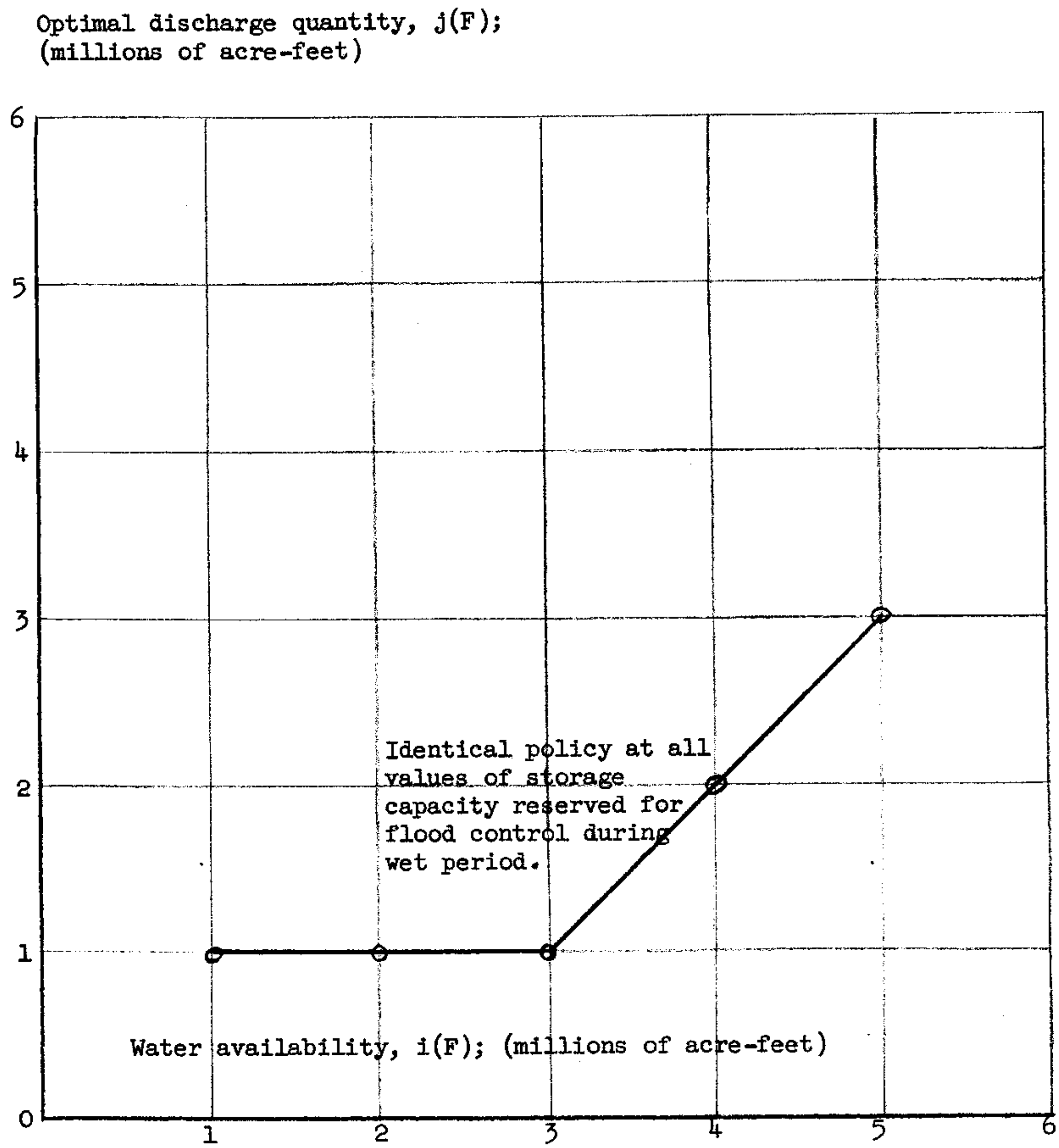
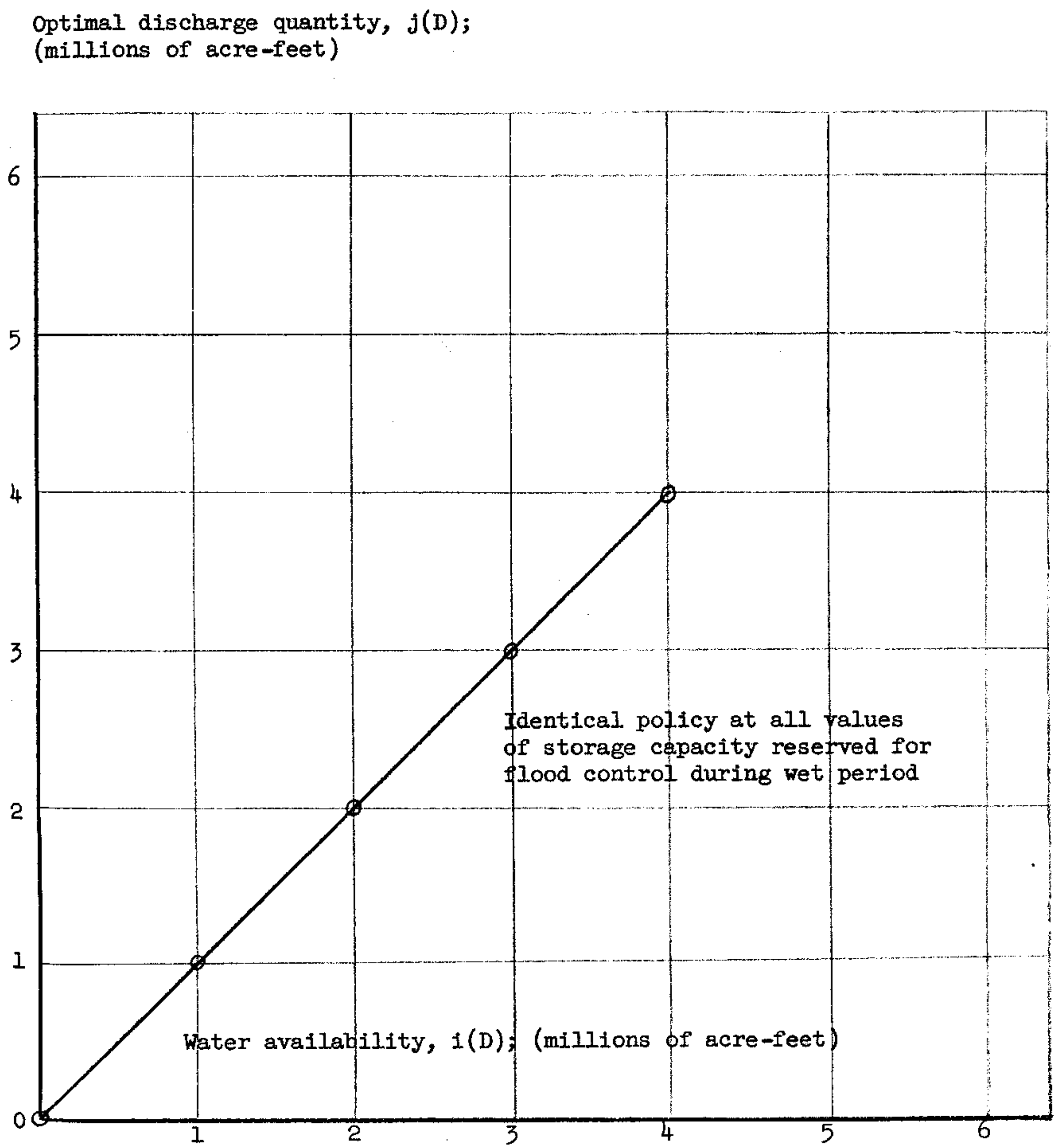


Figure 6. Optimal discharge policies; dry period, D.



the limiting difference between discounted benefits starting from availability position $i(t)$ and an arbitrary reference point, $i_{min}(t)$.* Because it does

* The reference point for this difference will always depend upon which availability level is selected as the redundant restriction -- in this example, $i_{min}(t)$.

not always happen that the level $i_{min}(t)$ occurs with positive probability, there are some "artificial" unknowns at a zero level in the optimal basis. Accordingly, the implicit prices have been transformed so as to correspond to the limiting difference in benefits between position $i(t)$ and the lowest availability level that occurs with a positive probability for season $\underline{t}$. For example, from Table 5, we may read off that when zero storage capacity is reserved for flood control purposes, the lowest availability level that occurs for season D is 1 unit, and that the implicit price associated with the restriction 1(D) is therefore arbitrarily taken as zero. Furthermore, if the reservoir managers were to begin season D with 2 units instead of 1, the limiting value of the additional discounted benefits would be $3.000 millions, the marginal value of one unit of interruptible power. With 4 units instead of 2, the difference in discounted benefits would be $4.200 millions; and so on.

These values for water during the dry season are fairly obvious. They follow immediately from knowing the direct benefits of water during that season, plus knowing that it doesn't pay to carry over anything into the succeeding wet season. The values for seasons W and F, however, are a little less obvious. It takes more than a back-of-the-envelope calculation to establish, for example, that if 3 units of water are available in season F, then the expected cost of meeting an additional demand for one unit of energy is $1.740 millions.

Table 5. Implicit prices;
no correction for head-height effect;
(millions of dollars).

Storage capacity reserved for flood control during wet period		0	1	2
period t	i(t)			
W	3	0	0	0
	4	1.200	1.200	.600
	5	2.382	1.800	1.200
	6	-	-	-
	7	-	-	-
F	1	-	-	0
	2	0	0	2.460
	3	1.740	1.740	4.200
	4	2.940	2.940	-
	5	3.540	-	-
D	0	-	-	-
	1	0	0	0
	2	3.000	3.000	3.000
	3	4.200	4.200	4.200
	4	5.400	5.400	5.400

This type of information on "efficiency prices" could conceivably be employed to bring electricity rate structures more closely into line with what would be suggested by the traditional free-market principle of marginal cost pricing. These prices could even be employed in designing a rate structure that varies both with the time of year $\underline{t}$, and also with $i(t)$, the quantity of water currently available to the reservoir managers. See Debreu [3, p. 99].

Head-height effects

In order to take account of head-height effects upon the amount of energy produced from a given amount of water, it is necessary to modify only the pay-off coefficients $c_{i(t),j(t)}$. In principle, in order to take account of these effects, one ought to allow for the fact that the reservoir level varies continuously during the course of a single time period, and hence that there is a systematic variation in the amount of power obtained from a given discharge rate. For present purposes, however, we shall make the inaccurate but handy assumption that the water-to-energy conversion factor depends only upon the *ending* reservoir level of the particular time period.

In order to avoid a direct translation into kilowatt-hours, we shall take as our energy unit the number of kilowatt-hours generated per million acre-feet of water discharge -- given an ending reservoir level of exactly one million acre-feet. In terms of this energy unit, it will be supposed that the water-to-energy conversion factor is 2/3, 1.0, or 4/3, depending whether the ending reservoir level happens to be zero, one, or two million acre-feet. With these factors , a release of, say, two units of water will generate either 4/3, 6/3, or 8/3 units of energy. From the original discharge-benefit curve of Figure 2, these energy quantities may be translated into direct economic benefits. During, say, the dry season, the benefits of a two unit discharge will be \$4.0, \$6.0, or \$6.8 millions, depending upon the ending level. These are the values entered as the payoff coefficients in the bottom row of Table 2 and in the columns bearing the identification numbers 32, 37, and 41 respectively.

The optimal solution may again be summarized by a product-mix curve: flood storage capacity versus hydroelectric benefits. (Figure 7) Again, there is a highly nonlinear relationship between flood protection and power benefits. Now, however, the first unit of reservoir capacity to be reserved for flood control purposes lowers the power benefits by over a million dollars annually, and the second unit leads to an additional drop of almost three millions! For comparative purposes, Figure 7 also includes the initial tradeoff curve, constructed ignoring head-height effects. The two curves have a considerably different shape, and they intersect each other. The initial calculation systematically understates the marginal cost of flood control in terms of hydroelectric benefits.

Tables 6 and 7 contain the details of the optimal activity levels and of the implicit prices for this second set of calculations. In several of these cases, the optimal solution indicates that it is worthwhile to plan on ending the dry season with a positive reservoir level. This qualitative feature differs considerably from the previous set of calculations, where such a carry-over was always undesirable. Here the risk of spillage during the wet season is outweighed by the risk of insufficient head during subsequent dry seasons.

Irrigation benefits

In order to insert downstream irrigation benefits into this model, all that is needed is a fairly minor modification of the payoff coefficients $c_{i(t),j(t)}$. These are now to be interpreted as the total of hydroelectric plus irrigation benefits generated during period t by a release of $j(t)$ units of water, given an initial availability $i(t)$. This interpretation

Figure 7. Product-mix alternatives:
flood control and electric power benefits

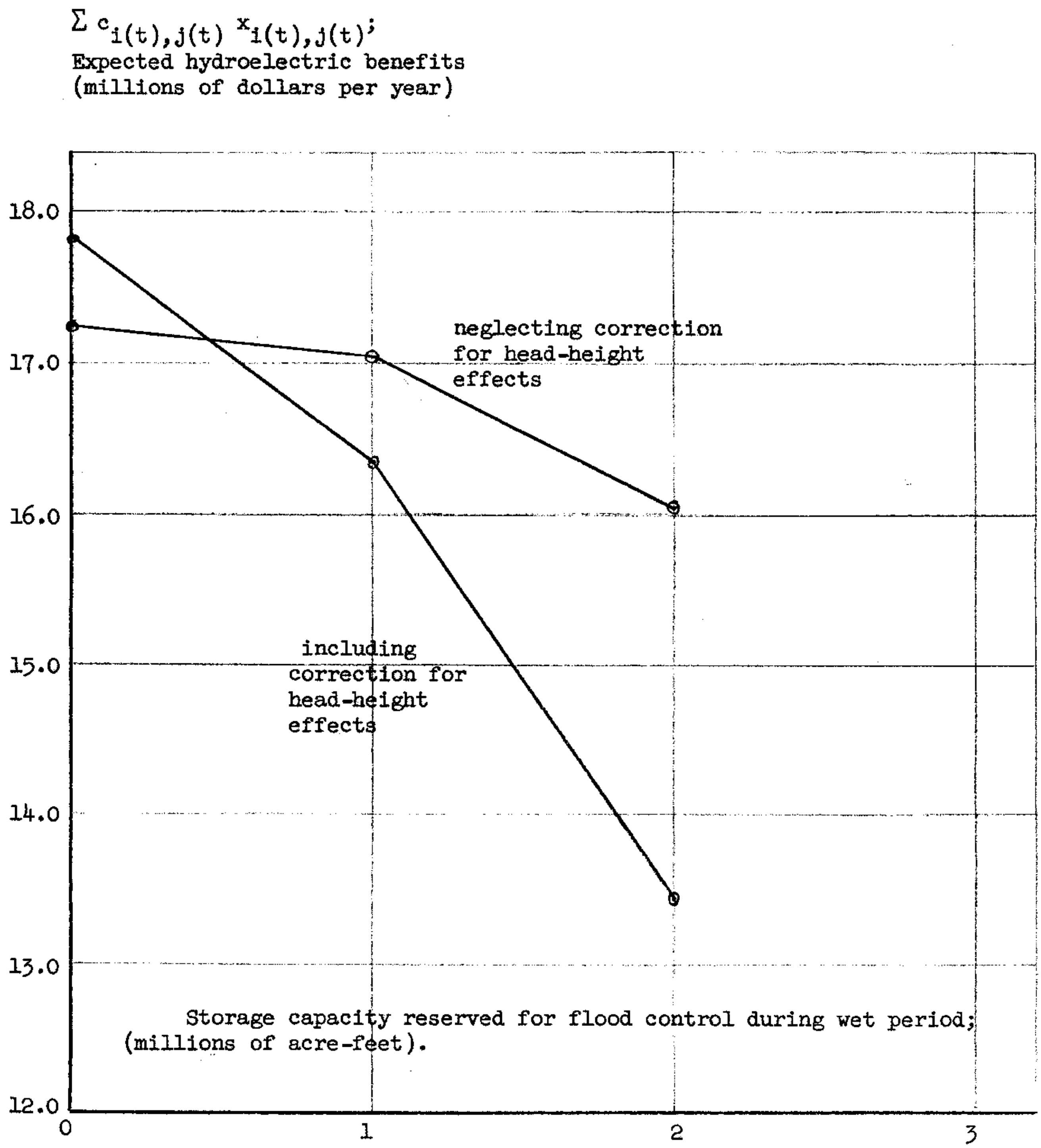


Table 6. Optimal activity levels; including correction for head-height effects

Storage capacity reserved for flood control during wet period		0		1		2	
Activities automatically excluded from simplex calculation		none		#11-15, #39-41		#6-15, #35-41	
period t	i(t)	j(t)	$x_{i(t),j(t)}$	j(t)	$x_{i(t),j(t)}$	j(t)	$x_{i(t),j(t)}$
W	3	-	-	2	.126	3	.300
	4	2	.210	3	.342	4	.400
	5	3	.370	4	.358	5	.300
	6	4	.330	5	.174	-	-
	7	5	.090	-	-	-	-
F	1	-	-	-	-	0	.300
	2	-	-	1	.300	1	.400
	3	1	.300	1	.400	1	.300
	4	2	.400	2	.300	-	-
	5	3	.300	-	-	-	-
D	0	-	-	-	-	-	-
	1	-	-	1	.090	1	.210
	2	1*	.300	2	.330	2	.370
	3	2*	.400	2*	.370	3	.330
	4	2*	.300	3*	.210	4	.090
Expected hydroelectric benefits, $\Sigma c_{i(t),j(t)} x_{i(t),j(t)}$ (millions of dollars per year)		17.818		16.344		13.452	

*Since $i(D) > j(D)$, this activity implies that water is to be carried over from the dry to the wet season.

Table 7. Implicit prices;
including correction for head-height effect;
(millions of dollars).

Storage capacity reserved for flood control during wet period		0	1	2
period t	i(t)			
W	3	-	0	0
	4	0	1.200	.800
	5	1.000	1.800	1.400
	6	1.800	2.400	-
	7	2.600	-	-
F	1	-	-	0
	2	-	0	3.000
	3	0	2.472	5.040
	4	1.200	3.672	-
	5	2.000	-	-
D	0	-	-	-
	1	-	0	0
	2	0	2.000	2.000
	3	3.000	4.780	4.000
	4	4.660	5.980	4.800

follows along much the same lines as a paper by Tolley and Hastings on scheduling a hydroelectric and irrigation system under conditions of deterministic inflows. [15]

If the irrigable land happens to lie downstream from the dam site, electric power is always available as a possible byproduct of the irrigation releases. That is, the same water can first be put through hydroelectric turbines, and then dispatched through irrigation canals to the agricultural producers.*

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* Similarly, byproduct electricity can be obtained from releases to downstream industrial and municipal users.

Suppose, for example, that a discharge of one million acre-feet generates an amount of electric power worth $3.0 millions in all seasons. Suppose furthermore that if the water is released during the crop growing season, it will irrigate enough downstream land to add $7.0 millions to the crop value. The payoff coefficient for combined power and irrigation benefits will then be $3.0 or $10.0 millions -- depending upon whether or not the release takes place during the crop growing season.

This does not mean that electric power is always necessarily available as a byproduct of irrigation releases. (1) For electric power purposes, the discharge may be needed during a non-growing season. And (2), if the irrigable land happens to be situated upstream from the dam site, it may take an <u>input</u> of power in order to pump the water uphill. In the case of irrigation upstream, there is a further complicating feature. A substantial fraction of the water consumed upstream may appear again, with a time lag, as an inflow to the reservoir. If the amount of this return flow were significant -- and if the lag required for recirculation did not exceed a single time period -- it would be both desirable and feasible to modify the transition probability matrix so as to take account of this feature.

Concluding remarks

The product-mix curves of Figure 7 constitute just one isolated type of relationship out of the many that are involved in the efficient utilization of water resources. In moving toward an overall optimization, an obvious next step would consist of examining the cost tradeoffs between area defenses (multi-purpose reservoir storage), local defenses (channel improvements, floodways, and levees), and flood plain rezoning. For examining these tradeoffs, it would be important to have reliable hydrological data on flood peaks and durations -- a formidable obstacle in view of the short records that are ordinarily available. The present model is concerned with just one aspect of the overall water management problem: reservoir capacity utilization. However, it is precisely this aspect -- joint utilization of a common facility -- that leads to significant interactions between the marginal cost of each of the main products derived from multi-purpose systems.

Appendix. Comparison of deterministic and probabilistic calculations

The tradeoff curves presented in Figures 3 and 7 were both calculated with the aid of a probabilistic optimizing model. It is of some interest to compare these overall results with those that would be calculated on the basis of two shortcut deterministic methods. Two sets of such calculations have been prepared -- the optimistic one based upon the assumption that each season's inflows always consist of the mean value recorded for that period. The pessimistic one is based upon the assumption that the water manager adjusts his releases so as to end up each time period with that amount of water which would be optimal if all future inflows consisted of the lowest value shown for individual seasons in Table 1. The optimistic shortcut overstates the average annual benefits consistent with a specified amount of reservoir capacity reserved for flood control purposes, and the pessimistic shortcut understates these benefits.

The comparison between the various methods has been constructed for the case in which head-height effects are neglected, and in which it is therefore possible to employ a particularly simple form of linear programming for the shortcut calculations: a "capacitated transportation" model. For more about this type of model, see [11a]. The "transportation" array is shown in Table 8. This table is set up for the optimistic case in which each period's inflow is set at its mean value, and in which one million acre-feet of storage are reserved for flood control during the wet season. The unknowns y_{W1}, y_{W2}, . . . y_{D3} represent various categories of water usage during seasons W, F, and D. The unknowns z_t represent the amount of water on hand at the end of period $\underline{t}$, and Z_t the amounts on hand at the beginning of $\underline{t}$. The row-sum equation for period W is therefore to be read:

Table 8. Allocation of deterministic inflows; array of unknowns in "transportation" form; one million acre-feet of storage capacity reserved for flood control; mean inflows of water; optimistic shortcut. (unit: millions of acre-feet)

	direct use of water			quantity of water carried over			
	service to interruptible customers	reduction of power output in old thermal plants	reduction of power output in new thermal plants	Period W to F	Period F to D	Period D to W	Mean inflow of water during period t
period W	\$3.00 y_{W1} 2	\$1.20 y_{W2} 1	\$.60 y_{W3} ∞	z_W 1	X	$-z_W$ 1	4
period F	\$3.00 y_{F1} 1	\$1.20 y_{F2} 1	\$.60 y_{F3} ∞	$-z_F$ 1	z_F 2	X	2
period D	\$3.00 y_{D1} 2	\$1.20 y_{D2} 2	\$.60 y_{D3} ∞	X	$-z_D$ 2	z_D 1	1
column sum restrictions	-	-	-	0	0	0	

direct use of water during period W	+	reservoir contents, end of period W	-	reservoir contents, beginning of period W	=	inflow during period W
$\overbrace{y_{W1} + y_{W2} + y_{W3}}$	+	$\overbrace{z_W}$	-	$\overbrace{Z_W}$	=	4

The column sum restrictions apply only to the variables z_t and Z_t. These equations specify that the reservoir contents at the end of one period be equal to the contents at the beginning of the next. For example, the right-most of these restrictions is to be read:

- reservoir contents, beginning of period W	+	reservoir contents, end of period D	=	0
$- Z_W$	+	z_D	=	0

All non-zero payoff coefficients are recorded in the upper left-hand corner of the boxes occupied by the unknowns $y_{W1}, y_{W2}, \ldots y_{D3}$. These coefficients measure the marginal benefit associated with each of the three alternative direct uses of hydroelectricity. A credit of \$3.00 per acre-foot is associated with any discharge provided to interruptible customers; \$1.20 for any discharge used to reduce power generation in the older thermal plants; and \$.60 for any discharge that reduces power generation in the newer thermal plants.

There are upper bounds placed upon all of the unknowns -- hence the expression a "capacitated" transportation model. The upper bounds are displayed in the lower right-hand corner of each box. During the wet period, for example, there is a bound of 2 million acre-feet placed upon the activity yielding a payoff of \$3.00; a bound of 1 million acre-feet upon the activity yielding \$1.20; and a bound of infinity (no upper bound) upon the activity yielding \$.60.

In addition, there is an upper bound associated with each of the unknowns Z_t and z_t -- respectively, the reservoir contents at the beginning and end of period $\underline{t}$. Since the particular tableau is constructed for the case in which there are one million acre-feet of storage capacity to be reserved for flood control throughout the wet season, there are upper bounds of one million acre-feet placed upon the reservoir contents both at the beginning and the end of the wet season. The only restriction placed upon reservoir contents at the beginning of the dry season consists of the two million acre-feet physical capacity limitation.

Three simplex calculations were performed for this optimistic case -- setting each season's inflow at its mean value -- and maximizing benefits subject to flood storage capacity limitations of 0, 1, and 2 million acre-feet.*

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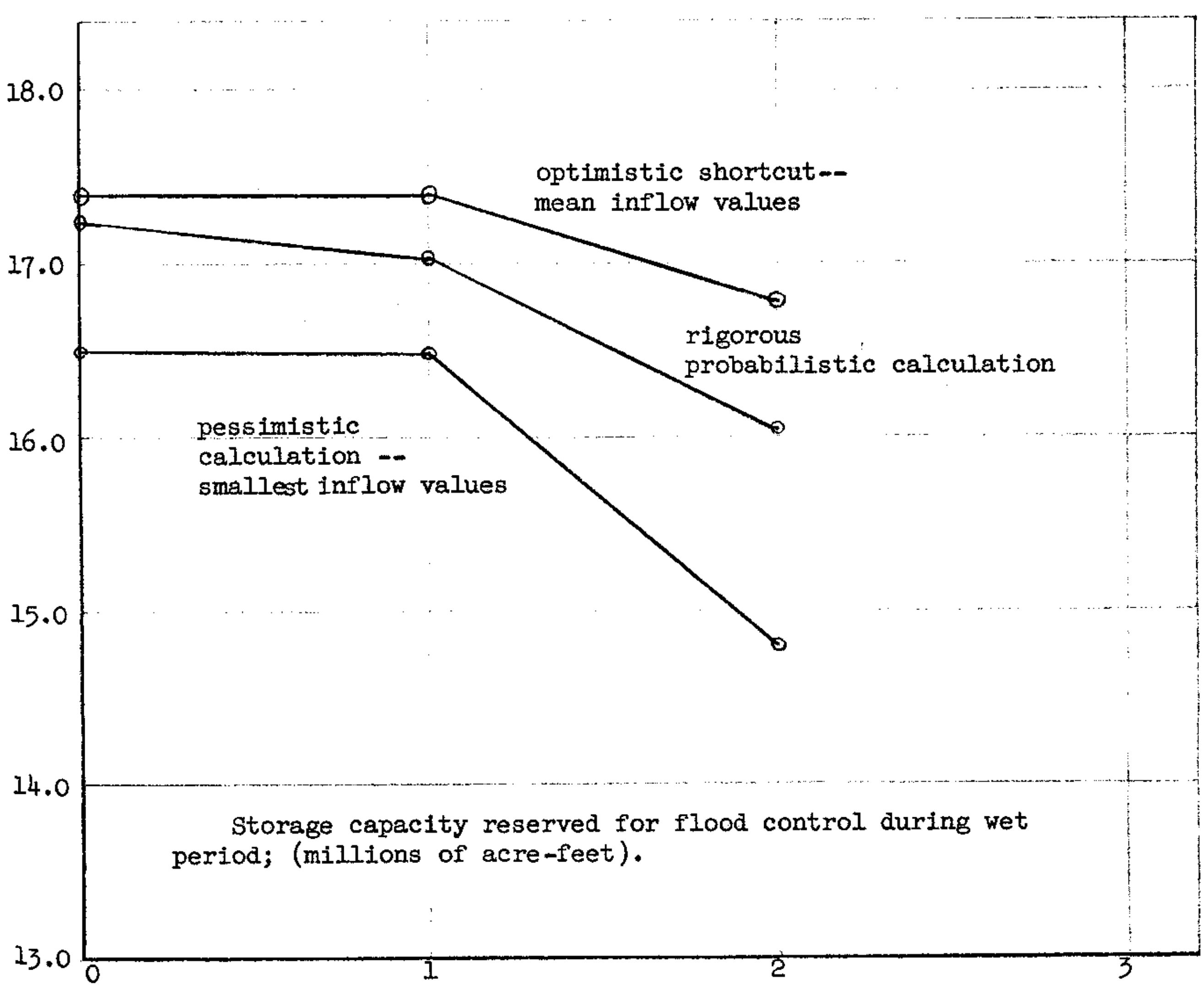
* These simplex calculations were performed virtually by inspection. It took considerably more time to construct the boxes in Table 8 than to carry through the optimization.

The overall results of these three calculations are shown in Figure 8 in the topmost curve labeled "optimistic shortcut -- mean inflow values." The middle tradeoff curve labeled "rigorous probabilistic calculation" is simply a repeat of that shown in Figures 3 and 7 above.

The bottom tradeoff curve in Figure 8, the one titled "pessimistic calculation -- smallest inflow values," is derived from three additional simplex optimizations of a "transportation" model -- also with flood storage capacity limitations of 0, 1, and 2 million acre-feet. Here, however, instead of the mean inflows shown in the right-hand column of Table 8, the lowest possible inflow values were utilized: 3, 1, and 0 million acre-feet for periods W, F, and D respectively. In addition to the direct benefits that correspond to the

Figure 8. Product-mix alternatives; comparison of deterministic and probabilistic models; neglecting correction for head-height effects.



minimum inflow values, the total amount for expected benefits includes a credit for the expected value of any water inflow above the minimum amount for each season. For example, when one million acre-feet of capacity are to be reserved for flood control, the simplex calculation indicates that $y_{W1} = 2$, and that $z_W = 1$. This is to be interpreted as indicating that the minimum water discharge during the wet season consists of 2 million acre-feet, and that 1 million acre-feet are always to be stored for subsequent use. The total expected benefits credited to the wet season are therefore calculated as follows, taking account of the probabilities that y_{W2} and y_{W3} will be assigned positive values in the event of water inflows exceeding the minimum value of 3 million acre-feet:

minimum expected benefits with inflow of 3 million acre-feet		($3.00)(2)
+		+
expected additional benefits with an inflow of 4 million acre-feet	=	.4($1.20)(1)
+		+
expected additional benefits with an inflow of 5 million acre-feet		.3($1.20)(1) + .3($.60)(1)
	=	$7.02 millions

According to Figure 8, the two types of shortcut calculations lead to estimates of total benefits that differ by less than 15% from each other, and by less than 10% from the rigorous solution based upon an optimal sequential decision rule. Such shortcut methods deserve further exploration -- particularly for the case of multi-reservoir systems where a rigorous optimization looks hopelessly expensive from the viewpoint of computational costs. It is important, however, to realize one clear defect of these shortcut methods: Whenever the head-height factor turns out to vary significantly with reservoir levels, the shortcut based upon a "transportation" model is no longer applicable. With significant head-height effects, nonconvexity is present; a local optimum is no longer necessarily a global optimum.

References

[1] Bernholtz, B., and L. J. Graham, "Hydro-Thermal Economic Scheduling Solution by Incremental Dynamic Programming," presented at the AIEE Winter General Meeting, New York, N.Y., January 31-February 5, 1960, CP60-280.

[2] Brudenell, R. N., and J. H. Gilbreath, "Economic Complementary Operation of Hydro Storage and Steam Power in the Integrated TVA System," Power Apparatus and Systems, A.I.E.E. Transactions, vol. 78, pt. 3, no. 42, June 1959, pp. 136-56.

[3] Debreu, G., Theory of Value: An Axiomatic Analysis of Economic Equilibrium, John Wiley and Sons, Inc., New York, 1959.

[4] Eckstein, O., Water-Resource Development, Harvard University Press, Cambridge, Massachusetts, 1958.

[5] Elliot, R. A., and L. R. Engstrom, "TVA Flood Control Experience: A 25-Year Backsight," presented at American Society of Civil Engineers, Hydraulics Division Conference, Atlanta, Georgia, August 22, 1958, (pamphlet available from TVA, Knoxville, Tennessee).

[6] d'Epenoux, F., "Sur un Problème de Production et de Stockage dans l'Aléatoire," Revue Française de Recherche Opérationnelle, 4, No. 14, 1960.

[7] Gessford, J., and S. Karlin, "Optimal Policy for Hydroelectric Operations" in K. J. Arrow, S. Karlin, and H. Scarf (eds.) Studies in the Mathematical Theory of Inventory and Production, Stanford University Press, Stanford, 1958.

[8] Kirchmayer, L. K., and G. H. McDaniel, "Transmission Losses and Economic Loading of Power Systems," General Electric Review, October, 1951.

[9] Koopmans, T.C., "Water Storage Policy in a Simplified Hydroelectric System," Proceedings of the First International Conference on Operations Research, Oxford, September, 1957.

[10] Little, J. D. C., "The Use of Storage Water in a Hydroelectric System," Journal of the Operations Research Society of America, May 1955.

[11a] Manne, A.S., "A Note on the Modigliani-Hohn Production Smoothing Model," Management Science, July 1957.

[11b] __________, "Linear Programming and Sequential Decisions," Management Science, April 1960.

[12] Moran, P. A. P., The Theory of Storage, Methuen and Co., Ltd., London, 1959.

[13] Ransmeier, J. S., The Tennessee Valley Authority, Vanderbilt University Press, Nashville, 1942.

[14] Samuelson, P. A., "Diagrammatic Exposition of a Theory of Public Expenditure," Review of Economics and Statistics, November 1955.